The 111^{th} Online Seminar of High Energy Nuclear Physics in China

Jet production and properties in heavy-ion collisions in a transport model

Weiyao Ke and Xin-Nian Wang, work in preparation

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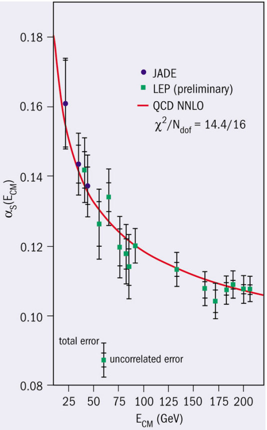
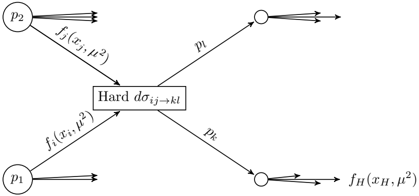
Short-distance QCD process on the collider



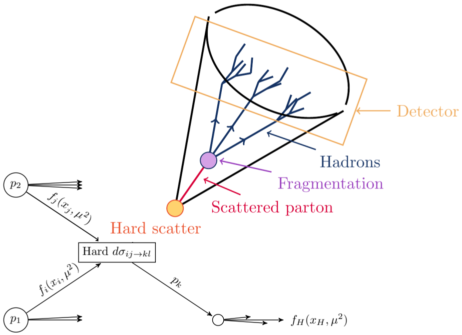
https://cerncourier.com/

- In high energy colliders, occasionally, one triggers a QCD scattering involves a large momentum transfer $Q \sim p_T \gg \Lambda_{QCD}$.
- The asymptotic freedom nature of QCD: coupling decreases with increasing energy scale $\alpha_s = a_0/\ln(Q^2/\Lambda^2_{QCD})$.
- One can understand the short distance process in terms of few-body & perturbative parton scattering $d\sigma[i+j \to k+l]$.
- But this is not what can be observed at larger distance.



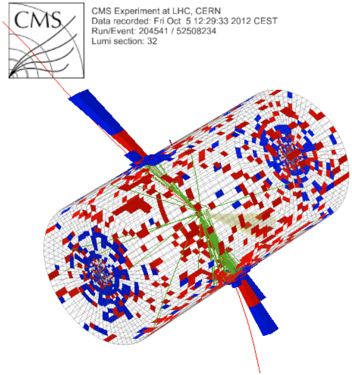
Evolution of the hard QCD process in vacuum

- Large scale Q parton tends to radiate soft & collinear partons $dP_{gg}^{g}, dP_{qg}^{q} \sim g^2 C_R \frac{dx}{x} \frac{dk_T^2}{k_T^2}$.
- Perturbative evolution starts from hard scale $Q \sim p_T$ down to $Q \gtrsim \Lambda_{QCD}$.



- Original hard partons evolve into parton showers.
- Parton shower undergoes hadronization when $Q \sim \Lambda_{\rm QCD}$, followed by hadronic decay.
- Final states: collimated bunch of particles.
- Theoretically, we want to define objects "jets" as close analog of the parton from the hard process.

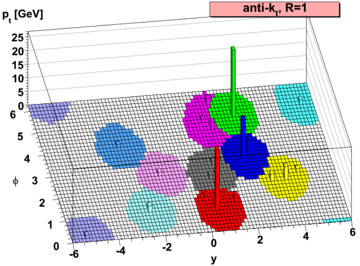
Jets in vacuum and definition



Event display from CMS

- Experimentally, one needs to identify jets from a list of "particles" (p^{μ}).
- Operational definition with jet finding algorithms.
  - Angular distance $\Delta r_{ij} = \sqrt{\Delta\phi^2 + \Delta\eta^2}$.
  - Define new distances $d_{ij} = \min(k_{T,i}^{2p}, k_{T,j}^{2p})\frac{\Delta r_{ij}}{R}$.
  - Iteratively group the four momentum of "nearest" "particles" into jets.
  - "R" is the jet distance parameter (radius).
- Insensitive to a soft or collinear splitting.

Jets in vacuum and definition



M. Cacciari et al JHEP04(2008)063

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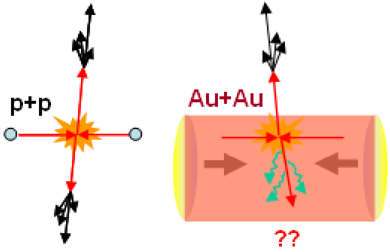
Why study jets in heavy-ion collisions?

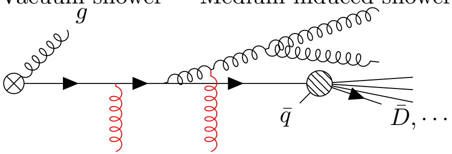
- Heavy-ion collision produces a quark-gluon plasma medium with color degrees of freedom.
- Medium displays near-equilibrium features. Introducing additional scales added to the problem: temperature, medium size, $\cdots$.



What to learn from jets in HIC

- Jets / high-p_T are expected to lose energy due to interactions with medium. Suppressed production yield, modified structure $\cdots$.
- Understand jet evolution in hot QCD medium from medium modifications to jet.
- What can be learned about the medium?

Single parton interacts with medium (in a weakly coupled picture)



Equilibrium distribution $f(p) \sim e^{-p\cdot u/T}$
Screening mass $m_D^2 = \left(\frac{N_c}{3} + \frac{N_f}{6}\right) g^2 T^2$.

