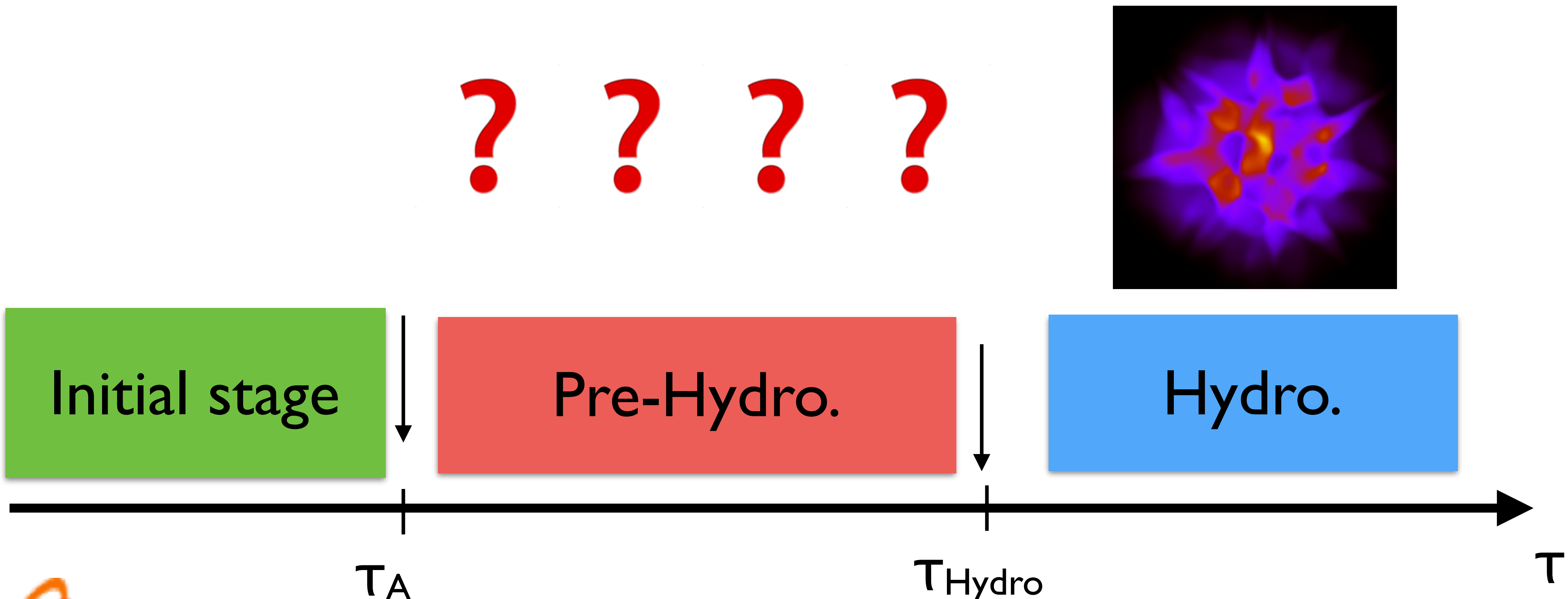
"Prehistory" of hydrodynamics in rapidly-expanding quark-gluon plasma



Yi Yin

Jasmine Brewer (MIT), Li Yan (Fudan U.) and YY, 1910.00021;
Weiyao Ke (LBNL) and YY, in progress.

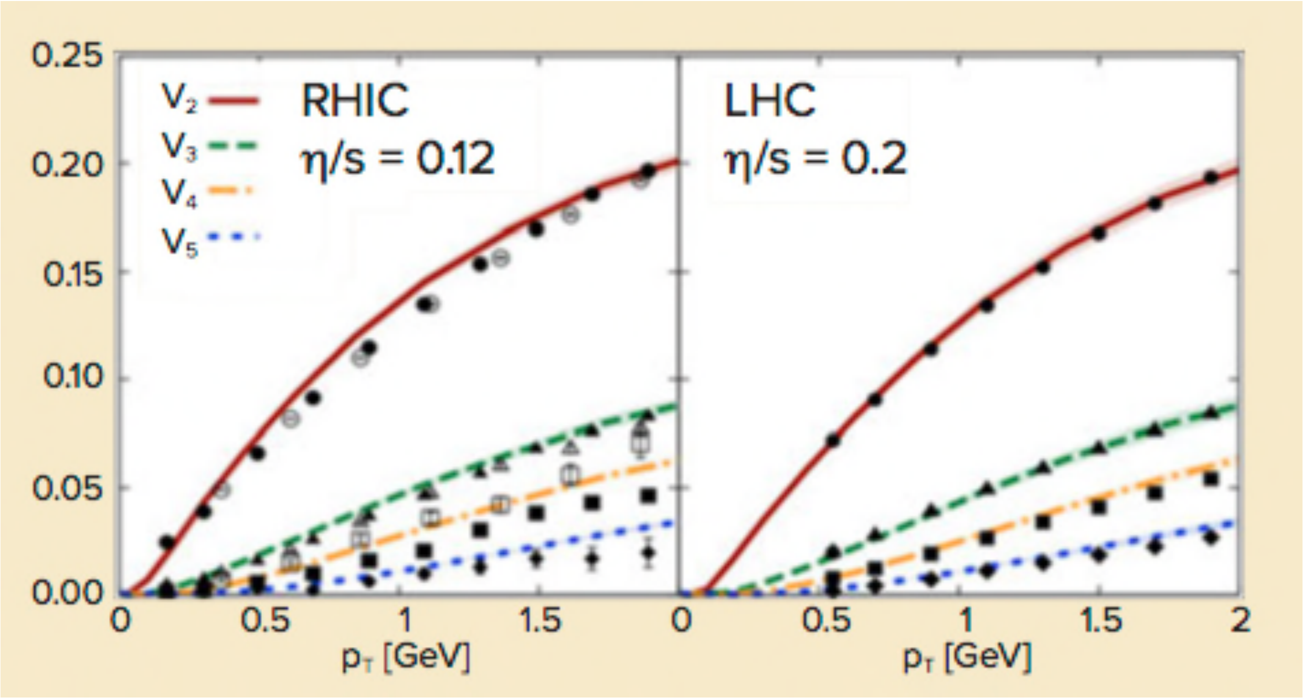
HEPNIC, Feb.13, 2020

Brewer, Grad. of MIT

Best wishes to the colleagues and students whose regular lives are impacted by COVID-19.

We should keep our passion to achieve scientific goals regardless of any difficulties.

Heavy-ion collisions create QGP liquid



(Hadron spectrum vs hydro. simulations by McGill Group)

Hydro. provides a quantitative description of QGP evolution in heavy-ion collisions (HIC).

This means after some time scale, τ_{Hydro} , the QGP becomes a fluid.

The discovery of QGP liquid in turn raises a number of deep and outstanding questions

The QCD phase diagram:

how does the properties of QGP liquid change as baryon density increases?

The emergence of QGP liquid.

How does strongly coupled fluid emerge from the asymptotic free QCD as resolution scale decreases?

Hydrodynamization: how does the fluid-like behavior emerges from highly an-isotropic and non-equilibrium quark-gluon matter at early time?.

see Romatschke-Romatschke, 1712.05815; Florkowski et al, Rept. Prog. Phys, for recent review.

This talk: proposing a new hydrodynamization scenario based on the notion of "the least relaxable state" and "adibaticity".

The least relaxable state

To motivate the idea, we consider the single particle distribution which is described by the standard kinetic equation

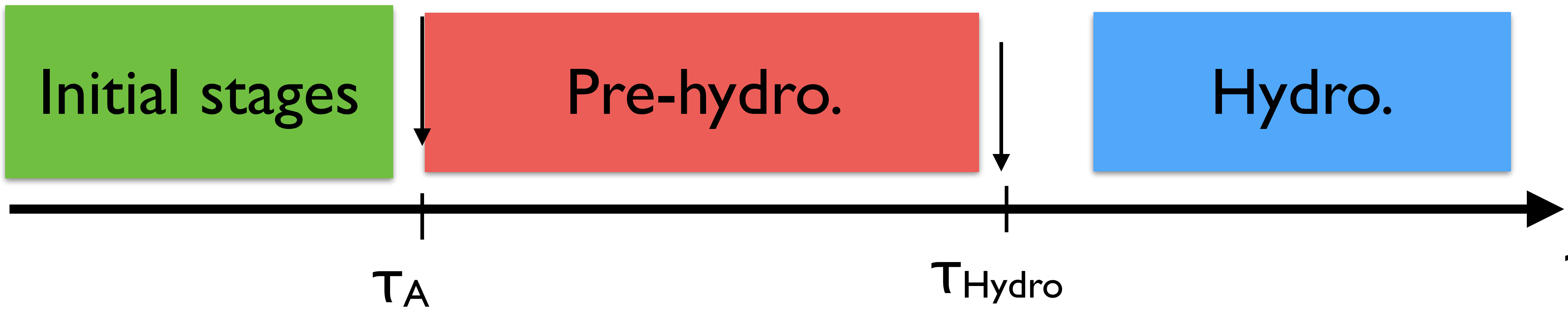
$$\partial_t f(t, \overrightarrow{x}, \overrightarrow{p}) = -\left(\overrightarrow{v}\partial_{\overrightarrow{x}} - \overrightarrow{F}\partial_{\overrightarrow{p}}\right) f(t, \overrightarrow{x}, \overrightarrow{p}) - \hat{C}[f]$$

In the absence of external force or expansion, the equilibrium distribution f_{eq} has the minimum change rate (zero): $\hat{C}[f_{eq}] = 0$

In the presence of external force or expansion, we define the distribution with the lowest possible change rate as the least relaxable distribution.

In general, one may define the state with the lowest possible change rate as *the least relaxable state*, which generalized the concept of the equilibrium to dynamical environment. Other states will be refered as "faster state"

A new scenario: "adiabatic hydrodynamization"



During the interval $\tau_A < \tau < \tau_{Hydro}$, there is a pre-hydro stage when the medium is (locally) in "the least relaxable state" and gradually evolves into hydro. stage.

The least relaxable state is in analogous to the instantaneous ground state of quantum mechanics.

The evolution of pre-hydro. stage is then in analogous to the adiabatic evolution of a quantum mechanical system governed by a time-dependent Hamiltonian.

