How vorticity generates spin polarization through nonlocal collisions

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INT program INT-20-1c

Criticality and Chirality: Novel Phenomena in Heavy-Ion Collisions

Institute for Nuclear Theory, University of Washington, Seattle, U.S.A., May 20, 2020

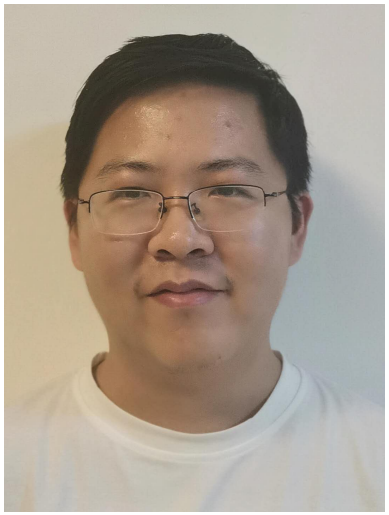
Talk based on:

N. Weickgenannt, X. L. Sheng, E. Speranza, Q. Wang, DHR,
"Kinetic theory for massive spin-1/2 particles from the Wigner-function formalism",
PRD 100 (2019) 056018

N. Weickgenannt, E. Speranza, X. L. Sheng, Q. Wang, DHR,
"Generating spin polarization from vorticity through nonlocal collisions",
[arXiv:2005.01506 [hep-ph]]



Nora



Xin-Li



Enrico



Qun

Hydrodynamics is effective theory for small wavenumbers and frequencies

$$k,\, \omega \sim \frac{1}{L} \ll \frac{1}{\lambda_{\text{mfp}}} \ll T,\, \mu$$

$\Longrightarrow$ can be derived from underlying microscopic theory by integrating out large wavenumbers and frequencies k, ω

E.g. microscopic theory: kinetic theory $\cong$ Boltzmann equation

$\Longrightarrow$ integrating out: method of moments (MoM)
G.S. Denicol, H. Niemi, E. Molnar, DHR, PRD 85 (2012) 114047

in fact, in MoM all momenta are integrated out!
usually ok, since integrals are dominated by $k \sim T$, μ

$\Longrightarrow$ conservation laws: dynamics of conserved quantities on scale L
relaxation equations: dynamics of dissipative quantities on scale λ_{mfp}

$\Longrightarrow$ however, not ok, if correlation length $\xi \gg \lambda_{\text{mfp}} \gg 1/T$, e.g. near critical point

$\Longrightarrow$ hydro+, hydro++, ...

see week-1 talks by Bazar, Heinz, Nahrgang, Schäfer, Teaney, Yin

MHD from microscopic theory

derive magnetohydrodynamics (MHD) from microscopic theory

$\Longrightarrow$ couple electromagnetic fields to Boltzmann equation: $\Longrightarrow$ Lorentz force!

$$(k \cdot \partial_x + \partial_k^\mu F_{\mu\nu} k^\nu) f_{\mathbf{k}} = C[f_{\mathbf{k}}]$$

$\Longrightarrow$ MoM gives relativistic second-order dissipative MHD for spin-0 particles

G.S. Denicol, X.G. Huang, E. Molnar, G.M. Monteiro, H. Niemi, J. Noronha, DHR, Q. Wang, PRD 98 (2018) 076009

G.S. Denicol, E. Molnar, H. Niemi, DHR, PRD 99 (2019) 056017

However: electric/magnetic dipole moment of particles neglected so far

spin $\cong$ dipole moment $\mathfrak{m}_{\alpha\beta}$ $\Longrightarrow$ Mathisson force!

$$\left[k \cdot \partial_x + \partial_k^\mu \left(F_{\mu\nu} k^\nu + \frac{1}{2} \mathfrak{m}_{\alpha\beta} \partial_{x\mu} F^{\alpha\beta}\right)\right] f_{\mathbf{k}} = C[f_{\mathbf{k}}]$$

$\Longrightarrow$ MoM gives relativistic second-order dissipative MHD for polarizable fluids

W. Israel, General Relativity and Gravitation 9 (1978) 451

D. Wagner, DHR, in preparation



Why MHD for polarizable fluids (spin-MHD)?

Early stage of non-central heavy-ion collisions: strong electromagnetic fields

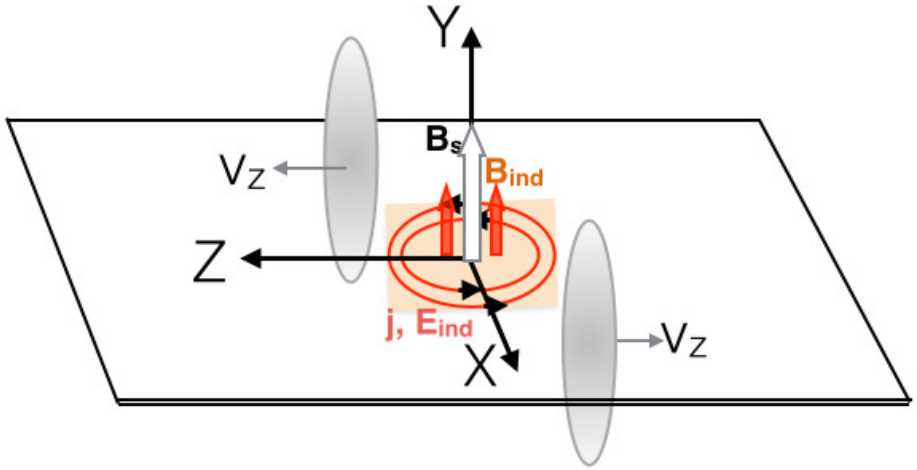


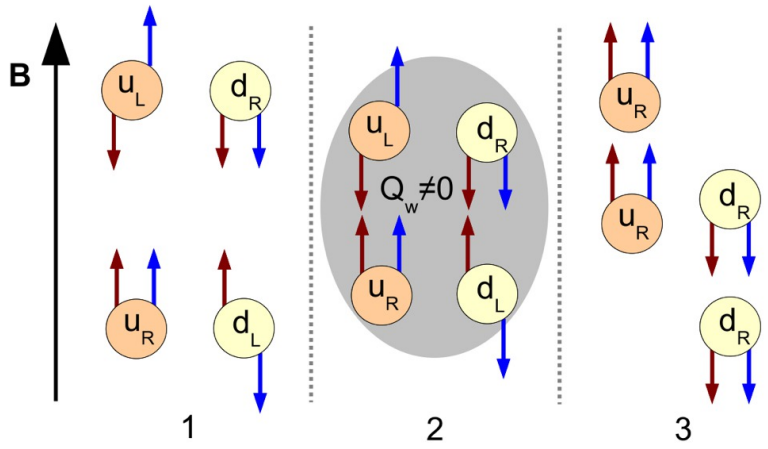
fig. from V. Roy, S. Pu, L. Rezzolla, DHR, PRC 96 (2017) 054909

$\Longrightarrow$ spins align in magnetic field

$\Longrightarrow$ non-vanishing polarization!