- **Hard**: $p \gg T$. **Elastic collisions**: direct momentum exchange between hard parton and medium constituents.
- Rate: number of collision per unit time,

$$\frac{dP}{dt dq_\perp^2} \propto T^3 \frac{\alpha_s^2}{q_\perp^2(q_\perp^2 + m_D^2)}$$

- A more physical quantity than rate is the so-called jet transport parameter $\hat{q}$, which measures the momentum broadening per unit time, directly related to medium properties.

$$\hat{q}_R = \frac{d\langle(\Delta p_\perp)^2\rangle}{dt} = \int q_\perp^2 \frac{dP}{dt dq_\perp^2} dq_\perp^2 = \alpha_s C_R T m_D^2 \ln \frac{Q_{\max}^2}{m_D^2}$$

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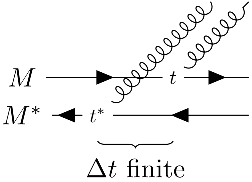


Medium-induced radiation

- Radiates of another parton due to collision with medium.
- Inelastic: energy is shared among two hard daughter partons.

Single radiation probability[1]. breaks into two pieces $dP = dP_{\mathrm{vac}} + dP_{\mathrm{med}}$,

$$\frac{dP_{\mathrm{med}}}{dtdx} = \frac{P_{gg}^{g}(x)}{2x(1-x)E}\int_0^t dt' dk_\perp^2 dq_\perp^2 \langle k_\perp | iV_3 e^{iH_3 t'} | q_\perp \rangle, \quad |k_\perp\rangle = \frac{\vec{k}_\perp}{k_\perp^2}$$

Radiation is not localized, $\Delta t^{-1} \sim$ average formation time in the medium $\langle \tau_f^{-1} \rangle$

$$H_3 \approx \frac{p_\perp^2}{2x(1-x)E} + i\frac{1}{2}\hat{q}_{\mathrm{eff}} b^2 + \cdots, \quad \begin{cases} H_3 = \Omega a^\dagger a + \cdots \\ \Omega = \sqrt{i2x(1-x)E\hat{q}_{\mathrm{eff}}} = \sqrt{i}\langle \tau_f^{-1} \rangle \end{cases}$$

[1] Zakharov JETP 63 952 and 65 615; Caron-Huot, Gale, PRC 82 064902

Multiple collisions & multiple medium-induced radiation

If subsequent interactions are independent ($\tau_f \ll \lambda$, mean-free-path), use a Boltzmann equation to include multiple collisions and multiple induced radiation.

$$p \cdot \partial f_H(t, x, \mathbf{p}) \quad = \quad -E\mathcal{C}[f_H, f_S], \quad \begin{cases} f_H \text{ distribution of hard particles } E \gg T \\ f_S = e^{-p\cdot u/T}, \text{soft sector obtain, e.g., from hydro} \end{cases}$$

$\mathcal{C}[f_H; T, u]$ contains elastic and radiation rates. For now, one also neglects:

- Back reaction from f_H to f_S.
- Hard-hard collisions.

Limitation of localized collision terms in treating medium-induced radiation

A closer look at the independent assumption: Elastic mean-free-path $\lambda_{\rm el} \sim 1/g^2 T$.

- Between elastic collisions: $\tau_f \sim 1/m_D \sim 1/gT \lesssim \lambda$. $(\checkmark)$
- Between induced radiation and elastic collision: $\langle \tau_f \rangle = \frac{2x(1-x)E}{\langle k_\perp^2 \rangle} \gg \lambda_{\rm el}$. (?)

  Especially, this leads to the LPM[2] effect in large & dense medium. Multiple collision reduces the radiation rate compared to the independent limit.

  $$R \sim \alpha_s P(x) \frac{1}{\langle \tau_f \rangle} \text{ v.s. } \alpha_s P(x) \frac{1}{\lambda_{\rm el}}.$$

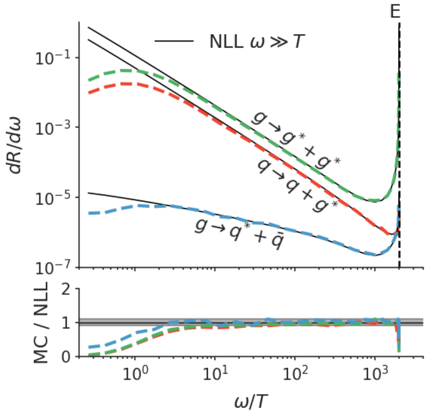
  Special treatment[3] needed in Boltzmann equation.
- Between induced radiations[4]: $\langle \tau_f \rangle < 1/R \sim \langle \tau_f \rangle / \alpha_s$. $(\checkmark)$

[2] The Landau-Pomeranchuk-Migdal effect

[3] For example, the LBT model uses a time-dependent radiation rate, Cao et al PRC 94 014909

[4] See works by Arnold, Iqbal, and Chang for efforts towards overlapped radiations, JHEP04(2015)070, ⋯

Modify the Boltzmann equation



Compare the single radiation rate to the theory in infinite medium

To be modified: Boltmzann equation with induced radiation rate in the independent collision limit. Then,

- Each radiation is suppressed by a probability [1] $\propto \frac{\lambda}{\tau_f(t)}$ accounting for the LPM effect.
- $\tau_f(t) = \frac{2x(1-x)E}{k_\perp^2(t)}$ is determined in simulation.
- Effect of delocalized multiple collisions is included in the broadening of $k_\perp^2(t)$.
- Good agreement with theory expectation in the infinite large static medium.

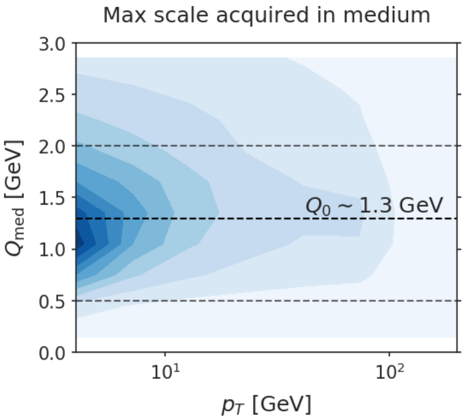
[1] An improved suppression factor with NLL correction Arnold and Dogan PRD 78 065008 is implemented in the simulation $\frac{\lambda_{\rm el}}{\tau_f}\left(\frac{\ln \hat{q}\tau_f/m_D^2}{\ln Q_{\max}^2/m_D^2}\right)^{1/2}$ WK et al PRC 100, 064911

Multiple radiation with both vacuum and medium-induced contribution

- Vacuum radiation was only subtracted at single radiation level in the transport equation $dP_{\rm med} = dP_1 - dP_{\rm vac} \to$ only treats multiple induced radiations of low scale partons.
- Multiple large-scale vacuum radiation should be summed in QCD evolution equation [5].
- In this work, we use a single scale parameter Q_0 to separate the two contribution. Q_0 should be the typical scale parton obtained from medium-induced radiation, $Q_0 \sim k_\perp$.

