Chiral transport and turbulence in supernovae

Naoki Yamamoto (Keio University)

"Chirality and Criticality: Novel Phenomena in Heavy-Ion Collisions"
May 18, 2020

Main topics

- Core-collapse supernovae
- Chiral radiation transfer theory of neutrinos
- Chiral plasma instability
- Chiral turbulence in supernovae

Units: $\hbar = c = k_{\rm B} = e = 1$

Core-collapse supernovae explosions

Core-collapse supernova explosions

- One of the most energetic phenomena in the Universe
- Transition to neutron stars & origin of heavy elements
- But explosion is difficult in conventional 3D hydrodynamic theory

One of the puzzles in astrophysics

http://www.riken.jp/pr/press/2009/20091211/

From micro to macro

Microscopic parity violation is reflected in macroscopic behavior:

Micro Chirality of fermions (e, ν) in Standard Model

↓

Chiral kinetic theory

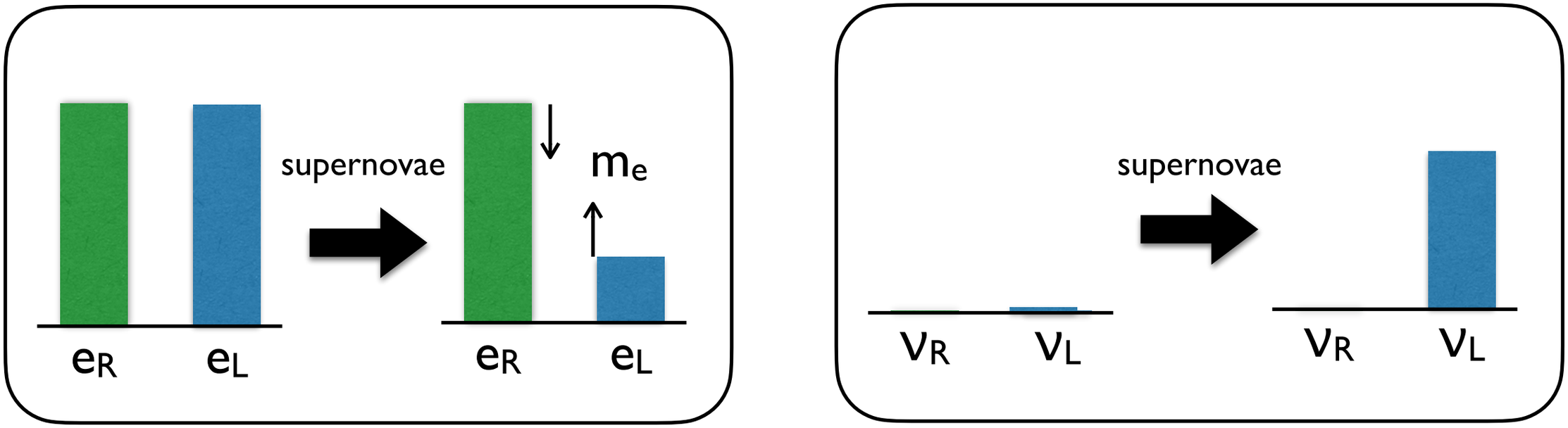
↓ Son, Yamamoto (2012); Stephanov, Yin (2012); J. W. Chen, S. Pu, Q. Wang, X. N. Wang (2012), …

Macro Evolution of core-collapse supernovae (giant P violation)

Yamamoto (2016)

Supernova = Giant Parity Breaker

$$p + \underline{e^{\mathrm{L}}} \rightarrow n + \underline{\nu_e^{\mathrm{L}}}$$