Chiral Magnetic Effect (CME)

$\Longrightarrow$ charge current induced by magnetic field and topology change



$$j_V^\mu \sim \mu_A B^\mu$$

D.E. Kharzeev, L.D. McLerran, H.J. Warringa, NPA 803 (2008) 227

K. Fukushima, D.E. Kharzeev, H.J. Warringa, PRD 78 (2008) 074033

see talk by Kharzeev

Rotation leads to polarization

- Condensed matter: **Barnett effect**

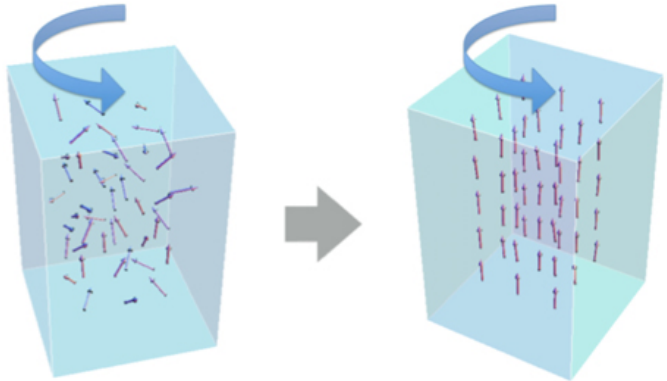


fig. by Mamoru Matsuo

$\Longrightarrow$ spins align under rotation (due to spin-orbit coupling)

$\Longrightarrow$ non-vanishing magnetization

- Non-central heavy-ion collisions: strong orbital angular momentum

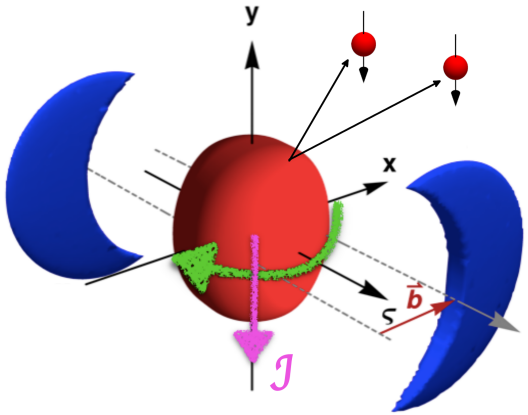


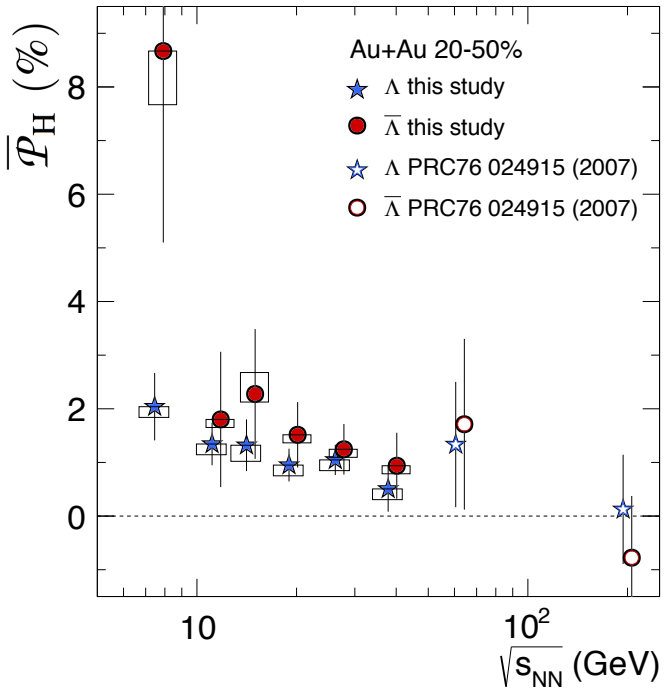
fig. by Radoslaw Ryblewski

Barnett effect in heavy-ion collisions?

Experimental observation - Λ polarization

Z.-T. Liang, X.-N. Wang, PRL 94 (2005) 102301
S.A. Voloshin [arXiv:nucl-th/0410089]

Measurement of Λ polarization along angular-momentum direction



L. Adamczyk et al. (STAR), Nature 548 (2017) 62

see talk by Voloshin

$\Longrightarrow$ QGP is "most vortical fluid ever observed"

$\omega \simeq (9+1) \times 10^{21} \mathrm{s}^{-1}$



For comparison:

- Great Red Spot of Jupiter $\omega \simeq 10^{-4}\,\mathrm{s}^{-1}$
- turbulent flow in superfluid He-II $\omega \sim 150\,\mathrm{s}^{-1}$
- superfluid nanodroplets $\omega \sim 10^{7}\,\mathrm{s}^{-1}$

Novel transport phenomena have been suggested in systems with spin-1/2 fermions:

- Chiral Magnetic Effect (CME): $j_V^\mu \sim \mu_A B^\mu$

  K. Fukushima, D.E. Kharzeev, H.J. Warringa, PRD 78 (2008) 074033

- Chiral Separation Effect (CSE): $j_A^\mu \sim \mu_V B^\mu$

  M.A. Metlitski, A.R. Zhitnitsky, PRD 72 (2005) 045011

- Chiral Magnetic Wave (CMW): interplay of CME and CSE

  D.E. Kharzeev, H.U. Yee, PRD 83 (2011) 085007

- Chiral Vortical Effect (CVE): $j_V^\mu \sim \omega^\mu$

  D.T. Son, P. Surowka, PRL 103 (2009) 191601

- ...

