HENPIC Seminar June 9, 2022

Spinodal Enhancement of Light Nuclei Yield Ratio in Relativistic Heavy-Ion Collisions

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Cyclotron Institute

June 8, 2022

Reference: K. J. Sun, W. H. Zhou, L. W. Chen, C. M. Ko, and F. Li, R. Wang, and J. Xu, arXiv:2205.11010(2022)

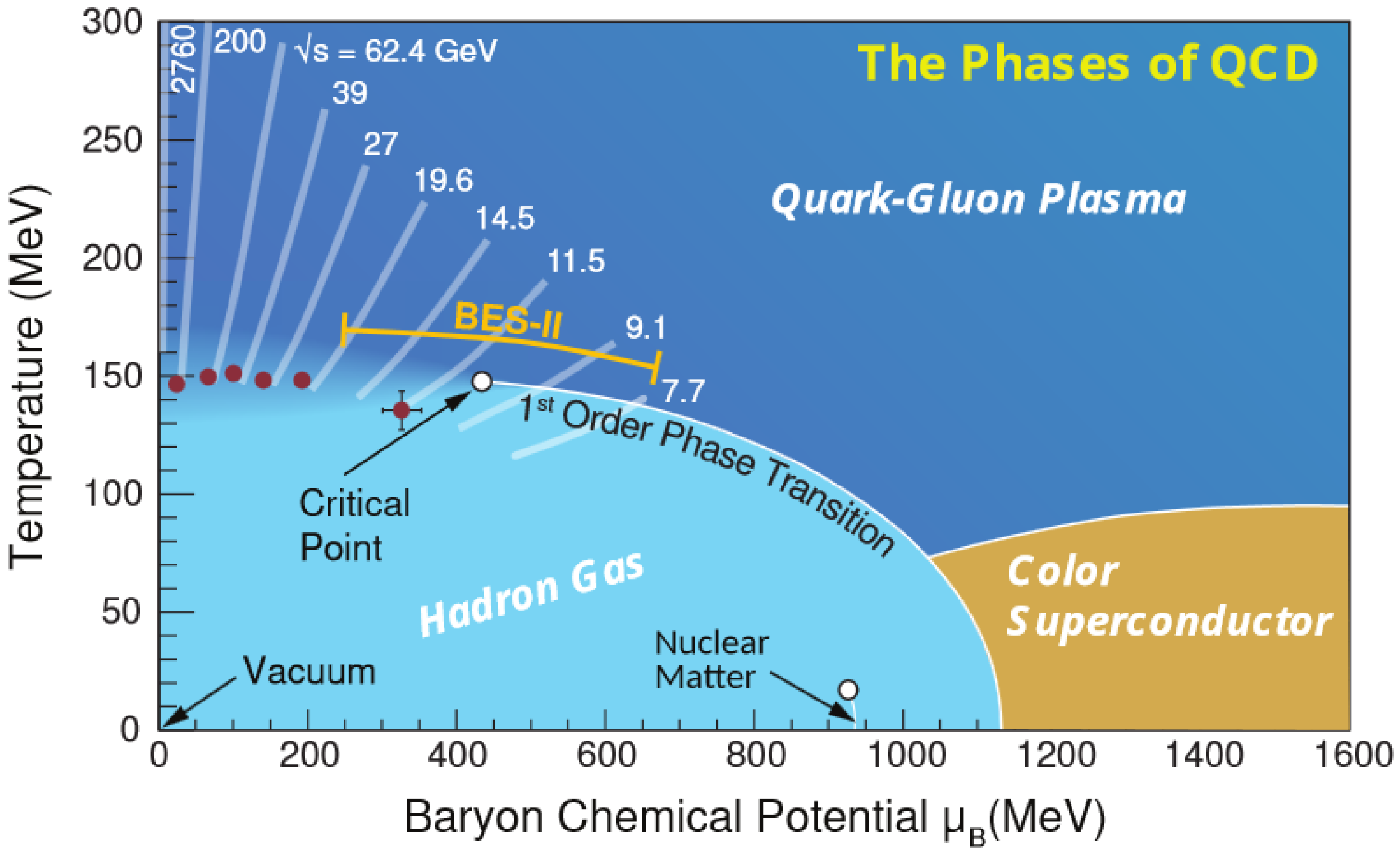
TEXAS A&M UNIVERSITY

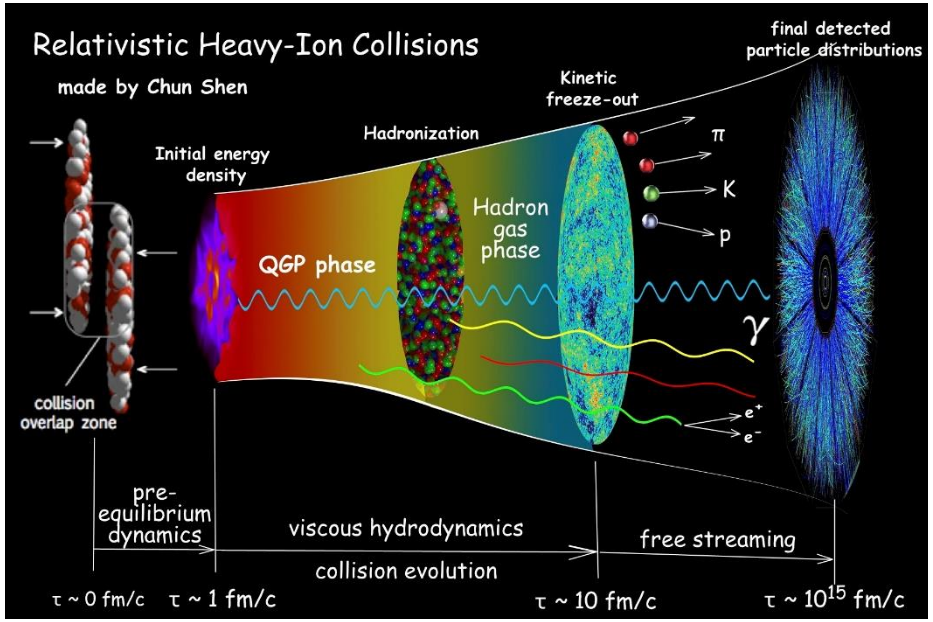
Outline

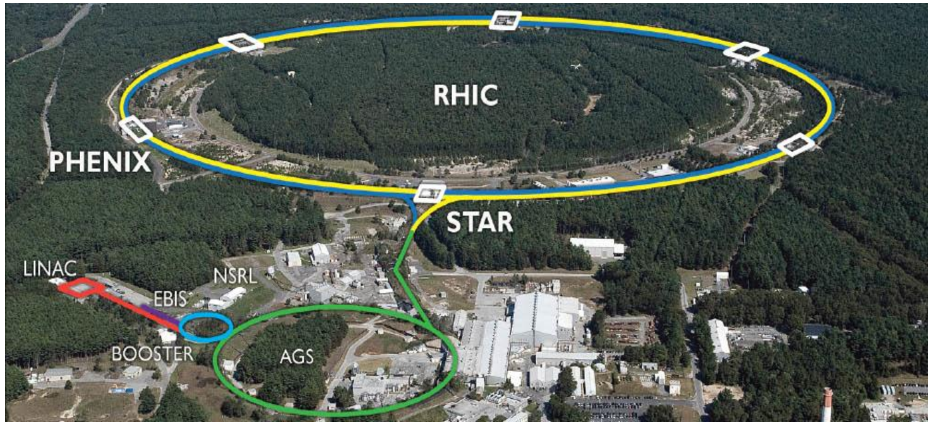
1. Motivation: Why light nuclei? Why $N_t N_p / N_d^2$ (tp/d^2)?
2. A novel transport model approach to the first-order QCD phase transition
3. Spinodal enhancement of tp/d^2 in central and peripheral collisions
4. Summary

Reference: K. J. Sun, W. H. Zhou, L. W. Chen, C. M. Ko, and F. Li, R. Wang, and J. Xu, arXiv:2205.11010(2022)

1.1 QCD phase diagram (1)







Critical Point: long-range correlation

First-order Phase Transition: Spinodal instability

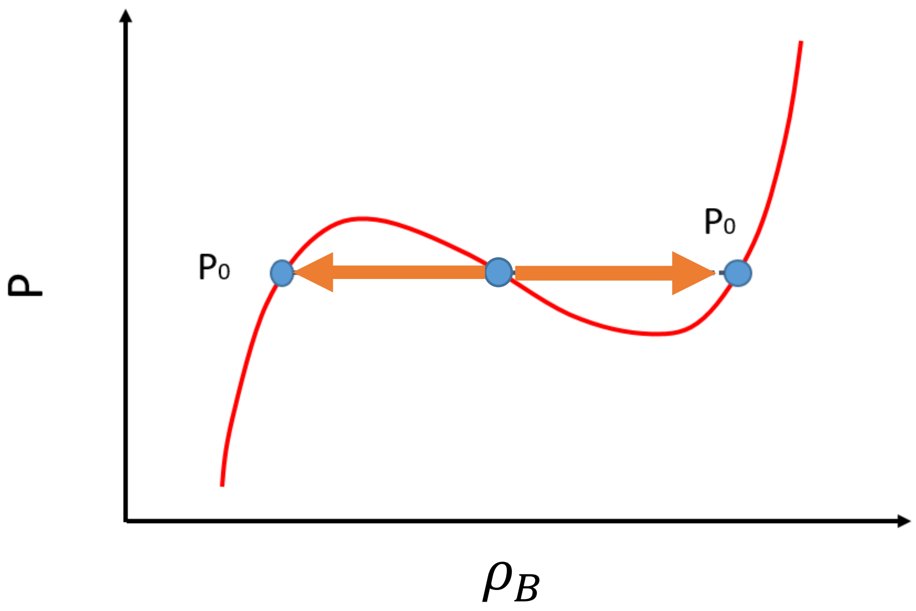
X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017)

