# QCD Criticality on Light Nuclei Production

## *in heavy-ion collisions*

*kjsun@tamu.edu*

**KaiJia Sun** (孙开佳)
Cyclotron Institute, TAMU

*In collaboration with Feng Li, Che-Ming Ko,*
*Lie-Wen Chen and Jun Xu*

HENPIC Seminar  August 27, 2020

**K. J. Sun, C. M. Ko, and F. Li, arXiv:2008.02325(2020)**

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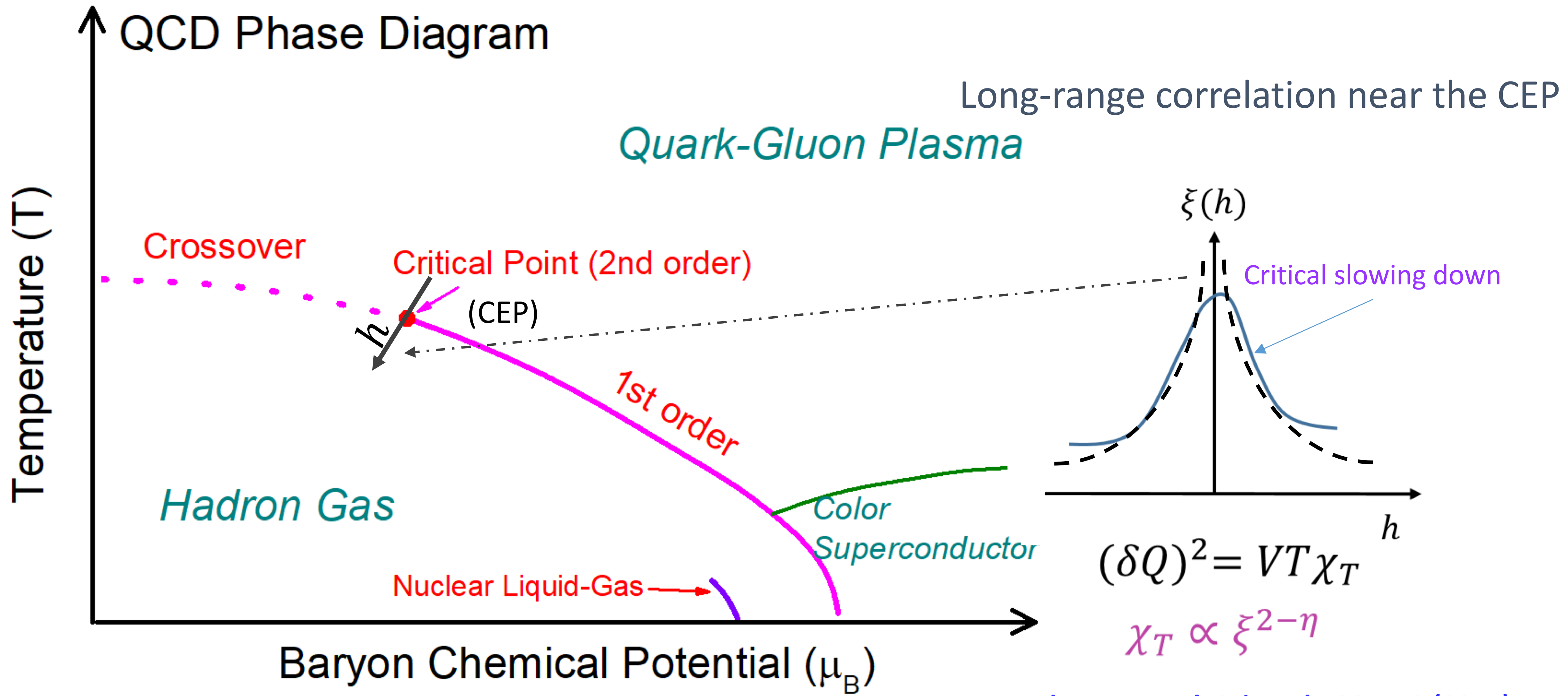
# 1. QCD phase diagram and heavy-ion collisions

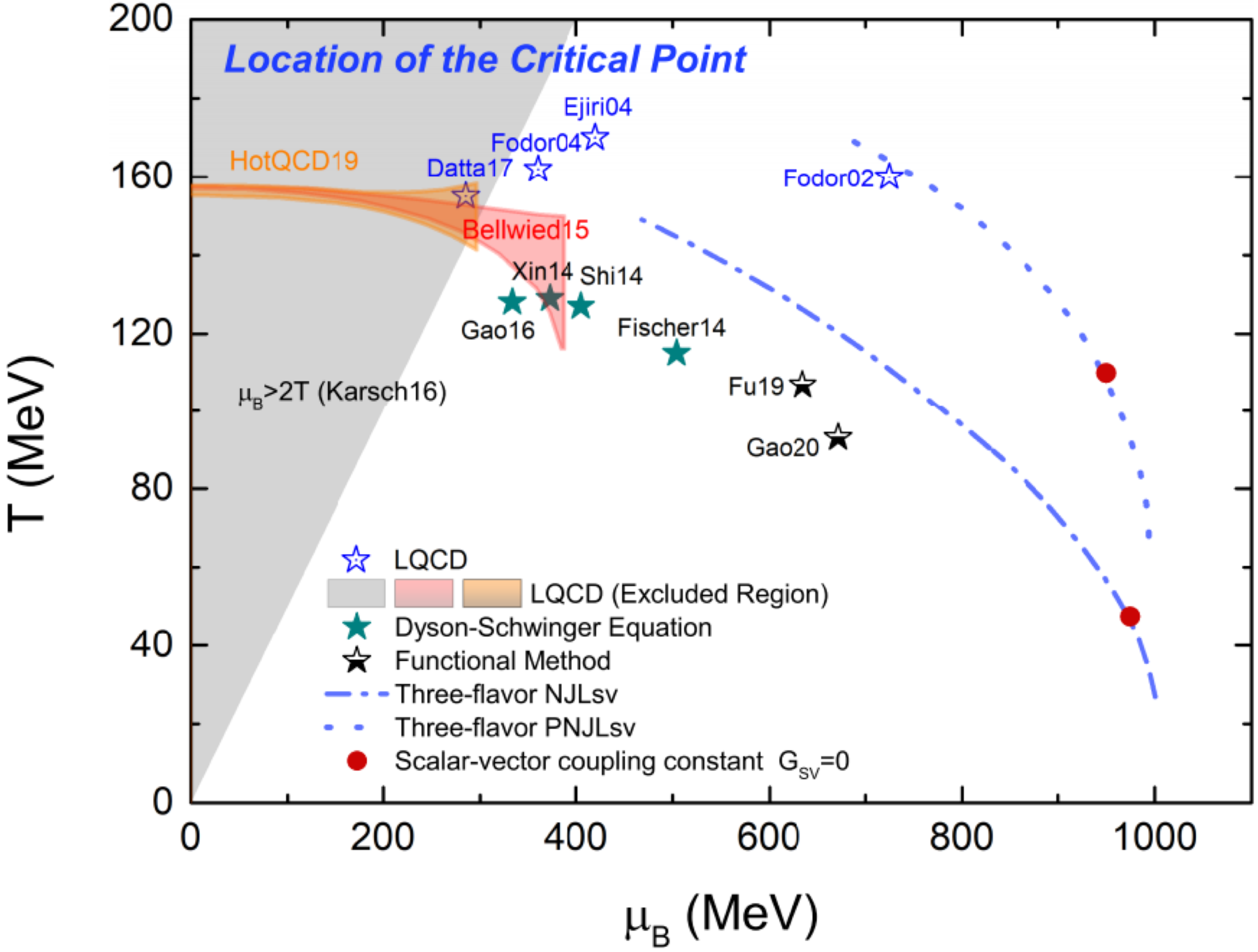





X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017)
A. Bzdak et al., Phys. Rept. 853, 1 (2020)
B. Berdnikov, K. Rajagopal, Phys. Rev. D 61 105017 (2000)

# 1. QCD phase diagram and heavy-ion collisions

(2)



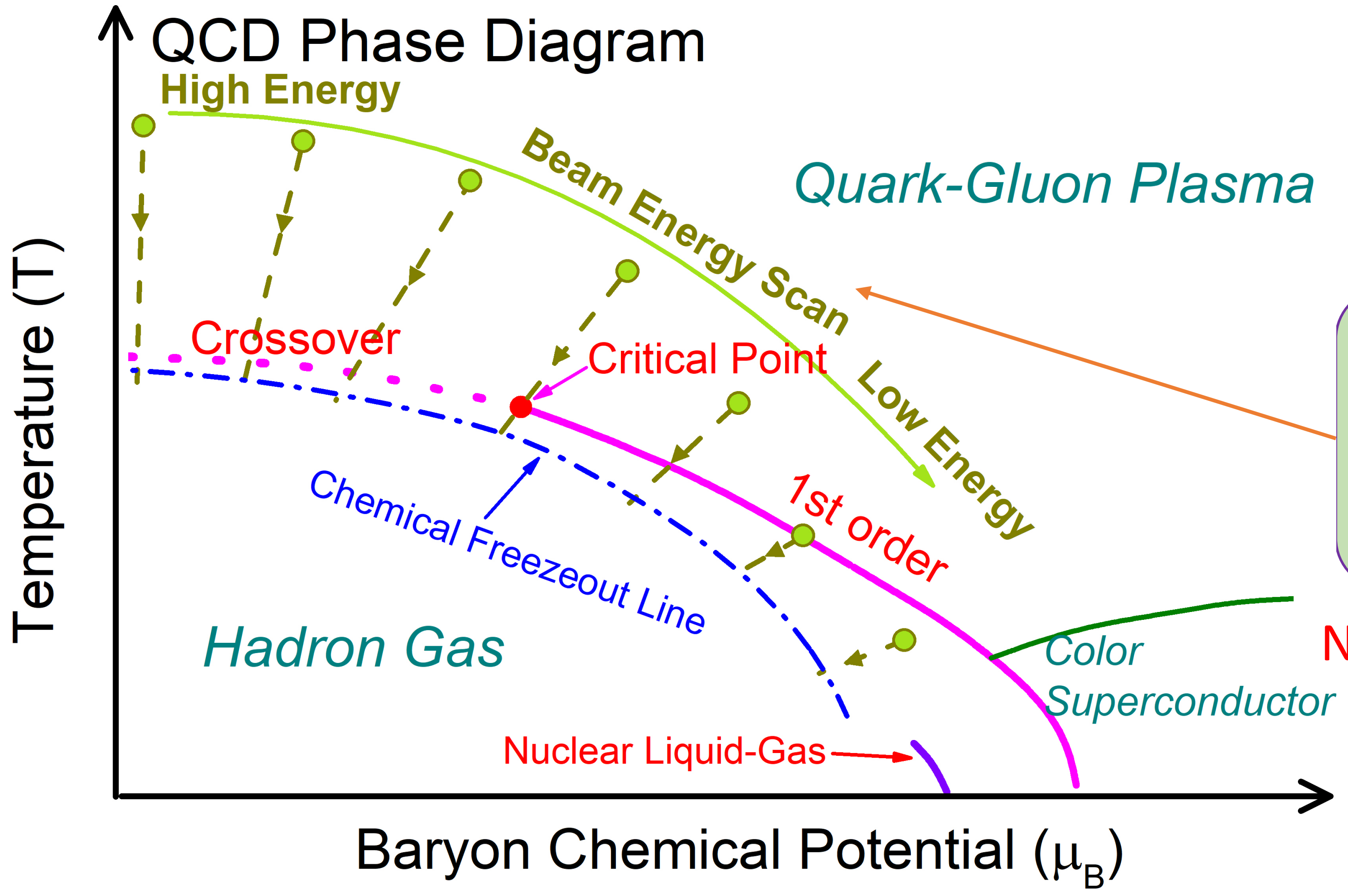

In theory, the location of QCD critical point(CEP) has large uncertainties!

