From Open Quantum System to Quarkonium Transport inside Quark-Gluon Plasma

Xiaojun Yao

HENPIC Online Talk
Feb. 6, 2020

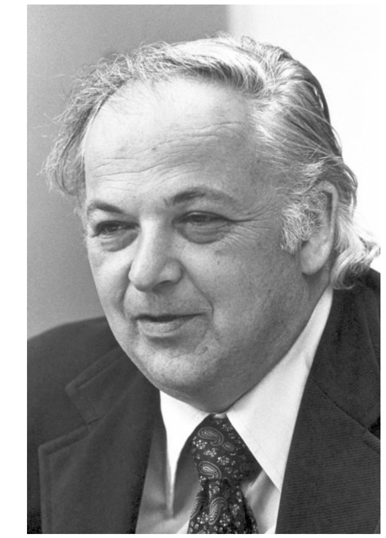
XY B.Müller, 1709.03529, 1811.09644

XY W.Ke Y.Xu S.Bass B.Müller, 1807.06199, 1812.02238

XY T.Mehen, 1811.07027

Introduction: Quarkonium

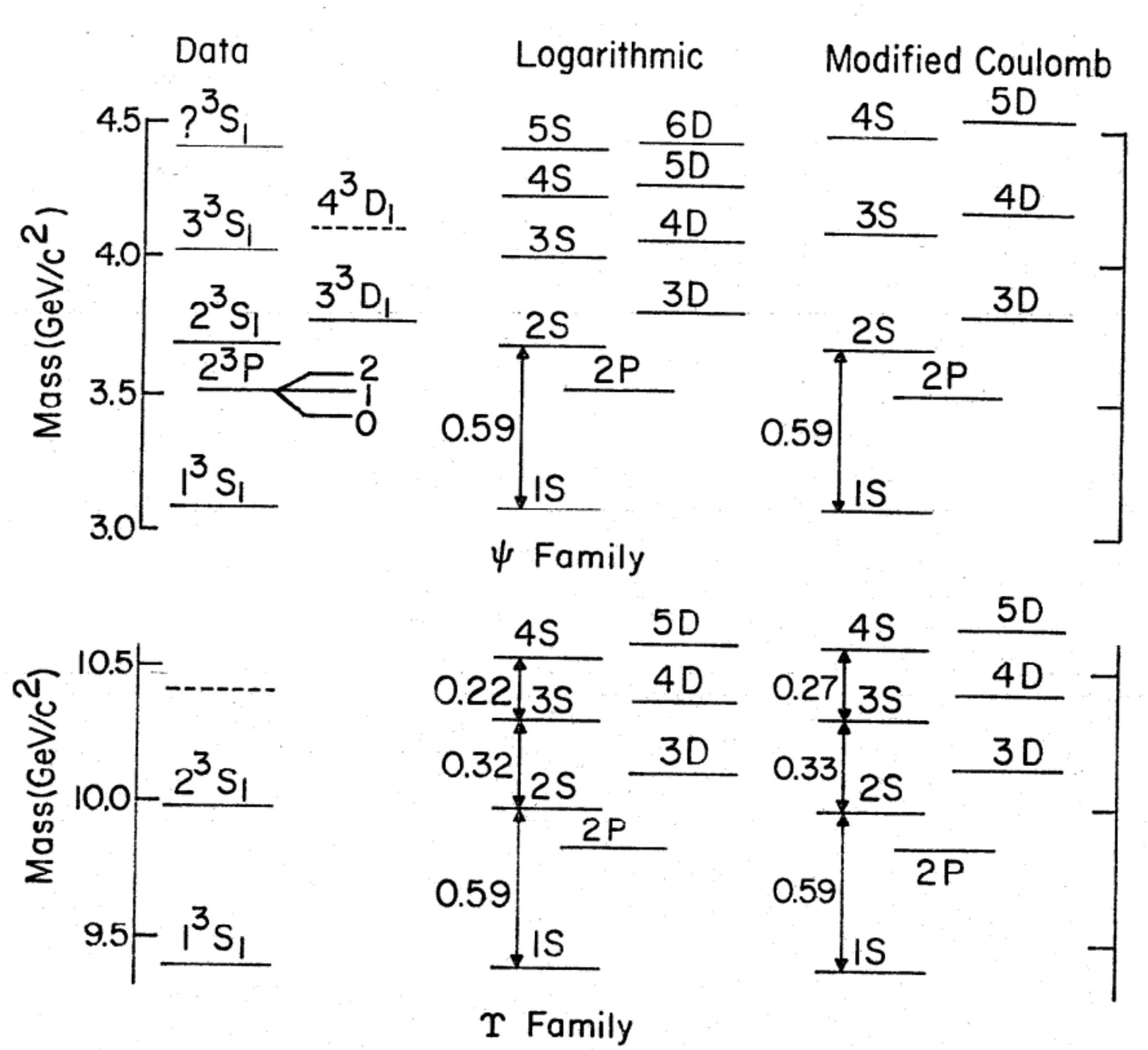
- 1974 discovery of J/Ψ at BNL and SLAC: bound state of charm anticharm —> Nobel prize in 1976



- Ground and lower excited states spectrum can be understood from nonrelativistic potential models:

Cornell potential (modified Coulomb)

$$V(r) = -\frac{A}{r} + Br$$

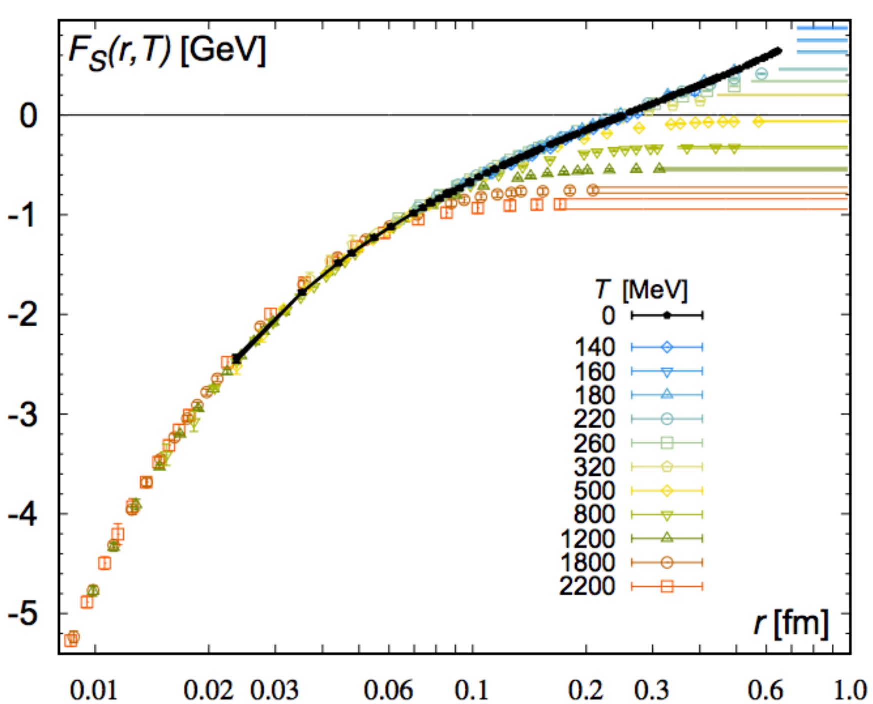


C.Quigg, J.L.Rosner Phys. Lett. 71B, 153 (1977)

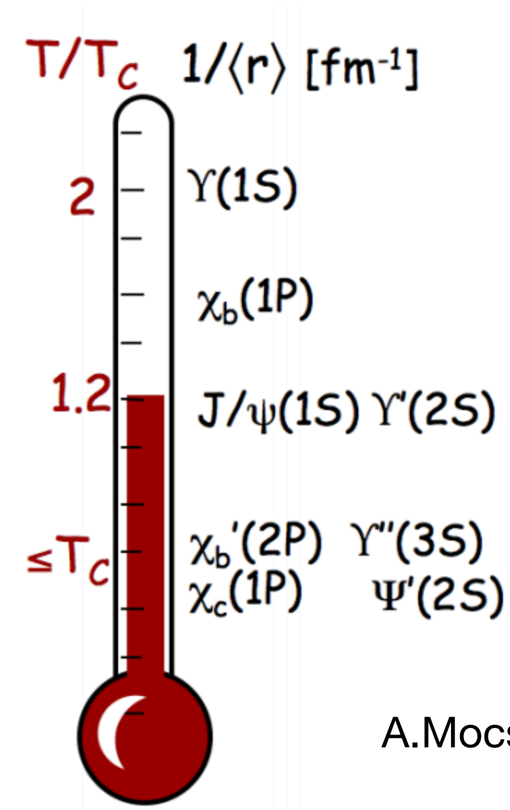
Quarkonium as Probe of Quark-Gluon Plasma

- **Static screening**: suppression of color attraction —> melting at high T —> reduced production —> thermometer

$$T=0: V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0:$$ Confining part flattened



1804.10600, A.Bazavov, N.Brambilla P.Petreczky, A.Vairo, J.H.Weber

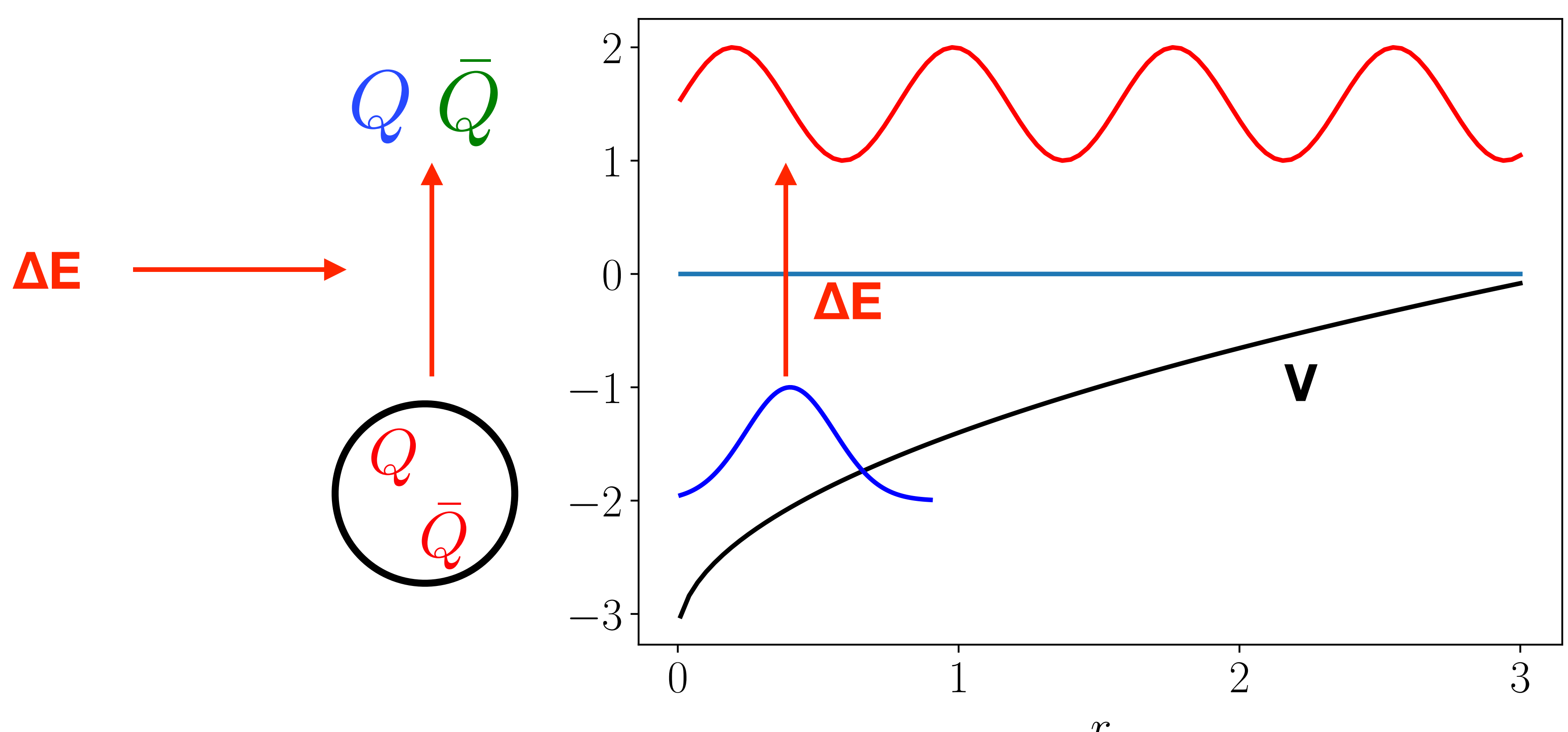


A.Mocsy, 0811.0337

Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer

- **Dynamical screening**: dissociation induced by dynamical process, imaginary potential