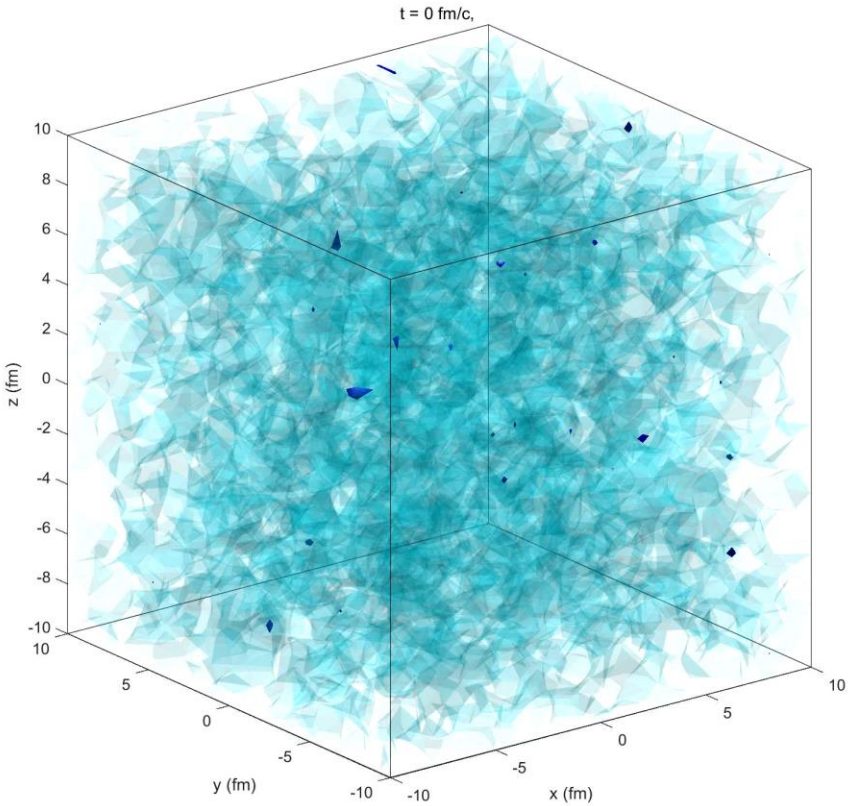
A. Bzdak et al., Phys. Rept. 853, 1 (2020)

W. J. Fu, J. M. Pawlowski, F. Renneke, Phys. Rev. D101, 054032 (2020)

1.2 Density fluctuation from the spinodal instability (2)

Phase separation, spinodal decomposition(SD)





Small irregularities will grow exponentially and soon the evolution becomes ’chaotic’.

In low-energy nuclear reactions, SD could lead to nuclear multifragmentation
(P. Chomaz, M. Clonna, and J. Randrup, Phys. Rep. 389, 263 (2004)).

What is the consequence for heavy ion collisions?

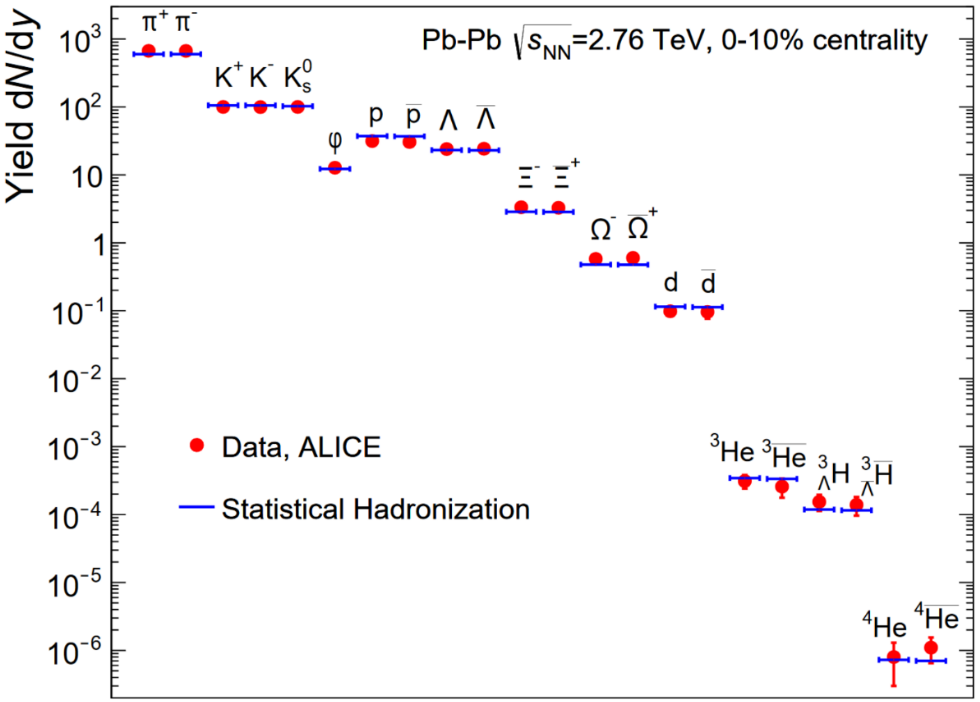
Hydro: J. Steinheimer and J. Randrup, PRL. 109, 212301 (2012); PRC79, 054911 (2009); K. Paech, A. Dumitru, PLB623, 200 (2005)
Chiral Fluid Dynamics: C. Herold, M. Nahrgang, I. Mishustin, and M. Bleicher, NPA 925, 14 (2014)
Transport: F. Li and C. M. Ko, PRC95, 055203 (2017); K. J. Sun et al., arXiv:2006.08929(2020)

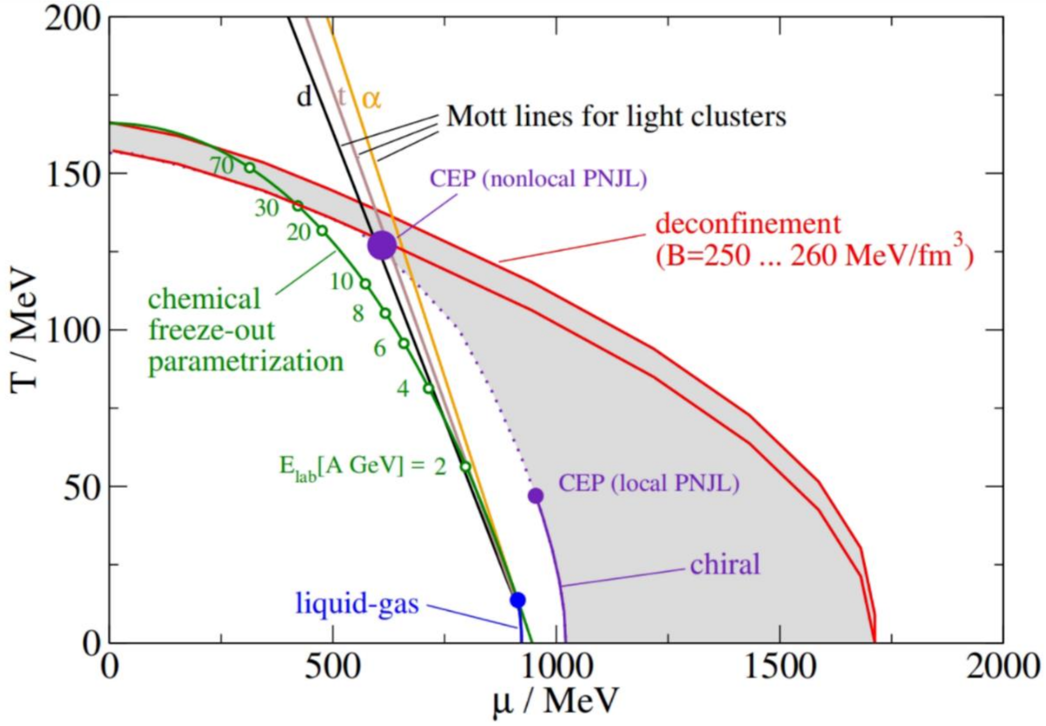
1.3 Existence of loosely bound nuclei (3)

An open question

$T \sim 150\text{MeV} \gg E_B \sim \text{a few MeV}$

arXiv:1711.05631





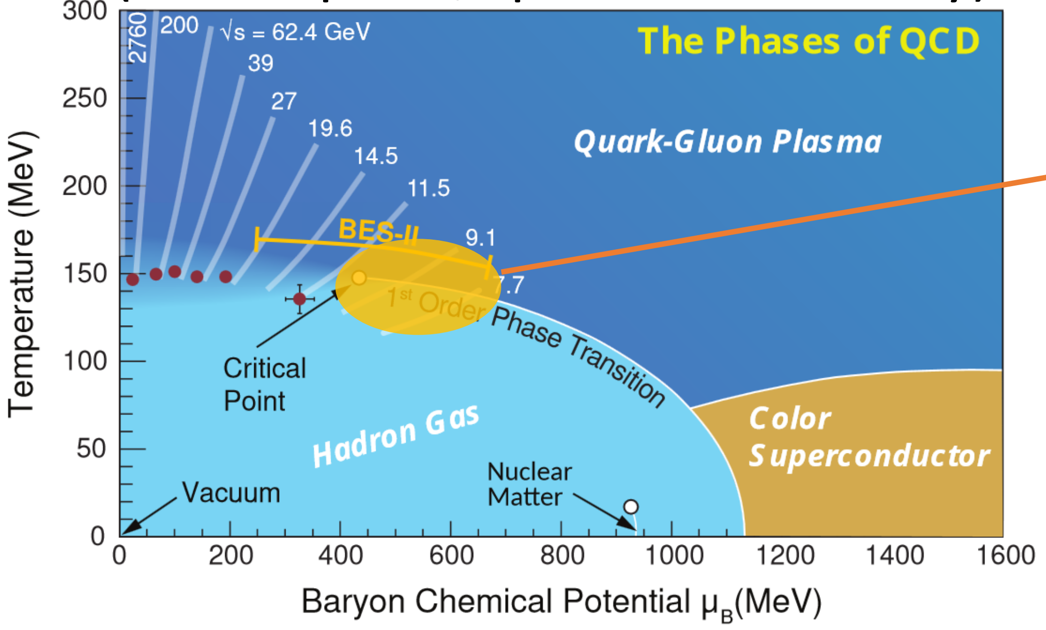
Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561, 321 (2018)
A. Andronic, P. Braun-Munzinger, J. Stachel, H. Stöcker, PLB 697, 203 (2011)
V. Vovchenko et al., PLB800, 135131 (2020)
H. Sato and K. Yazaki, PLB98, 153 (1981); E. Remler, Ann. Phys. 136, 293 (1981); M. Gyulassy, K. Frankel, and E. Remler, NPA402,596 (1983);
S. Mrowczynski, J. Phys. G 13, 1089 (1987); S. Leupold and U. Heinz, PRC50, 1110 (1994); R. Scheibl and U. W. Heinz, PRC59. 1585(1999);
X. Y. Zhao, Y. T. Feng, F. L. Shao, R. Q. Wang, J. Song, Phys. Rev. C 105. 054908 (2022); W. Zhao, K. J. Sun, C. M. Ko, and X. F. Luo, PLB 820, 136571 (2021)
A.Z. Mekjian, PRC17,1051 (1978); P. Danielewicz, G.F. Bertsch, NPA533, 712 (1991); P. Danielewicz and P. Schuck, PLB274, 268 (1992);
Y. Oh and C. M. Ko PRC76, 054910(2007);PRC80, 064902(2009); D. Oliinychenko, L. G. Pang, H. Elfner, and V. Koch, PRC99, 044907 (2019);