[5] The JETSCAPE collaboration develops a modular approach for this problem. Different evolution equations are applied to particles with different scale and energy.

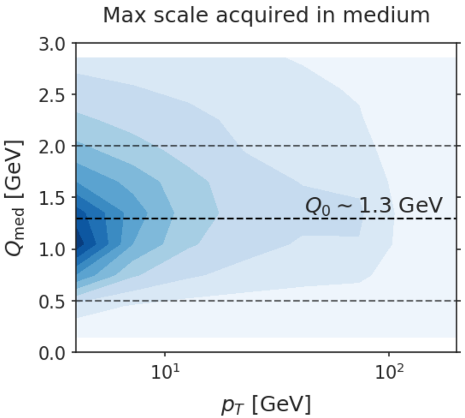
Multiple radiation with both vacuum and medium-induced contribution



Obtained from simulation using event with initial hard $p_T^{\rm hard} \approx 100$ GeV

- Hard process generated in `Pythia8`.
- Vacuum shower: $Q = p_T$ down to Q_0 (`Pythia8`). Q_0 estimates scales of induced radiation.
- Evolve parton shower by the transport equation.

Multiple radiation with both vacuum and medium-induced contribution



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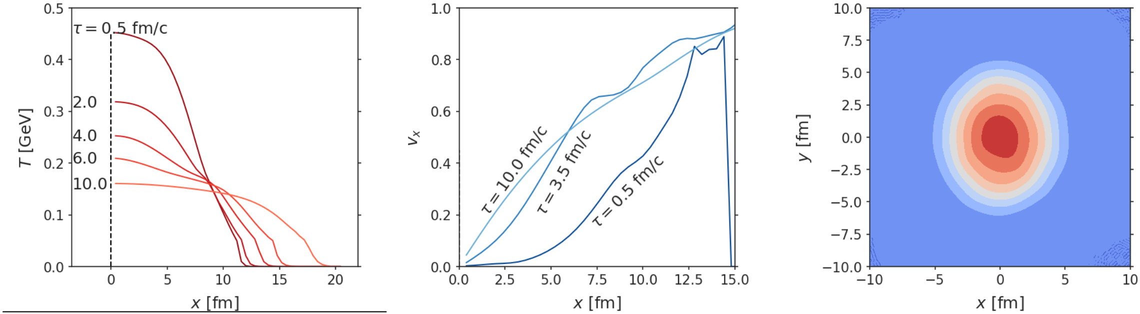
- Vacuum shower: $Q = p_T$ down to Q_0 (`Pythia8`). Q_0 estimates scales of induced radiation.
- Evolve parton shower by the transport equation.
- Outside hot medium, vacuum shower starting from scale acquired in medium, down to $Q_{\rm min} = 0.4$ GeV.
- Hadronization. `Pythia8` implementation of Lund string fragmentation.

Medium evolution

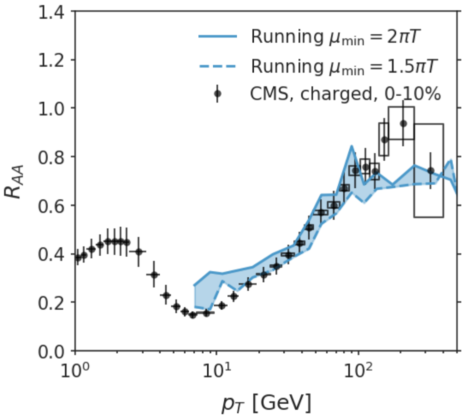
A hydrodynamic based medium simulation[6] provides space-time information of medium temperature (T) and flow velocity (v).

- Event-averaged initial condition + free-stream + (2+1) D viscous hydrodynamics.
- Hard production vertices sampled according binary collision density.
- Below: 0-10% central event for Pb+Pb @ 5.02 TeV.



[6] hic-eventgen, Bernhard arXiv:1804.06469

Energy loss and leading hadron production



CMS JHEP04(2017)039

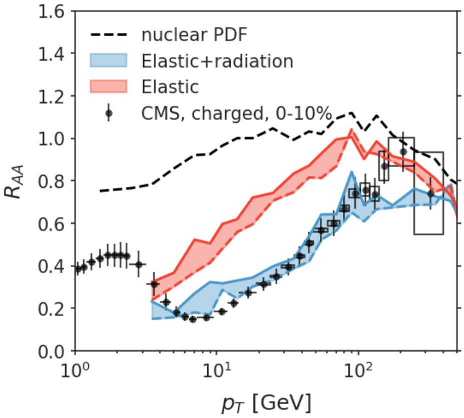
Nuclear modification factor: production yield of hard particles in AA relative to scaled pp.

$$R_{AA} = \frac{dN^{AA\to h}/dp_T}{N_{coll}\, dN^{pp\to h}/dp_T}$$

- Leading order running coupling [1] $\alpha_s = \frac{a_0}{\ln(Q^2/\Lambda^2)}$
- Running is truncated at medium scale μ_{min}: $\alpha_s = \alpha_s(\max\{Q, \mu_{min}\})$.
- μ_{min} controls in-medium coupling strength. Shown range $\mu_{min} \in [1.5\pi T, 2\pi T]$.