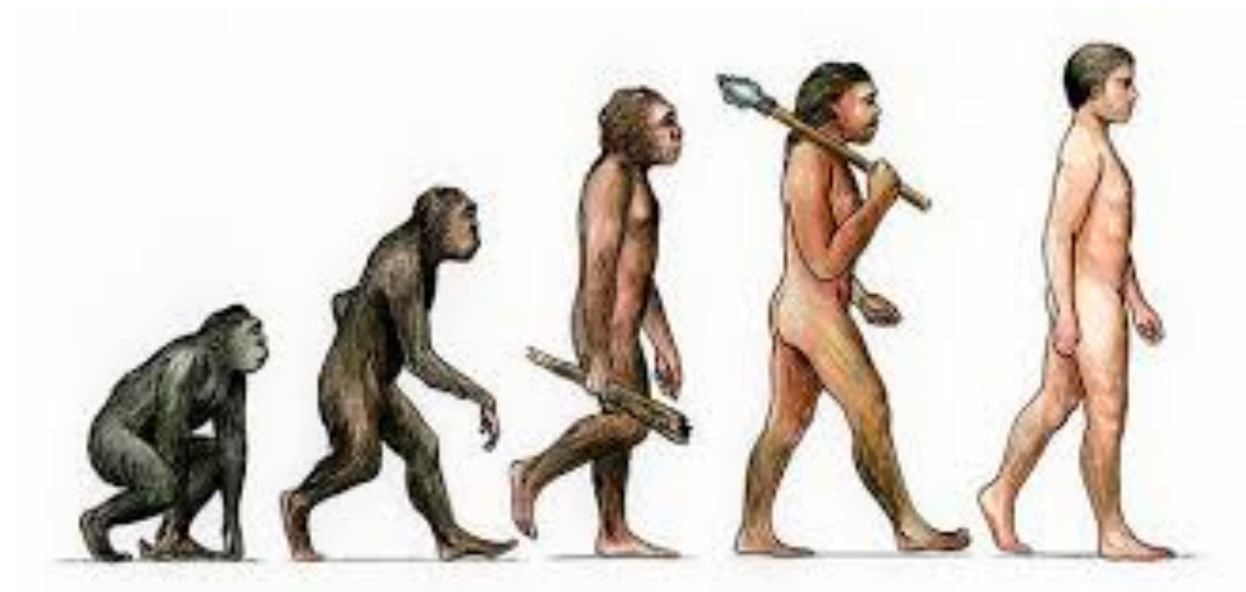
Pre-hydro modes and pre-hydrodynamic response.

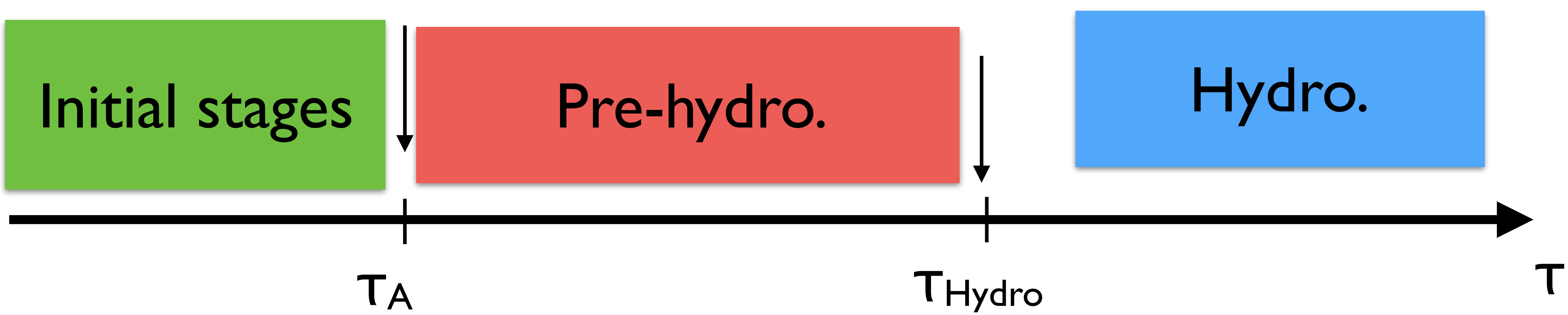
For a fluid, the response function in low frequency and low momentum limit are fully determined by the hydrodynamic modes (sound, diffusive modes).

We expect that the response of the medium during the pre-hydro. stage to external disturbance is described by a set of collective excitations, which will be referred as “pre-hydro. modes”. They are analogous to, but distinguishable from hydro. modes.

The collective modes would in general depend on the state of the medium. (E.g. in normal Fermi liquid, zero sounds have different physical characteristic than ordinary sounds)

A cartoonish summary: pre-hydro. c.f. pre-existing type of human.





In the remainder of this talk, we will demonstrate adiabatic hydrodynamization using a concrete microscopic description of expanding QGP .

The identification of "the least relaxable state" and the test of adiabatic hydrodynamization.

Longitudinal expansion and the emergence of τ_A .

The adiabaticity in the rapidly-expanding QGP

The hints for the pre-hydro. modes (or how mach-cone and diffusive wake is modified in pre-hydro. stage)

Discussion and outlook.

The least relaxable state (distribution) in kinetic theory

The microscopic model: kinetic theory

We use kinetic equation to describe the evolution of single particle distribution of gluons (quarks).

$$\partial_\tau f(\overrightarrow{p},\tau) = -\frac{p_z}{\tau}\partial_{p_z} f(\overrightarrow{p},\tau) - \hat{C}[f]$$

Simplification: assuming boost-invariance and homogeneity in transverse plane. (will be relaxed later)

The kinetic theory model captures the physics of the transition from free-streaming quark-gluon gas to quark-gluon liquid.

Limitation: assuming weak coupling throughout the evolution.

See Florkowski et al, Rept. Prog. Phys for references on studies based on holography.

Angle distribution:

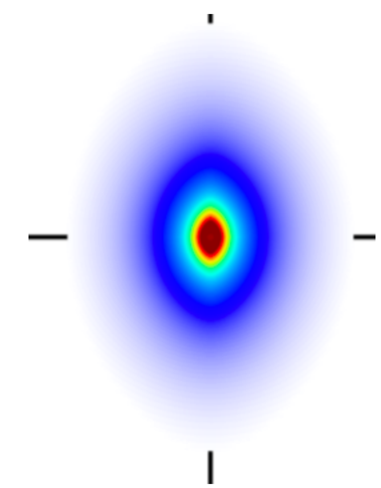
We introduce the angle distribution (ctanθ=p_z/p) function for gluons carrying most of the energy

$$F_\epsilon(\cos\theta) \equiv \frac{1}{2\pi^2}\int_0^\infty dp\, p^3\, f(p,\theta,\tau)\,, \qquad \epsilon = \int_{-1}^{1} d\cos\theta F_\epsilon(\cos\theta)$$

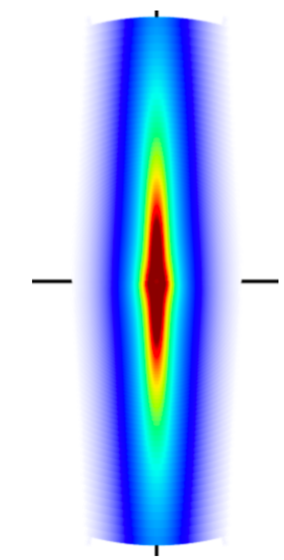
Consider the multipole expansion of F_ε

$$F_\epsilon(\cos\theta) = \epsilon(\tau) + \sum_{n=1} \frac{4n+1}{2} L_n(\tau) P_{2n}(\cos\theta)$$

$$\psi = \left(\epsilon, L_1, L_2, \ldots,\right)$$



$L_1/\varepsilon \ll 1$



$L_1/\varepsilon \sim 1$

(Figs from Kurkela et al, PRC 19')

Studying the evolution of ψ is equivalent to the studying that of angle distribution F_ε.

The evolution equation for ψ

$$\partial_y f(\vec{p},\tau) = -\frac{p_z}{\tau}\partial_{p_z} f(\vec{p},\tau) - \tau\,\hat{C}[f]$$

$$y \equiv \log(\tau/\tau_I)$$

a more convenient temporal variable

$$\times \int dp p^3$$

$$\partial_y F = \left(-4\cos^2\theta + \sin^2\theta\cos\theta\frac{1}{\partial\cos\theta}\right) F - \text{collisions}$$

Multipole expansion

$$\partial_y \psi = -H_F\,\psi - \text{ collisions}\,.$$

$$H_F = \begin{pmatrix} 4/3 & 2/3 & \ldots\ldots \\ 8/15 & 38/21 & \ldots\ldots \\ \ldots & \ldots & \ldots\ldots \end{pmatrix}$$

In what follows: we will consider the class of collision integrals that equation for ψ can be recast into the form:

$$\partial_y \psi = -\left(H_F + H_C(y)\right)\ \psi = -H(y)\,\psi\,.$$

The identification of the least relaxable mode

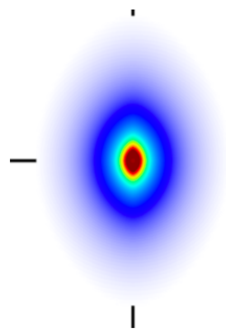
We consider the instantaneous eigenvalue of non-Hermitian matrix H(y) ordered by the real part (or the absolute value) of the eigenvalue

$$H(y)\phi_n(y) = E_n(y)\phi_n(y) \qquad \mathrm{Re}E_0(y) < \mathrm{Re}E_1(y) \leq \ldots$$

We identify instantaneous g.s. state as ϕ_0 the least relaxable state and refer E_0 as the minimum change rate.