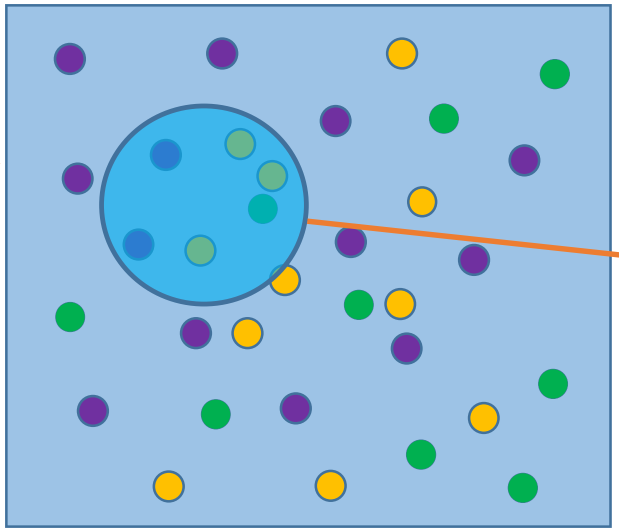
1.4 Why light nuclei? (4)

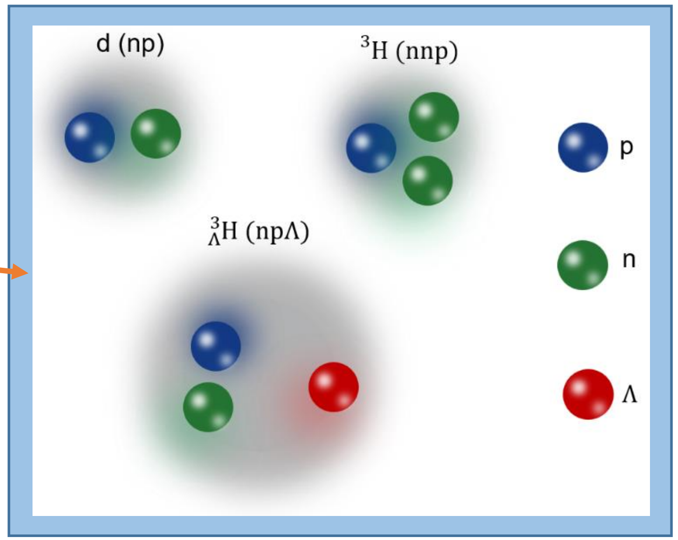
Equation of state
(critical point, spinodal instability)

Density fluctuation/correlation

Light nuclei production







Due to their composite structure:

Deuteron probes 2-body correlation function

Triton probes 3-body correlation function

1.4 Why light nuclei?

light nuclei production & QCD phase transition

	2012	2014	2015	2016	2017	2018	2019	2020	2021	2022

Other works

First-order phase transition & composite particle production

- J. Steinheimer et al. PRC 87, 054903 (2013)
- PRL 109, 212301 (2012)(Hydrodynamics)
- JHEP 12, 122(2019)(Machine learning)

Baryon clustering near the critical point

- E. Shuryak, J.M.Torres-Rincon et al., PRC 100, 024903(2019)
- PRC 101,034914(2020)
- EPJA 56, 241(2020)
- PRC 104,024908(2021)

Background effects

- S. Wu et al.,arXiv:2205.14302

Our works

Probing QCD phase transition with light nuclei production

- PLB 774, 103 (2017)

K. J. Sun, L. W. Chen, C. M. Ko, and Z. Xu

$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}[1 + \Delta\rho_n]$$

- PLB 781, 499 (2018)
- PLB 816, 136258 (2021)(criticality)

1st-order QCD phase transition

- PRD 103, 014006 (2021)
- EPJA 57, 313 (2021)(Transport)
- arxiv:2205.11010 (Transport) (First-order PT in BES)

1.4 Why $N_t N_p / N_d^2 (tp/d^2)$? (6)

Density matrix formulation

Phys. Lett. B 774, 103 (2017)
Phys. Lett. B 781, 499 (2018)
Phys. Lett. B 816, 136258 (2021)

$$N_d \propto Tr[\hat{\rho}_s \hat{\rho}_d]$$

Encodes many-body density fluctuation/correlation

Phase-space representation:

$$N_d = \frac{3}{4} \int d\Gamma\, f_{pn}(\vec{p}_1, \vec{r}_1, \vec{p}_2, \vec{r}_2) \times W_d(\vec{r}, \vec{p})$$

$$W_d(\vec{r}, \vec{p}) = \frac{1}{\pi\hbar} \int d\vec{r}' \psi_d^*(\vec{r} + \vec{r}') \psi_d(\vec{r} - \vec{r}') e^{2i\vec{p}\cdot\vec{r}'}$$

Wigner function(Gaussian):

$$W_d(r, k) = 8 \exp\left(-\frac{r^2}{\sigma_d^2} - \sigma_d^2 p^2\right) \qquad \sigma_d \approx 2.26 \text{ fm}$$

with density fluctuation and correlation:

$$f_{np}(x_1, p_1; x_2, p_2) = \rho_{np}(x_1, x_2)(2\pi m T)^{-3} e^{-\frac{p_1^2 + p_2^2}{2mT}}$$

$$\rho_{np}(x_1, x_2) = \rho_n(x_1)\rho_p(x_2) + C_2(x_1, x_2)$$

$$\rho_n(x) = \langle \rho_n \rangle + \delta\rho_n(x) \qquad \rho_p(x) = \langle \rho_p \rangle + \delta\rho_p(x)$$

$\delta\rho(x)$ denotes density fluctuation over space or inhomogeneity,

$$C_{np} = \langle \delta\rho_n(x)\delta\rho_p(x) \rangle / (\langle \rho_n \rangle \langle \rho_p \rangle)$$
$$\Delta\rho_n = \langle \delta\rho_n(x)^2 \rangle / \langle \rho_n \rangle^2$$

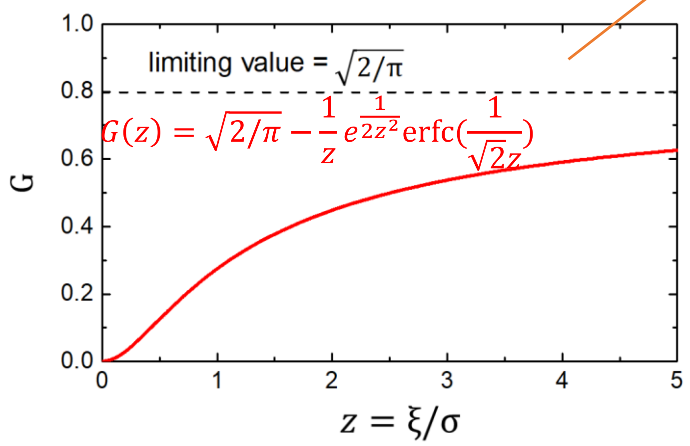
$$\langle \cdots \rangle \equiv \frac{1}{V} \int dx$$

$$C_2(x_1 - x_2) \approx \lambda \langle \rho_n \rangle \langle \rho_p \rangle \frac{e^{-|x_1 - x_2|/\xi}}{|x_1 - x_2|^{1+\eta}} \quad (\text{singular part only})$$

with ξ being the density − density correlation length

$$0 < \langle \delta N^2 \rangle \sim \int dx C_2(x) \sim \lambda \xi^2 \rightarrow \lambda > 0$$

$$\Rightarrow N_d \approx \frac{3}{\sqrt{2}} \left(\frac{2\pi}{mT}\right)^{\frac{3}{2}} N_p \langle \rho_n \rangle \left[1 + C_{np} + \frac{\lambda}{\sigma_d} G\left(\frac{\xi}{\sigma_d}\right)\right]$$



1.4 Why $N_t N_p / N_d^2 (tp/d^2)$?

$$N_d = \frac{3}{\sqrt{2}} \left(\frac{2\pi}{mT}\right)^{\frac{3}{2}} N_p \langle \rho_n \rangle \left[1 + C_{np} + \frac{\lambda}{\sigma_d} G\left(\frac{\xi}{\sigma_d}\right)\right]$$

$$N_t = \frac{3^{3/2}}{4} \left(\frac{2\pi}{mT}\right)^3 N_p \langle \rho_n \rangle^2 \left[1 + 2C_{np} + \Delta\rho_n + \frac{3\lambda}{\sigma_t} G\left(\frac{\xi}{\sigma_t}\right) + O(G^2)\right]$$

Pre-factors are thermal yields w/o density fluc./corr.