(see also A. Vilenkin, PRD 20 (1979) 1807; PRD 21 (1980) 2260; PRD 22 (1980) 3080)

Towards a theory of spin-MHD

so far, main focus on massless case:

- Chiral Kinetic Theory (CKT):
  M.A. Stephanov, Y. Yin, PRL 109 (2012) 162001
  D.T. Son, N. Yamamoto, PRL 109 (2012) 181602
  J.-Y. Chen, D.T. Son, M. Stephanov, PRL 115 (2015) 021601
  C. Manuel, J.M. Torres-Rincon, PRD 92 (2015) 074018
  Y. Hidaka, S. Pu, D.-L. Yang, PRD 95 (2017) 091901
  A. Huang, S. Shi, Y. Jiang, J. Liao, P. Zhuang, PRD 98 (2018) 036010
  J.-H. Gao, Z.-T. Liang, Q. Wang, X.-N. Wang, PRD 98 (2018) 036019
  S. Carignano, C. Manuel, J.M. Torres-Rincon, PRD 98 (2018) 076005
  ...
- Second-order dissipative chiral hydrodynamics:
  D.E. Kharzeev, H.U. Yee, PRD 84 (2011) 045025
  E.V. Gorbar, D.O. Rybalka, I.A. Shovkovy, PRD (2017) 096010

or on phenomenological approaches:

- "ideal" spin-hydrodynamics:
  W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (2018) 041901
  W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, E. Speranza, PRD 97 (2018) 116017
- dissipative spin-hydrodynamics from entropy-current analysis:
  K. Hattori, M. Hongo, X.G. Huang, M. Matsuo, H. Taya, PLB 795 (2019) 100

Spin-MHD for massive fermions

- goal: theory of spin-MHD for massive spin-1/2 particles
- starting point: QFT, Dirac equation
- strategy: use Wigner-function formalism to derive Boltzmann equation

  this talk: (i) with electromagnetic fields, no collisions

  (ii) with collisions, no electromagnetic fields
- future: – with collisions, with electromagnetic fields
  – use MoM to derive second-order dissipative fluid-dynamics

see also:

J.-H. Gao, Z.-T. Liang, PRD 100 (2019) 056021
K. Hattori, Y. Hidaka, D.-L. Yang, PRD 100 (2019) 096011
Z. Wang, X. Guo, S. Shi, P. Zhuang, PRD 100 (2019) 014015
D.-L. Yang, K. Hattori, Y. Hidaka [arXiv:2002.02612 [hep-ph]]
Y.-C. Liu, K. Mameda, X.-G. Huang [arXiv:2002.03753 [hep-ph]]

Wigner function:

- is quantum analogue of classical distribution function
- contains information about quantum state of system
- is Wigner transform of quantum-field theoretical two-point function
  H.-Th. Elze, M. Gyulassy, D. Vasak, Ann. Phys. 173 (1987) 462

$$W(x,p) = \int \frac{d^4 y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar} p\cdot y} \left\langle : \bar{\Psi}\left(x + \frac{y}{2}\right) \Psi\left(x - \frac{y}{2}\right) : \right\rangle$$

with $x = \frac{1}{2}(x_1 + x_2)$, $y = x_1 - x_2$

Wigner function:

- is quantum analogue of classical distribution function
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- is Wigner transform of quantum-field theoretical two-point function
  H.-Th. Elze, M. Gyulassy, D. Vasak, Ann. Phys. 173 (1987) 462

$$W(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y} \left\langle : \bar{\Psi}\left(x+\frac{y}{2}\right) U\left(x+\frac{y}{2},x\right) U\left(x,x-\frac{y}{2}\right) \Psi\left(x-\frac{y}{2}\right) : \right\rangle$$

with gauge link

$$U(b,a) \equiv P \exp\left[-\frac{i}{\hbar}\int_a^b dz^\mu A_\mu(z)\right]$$

to ensure gauge invariance

- in the following: A_μ external, classical gauge field

- in general: result of calculation of Wigner function directly from definition is not on-shell
- momentum variable of directly calculated Wigner function is physical (kinetic) momentum only at zeroth order in $\hbar$, resp. gradients
- Dirac equation implies transport equation for Wigner function
- idea: find general solutions for this transport equation by expanding in powers of $\hbar$ (equivalent to gradient expansion)
- convenient decomposition (Clifford algebra):

$$W = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)$$

- results will hold up to order $\mathcal{O}(\hbar)$
- notation: $W = W^{(0)} + \hbar W^{(1)} + \mathcal{O}(\hbar^2)$

- Dirac equation $\Longrightarrow$ equation of motion for Wigner function
  S.R. De Groot, W.A. van Leeuwen, C.G. van Weert, *Relativistic Kinetic Theory* (North-Holland, 1980)
  D. Vasak, M. Gyulassy, H.T. Elze, Ann. Phys. 173 (1987) 462

$$(\gamma \cdot K - m) W = \hbar \mathcal{C}[W]$$

with

$$\begin{aligned} K^\mu &\equiv \Pi^\mu + i\frac{\hbar}{2}\nabla^\mu , \\ \nabla^\mu &\equiv \partial_x^\mu - j_0(\Delta)\, F^{\mu\nu}\partial_{p\nu} , \\ \Pi^\mu &\equiv p^\mu - \frac{\hbar}{2} j_1(\Delta)\, F^{\mu\nu}\partial_{p\nu} , \end{aligned}$$

where $\Delta \equiv \frac{\hbar}{2}\partial_p \cdot \partial_x$, with ∂_x acting only on $F^{\mu\nu}$,
$j_0(x) = \sin x/x$, $j_1(x) = (\sin x - x\cos x)/x^2$ spherical Bessel functions

- exact quantum kinetic equation for Wigner function for massive spin 1/2-particles in external (in general inhomogeneous) electromagnetic fields!
- collision term $\mathcal{C}[W]$