In experiment, the location of CEP can be probed in heavy-ion collisions


Stephenov, Int. J. Mod. Phys. A20, 4387(2005)
Sun et al., arXiv:2004.05754

# 1. QCD phase diagram and heavy-ion collisions






Probing the QCD critical point through beam energy scan (BES) program on heavy-ion collisions

Non-monotonic behavior is expected!

X. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017)
A. Bzdak et al., Phys. Rept. 853, 1 (2020)

# 2. Signal of QCD phase transition: enhanced fluctuation



Classical ideal gas: $\langle (\delta Q_i)^2 \rangle \sim V$

V. Koch, arXiv:0810.2520 (2008)

Crossover: $\langle (\delta Q_i)^2 \rangle \sim V$

Second-order phase transition:

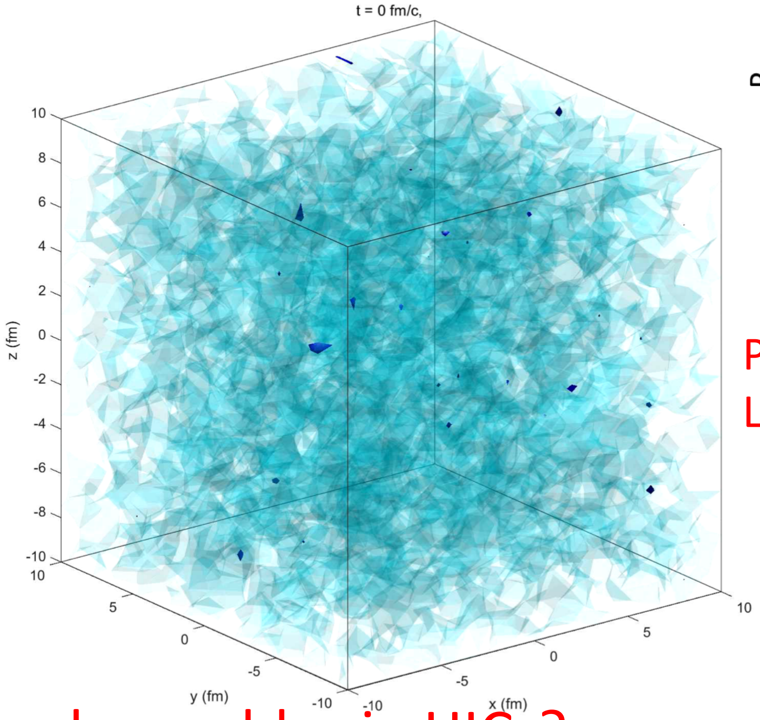
$$\langle (\delta Q_i)^2 \rangle \sim V^{5/3}$$

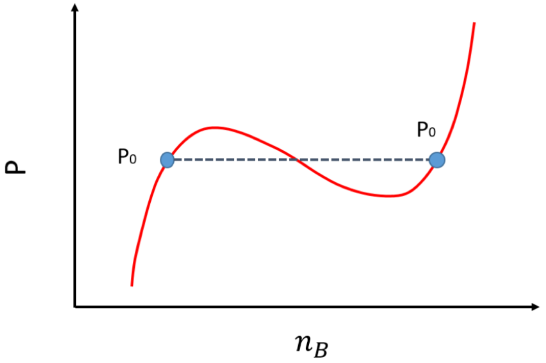
First-order phase transition:

$$\langle (\delta Q_i)^2 \rangle \sim V^2$$


Critical opalescence

Long-range correlation



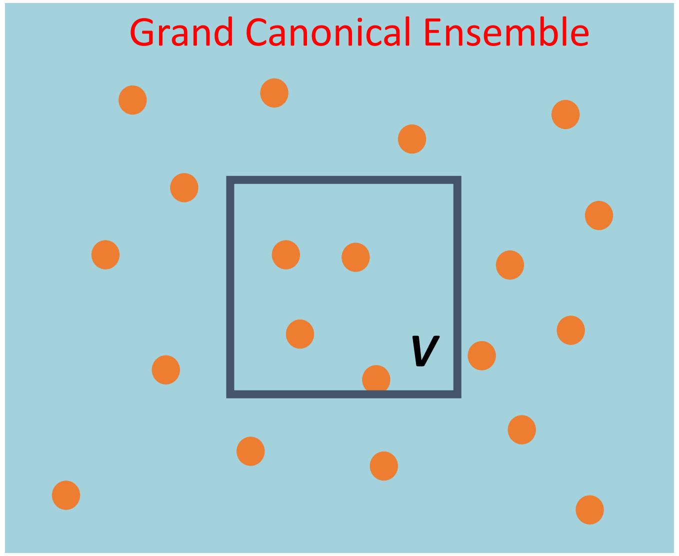




Phase separation

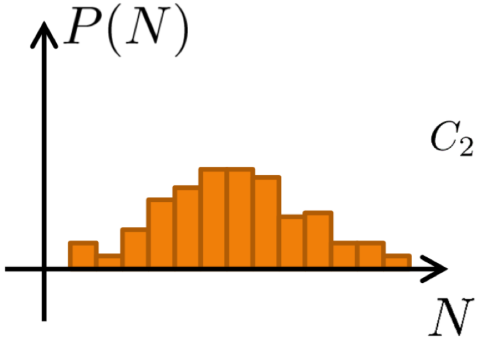
Large density inhomogeneity

What are the observables in HICs?

# 2. Observable 1: event-by-event fluctuation of conserved charges (5)




Fluctuation in a spatial volume *V*




In thermal equilibrium

$$C_2 = \langle N^2 \rangle_c = \langle N^2 \rangle - \langle N \rangle^2 = T^2 \frac{\partial^2 lnZ}{\partial \mu^2}$$

$$C_2 = VT^3 \chi_T \qquad \chi_T = \frac{1}{VT} \frac{\partial^2 lnZ}{\partial \mu^2}$$

$$\chi_T \propto \xi^{2-\eta}$$

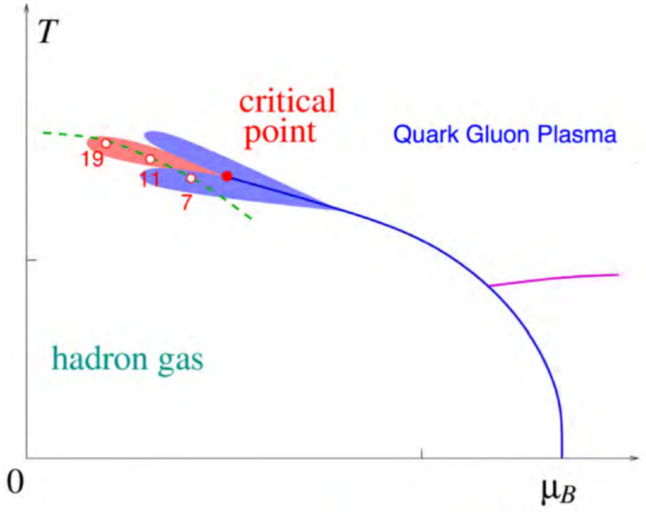
$$\frac{\chi_q^{(4)}}{\chi_q^{(2)}} = \kappa\sigma^2 = \frac{C_{4,q}}{C_{2,q}} \sim \xi^5$$

Higher-order cumulants are more sensitive

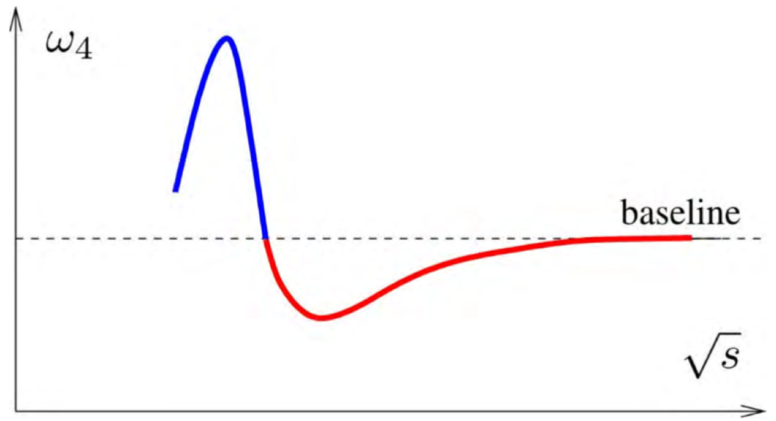
A well-known scenario:

$$\langle (\delta N)^4 \rangle_c = \langle N \rangle + \langle \sigma_V^4 \rangle_c \left( \frac{g\, d}{T} \int_p \frac{n_p}{\gamma_p} \right)^4 + \ldots,$$

$$\langle \sigma_V^4 \rangle_c = 6VT^3[2(\lambda_3 \xi)^2 - \lambda_4]\xi^8$$



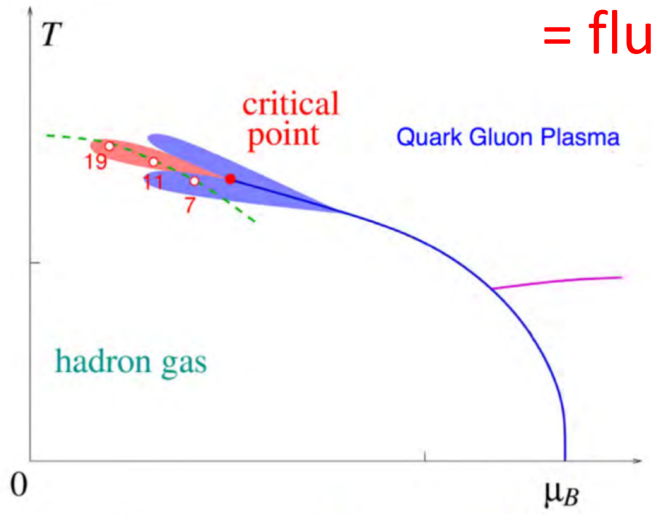

M. A. Stephanov, Phys. Rev. Lett. 107, 052301 (2011)
X. F. Luo and N. Xu, Nucl. Sci. Tech. 28, 112 (2017)

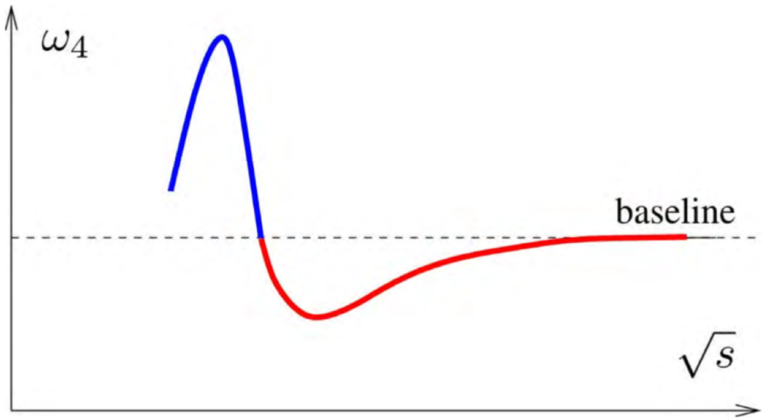


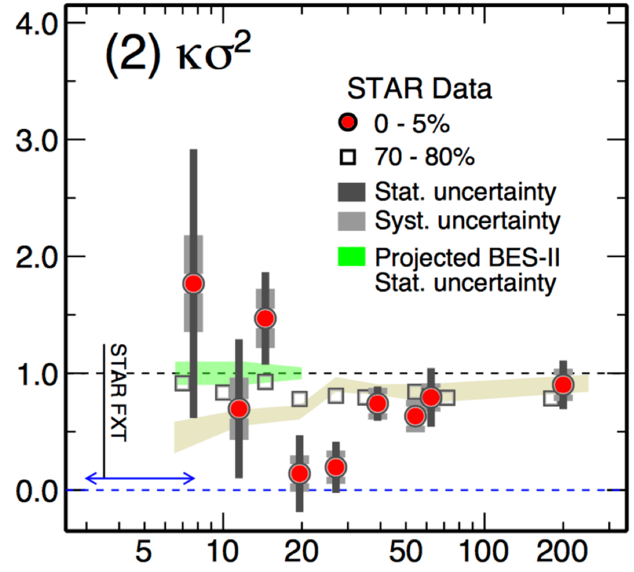
# 2. Observable 1: non-Gaussian fluctuation



