

Jets and flavor content of protons

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Jets for 3D imaging

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Jets for 3D imaging (II)

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Jets for hadron inner structures



One can replace a final-state hadron by a jet

Jet physics

- Jet evolution
 - display QCD over a wide range of energy scales, from colliding energy to the hadronization energy.
 - contain important signatures of exotic physics, such as top quarks.
- Most of the theoretical discussion on these aspects has taken place in the context of Monte Carlo simulation studies. However, MC analysis is not enough, and it is difficult to extract the key characteristics.
- With this motivation, it is imperative to understand jet observables from the first principle QCD.

From parton to hadrons

Parton (quark or gluon) fragmentation and hadronization

High-energy partons lead to collimated bunches of hadrons



Jets are not the same as partons

Jets inherit quantum property of partons

Jet evolution from perturbative QCD: Jet Mass



• Accounts for all terms $\sim \alpha_s^n L^{2n}$

 $L = \log(E_J^2/m_J^2)$

• All order results generally exponentiate

$$\sigma \sim \sigma_0 \exp(\alpha_s L^2 + \alpha_s L + \alpha_s^2 L + \dots)$$

Soft-collinear enhancement

Real & virtual graphs cancel exactly in soft approximation if the real emissions are integrated over without restriction



Weighting the real emissions induces a miscancellation and a logarithm

 $\alpha_s \int_0^Q \frac{\mathrm{d}k_T}{k_T} - \alpha_s \int_0^\mu \frac{\mathrm{d}k_T}{k_T} = \alpha_s \ln \frac{Q}{\mu}$ Virtual loop
Real emission phase space if emissions
forbidden above μ

NLO result for total hadronic cross section



Real and virtual corrections suffer from soft and collinear infrared divergences, e.g.

$$\sigma_{\text{virtual}} = \sigma_0 \frac{2\alpha_s}{4\pi} \left(\frac{\mu^2}{Q^2}\right)^{\epsilon} \left(-\frac{4}{\epsilon^2} - \frac{6}{\epsilon} - 16 + \frac{7\pi^2}{3}\right)$$

in $d=4-2\varepsilon$. Divergences cancel in the sum!

NLO result for dijet cross section

$$\begin{aligned} \sigma(\delta,\beta) &= \left| \begin{array}{c} & & \\ & & \\ & & \\ & & \\ & & \\ & = \sigma_0 \left\{ 1 + \frac{\alpha_s(\mu)}{3\pi} \left[-16\ln\delta\ln\beta - 12\ln\delta + 10 - \frac{4\pi^2}{3} + \mathcal{O}(\delta,\beta) \right] \right\} \end{aligned}$$



Sterman & Weinberg '77

IR finite, but problems for small β and/or δ :

1) Large logarithms can compensate α_s suppression: fixedorder perturbation theory becomes unreliable.

Soft limit

When particles with small energy and momentum are emitted, the amplitudes greatly simplify:



Soft emission

- factors from the rest of the amplitude.
- only depends on the direction $p\mu/E$
- sees charge, but not spin of emitting particle

Wilson lines

Multiple emissions can be obtained from

$$S_i = \mathbf{P} \exp\left[ig \int_0^\infty ds \, n_i \cdot A_s^a(sn_i) \mathbf{T}_i^a\right]$$

 $n_i^{\mu}=p_i^{\mu}/E$ is a vector in the direction of the energetic particle, and T_i^a is its color charge. **P** indicates that the color matrices are path ordered.