[1] $a_0 = \frac{12\pi}{(11N_c - 2N_f)}$ and $\Lambda_{QCD} = 0.2$ GeV

Energy loss and leading hadron production



CMS JHEP04(2017)039

What contributes to the single particle suppression?

- Elastic: frequent but small fraction of energy loss per collision.
- Radiation: rare, but efficient in carrying away large fraction of energy in a single splitting.
- Radiation is increasingly important at high p_T.

Hard parton dynamics with collision+ radiation does a good job for hard particle production in HIC. How about jets?

Transport of soft energy-momentum carrier

- Energy-momentum conservation is important for jet study. Transport equation only handles hard $E, Q \gg T$, loses track of soft particles.
- In principle, a self-consistent treatment requires coupled evolution of hard and soft. For example, the CoLBT model[7] uses a 3+1D hydrodynamic equation for medium evolution and medium excitation induced by hard parton.

$$
\begin{aligned}
p \cdot \partial f_H &= -E\mathcal{C}[f_H, f_S] \\
\partial_\mu T_S^{\mu\nu} &= \sum_i \delta(x^\mu - x_i^\mu(\tau))\Delta p_i^\nu
\end{aligned}
$$

The coupled evolutions are computationally intensive.

- In this work, we used a simpler method to
  1) Impose energy-momentum conservation.
  2) Qualitative behaviors of hydro-like excitations.

[7] Chen et al, PLB 777 86-90

Ansatz for medium excitation induced by jet: simplified hydrodynamics

Assumptions that make the problem manageable w/o numerical hydrodynamics,

- Energy-momentum deposition to soft sector is a perturbation $\delta e \ll e, \cdots$.
- Typical frequency & wave-number of the perturbations are much larger than those of background $\partial \ln \delta e(k) \gg \partial \ln e(k)$
- Speed-of-sound $c_s \approx$ constant. Drop viscous effects.
- Propagation in the η_s is small: $\Delta\eta_s \sim \frac{\Delta z}{\tau} \sim \frac{c_s \Delta\tau}{\tau} \sim c_s$.
- Neglect background radial flow in hydrodynamic equation.

Then in Bjorken frame, the energy density response to energy momentum deposition ΔP^μ is[8]

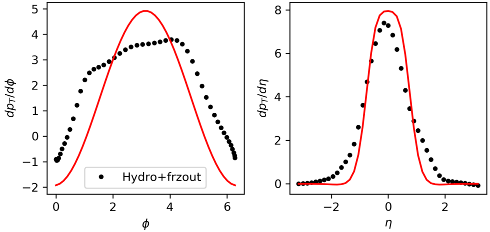
$$\Delta\tilde{G}^0 = \frac{\Delta P^0 k^0 + \delta\mathbf{P}\cdot\hat{\mathbf{k}}}{\omega^2 - c_s^2 k^2} \xrightarrow{\text{Angular distribution}} \frac{d\Delta G^0}{d\hat{\mathbf{k}}} \sim \frac{\Delta P^0 + \hat{\mathbf{k}}\cdot\Delta\mathbf{P}/c_s}{4\pi}$$

[8] Similar relation holds for momentum density perturbation $\Delta\mathbf{G}$

Ansatz for medium excitation induced by jet: freezeout effect

- Convert perturbations in energy-momentum density $\Delta G/\Delta V$ into change in distribution function (massless particles).
- Use a naïve freeze surface proportional to the velocity profile $\Delta\Sigma \sim \Delta V u^\mu$ with $v_{\hat{k}} = v_r \hat{k}$.

$$\frac{d\Delta p_T}{d\phi d\eta} = \int \Delta f(p) p_T^2 dp_T = \sum_{\text{sources } s} \int \frac{3}{(4\pi)^2} \frac{\frac{4}{3}(u_p\cdot u)u_\mu - u_{p,\mu}}{(u_p\cdot u)^4} \frac{d\Delta G_s^\mu(\hat{k})}{d\hat{k}'} d\hat{k}'$$



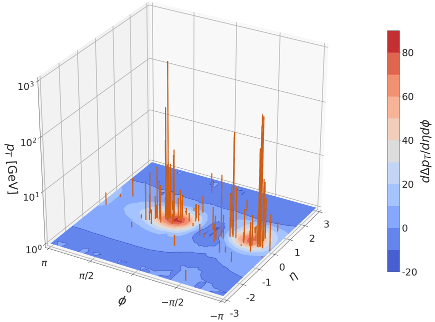
Comparison of a specific scenario with hydrodynamic solution provided by the CCNU group,

- A source depositing $dP^0/dt = dP^x/dt = 1$ GeV/fm, moving from origin to $x = 4$ fm
- Need more extensive tests + event averaging.

Jet definition in heavy-ion collisions

In our model: compute transverse energy towers in each $\Delta\eta$-$\Delta\phi$ bin, summing both hard particle and medium excitation contribution:

$$P_{ij}^{\mu} = \underbrace{\sum_{\Delta\eta\Delta\phi} p^{\mu}_{\text{hard}}}_{\text{Hard particles}} + \underbrace{\frac{d\Delta p^{\mu}}{d\phi d\eta}\Delta\eta\Delta\phi}_{\text{Medium excitation}}$$

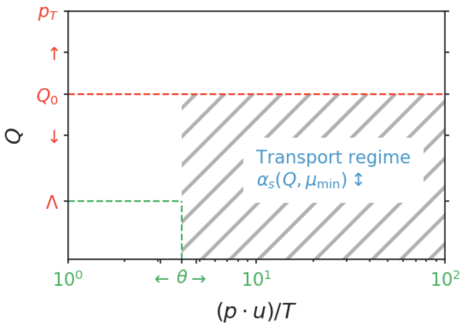


- Define jets using the grid P_{ij}^{μ} with anti-k_T algorithm as implemented in `FastJet`[1]
- The background is implicitly considered as the "unperturbed" medium.