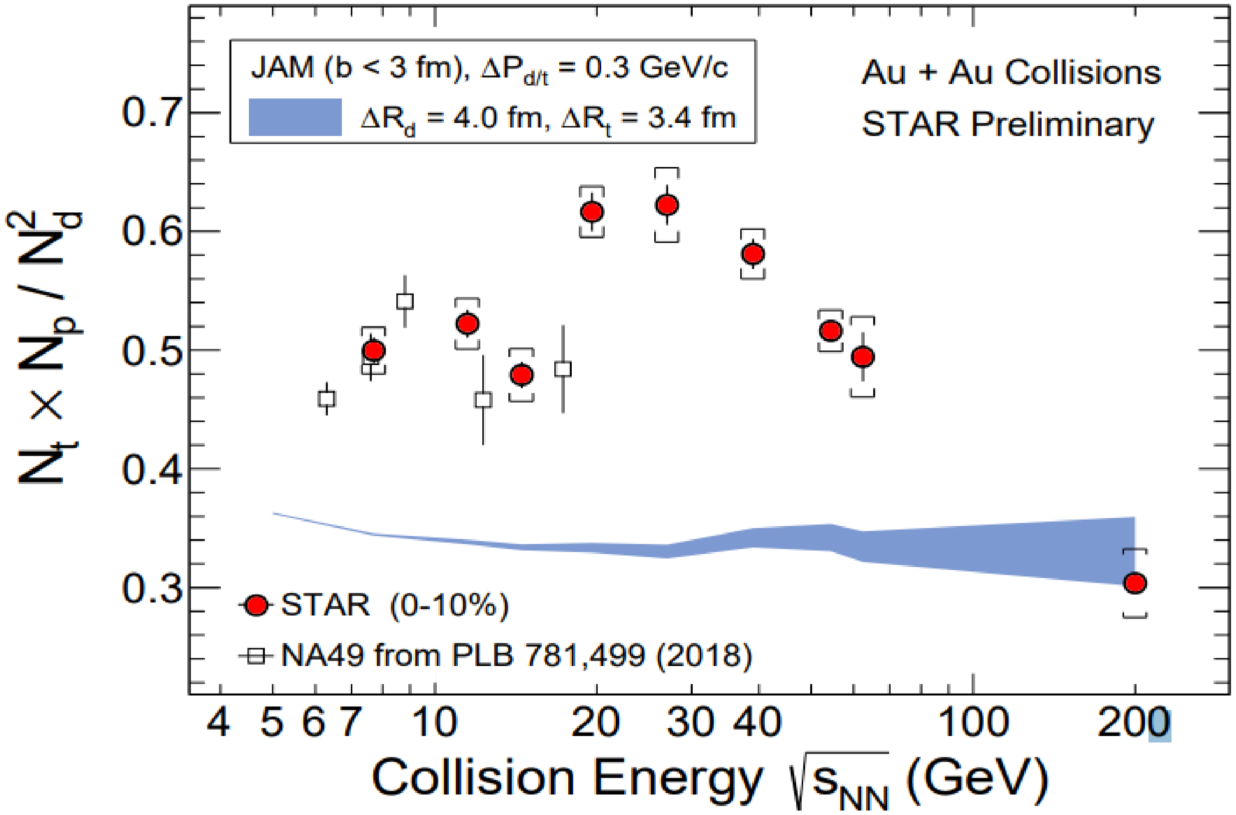
Ratio: $\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}} \left[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right], \quad \frac{3\ pairs}{2\ pairs} \sim 1\ pair, \quad \sigma \approx 2$ fm

Density fluctuation/correlation leads to enhancement of tp/d^2

We focus on the effects of density fluctuation generated from the first-order phase transition

K. J. Sun, C. M. Ko, and F. Li, Phys. Lett. B 816, 136258 (2021)

1.5 Energy dependence of tp/d^2 w/o 1st-order PT (8)



Model calculations without the inclusion of a first-order or second-order phase transition, e.g., JAM+COAL, AMPT+COAL, MUSIC+COAL, UrQMD+COAL, and SHM, all give monotonic or flat energy dependence

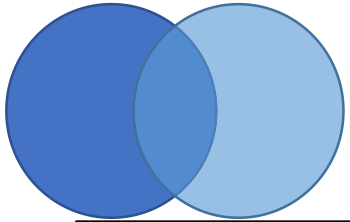
Data is preliminary and weak-decay correction is important

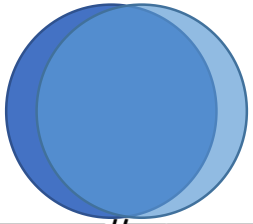
Dingwei Zhang(CCNU)for STAR Collaboration, QM2019

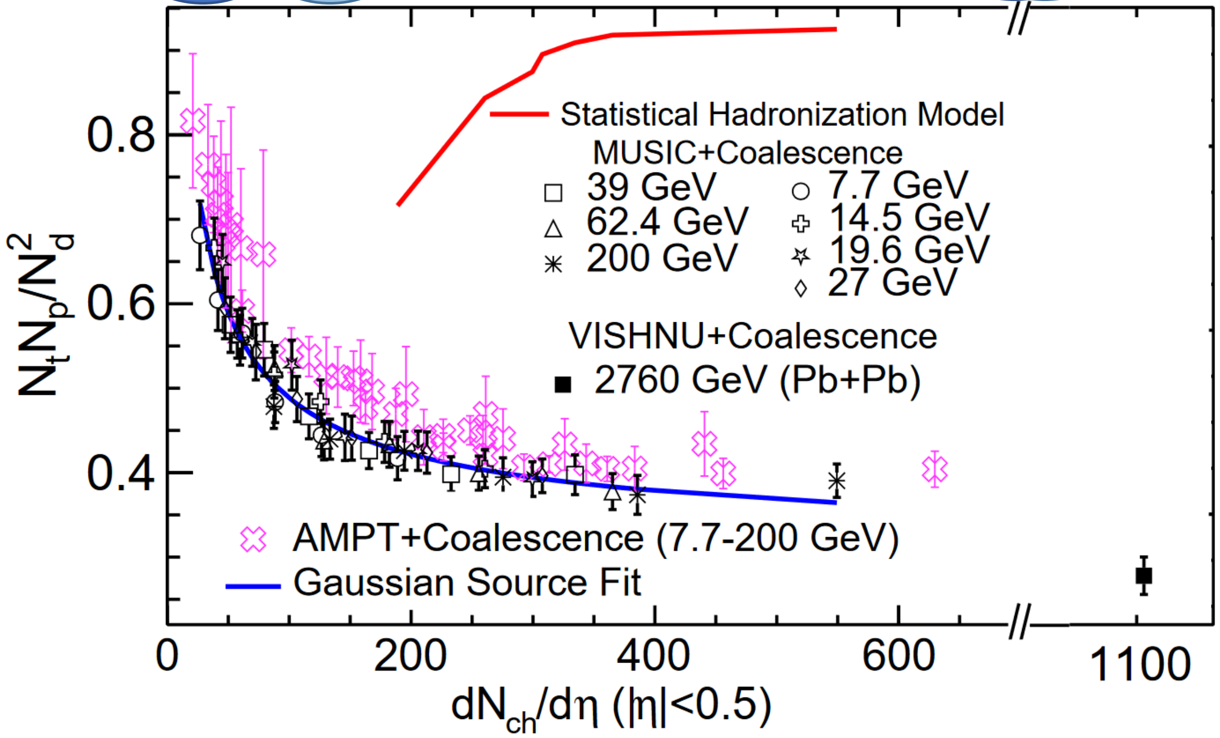
W. Zhao et al., arXiv:2009. 06959(2020)
D. Zhang (STAR), arXiv:2002.10677(2020)
H. Liu et al., Phys. Lett. B805, 135452 (2020)
V.Vovchenko et al., arXiv:2004.04411(2020)
K. J. Sun and C. M. Ko, arXiv:2005.00182(2020)

1.6 Centrality dependence of tp/d^2 w/o 1st-order PT(9)

This ratio increases in peripheral collisions due to the effects of finite nuclei sizes







Gaussian source:

$$\frac{N_t N_p}{N_d^2} = \frac{4}{9}\left(\frac{1+\frac{2r_d^2}{3R^2}}{1+\frac{r_t^2}{2R^2}}\right)^3 = \frac{4}{9}\left(1+\frac{\frac{4}{3}r_d^2 - r_t^2}{2R^2 + r_t^2}\right)^3$$

Wenbin Zhao, K. J. Sun, C. M. Ko, and X. F. Luo, *Phys.Lett.B* 820 (2021) 136571

S. Wu, K. Murase, S. Tang, and H. Song, arXiv:2205. 14302(2022)

How a first-order or second-order QCD phase transitions change the energy- and centrality-dependence of tp/d^2 ?

2. A novel transport model approach

K. J. Sun, W. H. Zhou, L. W. Chen, C. M. Ko, and F. Li, R. Wang, and J. Xu, arXiv:2205.11010(2022)

2.1 Extended NJL model

The eNJL provides a flexible equation of state (EoS) . The critical temperature can be easily changed by varying the strength of the scalar-vector interaction without affecting the vacuum properties.

