# Phase structure and thermal properties in continnum QCD

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# Outline

1. **Backgrounds**
2. **Framework of functional QCD method**
3. **Phase structure of QCD**
4. **QCD states**
   - Quark spectrum in sQGP
   - Thermal hadrons
5. **Summary**

## Quantum ChromoDynamics

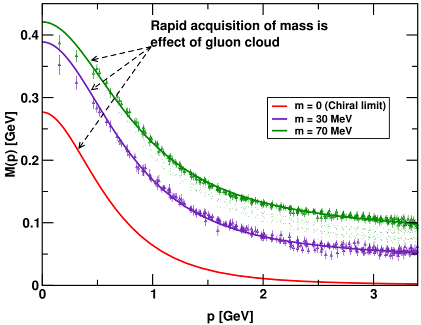
QCD is a non-Abelian gauge theory which describes the strong interaction between hadrons and also quarks, gluons inside.

- *Asymptotic free behavior in large energy scale*
- *In lower energy scale*
  1. *confinement*
  2. *dynamical mass generation*

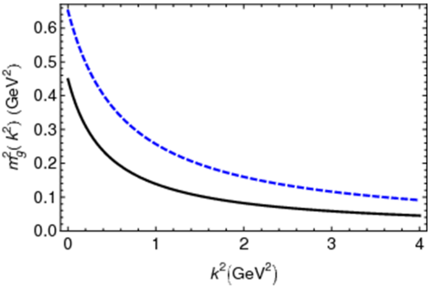
- Dynamics(Non-perturbative):
  (1) mass generation of quark (DCSB);
  (2)gluon mass generation might be related to confinement.
- Topology: special feature of non-Abelian gauge theory.

# dynamical mass generation

DCSB from quark propagator in fQCD approach.

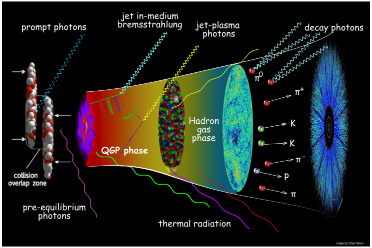


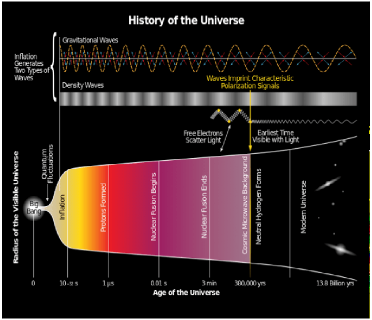

Gluon would also become massive



# phenomena

Mapping QCD in heavy ion collision and universe evolution:

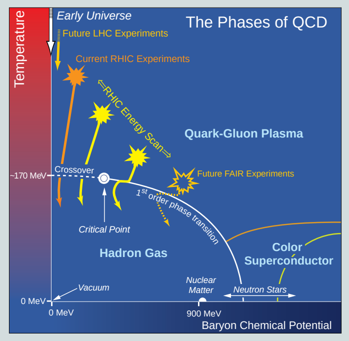


# Quantum ChromoDynamics

All these phenomena can be mapped into one phase diagram:

- quark-gluon plasma at high temperature and chemical potential
- Color-superconducting phase at low temperature and high baryon density
- The phase transition between the normal hadron phase and the quark-gluon plasma phase driven by different combination of temperature and chemical potential, and connected with crossover at low density by critical end point (CEP)




[1] The frontiers of nuclear science, A long range plan[J]. 2008.