In long time limit, the instantaneous ground state (g.s.) $\phi_0(y)$ will approach equilibrium:

$$f(y \to \infty) \to f_{\rm eq} \qquad \phi_0(y \to \infty) \to \phi_{eq} = (\epsilon, 0, \ldots)$$



The problem of hydrodynamization within the current model is reduced to understanding how ψ evolve into ϕ_{eq} for as described by

$$\partial_y \psi = -H(y)\,\psi\,.$$

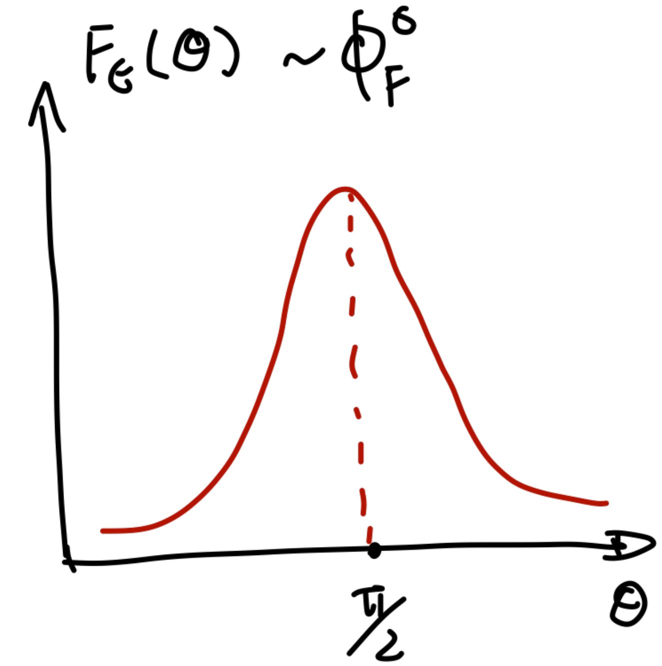
"Adiabatic hydrodynamization": $\psi(y) \sim \phi_0(y)$

The least relaxable state at early time limit and longitudinal expansion

Consider the evolution of ψ at very early time $\tau \ll \tau_{Coll}$

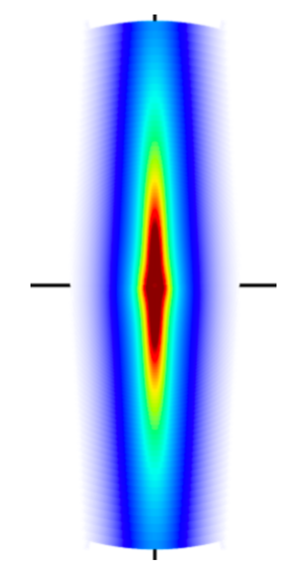
$$\partial_y \psi = -\left(H_F + H_{Coll}\right)\,\psi \approx -H_F \psi\,.$$

The least relaxable state at early times, ϕ_F, represents the most an-isotropic distribution (angle distribution sharply peak at $\theta=\pi/2$, i.e., characteristic p_z is much smaller than $p_\perp$.)



At free-streaming stage, distribution function f at given time τ can be related to the initial distribution by a boost along longitudinal direction.

c.f. P.Arnold, 0708.0812.



Therefore the less an-isotropic distribution ("faster states") evolves faster than ϕ_F.

The system is expected to reach the least relaxable state at early times.

There must exist an emergent scale τ_A around which the evolution is dominant by ϕ_F if $\tau_I \ll \tau_{\rm Coll}$

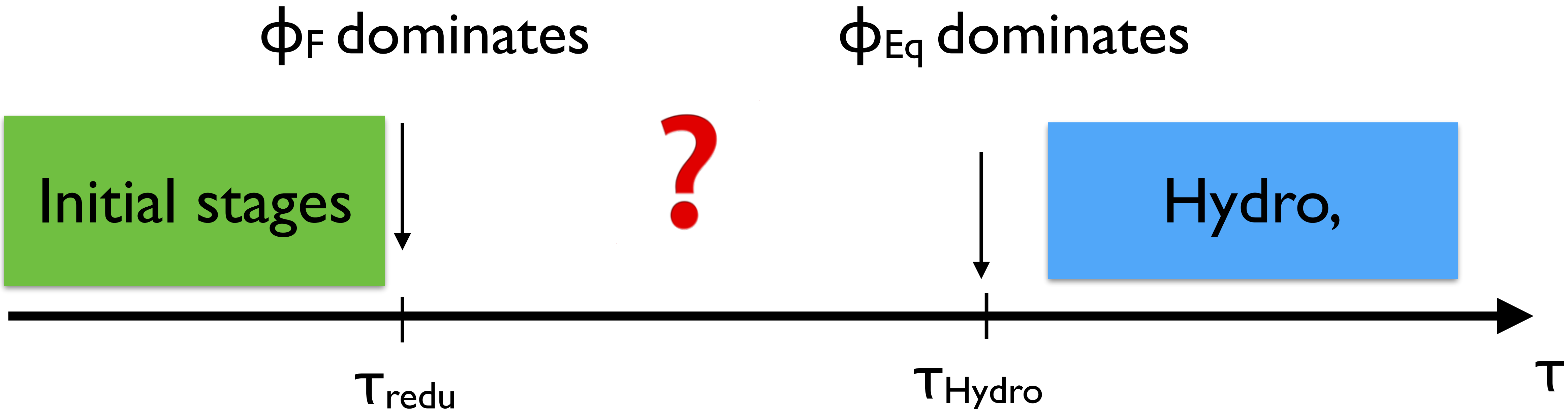
$$\lim_{\tau/\tau_I \to \infty} \lim_{\tau_{\rm Coll}/\tau \to 0} \psi(\tau) \sim \phi_0^F .$$

Indeed, the contribution from "faster state" will decay as power-law in τ:

$$\psi(\tau) = b_0(\tau)\phi_0^F + \sum_{n=1} b_n(\tau)\phi_n^F \qquad \frac{b_n(\tau)}{b_0(\tau)} \sim \frac{e^{-E_n^F y}}{e^{-E_0^F y}} \sim \left(\frac{\tau}{\tau_I}\right)^{-(E_n^F - E_0^F)},$$

Interestingly, τ_I and $\tau_{\rm Coll}$ is indeed separated parametrically for weakly coupled QGP in high energy density limit

$$\tau_I \sim Q_s^{-1} \ll \tau_{\rm coll} \sim \alpha_s^{-x} Q_s^{-1}, \qquad x > 0$$



The system reaches the least relaxable state at very early time for expanding QGP. This is due to not-trivial properties of QCD which describes the interaction among gluons (and quarks) and the non-trivial way that QGP is created in heavy-ion collision.

Next, how does ψ evolve from ϕ_F to ϕ_{eq} ?

We shall consider and demonstrate the scenario of adiabatic evolution, i.e. $\psi(y) \sim \phi_0(y)$

The test of Adiabatic Hydrodynamization

The least relaxable state and the properties of the medium.

The bulk properties of the pre-equilibrium medium can be related to the properties of the least relaxable state and the minimum change rate E_0 (its eigenvalue) under "adiabatic hydrodynamization".

In particular: consider the most important quantity characterizing the bulk evolution of a boost-invariant plasma undergoing Bjorken expansion, i.e., *the percentage rate of change of the energy density* ε

Since ε is the zeroth component of ψ, we then have the following non-trivial relation under adiabatic hydrodynamization:

$$(-\partial_y \epsilon / \epsilon) \approx E_0(y)$$

Likewise, the ratio $p_L(y)/\varepsilon(y)$ can also be related to the zeroth and first component of $\varphi_0(y)$, i.e. the "direction" of instantaneous g.s. under adiabatic hydrodynamization.

Relaxation time approximation (RTA)

(other studies of RTA model: Denicol& Noronha, 1608.07869; Heller et al, PRD 18'; Blaizot-Li, PLB 18'; Heller& Svensson, PRD 18';…)

We consider collision integral under relation time approximation (RTA):

$$\hat{C}\ \left[f(\overrightarrow{p},\tau)\right] = -\frac{\left(f(\overrightarrow{p},\tau) - f_{\rm eq}(p,\tau)\right)}{\tau_{\rm Coll}}$$

Then from matrix equation,

$$\partial_y \psi = -H_{RTA}(y)\,\psi\,.$$

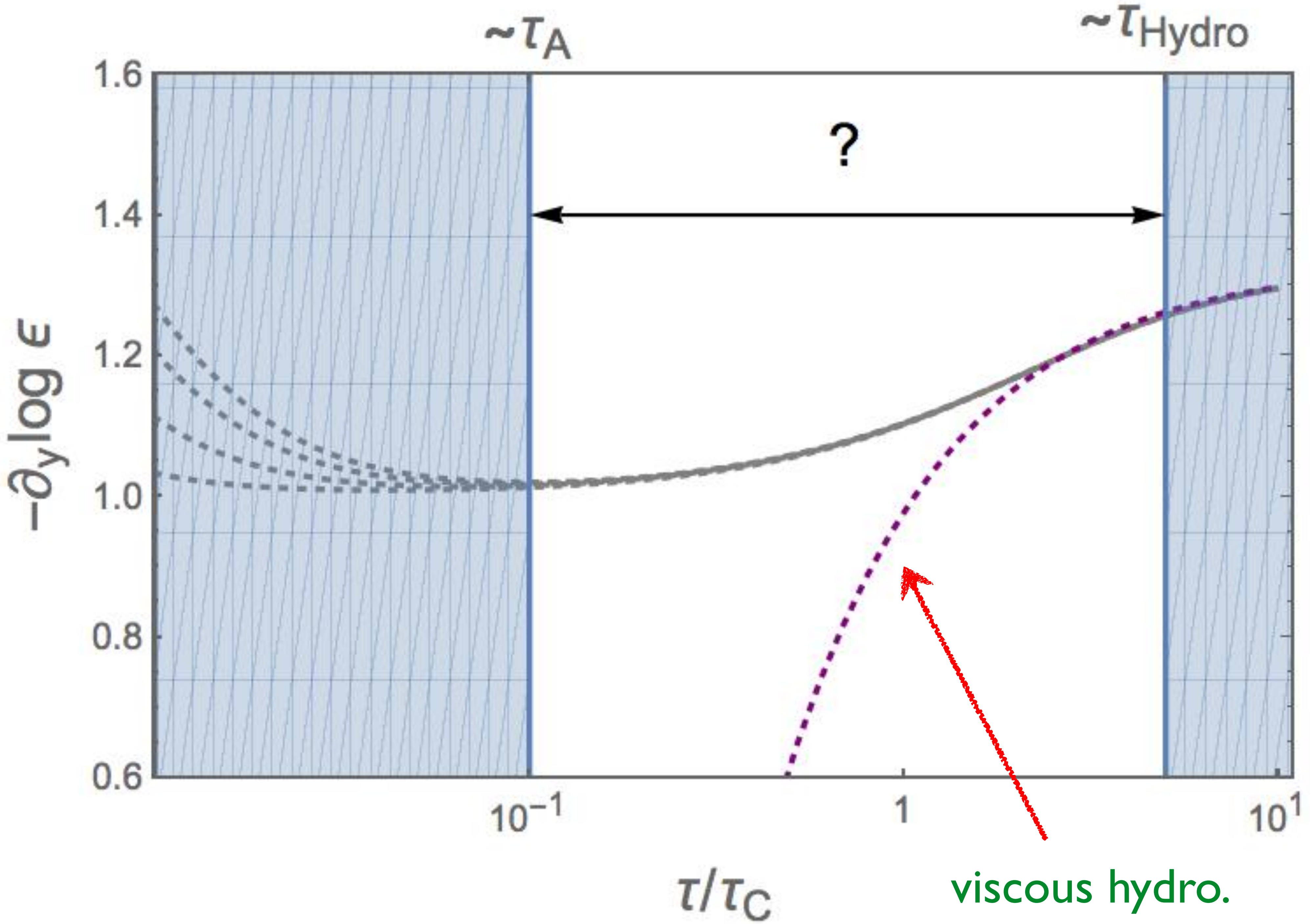
$$H_{RTA}(y) = H_F + (\frac{\tau}{\tau_{\rm Coll}})\,H_1$$

$$H_1 = \begin{pmatrix} 0 & 0 & \cdots\cdots \\ 0 & 1 & \cdots\cdots \\ \cdots & \cdots & \cdots\cdots \end{pmatrix}$$