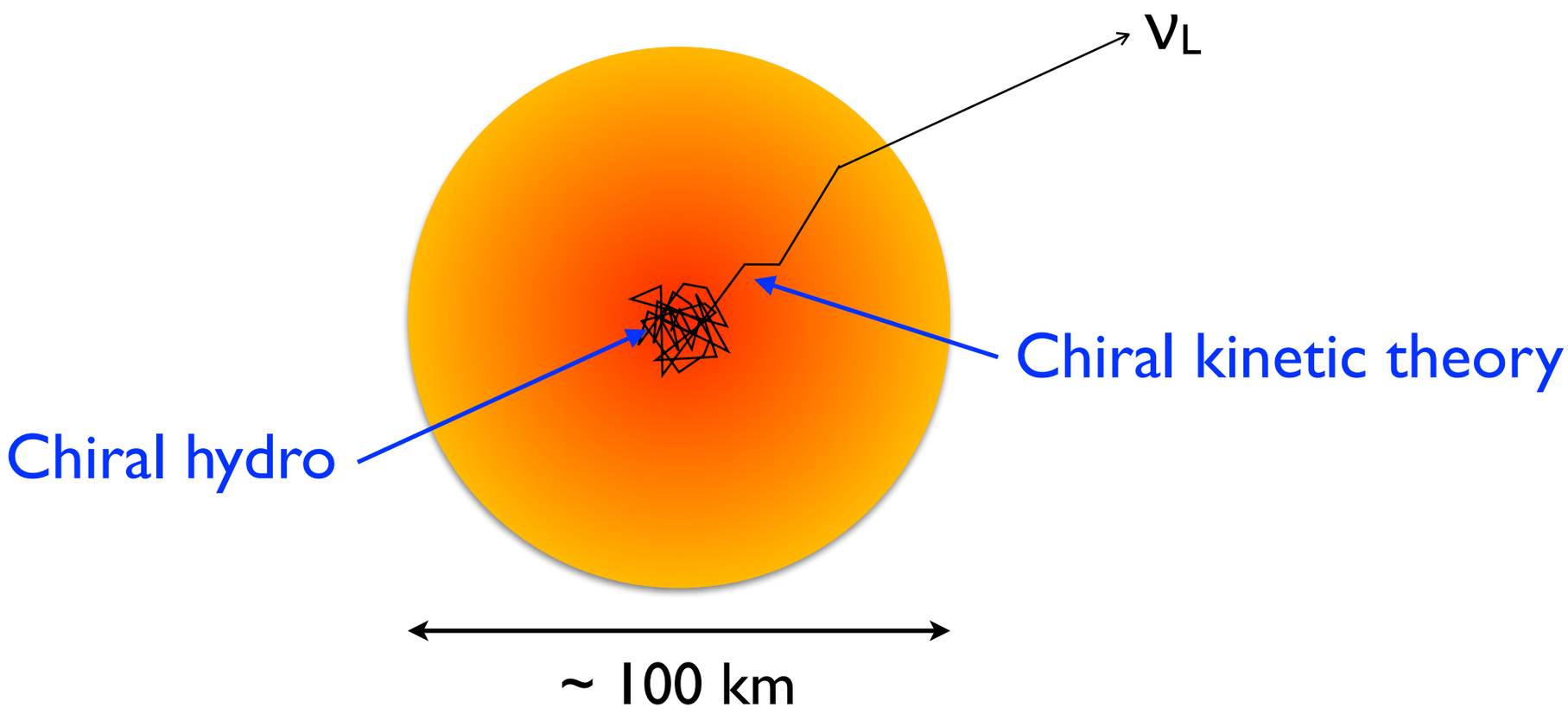


Ohnishi, Yamamoto (2014); Grabowska, Kaplan, Reddy (2015); Sigl, Leite (2016), …

Neutrino matter in supernovae

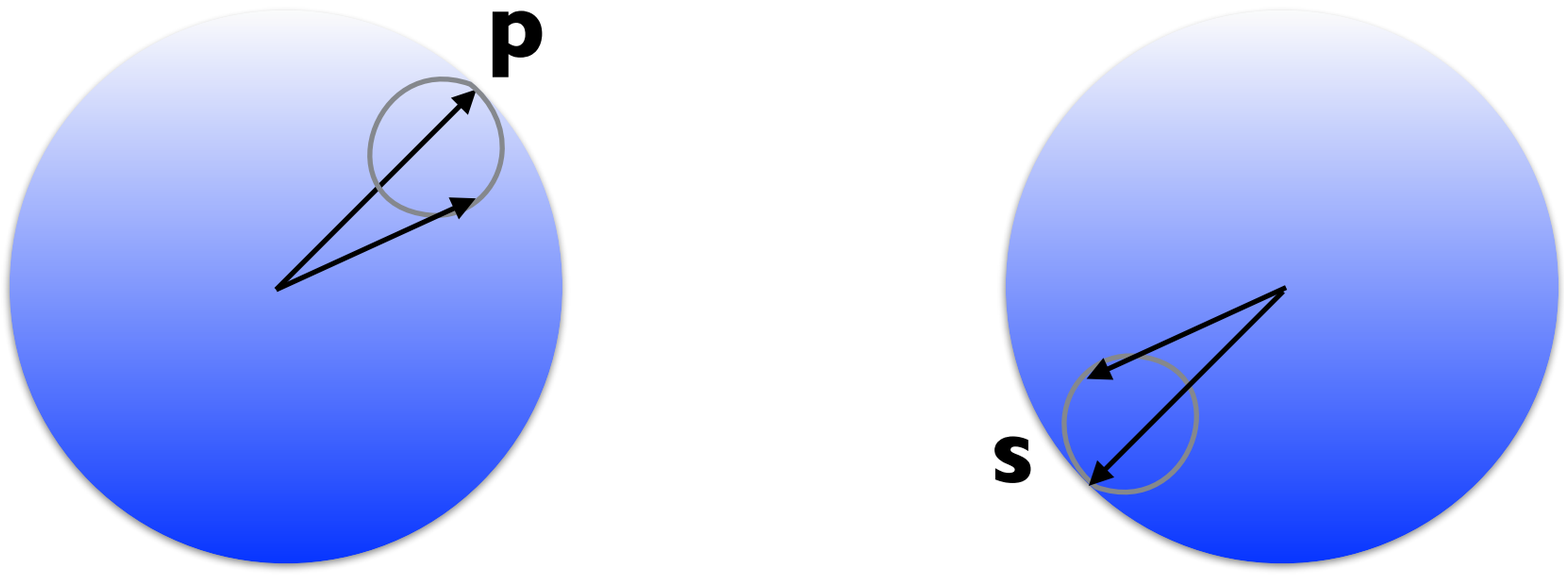
- At the core ($\rho_N \sim 10^{15}$ g/cm^3), neutrino mean free path ~ 1cm
- Neutrino matter = Chiral liquid ($\mu_\nu \sim 200$ MeV $\gg T \sim 10$ MeV at core)

  = 3D topological matter Yamamoto (2016)



Chirality and topology

Left-handed neutrinos
(spin-momentum locking)

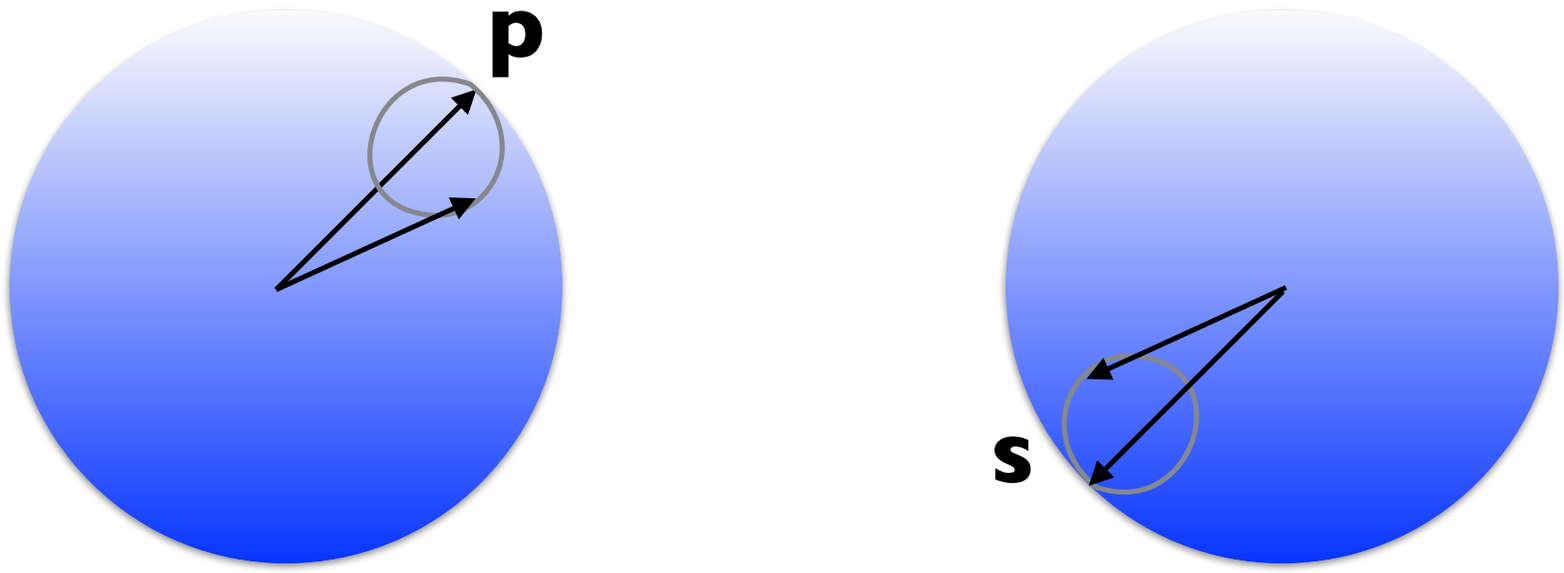


momentum space

spin space

Chirality and topology

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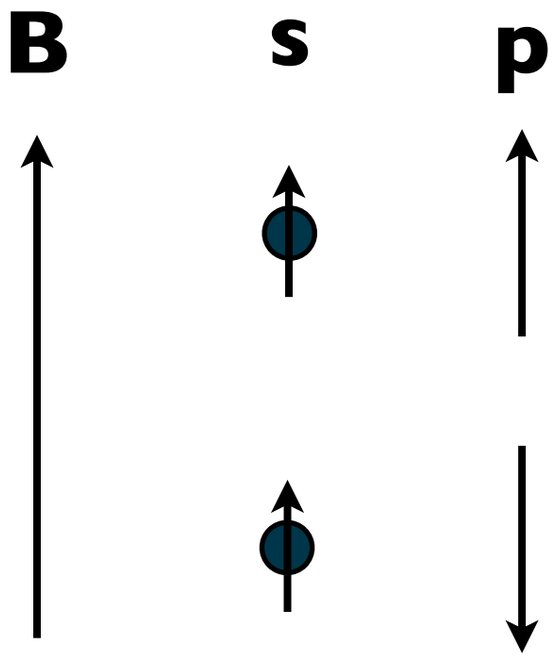


momentum space spin space

Mapping: S^2 (***p***-space) → S^2 (spin space) winding number -1

Neutrino matter = 3D topological matter

Chiral magnetic effect



right-handed $\quad \boldsymbol{j}_{\mathrm{R}} = \dfrac{\mu_{\mathrm{R}}}{4\pi^2}\boldsymbol{B}$

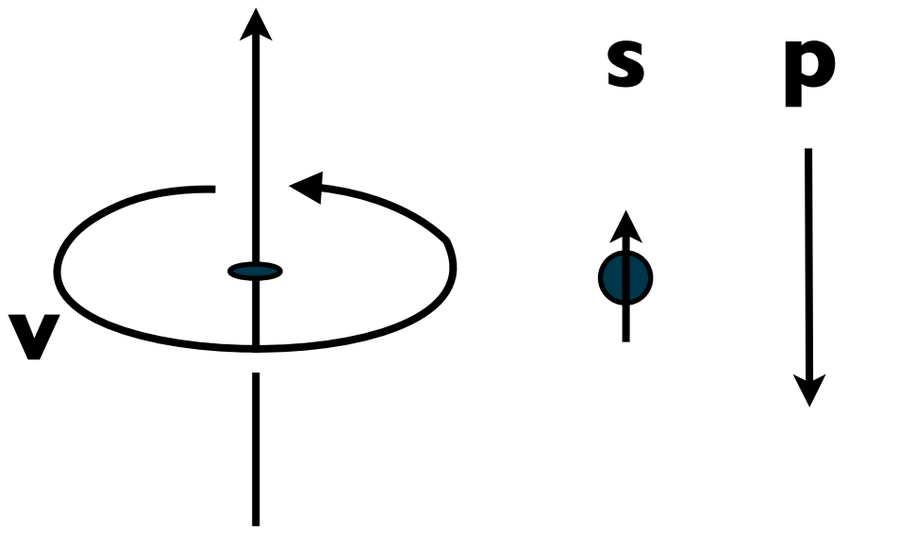
left-handed $\quad \boldsymbol{j}_{\mathrm{L}} = -\dfrac{\mu_{\mathrm{L}}}{4\pi^2}\boldsymbol{B}$

$$\boldsymbol{j} = \frac{\mu_{\mathrm{R}} - \mu_{\mathrm{L}}}{4\pi^2}\boldsymbol{B} \equiv \frac{\mu_5}{2\pi^2}\boldsymbol{B}$$

Vilenkin (1980); Nielsen, Ninomiya (1983); Fukushima, Kharzeev, Warringa (2008), …

