

Collectivity & QGP signals in Large and Small systems

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Illustration of heavy-ion collisions



QGP signatures in heavy-ion collisions



Hydrodynamics and Collectivity in A-A collision



C.Shen, Z.Qiu, H.Song, J.Bernhard, S.Bass and U.Heinz Comput. Phys. Commun, 199, 61 (2016). H.Song, S.A.Bass and U.Heinz, Phys. Rev. C 83, 024912 (2011).

Fluctuations and Correlations in heavy-ion collisions



• In heavy-ion collisions, hydrodynamics transform the initial state fluctuations to final state correlations.



Other flow observables

- Event-plane correlations
- Non-linear response coefficients
- Decorrelations at p_T direction.

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W. Zhao, H. j. Xu and H. Song, Eur. Phys. J. C 77, no. 9, 645 (2017).

• Hydrodynamic model does a great job in describing the hydrodynamic evolutions of heavy-ion collisions.

The predictions from hydrodynamics



- In heavy-ion collisions, hydrodynamics has the strong predictive power for the collective flow observables at low p_T range.
- It can constrain the initial models and extract the QGP transport coefficients from various bulk observables.

Extract the QGP transport coefficients by Bayesian global fitting



0.04

0.00

0.12

0.16

Temperature [GeV]

- Using Bayesian global fitting within the framework of TRENTo+iEBE-VISHNU to extract the QGP specific shear viscosity and bulk viscosity.
- LHC of Pb+Pb collisions flow data show good constraining power on the temperature dependence of QGP shear and bulk viscosity.
- The extracted η/s is close to the KSS bound of $1/4\pi$.

0.20

Collectivity & QGP signatures in small systems



Plot with TRENTo initial condition.

Collective flow? experimental observables in p-Pb collisons



Hydrodynamic simulations in p-Pb



ATLAS

• v₂{2}

 $\star v_{2}^{-}{4}$

120

 v_n {**4**}

Initial states correlations in small system



• Without QGP effects, initial state correlation also describe many "collective" features, such as multi-particle correlations and mass ordering in small systems

Initial state or Final state effects?

Initial state effects:

– Various Models interpolations

- K. Dusling and R. Venugopalan, PRL 2012, PRD2013, NPA 2014.
- A. Dumitru and A. V. Giannini, NPA 2015, A. Dumitru and V. Skokov PRD2015.
- B. Schenke, S. Schlichting, P. Tribedy, and R. Venugopalan, PRL2016.
- K. Dusling et al, Phys. Rev. Lett 120 042002 (2018)..
- C. Zhang, et al Phys. Rev. Lett. 122, no. 17, 172302 (2019).

Final state effects:

- P. Bozek, W. Broniowski, G. Torrieri, PRL2013.
- K. Werner, et. Al., PRL2014.
- G.-Y. Qin, B. Muller. PRC2014.
- A. Bzdak and G. L. Ma, PRL 113, 25, 252301 (2014).
- P. Bozek, A. Bzdak, and G.-L. Ma, PLB2015.
- P. Romatschke, Eur.Phys.J. C77 21(2017).
- W. Zhao, Y. Zhou, H. Xu, W. Deng and H. Song, Phys. Lett. B 780, 495 (2018).

Hybrid model with initial and final states correlations IP-Glasma +MUSIC+UrQMD



$$arepsilon_2 e^{i2\Psi_2^{\mathrm{PP}}} = -rac{\int d^2 \mathbf{r} \, r^2 s(r,\phi) e^{i2\phi}}{\int d^2 \mathbf{r} \, r^2 s(r,\phi)}.$$

• Initial state momentum anisotropy

$$\vec{\mathscr{E}}_{p} = \varepsilon_{p} e^{2i\psi_{2}^{p}} = \frac{\langle T^{xx} - T^{yy} \rangle + i\langle 2T^{xy} \rangle}{\langle T^{xx} + T^{yy} \rangle}$$

B.Schenke, C. Shen and P.Tribedy, PLB, 803, 135322 (2020). B.Schenke QM 2019.

- Below $dN_{ch}/d\eta \sim 10$, the initial momentum anisotropy contributes strongly to the final charged hadron elliptic anisotropy;
- Above $dN_{ch}/d\eta \sim 100$, everything is geometry driven.