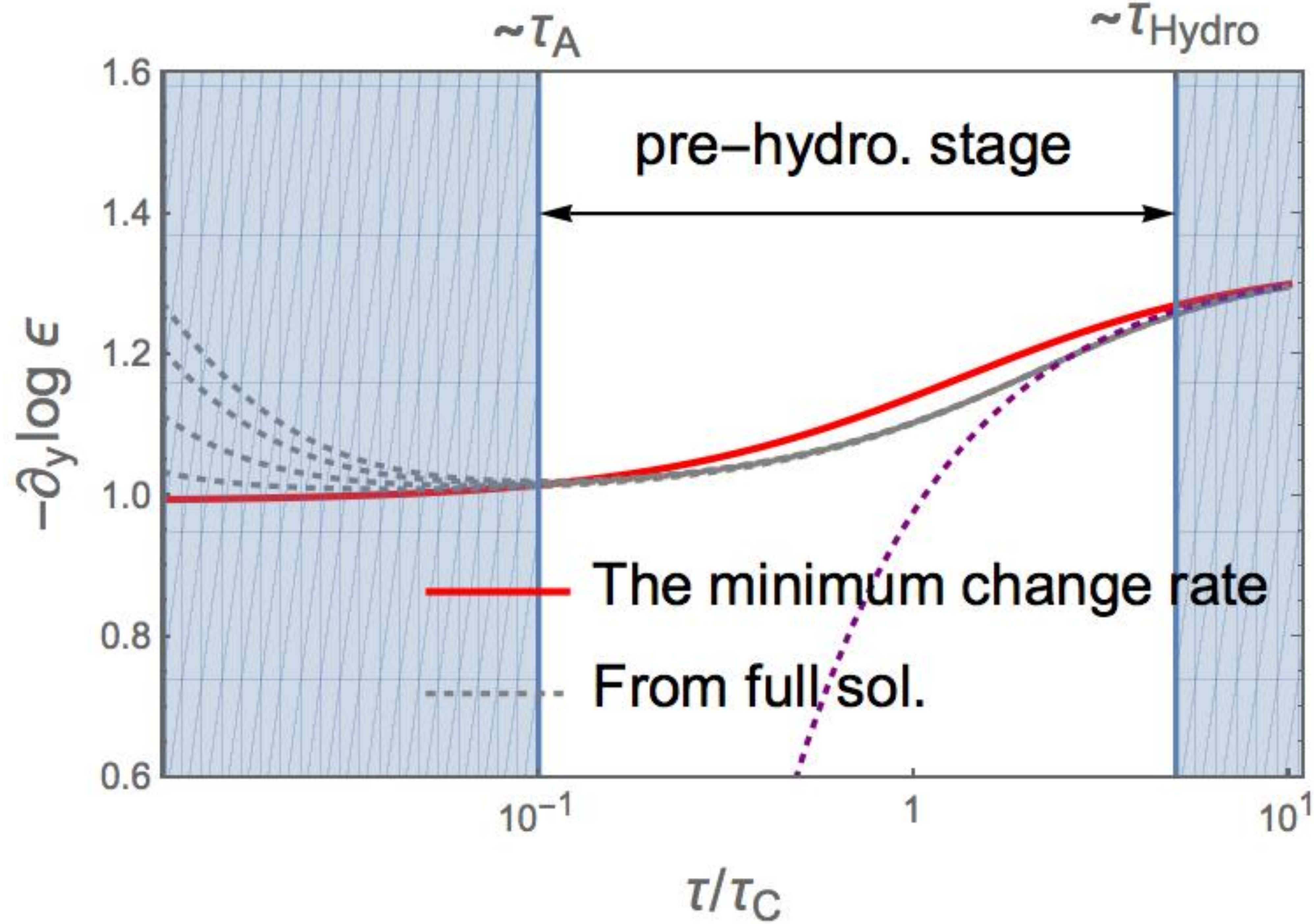
We can compute "the minimum change rate" at each snapshot of the evolution, and test the previous relation explicitly.

$$(-\partial_y \epsilon/\epsilon) = (-\partial_y \log \epsilon) \approx E_0(y)$$

Change rate of ε obtained from numerical solution of RTA kinetic equation



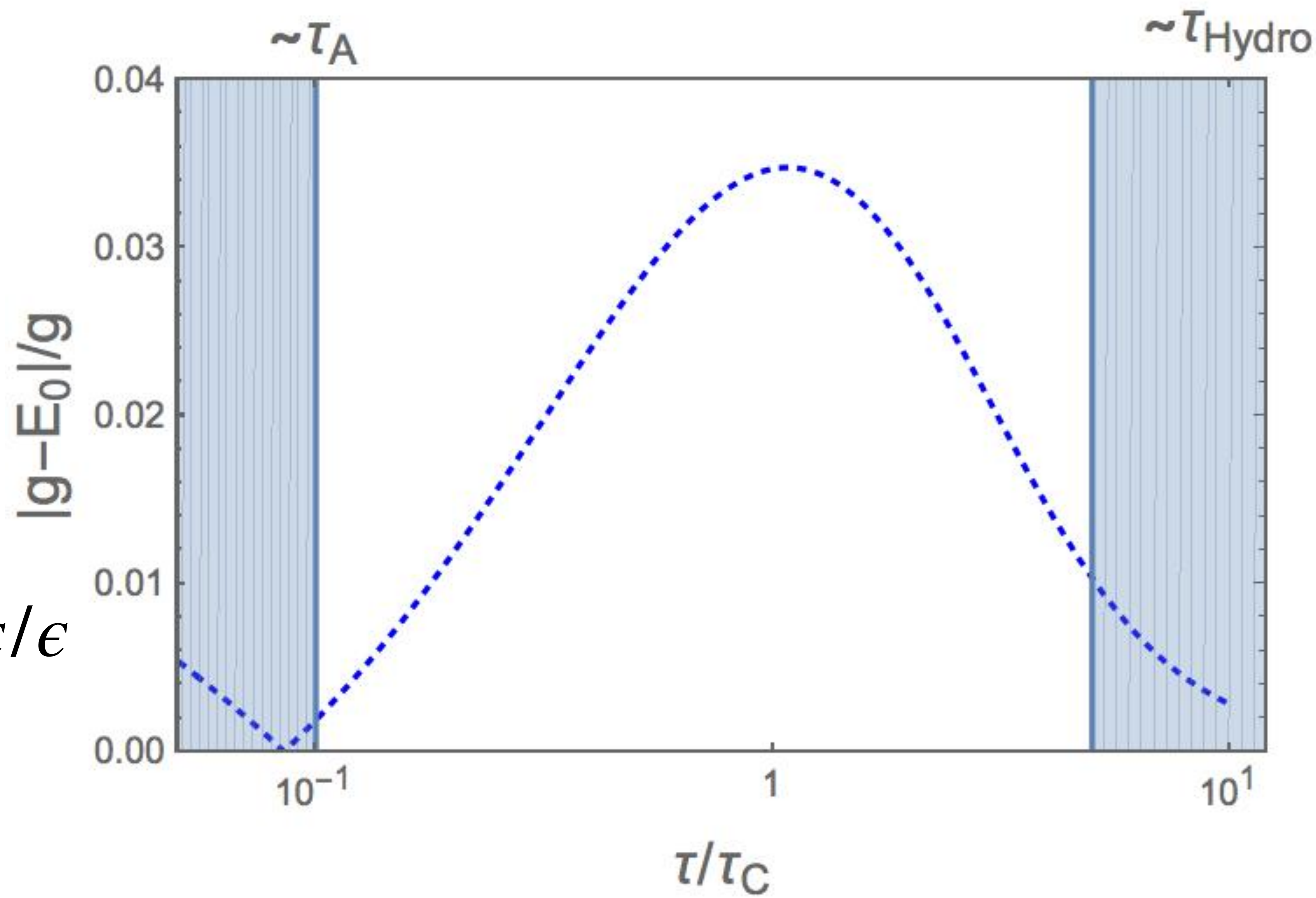
For RTA model, the change rate is approximated by the minimum charge rate since pre-hydro stage



J. Brewer, Li Yan and YY, 1910.00021

The contribution from faster states is suppressed during pre-hydrodynamic evolution

$g \equiv -\partial_y \epsilon / \epsilon$



In fact, we have identified a small parameter that controls the adiabaticity. By expanding in this parameter, the contributions from the "faster state" can also be accounted for quantitatively by generalizing adiabatic perturbation theory in QM.