# Framework of functional QCD method

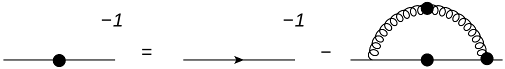
Dyson-Schwinger equations (DSEs) and functional renormailization group (fRG) approach are the nonperturbative approach in continuum QCD which contain the features of both confinement and chiral symmetry breaking.
They are both formulated as master equations for the effective action $\Gamma[\Phi]$ with the superfield $\Phi = (A_\mu, c, \bar{c}, q, \bar{q})$. The derivatives of $\Gamma$ with respect to the fields,

$$\Gamma^{(n)}_{\Phi_{i_1}\cdots\Phi_{i_n}}(p_1, ..., p_n) = \frac{\delta\Gamma[\Phi]}{\delta\Phi_{i_1}(p_1)\cdots\delta\Phi_{i_n}(p_n)}\,, \tag{1}$$

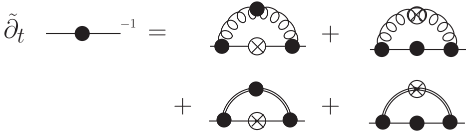
are the one particle irreducible correlation functions and are derived by taking field derivatives of the fRG or DSE for the effective action.

## quark gap equation

Taking the quark two point correlation function as an example, DSEs are basically integral equations, and can be exhibited as:

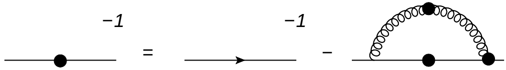


FRG are generally differential equations,



## solving the equation

Here we basically employ the framework of DSE, as shown above




To solve this equation, two ingredients need to be input:

- gluon propagator
- quark gluon vertex

To obtain the appropriate physical quantities:

- Symmetry constrained quark gluon vertex + modelling gluon propagator together with interaction kernel(RL, BC, ACM).
- FRG quark gluon vertex + FRG gluon propagator with no need for the interaction parameters in infrared region.

# Towards the realistic phase structure of QCD

lattice QCD simulation:

- solid computations at vanishing chemical potential,
- due to the sign problem, it is difficult for lattice QCD to reach large chemical potential region.

*Large chemical potential is accessible by functional QCD approaches.*

*For functional QCD method, it's quite straightforward to get the phase diagram of DCSB since the chiral phases could be identified by the scalar term in quark propagator.*

- chiral symmetry breaking phase: Nambu phase
- chiral symmetry preserved phase: Wigner phase

## QCD phase diagram

The Cornwall–Jackiw–Tomboulis (CJT) effective potential:

$$\Gamma(S) = -Tr[\ln(S_0^{-1}S) - S_0^{-1}S + 1] + \Gamma_2(S), \tag{2}$$

where $S_0$ and $S$ stands for the bare and full quark propagator,$\Gamma_2$ is the 2PI contribution. Calculating the variation respective to quark propagator, we have:

$$\frac{\partial^2\Gamma}{\partial S^2} = S^{-2} + \frac{\partial\Gamma_2(S)}{\partial S}. \tag{3}$$

combing with the derivative on the quark propagator DSE as:

$$-S^{-2}\frac{\partial S}{\partial T} = 1 + \frac{\partial\Gamma_2(S)}{\partial S}\frac{\partial S}{\partial T}, \tag{4}$$

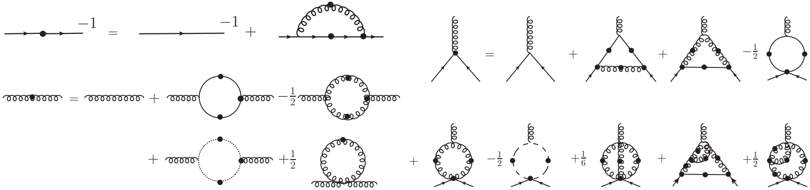
The criterion is then given by[1]:

$$\frac{\partial S}{\partial T} = -\frac{1}{\partial^2\Gamma/\partial S^2}. \tag{5}$$

[1] **Fei Gao**, Yu-xin Liu. Phys. Rev. D 94 (2016) 7, 076009.

## gap equation

To solve the gap equation[1,2]:




- **Input:** $N_f = 2$ gluon propagator and quark gluon vertex in vacuum from FRG.