Lagrangian density for eNJL

$$\begin{aligned}\mathcal{L} &= \bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m})\psi + G_S\sum_{a=0}^{8}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2] \\ &- K\{[\det[\bar{\psi}(1+\gamma_5)\psi] + \det[\bar{\psi}(1-\gamma_5)\psi]\} \\ &+ G_{SV}\left\{\sum_{a=1}^{3}[(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2]\right\} \\ &\times\left\{\sum_{a=1}^{3}[(\bar{\psi}\gamma^\mu\lambda^a\psi)^2 + (\bar{\psi}\gamma_5\gamma^\mu\lambda^a\psi)^2]\right\},\end{aligned}$$

Λ [MeV]	602.3	$M_{u,d}$ [MeV]	367.7
$G\Lambda^2$	1.835	M_s [MeV]	549.5
$K\Lambda^5$	12.36	$(\langle\bar{u}u\rangle)^{1/3}$ [MeV]	-241.9
$m_{u,d}$ [MeV]	5.5	$(\langle\bar{s}s\rangle)^{1/3}$ [MeV]	-257.7
m_s [MeV]	140.7		

Mean-field approximation

$$\begin{aligned}\mathcal{L} = \bar{u}(\gamma^\mu iD_{u\mu} - M_u)u + \bar{d}(\gamma^\mu iD_{d\mu} - M_d)d + \\ \bar{s}(\gamma^\mu iD_{s\mu} - M_s)s - 2G_S(\phi_u^2 + \phi_d^2 + \phi_s^2) + \\ 4K\phi_u\phi_d\phi_s - 3G_{SV}(\phi_u + \phi_d)^2(\rho_u + \rho_d)^2,\end{aligned}$$

Effective mass:

$$\begin{aligned}M_u &= m_u - 4G_S\phi_u + 2K\phi_d\phi_s \\ &\quad -2G_{SV}(\rho_u + \rho_d)^2(\phi_u + \phi_d), \\ M_d &= m_d - 4G_S\phi_d + 2K\phi_u\phi_s \\ &\quad -2G_{SV}(\rho_u + \rho_d)^2(\phi_u + \phi_d), \\ M_s &= m_s - 4G_S\phi_s + 2K\phi_u\phi_d\end{aligned}$$

$$\phi_i = -2N_c\int_0^\Lambda \frac{d^3p}{(2\pi\hbar)^3}\frac{M_i}{E_i}(1 - f_i - \bar{f}_i)$$

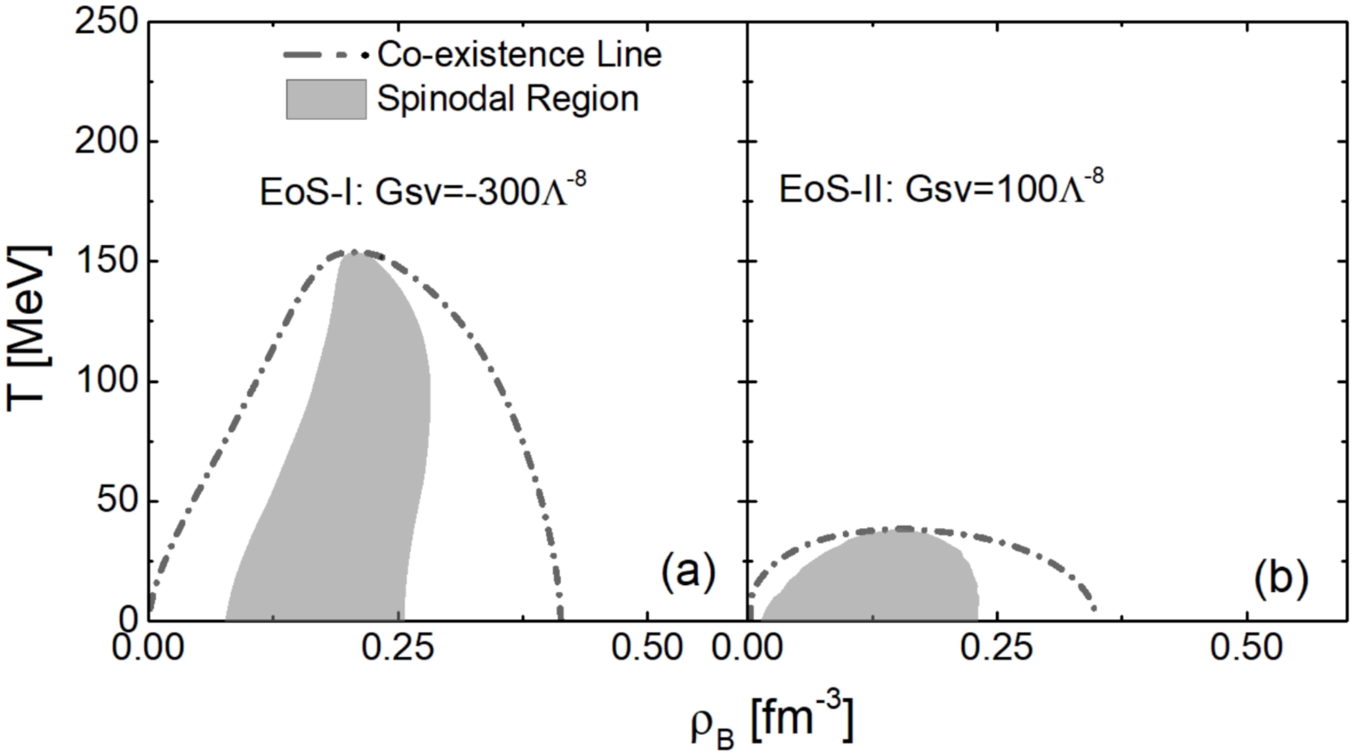
$$\rho_i = 2N_c\int_0^\Lambda \frac{d^3p}{(2\pi\hbar)^3}(f_i - \bar{f}_i)$$

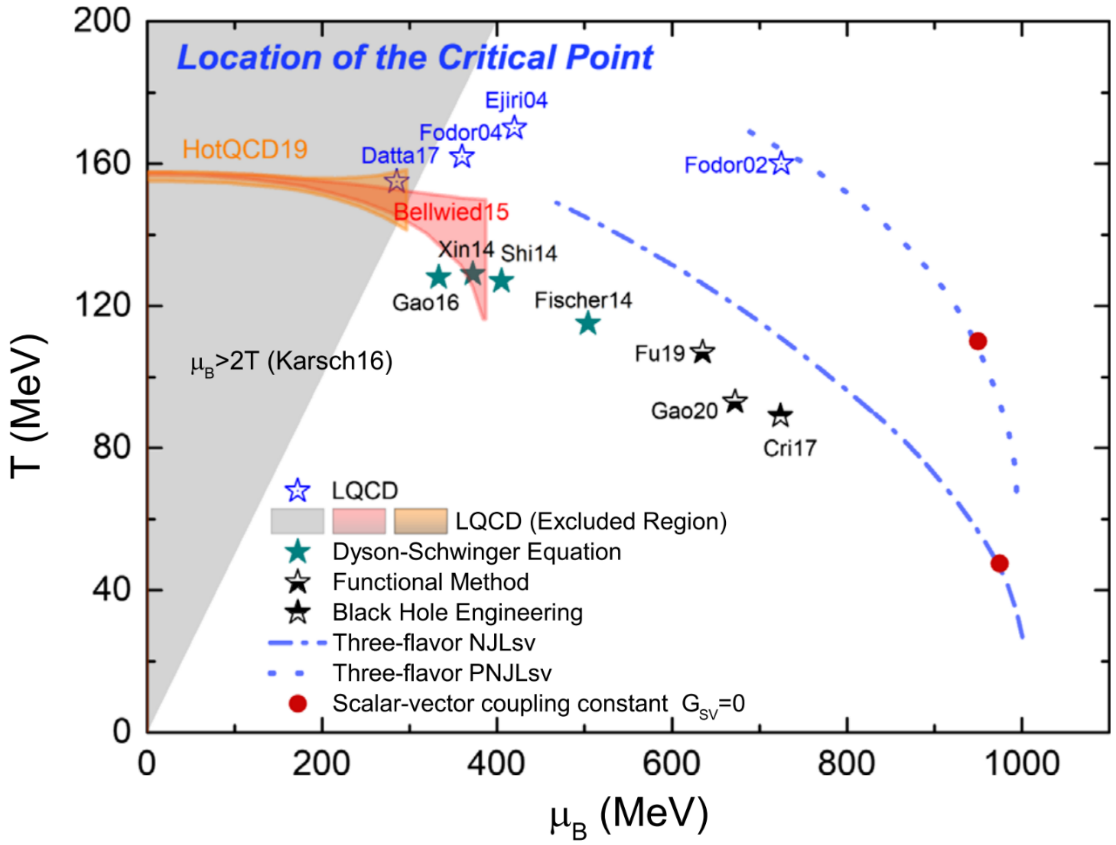
M. Buballa, Phys. Rept. 407, 205 (2005)

K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

2.2 Equation of state

The eNJL provides a flexible equation of state (EoS) . The critical temperature can be easily changed by varying the strength of the scalar-vector interaction without affecting the vacuum properties.