Assumption: fluctuation in coordinate space ($\eta_s$)
= fluctuation in momentum space ($y$)




STAR, arXiv:2001.02852(2020)

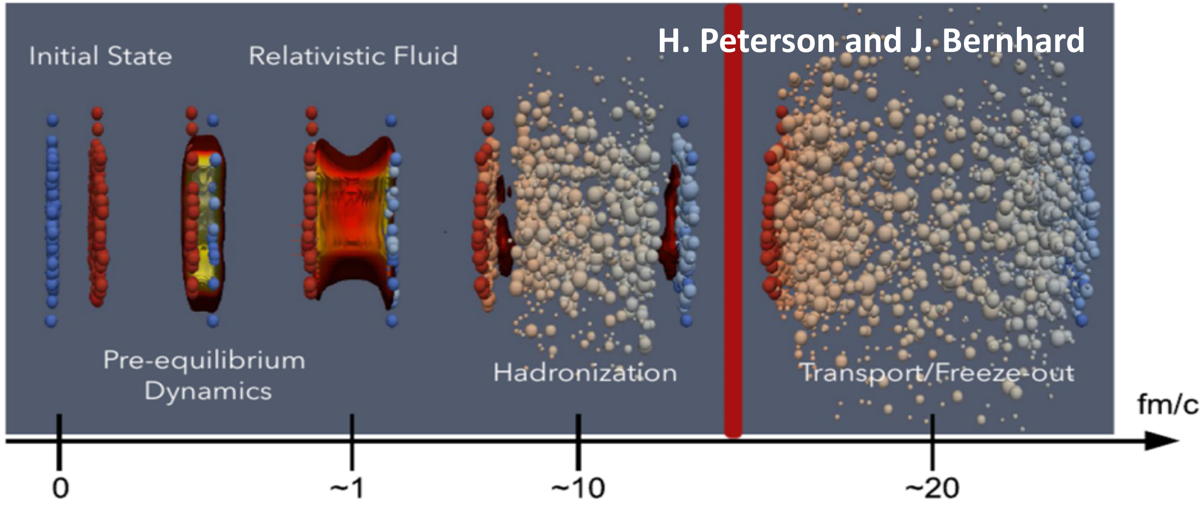
Difficulties:

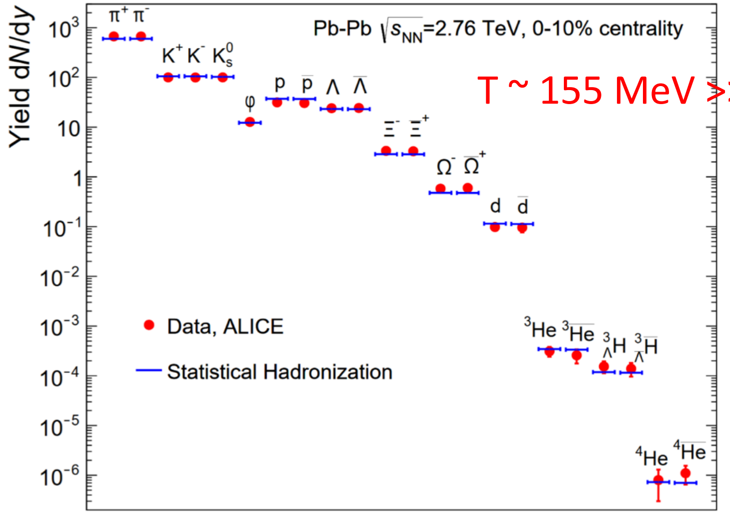
1. Breakdown of boost invariance at BES energies ($\eta_s \neq y$) M. Asakawa et al., Phys. Rev. C101, 034913 (2020)
2. Thermal smearing ($\eta_s \neq y$ even at LHC) Y. Ohnishi et al., Phys. Rev. C94, 044905 (2016)
3. Off-equilibrium effects S. Mukherjee et al., Phys. Rev. C92, 034912 (2015)

It is important to explore other observables sensitive to the long-range correlation!

# 3. Observable 2: light cluster production



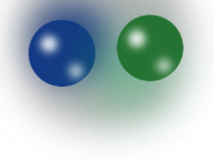



T ~ 155 MeV >> Binding energies

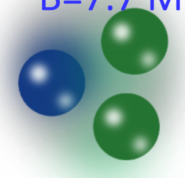
d (np)

B=2.2 MeV



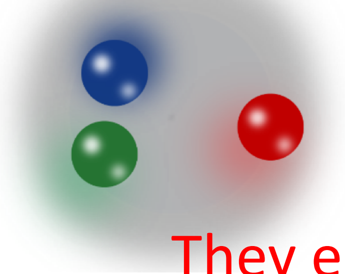
$^3$He (npp)

B=7.7 MeV



$^3_\Lambda$H (npΛ)

$B_{\Lambda d}$=0.13 MeV



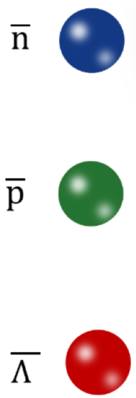

Deuteron (np)

Triton (nnp)

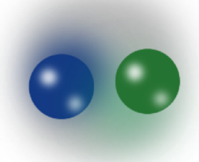
Helium-3 (npp)

Hypertriton (npΛ)

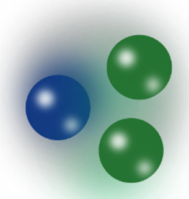
Helium-4 (nnpp)



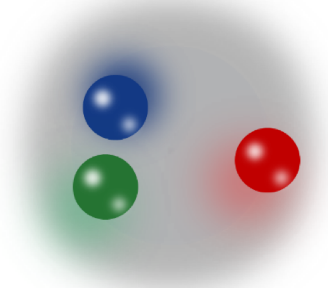

$\bar{d}$



$^3\overline{\text{He}}$



$^3_{\bar{\Lambda}}\bar{\text{H}}$

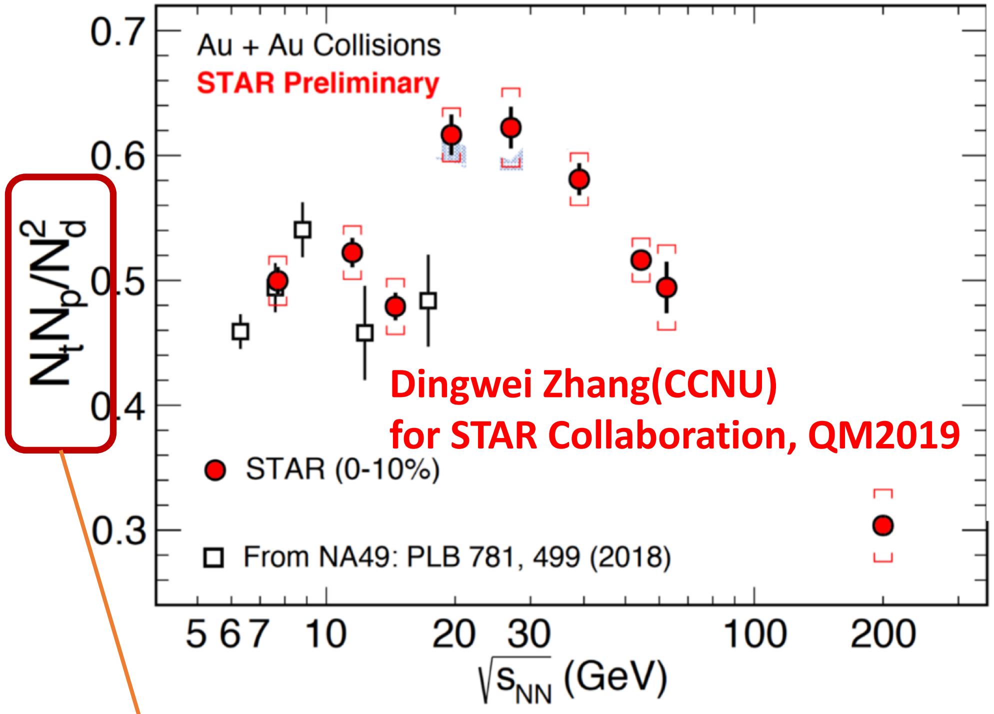


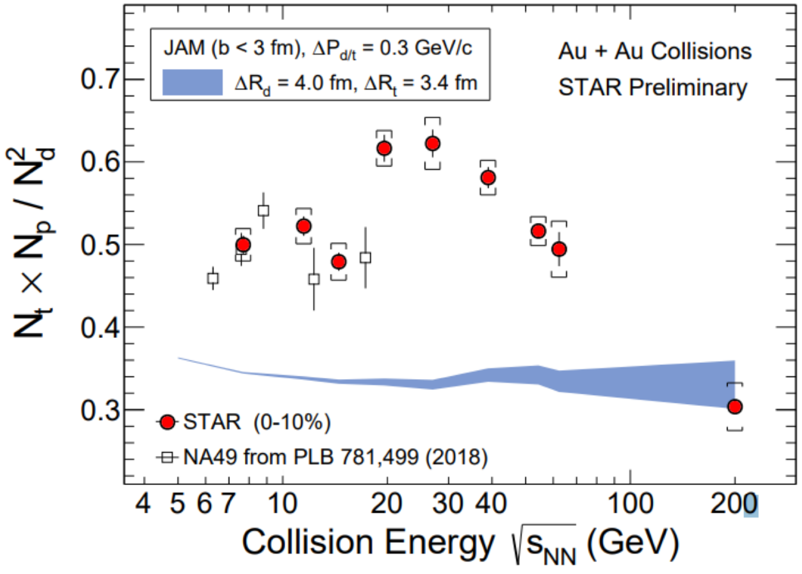
They encode the phase-space information of nucleons!

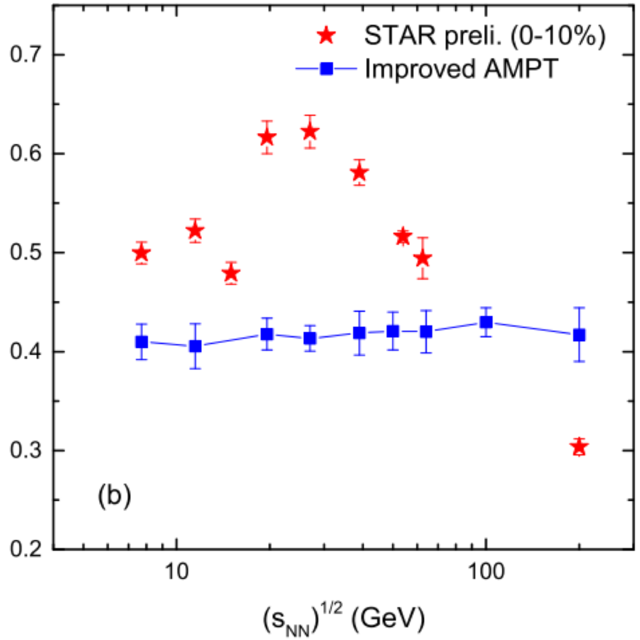
Andronic, Braun-Munzinger, Redlich, Stachel, Nature 561, 321 (2018)

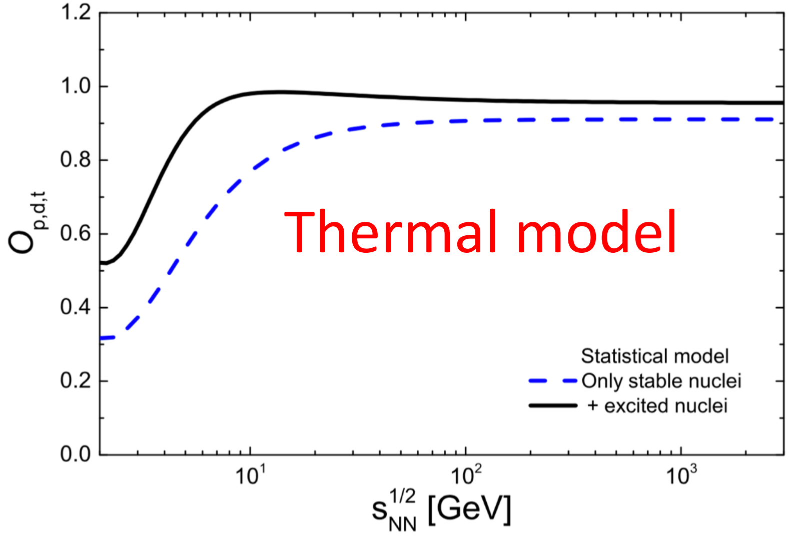
# 3. Experimental results of $tp/d^2$






No model can explain the data!