*Difference-DSE:* The gluon propagator and quark gluon vertex in finite temperature and chemical potential by computing the difference:

$$\Gamma^{(n)}(p)\Big|_{T,\mu_B,N_f} = \Gamma^{(n)}(p)\Big|_{0,0,2} + \Delta\Gamma^{(n)}(p)\,. \tag{6}$$

[1] **Fei Gao** and Jan M. Pawlowski, arXiv:2010.13705.

[2] **Fei Gao** and Jan M. Pawlowski, Phys. Rev. D 102 (2020) 3, 034027.

*Difference-DSE for gluon propagator:*

- *drop the ghost loop in the difference-DSE for the gluon propagator*
- *the full three-gluon vertex is approximated by its classical counterpart.*

- ghost-gluon correlation functions show a negligible dependence on quark-content, temperature and density.
- the ghost loop in the difference-DSE for the gluon propagator only shows a thermal dependence, which is negligible.
- the three-gluon vertex dressing of the classical tensor structure only runs mildly for momenta $p^2 \sim 1$GeV. For smaller momenta it drops rapidly and even turns negative (but stays small). In this regime the strange and density contributions from the three-gluon diagram are negligible, as well as the subleading thermal fluctuations.

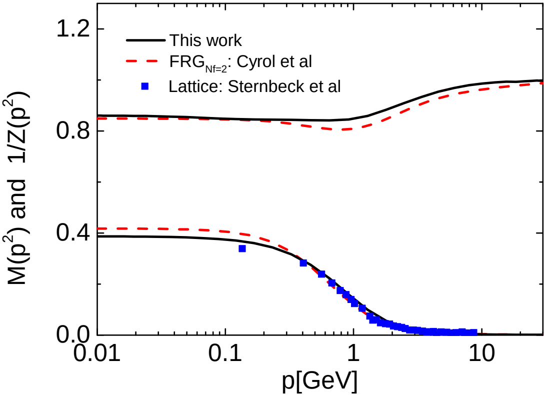
## *Difference-DSE for quark gluon vertex*

*We only compute the first two triangle diagrams in the first line within the difference-DSE for the quark-gluon vertex.*

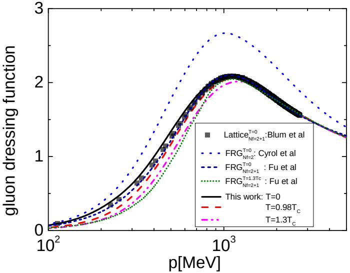
- The contribution of the ghost diagram can safely be neglected as well as the purely gluonic two-loop diagrams.
- The diagram with the four-quark vertex and the quark-gluon scattering vertex carry information about hadron resonances, which is the largest source for the systematic error in the present approximation:
  1. In the vacuum it is completely dominated by the sigma-pion resonance channel
  2. At finite density we expect a change of the dominance order to a diquark channel
  3. DSE-studies with resonance contributions shows that the contribution are subleading, though sizable.

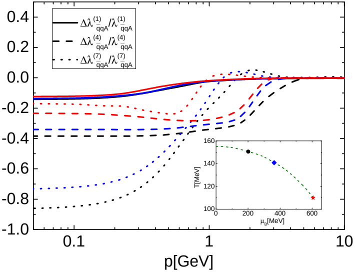
# consistency check

Consistency check between two methods:



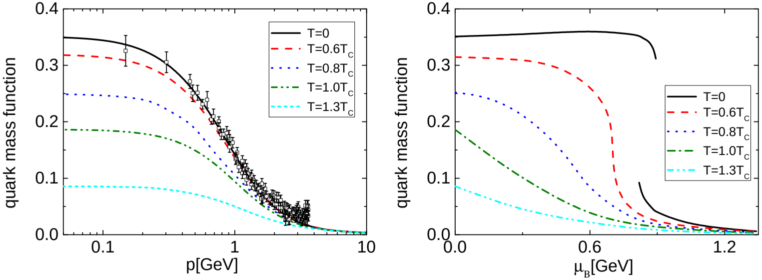
# Gluon propagator and Quark gluon vertex at finite T and $\mu_B$




- Medium effect includes a thermal mass scale in gluon propagator.
- Temperature correction enhances the infrared region of quark gluon vertex, Chemical potential effect gives non monotonous behaviour.