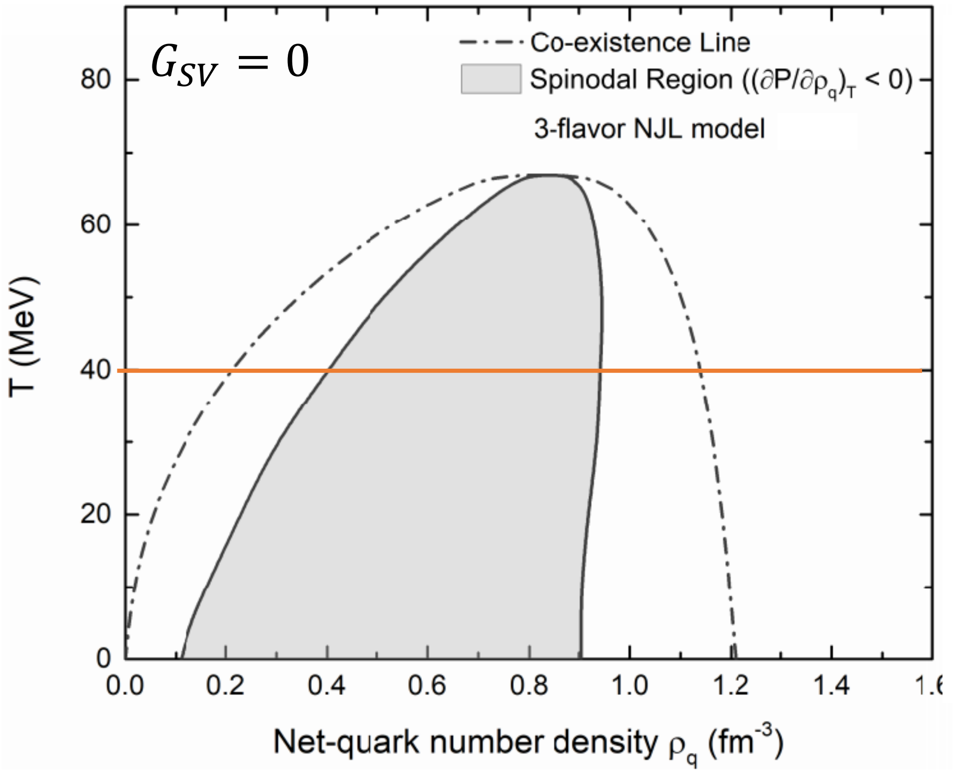


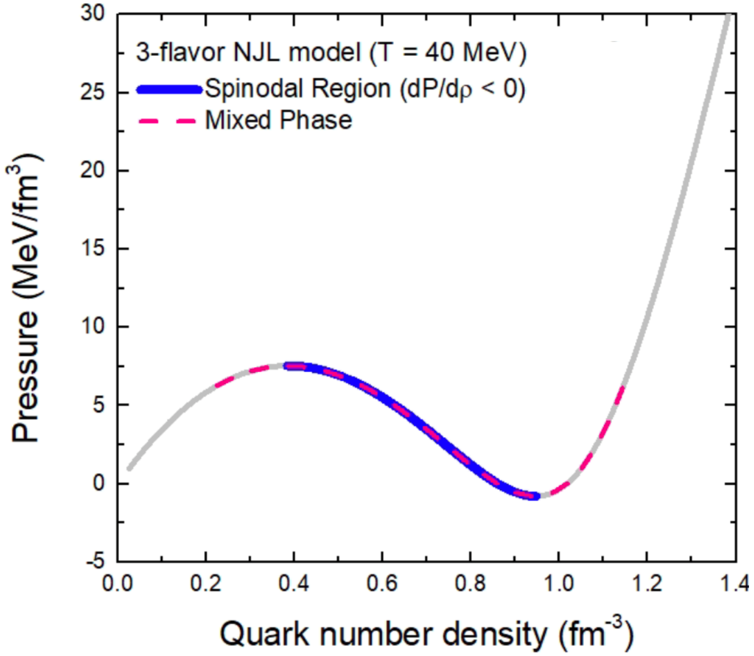


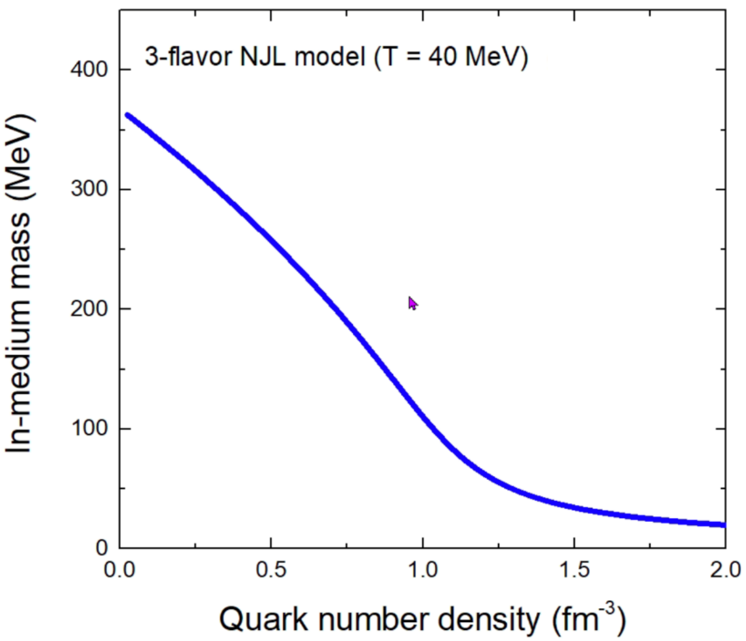
K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

2.3 Spinodal instability

$$\left(\frac{\partial p}{\partial \rho}\right)_T < 0$$







K. J. Sun, C. M. Ko, S. Cao, and F. Li., Phys. Rev. D 103, 014006 (2021)

2.4 Transport equation

Mean field + partonic scattering

$$\frac{\partial f_\pm}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}_r f_\pm + \left(-\frac{M}{E^*}\boldsymbol{\nabla}_r M \pm \mathbf{E} \pm \mathbf{v}\times\mathbf{B}\right)\cdot\nabla_p f_\pm$$

$$= \left(\frac{\partial f_\pm}{\partial t}\right)_{\rm coll}$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial \mathbf{t}} + \boldsymbol{\nabla}_\mathbf{r} \mathrm{A}_\mathbf{0}$$

$$\mathbf{B} = \boldsymbol{\nabla}_\mathbf{r} \times \mathbf{A}$$

$$A_{u\mu} = -2G_{SV}(\phi_u + \phi_d)^2(j_{u\mu} + j_{d\mu}),$$

$$A_{d\mu} = -2G_{SV}(\phi_u + \phi_d)^2(j_{u\mu} + j_{d\mu}),$$

$$A_{s\mu} = 0.$$

$$j_{i\mu} = \rho_i u_\mu$$

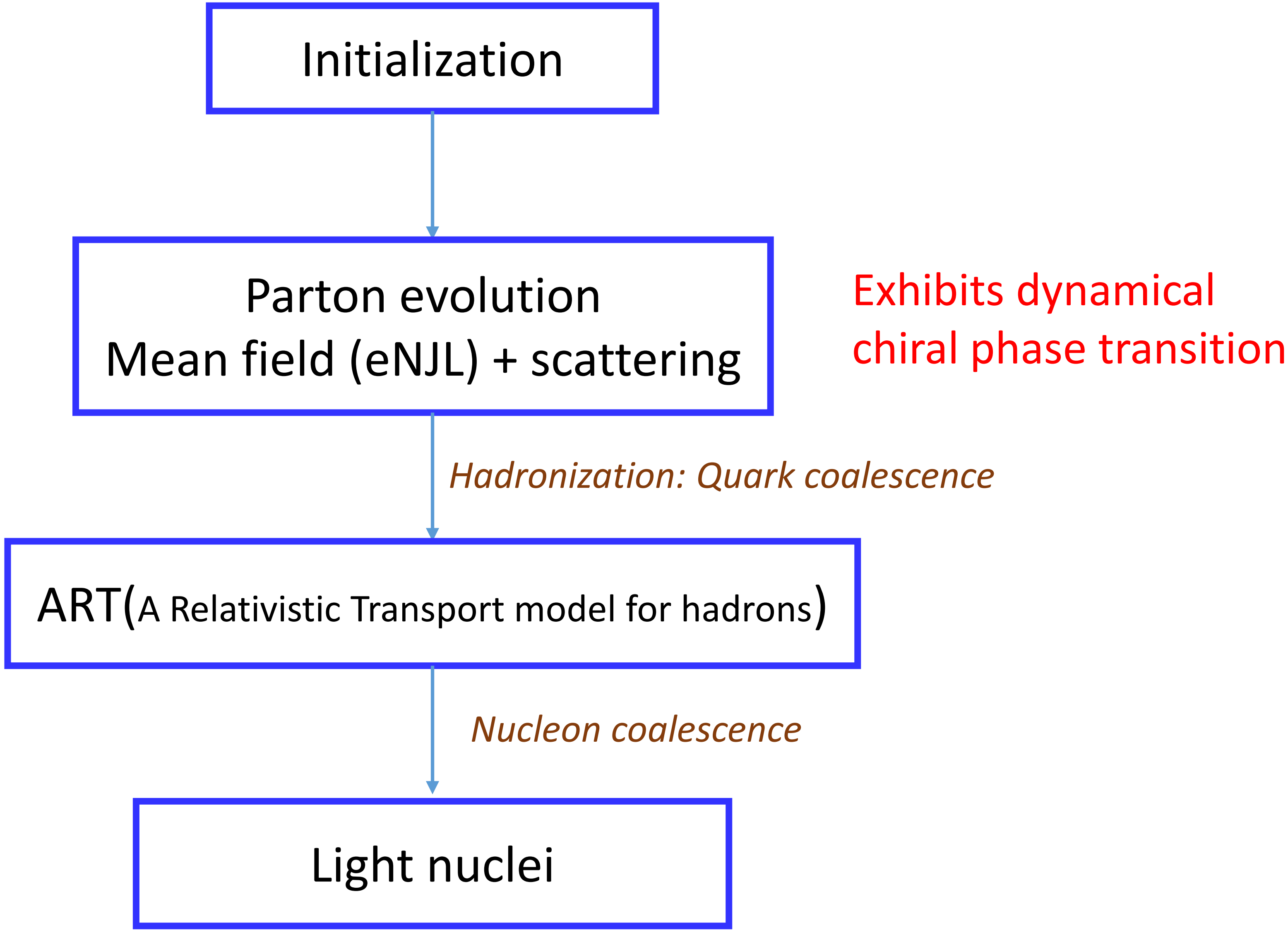
Test –particle method: J. Xu, arXiv:1904.00131 (2019)