Wilson line can be obtained by considering a point-like classical source moving along the line $x^{\mu} = sn^{\mu}$

$$\mathbf{X}^{\mu} = S n^{\mu}$$

From Wilson line to eikonal interaction

Consider one-gluon matrix element of Wilson line

$$\begin{split} \langle k, \lambda, b | \, \mathbf{S}_i \, | 0 \rangle &= i g_s \, \mathbf{T}^a \int_0^\infty ds \, \langle k, \lambda, b | n_i \cdot A^a(sn_i) | 0 \rangle + \mathcal{O}(g_s^2) \\ &= i g_s \, \mathbf{T}^a \int_0^\infty ds \, e^{i s n_i \cdot k} \langle k, \lambda, b | n_i \cdot A^a_\mu(0) | 0 \rangle \\ &= i g_s \, \mathbf{T}^b n_i \cdot \varepsilon(k, \lambda) \frac{e^{i s n_i \cdot k}}{i n_i \cdot k} \bigg|_0^\infty \qquad \text{need small imaginary} \\ &= -g_s \mathbf{T}^b \frac{n_i \cdot \varepsilon(k, \lambda)}{n_i \cdot k} = -g_s \mathbf{T}^b \frac{p_i \cdot \varepsilon(k, \lambda)}{p_i \cdot k} \end{split}$$

eikonal interaction

Collinear factorization



When partons become collinear, the amplitude factorizes into a lower-point amplitude times a splitting amplitude \mathbf{Sp} .

- Leading contribution to the squared amplitude does not involve interference with the other particles!
- Can be violated by Glauber phases for process with collinear in- and outgoing particles. Catani, de Florian, Rodrigo '11; Forshaw, Seymour, Siodmok '12; Schwartz, Yan, Zhu '17 ...

Two general approaches to evolution

- Top down: approximations of all-order factorization theorems. e.g. CSS, SCET, . . .
 - All-order structure manifest
 - Observable specific (but same structure for many)
- Bottom up: corrections to coherent branching. e.g. parton shower, . . .
 - Simplifications at a given accuracy (e.g. LL structure much simpler than full factorization theorem.)
 - Lends itself to automation and Monte Carlo implementation

Soft-Collinear Effective Theory

(Bauer, Pirjol, Stewart et.al. 2001,2002; Beneke, Diehl et.al. 2002; ...)

In collider processes, we have an interplay of three momentum regions

Hard high energy part Collinear low energy part

Correspondingly, EFT for such processes has two low energy modes:

- collinear fields describing the energetic partons propagating in each direction of large energy;
- soft fields which mediate long range interactions among them.

Region Expansion: a toy model

• Consider a integration:

$$I(a) = \int_0^\infty \frac{\cos t}{t+a} dt = -\cos a C_i(a) + \sin a \left[\frac{\pi}{2} - S_i(a)\right], \text{ with } C_i(a) = -\int_a^\infty \frac{\cos t}{t} dt$$

- If $a \ll 1$, we have $I(a) = -\gamma_E \ln a$
- Asymptotical expansion: not analytic in the expansion parameter because of presence of the logarithm.
- Our goal: obtain expanded results before carrying out the integral.
- Naive expansion before integration: $\frac{cc}{t}$

$$\frac{\cos t}{t+a} \xrightarrow{a \ll 1} \frac{\cos t}{t} \quad fail!!$$

Region Expansion: a toy model

Cut-off regularisation:

$$\int_0^\infty \to \int_0^\Lambda + \int_\Lambda^\infty$$

$$I_1(a) = \int_0^{\Lambda} dt \frac{\cos t}{t+a} = \int_0^{\Lambda} dt \left(\frac{1}{a+t} + \cdots\right) \approx -\ln a + \ln \Lambda$$

$$I_2(a) = \int_{\Lambda}^{\infty} dt \frac{\cos t}{t+a} = \int_{\Lambda}^{\infty} dt \left(\frac{\cos t}{t} + \cdots\right) \approx -\gamma_E - \ln \Lambda$$

• Dimensional regularisation:

$$I_1(a) = \int_0^\infty dt \, t^{-\epsilon} \frac{\cos t}{t+a} = \int_0^\infty dt \, t^{-\epsilon} \left(\frac{1}{a+t} + \cdots\right) \approx \frac{1}{\epsilon} - \ln a$$

$$I_2(a) = \int_0^\infty dt \, t^{-\epsilon} \frac{\cos t}{t+a} = \int_0^\infty dt \, t^{-\epsilon} \left(\frac{\cos t}{t} + \cdots\right) \approx -\frac{1}{\epsilon} - \gamma_E$$