[1] Cacciari and Salam, PLB 641 (2006) 57.

What do we want to see in the jet properties

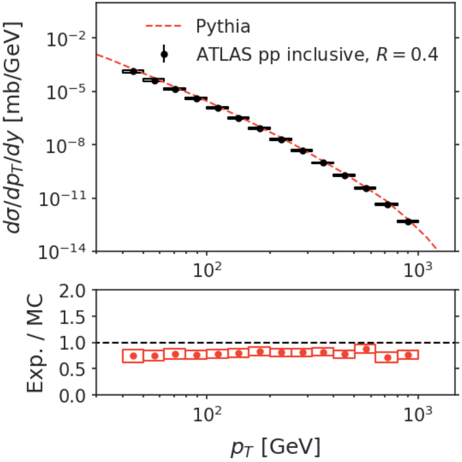
Currently, our model has three major parameters / sources of uncertainties



- In-medium coupling α_s, controlled by $\mu_{\min} \propto T$.
- A matching scale between vacuum parton shower evolution and the transport equation Q_0.
- A "thermal" energy θT, below which energy-momentum transport are modeled by a hydrodynamic driven ansatz.

- What causes medium modifications of jet properties, are they sensitive to these numbers?
- Do we have a consistent description of these observables.

Properties of inclusive jets: nuclear modification factor

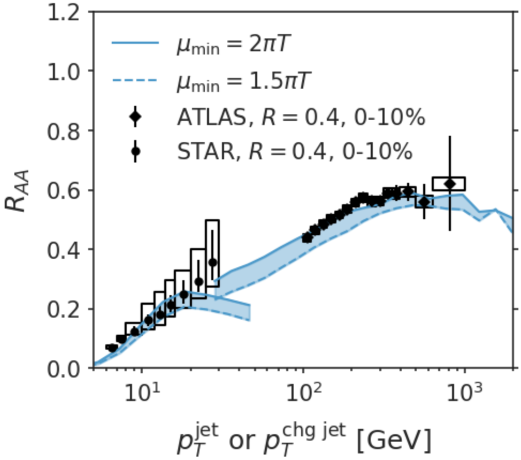


- ATLAS measurement in p+p @ 5.02 TeV. Inclusive jet cross-section with distance parameter $R = 0.4$, $|y| < 2.8$ ATLAS PLB 790 108-128.

$$\frac{1}{\Delta y}\frac{d\sigma}{dp_T}$$

- `Pythia8` simulation: CTEQ6.1 leading-order proton PDF. Hadron-level final state with particle decays.
- Jet cross-section shape well described by Pythia8 simulation.

Inclusive jets production: nuclear modification factor

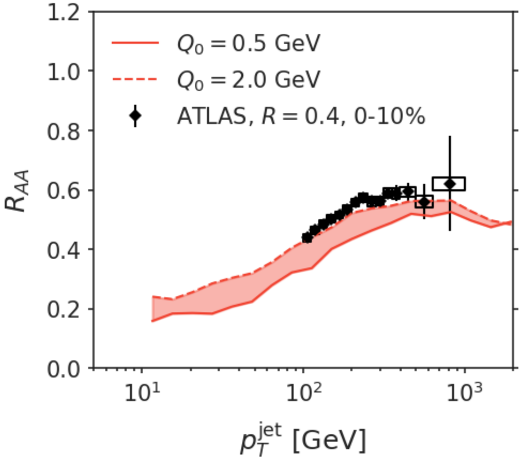


ATLAS PLB 790 108-128

STAR arXiv:2006.00582

- $R_{AA} = d\sigma_{AA}/N_{coll}d\sigma_{pp}$
  - ATLAS Pb+Pb @ 5 TeV, 0-10%
  - STAR Au+Au @ 200 GeV, 0-10%
- Sensitivity to coupling: $\mu_{min} \in [1.5\pi T, 2\pi T]$, $Q_0 = 1.3$ GeV(LHC), 0.8 GeV(RHIC), $\theta = 4T$.

Inclusive jets production: nuclear modification factor



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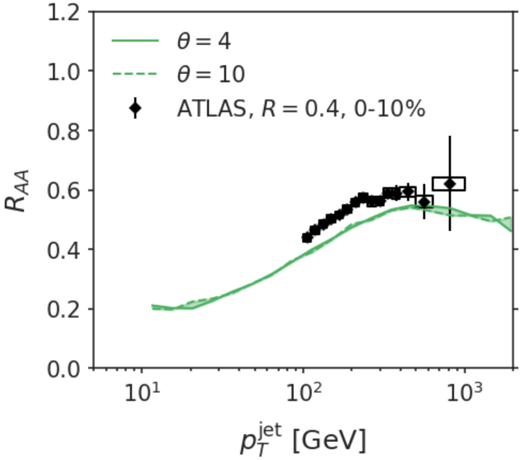

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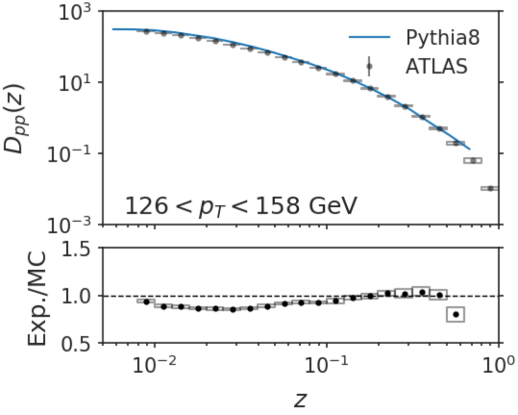
- Sensitivity to medium-response scale: $\mu_{\rm min} = 1.5\pi T$, $Q_0 = 1.3$ GeV, $\theta \in [4, 10]T$.