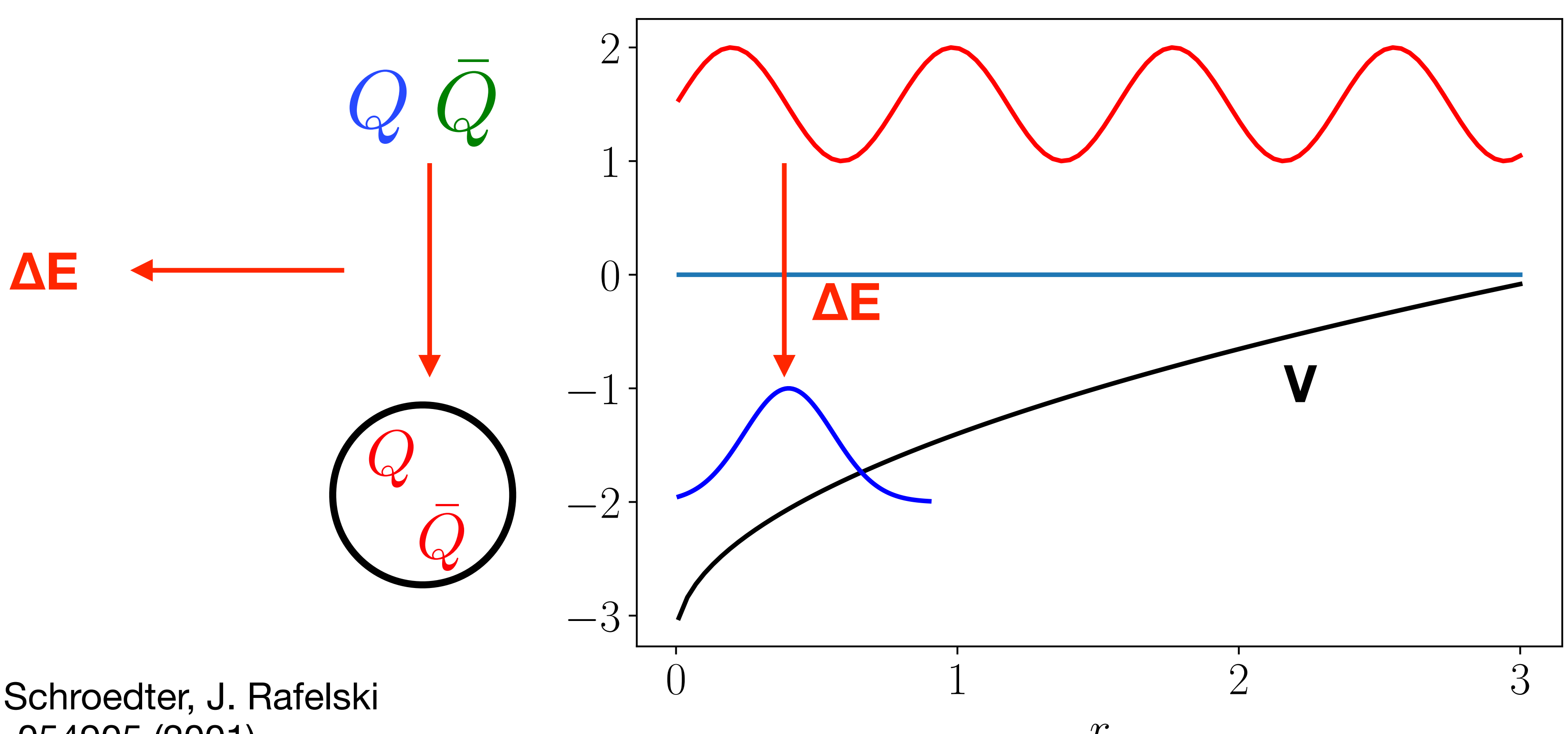


Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer

- Dynamical screening: dissociation induced by dynamical process, imaginary potential

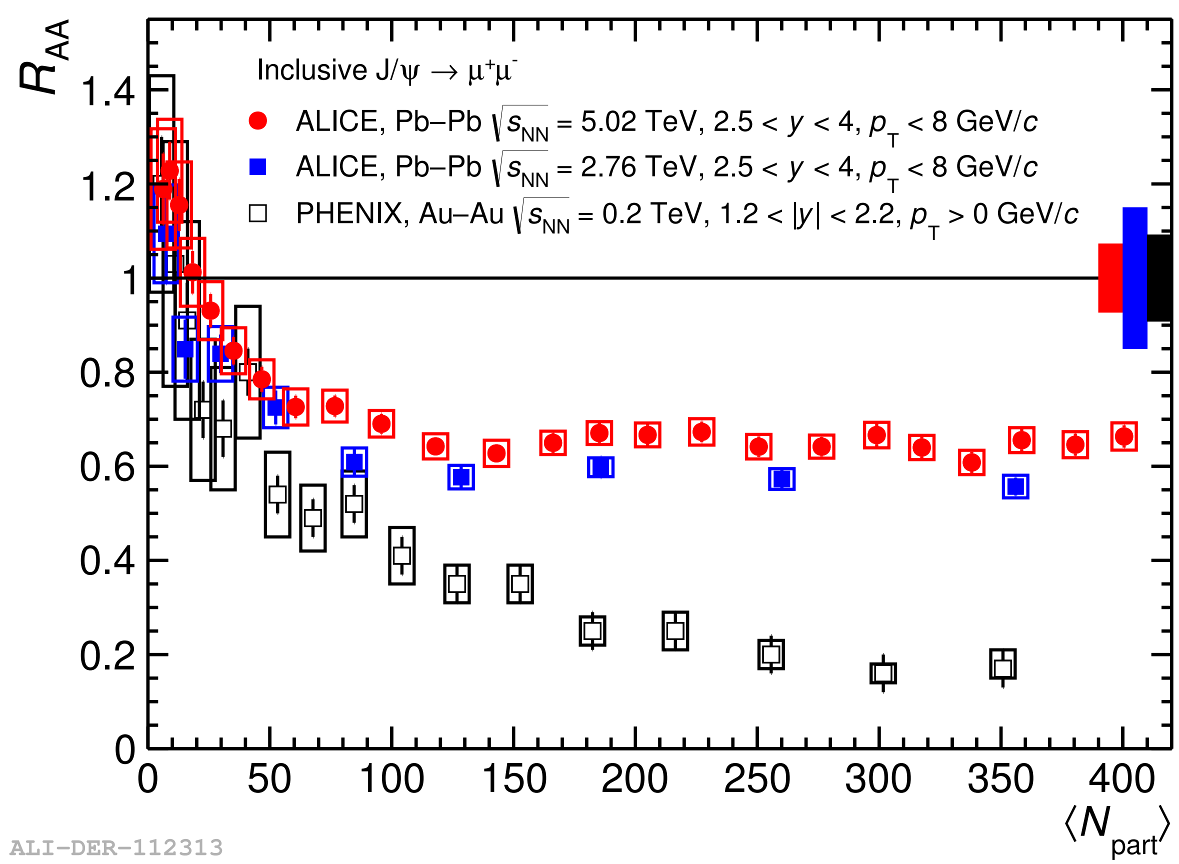
- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T



RL. Thews, M. Schroedter, J. Rafelski
Phys.Rev.C 63, 054905 (2001)

Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer
- Dynamical screening: dissociation induced by dynamical process, imaginary potential
- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T, **crucial for phenomenology** and theory consistency

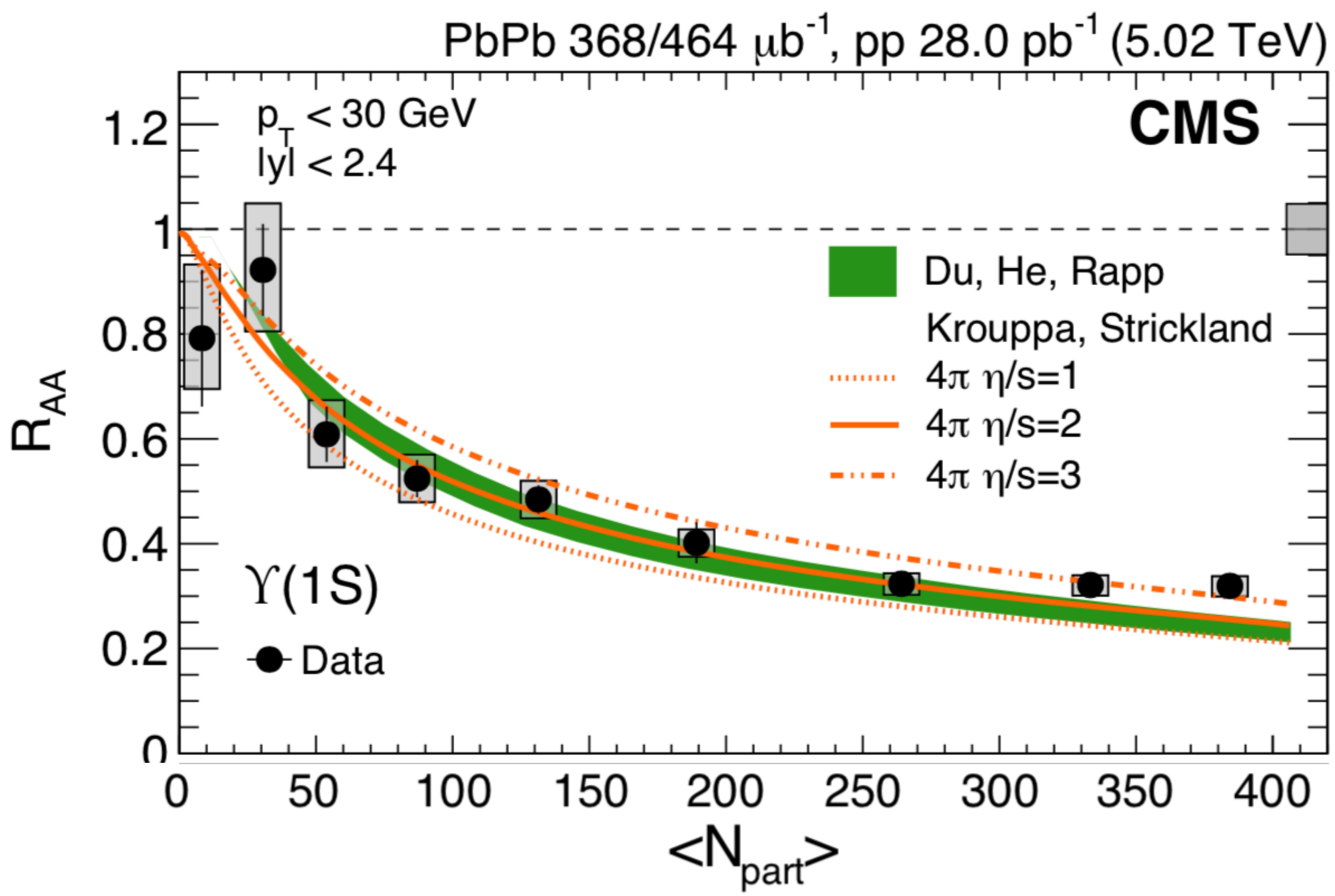


Phenomenological Success of Transport Theory

Evolution of distribution in phase space

$$(\partial_t + \boldsymbol{v}\cdot\nabla) f(\boldsymbol{x},\boldsymbol{p},t) = -C^{(-)}(\boldsymbol{x},\boldsymbol{p},t) + C^{(+)}(\boldsymbol{x},\boldsymbol{p},t)$$

Dissociation Recombination



X.Du, R.Rapp, M.He,1706.08670

B.Krouppa, M.Strickland,1605.03561

Why transport equation successful? Connection to QCD?