Soft-Collinear Factorization



For $M_1 \sim M_2 \ll Q$ the cross section $\sigma_{\text{phys}}(Q, M_1, M_2)$ factorizes:



All-order evolution equation

$$\mu \frac{d}{d\mu} \log \sigma_{\rm phys}(Q, M_1, M_2) = 0$$

Evaluate each part at its characteristic scale, evolve to common reference scale μ



Jets at the LHC



• Jets are produced copiously at the LHC



• At the LHC, 60 - 70 % of ATLAS & CMS papers use jets in their analysis!

Jets at the EIC

- low р_{т, ј}
- smaller jet multiplicity
- less contamination from underlying events and pileups



Different environment compared with the LHC new opportunities and new challenges!!!

Gluon TMDs and heavy flavor dijets at the EIC

Gluon Sivers function (GSF)

• Gauge link dependent gluon TMDs

$$\Gamma^{[U,U']}_{\mu\nu}(x,p_T;n) = \int \frac{d\xi \cdot P d^2 \xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P, S | F^{n\mu}(0) \ U_{[0,\xi]} F^{n\nu}(\xi) U'_{[\xi,0]} | P, S \rangle \Big|_{\rm LF}$$

- GSF: T-odd object; two gauge links; process dependence more involved
- For any process GSF can be expressed in terms of two functions:

$$f_{1T}^{\perp g[U]}\left(x,\mathbf{k}_{\perp}^{2}\right) = \sum_{c=1}^{2} C_{G,c}^{[U]} f_{1T}^{\perp g(Ac)}\left(x,\mathbf{k}_{\perp}^{2}\right)$$
(Buffing, Mukherjee, Mulders'13)
• $f_{1T}^{\perp g(f)}$ f-type, C-even
• $f_{1T}^{\perp g(d)}$ d-type, C-odd
 $\gamma^{*}g \rightarrow q\bar{q}$

$$\frac{\zeta_{T}}{\zeta_{-}}$$

$$gg \rightarrow \gamma\gamma$$

$$f_{1T}^{\perp g[e\,p^{\uparrow} \rightarrow e'\,Q\bar{Q}\,X]}\left(x,p_{T}^{2}\right) = -f_{1T}^{\perp g[p^{\uparrow}p \rightarrow \gamma\gamma\,X]}\left(x,p_{T}^{2}\right)$$

GSF and spin asymmetry in di-jet at the EIC

At the EIC , accessing of GSF via high-p_T dihadron, open di-charm, di-D-meson and dijet has been investigated using PYTHIA and reweighing methods in Zheng, Aschenauer, Lee, Xiao, Yin '18

They find that dijet process is the most promising channel

At the LO di-jet production in DIS involves two processes: $\gamma^*q o qg \qquad \gamma^*g o qar q$



- to distinguish different TMDs
 - jet charge tagging "different quark TMDs" Kang, Liu, Mantry, DYS '20 PRL)
 - Heavy-flavor tagging, where q-channel starts to contribute beyond the LO (Kang, Reiten, DYS, Terry '20 JHEP)

TMD factorization for heavy-flavor dijet production in DIS

(Kang, Reiten, DYS, Terry '20 JHEP)

 $e(\ell) + N(P, \mathbf{S}_T) \to e(\ell') + J_{\mathcal{Q}}(p_J) + J_{\bar{\mathcal{Q}}}(p_{\bar{J}}) + X$



In the Breit frame, the dijet imbalance is defined as $q_T = p_{JT} + p_{\overline{J}T}$

 $q_T R \ll q_T \lesssim m_Q \lesssim p_T R \ll p_T$

R: Jet radius M_Q: heavy quark mass

the factorized form of the spin-independent cross section

$$d\sigma^{UU} \sim H(Q, p_T) J_Q(p_T R, m_Q) J_{\bar{Q}}(p_T R, m_Q) S(\lambda_T) f_g(k_T) S_Q^c(l_{QT}) S_{\bar{Q}}^c(l_{\bar{Q}T}) \delta^{(2)}(k_T + \lambda_T + l_{QT} + l_{\bar{Q}T} - q_T)$$