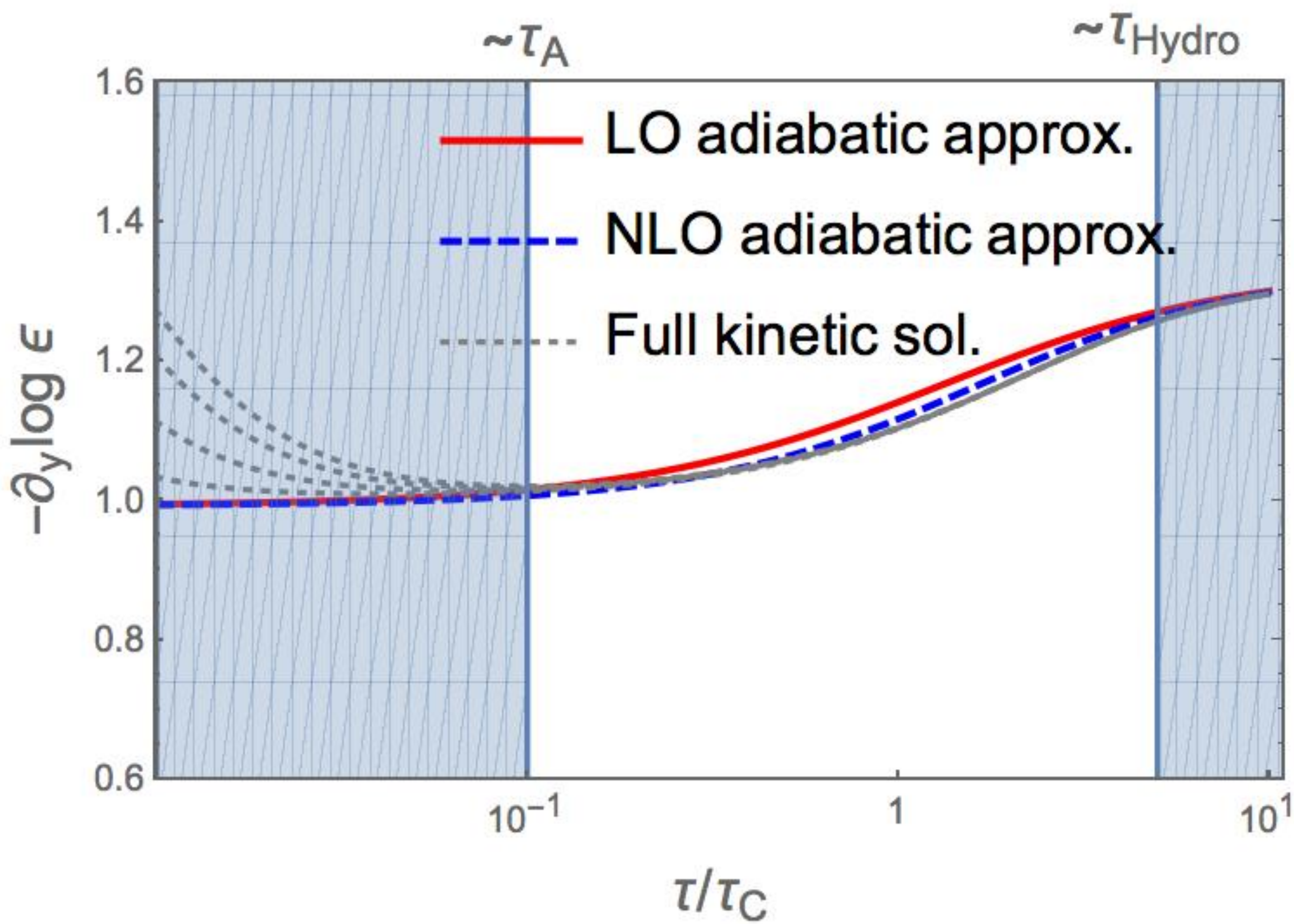
Adiabatic perturbation theory: from Landau-Zener problem to quenching through a quantum critical point

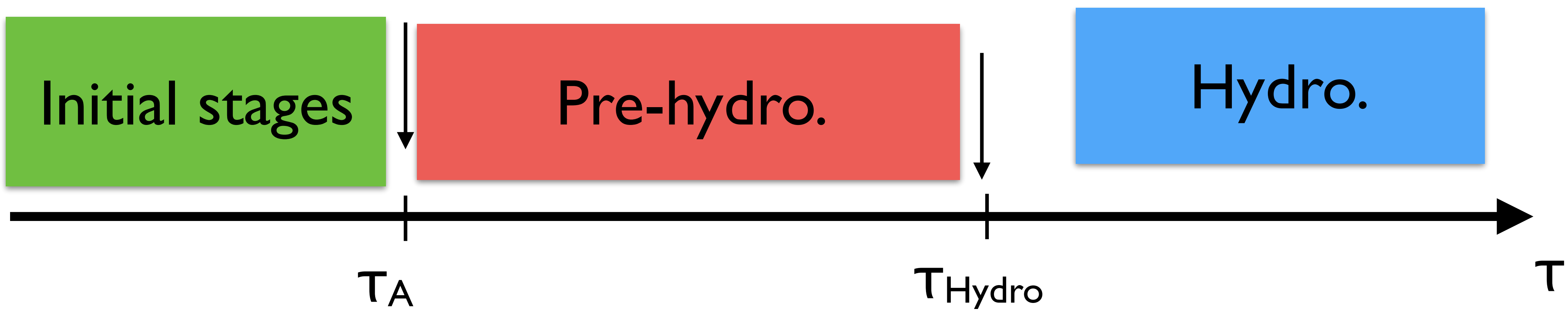
C. De Grandi and A. Polkovnikov
Department of Physics, Boston University, 590 Commonwealth Avenue, Boston, MA 02215, USA

We discuss the application of the adiabatic perturbation theory to analyze the dynamics in various systems in the limit of slow parametric changes of the Hamiltonian. We first consider a two-level system and give an elementary derivation of the asymptotics of the transition probability when the tuning parameter slowly changes in the finite range. Then we apply this perturbation theory to many-particle systems with low energy spectrum characterized by quasiparticle excitations. Within this approach we derive the scaling of various quantities such as the density of generated defects, entropy and energy. We discuss the applications of this approach to a specific situation where the system crosses a quantum critical point. We also show the connection between adiabatic and sudden quenches near a quantum phase transitions and discuss the effects of quasiparticle statistics on slow and sudden quenches at finite temperatures.

A review of modern formulation of adiabatic pTheory : De Grandi, A. Polkovnikov, 0910.2236

Systematic improvement with adiabatic perturbation theory





So, we have illustrated the adiabatic hydrodynamization within RTA kinetic theory.

But why does adiabaticity also apply to the violent expansion of the QGP in its early stages?

The criterion for adiabaticity in Quantum mechanics: suppression of the transition

Considering a prototype QM time-dependent Hamiltonian:

$$H_{\rm QM}(t) = H_{\rm QM,0} + \lambda_{\rm QM}(t) H_{\rm QM,1}$$

The transition rate between instantaneous g.s. and excited states is proportional to:

$$\text{transition rate} \propto \langle n, t \,|\, \lambda_{\rm QM} H_{\rm QM,1} \,|\, 0,1 \rangle \times \frac{\partial_t \lambda_{\rm QM}}{\Delta E(t)}$$

Adiabaticity means such transition is suppressed if

Either change rate is slow (slowly-quenching adiabaticity): $\dfrac{\partial_t \lambda_{\rm QM}}{\Delta E(t)} \ll 1$

or time-dependent part of the Hamiltonian is small in magnitude (fast-quenching adiabaticity):

$$\langle n, t \,|\, \lambda_{\rm QM} H_{\rm QM,1} \,|\, 0,1 \rangle \ll 1$$

The criterion for adiabaticity in expanding QGP

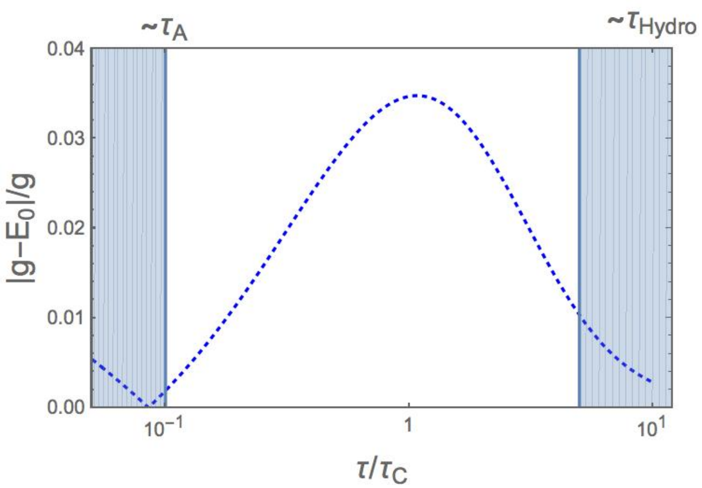
Slowly-quenching adiabaticity applies to the late stage

$$\frac{\partial_t \lambda_{\rm QM}}{\Delta E(t)} \ll 1 \quad \Longrightarrow \quad \frac{1/\tau}{1/\tau_{\rm C}} = \frac{\tau_{\rm C}}{\tau} \ll 1$$

Fast-quenching adiabaticity applies to the early stage (since collision rate is very rare.)

$$H_F \gg H_C(y)$$

Despite of the difference in quench rate, expanding QGP satisfies the adiabaticity criterion when: $\tau \ll \tau_{\rm C}$, and $\tau \gg \tau_{\rm C}$



How about the transition interval?

The transition interval when adiabaticity might be broken down is parametrically narrow for QCD in weakly coupled limit based on bottom-up thermalization scenario!

$$\alpha_s^{-2.5} Q_S^{-1} \le \tau \le \alpha_s^{-2.6} Q_S^{-1}$$

The adiabaticity is maintained during the pre-hydro stage for expanding QGP under RTA model. This scenario might be relevant to heavy-ion collisions, again, due to the non-trivial way that QGP is thermalized.

Pre-hydrodynamic modes

Weiyao Ke and YY, in progress

Going beyond boost-invariance

Consider RTA kinetic equation in the presence of an in-homogeneous source.