Electromagnetic fields, free-streaming limit

- free streaming: $\mathcal{C}[W] = 0$
  N. Weickgenannt, X.-L. Sheng, E. Speranza, Q. Wang, DHR, PRD 100 (2019) 056018
  J.-H. Gao, Z.-T. Liang, PRD 100 (2019) 056021
  K. Hattori, Y. Hidaka, D.-L. Yang, PRD 100 (2019) 096011
  Z. Wang, X. Guo, S. Shi, P. Zhuang, PRD 100 (2019) 014015
  Y.-C. Liu, K. Mameda, X.-G. Huang [arXiv:2002.03753 [hep-ph]]

- insert Clifford decomposition into equation for Wigner function
  $\Longrightarrow$ system of 32 coupled differential equations

- equations for $\mathcal{F}$ (scalar) and $\mathcal{S}_{\mu\nu}$ (tensor) decouple

- determine $\mathcal{V}_\mu$ (vector), $\mathcal{A}_\mu$ (axial-vector), $\mathcal{P}$ (pseudo-scalar) from $\mathcal{F}$, $\mathcal{S}_{\mu\nu}$

$$\begin{aligned}\mathcal{F} &= m\left[V\,\delta(p^2-m^2)-\frac{\hbar}{2}F^{\mu\nu}\bar{\Sigma}_{\mu\nu}\,\delta'(p^2-m^2)\right]+\mathcal{O}(\hbar^2)\,,\\ \mathcal{S}_{\mu\nu} &= m\left[\bar{\Sigma}_{\mu\nu}\delta(p^2-m^2)-\hbar F_{\mu\nu}V\delta'(p^2-m^2)\right]+\mathcal{O}(\hbar^2)\end{aligned}$$

where

$$\begin{aligned}V &\equiv V^{(0)}+\hbar V^{(1)}+\mathcal{O}(\hbar^2)\,,\\ \bar{\Sigma}^{\mu\nu} &\equiv \Sigma^{(0)\mu\nu}A^{(0)}+\hbar\bar{\Sigma}^{(1)\mu\nu}+\mathcal{O}(\hbar^2)\,,\\ p_\nu\bar{\Sigma}^{\mu\nu} &= \hbar\nabla^\mu V+\mathcal{O}(\hbar^2)\,,\end{aligned}$$

with dipole-moment tensor $\Sigma^{(0)\mu\nu}=-\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\alpha n^{(0)}_\beta$, $n^{(0)}_\beta$ spin-quantization direction

$\Longrightarrow$ From $\mathcal{F}$, $\mathcal{S}_{\mu\nu}$ follow:

$$\begin{aligned}\mathcal{P} &= \frac{\hbar}{4m}\epsilon^{\mu\nu\alpha\beta}\nabla_\mu\left[p_\nu\bar{\Sigma}_{\alpha\beta}\,\delta(p^2-m^2)\right]+\mathcal{O}(\hbar^2)\,,\\ \mathcal{V}_\mu &= p_\mu\left[V\delta(p^2-m^2)-\frac{\hbar}{2}F^{\alpha\beta}\bar{\Sigma}_{\alpha\beta}\delta'(p^2-m^2)\right]+\frac{\hbar}{2}\nabla^\nu\left[\bar{\Sigma}_{\mu\nu}\delta(p^2-m^2)\right]+\mathcal{O}(\hbar^2)\,,\\ \mathcal{A}_\mu &= -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu\bar{\Sigma}^{\alpha\beta}\,\delta(p^2-m^2)+\hbar\tilde{F}_{\mu\nu}p^\nu V\,\delta'(p^2-m^2)+\mathcal{O}(\hbar^2)\,,\end{aligned}$$

Taylor expansion of δ-function:

$$\mathcal{F} = \frac{2m}{(2\pi\hbar)^3} \sum_{s=\pm} \delta\left(p^2 - m^2 - s\,\frac{\hbar}{2}\,\Sigma^{(0)}_{\mu\nu} F^{\mu\nu}\right) f_s$$

$\Longrightarrow$ modified on-shell condition! (energy shift by dipole-field interaction),
f_s distribution function for particles with spin $s = \pm$, $V = f_+ + f_-$, $A^{(0)} = f^{(0)}_+ - f^{(0)}_-$

$\Longrightarrow$ generalized Boltzmann equation:

$$\sum_s \delta\left(p^2 - m^2\right)\left\{p\cdot\partial_x f_s + \partial_{p\mu}\left[F^{\mu\nu} p_\nu + s\,\frac{\hbar}{4}\,\Sigma^{(0)}_{\alpha\beta}\left(\partial^\mu_x F^{\alpha\beta}\right)\right] f_s\right\} = 0$$

$\Longrightarrow$ force on particle: Lorentz force plus Mathisson force, dipole moment $\mathfrak{m}_{\alpha\beta} \equiv s\,\frac{\hbar}{2}\Sigma^{(0)}_{\alpha\beta}$

$\Longrightarrow$ first Mathisson-Papapetrou-Dixon (MPD) equation!
describes particle with classical dipole moment $\mathfrak{m}_{\alpha\beta}$ in electromagnetic field
W. Israel, General Relativity and Gravitation 9 (1978) 451

$$m\frac{d}{d\tau}p^\mu = F^{\mu\nu} p_\nu + s\,\frac{\hbar}{4}\,\Sigma^{(0)}_{\alpha\beta}\left(\partial^\mu_x F^{\alpha\beta}\right)$$

where τ worldline parameter, $\frac{d}{d\tau} = \dot{x}^\mu \frac{\partial}{\partial x^\mu} + \dot{p}^\mu \frac{\partial}{\partial p^\mu}$

$\bar{\Sigma}_{\mu\nu}$ determined by kinetic equation for dipole moment:

$$\delta(p^2 - m^2)\left[p\cdot\nabla\bar{\Sigma}^{\mu\nu} - \bar{\Sigma}^{\lambda\nu}F^{\mu}_{\ \lambda} + \bar{\Sigma}^{\lambda\mu}F^{\nu}_{\ \lambda} + \frac{\hbar}{2}\left(\partial_{x\alpha}F^{\mu\nu}\right)\partial_{p\alpha}V\right] = 0$$

$\Longrightarrow$ to zeroth order in $\hbar$:

$$m\frac{d}{d\tau}\Sigma^{(0)\mu\nu} = \Sigma^{(0)\lambda\nu}F^{\mu}_{\ \lambda} - \Sigma^{(0)\lambda\mu}F^{\nu}_{\ \lambda}$$

$\Longrightarrow$ second MPD equation for dipole-moment tensor $\Sigma^{(0)}_{\mu\nu}$!