- Hard and soft functions are the same as light-jet cases, since $p_T >> m_Q$
- Jet and collinear-soft functions are new, which receive finite quark mass correction

Heavy quark mass corrections in the evolution equation

Anomalous dimension for the HF quark jet function:

$$\Gamma^{j_Q}(\alpha_s) = -C_F \gamma^{\text{cusp}}(\alpha_s) \ln \frac{m_Q^2 + p_T^2 R^2}{\mu^2} + \gamma^{j_Q}(\alpha_s) \qquad \qquad \gamma_0^{j_Q} = 2C_F \left(3 - \frac{2m_Q^2}{m_Q^2 + p_T^2 R^2}\right)$$

Anomalous dimension for the HF collinear-soft function

$$\Gamma^{cs_Q}(\alpha_s) = C_F \gamma^{\text{cusp}}(\alpha_s) \ln \frac{R^2 \mu_b^2}{\mu^2} + \gamma^{cs_Q}(\alpha_s) \qquad \gamma_0^{cs_Q} = -4C_F \left[2\ln\left[-2i\cos(\phi_b - \phi_J)\right] - \frac{m_Q^2}{m_Q^2 + p_T^2 R^2} - \ln\frac{m_Q^2 + p_T^2 R^2}{p_T^2 R^2} \right]$$

Heavy-quark mass dependence cancels out in

$$\Gamma^{j_Q} + \Gamma^{cs_Q} = \Gamma^{j_q} + \Gamma^{cs_q}$$

 $- \mu_j \sim p_T R$ $- \mu_{cs} \sim q_T R$

Heavy quark mass will contribute the RG evolution between jet and collinear-sot function

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RG evolution and resummation

• Resummation formula:

$$\begin{aligned} \frac{d\sigma^{UU}}{dQ^2 dy d^2 \boldsymbol{q}_T dy_J d^2 \boldsymbol{p}_T} = & H(Q, p_T, y_J, \mu_h) \int_0^\infty \frac{b db}{2\pi} J_0(b \, q_T) f_{g/N}(x_g, \mu_{b*}) \\ & \times \exp\left[-\int_{\mu_{b*}}^{\mu_h} \frac{d\mu}{\mu} \Gamma^h\left(\alpha_s\right) - 2 \int_{\mu_{b*}}^{\mu_j} \frac{d\mu}{\mu} \Gamma^{j_Q}\left(\alpha_s\right) - 2 \int_{\mu_{b*}}^{\mu_{cs}} \frac{d\mu}{\mu} \Gamma^{cs_Q}\left(\alpha_s\right)\right] \\ & \times \exp\left[-S_{\rm NP}(b, Q_0, n \cdot p_g)\right] \end{aligned}$$

- **b*-prescription to avoid Landau pole** $b_* = b/\sqrt{1 + b^2/b_{\text{max}}^2}$ $\mu_{b_*} = 2e^{-\gamma_E}/b_*$
- Non-perturbative model: $S_{\text{NP}}(b, Q_0, n \cdot p_g) = g_1^f b^2 + \frac{g_2}{2} \frac{C_A}{C_F} \ln \frac{n \cdot p_g}{Q_0} \ln \frac{b}{b_*}$

Sun, Isaacson, Yuan, Yuan '14

• Typical scales: $\mu_h \sim p_T$, $\mu_j \sim Rp_T$, $\mu_{cs} \sim R\mu_{b*}$

Spin dependent cross section

• Resummation formula:

$$\begin{aligned} \frac{d\sigma^{UT}(\boldsymbol{S}_{T})}{dQ^{2}dyd^{2}\boldsymbol{q}_{T}dy_{J}d^{2}\boldsymbol{p}_{T}} = \sin(\phi_{q} - \phi_{s}) H(Q, p_{T}, y_{J}, \mu_{h}) \int_{0}^{\infty} \frac{b^{2}db}{4\pi} J_{1}(b \, q_{T}) f_{1T,g/p}^{\perp}(x_{g}, \mu_{b*}) \\ \times \exp\left[-\int_{\mu_{b*}}^{\mu_{h}} \frac{d\mu}{\mu} \Gamma^{h}(\alpha_{s}) - 2 \int_{\mu_{b*}}^{\mu_{j}} \frac{d\mu}{\mu} \Gamma^{j}(\alpha_{s}) - 2 \int_{\mu_{b*}}^{\mu_{cs}} \frac{d\mu}{\mu} \Gamma^{cs}(\alpha_{s})\right] \\ \times \exp\left[-S_{\mathrm{NP}}^{\perp}(b, Q_{0}, n \cdot p_{g})\right] \end{aligned}$$

 Polarized hard function: For the polarized process, we must consider the attachment of an additional gluon from gauge link in GSF
 Oiu Vogelsange Yuan '07:

Qiu, Vogelsange, Yuan '07; Kang, Lee, DYS, Terry, '20 ...



polarized and unpolarized hard functions are the same $C_1 + C_2 = C_u$

f-type gluon Sivers function

Numerical results

Anti-k_T, R=0.6

C-jets: $5 \text{ GeV} < p_T < 10 \text{ GeV}, |\eta_J| < 4.5,$ b-jets: $10 \text{ GeV} < p_T < 15 \text{ GeV}, |\eta_J| < 4.5,$

$$d\sigma(\mathbf{S}_T) = d\sigma^{UU} + \sin(\phi_q - \phi_s) d\sigma^{UT}$$

$$A_{UT}^{\sin(\phi_q - \phi_s)} = \frac{d\sigma^{UT}}{d\sigma^{UU}} \qquad \begin{array}{l} \mathsf{GSF: SIDIS1 set} \\ \mathsf{D'Alesio, Murgia, Pisano '15} \end{array}$$



Heavy quark mass can give sizable corrections to the predicted asymmetry

Recoil-free azimuthal correlation and track jets

Electron-jet azimuthal correlation at the EIC

(Liu, Ringer, Vogelsang, Yuan '19)



$$\frac{d\sigma}{dy_{\ell}d^2k_{\ell\perp}d^2q_{\perp}} = \sigma_0 \int \frac{d^2b_{\perp}}{(2\pi)^2} e^{iq_{\perp}\cdot b_{\perp}} \tilde{W}_q(x,b_{\perp})$$

$$ilde{W}_q = x f_q(x, b_\perp, \zeta_c, \mu_F) S_J(b_\perp, \mu_F) H_{ ext{TMD}}(Q, \mu_F)$$

Nuclear P_T - broadening effects:

(Mueller, Wu, Xiao, Yuan '16,'17)

$$\tilde{W}_q \Rightarrow \tilde{W}_q e^{-\frac{\hat{q}Lb_{\perp}^2}{4}}$$

 $\hat{q}L$ represents the typical transverse momentum obtained by the quark through the cold nuclear matter



Jet radius and q_T joint resummation for boson-jet correlation

(Chien, DYS & Wu '19 JHEP)



Construction of the theory formalism

- Multiple scales in the problem
- Rely on effective field theory: SCET + Jet Effective Theory (Becher, Neubert, Rothen, DYS '16 PRL)

$$\frac{d\sigma}{d^2 q_T d^2 p_T d\eta_J dy_V} = \sum_{ijk} \int \frac{d^2 x_T}{(2\pi)^2} e^{i\vec{q}_T \cdot \vec{x}_T} \mathcal{S}_{ij \to Vk}(\vec{x}_T, \epsilon) \mathcal{B}_{i/N_1}(\xi_1, x_T, \epsilon) \mathcal{B}_{j/N_2}(\xi_2, x_T, \epsilon)$$
$$\times \mathcal{H}_{ij \to Vk}(\hat{s}, \hat{t}, m_V, \epsilon) \sum_{m=1}^{\infty} \langle \mathcal{J}_m^k(\{\underline{n}_J\}, R \, p_J, \epsilon) \otimes \mathcal{U}_m^k(\{\underline{n}_J\}, R \, \vec{x}_T, \epsilon) \rangle$$

(also see Sun, Yuan, Yuan '14; Buffing, Kang, Lee, Liu '18,...)