Chiral vortical effect

$$\boldsymbol{\omega} \equiv \frac{1}{2} \boldsymbol{\nabla} \times \boldsymbol{v}$$



$$\boldsymbol{j} = -\left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12} \right) \boldsymbol{\omega}$$

Vilenkin (1979); Erdmenger et al. (2009); Banerjee et al. (2011);
Son, Surowka (2009); Landsteiner et al. (2011)

Chiral radiation transfer theory for neutrinos

Chiral radiation transfer

- Hydrodynamics for ν is not applicable outside the core.
- Generally,

$$\nabla_\alpha (T^{\alpha\beta}_{\rm hyd} + T^{\alpha\beta}_{\nu}) = 0$$

Stress tensor for matter (Hydrodynamics)

Stress tensor for ν (Chiral kinetic theory)

- T_ν and collisions between ν and matter are corrected by chirality

Wigner function formalism

Hidaka, Pu, Yang (2017); A. Huang et al. (2018); Liu et al. (2019)

- Wigner function: $S^<(q,x) = \int_y \mathrm{e}^{-\mathrm{i}q\cdot y}\langle\psi^\dagger(x+y/2)\psi(x-y/2)\rangle \equiv \sigma^\mu \mathcal{L}^<_\mu$

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- Kadanoff-Baym eqs: $\mathcal{D}\cdot\mathcal{L}^< = 0,$

$$q\cdot\mathcal{L}^< = 0, \quad \mathcal{D}_\mu\mathcal{L}^<_\nu - \mathcal{D}_\nu\mathcal{L}^<_\mu = -2\epsilon_{\mu\nu\rho\sigma}q^\rho\mathcal{L}^{<\sigma}$$

where $\mathcal{D}_\mu\mathcal{L}^<_\nu \equiv \partial_\mu\mathcal{L}^<_\nu - \Sigma^<_\mu\mathcal{L}^>_\nu + \Sigma^>_\mu\mathcal{L}^<_\nu$

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- Solution: $\mathcal{L}^{<\mu} = 2\pi\delta(q^2)\big(q^\mu - S^{\mu\nu}_{(n)}\mathcal{D}_\nu\big)f^<$ where $S^{\mu\nu}_{(n)} = \dfrac{\epsilon^{\mu\nu\alpha\beta}q_\alpha n_\beta}{2q\cdot n}$

side-jump

frame vector

Chen, Son, Stephanov (2015)

ν chiral radiation transfer eq.

Yamamoto, Yang, arXiv:2020.11348

$$\left[q^\mu(\partial_\mu - \Gamma^\lambda_{\mu\rho} q^\rho \partial_{q\lambda}) - (D_\mu S^{\mu\nu}_{(n)})\partial_\nu + S^{\mu\nu}_{(n)} q^\rho R^\lambda_{\rho\mu\nu}\partial_{q\lambda}\right] f = (1-f)\Gamma^<_{(n)} - f\Gamma^>_{(n)}$$

emission absorption

where $D_\mu = \nabla_\mu - \Gamma^\lambda_{\mu\nu} q^\nu \partial_{q\lambda}$

$$\Gamma^{\lessgtr}_{(n)} = \left(q^\nu - D_\mu S^{\mu\nu}_{(n)}\right)\Sigma^{\lessgtr}_\nu$$

side-jump

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emission absorption

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$$\Gamma^{\lessgtr}_{(n)} = \left(q^\nu - D_\mu S^{\mu\nu}_{(n)}\right)\Sigma^{\lessgtr}_\nu = \left(q^\nu - S^{\mu\nu}_{(n)} D_\mu\right)\Sigma^{\lessgtr}_\nu$$

side-jump

- We take $R^\lambda_{\rho\mu\nu} = 0$ and $n^\mu = (1, \mathbf{0})$.

Collision term

Yamamoto, Yang, arXiv:2020.11348

- Assume that matter (except for ν) is in local thermal equilibrium.
- By symmetry, $\Gamma_q^{\lessgtr} \approx \Gamma_q^{(0)\lessgtr} + \Gamma_q^{(\omega)\lessgtr}(q\cdot\omega) + \Gamma_q^{(B)\lessgtr}(q\cdot B)$

  where $\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu\partial_\alpha u_\beta\,, \quad B^\mu = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}u_\nu F_{\alpha\beta}$

- We will be particularly interested in the process:

$$\nu_{\rm L}^{\rm e}(q) + {\rm n}(k) \rightleftharpoons {\rm e}_{\rm L}(q') + {\rm p}(k')$$

Absorption and emission rates

Yamamoto, Yang, arXiv:2020.11348

$$\Gamma_q^{\lessgtr} \approx \Gamma_q^{(0)\lessgtr} + \Gamma_q^{(\omega)\lessgtr}(q\cdot\omega) + \Gamma_q^{(B)\lessgtr}(q\cdot B)$$

- Based on the 4-Fermi theory of weak interaction,

$$\Gamma_q^{(0)>} \approx \frac{G_{\rm F}^2}{\pi}\left(g_{\rm V}^2+3g_{\rm A}^2\right)E_\nu^3(1-f_q^{(\rm e)})\left(1-\frac{3E_\nu}{M_{\rm N}}\right)\frac{n_{\rm p}-n_{\rm n}}{1-{\rm e}^{\beta(\mu_{\rm n}-\mu_{\rm p})}}$$

$$\Gamma_q^{(B)>} \approx \frac{G_{\rm F}^2}{2\pi M_{\rm N}}\left(g_{\rm V}^2+3g_{\rm A}^2\right)E_\nu(1-f_q^{(\rm e)})\left(1-\frac{8E_\nu}{3M_{\rm N}}\right)\frac{n_{\rm p}-n_{\rm n}}{1-{\rm e}^{\beta(\mu_{\rm n}-\mu_{\rm p})}}$$

$$\Gamma_q^{(\omega)>} \approx \frac{G_{\rm F}^2}{2\pi}\left(g_{\rm V}^2+3g_{\rm A}^2\right)E_\nu(1-f_q^{(\rm e)})\left(2+\beta E_\nu f_q^{(\rm e)}\right)\frac{n_{\rm p}-n_{\rm n}}{1-{\rm e}^{\beta(\mu_{\rm n}-\mu_{\rm p})}}$$

$\Gamma^{(0)}$ was previously computed in Reddy, Prakash, Lattimer (1998)

Physical consequences

Yamamoto (2016); Yamamoto, Yang, arXiv:2020.11348

$$\Gamma_q^{\lessgtr} \approx \Gamma_q^{(0)\lessgtr} + \Gamma_q^{(\omega)\lessgtr}(q\cdot\omega) + \Gamma_q^{(B)\lessgtr}(q\cdot B)$$

- ν propagating collinear to the flow generates fluid helicity $\boldsymbol{v}\cdot\boldsymbol{\omega}$ and cross helicity $\boldsymbol{v}\cdot\boldsymbol{B}$.

- Fluid helicity gives an analogue of CME (helical magnetic effect):

$$\boldsymbol{j} \sim (\boldsymbol{v}\cdot\boldsymbol{\omega})\boldsymbol{B}$$

Chiral plasma instability

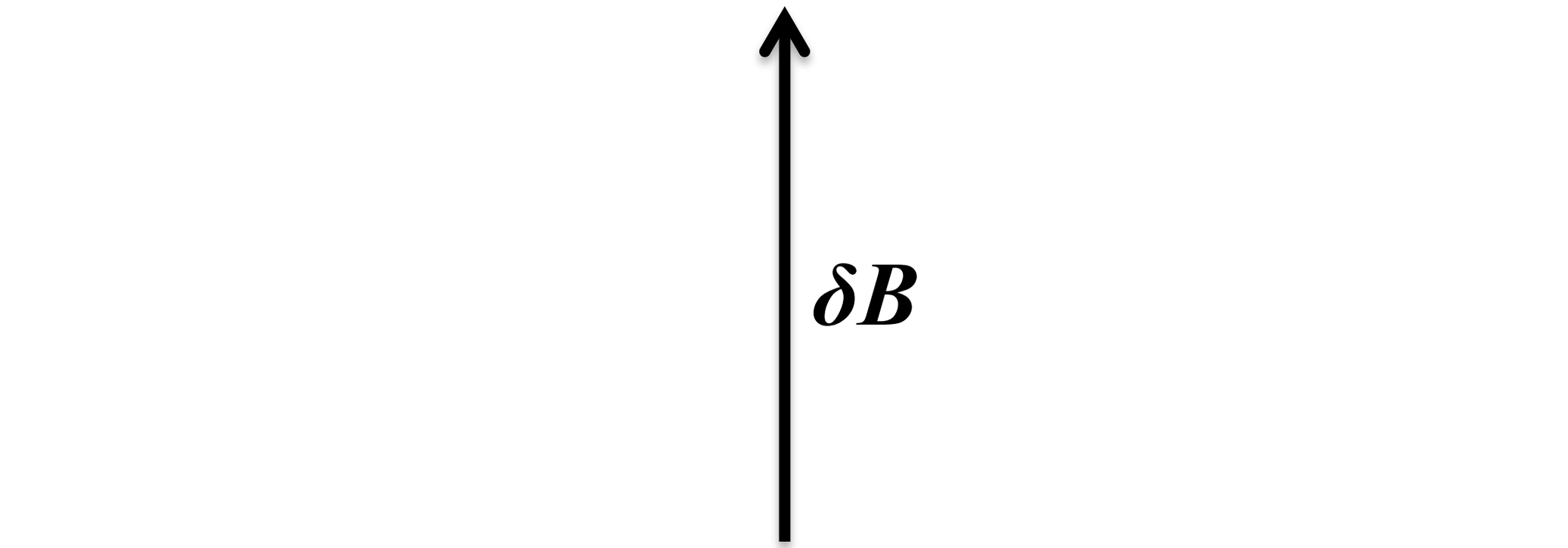
Electroweak theory Redlich, Wijewardhana (1985); Rubakov (1986); Laine (2005)