$$\frac{d\mathbf{r}}{dt} = \mathbf{v},$$

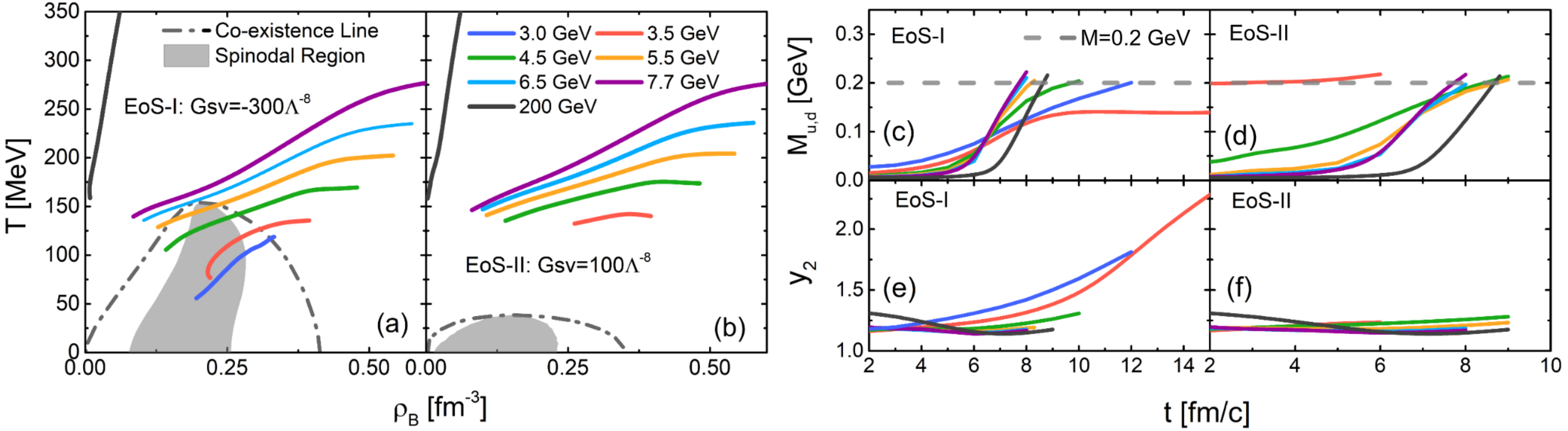
$$\frac{d\mathbf{p}}{dt} = -\frac{M}{E^*}\boldsymbol{\nabla}_r M \pm \mathbf{E} \pm \mathbf{v}\times\mathbf{B}$$

3. Spinodal enhancement of tp/d^2 in central and peripheral collisions

3.1 Framework



3.2 Trajectories in the phase diagram

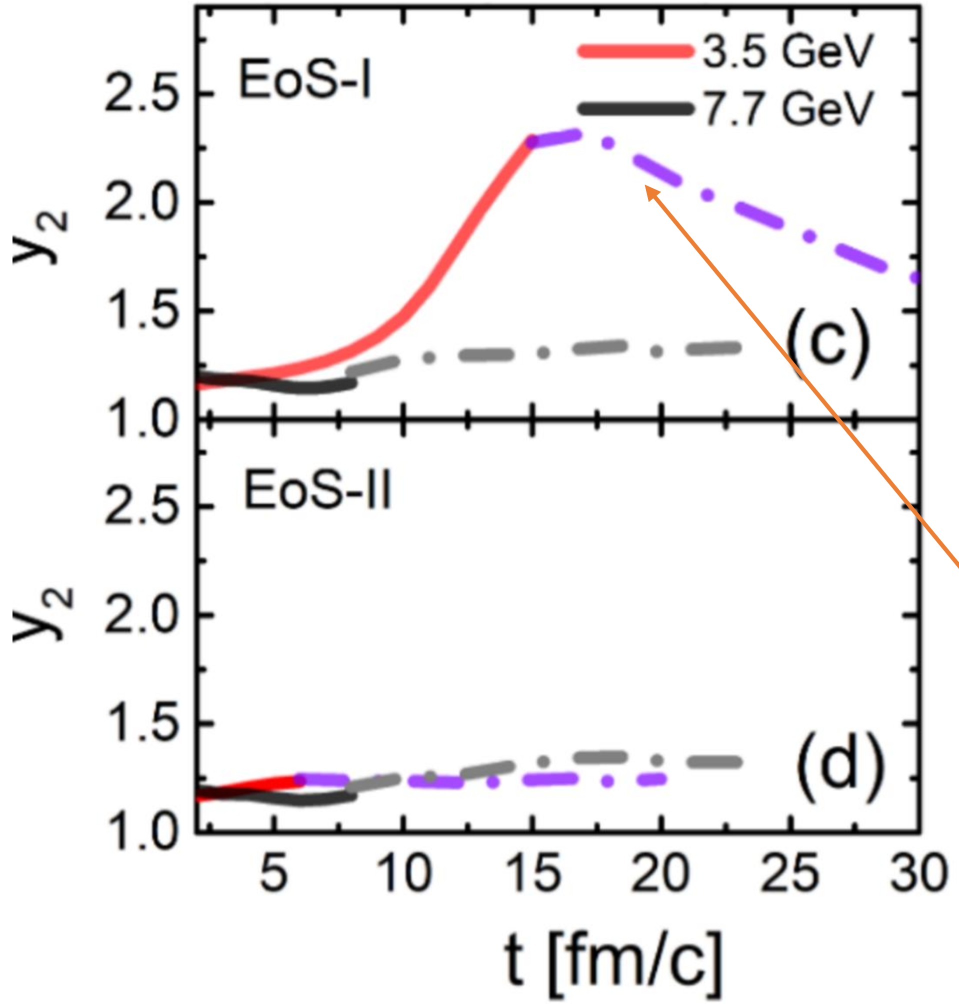


$$\overline{\rho^N} = \frac{\int d\mathbf{x}\rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x}\rho(\mathbf{x})}$$

$$y_2 = \frac{[\int d\mathbf{x}\rho(\mathbf{x})][\int \mathrm{d}\mathbf{x}\rho^{\mathbf{3}}(\mathbf{x})]}{[\int \mathrm{d}\mathbf{x}\rho^{\mathbf{2}}(\mathbf{x})]^{\mathbf{2}}}$$

3.3 Survival of density fluctuation in expanding fireball (16)

Off-equilibrium effects



Density moment:

$$\overline{\rho^N} = \frac{\int d\mathbf{x}\rho^{(N+1)}(\mathbf{x})}{\int d\mathbf{x}\rho(\mathbf{x})}$$

$$y_2 = \frac{[\int d\mathbf{x}\rho(\mathbf{x})][\int d\mathbf{x}\rho^{\mathbf{3}}(\mathbf{x})]}{[\int d\mathbf{x}\rho^{\mathbf{2}}(\mathbf{x})]^{\mathbf{2}}}$$

If the expansion is self-similar or scale invariant

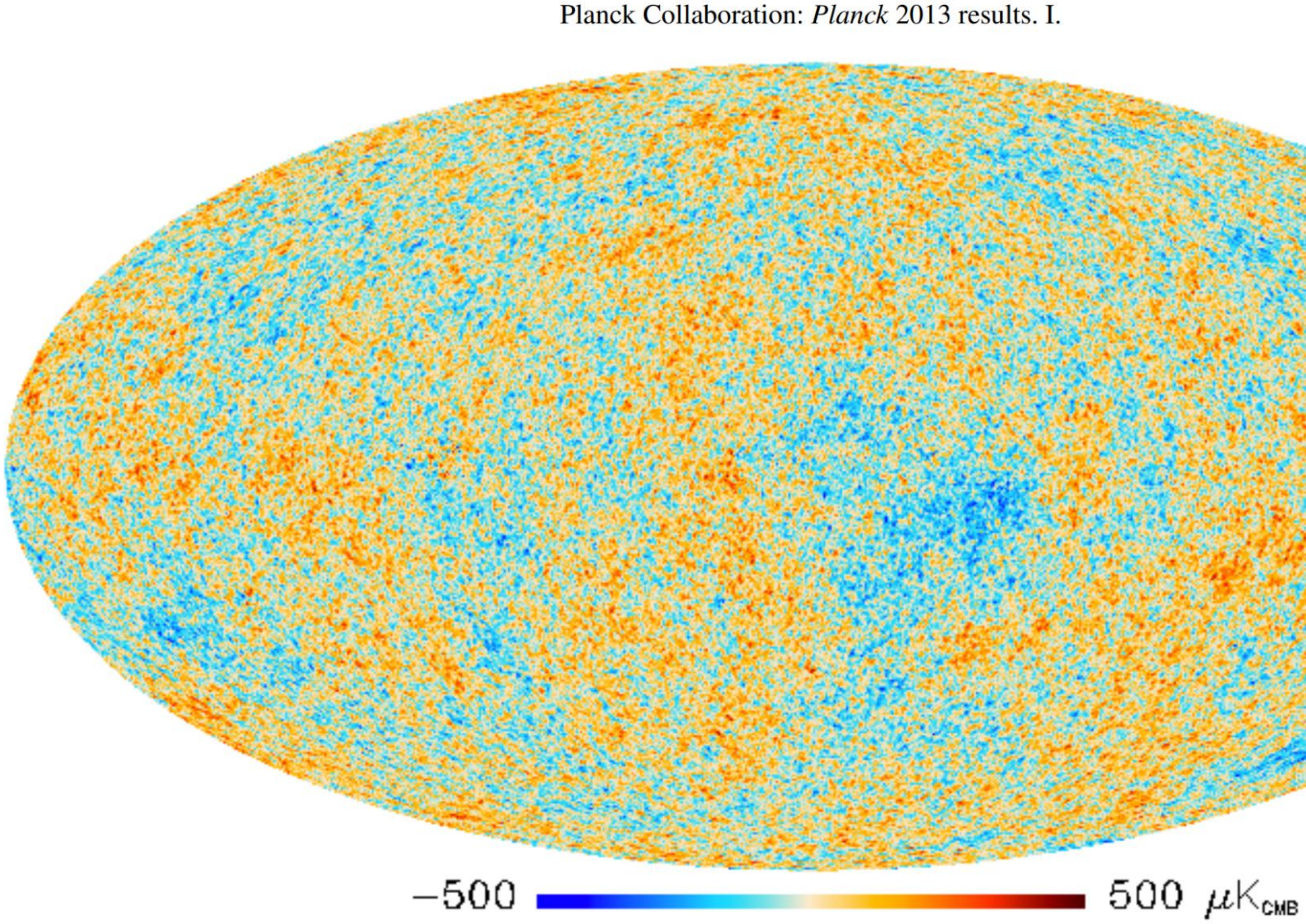
$$\rho(\lambda(t)x, t) = \alpha(t)\rho(x, t_h)$$