Phenomenological Success of Transport Theory

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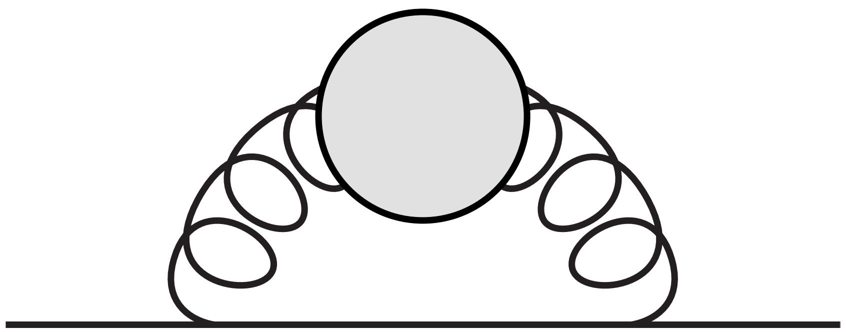
Dissociation **Recombination**

Two screening effects from thermal loops



quarkonium propagator in QGP

Real & imaginary parts —> static screening & dissociation

Recombination modeled, calculate from QCD consistently with dissociation?

Put screening and recombination into same framework?

Open Quantum System

$$H = H_S + H_E + H_I$$

$$\rho(t=0) = \rho_S \otimes \rho_E$$

System & environment

(Heavy quark pairs & QGP)

Unitary evolution

Time reversible

$$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$$

System & environment

Trace out (integrate out) environment

$$\rho_S(t=0)$$

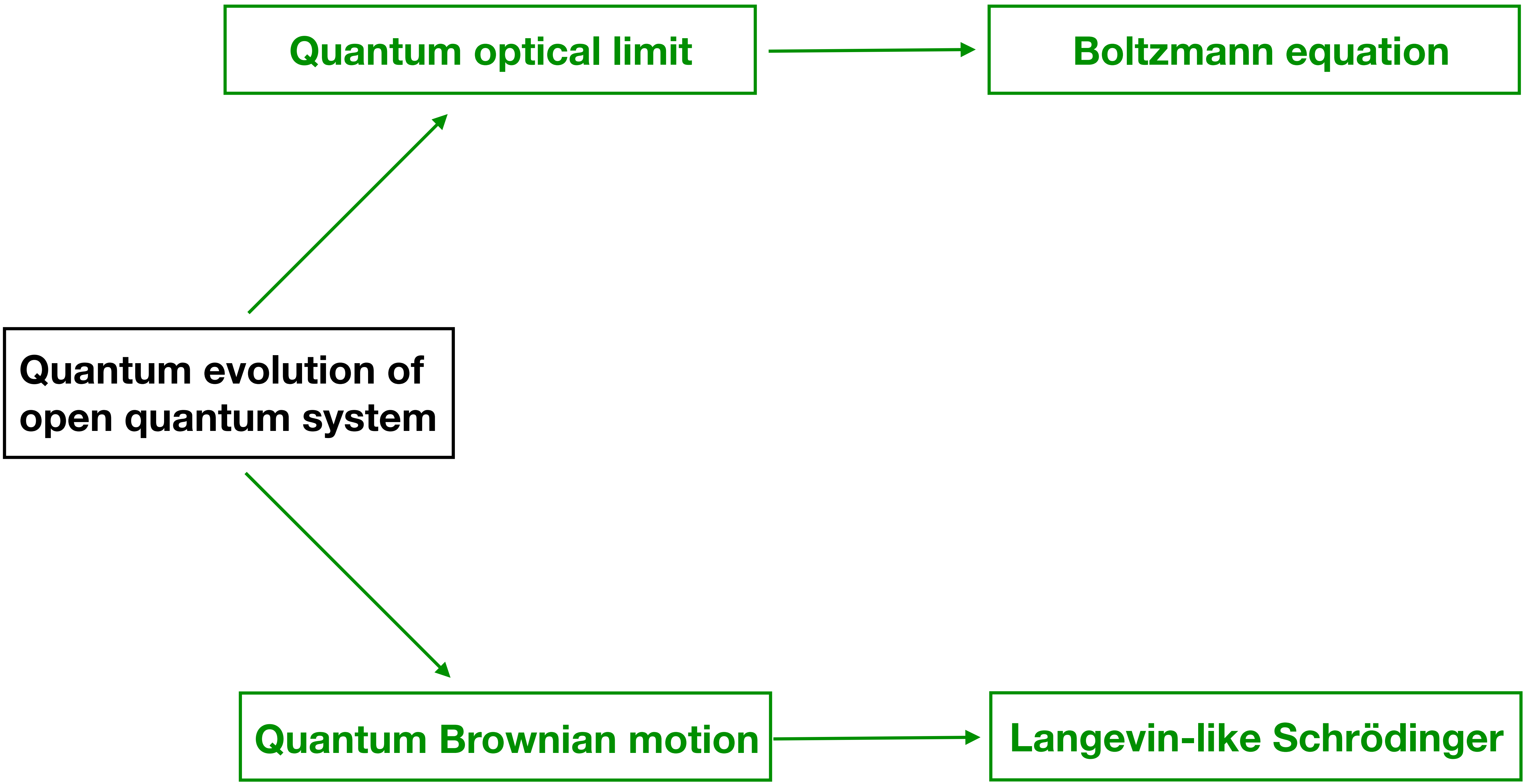
System(heavy quark pairs)

Non-unitary

Time irreversible

$$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$$

Two Limits of Open Quantum System Evolution



Two Limits of Open Quantum System Evolution

Quantum evolution of open quantum system

$\tau_R \gg \tau_E$

$\tau_R \gg \tau_S$

→ **Quantum optical limit** → **Boltzmann equation**

$\tau_R \gg \tau_E$

$\tau_S \gg \tau_E$

→ **Quantum Brownian motion** → **Langevin-like Schrödinger**

τ_S **System intrinsic time scale**

τ_E **Environment correlation time**

τ_R **System relaxation time**

Two Limits of Open Quantum System Evolution

Quantum evolution of open quantum system

→ **Quantum optical limit**: $\tau_R \gg \tau_E$, $\tau_R \gg \tau_S$

→ **Quantum Brownian motion**: $\tau_R \gg \tau_E$, $\tau_S \gg \tau_E$

$$\tau_R \gg \tau_E$$

During system relaxation, environment correlation has lost —> Markovian process

$$\tau_R \gg \tau_S$$

During system relaxation, heavy quark pair has revolved many periods, then it makes sense to use the concept of a well-defined bound state in the calculation

$$\tau_S \gg \tau_E$$

System only feels low frequency part of environment correlation

Separation of Scales

Separation of scales in vacuum $M \gg Mv \gg Mv^2$

QCD

M — perturbative matching

HQET/NRQCD

Heavy quark physics, A.Manohar, M.Wise
hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

Mv — perturbative matching / non-perturbative matching

potential NRQCD

$Mv^2 \sim 500\ \mathrm{MeV}$

hep-ph/9907240, N.Brambilla
A.Pineda, J.Soto, A.Vairo

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Inside QGP: thermal scales: T $\tau_E \sim \frac{1}{T}$ $\tau_S \sim \frac{1}{Mv^2}$

Case 1 $Mv \gg T$ **Quantum optical limit**

Case 2 $T \gg Mv^2$ **Quantum Brownian motion**

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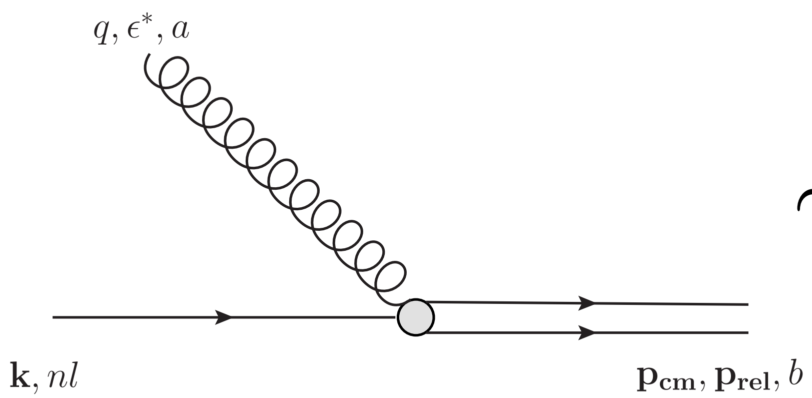
Quantum Optical Limit

Separation of scales $M \gg Mv \gg Mv^2 \gtrsim T$

NR & multipole expansions of QCD

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\, {\rm Tr}\Big({\rm S}^\dagger (i\partial_0 - H_s){\rm S} + {\rm O}^\dagger (iD_0 - H_o){\rm O} + \boxed{V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.})} + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, {\rm O}\} + \cdots \Big)$$

Dipole interaction $r \sim \dfrac{1}{Mv}$



$$\sim grT \sim g\frac{T}{Mv}$$

Weak coupling

Relaxation rate $(grT)^2 T \lesssim \alpha_s v^2 T \ll T \lesssim Mv^2$

So $\tau_R \gg \tau_E$

$\tau_R \gg \tau_S$

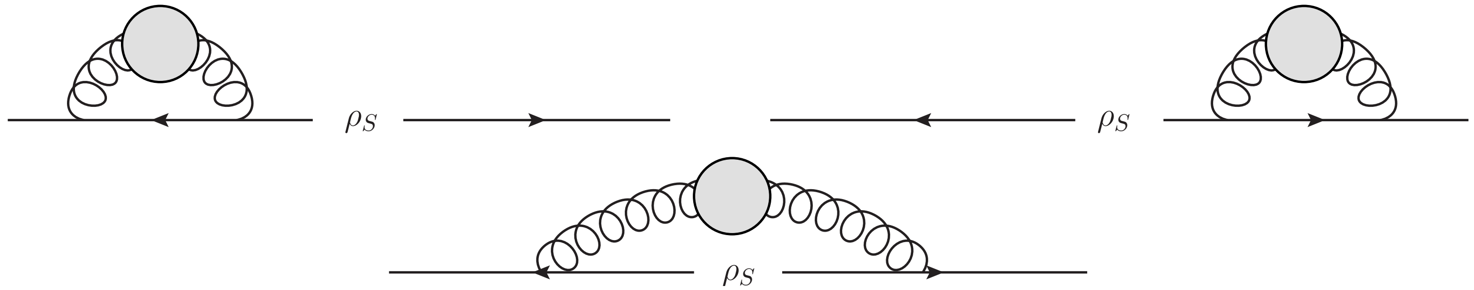
Arguments breakdown if (1) large log: Mv —> T, VA has no running at one loop
(2) large pT: medium boosted in rest frame of quarkonium, constrain to low pT

XY, B.Müller arXiv:1811.09644

From Open Quantum System to Transport Equation

Lindblad equation:

$$\rho_S(t) = \rho_S(0) - i\Big[tH_S + \sum_{a,b}\sigma_{ab}(t)L_{ab}, \rho_S(0)\Big] + \sum_{a,b,c,d}\gamma_{ab,cd}\Big(L_{ab}\rho_S(0)L^\dagger_{cd} - \frac{1}{2}\{L^\dagger_{cd}L_{ab}, \rho_S\}\Big)$$



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Markovian approximation

Wigner transform (smearing for positivity)

$$f_{nl}(\boldsymbol{x}, \boldsymbol{k}, t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \langle \boldsymbol{k} + \frac{\boldsymbol{k}'}{2}, nl, 1|\rho_S(t)|\boldsymbol{k} - \frac{\boldsymbol{k}'}{2}, nl, 1\rangle$$