Early Universe Joyce, Shaposhnikov (1997); Boyarsky et al. (2012); Tashiro et al. (2012), …

Quark-gluon plasma Akamatsu, Yamamoto (2013); Manuel, Torres-Rincon (2015);
Hirono, Kharzeev, Yin (2015); Tuchin (2016), …

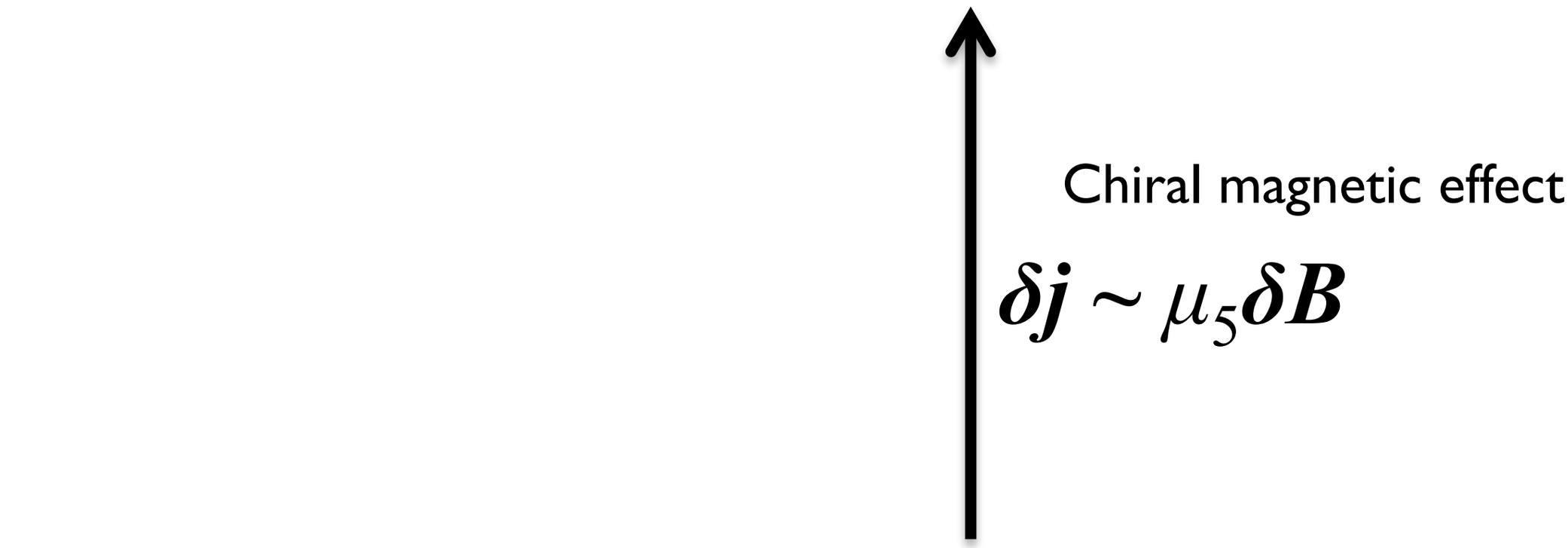
Neutron stars Ohnishi, Yamamoto (2014); Grabowska, Kaplan, Reddy (2015), …

