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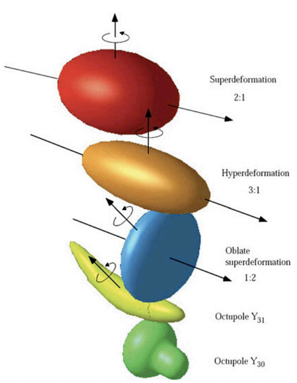
New opportunities to probe nuclear deformation using high-energy heavy-ion collisions

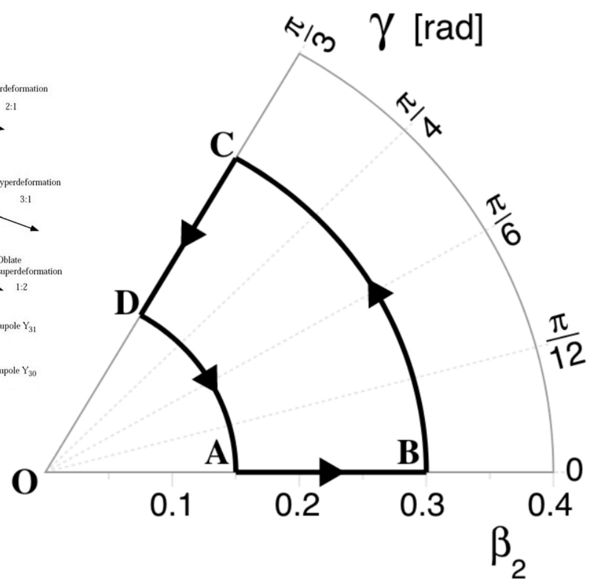
Jiangyong Jia

arXiv:2109.01631 arXiv:2105.05713
arXiv:2109.00604 arXiv:2105.01638
arXiv:2106.08768 arXiv:2102.08158

Collaborators: Giuliano Giacalone, Chunjian Zhang, Shengli Huang

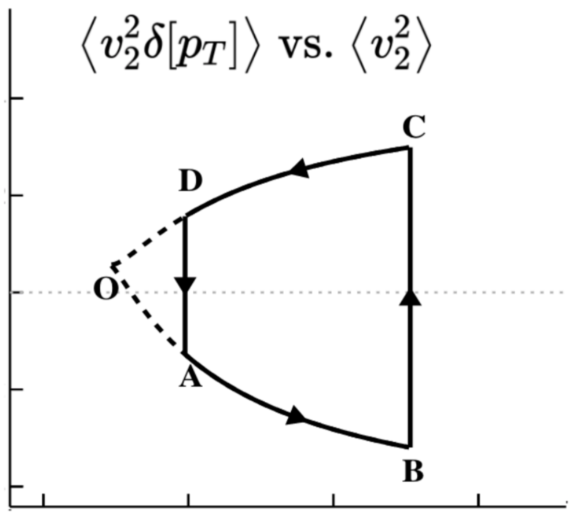
Nuclear structure



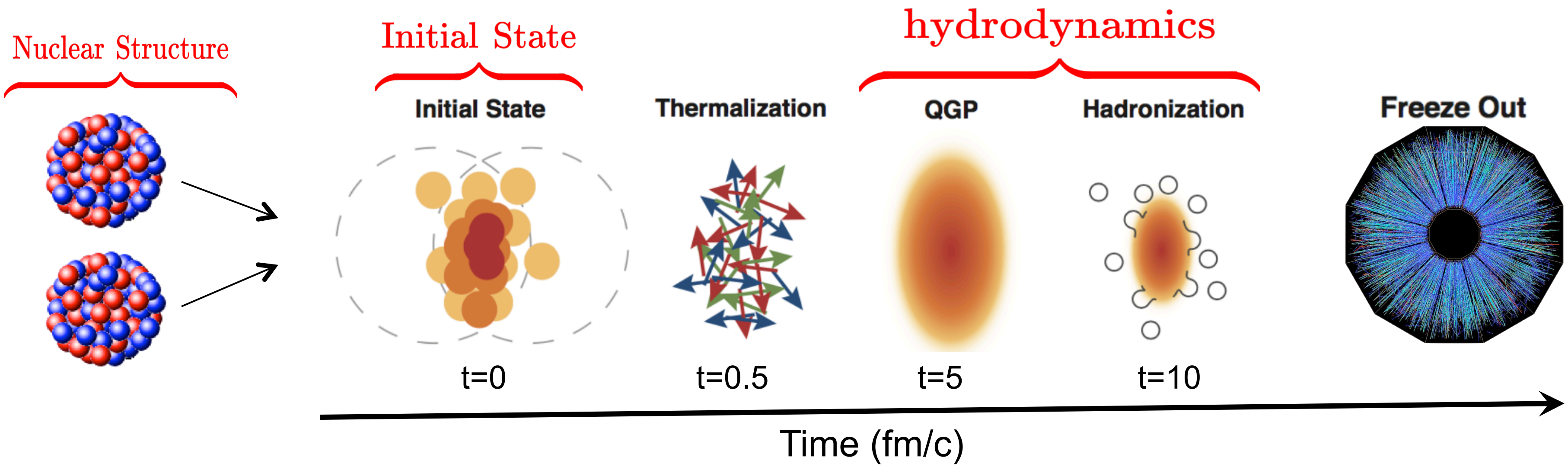


?

High-energy collisions



Heavy ion collisions and nuclear structure

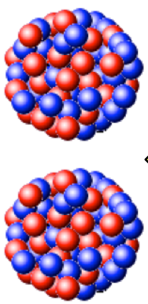


Space-time evolution of heavy ion collisions can be considered as a hydrodynamic response to the nucleon density distribution in the initial overlap region in the transverse plane, driven by pressure gradient forces.

The shape of the size of the overlap is directly controlled by the shape and radial profile of the colliding nuclei.

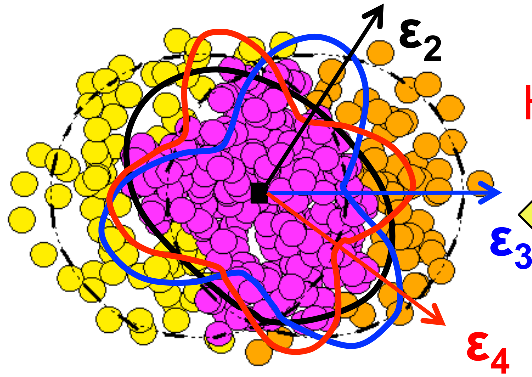
Hydrodynamic response to initial state

Nuclear Structure



Imaging?

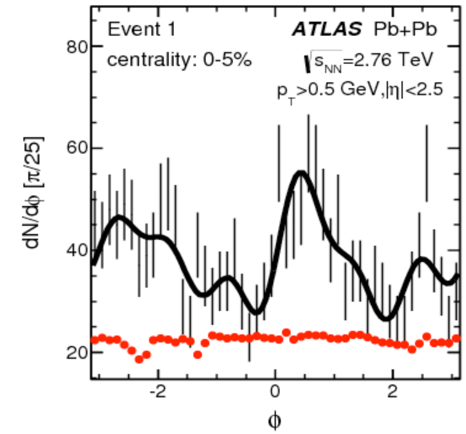
Initial State



Hydro-response

Space-time dynamics

Final Particle flow



$$\frac{\rho_0}{1+e^{(r-R_0(1+\sum_n \beta_n Y_n^0(\theta,\phi))/a_0}}$$

Initial Size | Initial Shape

$$R_\perp \propto \langle r_\perp^2 \rangle, \quad \mathcal{E}_n \propto \langle r_\perp^n e^{in\phi} \rangle$$

??

$$R_0 \quad a_0 \quad \beta_n$$

Radial Flow | Harmonic Flow

$$\frac{d^2N}{d\phi dp_T} = N(p_T)\left(\sum_n V_n\, e^{-in\phi}\right)$$

Approximate linear response in each event:

$$\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp} \qquad V_n \propto \mathcal{E}_n$$

arXiv:1206.1905

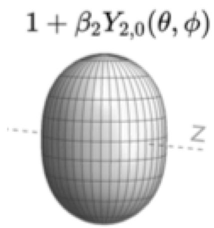
Shape of nuclei

Most ground state stable nuclei are deformed

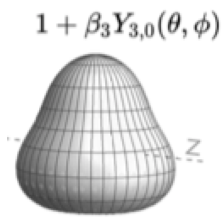
$$\rho(r,\theta,\phi) = \frac{\rho_0}{1 + e^{(r - R(\theta,\phi))/a_0}}$$

$$R(\theta,\phi) = R_0\left(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^{3} \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^{4} \alpha_{4,m} Y_{4,m}\right)$$

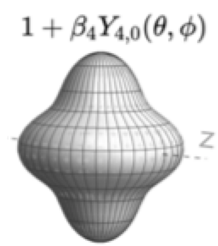
Quadrupole:



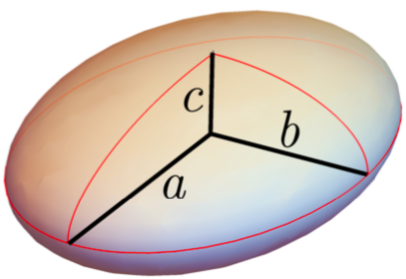
Octupole:



Hexadecapole:



Triaxial spheroid: $a \neq b \neq c$.