Semiclassical limit

Boltzmann transport equation

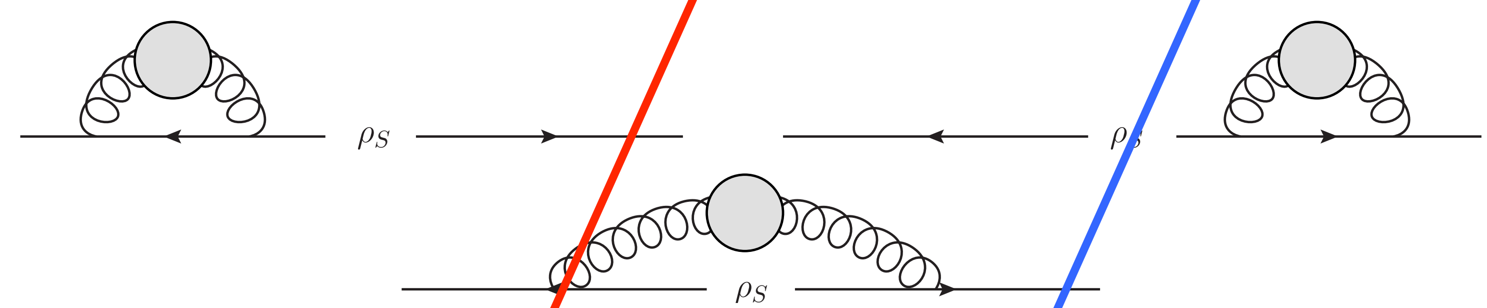
$$\frac{\partial}{\partial t} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) + \boldsymbol{v}\cdot\nabla_{\boldsymbol{x}} f_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) = \mathcal{C}^{(+)}_{nls}(\boldsymbol{x}, \boldsymbol{k}, t) - \mathcal{C}^{(-)}_{nls}(\boldsymbol{x}, \boldsymbol{k}, t)$$

XY T.Mehen,1811.07027

From Open Quantum System to Transport Equation

Lindblad equation: **Correction to Hamiltonian, static screening**

$$\rho_S(t) = \rho_S(0) - i\left[tH_S + \sum_{a,b}\sigma_{ab}(t)L_{ab}, \rho_S(0)\right] + \sum_{a,b,c,d}\gamma_{ab,cd}\left(L_{ab}\rho_S(0)L^{\dagger}_{cd} - \frac{1}{2}\{L^{\dagger}_{cd}L_{ab}, \rho_S\}\right)$$



Recombination **Dissociation**

Boltzmann transport equation

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XY T.Mehen,1811.07027

From Open Quantum System to Transport Equation

Success of transport equation in quarkonium phenomenology

↕

Separation of scales

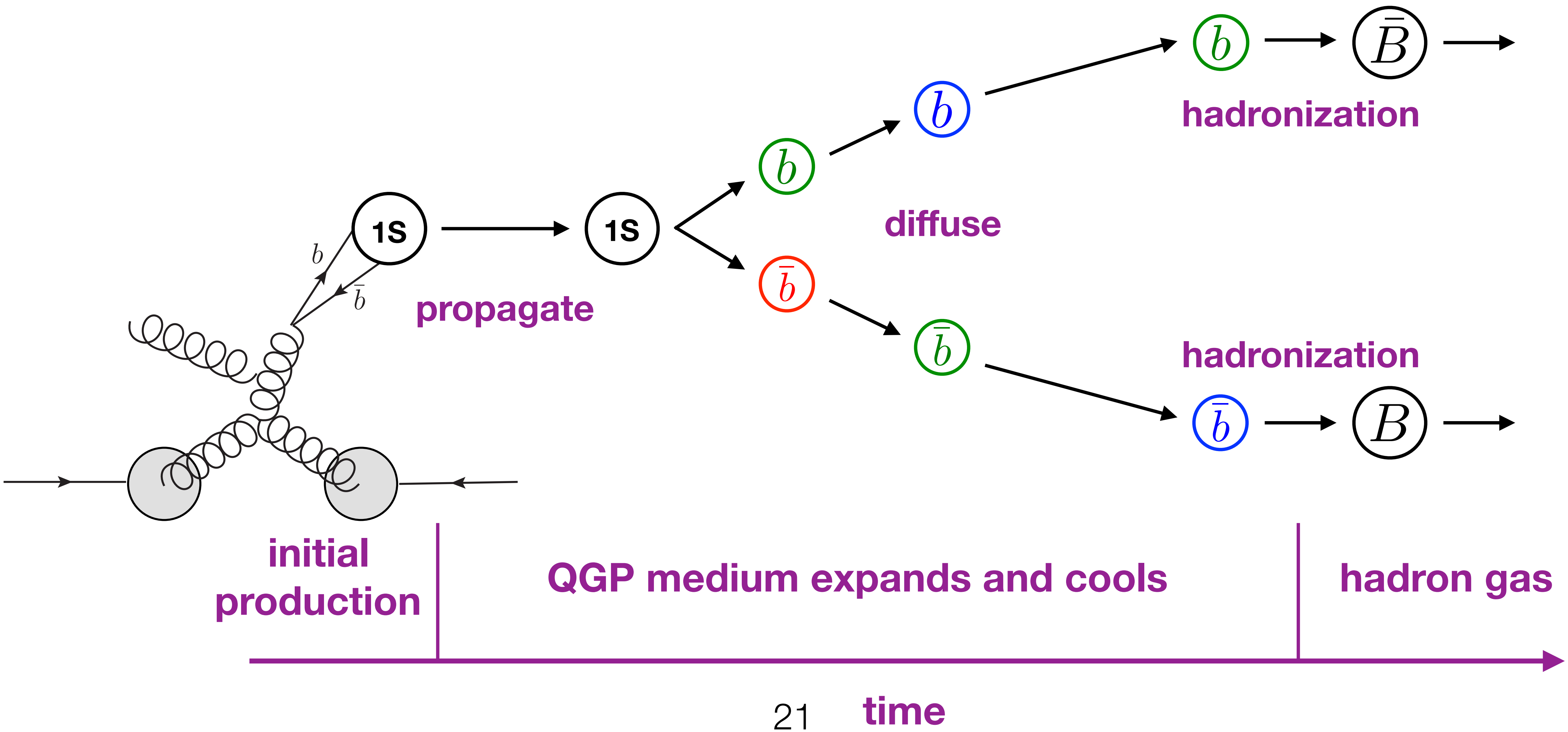
$$M \gg Mv \gg Mv^2 \gtrsim T$$

Coupled with Transport of Open Heavy Flavor

heavy quark $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_Q(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_Q - \mathcal{C}_Q^+ + \mathcal{C}_Q^-$$

anti-heavy quark $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{\bar{Q}}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_{\bar{Q}} - \mathcal{C}_{\bar{Q}}^+ + \mathcal{C}_{\bar{Q}}^-$$

each quarkonium state
nl = 1S, 2S,1P etc. $$(\frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla_{\boldsymbol{x}}) f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_{nls}^+ - \mathcal{C}_{nls}^-$$