W. Israel, General Relativity and Gravitation 9 (1978) 451

equivalent to Bargmann-Michel-Telegdi (BMT) equation

V. Bargmann, L. Michel, V.L. Telegdi, PRL 2 (1959) 435

$$m\frac{d}{d\tau}n^{(0)\mu} = F^{\mu\nu}n^{(0)}_{\nu}$$

with classical spin vector $n^{(0)\mu} = -\frac{1}{2m}\epsilon^{\mu\nu\alpha\beta}p_{\nu}\Sigma^{(0)}_{\alpha\beta}$

Massless limit

- non-relativistic dipole-moment tensor related to spin three-vector n^k:

$$\Sigma^{ij} = \epsilon^{ijk} n^k$$

- for massive particles: define spin in rest frame of particle $p^\mu_{LRF} = (m, \mathbf{0})^T$

$$\Sigma^{\mu\nu} = -\frac{1}{m} \epsilon^{\mu\nu\alpha\beta} p_\alpha n_\beta$$

U. Heinz, PLB 144 (1984) 228

- for massless particles: define spin in arbitrary frame with four-velocity u^μ
  $\Longrightarrow \quad p^\mu \to u^\mu$
  only projection of n^μ onto direction of p^μ ($\cong$ helicity) observable
  $\Longrightarrow \quad n^\mu \to p^\mu$

$$\Sigma^{\mu\nu}_u = -\frac{1}{p \cdot u} \epsilon^{\mu\nu\alpha\beta} u_\alpha p_\beta$$

J.-Y. Chen, D.T. Son, M. Stephanov, PRL 115 (2015) 021601

- massless limit: replace massive by massless dipole-moment tensor $\Sigma^{\mu\nu} \to \Sigma^{\mu\nu}_u$
- result agrees with previously known massless solution!
  Y. Hidaka, S. Pu, D.-L. Yang, PRD 95 (2017) 091901
  A. Huang, S. Shi, Y. Jiang, J. Liao, P. Zhuang, PRD 98 (2018) 036010
  J.-H. Gao, Z.-T. Liang, Q. Wang, X.-N. Wang, PRD 98 (2018) 036019
- more rigorous treatment: X.L. Sheng, Q. Wang, X.G. Huang [arXiv:2005.00204 [hep-ph]]

Equilibrium (I)

CRC-TR 211

Equilibrium ansatz for distribution function f_s

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338 (2013) 32

$$
\begin{aligned}
f_{s,eq}(x,p) &= \frac{1}{(2\pi\hbar)^3}\left[e^{g_s}+1\right]^{-1} \\
g_s &\equiv \beta(x)\cdot p - \alpha_s(x) - \frac{\hbar}{4}\Omega^{\mu\nu}(x)\Sigma^{(0)}_{\mu\nu}
\end{aligned}
$$

- $\alpha_s(x) \cong$ Lagrange multiplier for particle-number conservation (spin projection s)
- $\beta^\mu(x) \cong$ Lagrange multiplier for 4-momentum conservation
- spin potential $\Omega^{\mu\nu}(x) \cong$ Lagrange multiplier for **total** angular momentum conservation

$\Longrightarrow$ insert into transport equations, derive global equilibrium conditions:

$$
\begin{aligned}
\partial_x^\mu \beta^\nu + \partial_x^\nu \beta^\mu &= 0 \\
\partial_x^\mu \alpha_s &= -F^{\mu\nu}\beta_\nu \\
\Omega^{\mu\nu} = \varpi^{\mu\nu} \equiv -\frac{1}{2}\partial_x^{[\mu}\beta^{\nu]} &= \text{const.}
\end{aligned}
$$

where $a^{[\mu}b^{\nu]} \equiv a^\mu b^\nu - a^\nu b^\mu$

Insert $f_{s,eq}$ into $\mathcal{V}^\mu$, $\mathcal{A}^\mu$, expand to first order in $\hbar$ using

$$f_{s,eq} = f_s^{(0)} - s\frac{\hbar}{4}\varpi^{\mu\nu}\Sigma^{(0)}_{\mu\nu}\frac{\partial f_s^{(0)}}{\partial \alpha_s} + \mathcal{O}(\hbar^2)$$

where $f_s^{(0)} = \dfrac{1}{(2\pi\hbar)^3}\left[e^{g_s^{(0)}}+1\right]^{-1}$, $g_s^{(0)} = \beta(x)\cdot p - \alpha_s(x)$, to obtain

$$\begin{aligned}
\mathcal{V}^\mu &= 2\sum_s \left(\delta(p^2-m^2)\left\{p^\mu + s\frac{\hbar}{4}\epsilon^{\mu\nu\alpha\beta}\left[m\,\varpi_{\alpha\beta}\,n_\nu^{(0)}\frac{\partial}{\partial\alpha_s} + \frac{2}{m}p_\nu(\nabla_\alpha n_\beta^{(0)})\right]\right\}\right.\\
&\quad \left. + s\,m\,\hbar\tilde{F}^{\mu\nu}n_\nu^{(0)}\delta'(p^2-m^2)\right) f_s^{(0)} + \mathcal{O}(\hbar^2)\\
\mathcal{A}^\mu &= 2\sum_s \left(\delta(p^2-m^2)\left\{s\,m\,n^{(0)\mu} + \frac{\hbar}{4}\epsilon^{\mu\nu\alpha\beta}p_\nu\left[\varpi_{\alpha\beta}\frac{\partial}{\partial\alpha_s} - \Xi_{\alpha\beta}\right]\right\}\right.\\
&\quad \left. + \hbar\tilde{F}^{\mu\nu}p_\nu\delta'(p^2-m^2)\right) f_s^{(0)} + \mathcal{O}(\hbar^2)
\end{aligned}$$

with the analogues of CME, CVE, ACVE, CSE in the massive case

Polarization from vorticity through nonlocal collisions

N. Weickgenannt, E. Speranza, X.-L. Sheng, Q. Wang, DHR [arXiv 2005.01506 (2020)[hep-ph]]

- in the following: neglect electromagnetic fields
- idea: nonlocality of microscopic collision process couples orbital angular momentum to spin ("spin-orbit coupling")
- strategy: derive Boltzmann equation from Wigner-function formalism with $\mathcal{C}[W] \neq 0$