$0 \leq \gamma \leq \pi/3$

Prolate: a=b<c → β_2, $\gamma=0$

Oblate: a<b=c → β_2, $\gamma=\pi/3$ or $-\beta_2$, $\gamma=0$

Shape of nuclei

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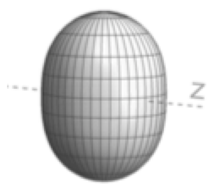
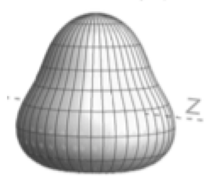
Quadrupole:

$1+\beta_2 Y_{2,0}(\theta,\phi)$



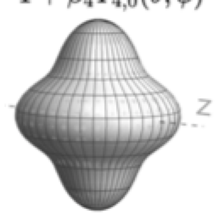
Octupole:

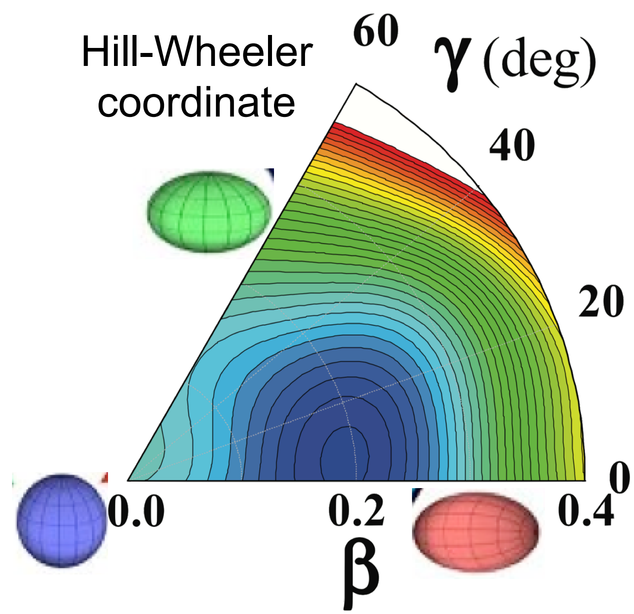
$1+\beta_3 Y_{3,0}(\theta,\phi)$



Hexadecapole:

$1+\beta_4 Y_{4,0}(\theta,\phi)$





Shape of nuclei

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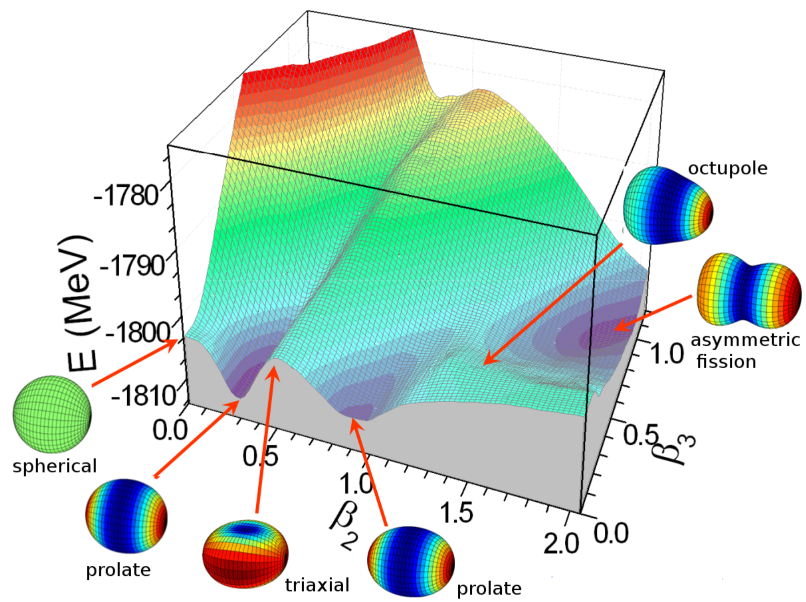
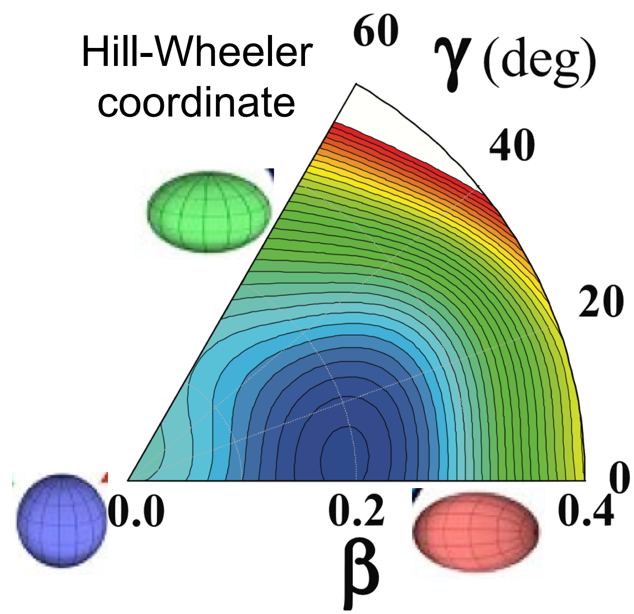
Shape determined by minimizing the potential energy surface



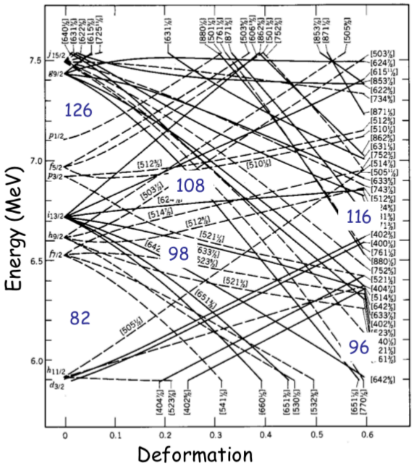


B.N. Lu, Meng Jie

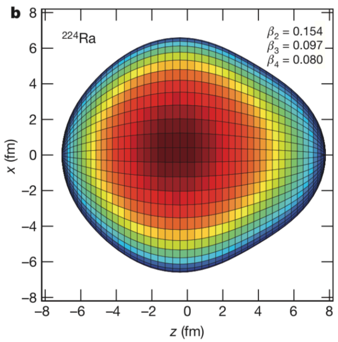
Main tool: transition rates B(En) among low lying states

Some topics in nuclear shape studies

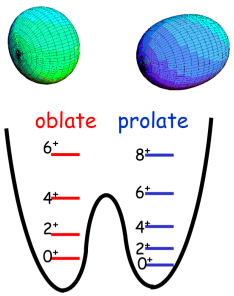
- Shape evolution: how the shape evolves along isotopic chain
  - Strong test on nuclear structure model

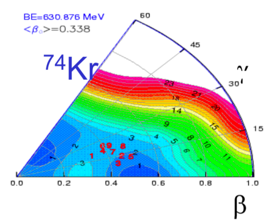


- Octuple (pear-shaped) deformation
  - Octupole correlation or static deformation
  - Strong test on EDM effects



- Trixaility : infers from γ-band, Chiral and Wobbling bands. Have large uncertainties.
  - shape coexistance



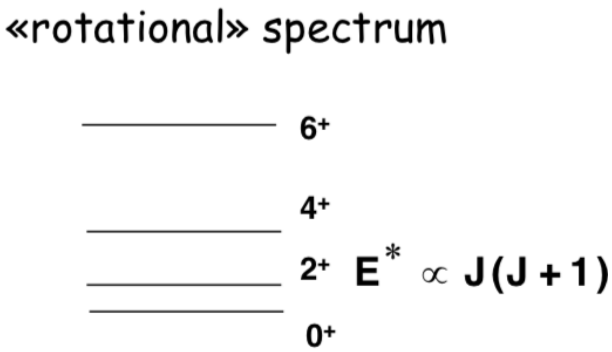


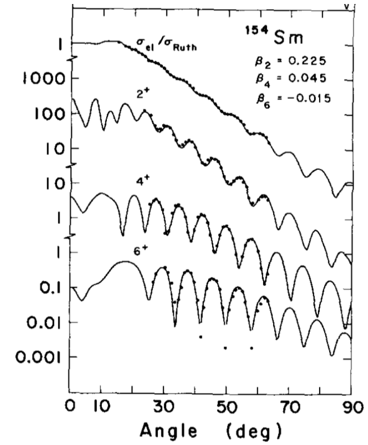
- Super-deformed nuclei, yzast-line etc.

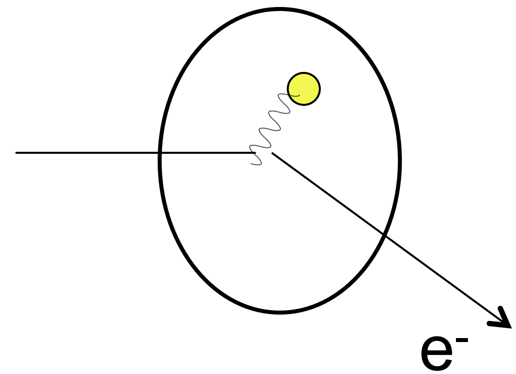
Use shape tomography in heavy-ion collision to help?

Nuclear structure vs HI method

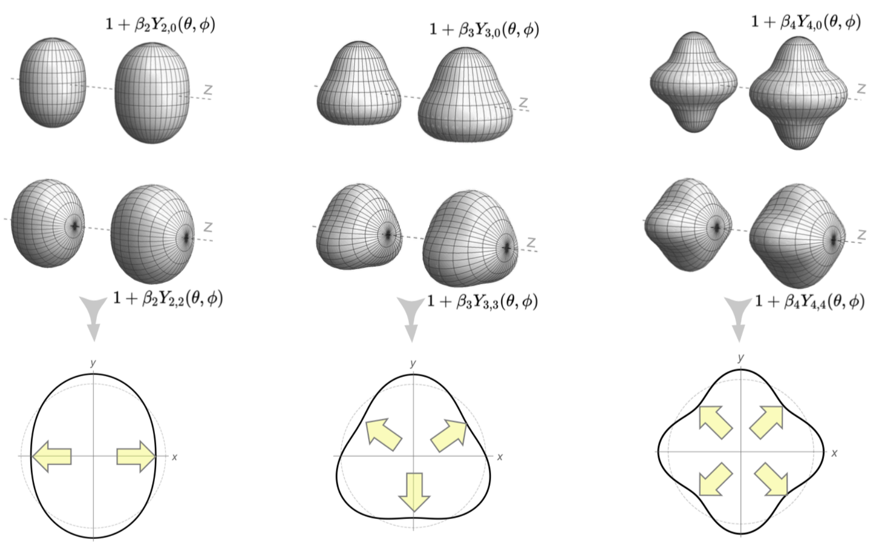
- Shape from B(En), radial profile from e+A or ion-A scattering

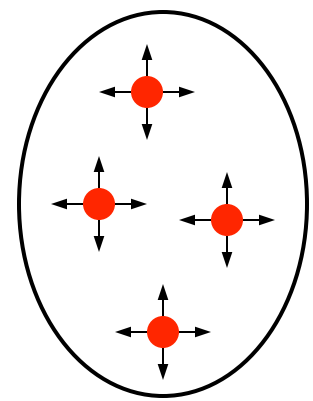






- Probe entire mass distribution: multi-point correlations





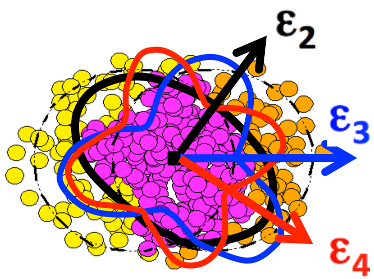
$$S(\mathbf{s}_1,\mathbf{s}_2) \equiv \langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2)\rangle$$
$$= \langle \rho(\mathbf{s}_1)\rho(\mathbf{s}_2)\rangle - \langle \rho(\mathbf{s}_1)\rangle\langle \rho(\mathbf{s}_2)\rangle.$$

collective flow response to the shape

Observables sensitive to deformation

Initial Shape

$$\mathcal{E}_n = \epsilon_n e^{in\Phi} \propto \langle r_\perp^n e^{in\phi} \rangle$$