System size scan of v_n at RHIC



IP-Glasma+MUSIC+UrQMD calculations: B.Schenke, C. Shen and P.Tribedy, PLB, 803, 135322 (2020). B.Schenke QM 2019. SATR data: Quark Matter 2019; PHENIX data:Nature Phys. 15, 214 (2019).

- The observed $v_2(p_T)$ is similar between STAR and PHENIX for all systems.
- The observed $v_3(p_T)$ from STAR is much larger than that from PHENIX, especially for p-Au and d-Au collisions.
- The hydro calculations with nucleonic substructure is consistent with the STAR data.

Collectivity in p-p Collisions at 13 TeV

2-particle correlations in p-p collisons

• "ridge" structures in p+p, p+Pb and Pb + Pb



• Mass ordering of $v_2(\mathbf{p}_T)$



- Similar double ridge structure, but with smaller magnitudes in p-p collisions.
- Observed the v₂{2} and v₃{3} in p-p collisions.
- Clear mass ordering of v_2 in high multiplicity p-p collisions.

• v_2 {2} and v_3 {2}



4-particle correlations in p-p collisons



$$\langle \langle 4 \rangle \rangle_{3sub} = \langle \langle \cos n(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4) \rangle \rangle$$
$$\langle \langle 2 \rangle \rangle^2_{3sub} = \langle \langle \cos n(\varphi_1 - \varphi_3) \rangle \rangle \langle \langle \cos n(\varphi_2 - \varphi_4) \rangle \rangle$$
$$\langle \langle 2 \rangle \rangle^2_{3sub} = \langle \langle \cos n(\varphi_1 - \varphi_4) \rangle \rangle \langle \langle \cos n(\varphi_2 - \varphi_3) \rangle \rangle$$
$$c_n \{4\}_{3sub} = \langle \langle 4 \rangle \rangle_{3sub} - 2 \cdot \langle \langle 2 \rangle \rangle^2_{3sub}$$

- The three-subevent can effectively suppress the non-flow and get the negative $c_2{4}$ in p-p collisions.
- The $c_2{4}$ obtained by three-subevent weakly depend on N_{ch}^{sel} in high multiplicity in p-p collisions.

Hydrodynamic Collectivity in p+p collisions at 13 TeV



HIJING initial condition

X. N.Wang and M.Gyulassy, Phys. Rev. D 44, 3501 (1991). W. Zhao, Y. Zhou, H. Xu, W. Deng and H. Song, Phys. Lett. B 780, 495 (2018).

-produced jets pairs & excited nucleus \rightarrow independent strings

-strings break into partons \rightarrow form hot spots for succeeding hydro.

1) The center positions of strings (xc , yc) are sampled by Saxon-Woods distribution

2) positions of partons within the strings are sampled by exp $\left(-\frac{(x-x_c)^2+(y-y_c)^2}{2\sigma^2}\right)$

3) Energy decompositions of individual partons with a Gaussian smearing:

$$\epsilon = K \sum_{i} \frac{E_i^*}{2\pi\sigma^2\tau_0 \Delta\eta_s} \exp\left(-\frac{(x-x_i)^2 + (y-y_i)^2}{2\sigma^2}\right),$$



2-particle correlation in p-p collisions



- iEBE-VISHNU + HIJING, use three set-ups to fit CMS v_2 {2}, one to fit ATLAS v_2 {2}.
- In general, iEBE-VISHNU + HIJING can describe the $v_2\{2\}$ and $v_4\{2\}$ from ATLAS and CMS. But iEBE-VISHNU + HIJING tend to overestimate the observed $v_3\{2\}$.
- v_2 {2} calculated by iEBE-VISHNU + HIJING increase slowly as a function of multiplicity.

Differential elliptic flow



W. Zhao, Y. Zhou, H. Xu, W. Deng and H. Song, Phys. Lett. B 780, 495 (2018)

- In general, iEBE-VISHNU + HIJING can describe the $v_2(p_T)$ from ATLAS and CMS well.
- Hydrodynamics can well reproduce the mass ordering of $v_2(p_T)$ among K_s^0 and Λ as observed in experimental data in high multiplicity p-p collisions.

4-particle correlation by hydrodynamic simulations in p-p



$$C_{2}\{4\} = -2\langle v_{2}^{2} \rangle^{2} + \langle v_{2}^{4} \rangle$$

= $-\langle v_{2} \rangle^{4} + 2\sigma_{v2}^{2} \langle v_{2}^{2} \rangle^{2} + \sigma_{v2}^{2}$
 $(v_{2} = \langle v_{2} \rangle + \sigma_{v2})$
 $v_{2}\{4\} = (-c_{2}\{4\})^{\frac{1}{4}}$

• To get the real value of v_2 {4}, c_2 {4} should be negative.