Properties of inclusive jets: fragmentation function



ATLAS PRC 98 024908

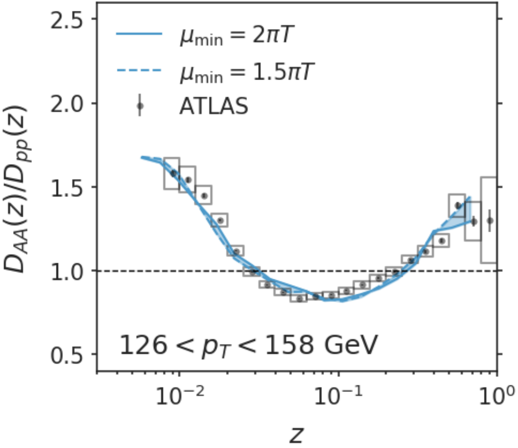
Histogram jet constituents according to its momentum projection to jet:

$$D(z) = \frac{1}{N_{\rm jet}} \frac{dN_{\rm ch}}{dz}, z = \frac{p_T \cos(\Delta R)}{p_T^{\rm jet}}$$

Not soft/collinear safe, sensitive to hadronization and decays.

- Baseline calculation using Pythia8 simulation. Charged particles only.
- Need to double check high-z. Otherwise agreement within 20%.

Properties of inclusive jets: fragmentation function



ATLAS PRC 98 024908

- Modifications $D_{AA}(z)/D_{pp}(z)$. **Remark**: jets in AA + energy loss are compared to jets with similar p_T in pp.
- Varying coupling strength: $\mu_{\rm min} \in [1.5\pi T, 2\pi T]$, $Q_0 = 1.3$ GeV, $\theta = 4T$.

[1] $z = 0.01 \to p_T \approx 1$ to 1.5 GeV. Particle production associated to medium excitation are assumed as massless pions.

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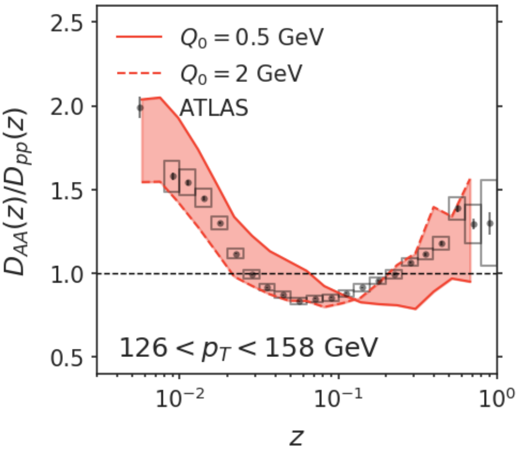

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- Varying coupling strength: $\mu_{\min} \in [1.5\pi T, 2\pi T]$, $Q_0 = 1.3$ GeV, $\theta = 4T$.
- Very sensitivity to matching scale: $\mu_{\min} = 1.5\pi T$, $Q_0 \in [0.5, 2.0]$ GeV, $\theta = 4T$.

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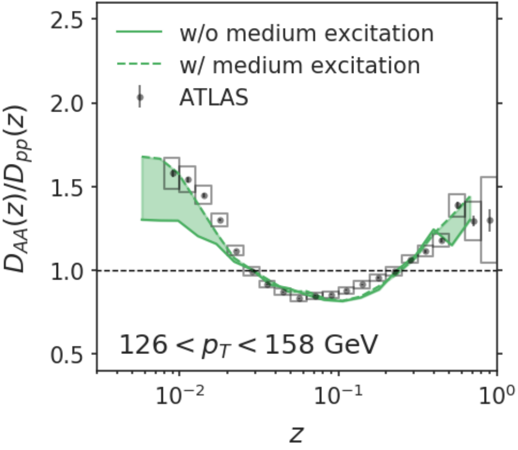
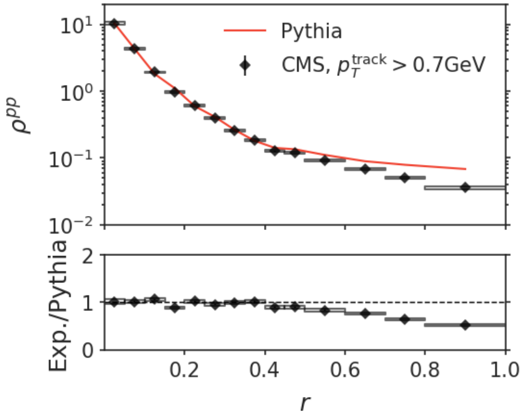


ATLAS PRC 98 024908

- Modifications $D_{AA}(z)/D_{pp}(z)$. **Remark**: jets in AA + energy loss are compared to jets with similar p_T in pp.
- Varying coupling strength: $\mu_{\min} \in [1.5\pi T, 2\pi T]$, $Q_0 = 1.3$ GeV, $\theta = 4T$.
- Very sensitivity to matching scale: $\mu_{\min} = 1.5\pi T$, $Q_0 \in [0.5, 2.0]$ GeV, $\theta = 4T$.
- Importance of energy-momentum carried by medium excitation at low-z region[1].

[1] $z = 0.01 \rightarrow p_T \approx 1$ to 1.5 GeV. Particle production associated to medium excitation are assumed as massless pions.

Properties of inclusive jets: jet shape



CMS JHEP05(2018)006

Histogram the jet energy by the angular distance from the center of jet $r = \sqrt{\Delta\phi^2 + \Delta\eta^2}$,

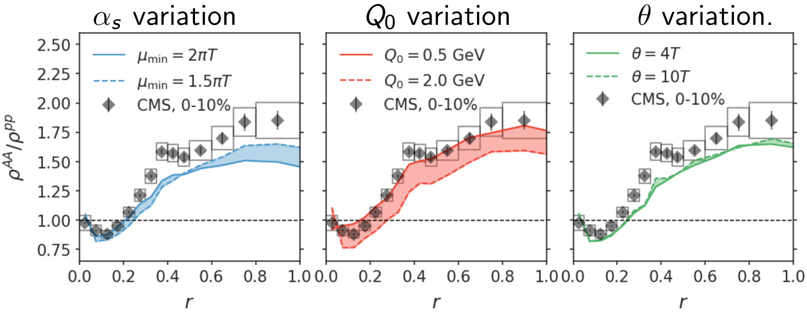
$$\rho(r) = \frac{1}{\sum_{r<1} \Delta p_T} \frac{\Delta p_T}{\Delta r}$$

Remark: self normalized object.