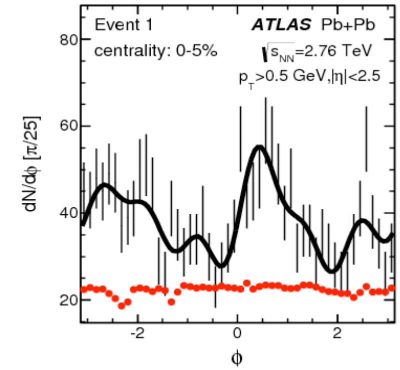
Hydro-response

Space-time dynamics

$$\epsilon_\mathrm{n} \rightarrow v_\mathrm{n}$$

Harmonic flow

$$\frac{dN}{d\phi} \propto 1 + 2\sum_n v_n \cos n(\phi - \Phi_n)$$

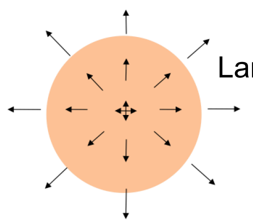


Initial Size

Radial flow [p_T]

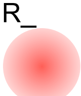
Small R_

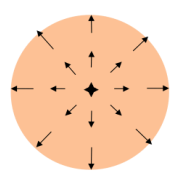
Hydro-response



Large [p_T]

Large R_

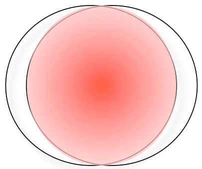




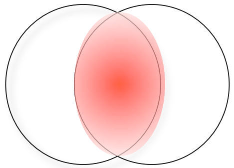
Small [p_T]

$$d_\perp = \frac{1}{R_\perp} \rightarrow [p_\mathrm{T}]$$

Correlated fluctuations in shape & size
→Correlated fluctuations in v_n and [p_T]



small ε_2, larger $R_\perp$



large ε_2, small $R_\perp$

$$\left\langle \epsilon_\mathrm{n}^2 \frac{1}{R_\perp} \right\rangle \rightarrow \langle v_\mathrm{n}^2 p_\mathrm{T} \rangle$$

Infer shape & size fluctuations from $p(v_n)$, $p([p_T])$, and $p(v_n,[p_T])$

Evidence of deformation in U+U vs Au+Au

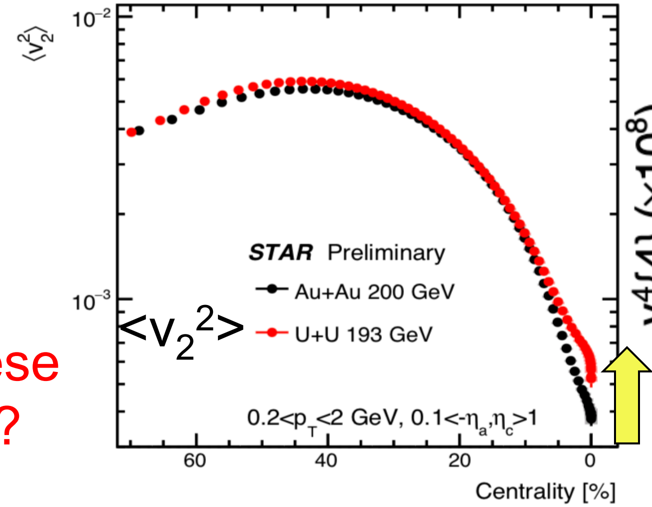
Collisions at $\sqrt{s_{NN}}$=193-200 GeV

Large deformation in ^{238}U relative to ^{197}Au strongly influence flow signals

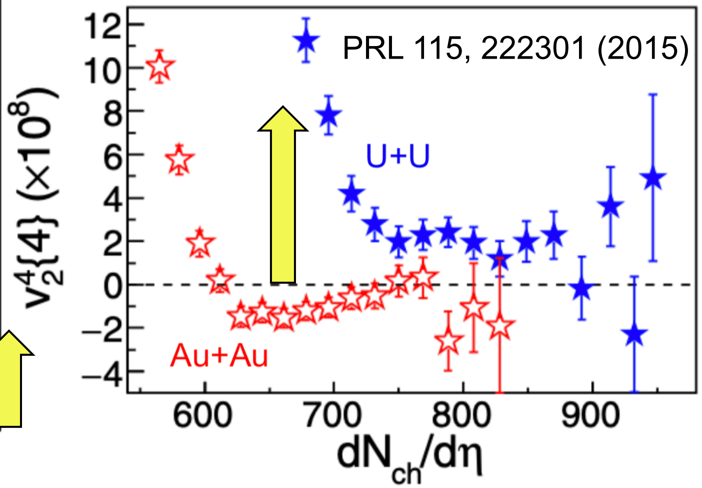
$\beta_{2U} \sim 0.28$
$\beta_{2Au} \sim 0.13?$

Can and how to turn these into a quantitative tool?

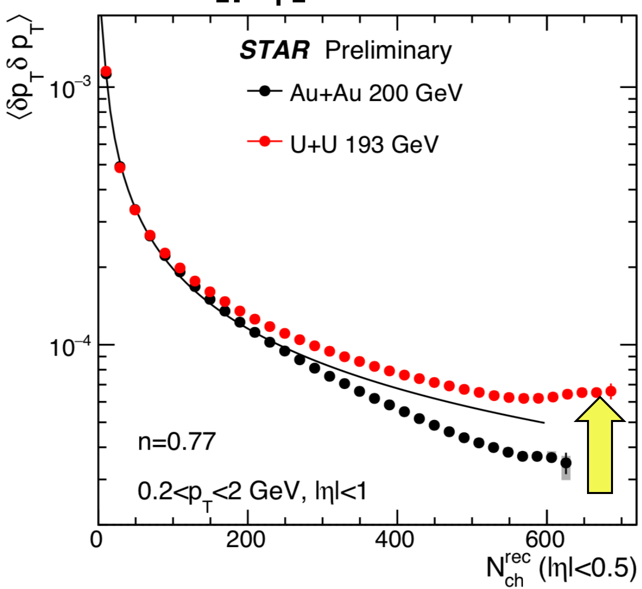
v_2 variance



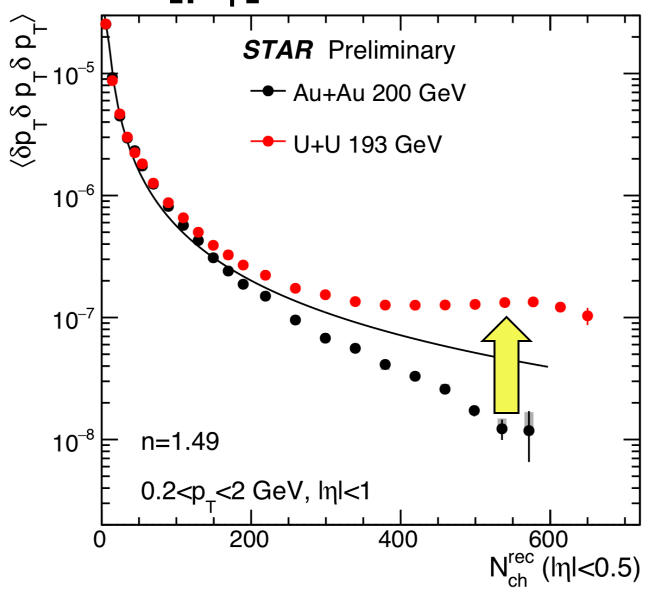
v_2 kurtosis



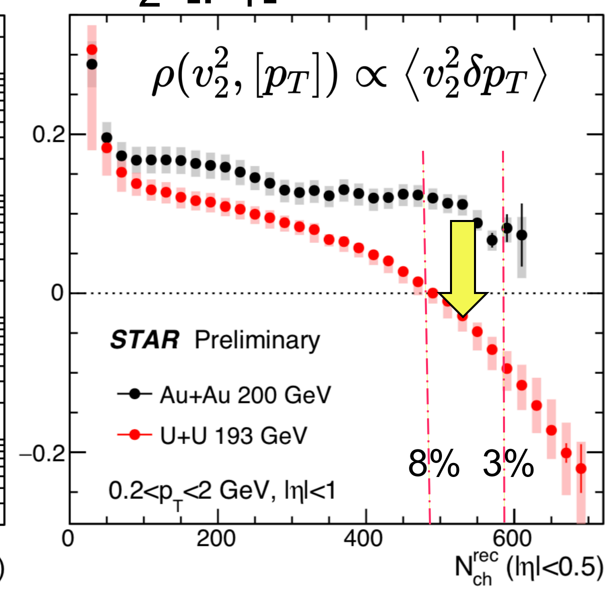
$[p_T]$ variance



$[p_T]$ skewness



v_2-$[p_T]$ covariance



Influence of shape fluctuations in relativistic heavy ion collisions

A. Rosenhauer, H. Stöcker, J. A. Maruhn, and W. Greiner
Phys. Rev. C **34**, 185 – Published 1 July 1986

Article | References | No Citing Articles | PDF | Export Citation

High energy collisions of strongly deformed nuclei: An old idea with a new twist

E. V. Shuryak
Phys. Rev. C **61**, 034905 – Published 22 February 2000

Article | References | Citing Articles (26) | PDF | Export Citation

Uranium on uranium collisions at relativistic energies

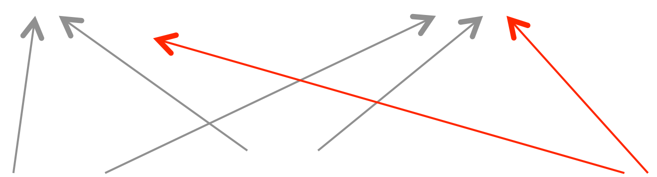
Bao-An Li
Phys. Rev. C **61**, 021903(R) – Published 12 January 2000

Article | References | Citing Articles (25) | PDF | Export Citation

Mostly for low energy, where one lacks initial state quantities with simple linear response to the final state observables