W. Zhao, Y. Zhou, H. Xu, W. Deng and H. Song, Phys. Lett. B 780, 495 (2018)

• iEBE-VISHNU + HIJING cannot obtain the negative c_2 {4}.

$P(v_2)$ and $P(\varepsilon_2)$ distributions: from $c_2^{\varepsilon}{4}$ to $c_2^{\nu}{4}$



Linear term + cubic term fit: $|v_2| = 0.110 \times |\varepsilon_2| + 0.105 \times |\varepsilon_2|^3$ W. Zhao, Y. Zhou, K. Murase, and H. Song, arXiv:2001.06742.

- Small ε_2 , linear response is good; at large ε_2 , the cubic response is important.
- Certain deviations between $P(v_2/\langle v_2 \rangle)$ and $P(\varepsilon_2/\langle \varepsilon_2 \rangle)$.
- Leading to positive $C_2^{\nu}{4}$ in final states even with negative $C_2^{\varepsilon}{4}$ in initial state change.

Other initial models: HIJING, super-MC and TRENTo



W. Zhao, Y. Zhou, K.Murase, and H. Song, arXiv:2001.06742.
K. Welsh, J. Singer, and U. W. Heinz, PRC 94,024919 (2016).
J. S.Moreland, J.E.Bernhard and S.A.Bass, PRC 101, 024911 (2020).

- Different shapes of $p(\varepsilon_2)$ of HIJING, super-MC and TRENTo initial models.
- Some initial models get the negative $c_2^{\varepsilon}\{4\} < 0$ in the initial state.

v_n {2} calculated by HIJING, super-MC and TRENTo



 $v_n\{2\} = \sqrt{\langle v_n^2 \rangle} = \sqrt{\langle v_n \rangle^2 + \sigma_{vn}^2}, (v_n = \langle v_n \rangle + \sigma_{vn})$ W. Zhao, Y. Zhou, K.Murase, and H. Song, arXiv:2001.06742. ATLAS, Phys. Lett. B 789, 444 (2019).

- With properly turned parameters, iEBE-VISHNU + HIJING, super-MC and TRENTo can well describe the $v_2\{2\}$ and $v_4\{2\}$ of high multiplicity events in p-p system.
- Again, hydro simulations tend to overestimate the $v_3\{2\}$, which needs to be understood.

c_2 {4} from hydro with various initial conditions on market



$$C_{2}\{4\} = -2\langle v_{2}^{2} \rangle^{2} + \langle v_{2}^{4} \rangle$$

= $-\langle v_{2} \rangle^{4} + 2\sigma_{v2}^{2} \langle v_{2}^{2} \rangle^{2} + \sigma_{v2}^{2}$
 $(v_{2} = \langle v_{2} \rangle + \sigma_{v2})$
 $v_{2}\{4\} = (-c_{2}\{4\})^{\frac{1}{4}}$

• To get the real value of v_2 {4}, c_2 {4} should be negative.

c_{2} {4} puzzle in p-p collisions

- iEBE-VISHNU with various initial conditions cannot describe the negative *c*₂{4}.
- MUSIC with IP-Glasma also give positive c2{4} in pp collisions.

Short Summary for the collectivity in small systems at LHC



Is QGP formed in the small systems? (p-Pb collisions)

Reminder: QGP signatures in heavy-ion collisions



Related signatures in p-Pb system

- Collective flow: Hydrodynamics or CGC initial states correlations?
- Hard Probes: no longer leave obvious hints due to the limited size and lifetime.



- R_{pA} of light hadrons and heavy flavor are consistent with one and compatible with cold nuclear effect in p-Pb collsions.
- $v_2(p_T)$ of heavy quark is also compatible with the CGC model calculations.

NCQ scaling in small system



• Observe the approximately NCQ scaling of v_2 at intermediate p_T in high multiplicity events of p-Pb collision in data.