  $\Longrightarrow$ obtain explicit form of nonlocal collision term
- study impact on spin-hydrodynamics

$$
\begin{aligned}
(\gamma\cdot K - m)W &= \hbar\,\mathcal{C}[W]\,, \\
\mathcal{C}[W] &= \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p\cdot y}\left\langle : \rho\left(x-\frac{y}{2}\right)\bar{\Psi}\left(x+\frac{y}{2}\right) :\right\rangle
\end{aligned}
$$

with $\rho \equiv -(1/\hbar)\partial\mathcal{L}_I/\partial\bar{\Psi}$, $\mathcal{L}_I$ interaction Lagrangian

- Clifford decomposition results (again) in 32 (coupled) differential equations
- expand $\mathcal{F}$, $\mathcal{P}$, $\mathcal{V}_\mu$, $\mathcal{A}_\mu$, $\mathcal{S}_{\mu\nu}$ in powers of $\hbar$
- assume that spin effects are $\mathcal{O}(\hbar)$

  $\Longrightarrow \quad \mathcal{V}^\mu = \frac{1}{m}p^\mu\bar{\mathcal{F}} + \mathcal{O}(\hbar^2)$, where $\bar{\mathcal{F}} \equiv \mathcal{F} - \hbar\, p^\mu \mathrm{ReTr}(\gamma_\mu\mathcal{C})/m^2 + \mathcal{O}(\hbar^2)$
- relevant transport equations:

$$
\begin{aligned}
p\cdot\partial\bar{\mathcal{F}} &= m\,C_F \\
p\cdot\partial\mathcal{A}^\mu &= m\,C_A^\mu
\end{aligned}
$$

where $\quad C_F \equiv 2\,\mathrm{Im\,Tr}(\mathcal{C})\,, \quad C_A^\mu \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\nu\,\mathrm{Im\,Tr}(\sigma_{\alpha\beta}\mathcal{C})$

Spin as phase-space variable

In order to account for spin dynamics enlarge phase space

J. Zamanian, M. Marklund, G. Brodin, NJP 12 (2010) 043019

W. Florkowski, R. Ryblewski, A. Kumar, Prog. Part. Nucl. Phys. 108 (2019) 103709

$\Longrightarrow$ introduce new phase-space variable $\mathfrak{s}^\mu$ by defining

$$\mathfrak{f}(x,p,\mathfrak{s}) \equiv \frac{1}{2}\left[\bar{\mathcal{F}}(x,p) - \mathfrak{s}\cdot\mathcal{A}(x,p)\right]$$

$\Longrightarrow$ obtain $\bar{\mathcal{F}}$, $\mathcal{A}^\mu$ via

$$\bar{\mathcal{F}} = \int dS(p)\,\mathfrak{f}(x,p,\mathfrak{s})\,, \quad \mathcal{A}^\mu = \int dS(p)\,\mathfrak{s}^\mu \mathfrak{f}(x,p,\mathfrak{s})$$

where $dS(p) \equiv \dfrac{\sqrt{p^2}}{\sqrt{3}\pi} d^4\mathfrak{s}\,\delta(\mathfrak{s}^2+3)\,\delta(p\cdot\mathfrak{s})$

$\Longrightarrow$ Boltzmann equation

$$p\cdot\partial\,\mathfrak{f}(x,p,\mathfrak{s}) = m\,\mathfrak{C}[\mathfrak{f}]\,, \quad \mathfrak{C}[\mathfrak{f}] \equiv \frac{1}{2}(C_F - \mathfrak{s}\cdot C_A)$$

$\Longrightarrow$ all dynamics in one scalar equation!

- compute collision term $\mathfrak{C}[\mathfrak{f}]$ following
  S.R. De Groot, W.A. van Leeuwen, C.G. van Weert, *Relativistic Kinetic Theory* (North-Holland, 1980) $\Longrightarrow$ full quantum field-theoretical calculation!
- expand up to first order in gradients $\Longrightarrow$ $\mathfrak{C}[\mathfrak{f}] = \mathfrak{C}_l[\mathfrak{f}] + \hbar\,\mathfrak{C}_{nl}[\mathfrak{f}] + \mathcal{O}(\hbar^2)$

  where $\mathfrak{C}_l[\mathfrak{f}] \equiv \mathfrak{C}_l^{(0)}[\mathfrak{f}] + \hbar\,\mathfrak{C}_l^{(1)}[\mathfrak{f}]$ is **local** contribution and $\mathfrak{C}_{nl}[\mathfrak{f}]$ is **nonlocal** contribution
- after lengthy calculation, **full** collision term can be written in very intuitive form:

$$\begin{aligned}\mathfrak{C}[\mathfrak{f}] \;=\; & \int d\Gamma_1 d\Gamma_2 d\Gamma'\, \mathcal{W}\,\big[\mathfrak{f}(x+\Delta_1,p_1,\mathfrak{s}_1)\,\mathfrak{f}(x+\Delta_2,p_2,\mathfrak{s}_2) \\ & \qquad - \mathfrak{f}(x+\Delta,p,\mathfrak{s})\,\mathfrak{f}(x+\Delta',p',\mathfrak{s}')\big] \\ + & \int d\Gamma_2\, dS_1(p)\,\mathfrak{W}\,\mathfrak{f}(x+\Delta_1,p,\mathfrak{s}_1)\,\mathfrak{f}(x+\Delta_2,p_2,\mathfrak{s}_2)\end{aligned}$$

  where $d\Gamma \equiv d^4p\, dS(p)$
- structure: momentum and spin exchange $(p_1,\mathfrak{s}_1),(p_2,\mathfrak{s}_2) \to (p,\mathfrak{s}),(p',\mathfrak{s}')$
  $+$ spin exchange only $(p,\mathfrak{s}_1),(p_2,\mathfrak{s}_2) \to (p,\mathfrak{s}),(p_2,\mathfrak{s}')$
- collision nonlocal, particle positions displaced by $\Delta \sim \mathcal{O}(\hbar)$
- displacements Δ are functions of $p,\mathfrak{s}$
- transition amplitudes $\mathcal{W}$, $\mathfrak{W}$ depend on phase-space spin variables, assumed to be constant over scales $\sim \Delta$ (low-density approximation)