Relating initial state to deformation

Initial state : $R_\perp \propto \langle r_\perp^2 \rangle, \quad \mathcal{E}_n \propto \langle r_\perp^n e^{in\phi} \rangle$



$$R_0 \quad a_0 \quad \beta_n$$

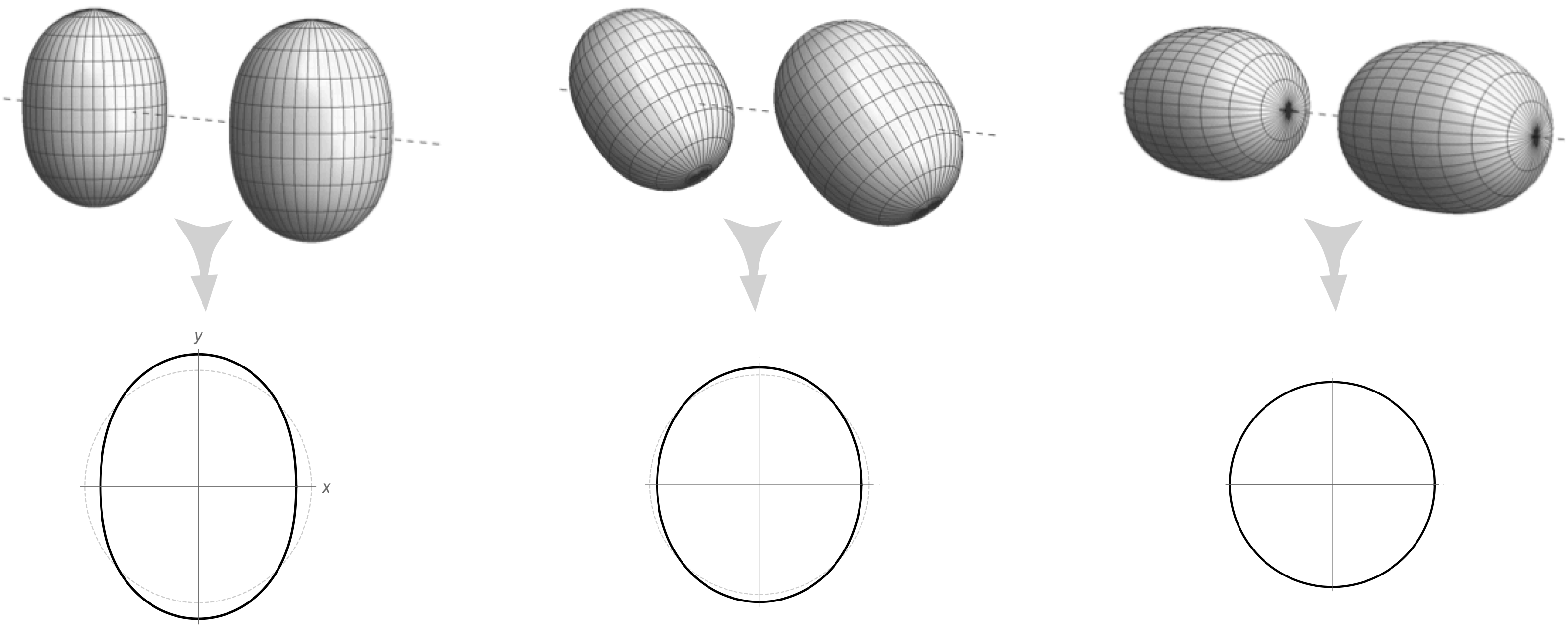
Nuclear structure : $$\frac{\rho_0}{1 + e^{(r - R_0(1+\sum_n \beta_n Y_n^0(\theta,\phi))/a_0}}$$

Shape of the initial state in HI

Body-Body

$$\mathcal{E}_n = \epsilon_n e^{in\Phi} \propto \left\langle r_\perp^n e^{in\phi} \right\rangle$$

Tip-Tip



$\varepsilon_2 \sim 0.95\beta_2$ $\varepsilon_2 \sim 0.48\beta_2$ $\varepsilon_2 \sim 0$

The ε_2 of overlap depends on the orientation: Euler angle $\Omega = \varphi\theta\psi$

Ultra-central → events $\Omega_1 \approx \Omega_2$ → shape of overlap =

Shape of nucleon density projected along Ω

Connecting shape ε_n and size R to β_n

$$R(\theta,\phi) = R_0\left(1+\beta_2[\cos\gamma Y_{2,0}+\sin\gamma Y_{2,2}]\right)$$

See 2109.00604

- ε_n is just shape of Y_n^n projected to the transverse plane

$$\mathcal{E}_n = -\frac{\langle r_\perp^n e^{in\phi}\rangle}{\langle r_\perp^n\rangle} \propto \langle Y_n^n\rangle = \underbrace{\epsilon_{n;0}}_{\text{undeformed}} + p_n\underbrace{(\Omega_1,\Omega_2,\gamma)}_{\text{phase factor}}\beta_n + \mathcal{O}(\beta_n^2)$$

γ only appear here, since when β=0, γ doesn't matter, must in the form of cos3γ, cos6γ, cos9γ…

- $R_\perp$ is related to Y_2^0 projected to the transverse plane

$$d_\perp \equiv 1/R_\perp$$

$$R_\perp^2 = \langle x^2\rangle + \langle y^2\rangle = \frac{2}{3}\left\langle 1-2\sqrt{\frac{\pi}{5}}Y_2^0\right\rangle \implies \frac{\delta R_\perp^2}{R_\perp^2} = -2\sqrt{\frac{\pi}{5}}\langle Y_2^0\rangle \implies \frac{\delta d_\perp}{d_\perp} = \sqrt{\frac{\pi}{5}}\langle Y_2^0\rangle$$

$$\frac{\delta d_\perp}{d_\perp} = \delta_d + p_0(\Omega_1,\Omega_2,\gamma)\beta_2 + \mathcal{O}(\beta_2^2)$$

- Again, linear response:

$$v_n \propto \varepsilon_n \qquad \frac{\delta[p_T]}{[p_T]} \propto \frac{\delta d_\perp}{d_\perp}$$

Get deformation from cumuants of

$p(\varepsilon_n)$ and $p(\delta d_\perp/d_\perp)$

Connecting shape ε_2 and size R to β_2

Single event

See 2109.00604

$$\frac{\delta d_\perp}{d_\perp} = \delta_d + p_0(\Omega_1,\Omega_2,\gamma)\beta_2 + \mathcal{O}(\beta_2^2)\ ,\ \boldsymbol{\epsilon}_2 = \boldsymbol{\epsilon}_0 + \boldsymbol{p}_2(\Omega_1,\Omega_2,\gamma)\beta_2 + \mathcal{O}(\beta_2^2)$$

fluctuation of δ_d ($\boldsymbol{\epsilon}_0$) is uncorrelated with p_0 ($\boldsymbol{p}_2$)

Variances

$$\left\langle(\delta d_\perp/d_\perp)^2\right\rangle = \left\langle\delta_d^2\right\rangle + \left\langle p_0(\Omega_1,\Omega_2,\gamma)^2\right\rangle\beta_2^2\ ,\ \left\langle\varepsilon_2^2\right\rangle = \left\langle\varepsilon_0^2\right\rangle + \left\langle\boldsymbol{p}_2(\Omega_1,\Omega_2,\gamma)\boldsymbol{p}_2^*(\Omega_1,\Omega_2,\gamma)\right\rangle\beta_2^2$$

$$\propto \left\langle(\delta[p_\mathrm{T}]/[p_\mathrm{T}])^2\right\rangle \qquad \propto \left\langle v_2^2\right\rangle$$

Sknewness

$$\left\langle(\delta d_\perp/d_\perp)^3\right\rangle = \left\langle\delta_d^3\right\rangle + \left\langle p_0^3\right\rangle\beta_2^3 \qquad \left\langle\varepsilon_2^2\delta d_\perp/d_\perp\right\rangle = \left\langle\varepsilon_0^2\delta_d\right\rangle + \left\langle p_0\boldsymbol{p}_2\boldsymbol{p}_2^*\right\rangle\beta_2^3$$

$$\propto \left\langle(\delta[p_\mathrm{T}]/[p_\mathrm{T}])^3\right\rangle \qquad \propto \left\langle v_2^2\delta[p_\mathrm{T}]/[p_\mathrm{T}]\right\rangle$$

Kurtosis

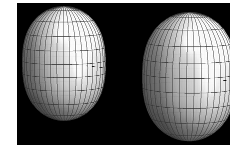
$$\left\langle(\delta d_\perp/d_\perp)^4\right\rangle - 3\left\langle(\delta d_\perp/d_\perp)^2\right\rangle^2 = \left\langle\delta_d^4\right\rangle - 3\left\langle\delta_d^2\right\rangle^2 + (\langle p_0)^4\rangle - 3\left\langle p_0^2\right\rangle^2)\beta_2^4$$

$$\propto \left\langle(\delta[p_\mathrm{T}]/[p_\mathrm{T}])^4\right\rangle - 3\left\langle(\delta[p_\mathrm{T}]/[p_\mathrm{T}])^2\right\rangle^2$$

$$\left\langle\varepsilon_2^4\right\rangle - 2\left\langle\varepsilon_2^2\right\rangle^2 = \left\langle\varepsilon_0^4\right\rangle - 2\left\langle\varepsilon_0^2\right\rangle^2 + \left(\left\langle\boldsymbol{p}_2^2\boldsymbol{p}_2^{*2}\right\rangle - 2\left\langle\boldsymbol{p}_2\boldsymbol{p}_2^*\right\rangle^2\right)\beta_2^4 \quad \propto \left\langle v_2^4\right\rangle - 2\left\langle v_2^2\right\rangle^2$$

Liquid drop model estimate for head-on collisions

Nucleus with a sharp surface: $\rho(r,\theta,\phi) = \begin{cases} 1 & r < R(\theta,\phi) \\ 0 & r > R(\theta,\phi) \end{cases}$



$$\frac{\delta d_\perp}{d_\perp} = \sqrt{\frac{5}{16\pi}}\beta_2\left(\cos\gamma D^2_{0,0} + \frac{\sin\gamma}{\sqrt{2}}\left[D^2_{0,2}+D^2_{0,-2}\right]\right)\ ,\ \epsilon_2 = -\sqrt{\frac{15}{2\pi}}\beta_2\left(\cos\gamma D^2_{2,0} + \frac{\sin\gamma}{\sqrt{2}}\left[D^2_{2,2}+D^2_{2,-2}\right]\right)$$

$$\alpha_{2,0} \equiv \cos\gamma,\ \alpha_{2,\pm 2} \equiv \frac{\sin\gamma}{\sqrt{2}}$$

Variances

$$\left\langle \varepsilon_2^2 \right\rangle = \beta_2^2 \frac{15}{2\pi} \int \left(\sum_m \alpha_{2,m} D^2_{2,m}\right)\left(\sum_m \alpha_{2,m} D^2_{2,m}\right)^* \frac{d\Omega}{8\pi^2} = \frac{3}{2\pi}\beta_2^2$$

$$\left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^2 \right\rangle = \beta_2^2 \frac{5}{16\pi} \int \left(\sum_m \alpha_{2,m} D^2_{0,m}\right)^2 \frac{d\Omega}{8\pi^2} = \frac{1}{16\pi}\beta_2^2$$

do not depend on γ

Values reduce when consider $\Omega_1 \neq \Omega_2$

Skewness

$$\left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^3 \right\rangle = \beta_2^3 \left(\frac{5}{16\pi}\right)^{3/2} \int \left(\sum_m \alpha_{2,m} D^2_{0,m}\right)^3 \frac{d\Omega}{8\pi^2} = \frac{\sqrt{5}}{224\pi^{3/2}}\cos(3\gamma)\beta_2^3$$

opposite sign

$$\left\langle \varepsilon_2^2 \frac{\delta d_\perp}{d_\perp} \right\rangle = \beta_2^3 \frac{15}{2\pi}\sqrt{\frac{5}{16\pi}} \int \left(\sum_m \alpha_{2,m} D^2_{2,m}\right)\left(\sum_m \alpha_{2,m} D^2_{2,m}\right)^* \left(\sum_m \alpha_{2,m} D^2_{0,m}\right) \frac{d\Omega}{8\pi^2} = -\frac{3\sqrt{5}}{28\pi^{3/2}}\cos(3\gamma)\beta_2^3$$