Chiral plasma instability

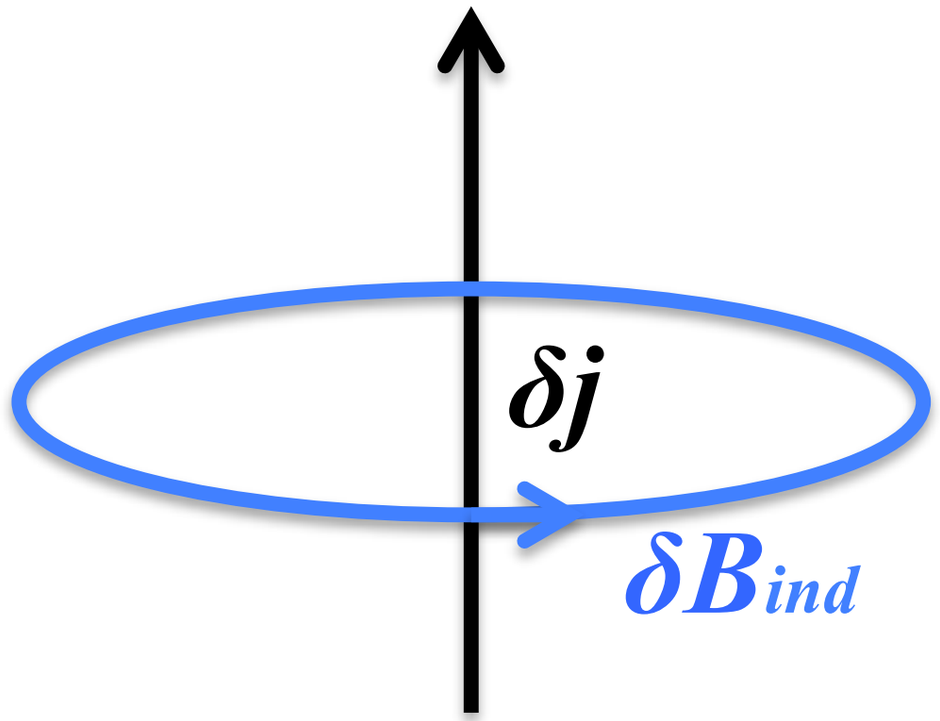


Assume nonvanishing $\mu_5 \equiv \mu_{\rm R} - \mu_{\rm L}$ or fluid helicity

Chiral plasma instability



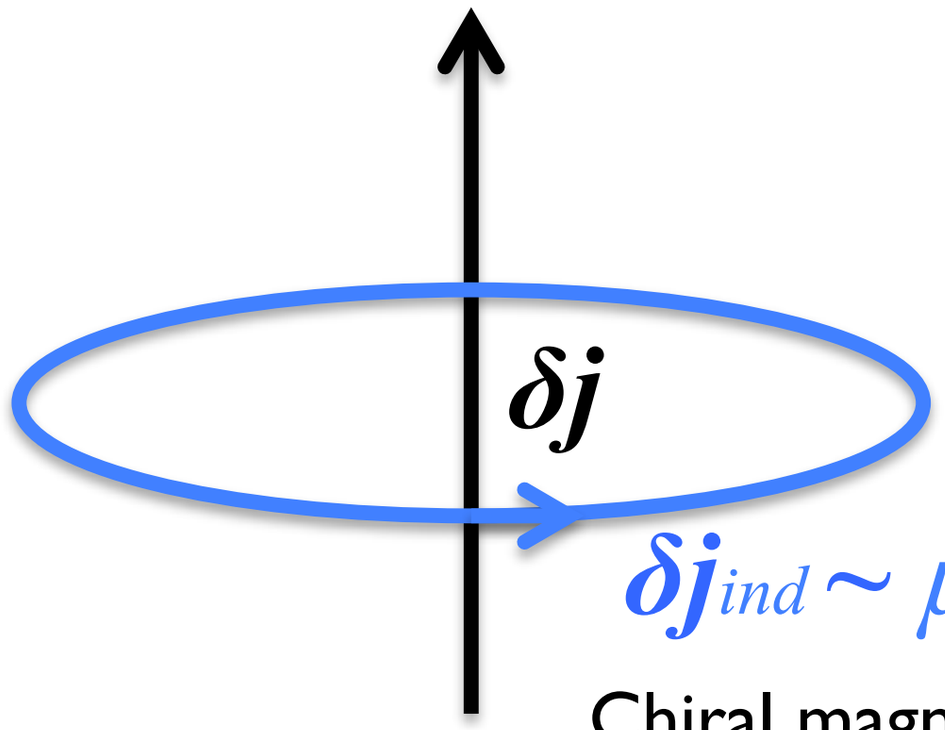
Chiral plasma instability



Ampere's law

$$\nabla \times B = j$$

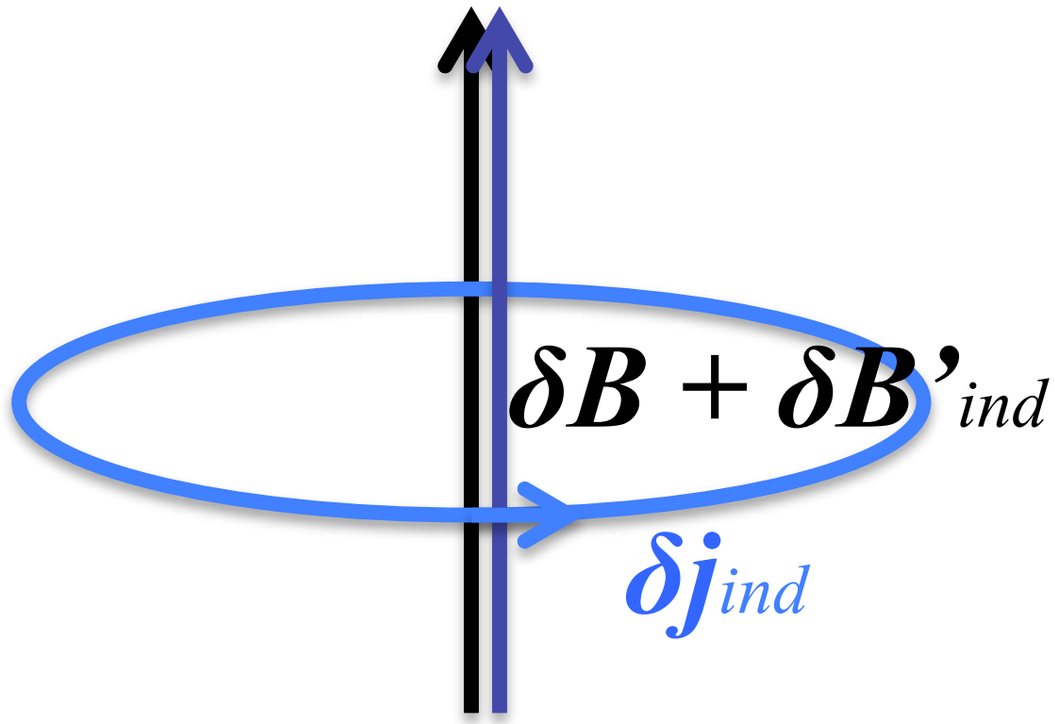
Chiral plasma instability



$\delta j_{ind} \sim \mu_5 \delta B_{ind}$

Chiral magnetic effect

Chiral plasma instability

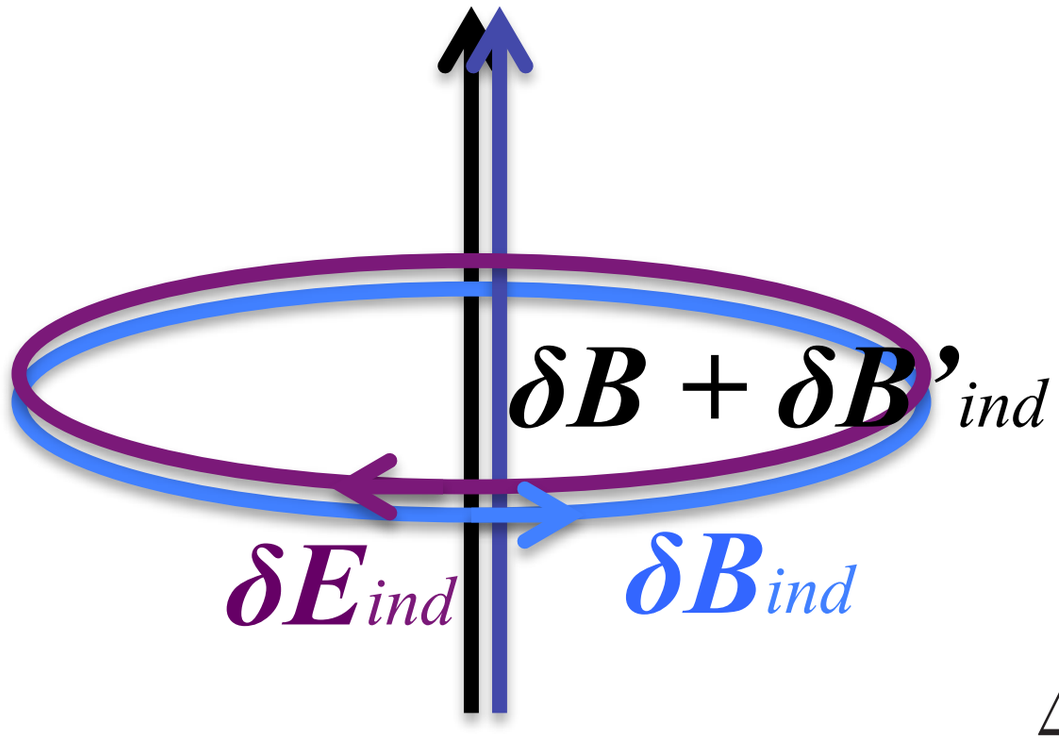


Ampere's law

$$\nabla \times B = j$$

Positive feedback: instability

Chiral plasma instability



Faraday's law

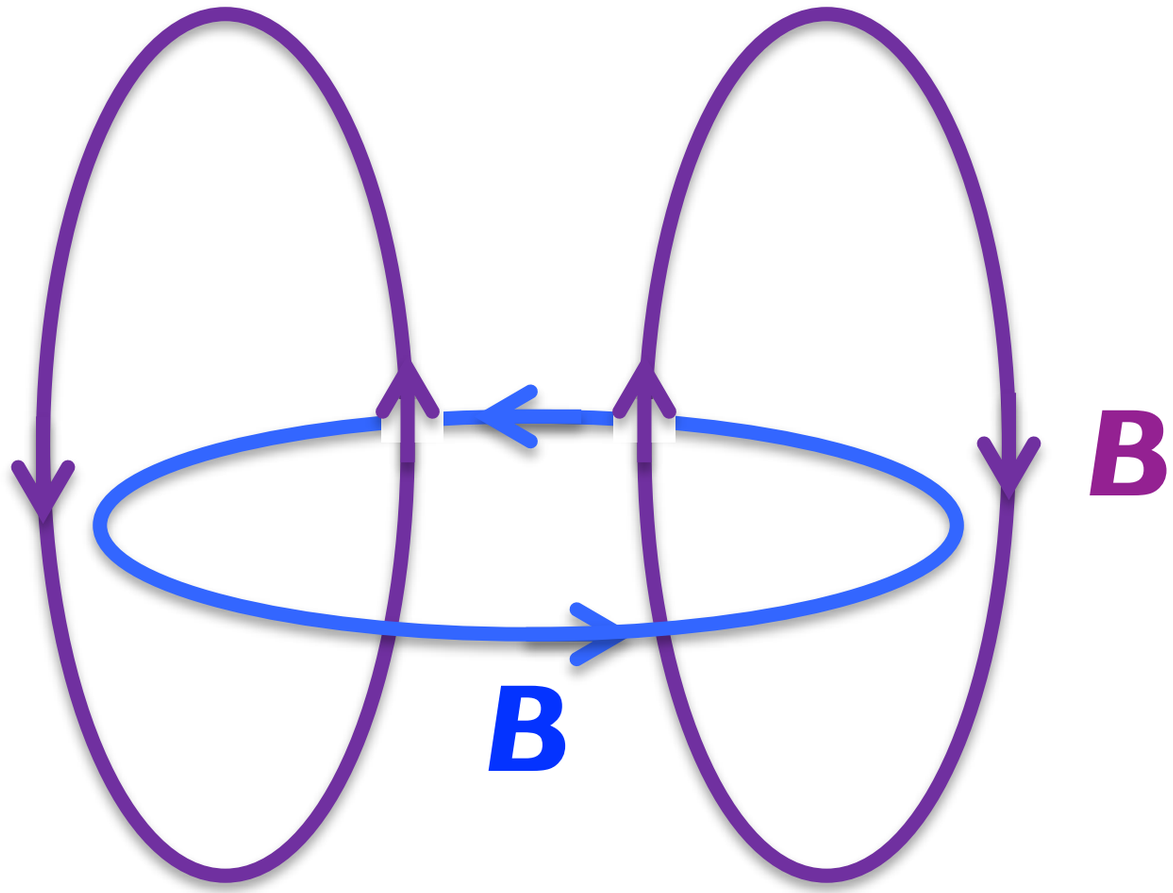
$$\nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}$$