Simple coalescence and NCQ scaling

- If hadrons' wave function has the form: $W_{M/B} \sim \delta(\mathbf{P} \mathbf{p}_1 \mathbf{p}_2)$ and quark exhibits the same elliptic flow: $f_a(\mathbf{p}_T) = \overline{f}_a(p_T) (1 + 2v_{2,q}(p_T) \cos 2\phi)$
- the meson's elliptic flow: $v_2^M(p_T) = \frac{2v_{2,q}(p_T/2)}{1+2v_{2,q}^2(p_T/2)} \sim 2v_{2,q}(p_T/2)$ the baryon's elliptic flow: $v_2^B(p_T) = \frac{3v_{2,q}(p_T/3)}{1+6v_{2,q}^2(p_T/3)} \sim 3v_{2,q}(p_T/3)$





• NCQ scaling is important signal to probe partonic degree of freedom in small systems.

V.Greco, C. M. Ko and P. Levai, PRL 90, 202302 (2003). R.J.Fries, B. Muller, C. Nonaka and S. A. Bass, PRL 90,202303 (2003). D.Molnar and S.A.Voloshin, PRL 91, 092301 (2003).

Sophisticated Coalescence model

• Coalescence model $\frac{dN_{M}}{d^{3}\mathbf{P}_{M}} = g_{M} \int d^{3}\mathbf{x}_{1} d^{3}\mathbf{p}_{1} d^{3}\mathbf{x}_{2} d^{3}\mathbf{p}_{2} f_{q}(\mathbf{x}_{1}, \mathbf{p}_{1}) f_{\bar{q}}(\mathbf{x}_{2}, \mathbf{p}_{2}) d_{0} d_{0} \\ \times W_{M}(\mathbf{y}, \mathbf{k}) \delta^{(3)}(\mathbf{P}_{M} - \mathbf{p}_{1} - \mathbf{p}_{2}), \qquad H_{1} \\ \frac{dN_{B}}{d^{3}\mathbf{P}_{B}} = g_{B} \int d^{3}\mathbf{x}_{1} d^{3}\mathbf{p}_{1} d^{3}\mathbf{x}_{2} d^{3}\mathbf{p}_{2} d^{3}\mathbf{x}_{3} d^{3}\mathbf{p}_{3} f_{q_{1}}(\mathbf{x}_{1}, \mathbf{p}_{1}) t_{0} \\ \times f_{q_{2}}(\mathbf{x}_{2}, \mathbf{p}_{2}) f_{q_{3}}(\mathbf{x}_{3}, \mathbf{p}_{3}) W_{B}(\mathbf{y}_{1}, \mathbf{k}_{1}; \mathbf{y}_{2}, \mathbf{k}_{2}) \\ \times \delta^{(3)}(\mathbf{P}_{B} - \mathbf{p}_{1} - \mathbf{p}_{2} - \mathbf{p}_{3}), \qquad 0.1$

Thermal & hard Partons:

- Thermal partons generated by hydro
- Hard partons generated by PYTHIA8, then suffered with energy loss by LBT

Coalesence processes:

- thermal thermal parton coalescence
- thermal hard parton coalescence
- hard hard parton coalescence

 $g_{B(M)}$ is statistic factor, $f_{q/\bar{q}}$ is the phase-space distribution of (anti)quarks, $W_{M/B}$ is wigner function of meson(baryon).

Here, we use the harmonic oscillator for wave functions of hadrons, then do the Wigner transform to get the $W_{M/B}$.





Hydro-Coal-Frag Hybrid Model

Thermal hadrons (VISH2+1):

- generated by hydro. with Cooper-Frye. Meson: $p_T < 2p_{T1}$; baryon: $p_T < 3p_{T1}$.

Coalescence hadrons (Coal Model):

-generated by coalescence model including thermal-thermal, thermal-hard & hard-hard parton coalescence.

Fragmentation hadrons :

-the remnant hard quarks feed to fragmentation .

UrQMD afterburner:

-All hadrons are feed into UrQMD for hadronic evolution, scatterings and decays

Zhao, Ko, Liu, Qin and Song, arxiv:1911,00826.



Fixed by the p_T spectra, with $p_{T_1} = 1.6 \text{GeV}$ and $p_{T_2} = 2.6 \text{GeV}$

Spectra, and Hydro. Coal. and Frag. contributions



- Hydro-Coal-Frag hybrid model well describe the spectra and P/ π at 0-8 GeV.
- Hydro. dominates at low p_T , at inter-mediate p_T , coal. and frag. both have contributions. Fragmentation dominates high p_T . Three processes smoothly merge at intermediate p_T .
- Coalescence hadrons: Thermal-thermal coalescence dominates at intermediate p_T .