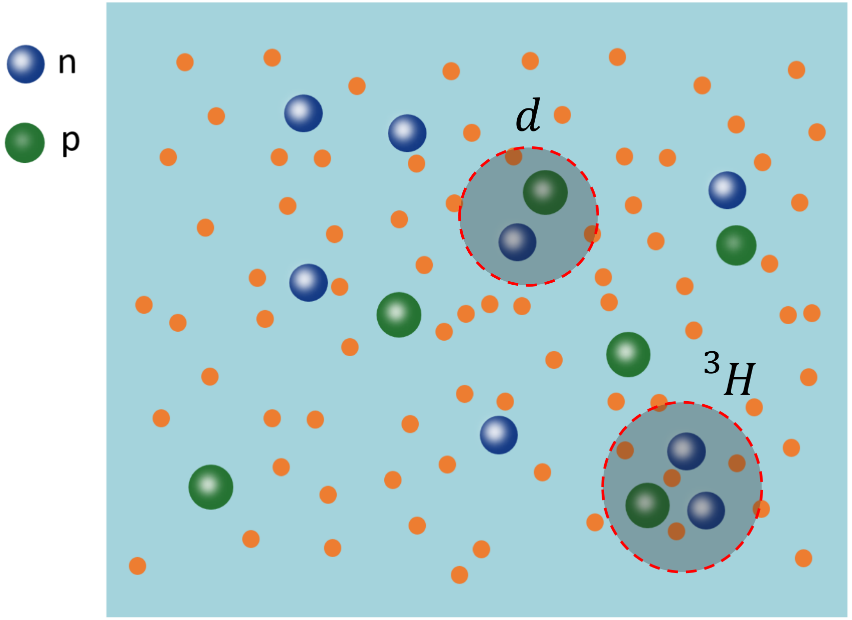
This double ratio was first proposed in

K. J. Sun et al., Phys. Lett. B 774, 103 (2017)
K. J. Sun et al., Phys. Lett. B 781, 499 (2018)

D. Zhang (STAR), arXiv:2002.10677(2020)
H. Liu et al., Phys. Lett. B805, 135452 (2020)
V.Vovchenko et al., arXiv:2004.04411(2020)
K. J. Sun and C. M. Ko, arXiv:2005.00182(2020)

# 3. Observable 2: light nuclei production






To study the effect of long-range correlation on light nuclei production, we adopt the quantum coalescence model:

$N_C \propto Tr(\widehat{\rho_i}\widehat{\rho_f})$

R. Scheibl and U. W. Heinz, Phys. Rev. C59. 1585(1999)

$$N_d = g_d \int dx_1 dx_2 dp_1 dp_2 f_{np}(x_1, p_1; x_2, p_2) \times W_d\left(\frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}}\right)$$

$$N_t = g_t \int dx_1 dx_2 dx_3 dp_1 dp_2 dp_3 f_{nnp}(x_1, p_1; x_2, p_2; x_3, p_3)$$
$$\times W_t\left(\frac{x_1 - x_2}{\sqrt{2}}, \frac{p_1 - p_2}{\sqrt{2}}, \frac{x_1 + x_2 - 2x_3}{\sqrt{6}}, \frac{p_1 + p_2 - 2p_3}{\sqrt{6}}\right)$$

Wigner function: $W_d(r, k) = 8\exp\left(-\frac{r^2}{\lambda^2 \sigma_d^2} - \sigma_d^2 k^2\right)$ $\sigma_d \approx 2.26$ fm

$$W_t(\rho, \lambda, k_\rho, k_\lambda) = 8^2 \exp\left(-\frac{\rho^2}{\sigma_t^2} - \frac{\lambda^2}{\sigma_t^2} - \sigma_t^2 k_\rho^2 - \sigma_t^2 k_\lambda^2\right)$$ $\sigma_t \approx 1.59$ fm

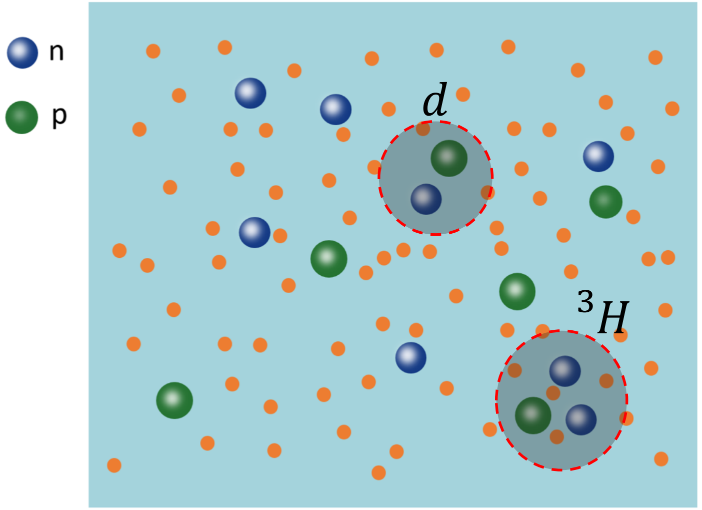
K. J. Sun, C. M. Ko, and F. Li, arXiv:2008.02325(2020)

Recent studies on light nuclei production:

J. Chen et al., Phys. Rept. 760,1 (2018)
P. Braun-Munzinger and B. Donigus, Nucl. Phys. A987, 144(2019)
B. Donigus, Int. J. Mod. Phys. E29, 2040001 (2020)
D. Oliinychenko, arXiv:2003,05476(2020)
S. Bazak et al., Mod. Phys. Lett. A3, 1850142 (2018)
W. Zhao et al., Phys. Rev. C98,054905 (2018)
F. Bellini et al., Phys. Rev. C99,054905 (2019)
K.J.Sun and C. M. Ko, Phys. Lett. B792, 132(2019)
D. Oliinychenko et al., Phys. Rev. C99, 044907(2019)
X. Xu and R. Rapp, Eur.Phys.J. A55,68(2019)
Y. Cai et al., Phys.Rev. C100 , 024911 (2019)
V. Vovchenko et al., arXiv:2004.04411(2020)
S. Mrowczynski, arXiv:2004.07029(2020)
K. Blum and M. Takimoto, Phys.Rev.C99, 044913(2019) ……

# 3. QCD criticality on light nuclei production






Joint distribution function in phase space:

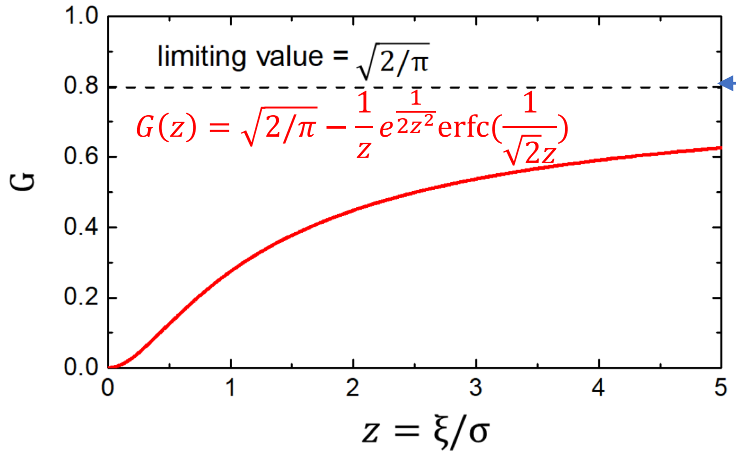
$$f_{np}(x_1,p_1;x_2,p_2) = \rho_{np}(x_1,x_2)(2\pi mT)^{-3} e^{-\frac{p_1^2+p_2^2}{2mT}}$$

$$\rho_{np}(x_1,x_2) = \underline{\rho_n(x_1)\rho_p(x_2)} + \underline{C_2(x_1,x_2)}$$

$$C_2(x_1-x_2) \approx \lambda\langle\rho_n\rangle\langle\rho_p\rangle \frac{e^{-|x_1-x_2|/\xi}}{|x_1-x_2|^{1+\eta}} \quad (singular\ part\ only)$$

*with $\xi$ being the density − density correlation length*

$$0 < \langle\delta N^2\rangle \sim \int dx C_2(x) \sim \lambda\xi^2 \rightarrow \lambda > 0$$




$$N_d = \frac{3}{\sqrt{2}}\left(\frac{2\pi}{mT}\right)^{\frac{3}{2}} N_p\langle\rho_n\rangle\left[1 + C_{np} + \frac{\lambda}{\sigma_d} G\left(\frac{\xi}{\sigma_d}\right)\right]$$

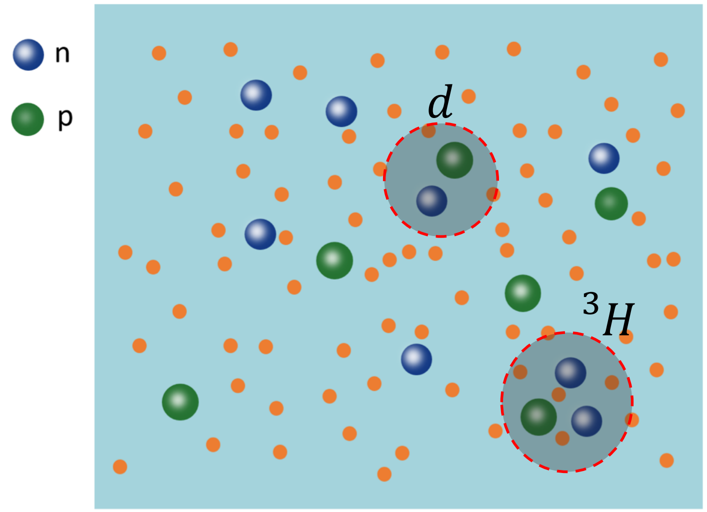
$$\rho_n(x) = \langle\rho_n\rangle + \delta\rho_n(x) \qquad C_{\rm np} = \langle\delta\rho_n(x)\delta\rho_p(x)\rangle/(\langle\rho_n\rangle\langle\rho_p\rangle)$$

$$\rho_p(x) = \langle\rho_p\rangle + \delta\rho_p(x) \qquad \Delta\rho_{\rm n} = \langle\delta\rho_n(x)^2\rangle/\langle\rho_n\rangle^2$$

K. J. Sun et al., Phys. Lett. B 774, 103 (2017)
K. J. Sun et al., Phys. Lett. B 781, 499 (2018)
K. J. Sun, C. M. Ko, and F. Li, arXiv:2008.02325(2020)