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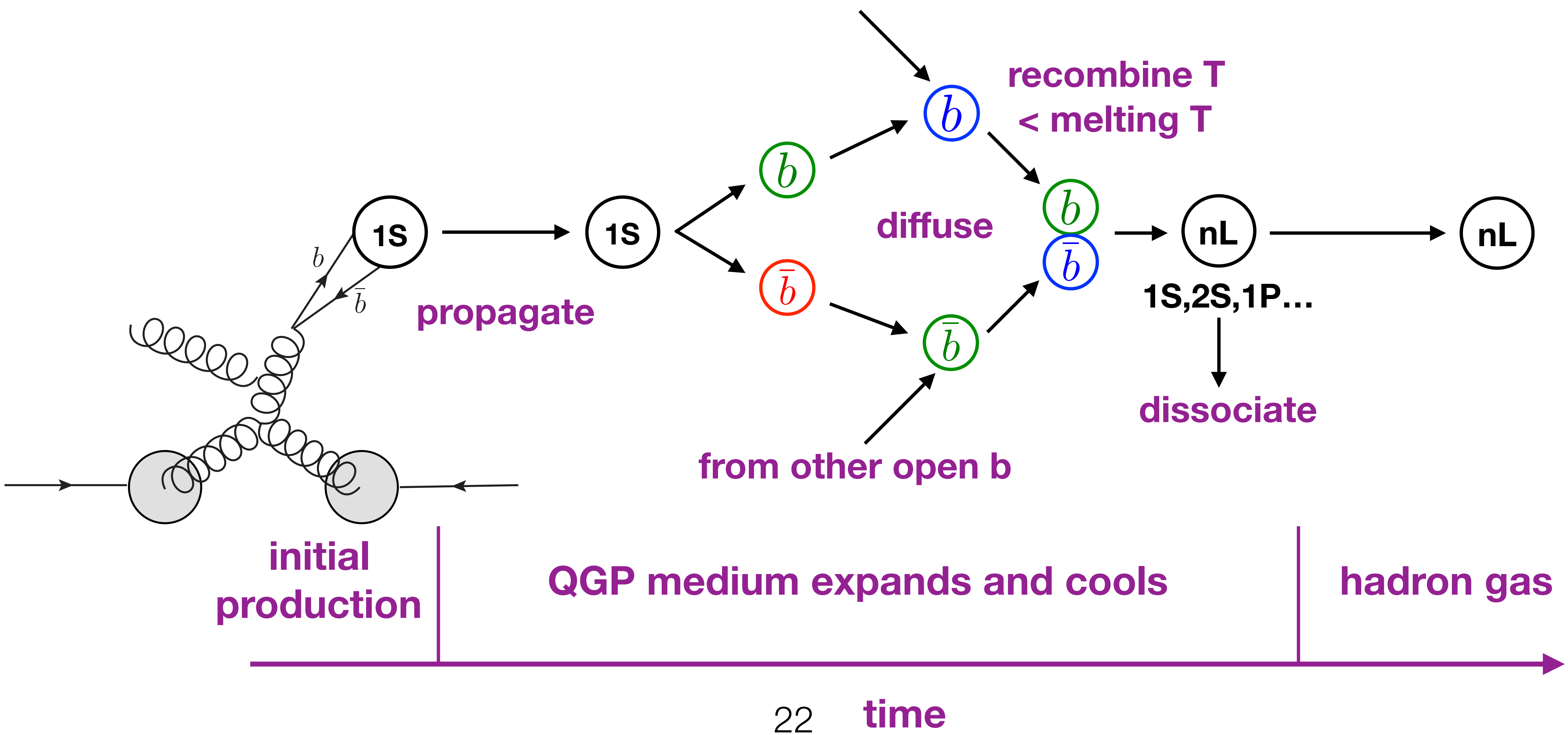
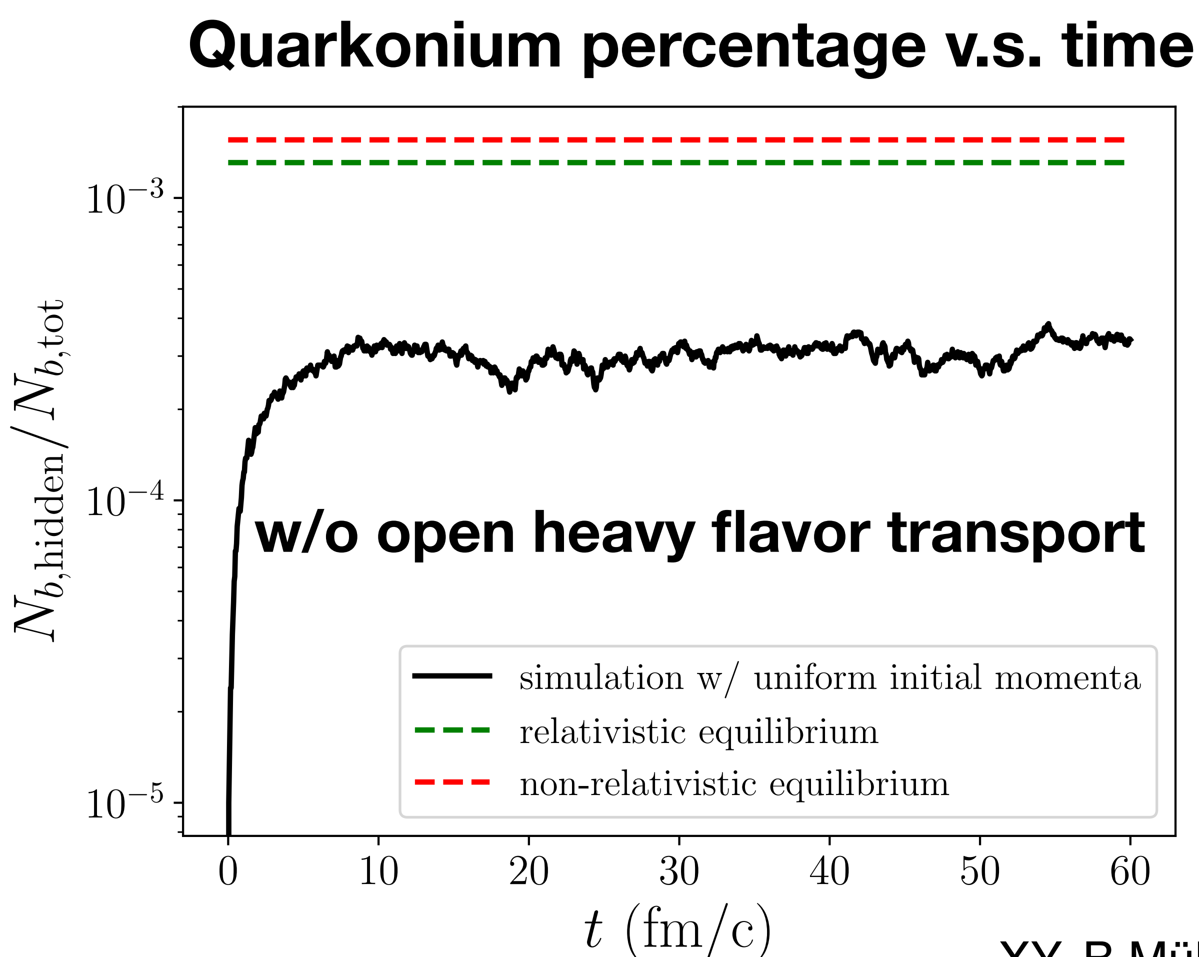
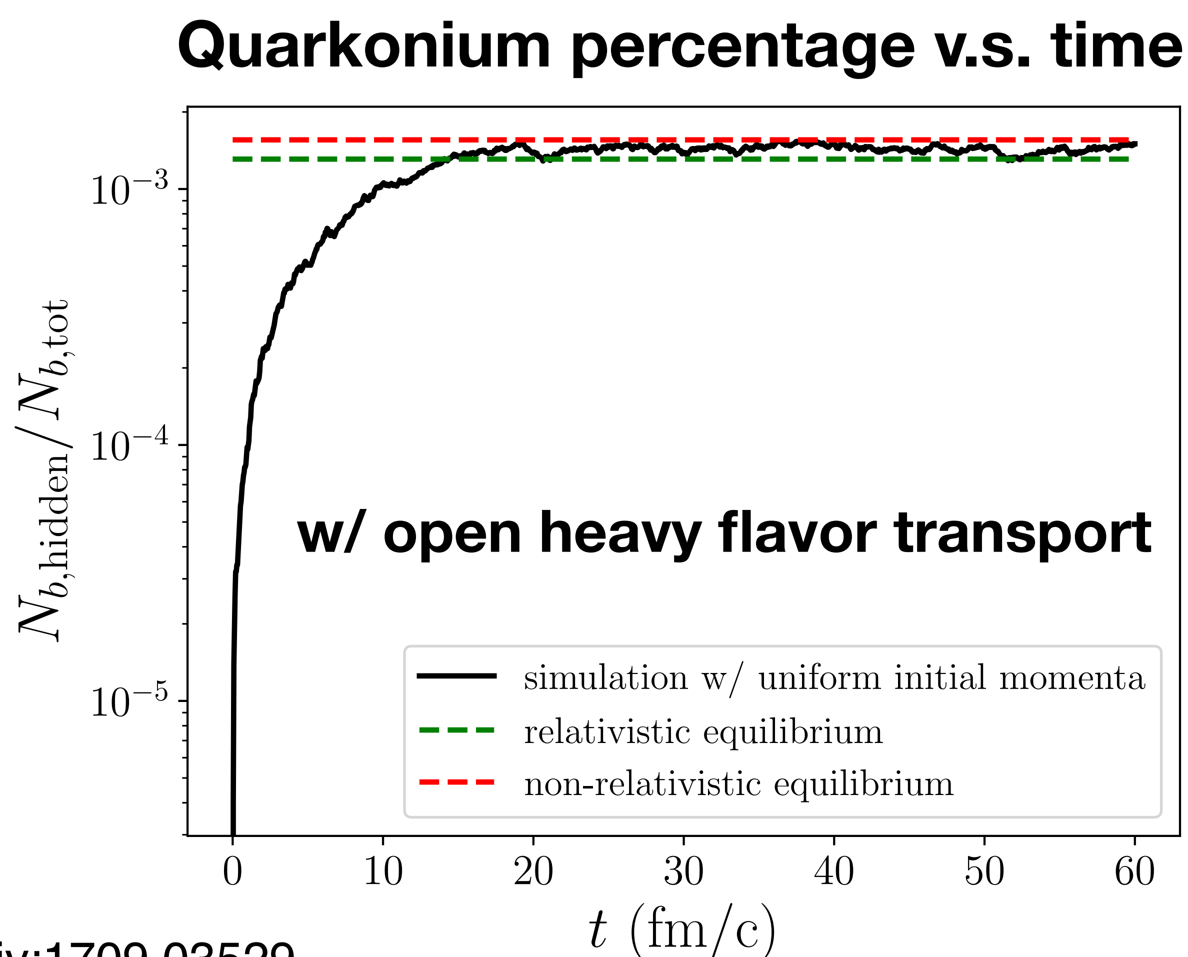


Detailed Balance and Thermalization

Setup:
- QGP box w/ const T=300 MeV, 1S state & b quarks, total b flavor = 50 (fixed)
- Initial momenta sampled from uniform distributions 0-5 GeV
- Turn on/off open heavy quark transport





XY, B.Müller arXiv:1709.03529

Dissociation-recombination interplay drives to detailed balance

Heavy quark energy gain/loss necessary to drive kinetic equilibrium of quarkonium

Collision Event Simulation

- Initial production:

  PYTHIA 8.2: NRQCD factorization

  Sjostrand, et al, Comput. Phys.Commun.191 (2015) 159
  Bodwin, Braaten, Lepage Phys. Rev. D 51, 1125 (1995)

  Nuclear PDF: EPS09 (cold nuclear matter effect) Eskola, Paukkunen, Salgado, JHEP 0904 (2009) 065

  Trento, sample position, hydro. initial condition

  Moreland, Bernhard, Bass, Phys. Rev. C 92, no. 1, 011901 (2015)

- Medium background: 2+1D viscous hydrodynamics (calibrated)

  Song, Heinz, Phys.Rev.C77,064901(2008)
  Shen, Qiu, Song, Bernhard, Bass, Heinz, Comput. Phys. Commun.199,61 (2016)
  Bernhard, Moreland, Bass, Liu, Heinz, Phys. Rev. C 94,no.2,024907(2016)

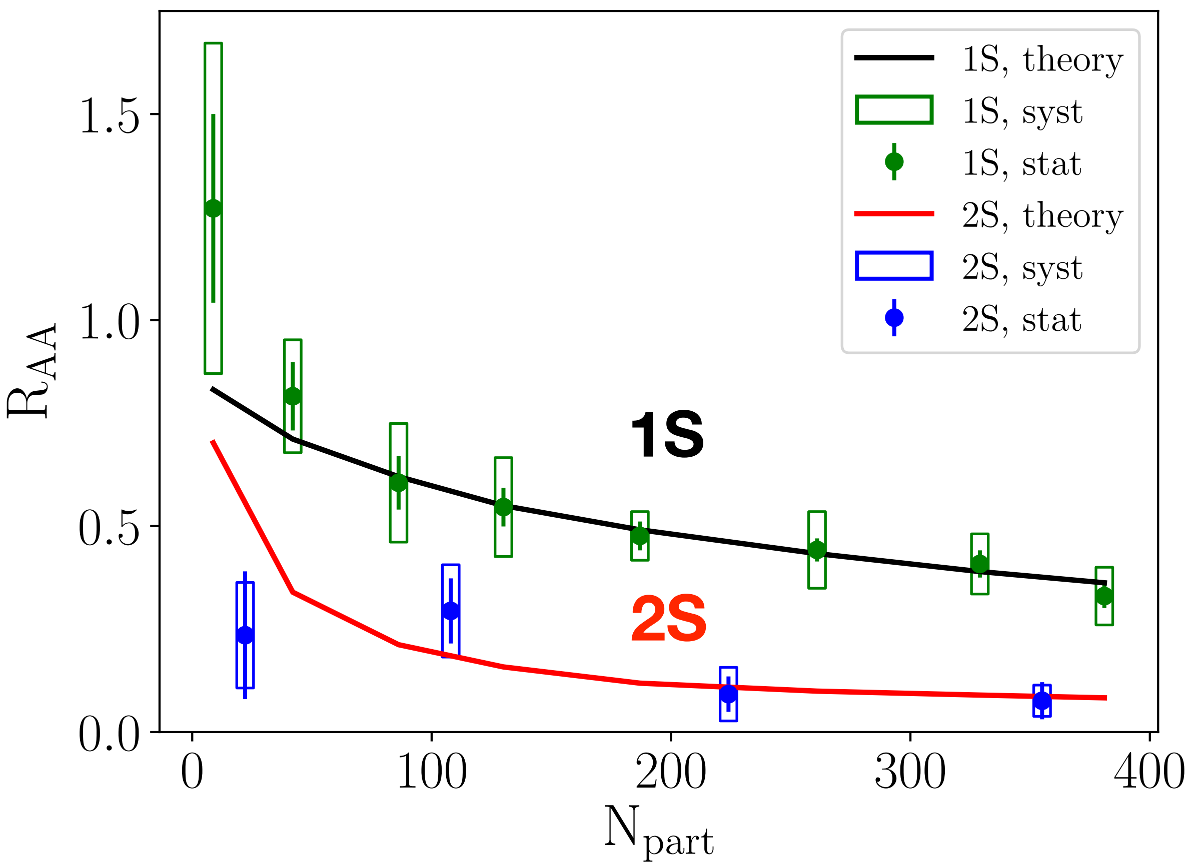
- Study bottomonium (larger separation of scales); include 1S 2S; ~26% 2S feed-down to 1S in hadronic phase (from PDG); initial production ratio 1S : 2S ~ between 3:1 to 4:1 (PYTHIA)