Equilibrium (I)

equilibrium ansatz for distribution function $\mathfrak{f}$

F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338 (2013) 32

W. Florkowski, R. Ryblewski, A. Kumar, Prog. Part. Nucl. Phys. 108 (2019) 103709

$$\mathfrak{f}_{eq}(x,p,\mathfrak{s}) = \frac{m}{(2\pi\hbar)^3}\exp\left[-\beta(x)\cdot p + \frac{\hbar}{4}\Omega_{\mu\nu}(x)\Sigma^{\mu\nu}_{p,\mathfrak{s}}\right]\delta(p^2-M^2)$$

- for sake of simplicity, no Lagrange multiplier for particle-number conservation
- $\beta^\mu(x) \cong$ Lagrange multiplier for 4-momentum conservation
- spin potential $\Omega^{\mu\nu}(x) \cong$ Lagrange multiplier for **total** angular momentum conservation
- $M^2 = m^2 + \hbar\,\delta m^2$, δm^2 mass correction from interactions
- dipole-moment tensor $\Sigma^{\mu\nu}_{p,\mathfrak{s}} \equiv -\frac{1}{m}\epsilon^{\mu\nu\alpha\beta}p_\alpha \mathfrak{s}_\beta$

$\Longrightarrow$ insert into $\mathfrak{C}[\mathfrak{f}]$ and expand up to first order in $\hbar$

$\Longrightarrow$ zeroth-order collision term vanishes due to momentum conservation

At first order in $\hbar$:

$$\mathfrak{C}[f_{eq}] = -\int d\Gamma_1 d\Gamma_2 d\Gamma' \, \mathcal{W} \, e^{-\beta\cdot(p_1+p_2)}$$
$$\times \left[\partial_{x\mu}\beta_\nu \left(\Delta_1^\mu p_1^\nu + \Delta_2^\mu p_2^\nu - \Delta^\mu p^\nu - \Delta'^\mu p'^\nu\right) - \frac{\hbar}{4}\Omega_{\mu\nu}\left(\Sigma^{\mu\nu}_{p_1,s_1} + \Sigma^{\mu\nu}_{p_2,s_2} - \Sigma^{\mu\nu}_{p,s} - \Sigma^{\mu\nu}_{p',s'}\right)\right]$$
$$-\int d\Gamma_2 \, dS_1(p) dS'(p_2) \, \mathfrak{W} \, e^{-\beta\cdot(p+p_2)}$$
$$\times \left\{\partial_{x\mu}\beta_\nu \left[(\Delta_1^\mu - \Delta^\mu)p^\nu + (\Delta_2^\mu - \Delta'^\mu)p_2^\nu\right] - \frac{\hbar}{4}\Omega_{\mu\nu}(\Sigma^{\mu\nu}_{p,s_1} + \Sigma^{\mu\nu}_{p_2,s_2} - \Sigma^{\mu\nu}_{p,s} - \Sigma^{\mu\nu}_{p_2,s'})\right\}$$

total angular momentum (orbital + spin) $\quad J^{\mu\nu} = \Delta^{[\mu} p^{\nu]} + \frac{\hbar}{2}\Sigma^{\mu\nu}_{p,s}$

$\Longrightarrow$ conserved in a microscopic collision $\quad J_1^{\mu\nu} + J_2^{\mu\nu} = J^{\mu\nu} + J'^{\mu\nu}$

$\Longrightarrow$ collision term at first order vanishes, if:

$$\begin{aligned} \partial_x^\mu \beta^\nu + \partial_x^\nu \beta^\mu &= 0 \\ \Omega^{\mu\nu} = \varpi^{\mu\nu} \equiv -\frac{1}{2}\partial_x^{[\mu}\beta^{\nu]} &= \text{const.} \end{aligned}$$

where $a^{[\mu} b^{\nu]} \equiv a^\mu b^\nu - a^\nu b^\mu$

Equilibrium (III)

Discussion:

- $\mathfrak{C}[\mathfrak{f}_{eq}] = 0$ in global equilibrium!
- $\exists$ local equilibrium with nonlocal collisions? $\Longrightarrow$ (probably) no!
- confirms result from statistical quantum field theory:
  in equilibrium, spin potential equal to thermal vorticity
  F. Becattini, PRL 108 (2012) 244502
- interpretation: when approaching equilibrium, non-vanishing vorticity converts orbital angular momentum into spin through nonlocal collisions
  $\Longrightarrow$ initially unpolarized fluid gets polarized, Barnett effect!

- additional equations of motion from conservation of total angular momentum of fluid

$$J^{\lambda,\mu\nu} \equiv x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu} + \hbar S^{\lambda,\mu\nu}$$

  where $T^{\mu\nu}$ energy-momentum tensor, $S^{\lambda,\mu\nu}$ spin tensor

- equations of motion:

$$\partial_\mu T^{\mu\nu} = 0 \,, \qquad \hbar \partial_\lambda S^{\lambda,\mu\nu} = T^{[\nu\mu]}$$

  $\Longrightarrow$ 4 due to energy-momentum conservation
  + 6 due to total angular momentum conservation

- in f_{eq}: 10 unknowns, 4 components of β^μ + 6 independent components of $\Omega^{\mu\nu}$
  (however: additional unknowns enter in second-order dissipative fluid dynamics
  $\Longrightarrow$ additional equations required!)