# 3. QCD criticality on light nuclei production



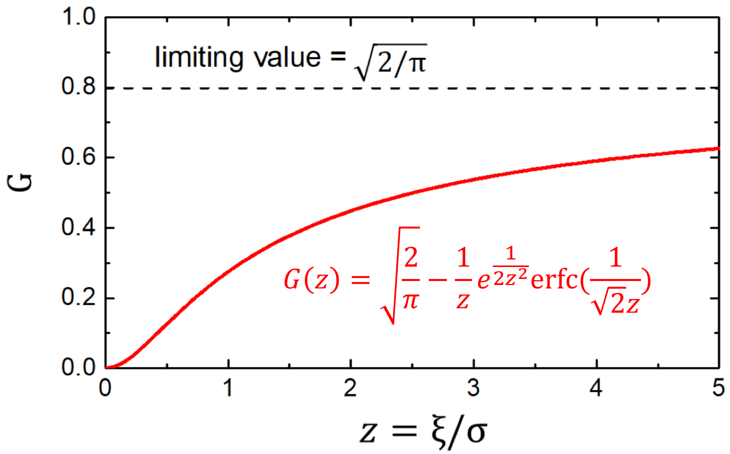



Joint distribution function in phase space:

$$f_{nnp}(x_1,p_1;x_2,p_2;x_3,p_3) = \rho_{nnp}(x_1,x_2,x_3)\,(2\pi mT)^{-\frac{9}{2}} e^{-\frac{p_1^2+p_2^2+p_3^2}{2mT}}$$

$$\rho_{nnp}(x_1,x_2,x_3) = \rho_n(x_1)\rho_n(x_2)\,\rho_p(x_3) + \rho_n(x_1)C_2(x_2,x_3) + \rho_n(x_2)C_2(x_1,x_3) + \rho_p(x_3)C_2(x_1,x_2) + C_3(x_1,x_2,x_3)$$

$$C_3(x_1,x_2,x_3) \sim \frac{\lambda'\langle\rho_n\rangle^2\langle\rho_p\rangle e^{-\frac{|x_1-x_2|+|x_2-x_3|}{\xi}}}{|x_1-x_2||x_2-x_3|} + (1\to 2, 2\to 3) + (1\to 3, 2\to 1)$$




$$N_t = \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^3 N_p\langle\rho_n\rangle^2\left[1 + 2C_{np} + \Delta\rho_n + \frac{3\lambda}{\sigma_d} G\left(\frac{\xi}{\sigma_t}\right) + O(G^2)\right]$$

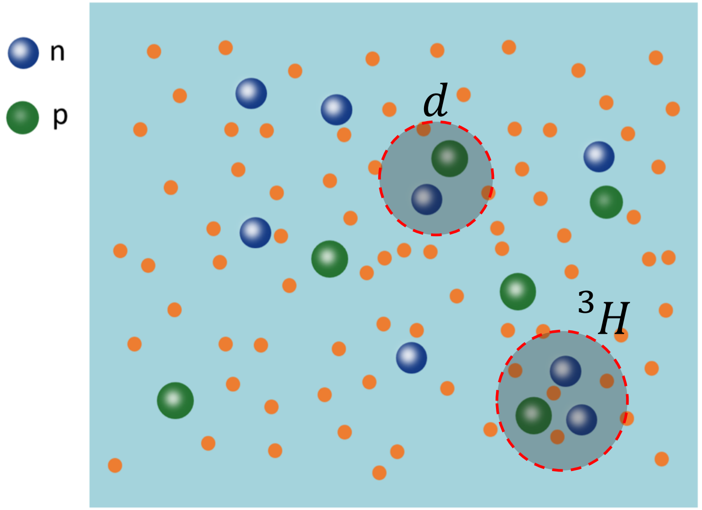
$$\rho_n(x) = \langle\rho_n\rangle + \delta\rho_n(x) \qquad C_{\mathrm{np}} = \langle\delta\rho_n(x)\delta\rho_p(x)\rangle/(\langle\rho_n\rangle\langle\rho_p\rangle)$$

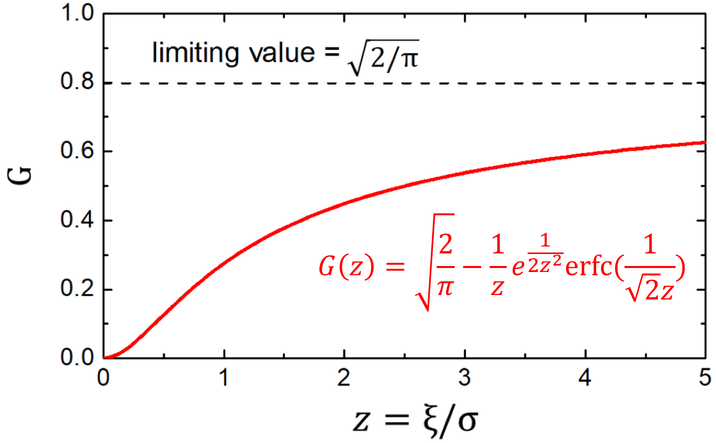
$$\rho_p(x) = \langle\rho_p\rangle + \delta\rho_p(x) \qquad \Delta\rho_{\mathrm{n}} = \langle\delta\rho_n(x)^2\rangle/\langle\rho_n\rangle^2$$

K. J. Sun, C. M. Ko, and F. Li, arXiv:2008.02325(2020)

# 3. QCD criticality on light nuclei production






$$N_d = \frac{3}{\sqrt{2}}\left(\frac{2\pi}{mT}\right)^{\frac{3}{2}} N_p \langle\rho_n\rangle \left[1 + C_{np} + \frac{\lambda}{\sigma_d} G\left(\frac{\xi}{\sigma_d}\right)\right]$$

$$N_t = \frac{3^{3/2}}{4}\left(\frac{2\pi}{mT}\right)^{3} N_p \langle\rho_n\rangle^2 \left[1 + 2C_{np} + \Delta\rho_n + \frac{3\lambda}{\sigma_d} G\left(\frac{\xi}{\sigma_d}\right) + O(G^2)\right]$$

Pre-factors are thermal yields w/o density fluc./corr.

Ratio: $$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}\left[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right]$$ $\sigma \approx 2$ fm

Enhancement of $\xi$ leads to enhancement of $tp/d^2$

A novel phenomenon of criticality different from the critical opalescence!

Heavier nucleus:

$$\frac{N_\alpha N_p}{N_{^3He} N_d} \approx \frac{2\sqrt{2}}{9\sqrt{3}}\left[1 + C_{np} + \Delta\rho_n + \frac{2\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right] \quad \frac{N_\alpha N_t N_p^2}{N_{^3He} N_d^3} \approx \frac{1}{27\sqrt{2}}\left[1 + C_{np} + 2\Delta\rho_n + \frac{3\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right]$$

K. J. Sun, C. M. Ko, and F. Li, arXiv:2008.02325(2020)

# 3. QCD criticality on light nuclei production



$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}\left[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right]$$

First-order phase transition

Second-order phase transition

Phase I: $\rho_I$

Phase II: $\rho_{II}$

Co-existence

small $\xi$, large density

Inhomogeneity $\Delta\rho_n$

large $\xi$, small density

Inhomogeneity $\Delta\rho_n$

**K. J. Sun, C. M. Ko, and F. Li, arXiv:2008.02325(2020)**

# 3. QCD criticality on light nuclei production

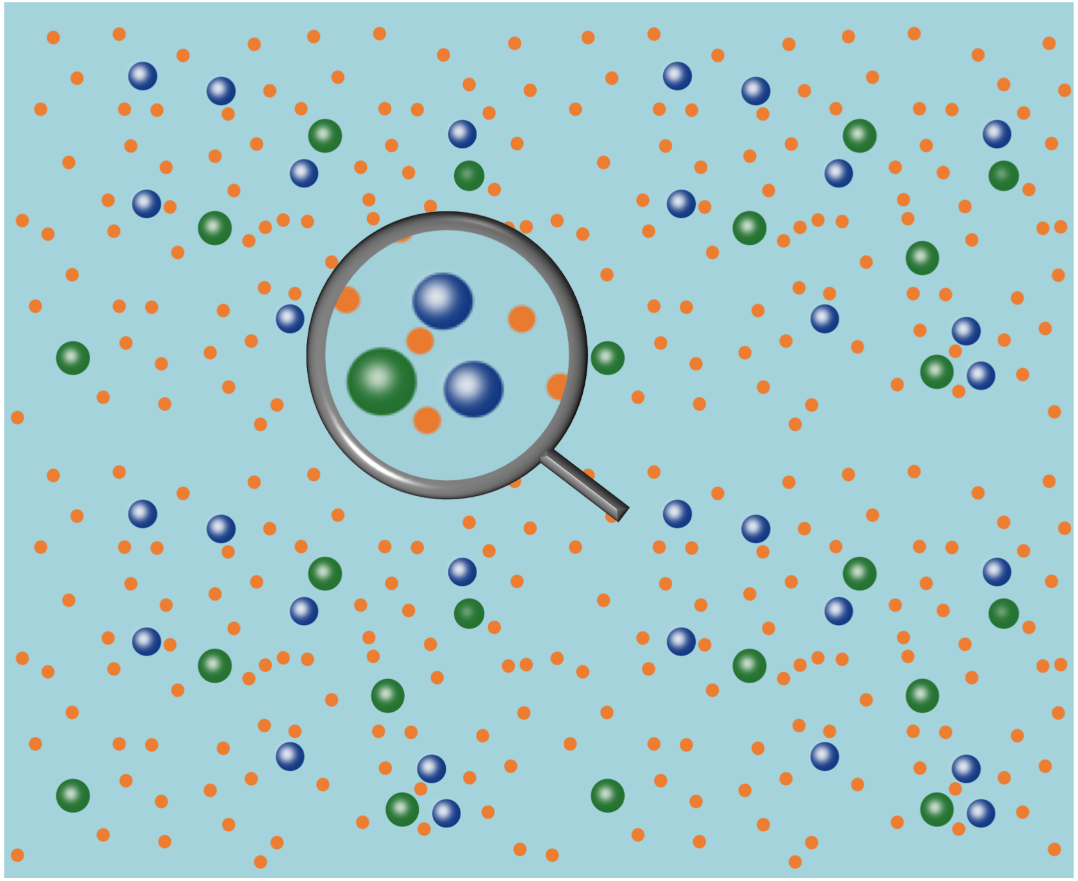


$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}\left[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right]$$

An unique feature:

The size of light nuclei provides a natural resolution scale $\sigma$ as small as 2 fm which is comparable or smaller than the correlation length $\xi$ that can be generated in realistic heavy-ion collisions near the CEP.

The light nuclei $(d, t)$, like a microscope, can 'see' the baryon density fluctuation and correlation.



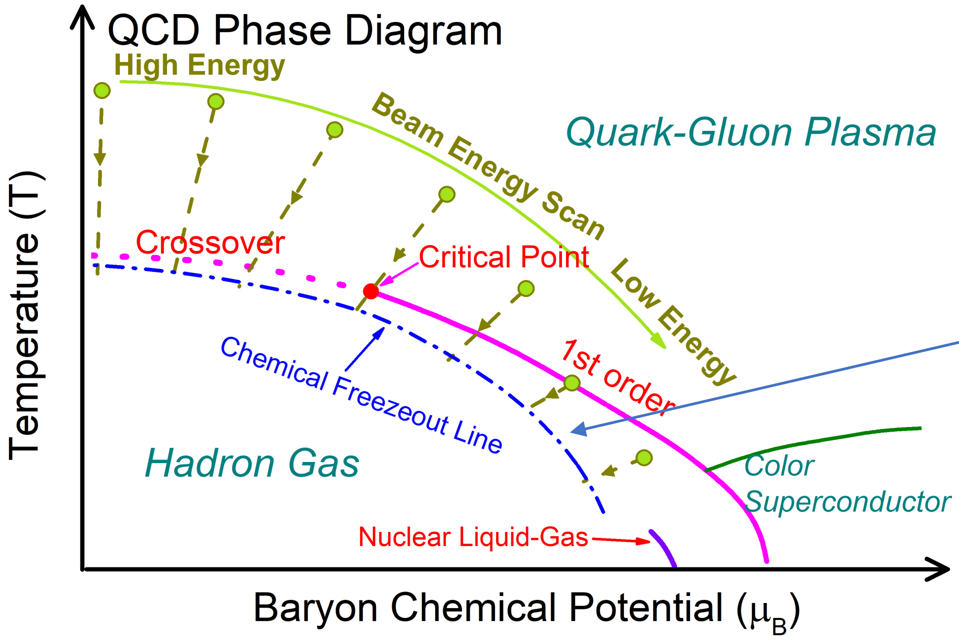
K. J. Sun, C. M. Ko, and F. Li, arXiv:2008.02325(2020)