Chiral anomaly

$$\Delta n_5 = C\boldsymbol{E} \cdot \boldsymbol{B} < 0$$

Chiral anomaly tends to reduce μ_5

Chiral plasma instability

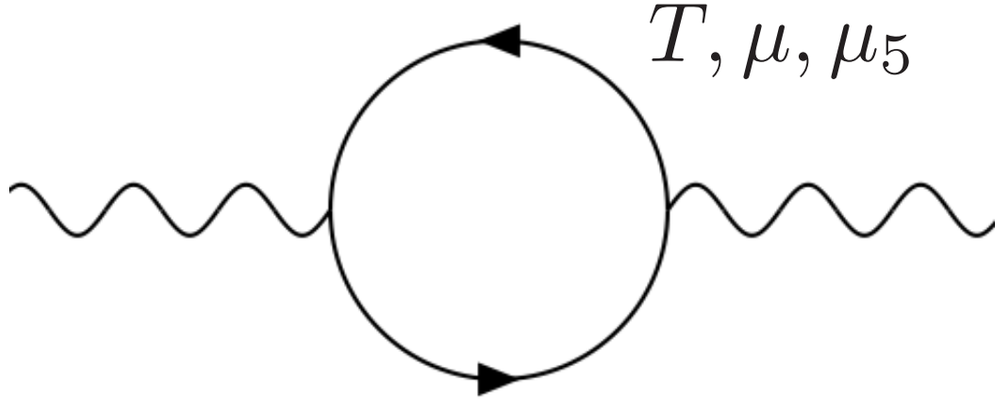


Chirality imbalance or helical flow field

→ Helical magnetic field

Polarization tensor with μ_5

$$\Pi^{\mu\nu}(k) =$$



- $\Pi^{ij}(k)$ can be decomposed by the tensors ($A_0 = 0$):

$$\hat{k}^i \hat{k}^j, \ \delta^{ij} - \hat{k}^i \hat{k}^j, \ i\epsilon^{ijk}\hat{k}^k$$

$$\Pi_{-}^{ij}(k) = \frac{e^2\mu_5}{4\pi^2} i\epsilon^{ijk}\left(1 - \frac{\omega^2}{|\boldsymbol{k}|^2}\right)\left(1 - \frac{\omega}{2|\boldsymbol{k}|}\ln\frac{\omega + |\boldsymbol{k}|}{\omega - |\boldsymbol{k}|}\right)k^k$$

T=0 [Son, Yamamoto (2013)] T≠0 [Manuel, Torres-Rincon (2014)]

Collective modes with μ_5

- Maxwell equation: $\partial_\nu F^{\nu\mu} = j^\mu$
- Linear response: $j^\mu(k) = \Pi^{\mu\nu}(k) A_\nu(k)$

$$[k^2\delta^{ij} - k^i k^j + \Pi^{ij}(k)]A^j = 0$$

($A_0 = 0$ gauge)

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Focus on transverse gauge fields:

- For μ_5=0, $\omega = -\dfrac{4ik^3}{\pi m_D^2} \rightarrow e^{-i\omega t} \sim e^{-\gamma(k)t}$ Landau damping

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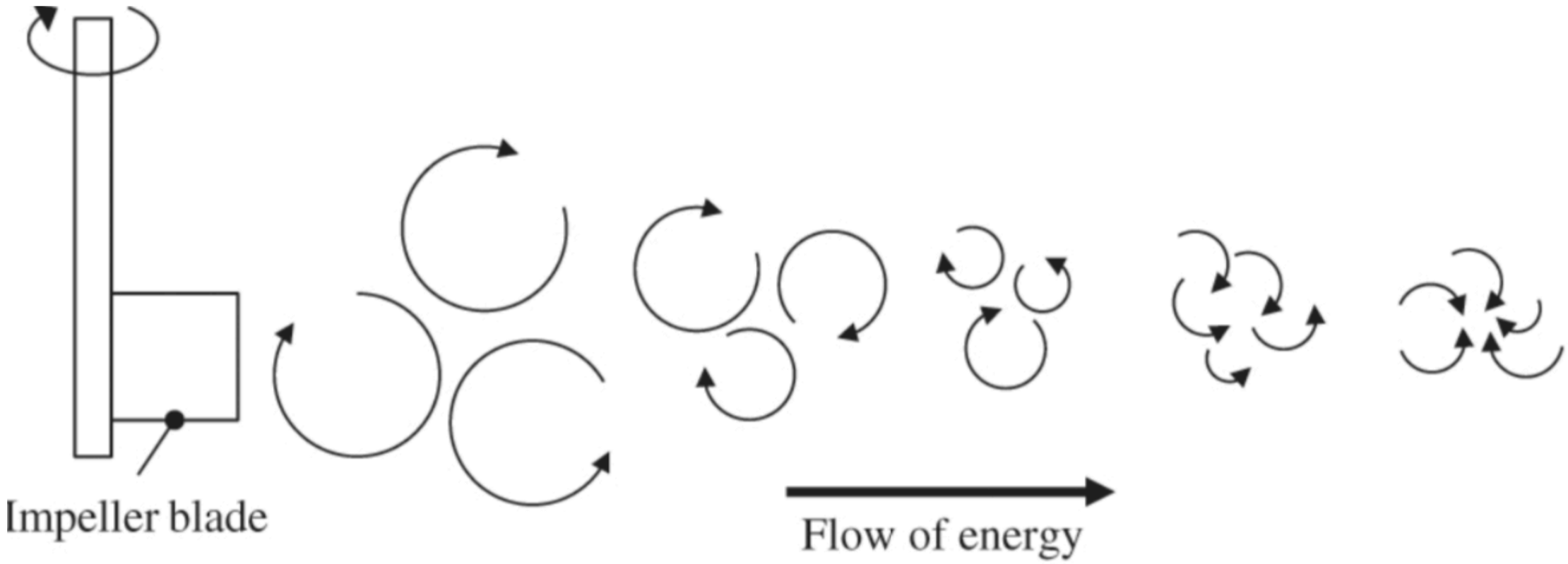
- For μ_5=0, $\omega = -\dfrac{4ik^3}{\pi m_D^2} \rightarrow e^{-i\omega t} \sim e^{-\gamma(k)t}$ Landau damping
- For $\mu_5 \neq 0$, $\omega = \dfrac{4ik^2}{\pi m_D^2}\left(\dfrac{e^2|\mu_5|}{4\pi^2} - k\right) \rightarrow e^{-i\omega t} \sim e^{\gamma(k)t}$ for small k

Chiral plasma instability

Akamatsu, Yamamoto (2013)

Chiral MHD turbulence in supernovae

Turbulence and cascade



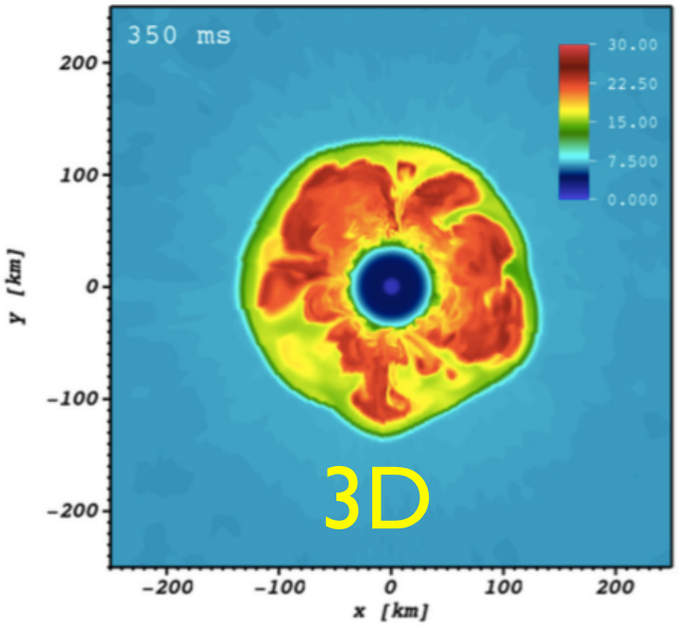
https://doi.org/10.1515/htmp-2016-0043

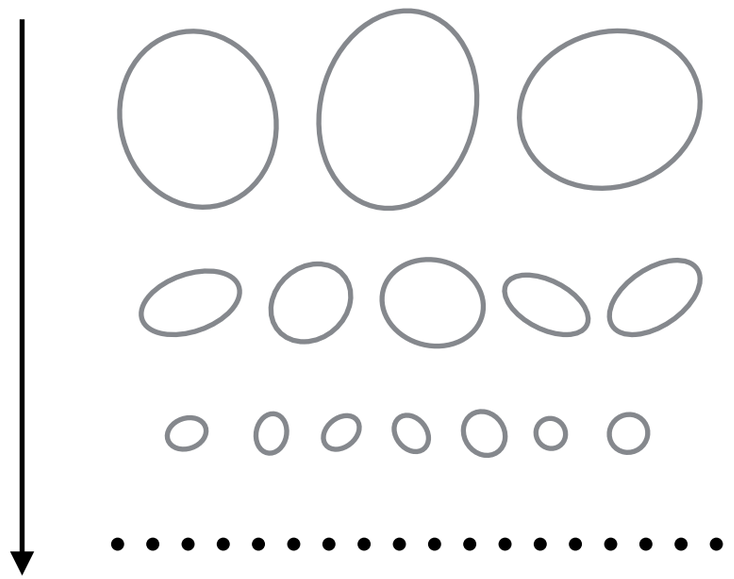
- The structure becomes smaller, and eventually dissipates (direct cascade)
- Similar in magneto-hydrodynamics (MHD)