- CMS measurement of p+p @ 5.02 TeV, $p_{T,\mathrm{jet}} > 120$ GeV $R = 0.4$.
- Good agreement between Pythia8 baseline inside the jet cone $r < R$. Overshoot data for $r > R$.

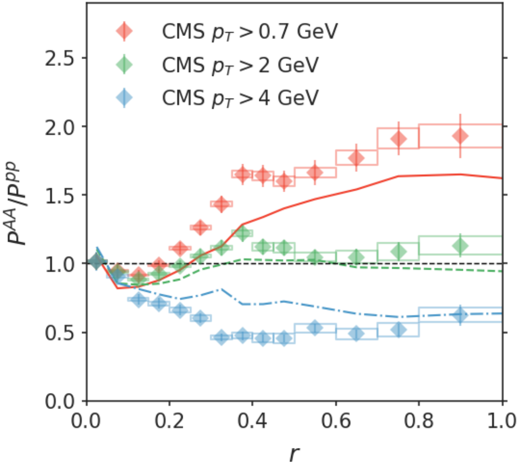
Properties of inclusive jets: jet shape

- Nuclear modification in central Pb+Pb from ATLAS JHEP05(2018)006.
- At similar p_T, jet in AA has a wider[9] radial distribution compared to jets pp..
- Again, significant uncertainty from the Q_0 parameter.



[9] Note that $\rho(r)$ is self normalized which introduces auto correlation.

Even more differential measurement



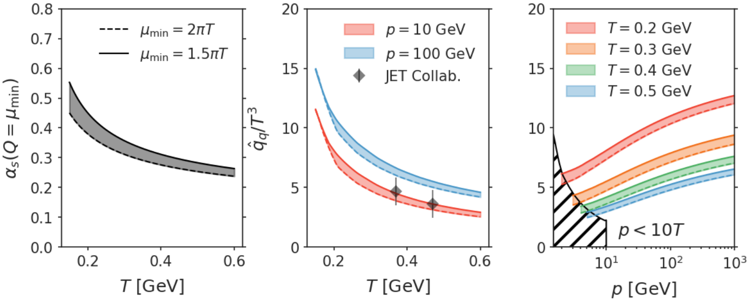
CMS JHEP05(2018)006

Construct jet shape with different minimum particle track p_T cut.

- Increasing p_T cut, the modification at larger r change from "excess" to "depletion".
- From the fragmentation function study, an increased $p_{T,\min}$ quickly removes medium excitation contributions, which is suppressed as $\int_{p_{T,\min}} e^{-p\cdot u/T} p_T^2 dp_T$.
- Potentially strong constraining power to model, but it is also sensitive to hadronization and decays.

Implication on the $\hat{q}$ parameter and transport mechanism

- Shown variation for $\mu_{\min}$ only. Need to include uncertainties from $Q_0, \theta, \cdots$ to improve this simple "eyeball fit". Can we constrain Q_0 as function of centrality & collision energy?
- The possibility to constrain the transport coefficients at intermediate p_T by studying jet modifications at low-z and large r.
- Can we put a limit on the parton momentum where its transport is more hydro /many-body like than quasi-particle like?



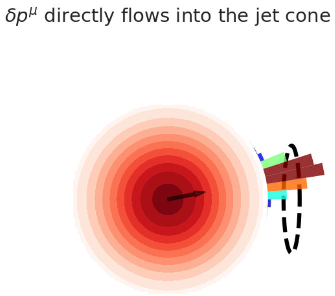
Summary

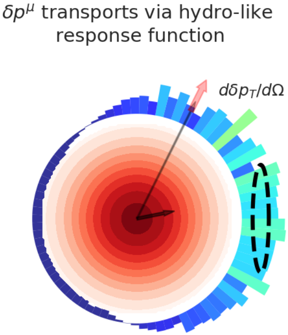
- A transport model approach to jets in a QGP medium:
  - Hard partons: elastic collisions + medium induced radiations.
  - Soft partons: hydrodynamic-like medium excitations.
- Reasonable agreement with inclusive hadron suppression and jet suppression, and jet properties (FFs, jet shapes).
- Major uncertainty from in-medium coupling constant, and the matching scaling between vacuum shower and transport evolution.
- Detailed jet properties measurements help map out an energy and scale dependent picture of parton transport inside the QGP.

Back-up: a distinct way of energy-momentum angular redistribution

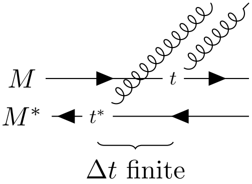
Both conserves energy momentum if summed over 4π, but very different angular distribution.

Free-particle-like propagation **hydrodynamic-like propagation**





Single parton interacts with medium (in a weakly coupled picture)



Medium-induced radiation

- Radiates of another parton due to collision with medium.
- Inelastic: energy is shared among two hard daughter partons.