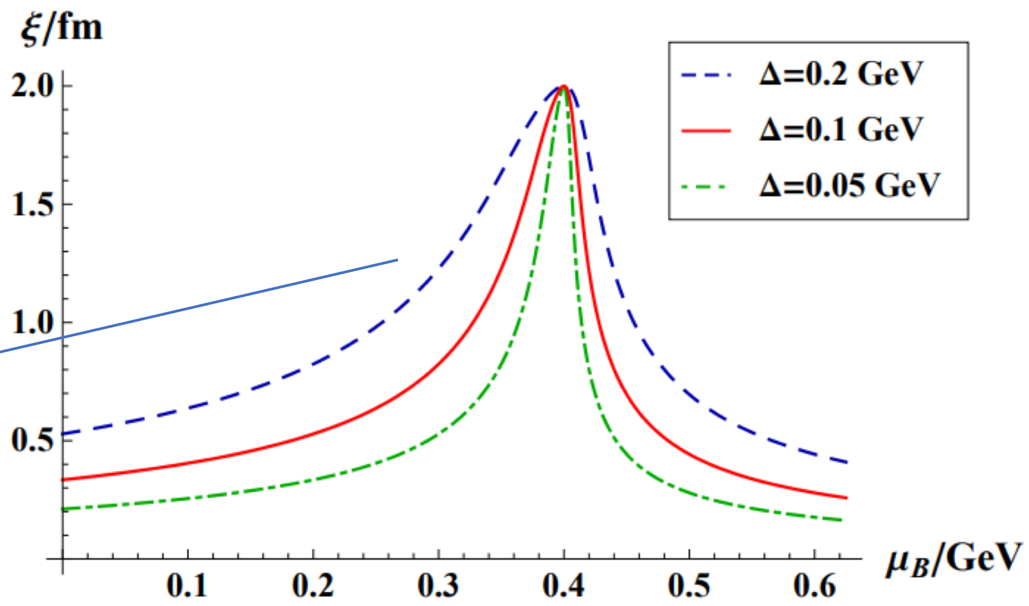
# 4. Enhancement of $tp/d^2$ near the critical point



$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)]$$

C. Athanasion, K. Rajagopal, and M. Stephanov, Phys. Rev. D82, 074008 (2010)




$$\xi(\mu_B) = \frac{\xi_{max}}{[1 + \frac{(\mu_B - \mu_B^c)^2}{W(\mu_B)^2}]^{1/3}}$$

$$W(\mu_B) = \mathrm{W} + \delta W \tanh\left(\frac{\mu_B - \mu_B^c}{w}\right) \qquad \mathrm{W} \approx 2.2\,\delta W$$

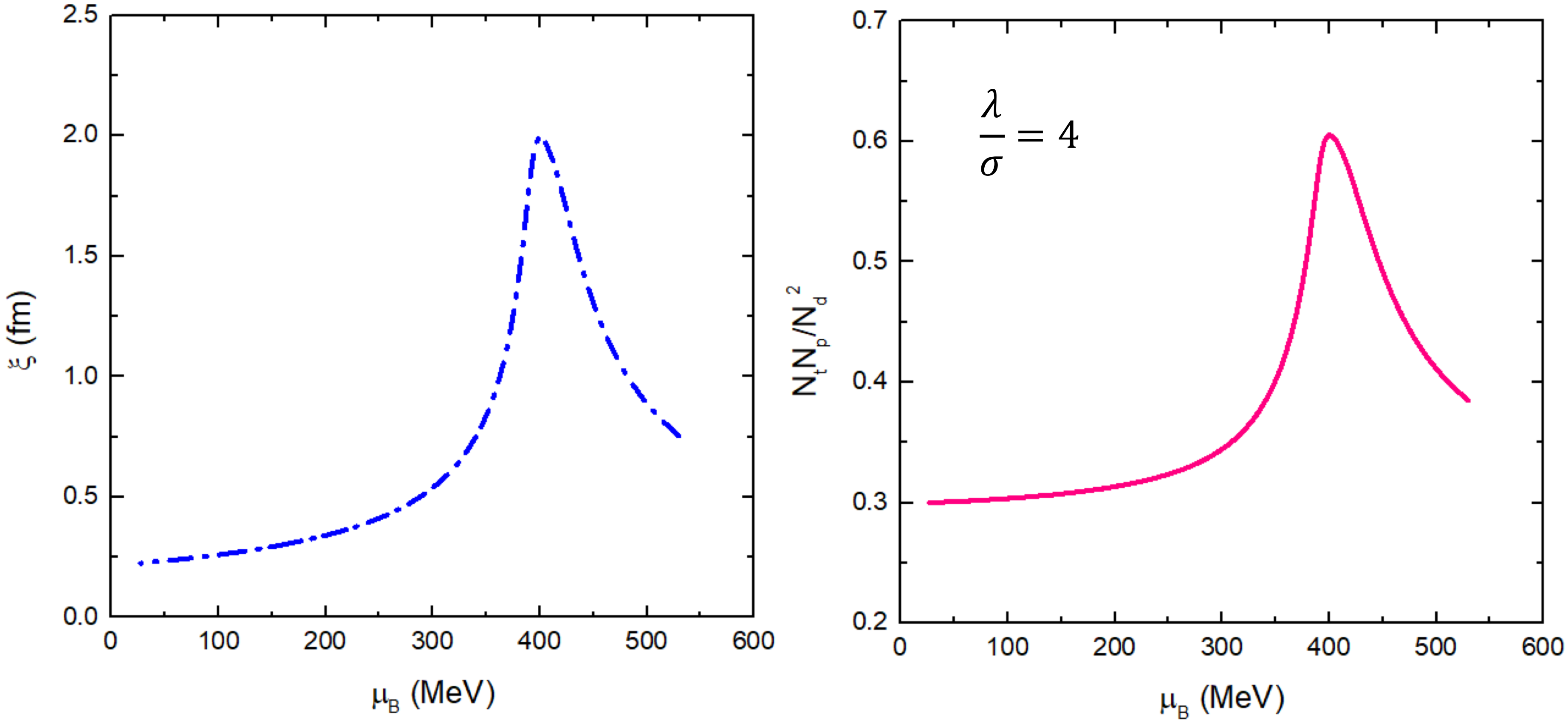
Peak of $\xi$ near the critical point

# 4. Enhancement of $tp/d^2$ near the critical point



$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}\left[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)\right]$$

$$G(z) = \sqrt{\frac{2}{\pi}} - \frac{1}{z} e^{\frac{1}{2z^2}} \mathrm{erfc}\left(\frac{1}{\sqrt{2}z}\right)$$




Peak of $\xi$ leads to peak of $tp/d^2$

# 4. Enhancement of $tp/d^2$ and first-order phase transition



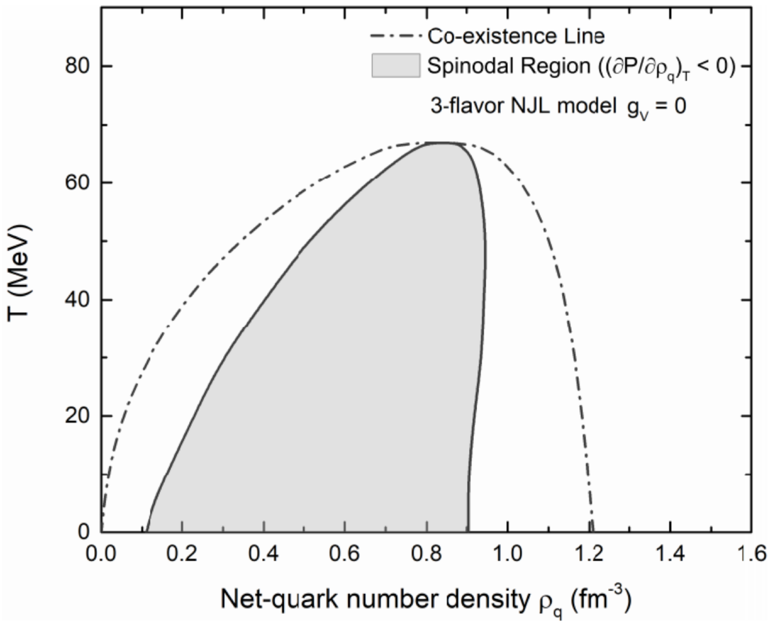
K. J. Sun et al., arXiv:2006.08929(2020)
M. Buballa, Phys. Rept. 407, 205 (2005)

3-flavor NJL model:

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_S + \mathcal{L}_V + \mathcal{L}_{det},$$

with

$$\begin{aligned}
\mathcal{L}_0 &= \bar{\psi}(i\gamma^\mu \partial_\mu - \hat{m})\psi, \\
\mathcal{L}_S &= G_S \sum_{a=0}^{8} [(\bar{\psi}\lambda^a\psi)^2 + (\bar{\psi}i\gamma_5\lambda^a\psi)^2], \\
\mathcal{L}_V &= -g_V(\bar{\psi}\gamma^\mu\psi)^2, \\
\mathcal{L}_{det} &= -K[\det\bar{\psi}(1+\gamma_5)\psi + \det\bar{\psi}(1-\gamma_5)\psi]
\end{aligned}$$