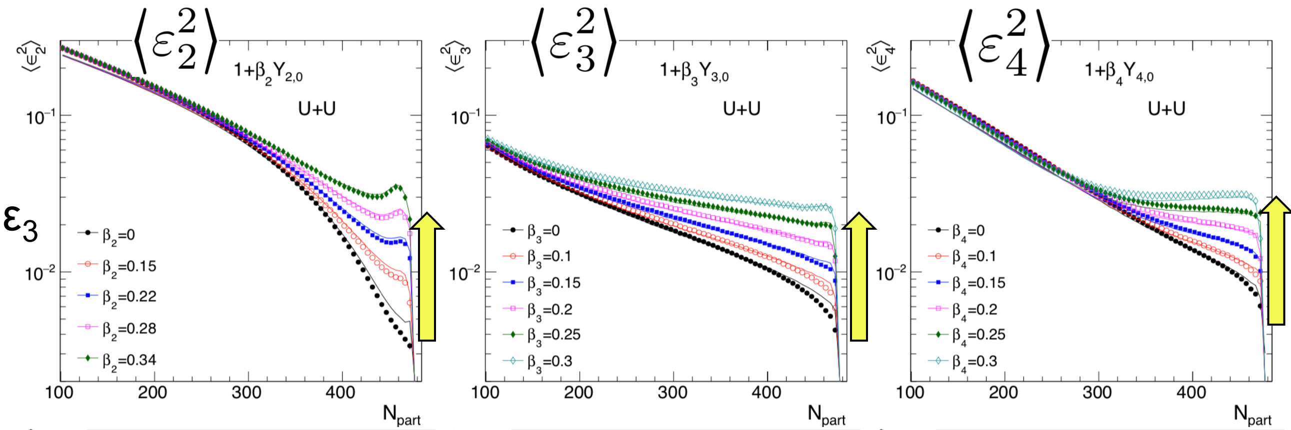
Expect a leading-order cos3γ dependence

Monte Carlo Glauber model results

See 2106.08768

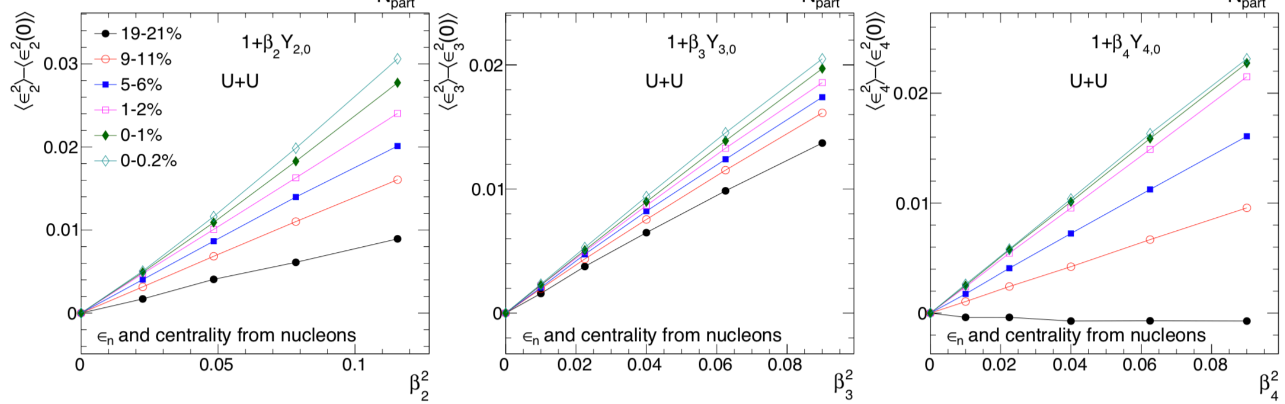
Clear enhancements in UCC

β_3 affects wide cent. range for ε_3



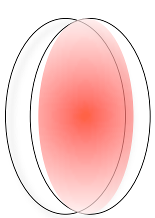
Dependence as expected

$$\left\langle \varepsilon_n^2 \right\rangle \approx a'_n + b'_n \beta_n^2$$

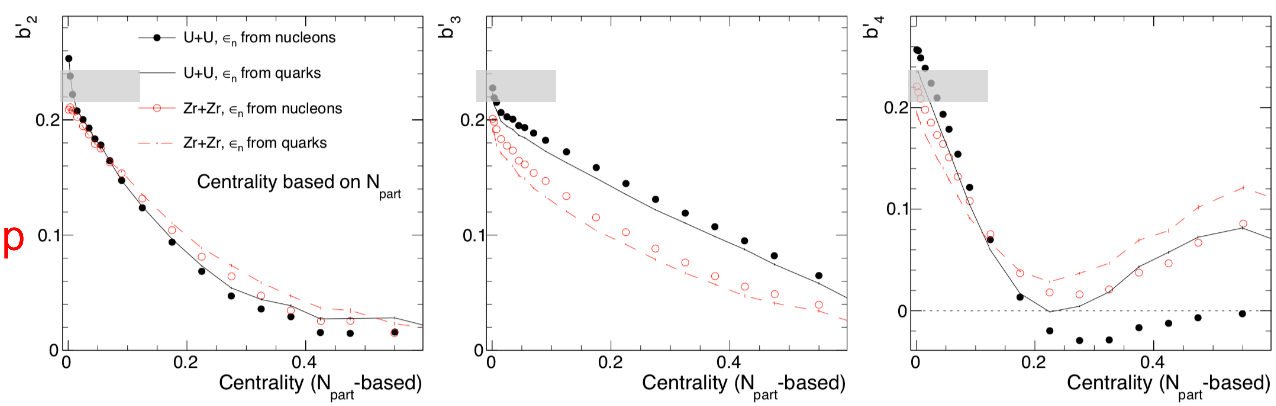


b_n' coefficients are indep. of system size, same for nucleon Glauber and quark Glauber.

Influence of deformation is a Global geometry effects, not affected by nucleon fluct.s



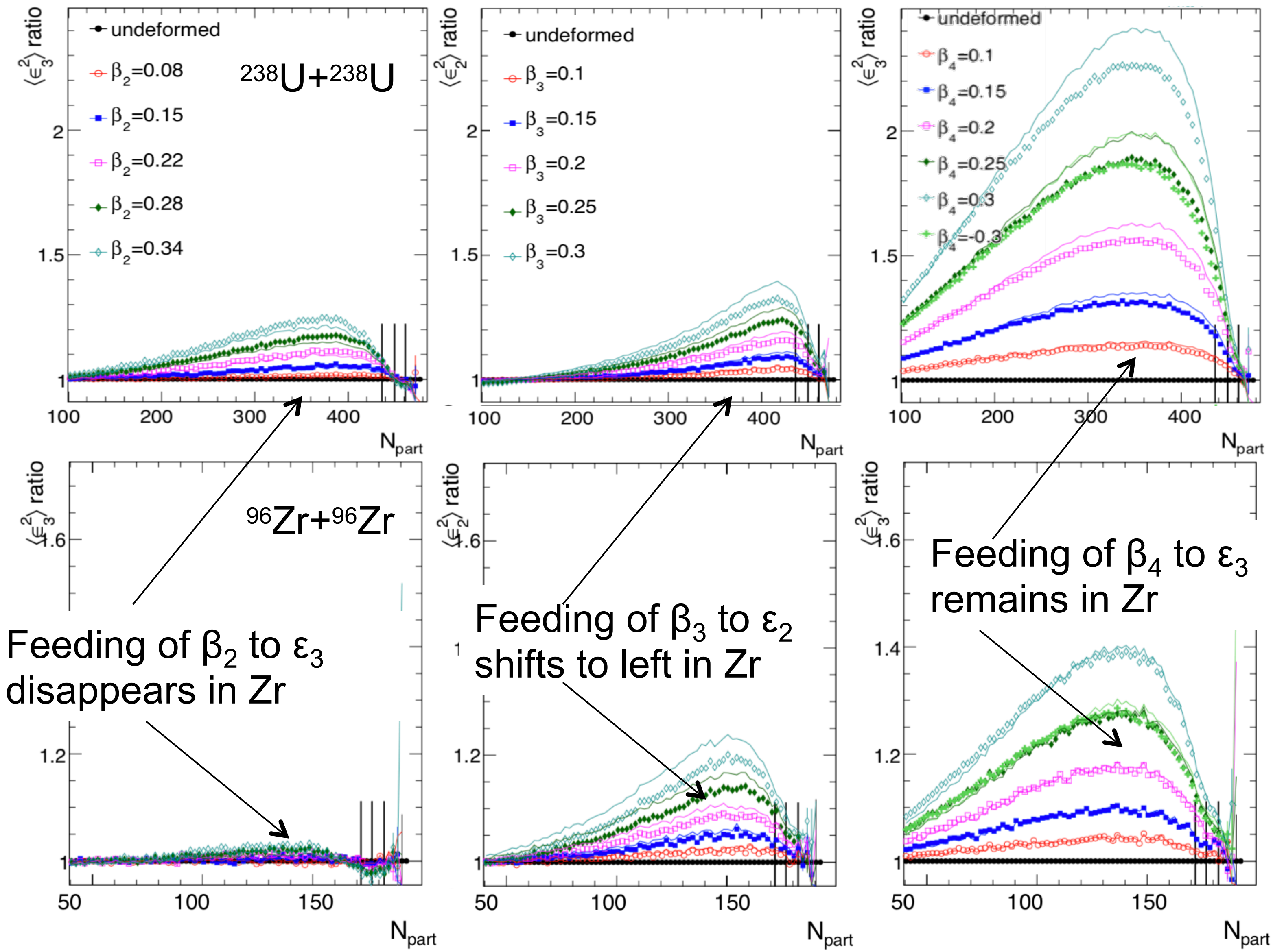
Agree with liquid drop model prediction in ultra-central region



Does β_n influence ε_m? $m \neq n$

See 2106.08768

$$\langle \varepsilon_3^2 \rangle = a'_3 + b'_{3,2}\beta_2^2 \qquad \langle \varepsilon_2^2 \rangle = a'_2 + b'_{2,3}\beta_3^2 \qquad \langle \varepsilon_3^2 \rangle = a'_3 + b'_{3,4}\beta_4^2$$



Application: variances

$$\langle \varepsilon_n^2 \rangle \qquad \langle v_n^2 \rangle$$

$$R(\theta,\phi) = R_0\left(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^{3} \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^{4} \alpha_{4,m} Y_{4,m}\right)$$