$v_2(p_T)$ and NCQ scaling



• Combine hydro and jet with coalescence and fragmentation, Hydro-Coal-Frag model can well describe the $v_2(p_T)$ of π , K and P within p_T range of 0-6 GeV.

• At intermediate p_T , Hydro-Coal-Frag model can get the approximately NCQ scaling of π , K and P of v_2 as the data shown. 37



Zhao, Ko, Liu, Qin and Song, arxiv:1911,00826.

The importance of quark coalescence in p-Pb collisions

- Without coalescence, Hydro-Frag greatly underestimates the $v_2(p_T)$ at intermediate p_T
- Without coalescence, Hydro-Frag will also greatly violate the NCQ scaling at intermediate p_T , with the deviation of NCQ scaling at the level of $\pm 50\%$.

Summary Pb+Pb Collisions at the LHC

- Hydrodynamics has the strong predictive power for the various flow data in heavy-ion collisions.
- The η/s and ζ/s have been extracted by the Bayesian global fitting.

p+Pb Collisions at the LHC

- Many flow observables have been quantitatively/qualitatively described by hydro, supporting the collective expansion in p-Pb collisions.
- Coalescence model calculations nicely described NCQ scaling of v_2 at intermediate p_T , strongly hint the partonic degrees of freedom in high multiplicity p-Pb collision.

p+p Collisions at the LHC

- The sign of $c_2{4}$ is still a puzzle for hydro with various initial conditions on market.
- More flow observables are still needed to be measured

It is important to investigate why hydro works & when and where it is works ?

Thanks

THAIRS

Back Up

More comments: non-linear response's effects on Symmetric-Cumulant



- In hydro simulations, $nsc_{2,3}^{\nu}\{4\}$ in the final states keep the same sign of the initial states correlations ns_{23}^{ε} {4}. But the hierarchy changes, Para-I> Para-II \approx Para-III for initial states, but Para-III> Para-II> Para-I for the final states. This is caused by different non-linear response effects with different shapes of $\mathbf{P}(\boldsymbol{\varepsilon}_2)$. 42

Check α_s effects in p-Pb collisions



- The effective coupling constant α_s in the LBT model controls the energy loss effect when hard parton traversing the medium.
- Changing $\alpha_s = 0.15$ to $\alpha_s = 0.0$ increase p_T -spectra of π , K and P by about 40% for $p_T > 3$ GeV and has negligible effects for $p_T > 8$ GeV. It also decreases the $v_2(p_T)$ of π , K and P for $p_T > 3$ GeV, where the fragmentation contribution gradually becomes important.

Smooth transition of p_{T1} and p_{T2}



• Using a smooth transitions in the p_T spectra for thermal and hard partons gives the same results with a sharp cut.

Zhao, Ko, Liu, Qin and Song, arxiv:1911,00826. Zhao, Ko, Liu, Qin and Song, in preparation.



Comparison of PHENIX and STAR v_n (p_T) measurements for p/d/³He+Au



RC.58, 1671 (1998) 1 1

• p-Au, 0-5%,
$$\left(\frac{dN_{ch}}{d\eta}\right) = 12.3 \pm 1.7$$

• d-Au, 0-5%, $\left(\frac{dN_{ch}}{d\eta}\right) = 18.6 \pm 1.5$

³He+Au, 0-5%,
$$\left(\frac{dN_{ch}}{d\eta}\right)$$
=23.6±2.5
PHENIX, PRL 121, 222301 (2018).

- The STAR measurements are consistent with the important role of "size" (N_{ch}) in addition to the fluctuations-driven eccentricity ($\epsilon_{2,3}$)
 - This observation is consistent with recent hydrodynamic calculations which incorporates nucleonic substructure

Roy A. Lacey, Quark Matter, Wuhan, China, Nov. 2019

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- \blacktriangleright The STAR and PHENIX measurements for v₂{2} are in reasonable agreement for all systems
 - ✓ System-dependent trends consistent with "shape-size" dependencies
 - \blacktriangleright The STAR and PHENIX v₃{2} measurements for p/d+Au differ by more than a factor of 3
 - \checkmark System independent STAR v₃
 - \checkmark System dependent PHENIX v₃

PHENIX: PRC95 034910 Nature Physics 15 214-220

PHENIX EP	³ He+Au	d+Au	p+Au
(ψ_2^{BBCS})	0.110±0.0050	0.1073 ±0.0003	0.062±0.003
(ψ_3^{BBCS})	0.034±0.0051	0.0565 ± 0.0097	0.067±0.009

Note:

 \checkmark EP resolution is proportional to v_n and \sqrt{N} !