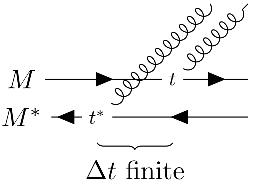
Single radiation probability for a parton moving in a medium of length L[1].

$$M^g_{gg} = \int_0^L dt' dk_\perp^2 \overbrace{\langle xp, (1-x)p, k_\perp^2 | e^{i\int_{t'}^{L} \hat{H}_{xp}+\hat{H}_{(1-x)p} dt}}^{\text{Evolution of 2-particle state to } t=L} \overbrace{\sqrt{P^g_{gg}(x)}\frac{k_\perp \cdot \epsilon}{k_\perp^2}}^{\text{splits into two}} \overbrace{e^{i\int_0^{t'} H_p dt} | p\rangle}^{\text{1 particle at t'}}$$

$$\frac{dP}{dx} = \langle M^{a*}_{bc} M^{a}_{bc} \rangle_{\text{ensemble avg.}}$$

[1] Zakharov JETP 63 952 and 65 615; Caron-Huot, Gale, PRC 82 064902; Arnold, Iqbal, JHEP04(2015)070

Single parton interacts with medium (in a weakly coupled picture)



Medium-induced radiation

- Radiates of another parton due to collision with medium.
- Inelastic: energy is shared among two hard daughter partons.

Single radiation probability breaks into two pieces $dP = dP_{\rm vac} + dP_{\rm med}$,

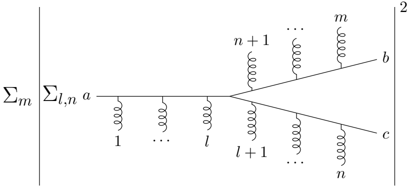
$$\frac{dP_{\rm med}}{dtdx} = \frac{P_{gg}^{g}(x)}{2x(1-x)E}\int_0^t dt' dk_\perp^2 dq_\perp^2 \langle k_\perp | iV_3 e^{iH_3 t'} | q_\perp \rangle, \quad |k_\perp\rangle = \frac{\vec{k}_\perp}{k_\perp^2}$$

Radiation is not localized, $\Delta t^{-1} \sim$ average formation time in the medium $\langle \tau_f^{-1} \rangle$

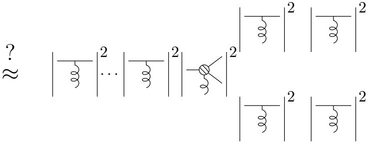
$$H_3 \approx \frac{p_\perp^2}{2x(1-x)E} + i\frac{1}{2}\hat{q}_{\rm eff} b^2 + \cdots, \quad \begin{cases} H_3 = \Omega a^\dagger a + \cdots \\ \Omega = \sqrt{i2x(1-x)E\hat{q}_{\rm eff}} = \sqrt{i}\langle \tau_f^{-1} \rangle \end{cases}$$

Modify the Boltzmann equation

The theoretical picture: multiple interfering collisions induce a radiation.



What a semi-classical equation solves: independent elastic collision and radiations.



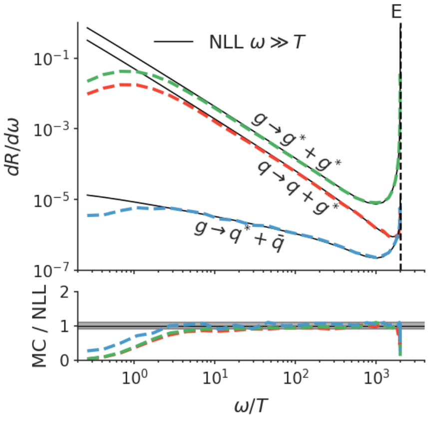
- We modify the single particle evolution Monte Carlo to correction for this difference.
  1. At t_0, an induced radiation is sampled according to the non-interfering rate ($\sim \alpha_s P(x)/\lambda$)
  2. The daughter partons are not treated as physical until $t - t_0 > \tau_f(t)$.
  3. This radiation is only accepted with probability[10] $\frac{\lambda}{\tau_f(t)}$ to become physical ($\sim \alpha_s P(x)\lambda/\tau_f$).
- This method is implemented in the LIDO transport model[11] used for this work.

[10] This probability is further improved to match NLL calculation, Arnold and Dogan PRD 78 065008

[11] Ke et al, PRC 100 064911

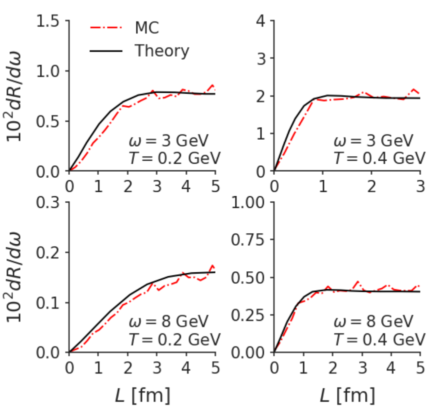
Single radiation rate: simulation compared to theory in special cases

Infinite static medium:
simulation from transport equation compared to next-to-leading-log solution of the rate in infinite limit.

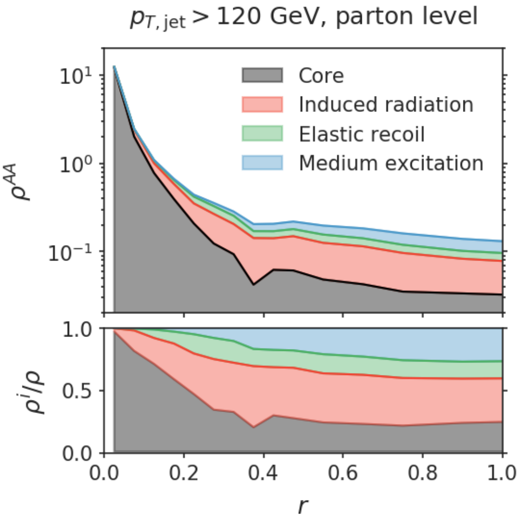


Finite size effect:
path-length dependence of the radiation rate, simulation compared to numerical solution of the rate in finite medium

$E = 16$ GeV, $\alpha_s = 0.3$



A closer look at the energy transport in the radial direction



Different contributions to the jet shape

- Core: the harder parton produced in radiation.
- Induced radiation: the softer parton in radiation.
- Elastic recoil: hard recoiled medium parton.
- Medium excitation: energy deposition to the medium.

Radiative processes still dominates within the jet cone.