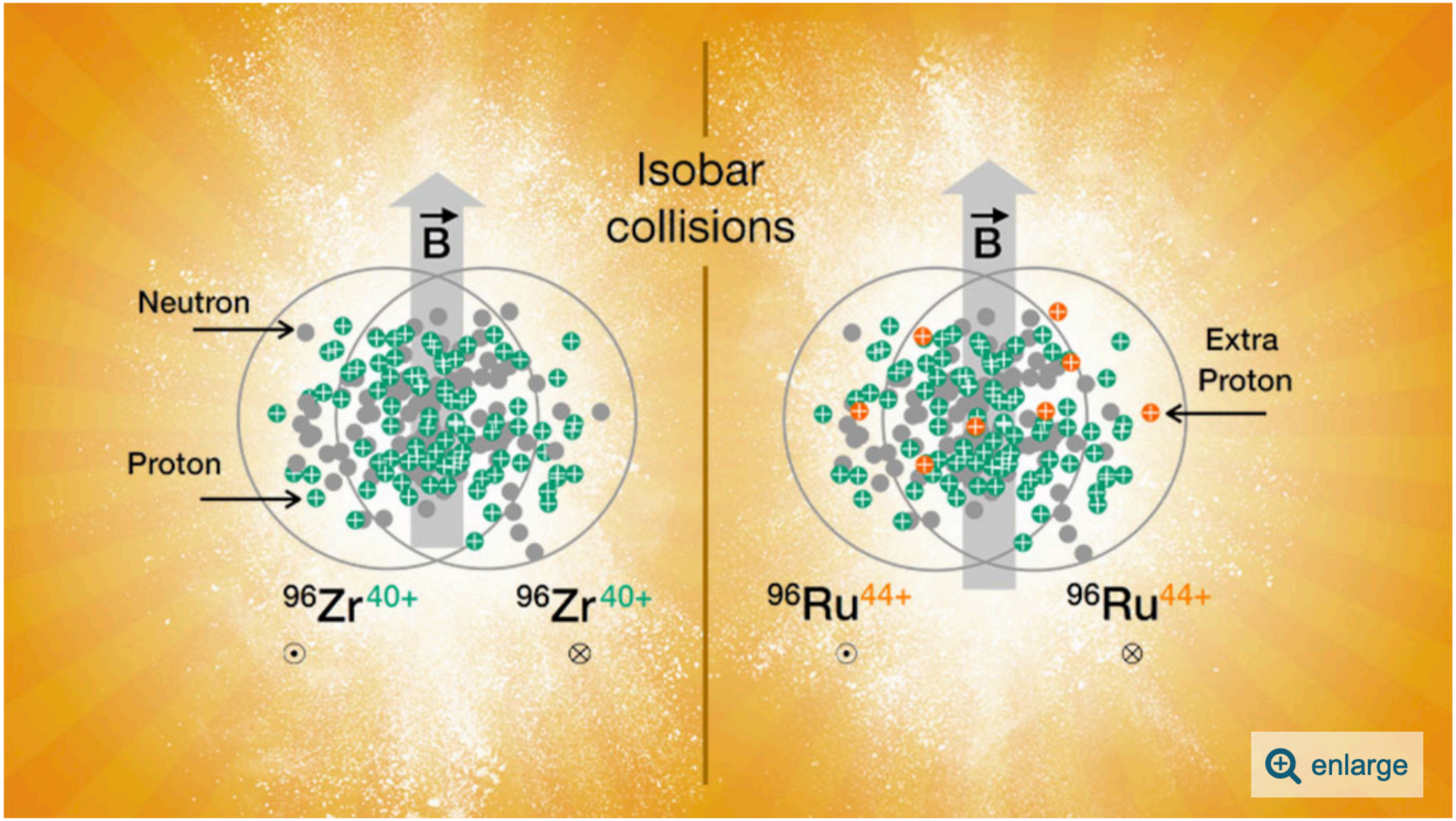
Results from Search for 'Chiral Magnetic Effect' at RHIC

Collisions of 'isobars' test effect of magnetic field, searching for signs of a broken symmetry

August 31, 2021

No CME yet, but a precision down to 0.4% is achieved in ratio of observables between the two isobar systems.

arXiv:2109.00131



Physicists compared collisions of two different sets of isobars, which are ions that have the same overall mass but different numbers of protons—zirconium (^{96}Zr), with 40 protons, and ruthenium (^{96}Ru) with 44 protons. The higher proton number (and thus electric charge) in ruthenium should generate a stronger magnetic field during collisions than zirconium (indicated by size of gray arrows). Scientists expected the stronger magnetic field of ruthenium collisions to result in greater separation of charged particles emerging from those collisions than seen in zirconium collisions.

Nuclear deformation in isobar collision

- Isobar systems, i.e. 96Ru+96Ru and 96Zr+96Zr

arXiv:2102.08158

Question: $$\frac{v_{n,\mathrm{Ru+Ru}}}{v_{n,\mathrm{Zr+Zr}}} \stackrel{?}{=} 1$$

- Nuclear structure data on Zr/Ru deformation

β_2 from ADNDT107,1(2016)

β_3 from ADNDT80,35(2002)

	β_2	$E_{2_1^+}$ (MeV)	β_3	$E_{3_1^-}$ (MeV)
^{96}Ru	0.154	0.83	-	3.08
^{96}Zr	0.062	1.75	0.202,0.235,0.27	1.90

Conversion from B(En) to β_n via: $\beta_2 = \frac{4\pi}{3ZR_0^2}\sqrt{\frac{B(E2)\uparrow}{e^2}}$, $\beta_3 = \frac{4\pi}{3ZR_0^3}\sqrt{\frac{B(E3)\uparrow}{e^2}}$

PHYSICAL REVIEW C VOLUME 42, NUMBER 3 SEPTEMBER 1990

Strong octupole and dipole collectivity in ^{96}Zr: Indication for octupole instability in the $A=100$ mass region

^{96}Zr has very large octupole collectivity/deformation from $B(E3; 0_1^+ \to 3_1^-)$

Three measurements all give large yet inconsistent values

Nuclear deformation in isobar collision

- Isobar systems, i.e. 96Ru+96Ru and 96Zr+96Zr

arXiv:2102.08158

Question: $$\frac{v_{n,\mathrm{Ru+Ru}}}{v_{n,\mathrm{Zr+Zr}}} \stackrel{?}{=} 1$$

- Glauber model suggest:

$$\varepsilon_2^2 = a_2' + b_2'\beta_2^2 + b_{2,3}'\beta_3^2, \quad \varepsilon_3^2 = a_3' + b_3'\beta_3^2$$

$$v_2^2 = a_2 + b_2\beta_2^2 + b_{2,3}\beta_3^2, \quad v_3^2 = a_3 + b_3\beta_3^2$$

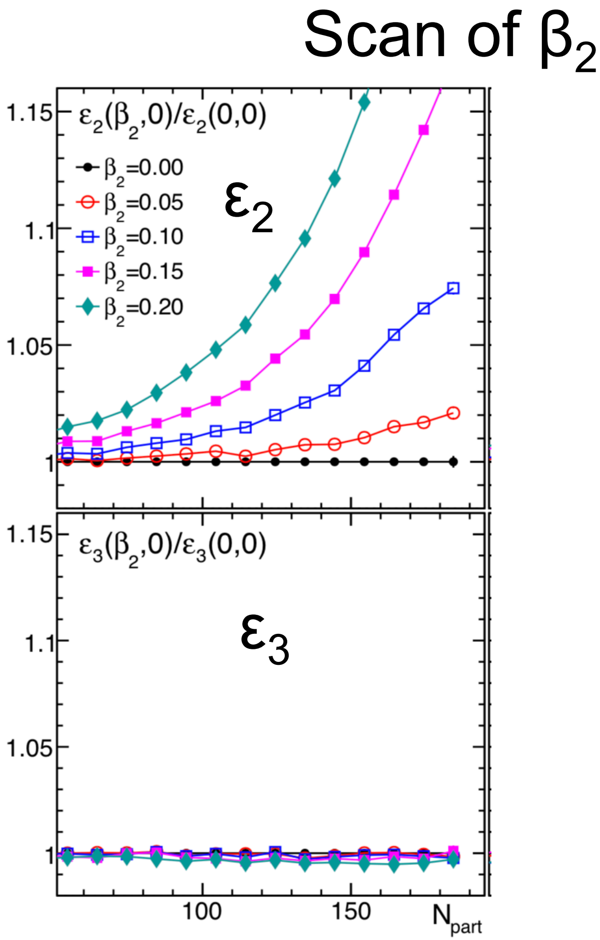
$$\frac{v_{2,\mathrm{Ru}}^2}{v_{2,\mathrm{Zr}}^2} \approx 1 + \frac{b_2}{a_2}(\beta_{2,\mathrm{Ru}}^2 - \beta_{2,\mathrm{Zr}}^2) - \frac{b_{2,3}}{a_2}\beta_{3,\mathrm{Zr}}^2$$

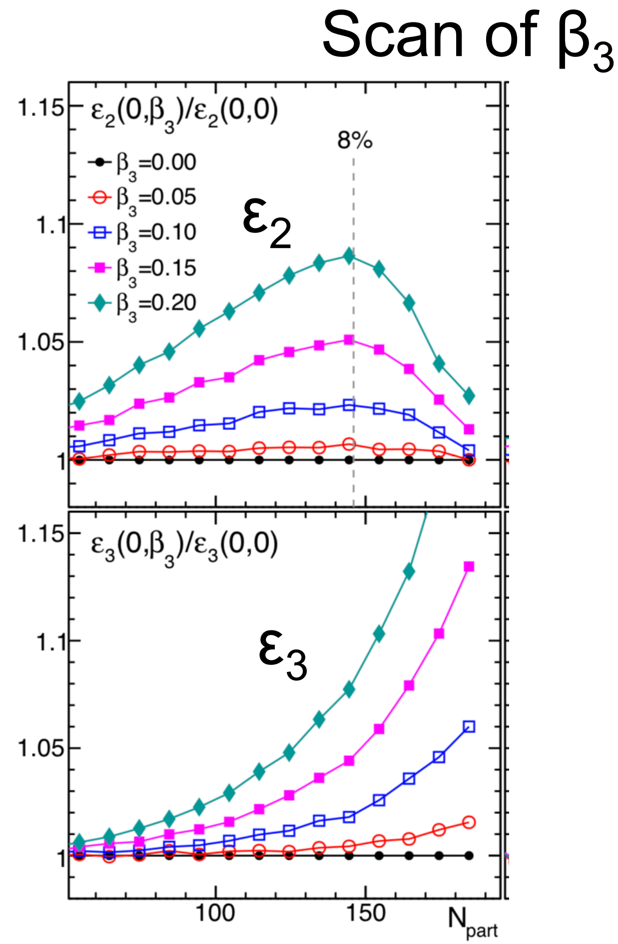
$$\frac{v_{3,\mathrm{Ru}}^2}{v_{3,\mathrm{Zr}}^2} \approx 1 - \frac{b_3}{a_3}\beta_{3,\mathrm{Zr}}^2.$$