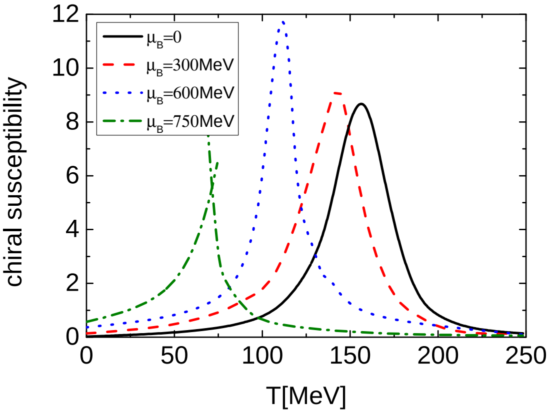
# The quark mass function at finite T and $\mu_B$




- the key feature of QCD is mass running, which leads to the possible phase transition.
- one usually chooses the mass at zero momentum as the typical scale to identify the quark.
- the strict definition needs to analyze the spectral function of quark.

## phase diagram

Scanning the susceptibility in the whole temperature-chemical potential plane:



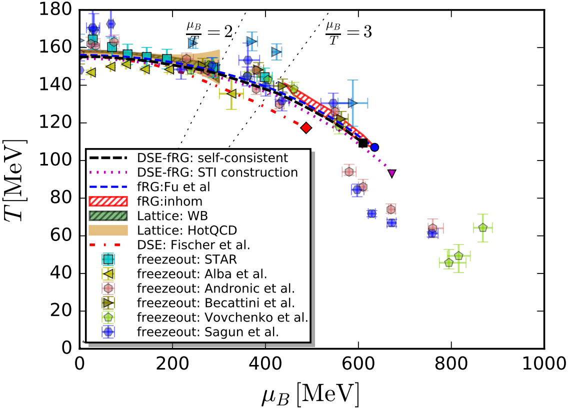

- low chemical potential, crossover
- large chemical potential, first order phase transition

| reference \ curvature $\kappa$ | $N_f = 2$ | $N_f = 2+1$ |
|---|---|---|
| fRG-DSE: this work | 0.0179(8) | 0.0150(7) |
| fRG: [19] | 0.0176(1) | 0.0142(2) |
| lattice: [36] | —— | 0.0149(21) |
| lattice: [39] | —— | 0.0144(26) |
| lattice: [41] | —— | 0.015(4) |
| DSE: [22] | —— | 0.0238 |
| DSE: [24] | —— | 0.038 |
| lattice: [65] | 0.0078(39) | —— |
| lattice: [66] | 0.0056(6) | —— |

TABLE I. Curvature coefficients $\kappa$, see (33): fRG-DSE: this work; fRG: [19]; Lattice collaborations: [36] (WB), [39] (Bonati *et al.*), [41] (hotQCD), [65] (Allton *et al.*), [66] (Forcrand and Philipsen), Lattice overviews [67, 68]; DSE: [22] (Fischer *et al.*), [24] (Gao *et al.*), DSE overview [8].

# QCD phase diagram

Phase diagram in temperature-chemical potential region for 2+1 flavour QCD




- $T_c = 155\,\text{MeV}$
- $(T_{CEP}, \mu_{B_{CEP}}) = (109, 610)\,\mathrm{MeV}$

- with no model parameters included
- subleading term: the hadron resonance channel

## strongly coupling quark gluon plasma

# Zero mode in strongly coupling quark gluon plasma [1]

After having a whole picture in the T-$\mu$ plane, now we focus on a short temperature range nearly above phase transition temperature $T_c$ at $\mu = 0$.