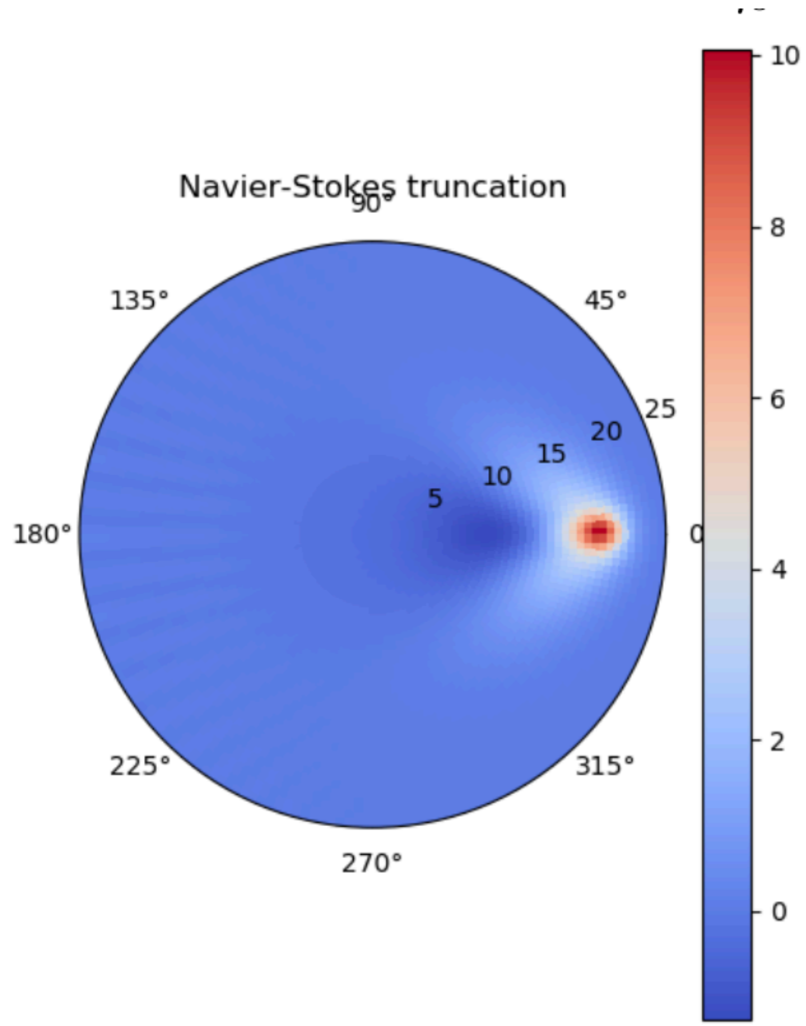
The medium response can be studied by introducing the expansion in spherical Harmonics.

$$F_\epsilon(\cos\theta, \varphi; \tau, \overrightarrow{x}_\perp, \eta) \sim \epsilon + \sum_{l>0,m} \frac{4n+1}{2} \mathscr{L}_{l,m}(\tau, \overrightarrow{x}_\perp, \eta)\, Y_{lm}(\cos\theta, \varphi)\,.$$

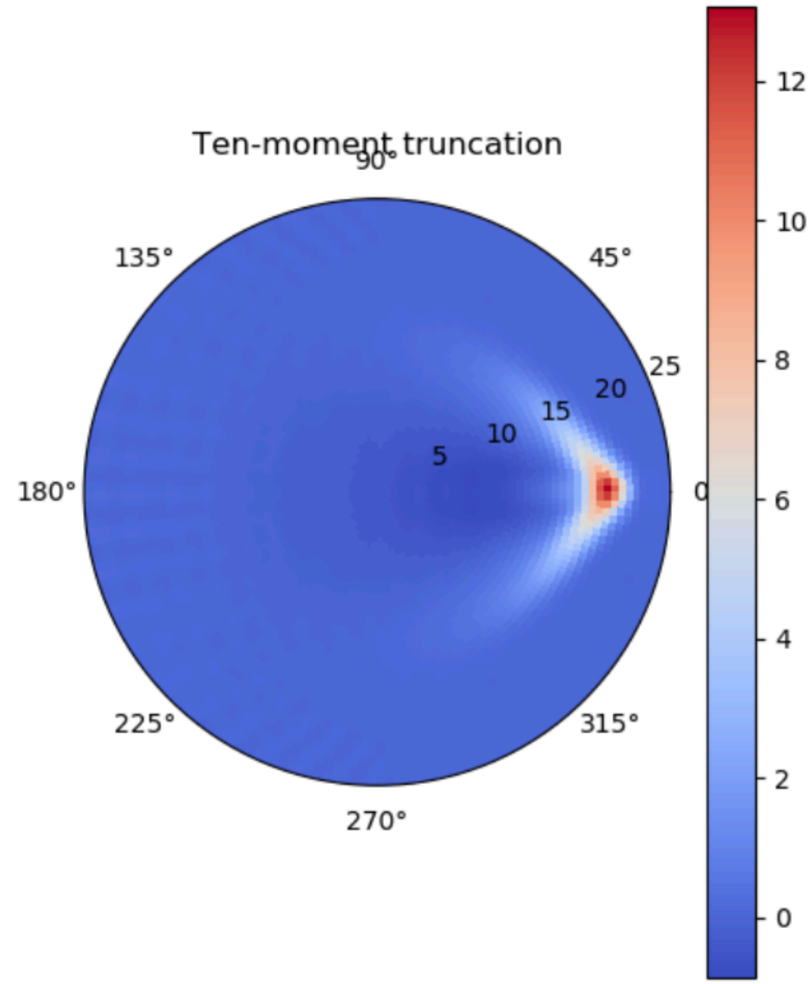
$$\psi \to \psi' \equiv \left(\epsilon, \mathscr{L}_{10}, \mathscr{L}_{11}, \mathscr{L}_{1-1}, \ldots\right) \cdot \quad H_F + H_C \to i\,\overrightarrow{k}\,\overrightarrow{H} + H'_F + H_C$$

In the limit $\tau_C/\tau << 1$ and k $\tau_C << 1$, we reproduce the linearized hydro. equation around Bjorken background.

At early times, $\tau_C/\tau >> 1$ and k $\tau << 1$ while k $\tau_C >> 1$, the preliminary analytic study suggests the presence of new collective excitations which might be interpreted as the pre-hydrodynamic modes.



Using linearized hydro.



Using RTA kinetic equation

(Preliminary)We observe the modification of the Mach cone and the diffusive wake. Hints for pre-hydro. modes?

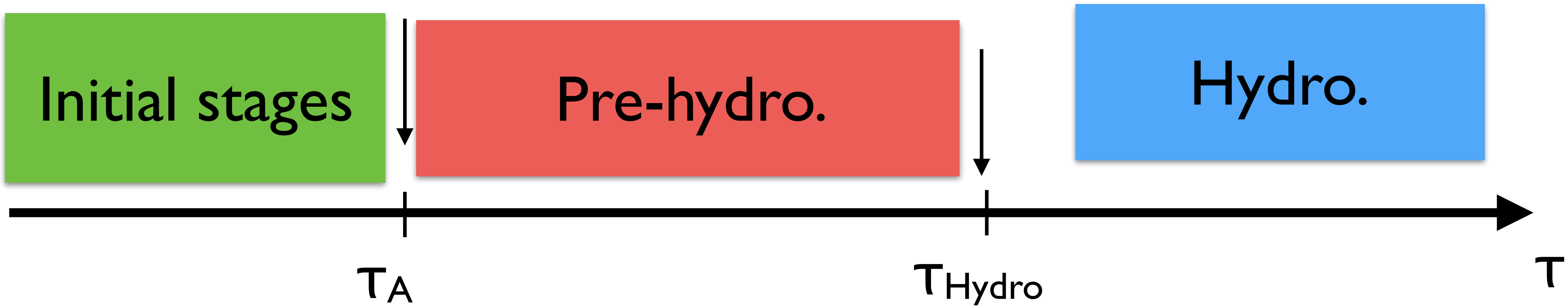
Discussion and outlook

Discussion and comments

On “attractor”: the system would evolve into the least relaxable state at very early time, but this does not necessarily imply the adiabaticity

On “an-isotropic hydro.”: the an-isotropic distribution in the formulation of “an-isotropic hydro.” might be viewed as an ansartz for the least relaxable distribution.

Conclusion and outlook



We propose a new hydrodynamization scenario, "adiabatic hydrodynamization" (AH), for a weakly-coupled, longitudinally-expanding QGP. ➡More studies are necessary to test this scenario!

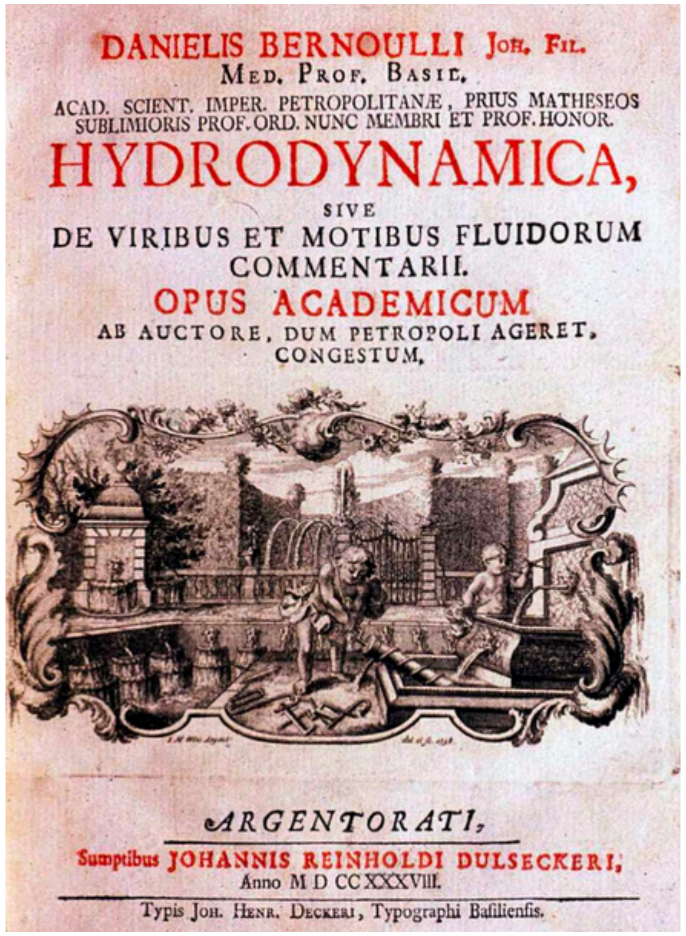
The concept of the least relaxable state and AH is expected to be relevant more broadly, e.g, strongly coupled QGP-like theories, condensed matter systems.

A more systematically way to identify pre-hydrodynamic modes is needed, e.g., studying pole structure of off-equilibrium two-point functions?