cancelation expected in non-central collisions

AMPT results

arXiv:2109.01631



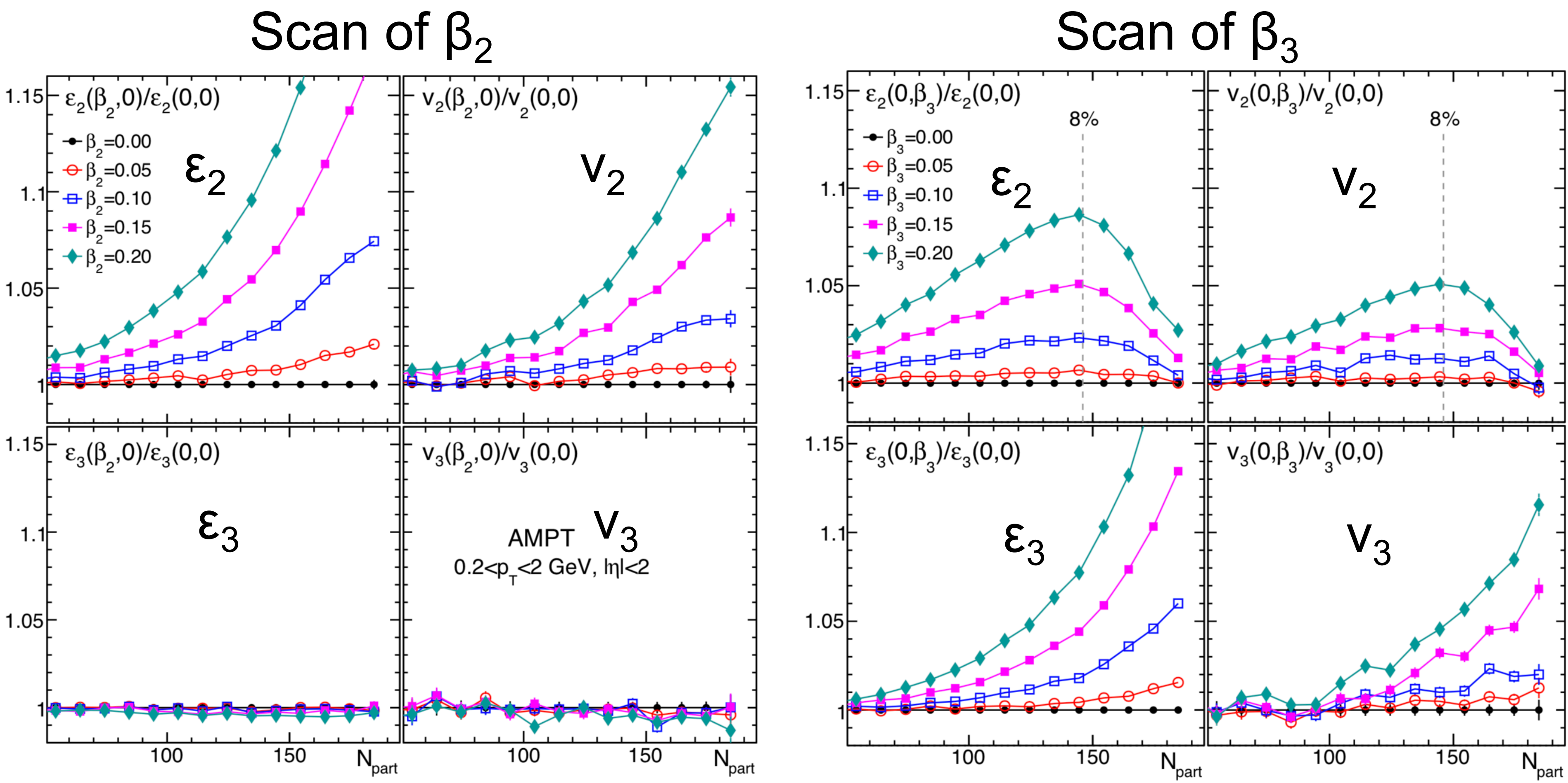


$$\varepsilon_2^2 = a_2' + b_2'\beta_2^2 + b_{2,3}'\beta_3^2, \quad \varepsilon_3^2 = a_3' + b_3'\beta_3^2$$

confirmed!

AMPT results

arXiv:2109.01631



- Dependence is weaker for v_n than ε_n, but identical trends

$$\frac{v_{2,\mathrm{Ru}}^2}{v_{2,\mathrm{Zr}}^2} \approx 1 + \frac{b_2}{a_2}(\beta_{2,\mathrm{Ru}}^2 - \beta_{2,\mathrm{Zr}}^2) - \frac{b_{2,3}}{a_2}\beta_{3,\mathrm{Zr}}^2$$

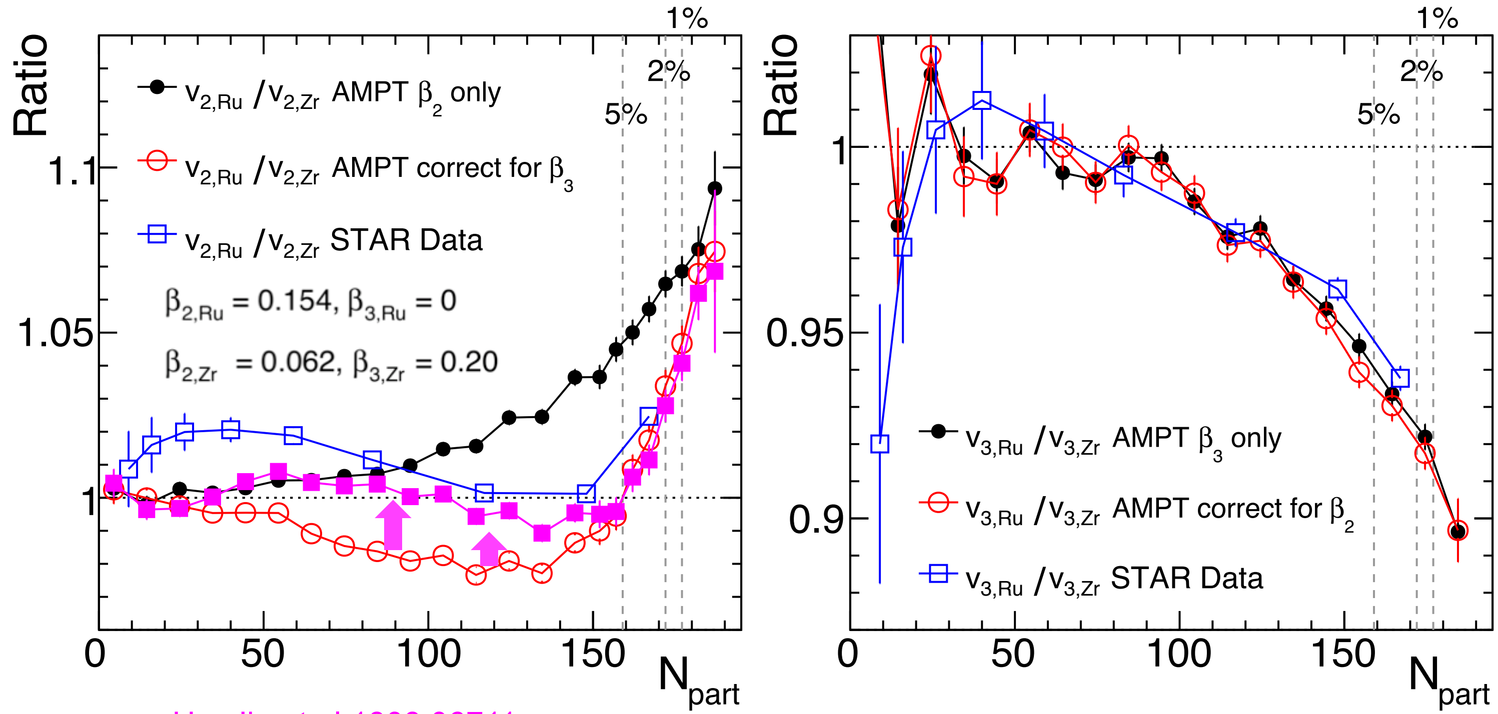
$$\frac{v_{3,\mathrm{Ru}}^2}{v_{3,\mathrm{Zr}}^2} \approx 1 - \frac{b_3}{a_3}\beta_{3,\mathrm{Zr}}^2.$$

$$\frac{b_n}{a_n} < \frac{b'_n}{a'_n} \implies \frac{b_n}{b'_n} < \frac{a_n}{a'_n}$$

Hydro response to deformation b_n/b_n' is weaker than hydro response to undeformed case a_n/a_n'

Predicted ratio

arXiv:2109.01631

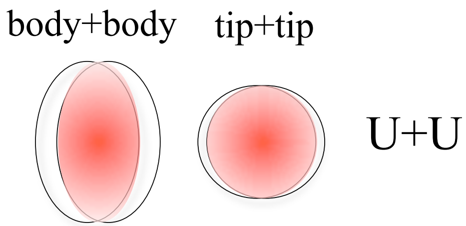


HaoJie et.al 1808.06711

- v_2-ratio: Negative contribution from β_{3zr} → sharper decrease in UCC
- v_3-ratio: strong decrease in UCC from β_{3zr}.
- Residual difference due to neutron skin of Zr? What about in >40%?
  - Get β_{3zr} ~ 0.2, prefers lower end of NS measurements
  - Measurement to be improved with finer bins e.g. 0-1%

Application in ^{197}Au+^{197}Au vs ^{238}U+^{238}U

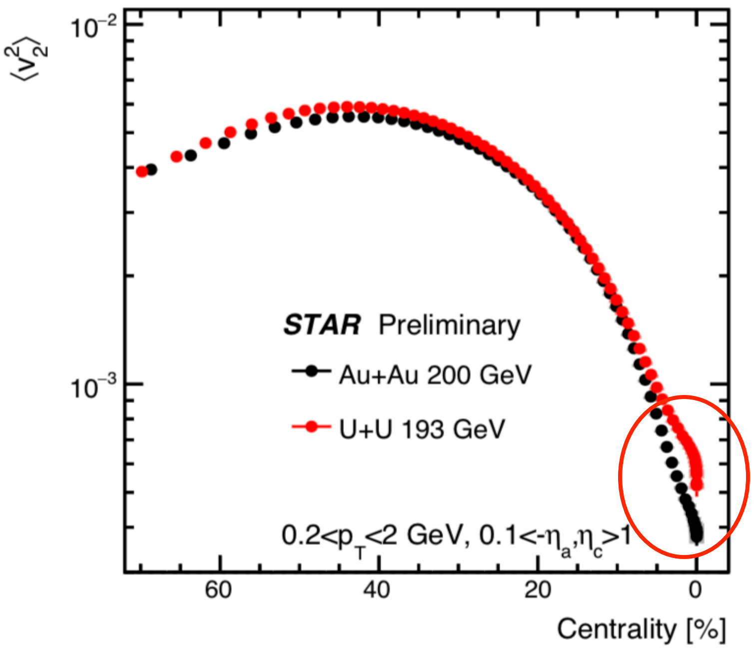
Collisions at $\sqrt{s_{NN}}$=193-200 GeV See:arXiv:2105.01638