*In this region, the QCD matters are strongly coupled quark gluon plasma (sQGP), and will get an interesting feature from its nonperturbative property different from the perturbative results shown in the propagators' spectral.*

[1] FG et al, Phys. Rev. D 89 (2014)7, 076009.

## strongly coupling quark gluon plasma

The quark spectral could be connected with the quark propagator as following.

$$\begin{aligned} D_{\pm}(\tau, |\vec{p}|) &= T \sum_{n} e^{-i\omega_n \tau} S_{\pm}(i\omega_n + \mu, |\vec{p}|) \\ &= \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} \rho_{\pm}(\omega, |\vec{p}|) \left( \frac{e^{-(\omega+\mu)\tau}}{1 + e^{-(\omega+\mu)/T}} \right) . \end{aligned} \tag{7}$$

*It is an ill-posed problem to obtain the spectral since its degrees of freedom are much more than the dimension of our data. The solution cannot been obtained uniquely.*

# Maximum Entropy Method

The probability of quark spectral density $\rho$ can be obtained through Maximum Entropy Method (MEM) when we have known the information of quark propagator D and some prior information H.
MEM is based on the Bayes' theorem

$$P[\rho|DH] = \frac{P[D|\rho H]P[\rho|H]}{P[D|H]}\,, \tag{8}$$

where $P[D|\rho H]$ is the likelihood probability and $P[\rho|H]$ is the prior probability, $P[D|H]$ is just a normalization constant.

$$P[D|\rho H] = \frac{1}{Z_L}e^{-L}, \tag{9}$$

$$L = \sum_{i}^{N_{data}} \frac{(D_{data}(\tau_i) - D_\rho(\tau_i))^2}{2\sigma_i^2}\,. \tag{10}$$

## Maximum Entropy Method

$$P[\rho|H\alpha m] = \frac{1}{Z_S} e^{\alpha S}, \tag{11}$$

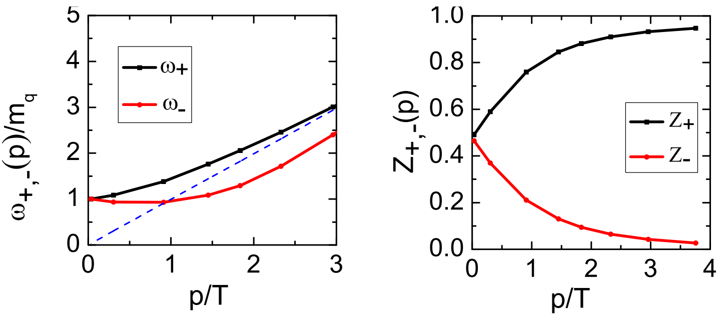
$$S[\rho] = \int_{-\infty}^{+\infty} d\omega \left[ \rho(\omega) - m(\omega) - \rho(\omega) \log \frac{\rho(\omega)}{m(\omega)} \right], \tag{12}$$

the quantity $m(\omega)$ is the "default model" which is simply chosen to be $m(\omega) = m_0 \theta(\Lambda^2 - \omega^2)$. The results should not depend on the choices for $m_0$, $\Lambda$.

*It can be proved that after setting this, the maximum solution for spectral $\rho$ is unique.*

## sQGP

We can then extract the quark spectral density.
At temperature T=3$T_c$, spectral of QGP return to the perturbative case.