# 4. Enhancement of $tp/d^2$ and first-order phase transition 

K. J. Sun et al., arXiv:2006.08929(2020)
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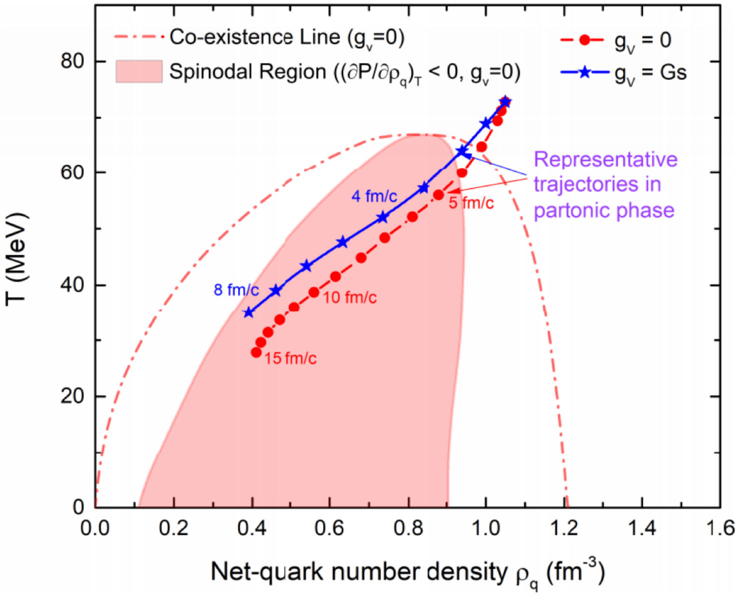
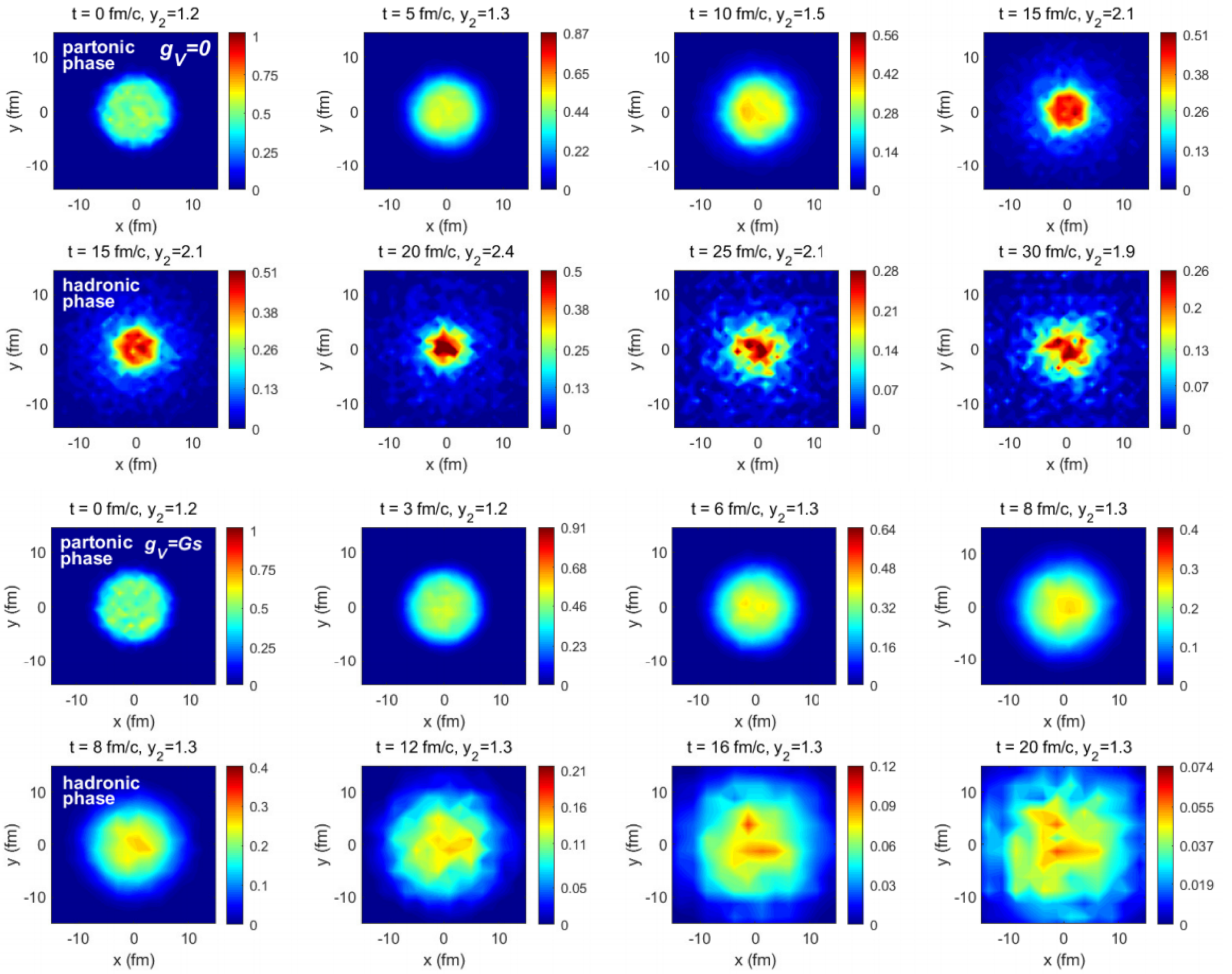
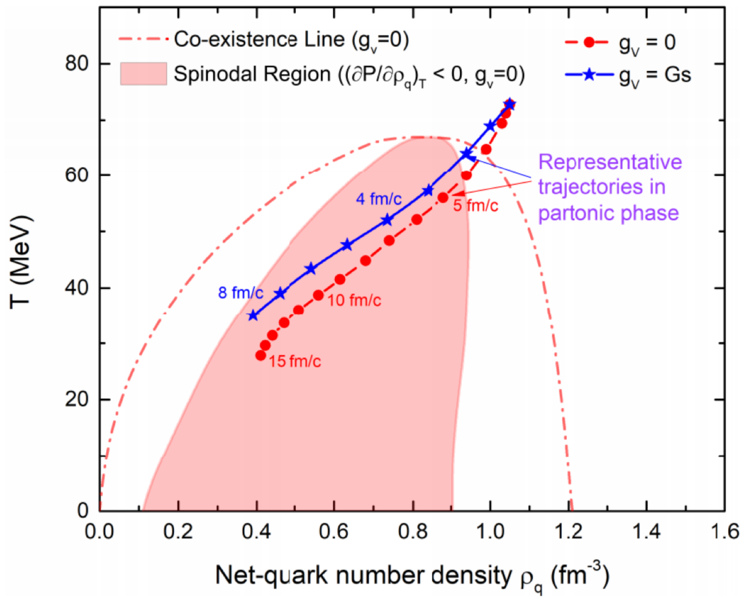


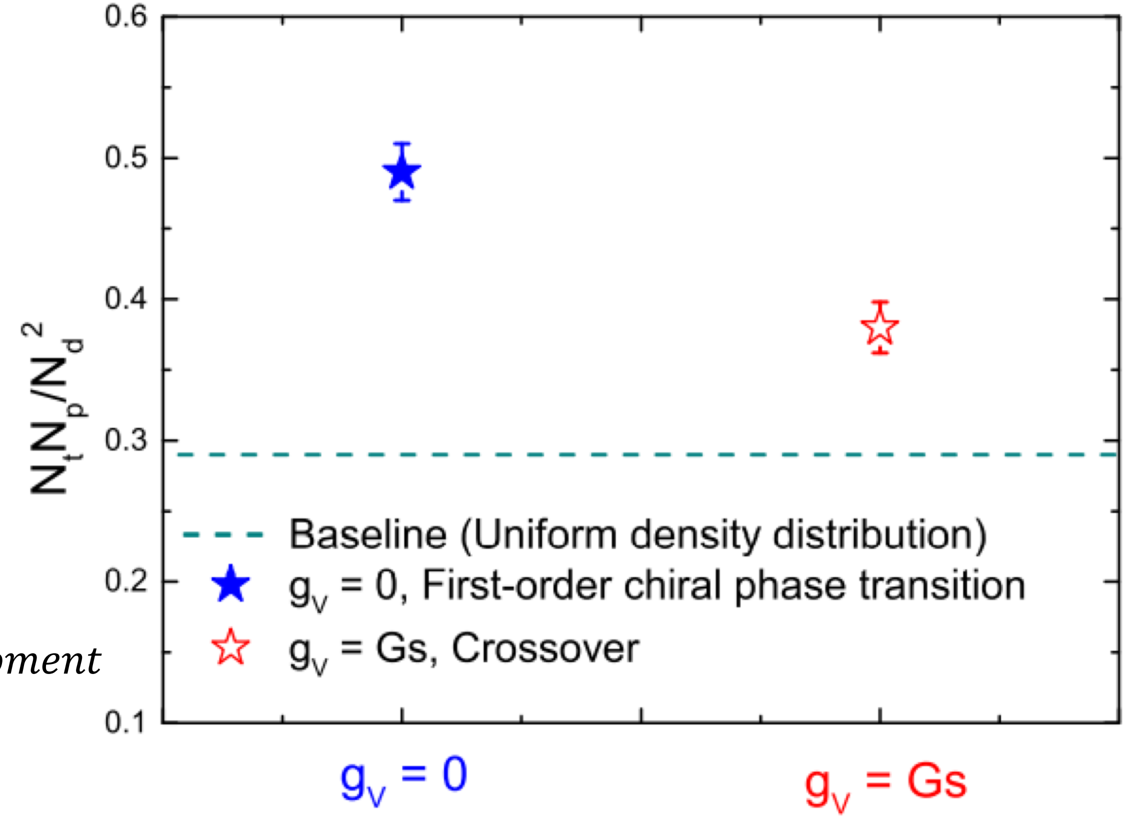



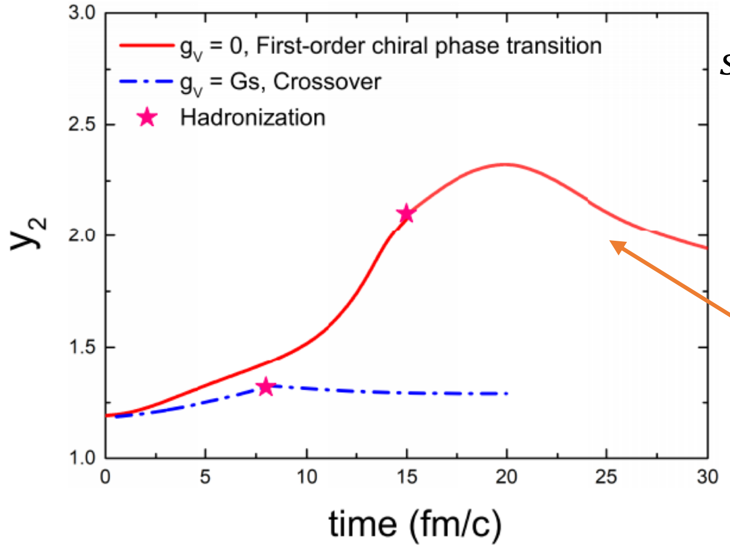
# 4. Enhancement of $tp/d^2$ and first-order phase transition






K. J. Sun et al., arXiv:2006.08929(2020)




*scaled density moment*
$y_2 \approx 1 + \Delta\rho_n$

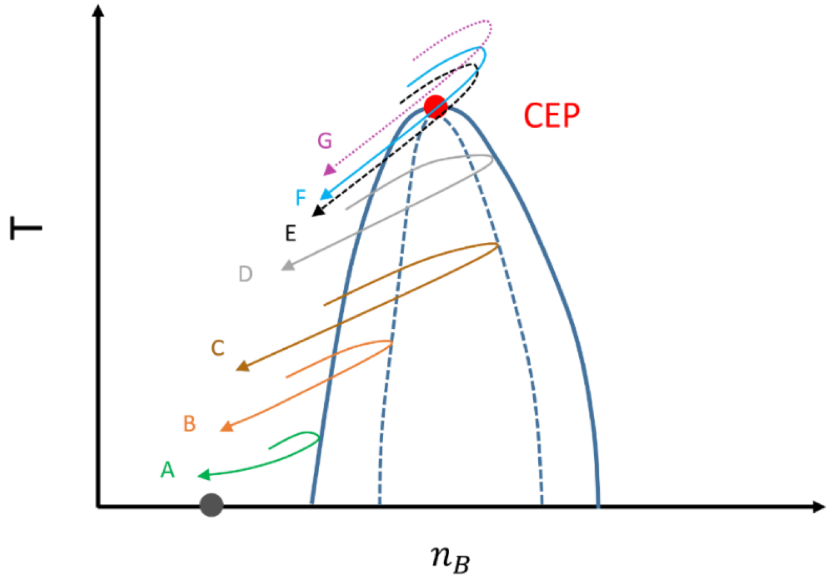
Further increasing $g_v$, this ratio remains unchanged

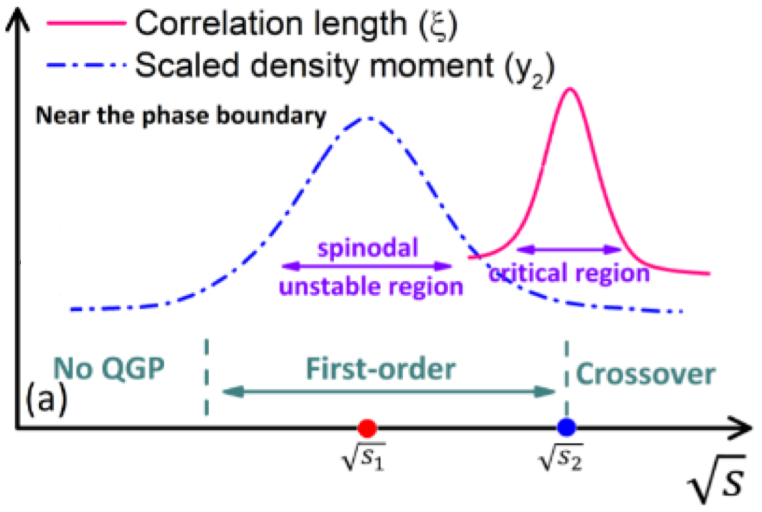
Large density inhomogeneity survives to kinetic freezeout