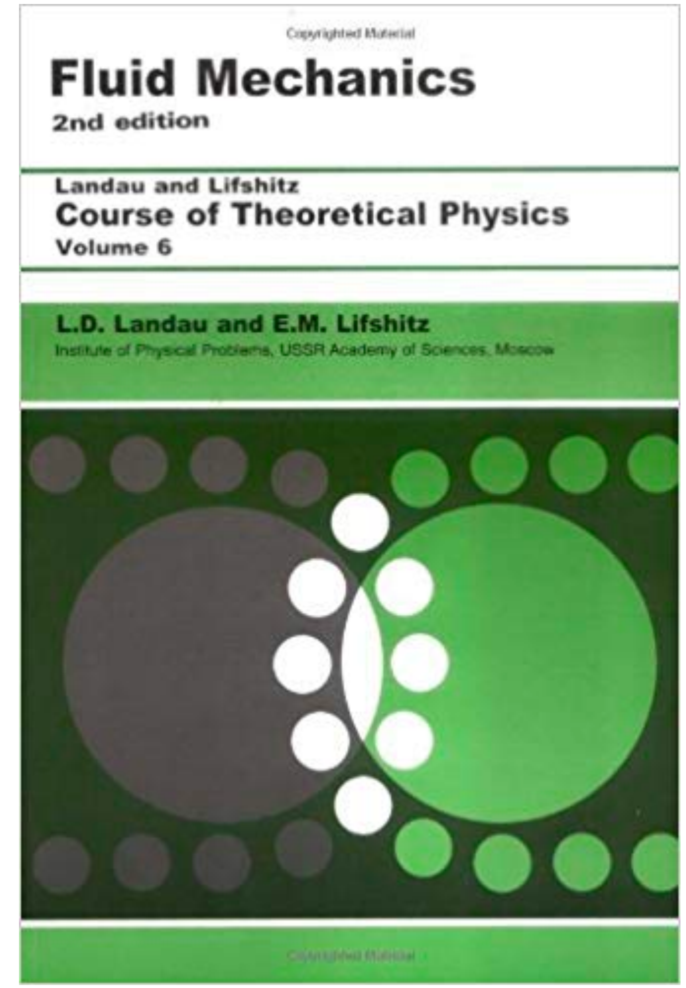
Future: effective theory with pre-hydro. modes as relevant d.o.f. .

Back-up

The development of hydrodynamics has a long history



Bernoulli, "Hydrodynamica", 1738

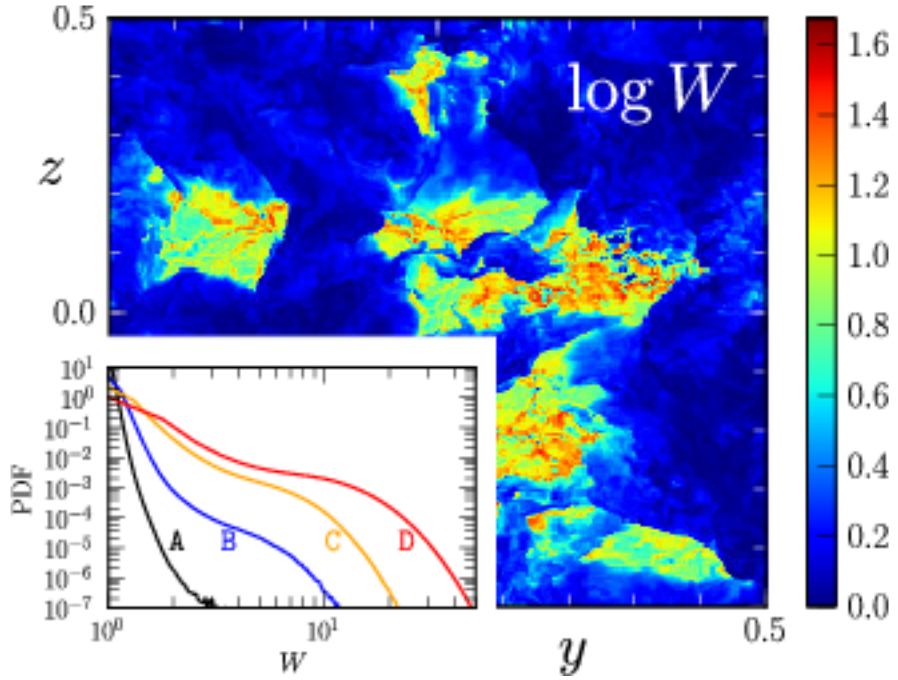


Landau-Lifshitz, "Fluid mechanics", 1950s

(Relativistic) Hydrodynamics: broad applications

Astrophysical scale:

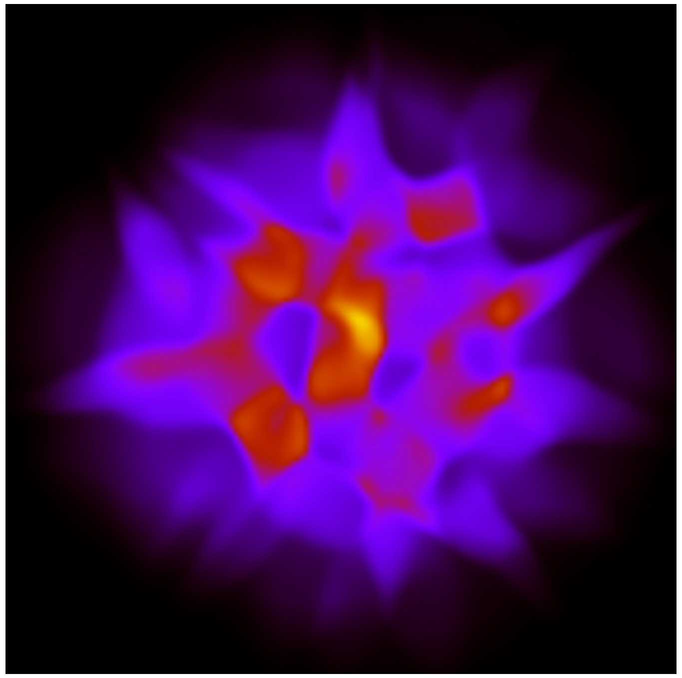
1 parsecs ~ 10^{17}m



The turbulence of relativistic plasma from hydro. simulation(arXiv: 1209.2936v2)

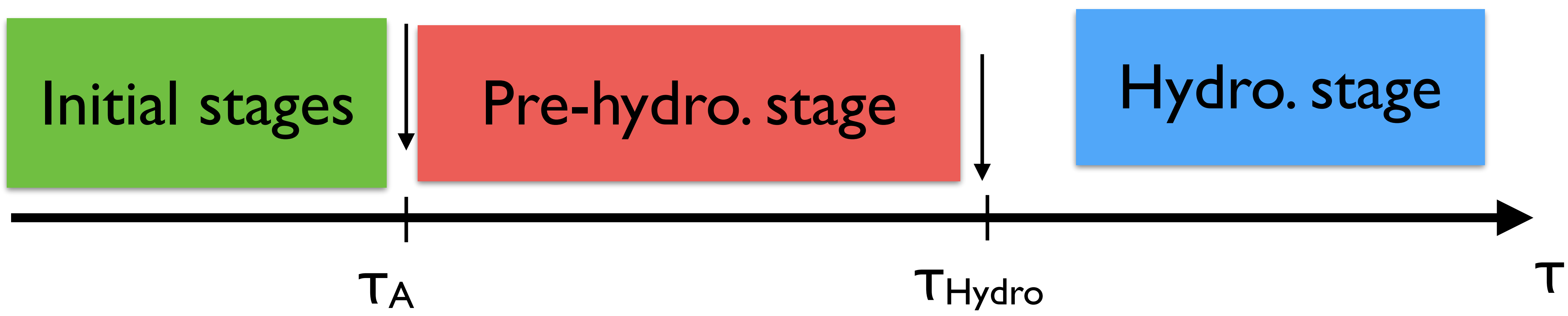
Typical size of quark-gluon mater cerated in heavy-ion collisions:

10 fm ~ 10^{-14} m



Hydro. simulation for heavy-ion collisions (by Schenke)

A new scenario: "adiabatic hydrodynamization"



During adiabatic hydrodynamization, the system is evolved gradually from pre-hydro. stage in hydro. stage.

τ_{redu} is an emergent scale that is parametrically shorter than τ_{Hydro}.

The pre-history of hydrodynamic modes is (almost) the history of pre-hydrodynamic modes!

The least relaxable state at early time limit (or collision-less limit)

Consider the evolution of ψ at very early time $\tau \ll \tau_{Coll}$

$$\partial_y \psi = -\left(H_F + H_{Coll}\right)\psi \approx -H_F \psi\,.$$

We expanding in terms of eigenfunctions of matrix H_F:

$$\psi(\tau) = b_0(\tau)\phi_0^F + \sum_{n=1} b_n(\tau)\phi_n^F$$

Then,

$$\frac{b_n(\tau)}{b_0(\tau)} \sim \frac{e^{-E_n^F y}}{e^{-E_0^F y}} \sim \left(\frac{\tau}{\tau_I}\right)^{-(E_n^F - E_0^F)}, \qquad (E_n^F - E_0^F) \sim \mathcal{O}(1)$$

There must exist an emergent scale τ_A around which the evolution is dominant by ϕ_F :

$$\lim_{\tau/\tau_I \to \infty} \lim_{\tau_{\rm Coll}/\tau \to 0} \psi(\tau) \sim \phi_0^F\,.$$

The separation of scale for weakly coupled QGP in high energy density regime

So far, we have assumed the separation of scale between the initial time and typical collision time (or mean free time), i.e,

$$\tau_I \ll \tau_{\mathrm{Coll}}$$

In many cases, τ_I is a matter of choice, but for weakly coupled QGP in high energy density limit, τ_I is set by an emergent scale, i.e. saturation scale Q_s (typical momentum of gluon produced after a collision which is much larger than Λ_{QCD}):

$$\tau_I \sim Q_s^{-1}$$

see e.g. McLerran-Venugopalan, PRD 94'; Blaizot-Mueller, Nucl.Phys.B 87'

Meanwhile, the collision time is inversely proportional to the cross-section of scattering process, therefore we see the parametrically separation between τ_I and τ_{Coll} .

$$\tau_I \sim Q_s^{-1} \ll \tau_{\mathrm{coll}} \sim \alpha_s^{-x} Q_s^{-1}\,, \qquad x > 0$$

The stages of pre-equilibrium evolution of weakly coupled QGP has been delineated parametrically by Baier et al in 2001. Based on their analysis, we could discuss the applicability of adiabaticity:

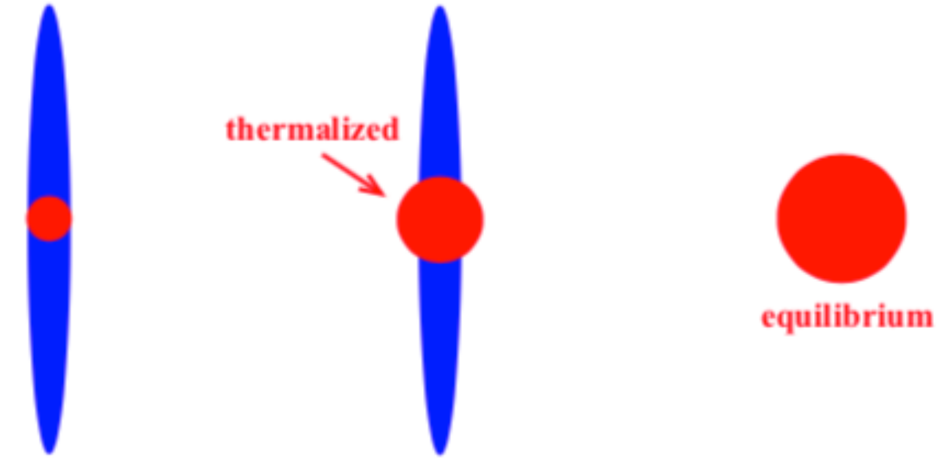


Figure adapted from Arnold, 0708.0812.