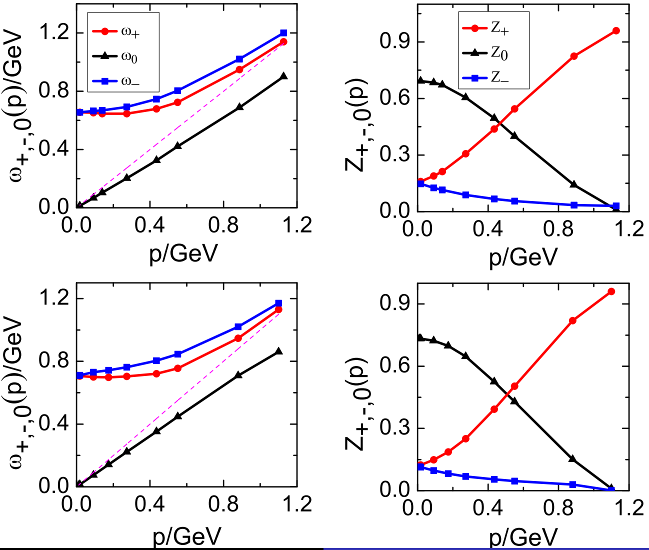



There're two collective modes, normal mode and plasmino.

*However, there will occur another mode nearly above $T_c$.*

## sQGP

The dispersion relation at $T = 1.1\,T_c$

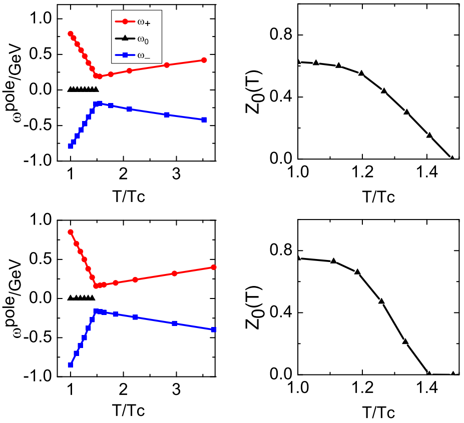


The new mode near above $T_c$

- new mode different from the normal and plasmino mode in perturbative computation
- zero mode (no thermal mass scale) and spacelike: first sound
  1. indicates the state is strongly-coupled quark gluon plasma
  2. in the hydrodynamics region

# sQGP

The mass poles of three collective modes with respect to temperature.




| | bare | BC | ACM |
|---|---|---|---|
| MT | $R_{MT}$ | $BC_{MT}$ | $DB_{MT}$ |
| QC | $R_{QC}$ | $BC_{QC}$ | $DB_{QC}$ |

All six cases shown similar results:

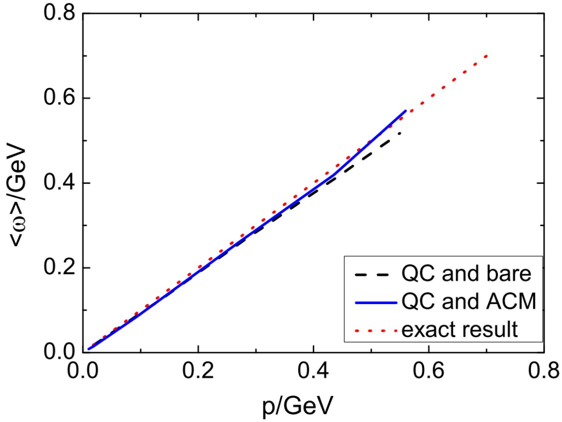
- nearly above $T_c$, the third mode occur;
- back to the perturbative case at high temperature.

# sQGP

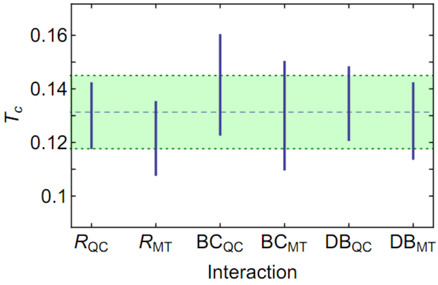
$$Z_2 \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \rho_{\pm}(|\vec{p}|, \omega') = 1,$$