Cascade and explosion

Direct cascade
(3D usual matter)

explosion difficult

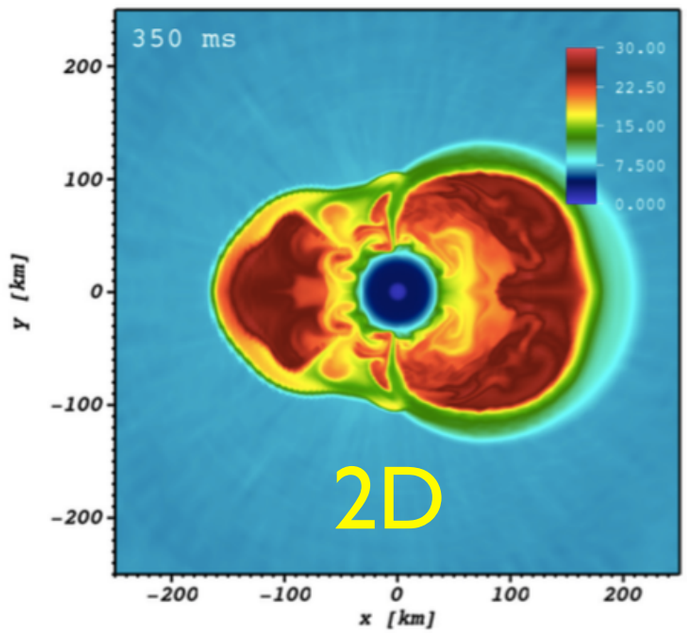




F. Hanke (2014)

Inverse cascade
(2D usual matter)

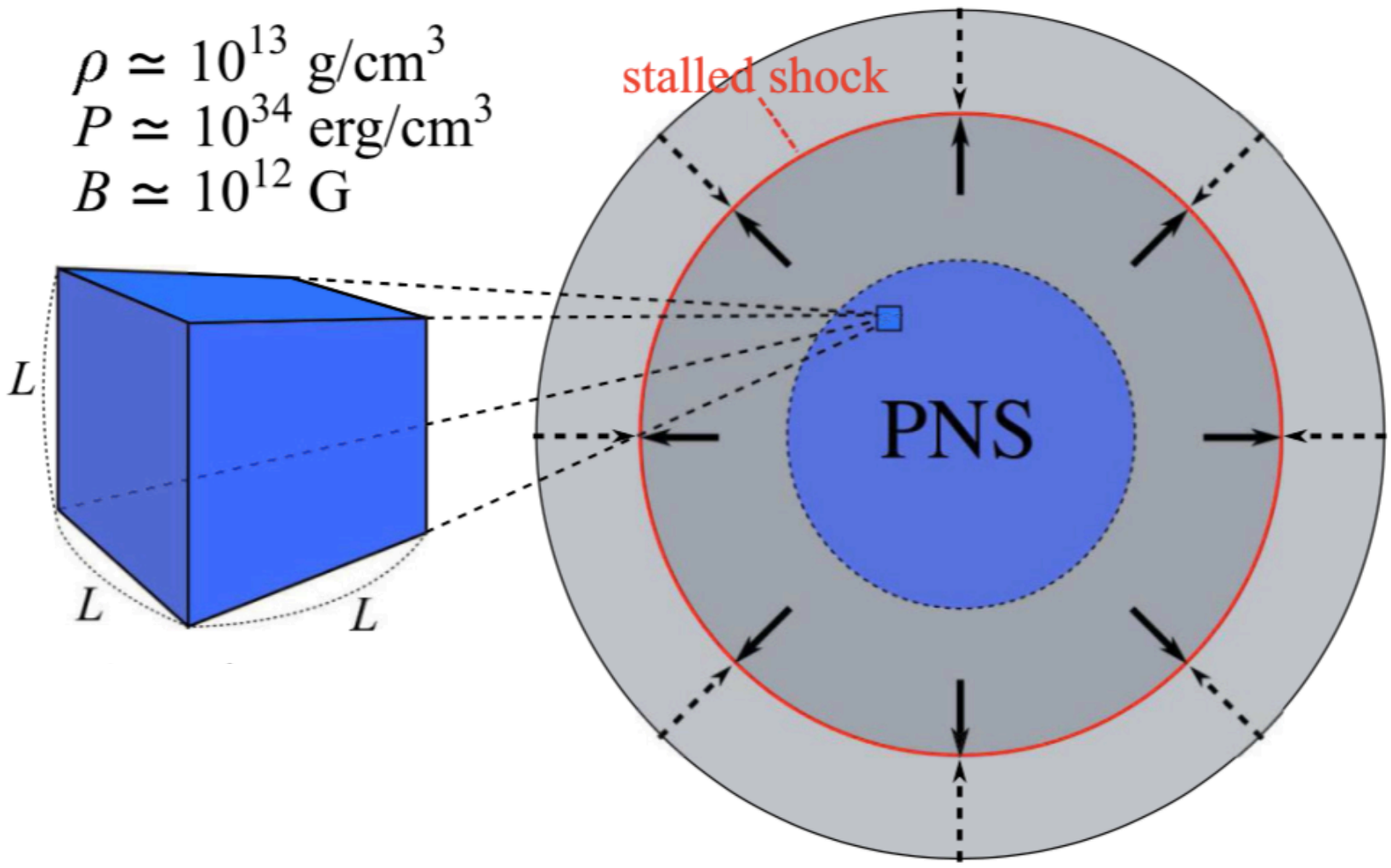
explosion easier



What about 3D chiral matter?

Chiral MHD for supernovae

Masada, Kotake, Takiwaki, Yamamoto, arXiv:1805.10419



Proto-neutron star (PNS)

Chiral MHD for supernovae

Masada, Kotake, Takiwaki, Yamamoto, arXiv:1805.10419

- Chiral MHD w/o vorticity at the core (proton and electrons):

$$\partial_t \rho + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0$$

$$\partial_t (\rho \boldsymbol{v}) + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v} \boldsymbol{v}) = -\boldsymbol{\nabla} P + \boldsymbol{J} \times \boldsymbol{B} + \text{(dissipation)}$$

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \boldsymbol{\nabla}^2 \boldsymbol{B} + \eta \boldsymbol{\nabla} \times (\underline{\xi_B \boldsymbol{B}})$$

CME or helical magnetic effect

$$\partial_t n_5 = \underline{\frac{\eta}{2\pi^2} (\boldsymbol{\nabla} \times \boldsymbol{B} - \xi_B \boldsymbol{B}) \cdot \boldsymbol{B}}$$