then $y_2(t) = y_2(t_h)$, i.e., remains a constant

'Memory effects': Large density inhomogeneity survives to kinetic freezeout

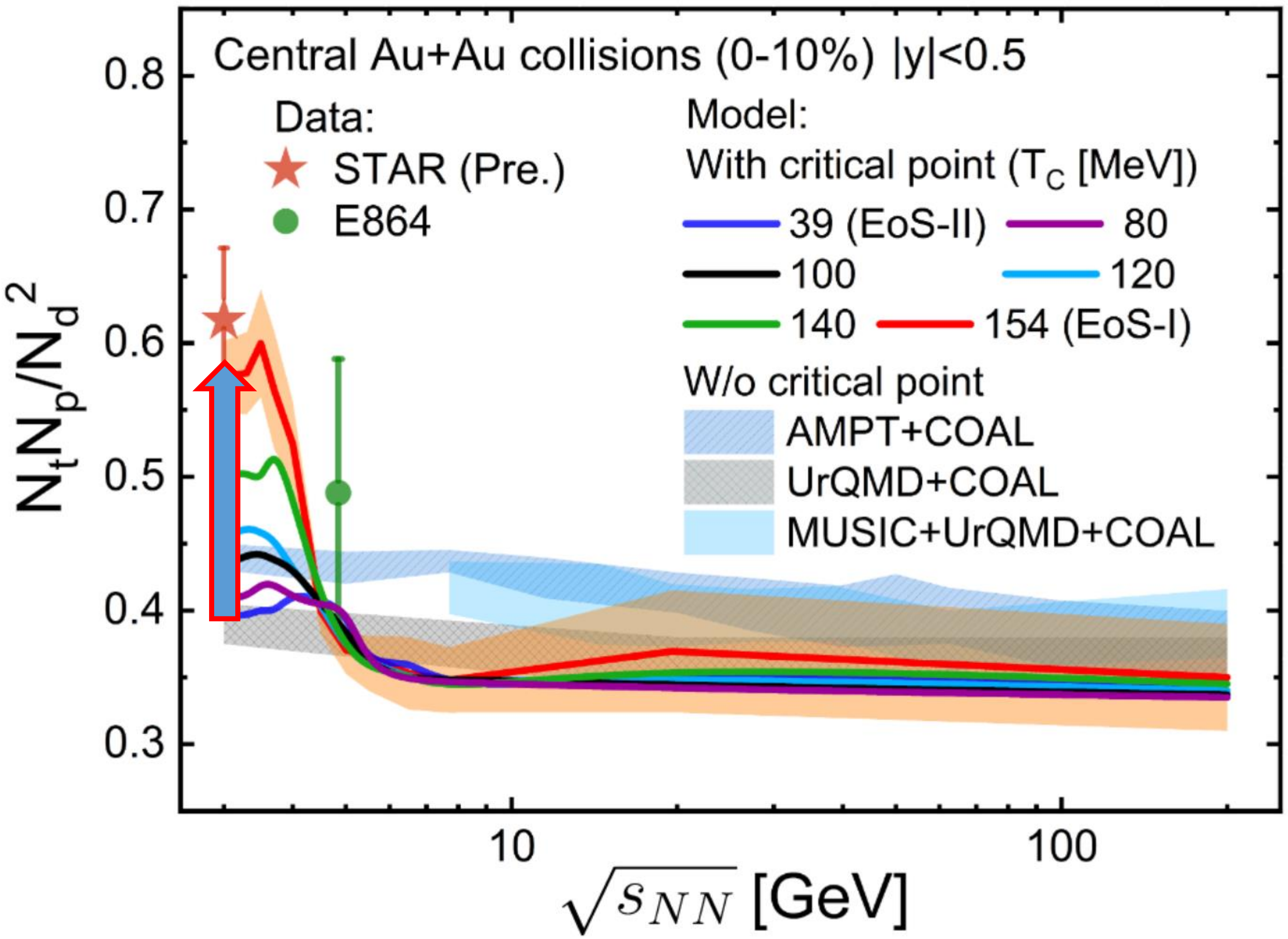
3.3 Survival of density fluctuation in expanding universe (17)

A similar example is the observed temperature anisotropy in the cosmic microwave background, which is now considered as the remnant of quantum fluctuations in the primordial Universe during the rapid inflation epoch.



Plank Collaboration, Astron. Astrophys. 571, A1 (2014)

3.3 Collision energy dependence



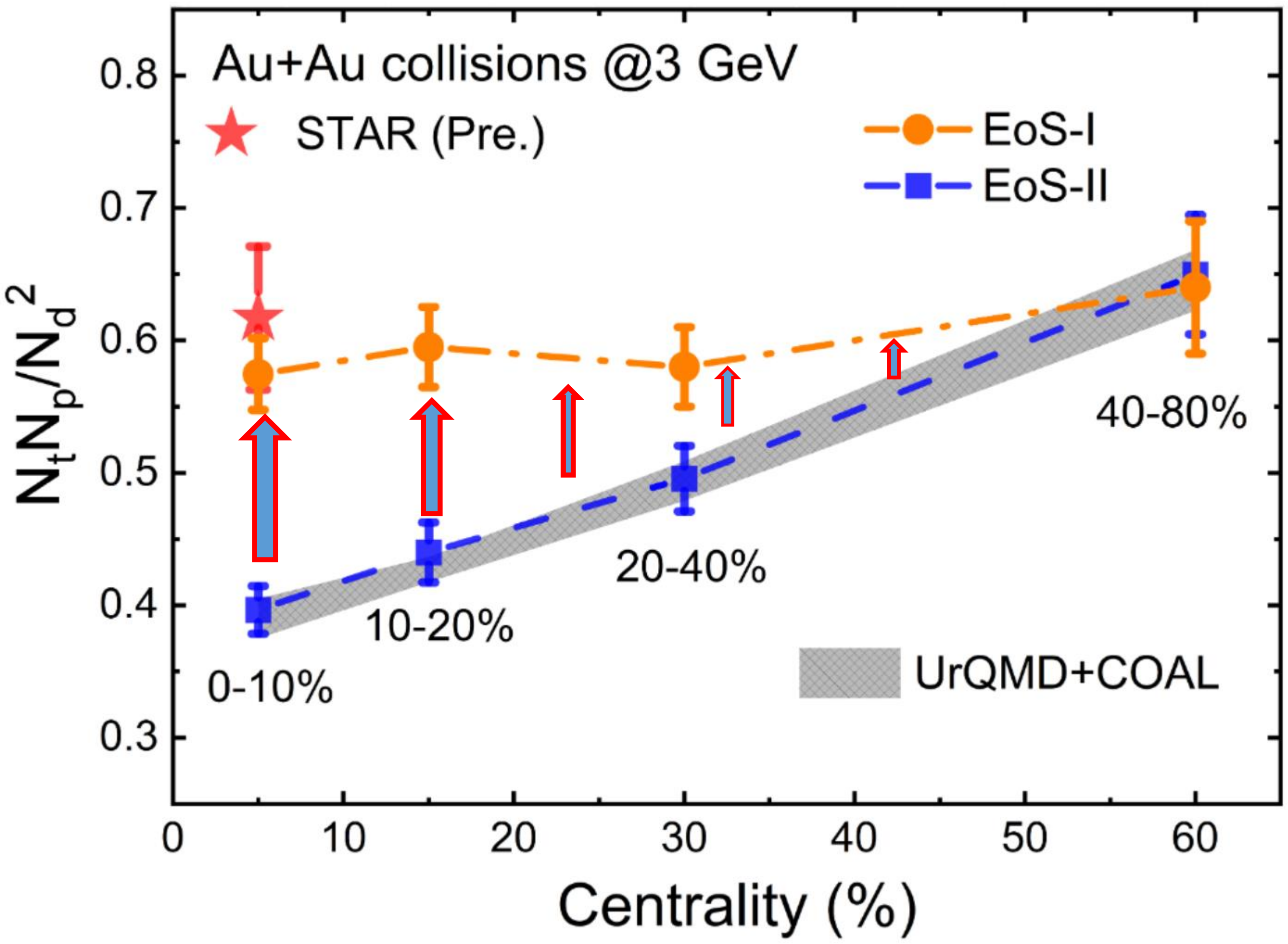
1. Without a critical point:
   The energy dependence of tp/d^2 is almost flat.

2. With a first-order phase transition:
   The spinodal instability induced enhancement of tp/d^2 during the first-order phase transition increases as increasing the critical temperature.

Hui Liu (STAR), QM2022

T. A. Armstrong et al. (E864), Phys. Rev. C 61, 064908 (2000).

3.4 Centrality dependence



The spinodal enhancement of tp/d^2 subsides with increasing collision centrality because of smaller fireball lifetime in more peripheral collisions.

The slope with EoS-I is 5 times smaller

4. Summary

Main findings:

1. With scans of the collision energy and centrality as well as the equation of state using a novel transport model, we find that large density inhomogeneities generated by the spinodal instability during the first-order QCD phase transition can survive the fast expansion of the subsequent hadronic matter and lead to an enhanced tp/d^2 in central Au+Au collisions at $\sqrt{s_{NN}}$ =3 – 5 GeV for $T_c \geq 80$ MeV, which is in accordance with the experimental measurements.

2. We also find that the spinodal enhancement of tp/d^2 subsides with increasing collision centrality because of the shortening of fireball lifetime, and this effect results an almost flat centrality dependence of tp/d^2 at $\sqrt{s_{NN}}$ =3 GeV, which can also be used as a signal for the occurrence of a first-order phase transition.

Future developments:

1. Incorporation of Polyakov loop
2. Inclusion of long-range correlation