Numerical results

(Chien, DYS & Wu '19)



- All-order resummation result is consistent with CMS data
- Next-to-leading logarithms result has 20-30% scale uncertainties.
- Direct formula for $\Delta \phi$? $\vec{q}_T = q_T(\sin \phi_q, \cos \phi_q)$
- Better angular resolution?
- Reduce contamination?
- Higher accuracy?

Jet definition

Which particles get put together?

How to combine their momenta?

Jet definition

Which particles get put together?

Jet algorithm

How to combine their momenta?

Recombination scheme

Jet definition with clustering algorithms

- Determine distances between "particles"
- Recombine nearest "particles": $p_i^{\mu}, p_j^{\mu} \rightarrow p_i^{\mu} + p_j^{\mu}$
- Repeat until distances larger than jet radius R



Recoil and the jet axis



Jet axis is along jet momentum: recoiled by soft radiation in jet

- TH challenge: Non-linear evolution (Non-global logs)
- EX challenge: Contamination

Recoil absent for the p_T-weighted recombination (Ellis, Soper '93)

$$p_{t,r} = p_{t,i} + p_{t,j},$$

$$\phi_r = (w_i \phi_i + w_j \phi_j) / (w_i + w_j) \qquad w_i = p_t^n$$

$$y_r = (w_i y_i + w_j y_j) / (w_i + w_j)$$

 $n \rightarrow \infty$ (Winner-take-all scheme) (Bertolini, Chan, Thaler '13)

Recoil-free azimuthal angle for boson-jet correlation

(Chien, Rahn, Schrignder, DYS, Waalewijn & Wu '21 PLB)



 $\pi - \Delta \phi \equiv \delta \phi \approx \sin(\delta \phi) = |p_{x,V}|/p_{T,V}$

Following the standard steps in SCET₂ we obtain the following factorization formula

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_{x,V}\,\mathrm{d}p_{T,J}\,\mathrm{d}y_V\,\mathrm{d}\eta_J} = \int \frac{\mathrm{d}b_x}{2\pi} \,e^{\mathrm{i}p_{x,V}b_x} \sum_{i,j,k} B_i(x_a, b_x) B_j(x_b, b_x) S_{ijk}(b_x, \eta_J) H_{ij\to Vk}(p_{T,V}, y_V - \eta_J) J_k(b_x)$$

TMD jet function



At one-loop: WTA axis along most energetic particle.

$$J_i^{\text{WTA}}(\vec{q}_T, z, QR) \to \delta(1-z)\mathcal{J}_i(\vec{q}_T)$$
$$J_i^{\text{SJA}}(\vec{q}_T, z, QR) \to \delta(1-z)\delta^2(\vec{q}_T)J_i(QR)$$

Track-based jet definition

- The angular resolution of jet measurements is about 0.1 radians, limiting access to the back-to-back region
- This can be overcome by measuring the jet using only charged particles, exploiting the superior angular resolution of the tracking systems at the LHC.



Numerical results



- first N²LL resummation including full jet dynamics
- Pythia agrees well
- Our work serves as a baseline for pinning down the inner workings of hot and cold dense nuclear matter using hard probes

Conclusion

- Jets and jet substructures are useful tools to understand nucleon inner structures and dense nuclear matter property
- Theory tools: EFT of QCD in collinear and soft limit (e.g. SCET)
- The heavy flavor flavor tagging provides a novel probe of gluon contribution in the nucleon spin program at the EIC
 - Heavy quark mass can give sizable corrections to the predicted asymmetry
- Recoil-free azimuthal correlation serves as a baseline for pinning down the inner workings of hot and cold dense nuclear matter using hard probes
 - Track jets provide superior angular resolution

Thank you