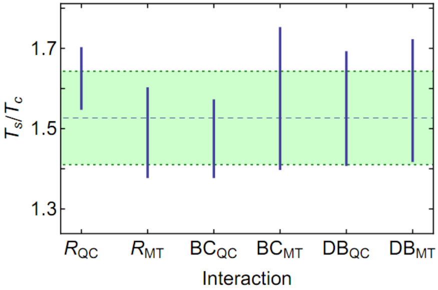
| $\lvert\vec{p}\rvert/GeV$ | $Z_+$ | $Z_0$ | $Z_-$ | $\sum_{i=+,0,-} Z_i$ |
|---|---|---|---|---|
| 0.10 | 0.13 | 0.75 | 0.11 | 0.99 |
| 0.27 | 0.21 | 0.71 | 0.09 | 1.01 |
| 0.57 | 0.45 | 0.50 | 0.06 | 1.01 |

$$\frac{Z_2^2}{Z_2^A} \int_{-\infty}^{\infty} \frac{d\omega'}{2\pi} \omega' \rho_{\pm}(|\vec{p}|, \omega') = |\vec{p}|$$



# sQGP




It shown that the variation domains for each interaction overlap considerably and hence it is sensible to list their averages as conservative measures of the critical temperature and zero mode survival domain: $T_s/T_c = 1.53 \pm 0.12$

## Thermal hadrons

# Hadrons at finite temperature[1]

The study of in-medium hadron properties has a long history with many questions of interest however still unsolved.

- the real-time properties of strongly interacting systems still defy a direct determination on nonperturbative approaches.
- it shed light on the physics, probed in current and upcoming collider facilities.

[1] **Fei Gao** and Minghui Ding. Eur.Phys.J.C 80 (2020)12, 1171.

# Pole and screening mass of hadron at finite temperautre

The practical way of computing BSEs for mesons at finite temperature is through the imaginary time formula which is simply to change the fourth component of all the momentum in Euclidean space to Matsubara frequency:

$$\begin{aligned}\lambda(\vec{P}^2,\Omega_m^2)\Gamma^{ab}_{\pi,\rho}(k;P) &= T\sum_n\int\frac{d^3q}{(2\pi)^3}g^2D_{\mu\nu}(k-q;T)\\ &\quad\times\gamma_\mu\chi^{ab}_{\pi,\rho}(q;P)\gamma_\nu\,, \qquad (13)\end{aligned}$$

The eigenvalue of the homogeneous BSE becomes 1 when the meson propagator is on shell, i.e.,

$$\vec{P}^2+P_0^2+M^2(\vec{P}^2,P_0^2)=0\,,$$

where $M(\vec{P}^2,P_0^2)$ is the meson mass.

# Pole and screening mass of hadron at finite temperautre

- screening mass $M_{\rm scr}$ via putting $\Omega_m^2 = 0$, extending $\vec{P}$ into complex plane and then locating the screening mass at $\lambda(-M_{\rm scr}^2, 0) = 1$.
- the pole mass is in principle difficult to define since an analytic continuation of the Matsubara frequency in the form of spectral representation is required:

$$\frac{1}{\Omega_m^2 + \vec{P}^2 + M^2(\vec{P}^2, \Omega_m^2)} = \int_{-\infty}^{\infty} d\omega \frac{\rho(\vec{P}, \omega)}{\omega - i\Omega_m} \,. \tag{14}$$

The pole mass is located at $\lambda(\vec{P} = 0, P^2 = -M_{\rm pole}^2) = 1$ through replacing $i\Omega_m$ with $M_{\rm pole}$ in the above spectral representation. Therefore, if people try to obtain the pole mass directly, the BSE in real time formula with the spectral representation is required.