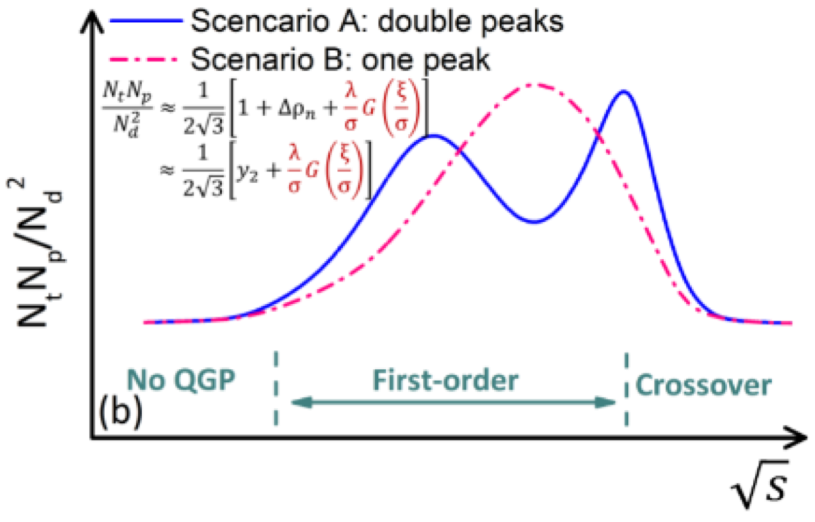
We expect long-range correlation also survives

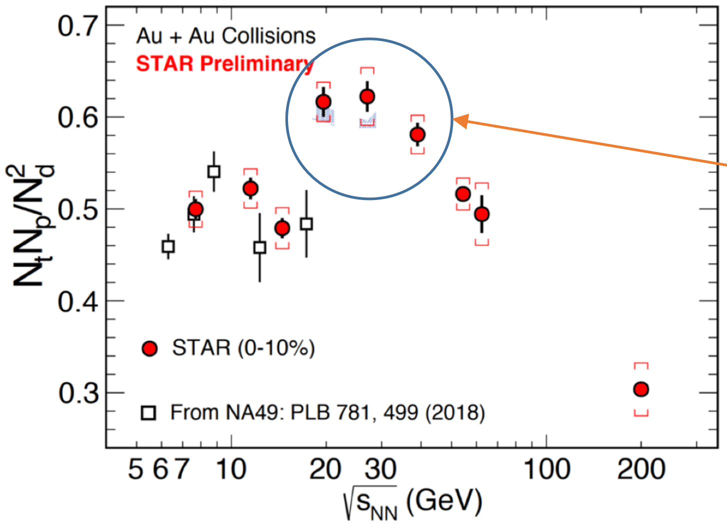
# 4. Collision energy dependence of $tp/d^2$





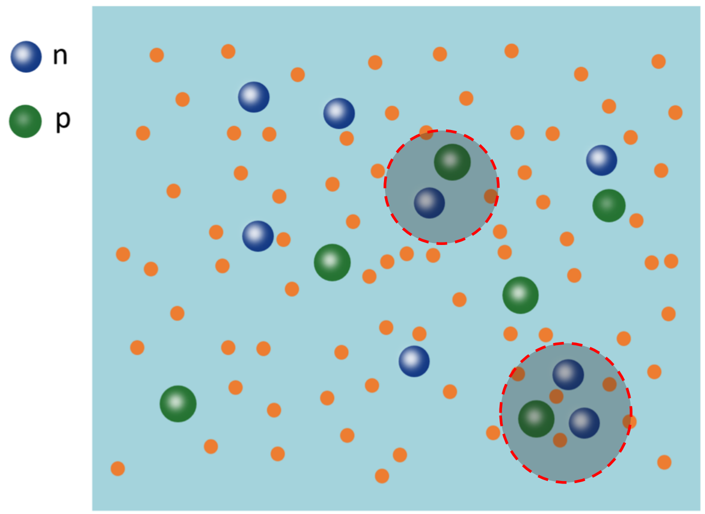




Signal from critical point?

*Realistic dynamical modeling of the non-smooth quark-hadron phase transition within transport or hydro approaches is indispensable!*

K. J. Sun et al., Phys. Lett. B 774, 103 (2017)
K. J. Sun et al., Phys. Lett. B 781, 499 (2018)
K. J. Sun et al., arXiv: 2006.08929(2020)
K. J. Sun, C. M. Ko, and F. Li, arXiv:2008.02325(2020)

# 5. Baryon clustering near the QCD critical point

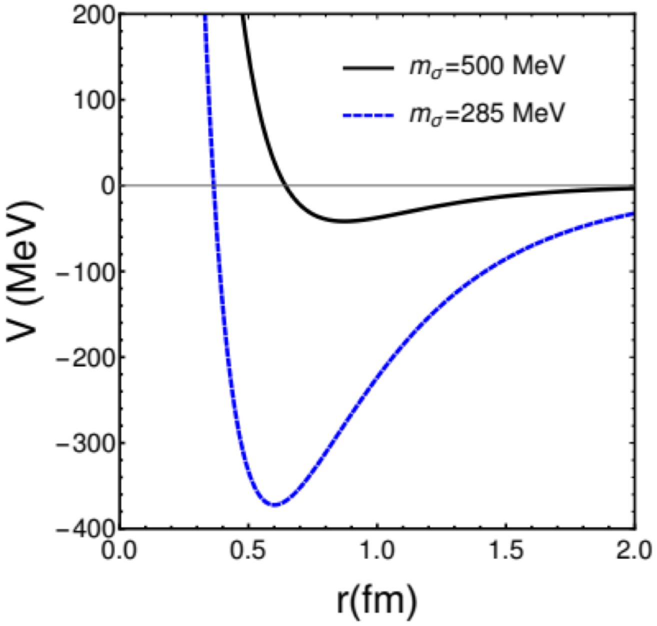



Nucleon-Nucleon potential:

$$V_A(r) = -\frac{g_\sigma^2}{4\pi r}e^{-m_\sigma r} + \frac{g_\omega^2}{4\pi r}e^{-m_\omega r}$$

$$g_\sigma^2 = 267.1\left(\frac{m_\sigma^2}{m_N^2}\right), \qquad g_\omega^2 = 195.9\left(\frac{m_\omega^2}{m_N^2}\right)$$

Near the critical point, the mass of sigma meson is reduced




1. Precluster formation is related to $\exp(-V(r_{min})/T)$, thus the modified NN potential leads to stronger baryon clustering.
2. Preclusters decay into bound nuclei which are observed in experiments.

$$\mathcal{O}_{tpd} \simeq 0.29\frac{\langle e^{-3V/T}\rangle}{\langle e^{-V/T}\rangle^2} \qquad \mathcal{O}_{\alpha p^3\mathrm{He}d} \equiv \frac{N_\alpha N_p}{N_{^3\mathrm{He}}N_d} \simeq 0.18\frac{\langle e^{-6V/T}\rangle}{\langle e^{-3V/T}\rangle\langle e^{-V/T}\rangle} \qquad \mathcal{O}_{\alpha t p^3\mathrm{He}d} \equiv \frac{N_\alpha N_t N_p^2}{N_{^3\mathrm{He}}N_d^3} \simeq 0.05\frac{\langle e^{-6V/T}\rangle}{\langle e^{-V/T}\rangle^3}$$

From thermal model, modified NN potential leads to larger binding energy, thus large yields of light nuclei

E. Shuryak and J. M. Torres-Rincon, arXiv:1805.04444(2018)
E. Shuryak and J. M. Torres-Rincon, arXiv:1910.08119(2019)
E. Shuryak and J. M. Torres-Rincon, arXiv:2005.14216(2020)

# 6. Summary



1. The long-range correlation near the QCD critical point leads to enhancement of light nuclei production and their yield ratios.

$$\frac{N_t N_p}{N_d^2} \approx \frac{1}{2\sqrt{3}}[1 + \Delta\rho_n + \frac{\lambda}{\sigma} G\left(\frac{\xi}{\sigma}\right)]$$

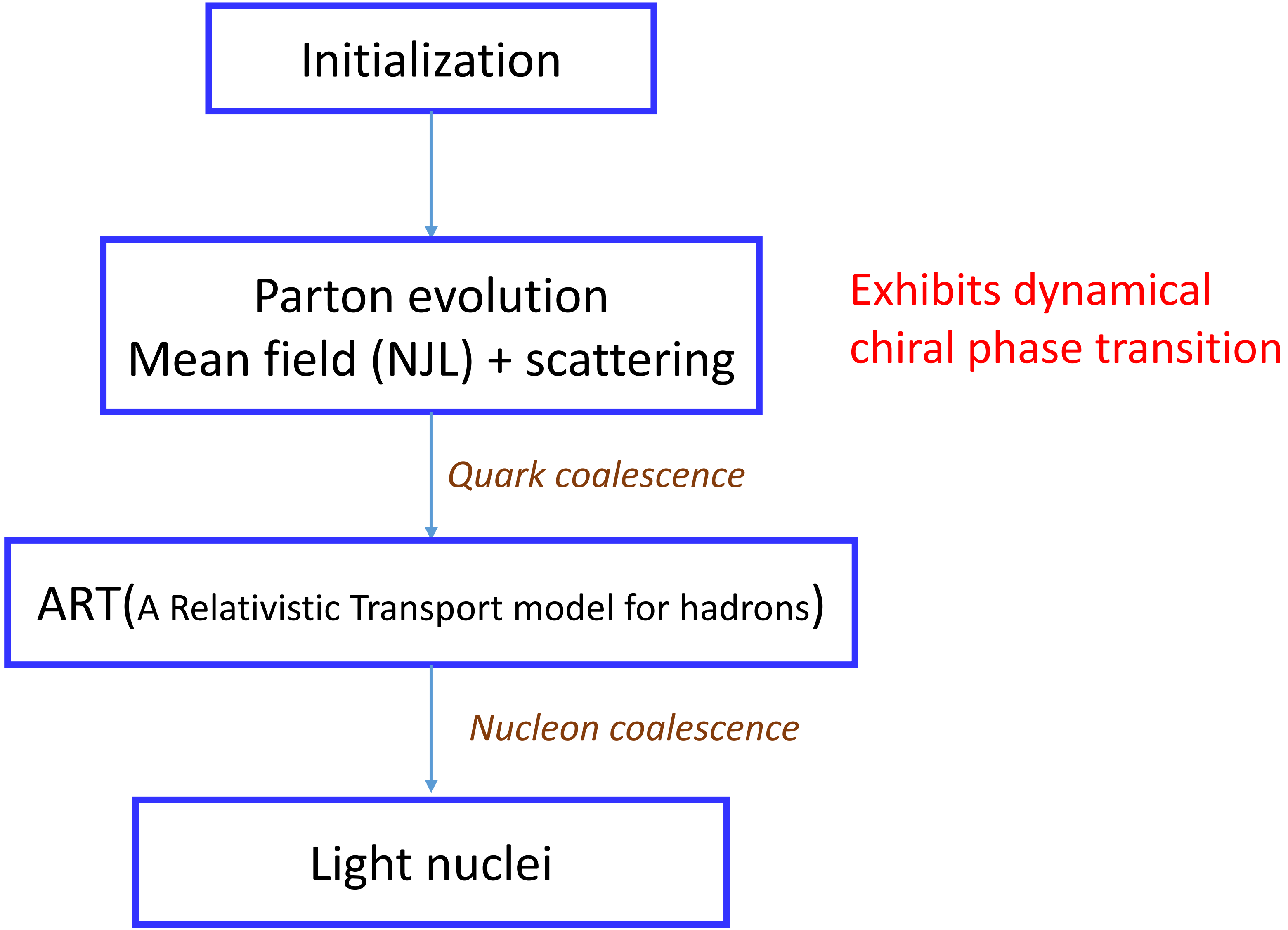
2. This novel phenomena of criticality opens up new possibilities to probe the QCD critical point with light nuclei production in relativistic heavy-ion collisions.

3. The observed non-monotonic behavior of $tp/d^2$ could be due to the non-smooth phase transitions from QGP to hadronic gas.

4. To better understand the experimental results and locate the phase boundary in QCD phase diagram, we need better understanding of light nuclei production and better modeling of quark-hadron phase transition with transport or hydro approaches.

Thank you very much!

# 7. Backup


Initialization
Parton evolution
Mean field (NJL) + scattering
Exhibits dynamical
chiral phase transition
Quark coalescence
ART(A Relativistic Transport model for hadrons)
Nucleon coalescence
Light nuclei