W. Florkowski, B. Friman, A. Jaiswal, E. Speranza, PRC 97 (2018) 41901
W. Florkowski, B. Friman, A. Jaiswal, R. Ryblewski, E. Speranza, PRD 97 (2018) 116017
W. Florkowski, F. Becattini, E. Speranza, Act. Phys. Pol. B 49 (2018) 1409
W. Florkowski, F. Becattini, E. Speranza, PLB 789 (2019) 419
K. Hattori, M. Hongo, X.-G. Huang, M. Matsuo, H. Taya, PLB 795 (2019) 100

- definition of $T^{\mu\nu}$ and $S^{\lambda,\mu\nu}$ depends on choice of pseudo-gauge
  F.W. Hehl, Rept. Math. Phys. 9 (1976) 55
  E. Leader, C. Lorce, Phys. Rept. 541 (2014) 163
  F. Becattini, W. Florkowski, E. Speranza, PLB 789 (2019) 419
- canonical choice: applying Noether's theorem to Dirac equation
  $\Longrightarrow$ spin tensor not conserved for free fields
  $\Longrightarrow$ does not lead to covariant description of spin
- solution: apply Noether's theorem to Klein-Gordon equation for spinors
  J. Hilgevoord, S. Wouthuysen, NP 40 (1963) 1
  $\Longrightarrow$ connection to Wigner function:

$$T^{\mu\nu}_{\rm HW} = \frac{1}{m}\int d\Gamma\, p^\mu p^\nu\, \mathfrak{f}(x,p,\mathfrak{s}) + \mathcal{O}(\hbar^2)$$

$$S^{\lambda,\mu\nu}_{\rm HW} = \frac{1}{m}\int d\Gamma\, p^\lambda \left(\frac{1}{2}\Sigma^{\mu\nu}_{p,\mathfrak{s}} - \frac{\hbar}{4m^2}p^{[\mu}\partial^{\nu]}\right)\mathfrak{f}(x,p,\mathfrak{s}) + \mathcal{O}(\hbar^2)$$

- using Boltzmann equation

$$\partial_\mu T^{\mu\nu}_{\mathrm{HW}} = \int d\Gamma\, p^\nu\, \mathfrak{C}[\mathfrak{f}] = 0$$

$$\hbar\, \partial_\lambda S^{\lambda,\mu\nu}_{\mathrm{HW}} = \int d\Gamma\, \frac{\hbar}{2} \Sigma^{\mu\nu}_{p,\mathfrak{s}}\, \mathfrak{C}[\mathfrak{f}] = T^{[\nu\mu]}_{\mathrm{HW}}$$

- energy and momentum conserved in a collision
- spin not conserved in nonlocal collisions $\Longrightarrow$ $T^{[\nu\mu]}_{\mathrm{HW}} \neq 0$

  $\Longrightarrow$ spin is converted into orbital angular momentum (and vice versa)
- $T^{[\nu\mu]}_{\mathrm{HW}} = 0$ (i) for local collisions, as spin is collisional invariant
  (ii) in global equilibrium, as collision term vanishes
- with nonlocal collisions away from global equilibrium:
  dynamics (most probably) always dissipative!

- $p^\mu \to m(1,\boldsymbol{v})^T$, $\Sigma^{\mu\nu}_{p,\mathfrak{s}} \to \epsilon^{ijk}\mathfrak{s}^k$

$$T^{[ji]}_{\rm HW} = m\epsilon^{ijk}\partial^0\left\langle\frac{\hbar}{2}\mathfrak{s}^k\right\rangle + m\epsilon^{ijk}\partial^l\left\langle v^l\frac{\hbar}{2}\mathfrak{s}^k\right\rangle$$

with $\langle\ldots\rangle \equiv (m^2/2\pi\sqrt{3})\int d^3v\, d^3\mathfrak{s}\,\delta(\mathfrak{s}^2-3)\,(\ldots)f$

- comparison with micropolar fluids

G. Lukaszewicz, *Micropolar Fluids, Theory and Applications* (Birkhäuser, Boston, 1999)

$$\rho\left(\partial^0 + u^j\partial^j\right)\ell^i = \partial^j C^{ji} - \epsilon^{ijk}T^{jk}$$

with mass density ρ, fluid velocity $\boldsymbol{u}$, internal angular momentum

$$\rho\,\ell^i = m\left\langle\frac{\hbar}{2}\mathfrak{s}^i\right\rangle,$$

and couple-stress tensor

$$C^{ji} = -\left\langle\frac{\hbar}{2}\mathfrak{s}^i p^j\right\rangle + m\left\langle\frac{\hbar}{2}\mathfrak{s}^i\right\rangle u^j$$

- applications: spintronics, chiral active fluids, ...

R. Takahashi, M. Matsuo, M. Ono, K. Harii, H. Chudo, S. Okayasu, J. Ieda, S. Takahashi, S. Maekawa, E. Saitoh, Nature Physics 12 (2016) 52

D. Banerjee, A. Souslov, A.G. Abanov, V. Vitelli, Nature comm. 8 (2017) 1

Conclusions and Outlook

Derived transport equations for massive spin-1/2 particles

– with electromagnetic fields, no collisions:

- recovered classical equations of motion
- solution agrees with previously known solution in massless limit
- solution of collisionless Boltzmann equation: global equilibrium

– with collisions, no electromagnetic fields:

- derived nonlocal collision term
- collision term vanishes in global (but not necessarily local) equilibrium!
- local collisions $\Longrightarrow$ "ideal" spin-hydrodynamics possible
  nonlocal collisions $\Longrightarrow$ (most probably) always dissipative dynamics
- HW pseudo-gauge:
  - $T^{[\mu\nu]}_{\mathrm{HW}} \neq 0 \Longrightarrow$ spin is converted into orbital angular momentum (and vice versa)
  - $T^{[\mu\nu]}_{\mathrm{HW}} = 0$ with local collisions or in global equilibrium
- correct nonrelativistic limit

Outlook:

- with collisions, with electromagnetic fields
- derive second-order dissipative spin-MHD using MoM