# A novel method of computing pole mass

*A novel method of computing pole mass:*

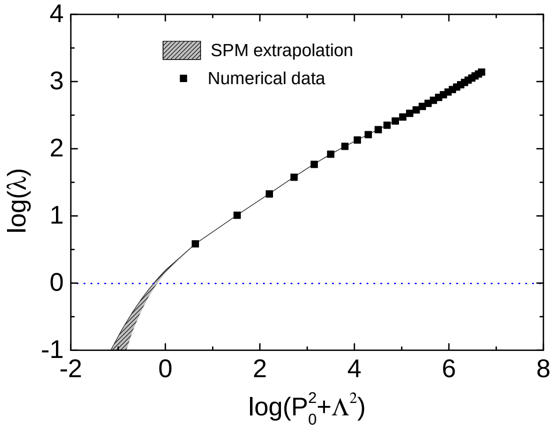
We compute the eigenvalues $\lambda(P^2 = \Omega_m^2)$ at each $\Omega_m$ with $m = 1, 2, ..., m_{\mathrm{Max}}$, and extrapolate them to obtain the pole mass of the meson $M_{\pi,\rho}$ at $\lambda(P^2 = -M_{\pi,\rho}^2) = 1$.

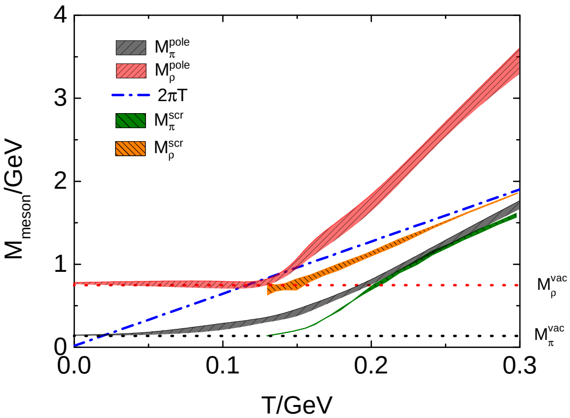
*The referee reports:*

The authors propose to compute the Bethe-Salpeter equation here in Euclidean time, a standard procedure and then extrapolate their result into the complex plane. This is very reminiscent of the imaginary chemical potential approach deployed in the lattice community.

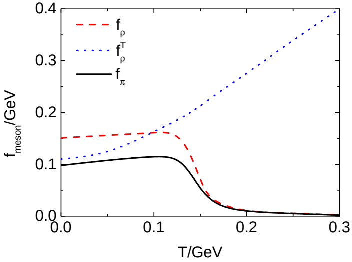
Based on the similar consideration to apply this extrapolation, ie, the extrapolation can work because we know the eigenvalue of Bethe-Salpeter equation increases smoothly and monotonously from 0 to 1, as $P_0^2$ goes from positive to the on-shell pole mass $P_0^2 = -M_{\text{pole}}^2$.

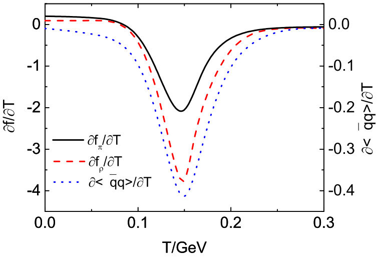
# An example of extrapolation for $\pi$ pole mass at $T = 150$ MeV






- At low temperature, $\pi$ pole mass increases monotonously , $\rho$ pole mass mass at $T_s$ has reached the vacuum value.
- At high temperature, $\pi$ pole mass gets close to the free field limit above the critical temperature, while the $\rho$ meson pole mass is twice as large as this limit.




- monotonous increase for $f_\rho^T$
- similar behavior for $f_\pi$ and $f_\rho$
- $f_\pi$ and $f_\rho$ can be considered as the criterion of phase transition.

## Summary and future:

- Phase diagram: obtained for the first time, QCD phase diagram without further infrared parameters in functional QCD method.
- The improvement at higher chemical potential needs the hadron resonance channel.
- Thermal states:
  1. Quark spectrum in sQGP phase
  2. Thermal hadron

Thank you !