Upsilon in 2760 GeV PbPb Collision

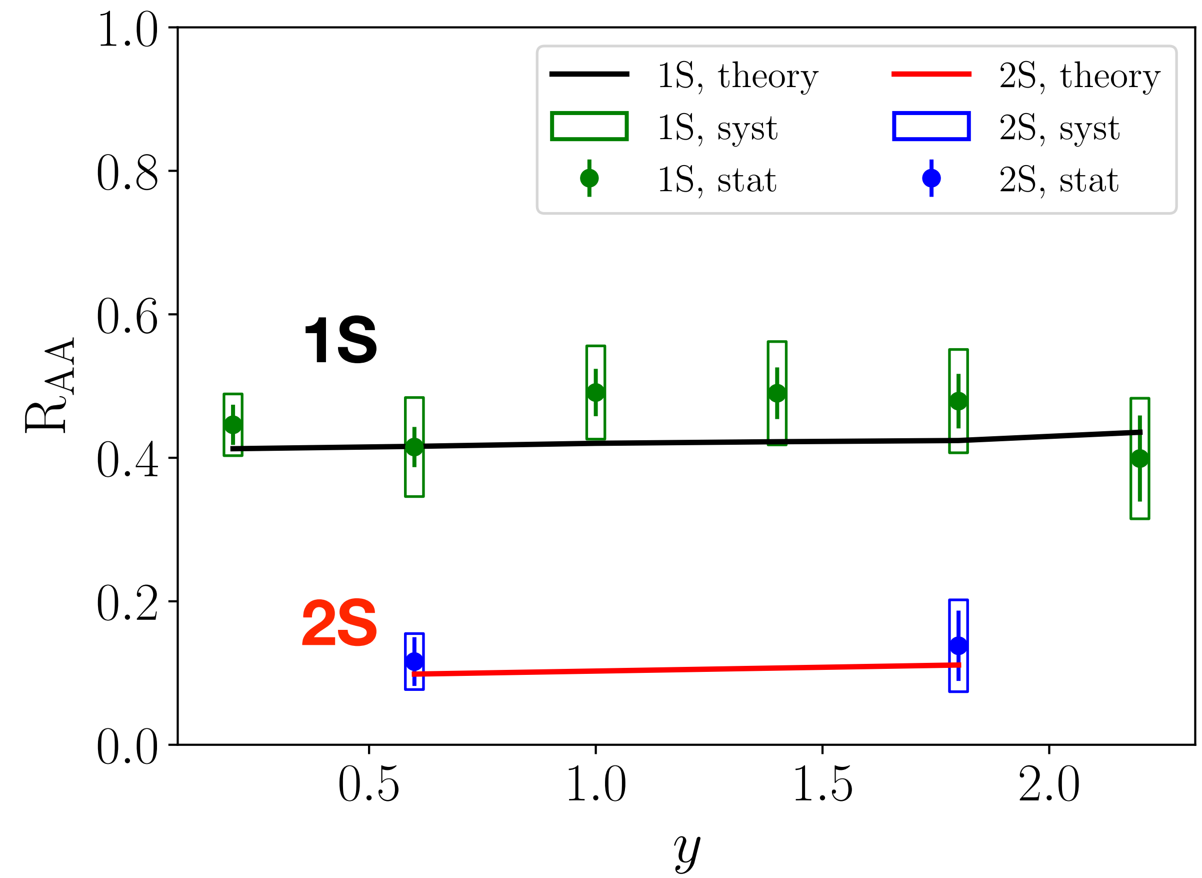
Fix $\alpha_s = 0.3$

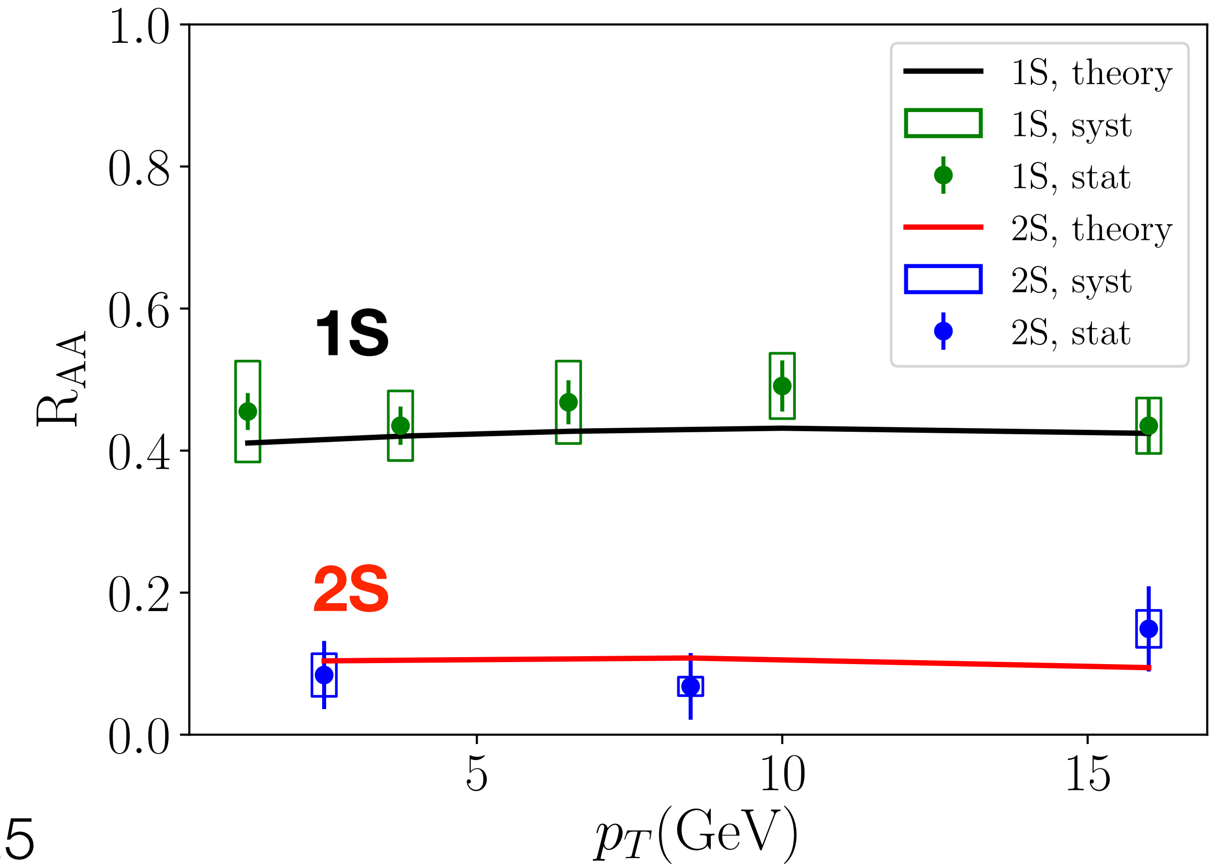
Tune $T_{\mathrm{melt}}(2S) = 210\ \mathrm{MeV}$