“Fast quench picture” applies to the period when ψ represents the angular distribution of hard gluons (with typical energy Q_s) that rarely collide with one another, i.e when

$$\tau_{\mathrm{redu}} \leq \tau \leq \alpha_s^{-5/2} Q_S^{-1}$$

“Slow quench picture” applies to the period when ψ represents the angular distribution of soft gluons (with typical energy T) that are already in thermal equilibrium, i.e when

$$\tau \geq \alpha_s^{-13/5} Q_S^{-1}$$

The transition interval when adiabaticity might be broken down is parametrically narrow for QCD in weakly coupled limit!

$$\alpha_s^{-2.5} Q_S^{-1} \leq \tau \leq \alpha_s^{-2.6} Q_S^{-1}$$

One fluid to rule them all: Viscous hydrodynamic description of event-by-event central p+p, p+Pb and Pb+Pb collisions at $\sqrt{s} = 5.02$ TeV

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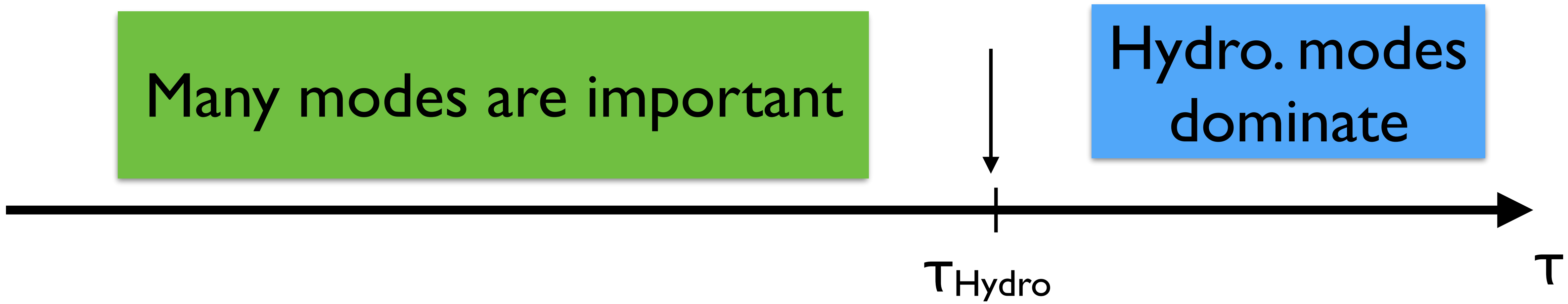
Fluid: the medium which exhibits hydrodynamic behavior. Then, what are the defining attributes of hydrodynamic behavior?

Attributes of a fluid

"Local thermal equilibrium" (the principle of the least entropy production): change rate of the local entropy density is bounded from below.

"Gradient expansion" (the existence of a small parameter suppressing the transition to non-equilibrium state): the expectation value of bulk quantities are expressible in terms of conserved densities and their derivatives.

The common view of "hydrodynamization"



Conventional scenario: hydrodynamization as the decay of non-hydrodynamic modes.

For example, the decay of disturbances for fluids in a slow varying environment.

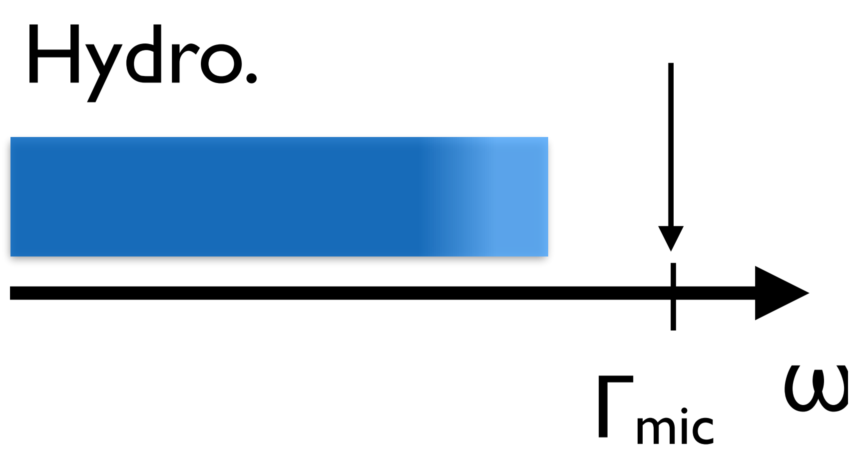
Note, this scenario does not rely on specifics of the underlying microscopic theory.

We propose a new hydrodynamization scenario in which the longitudinal expansion and some intrinsic hierarchy in time scale of QCD matter at high energy density regimes play an essential role.

The modern view of hydro. : a low-energy effective theory

Hydro. can be considered as an effective theory of many-body interacting systems in long time and long wavelength limit.

Hydro. d.o.f.: conserved densities, e.g, energy density ε and momentum density (related to flow velocity u^μ) which evolves slowly near thermal equilibrium.



Small parameter: gradient times mean free path and/or frequency of hydro. modes times mean free time.

Hydro. equation: conservation laws together with the constitutive relation obtained by gradient expansion.

$$\partial_\mu T^{\mu\nu} = 0\,. \qquad T^{\mu\nu} = \epsilon\, u^\mu\, u^\nu + p(\epsilon)\left(g^{\mu\nu} + u^\mu u^\nu\right) + \mathcal{O}(\partial)$$

“attractor”

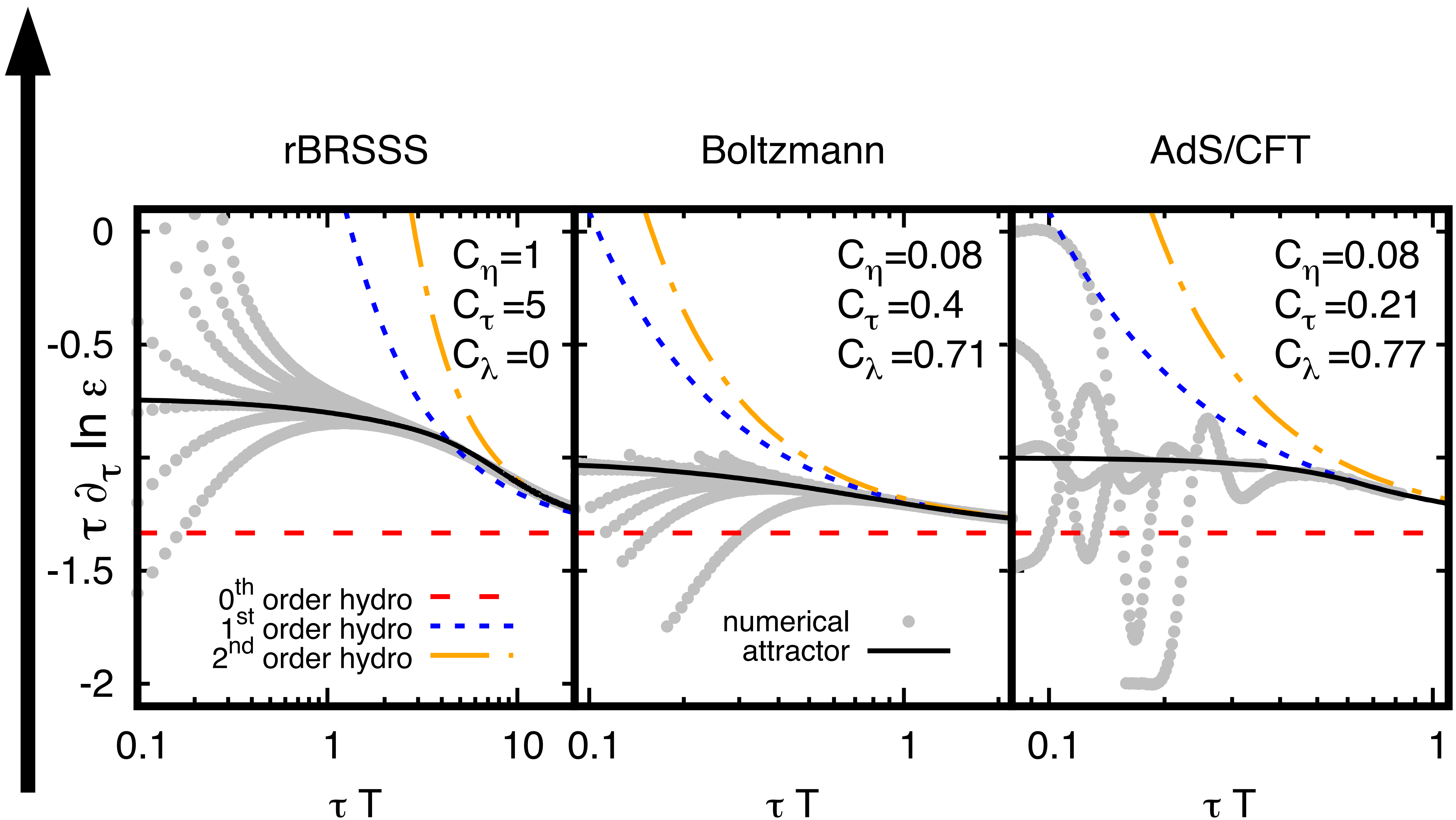


Figure adapted from Romatschke, 1704.08699, PRL 17’.