2-particle correlation in p-p collisions



• Two-particle correlations in p-p:



- Similar double ridge structure, but with smaller magnitudes in p-p collisions.
- Peripheral subtraction (CMS): $v_{n,n}^{peri} \approx 0$
- Template fit (ATLAS): $v_{n,n}^{cent} \approx v_{n,n}^{peri}$

Beyond fluid dynamics?

Flow in AA and pA as an interplay of fluid-like and non-fluid like excitations

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To study the microscopic structure of quark-gluon plasma, data from hadronic collisions must be confronted with models that go beyond fluid dynamics. Here, we study a simple kinetic theory model that encompasses fluid dynamics but contains also particle-like excitations in a boost invariant setting with no symmetries in the transverse plane and with large initial momentum asymmetries. We determine the relative weight of fluid dynamical and particle like excitations as a function of system size and energy density by comparing kinetic transport to results from the 0th, 1st and 2nd order gradient expansion of viscous fluid dynamics. We then confront this kinetic theory with data on azimuthal flow coefficients over a wide centrality range in PbPb collisions at the LHC, in AuAu collisions at RHIC, and in pPb collisions at the LHC. Evidence is presented that nonhydrodynamic excitations make the dominant contribution to collective flow signals in pPb collisions at the LHC and contribute significantly to flow in peripheral nucleus-nucleus collisions, while fluidlike excitations dominate collectivity in central nucleus-nucleus collisions at collider energies.

A. Kurkela, U. A. Wiedemann and B. Wu, EPJC 79, no. 11, 965 (2019). Jamie Nagle QM 2019.

• Non-hydrodynamic excitations dominate in small systems?

Wigner functions of hadrons

To guarantee positive value of Wigner function for stable Monto Carlo sampling, the Wigner function replaced by the overlap of hadron Wigner function W_M with parton's Wigner function, $W_{q,\bar{q}}$:

$$\overline{W}_{M}(\mathbf{y},\mathbf{k}) = \int d^{3}\mathbf{x}_{1}' d^{3}\mathbf{k}_{1}' d^{3}\mathbf{x}_{2}' d^{3}\mathbf{k}_{2}'$$

$$\times W_{q}(\mathbf{x}_{1}',\mathbf{k}_{1}') W_{\bar{q}}(\mathbf{x}_{2}',\mathbf{k}_{2}') W_{M}(\mathbf{y}',\mathbf{k}'). \qquad (3)$$

Using harmonic oscillator for wave functions of excited stated of hadrons,

$$\phi_n(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(\xi) e^{-\xi^2/2},\tag{4}$$

 $\xi = \sqrt{\frac{m\omega}{\hbar}} x$, $H_n(\xi)$ are Hermite polynomials, ω is the oscillator frequency. K. C. Han, R. J. Fries and C. M. Ko, Phys. Rev. C 93, no. 4, 045207 (2016).

Wigner functions of hadrons

The quark wave function to be Gaussian wave packet, the wigner function of a meson in *n*-th excited state is

$$\overline{W}_{M,n}(\mathbf{y},\mathbf{k}) = \frac{v^n}{n!} e^{-v}.$$
(5)

with

$$\mathbf{v} = \frac{1}{2} \left(\frac{\mathbf{y}^2}{\sigma_M^2} + \mathbf{k}^2 \sigma_M^2 \right).$$
 (6)

Similarly, the Gaussian smeared Wigner function for baryon is:

$$\overline{W}_{B,n_1,n_2}(\mathbf{y}_1,\mathbf{k}_1;\mathbf{y}_2,\mathbf{k}_2) = \frac{v_1^{n_1}}{n_1!}e^{-v_1}\cdot\frac{v_2^{n_2}}{n_2!}e^{-v_2},\tag{7}$$

with

$$v_i = \frac{1}{2} \left(\frac{\mathbf{y}_i^2}{\sigma_{Bi}^2} + \mathbf{k}_i^2 \sigma_{B_i^2} \right), \quad i = 1, 2.$$
 (8)

K. C. Han, R. J. Fries and C. M. Ko, Phys. Rev. C 93, no. 4, 045207 (2016).