Tune $V_s = -C_F \frac{0.42}{r}$



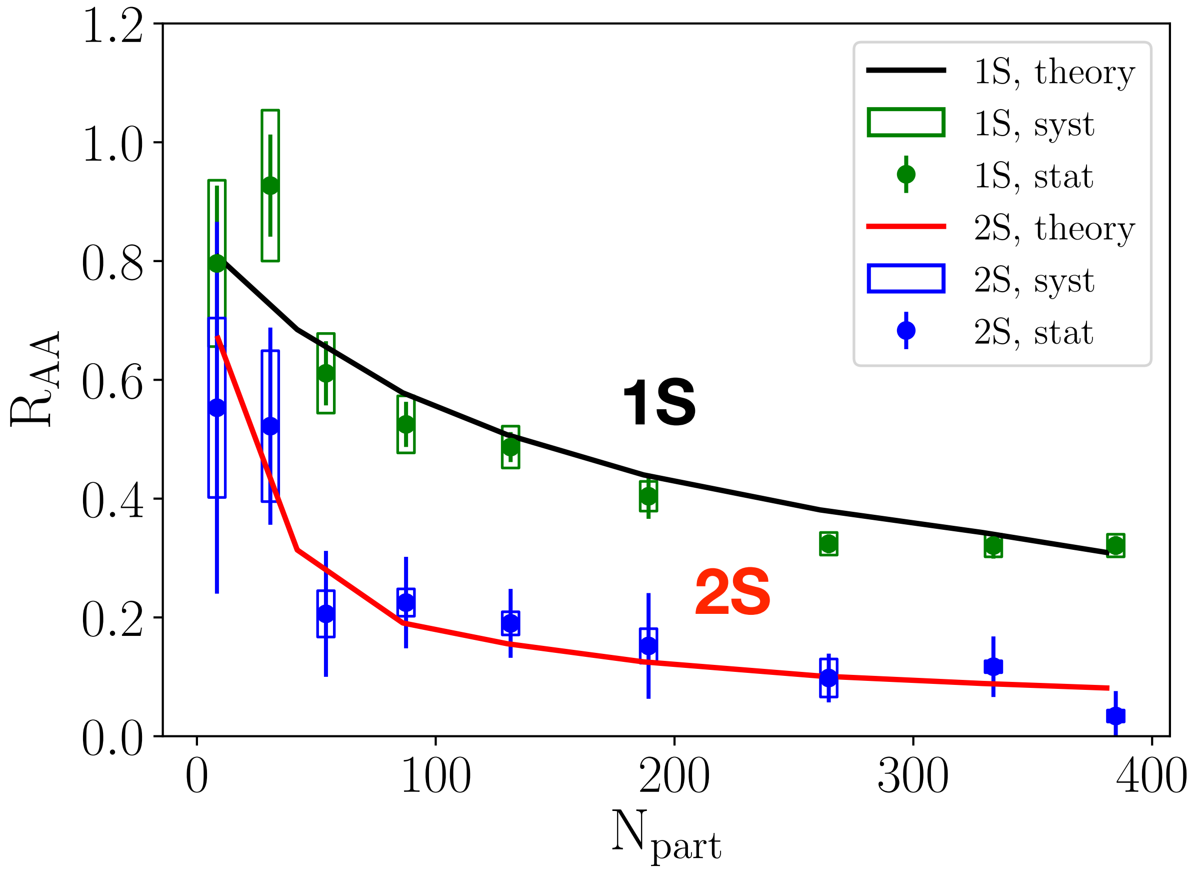
CMS Phys.Lett. B
770 (2017) 357-379



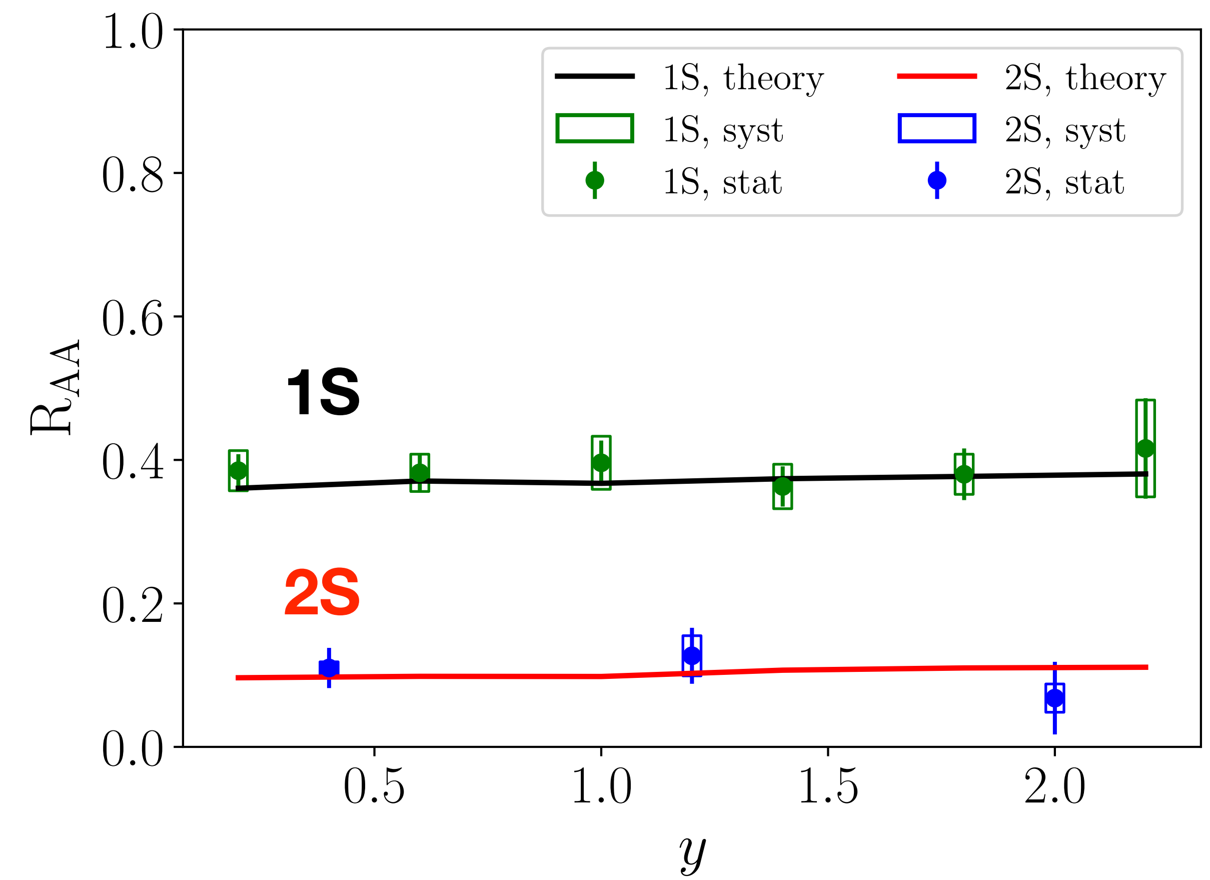


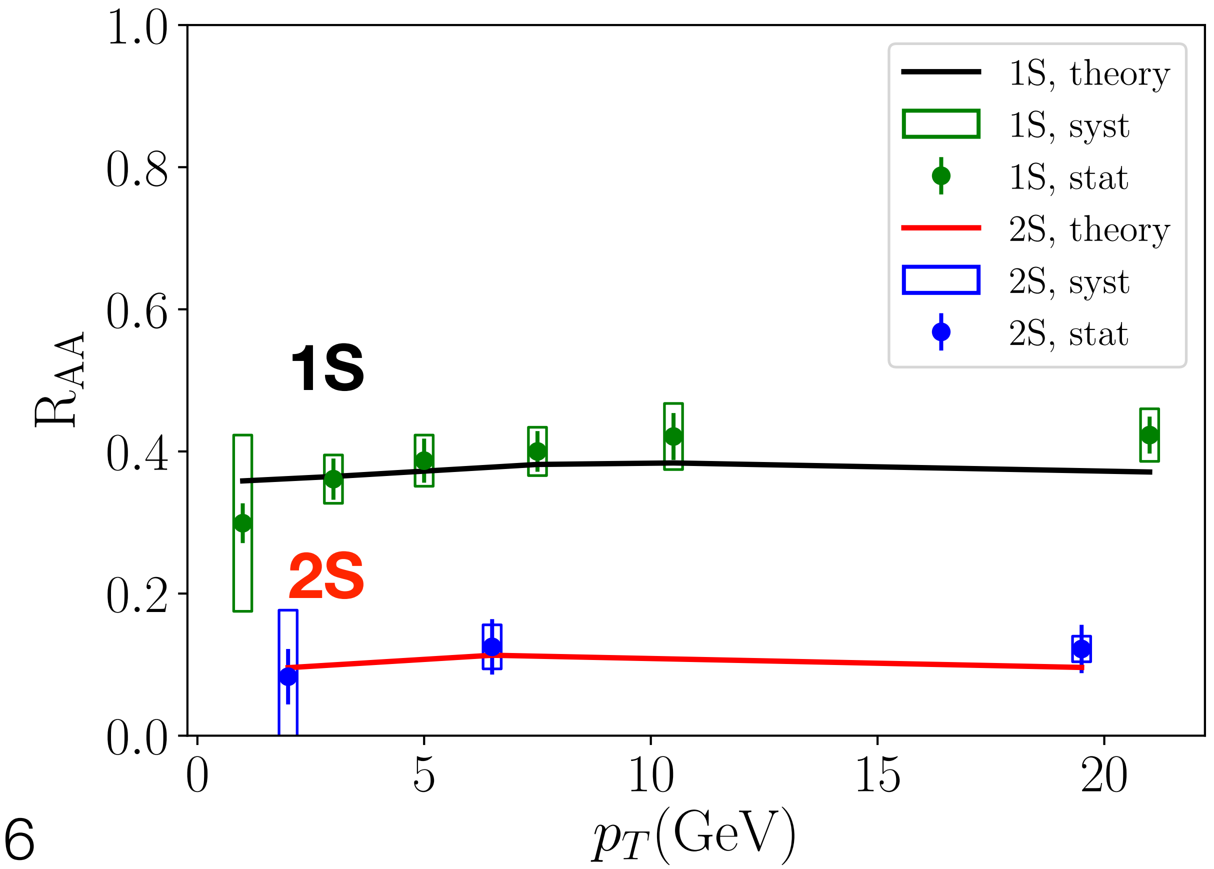
Upsilon in 5020 GeV PbPb Collision

Use same set of parameters



CMS arXiv:1805.09215

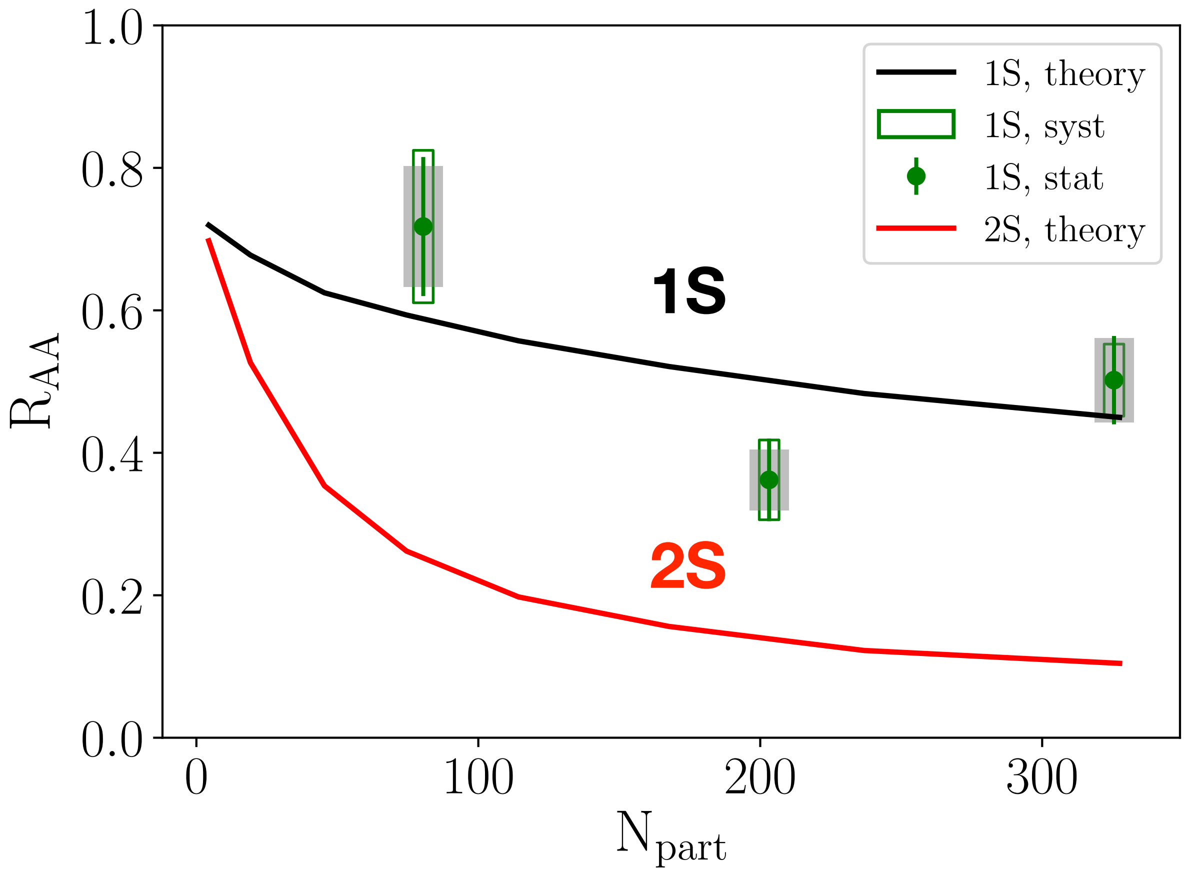


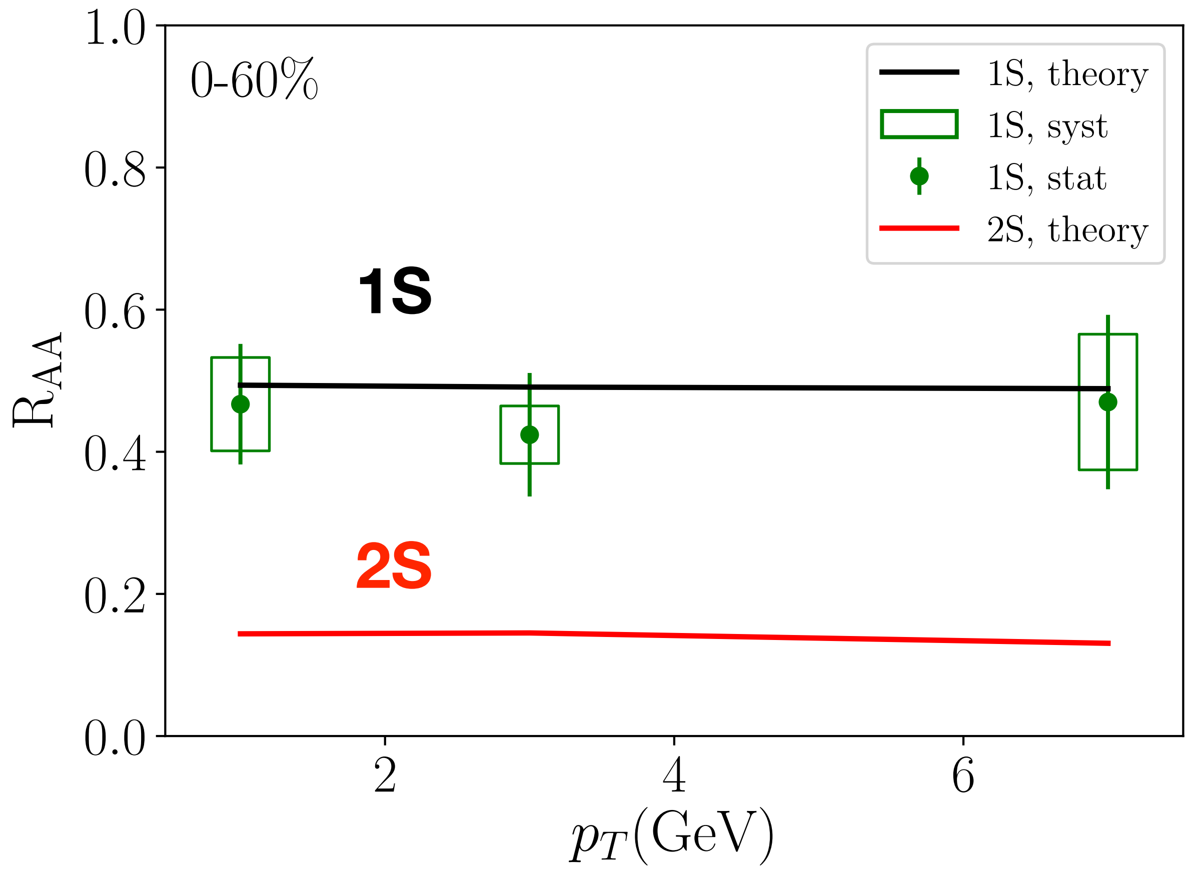


Upsilon in 200 GeV AuAu Collision

Use same set of parameters

Cold nuclear matter effect ~ 0.72 (use p-Au data of STAR)