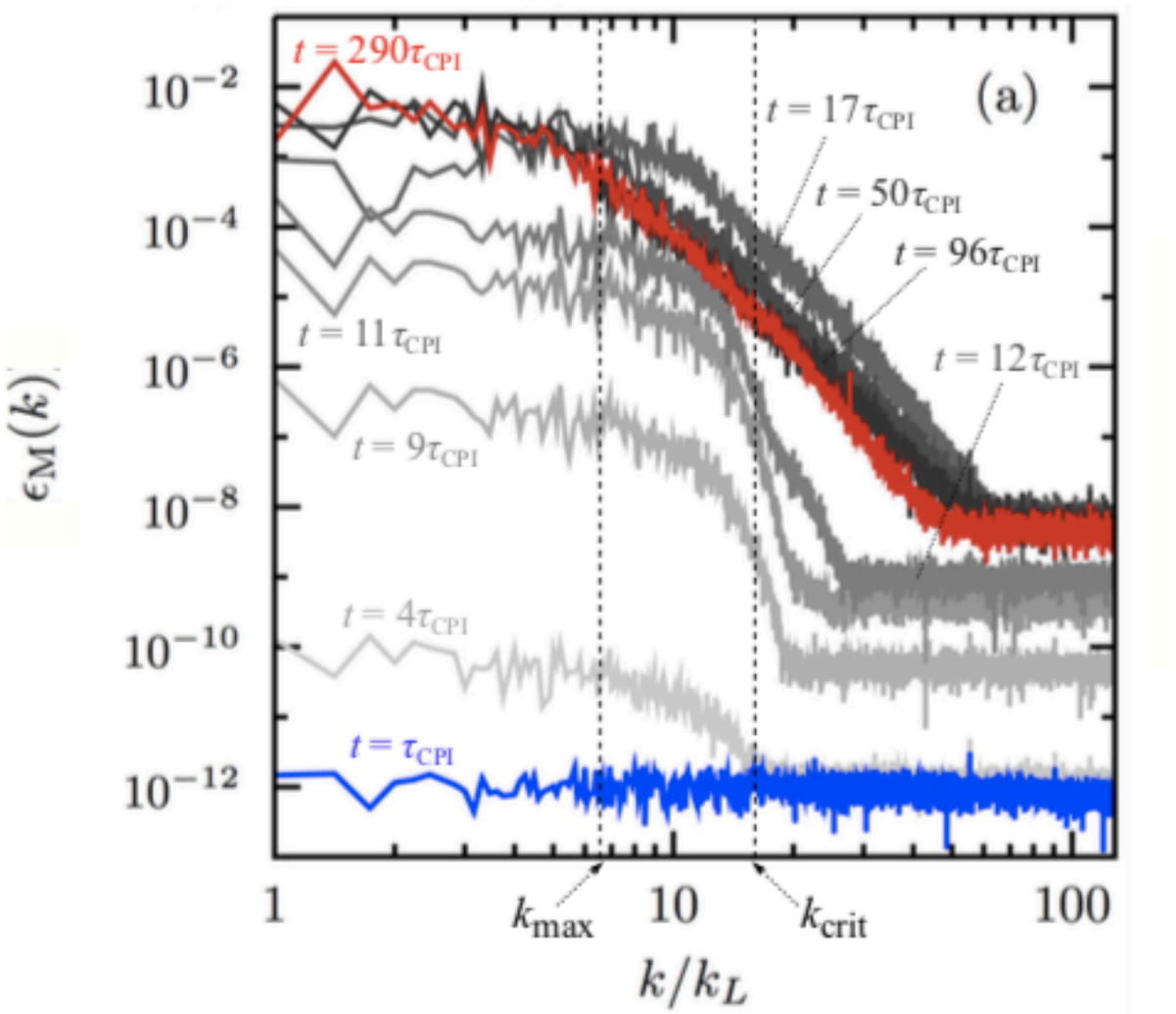
chiral anomaly

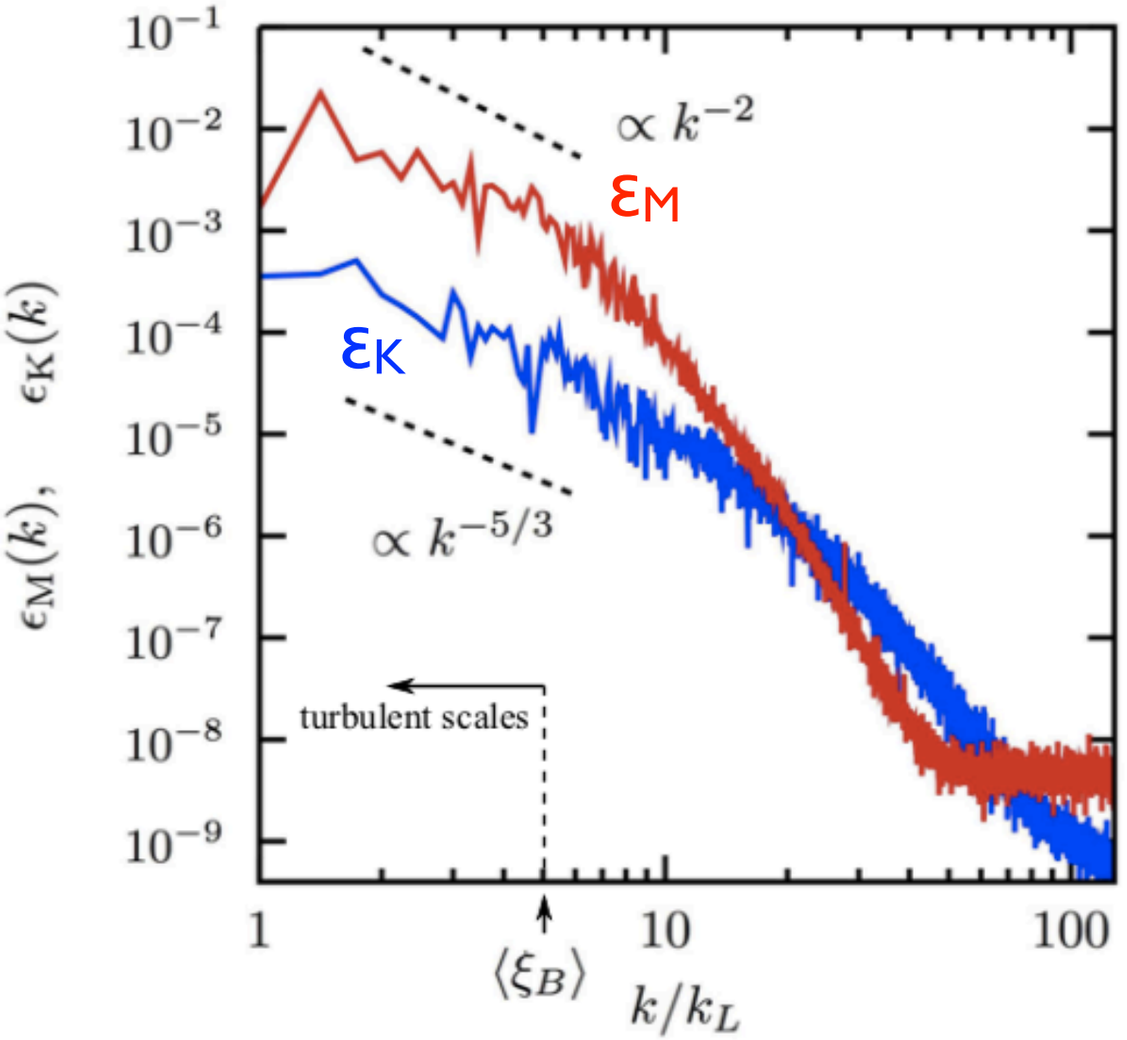
- Setup for proto-neutron stars (100 MeV = 1):

$$\rho_0 = 5.0, \ P_0 = 1.0, \ \xi_{B0} = 4.2 \times 10^{-3}, \ \eta = 100.0$$

Masada et al., arXiv:1805.10419; movie available at http://www.kusastro.kyoto-u.ac.jp/~masada/movie.mp4

Energy spectra





Chiral effects lead to the inverse cascade, which may be favorable for explosions

Masada et al., arXiv:1805.10419; see also Brandenburg et al., arXiv:1707.03385; Mace et al., arXiv:1910.01654

Inverse Energy Cascade in Three-Dimensional Isotropic Turbulence

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(Received 2 November 2011; published 20 April 2012)

We study the statistical properties of homogeneous and isotropic three-dimensional (3D) turbulent flows. By introducing a novel way to make numerical investigations of Navier-Stokes equations, we show that all 3D flows in nature possess a subset of nonlinear evolution leading to a reverse energy transfer: from small to large scales. Up to now, such an inverse cascade was only observed in flows under strong rotation and in quasi-two-dimensional geometries under strong confinement. We show here that energy flux is always reversed when mirror symmetry is broken, leading to a distribution of helicity in the system with a well-defined sign at all wave numbers. Our findings broaden the range of flows where the inverse energy cascade may be detected and rationalize the role played by helicity in the energy transfer process, showing that both 2D and 3D properties naturally coexist in all flows in nature. The unconventional numerical methodology here proposed, based on a Galerkin decimation of helical Fourier modes, paves the road for future studies on the influence of helicity on small-scale intermittency and the nature of the nonlinear interaction in magnetohydrodynamics.

3D Navier-Stokes eq. shows inverse cascade if parity is maximally broken

Conclusion

- Conventional theory for supernovae misses two key physics: symmetry and topology
- Chiral effects can reverse the turbulent behavior from direct to inverse cascade, which may help the explosion.
- Future applications of ν chiral radiation transfer theory