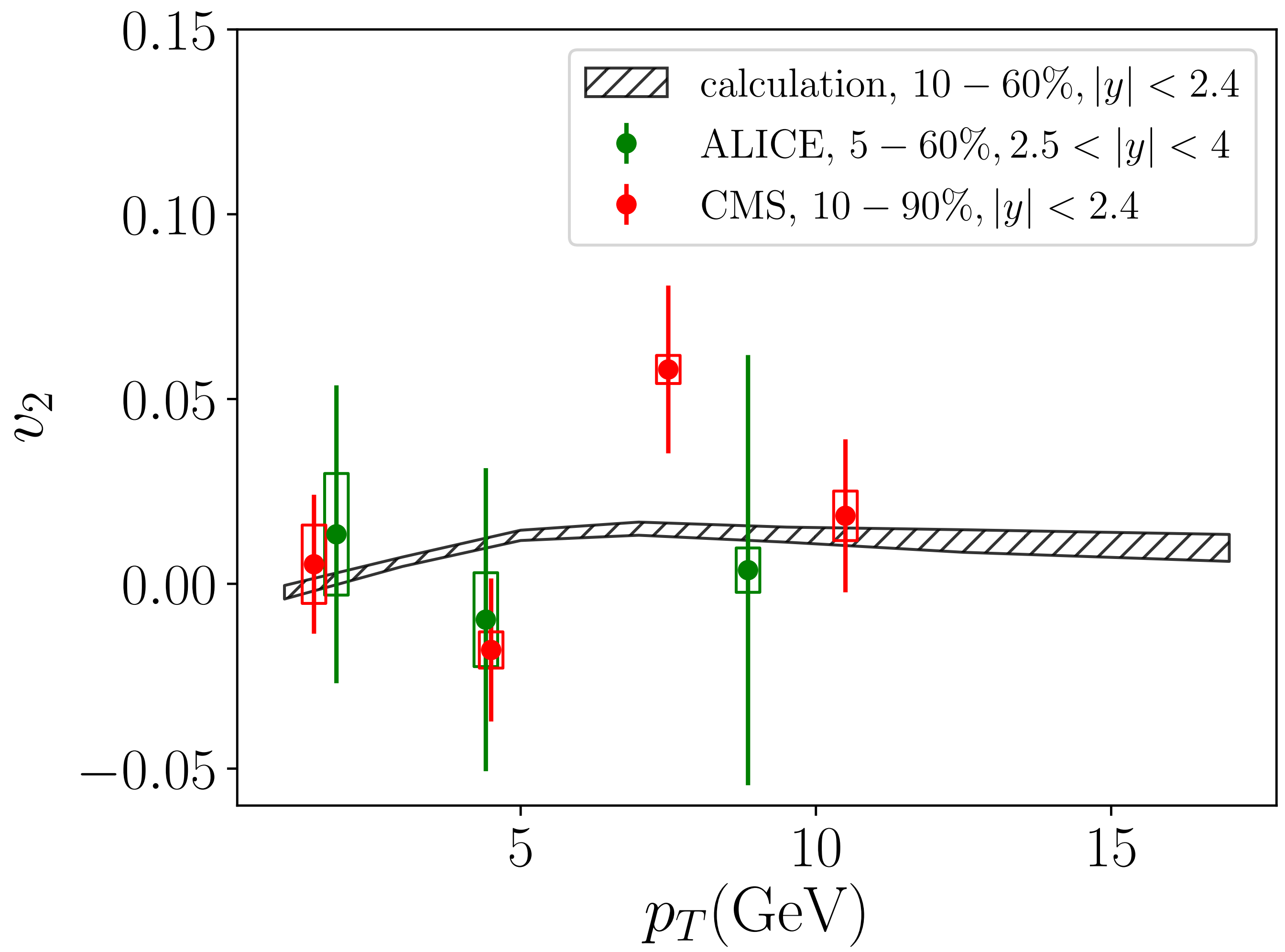


STAR measures 2S+3S

STAR Talks at QM 17&18

Upsilon(1S) Azimuthal Anisotropy in 5020 GeV PbPb

$$E\frac{d^3N}{dp^3} = \frac{1}{2\pi}\frac{d^2N}{p_T dp_T dy}(1 + 2v_2\cos(2\phi) + \cdots)$$



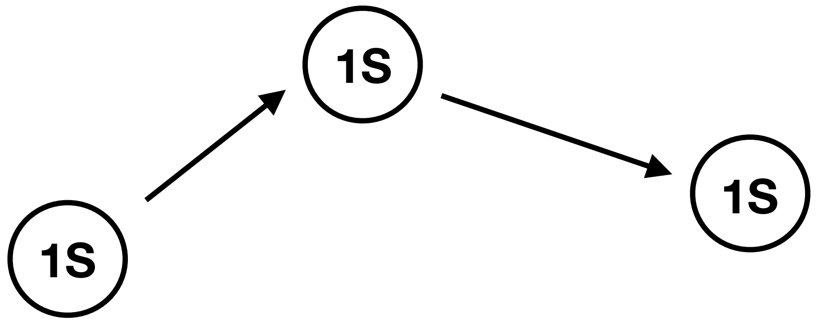
ALICE arXiv:1907.03169
CMS-PAS-HIN-19-002

v2 from path dependence
recombination from uncorrelated b-quarks negligible (different for charm)

Diffusion of Quarkonium

Elastic scattering

$$\frac{\partial}{\partial t} f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) + \boldsymbol{v} \cdot \nabla_{\boldsymbol{x}} f_{nls}(\boldsymbol{x}, \boldsymbol{p}, t) = \mathcal{C}_{nls}^{(+)}(\boldsymbol{x}, \boldsymbol{p}, t) - \mathcal{C}_{nls}^{(-)}(\boldsymbol{x}, \boldsymbol{p}, t) + \boxed{\mathcal{C}_{nls}(\boldsymbol{x}, \boldsymbol{p}, t)}$$



Second order in r : neglected in numerical calculations

Diffusion coefficient: square of momentum transferred per unit time

$$\frac{\kappa}{T^3} < 0.1$$

XY, B.Müller arXiv:1811.09644

Conclusion

- Open quantum system approach for quarkonium inside QGP
  - Quantum optical limit
    - **Separation of scales explains why transport equation works** $M \gg Mv \gg Mv^2 \gtrsim T$
    - Lindblad equation —> Boltzmann transport equation
  - Quantum Brownian motion limit

Y.Akamatsu, M.Asakawa, A.Rothkopf…
J-P Blaizot, M.A.Escobedo…
N.Brambilla, M.A.Escobedo, A.Vairo…
R.Katz, P-B Gossiaux…

- Phenomenological results from **coupled transport equations**
- Future: add 1P, 2P, 3S with more complete feed-down network

XY W.Ke Y.Xu S.Bass B.Müller, in preparation

Quantum Brownian Motion Limit

Quantum evolution of heavy quark pair density matrix (not necessarily Lindblad)

$$\frac{d}{dt}\rho_{Q\bar{Q}}(t) = -i\left[H_{Q\bar{Q}}, \rho_{Q\bar{Q}}\right] + \sum_{i=1}^{N_{LB}} \gamma_i \left(\hat{L}_i \rho_{Q\bar{Q}} \hat{L}_i^\dagger - \frac{1}{2}\hat{L}_i \hat{L}_i^\dagger \rho_{Q\bar{Q}} - \frac{1}{2}\rho_{Q\bar{Q}} \hat{L}_i \hat{L}_i^\dagger\right)$$

Only real potential — **Encoding imaginary potential**

Quantum state diffusion method

Directly solve density matrix evolution

Saclay: color singlet and octet

J-P Blaizot, M.A.Escobedo…

TUM: pNRQCD, color singlet and octet

N.Brambilla, M.A.Escobedo, A.Vairo…

Map onto Langevin-like Schrödinger

$$i\frac{\partial}{\partial t}\psi_{Q\bar{Q}} = (H_{Q\bar{Q}} - i\gamma + \xi)\psi_{Q\bar{Q}}$$

Dissipation — **Fluctuation**

Nantes R.Katz, P-B Gossiaux…

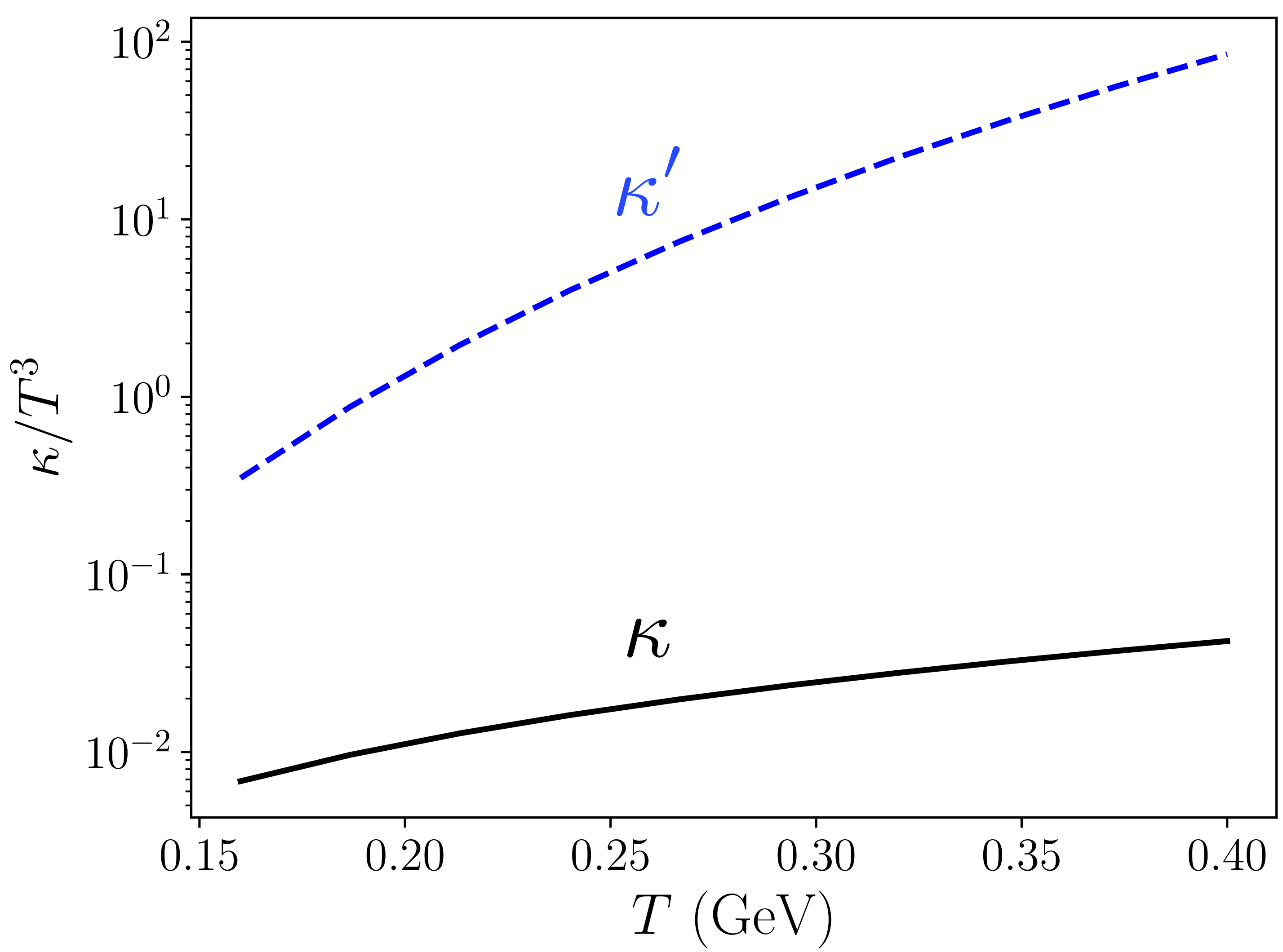
Stavanger/Osaka

Y.Akamatsu, M.Asakawa, A.Rothkopf…

Diffusion Coefficient of Upsilon(1S)

$$\kappa = \frac{32}{729\pi^5}\alpha_s^2 \int dq\, q^8 n_B(q)(1+n_B(q))\left(\mathcal{P}\int dp_{\rm rel} \frac{p_{\rm rel}^2 |\langle \Psi_{\boldsymbol{p}_{\rm rel}}|\boldsymbol{r}|\psi_{nl}\rangle|^2 (|E_{nl} + \frac{\boldsymbol{p}_{\rm rel}^2}{M}|)}{(|E_{nl} + \frac{\boldsymbol{p}_{\rm rel}^2}{M}|)^2 - q^2}\right)^2$$

Neglect q^2 in denominator $\kappa'(1\mathrm{S}) = \frac{T^3(\pi T a_B)^6}{N_c^2}\frac{50176\pi}{1215}\frac{2}{C_F^2}$



arXiv:0808.0957, K.Dusling, J.Erdmenger, M.Kaminski, F.Rust, D.Teaney and C.Young

Expansion in q asymptotic