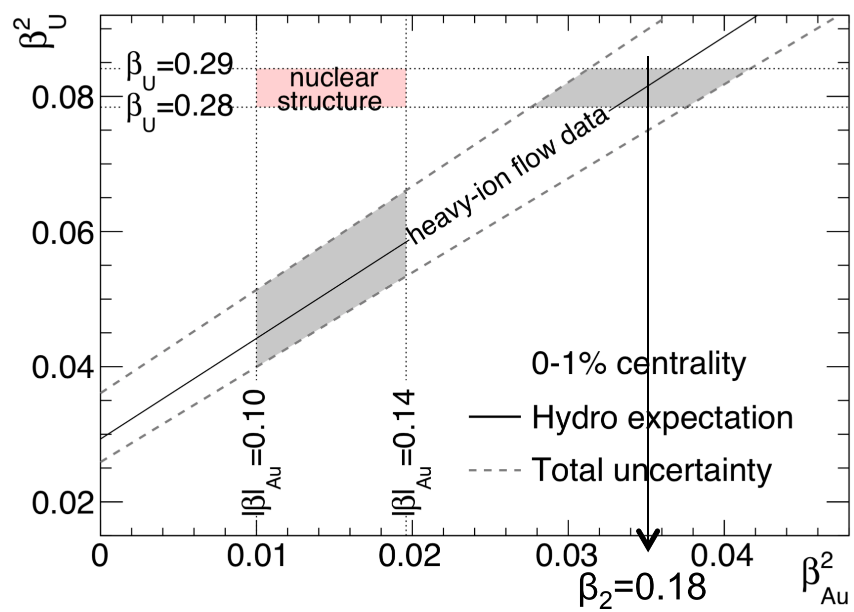


$$v_{2,\mathrm{Au}}^2 = a_{\mathrm{Au}} + b\beta_2^2 \qquad v_{2,\mathrm{U}}^2 = a_{\mathrm{U}} + b\beta_2^2$$

Need to correct for slightly different size: $a \propto 1/A\,,\quad r_a = \frac{a_{\mathrm{Au}}}{a_{\mathrm{U}}} = \frac{238}{197} = 1.21$

A simple relation for β_{2U} and β_{2Au} : $\beta_{\mathrm{U}}^2 = \frac{r_{v_2^2} r_a - 1}{b/a_{\mathrm{U}}} + r_{v_2^2}\beta_{\mathrm{Au}}^2 \quad r_{v_2^2} = \frac{v_{2,\mathrm{U}}^2}{v_{2,\mathrm{Au}}^2}$





Suggests $|\beta_2|_{Au}$~0.18+-0.02, larger than NS model of 0.13+-0.02
Note: 197Au is a odd-mass nuclei, β_2 not measured!

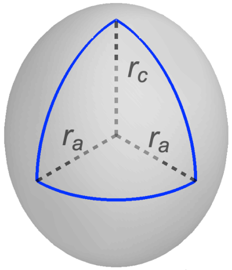
Application: skewness

$$\left\langle\left(\frac{\delta d_\perp}{d_\perp}\right)^3\right\rangle = \frac{\sqrt{5}}{224\pi^{3/2}}\cos(3\gamma)\beta_2^3 \qquad \left\langle\varepsilon_2^2\frac{\delta d_\perp}{d_\perp}\right\rangle = -\frac{3\sqrt{5}}{28\pi^{3/2}}\cos(3\gamma)\beta_2^3$$

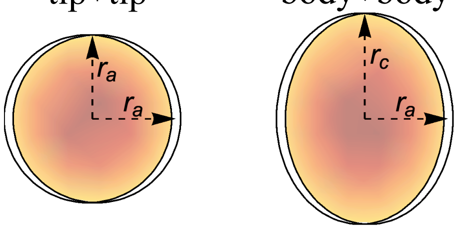
Triaxiality γ: $R(\theta,\phi) = R_0\left(1 + \beta_2[\cos\gamma Y_{2,0} + \sin\gamma Y_{2,2}]\right.$

Prolate

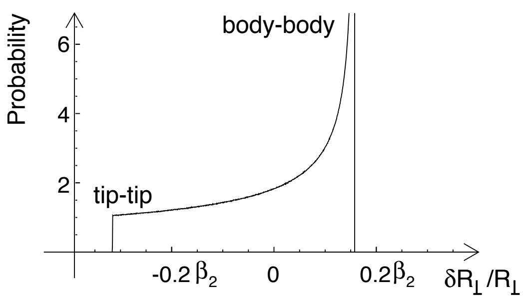
$\beta_2 = 0.25, \cos(3\gamma) = 1$



tip+tip body+body



$\epsilon_2 \downarrow, R_\perp \downarrow$ $\epsilon_2 \uparrow, R_\perp \uparrow$

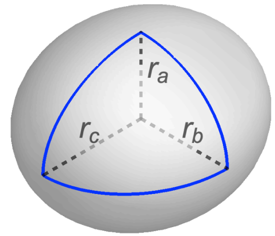


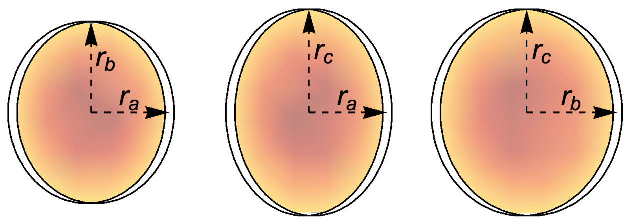
$$\mathrm{cov}\left(\epsilon_2^2, R_\perp\right) > 0$$

$$\left\langle(\delta R_\perp)^3\right\rangle < 0$$

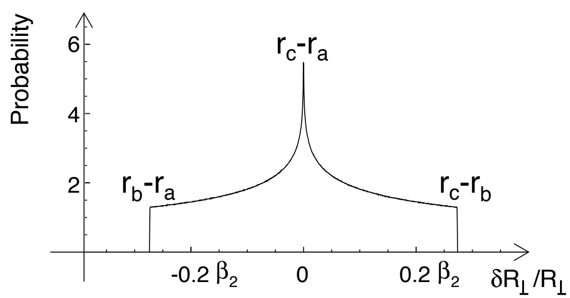
Triaxial

$\beta_2 = 0.25, \cos(3\gamma) = 0$





$\epsilon_2, R_\perp$ no linear correlation

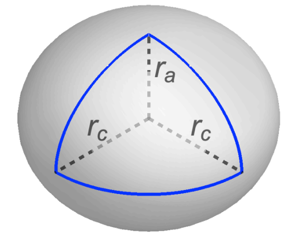


$$\mathrm{cov}\left(\epsilon_2^2, R_\perp\right) = 0$$

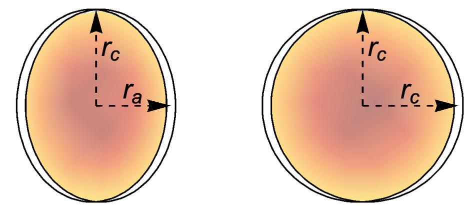
$$\left\langle(\delta R_\perp)^3\right\rangle = 0$$

Oblate

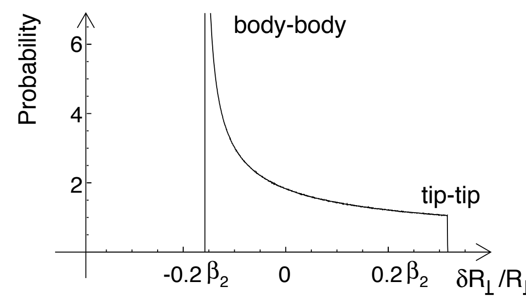
$\beta_2 = 0.25, \cos(3\gamma) = -1$



body+body tip+tip



$\epsilon_2 \uparrow, R_\perp \downarrow$ $\epsilon_2 \downarrow, R_\perp \uparrow$



$$\mathrm{cov}\left(\epsilon_2^2, R_\perp\right) < 0$$

$$\left\langle(\delta R_\perp)^3\right\rangle > 0$$

$\mathrm{cov}\left(\epsilon_2^2, d_\perp\right) < 0$, $\left\langle(\delta d_\perp)^3\right\rangle > 0$

$[p_T] \sim 1/R_\perp = d_\perp$

$\mathrm{cov}\left(\epsilon_2^2, d_\perp\right) = 0$, $\left\langle(\delta d_\perp)^3\right\rangle = 0$

$\mathrm{cov}\left(\epsilon_2^2, d_\perp\right) > 0$, $\left\langle(\delta d_\perp)^3\right\rangle < 0$

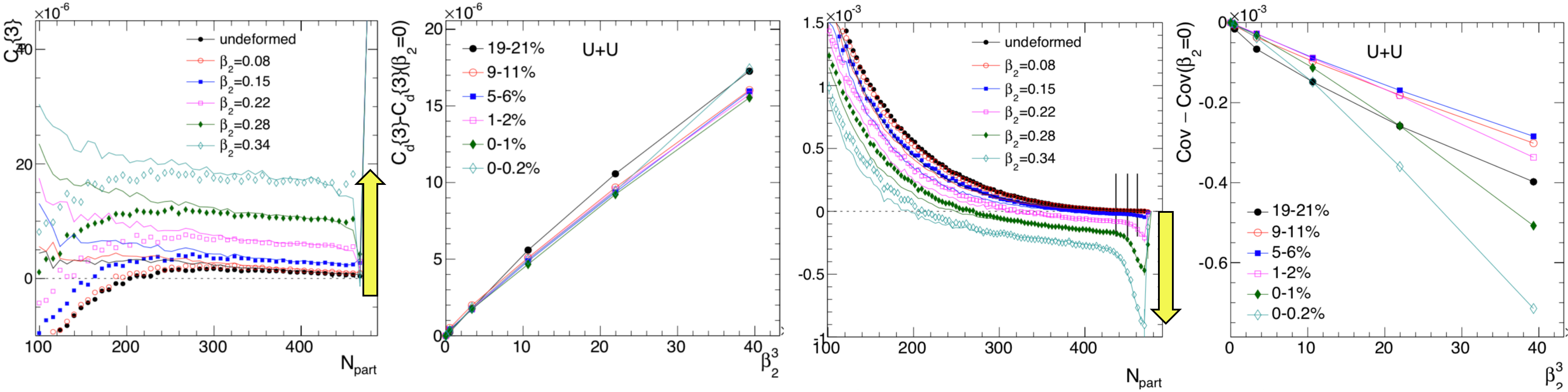
β_2 and γ dependence for skewness

Glauber model

See 2109.00604

$$\left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^3 \right\rangle \qquad \left\langle \varepsilon_2^2 \frac{\delta d_\perp}{d_\perp} \right\rangle$$

β_2 dependence



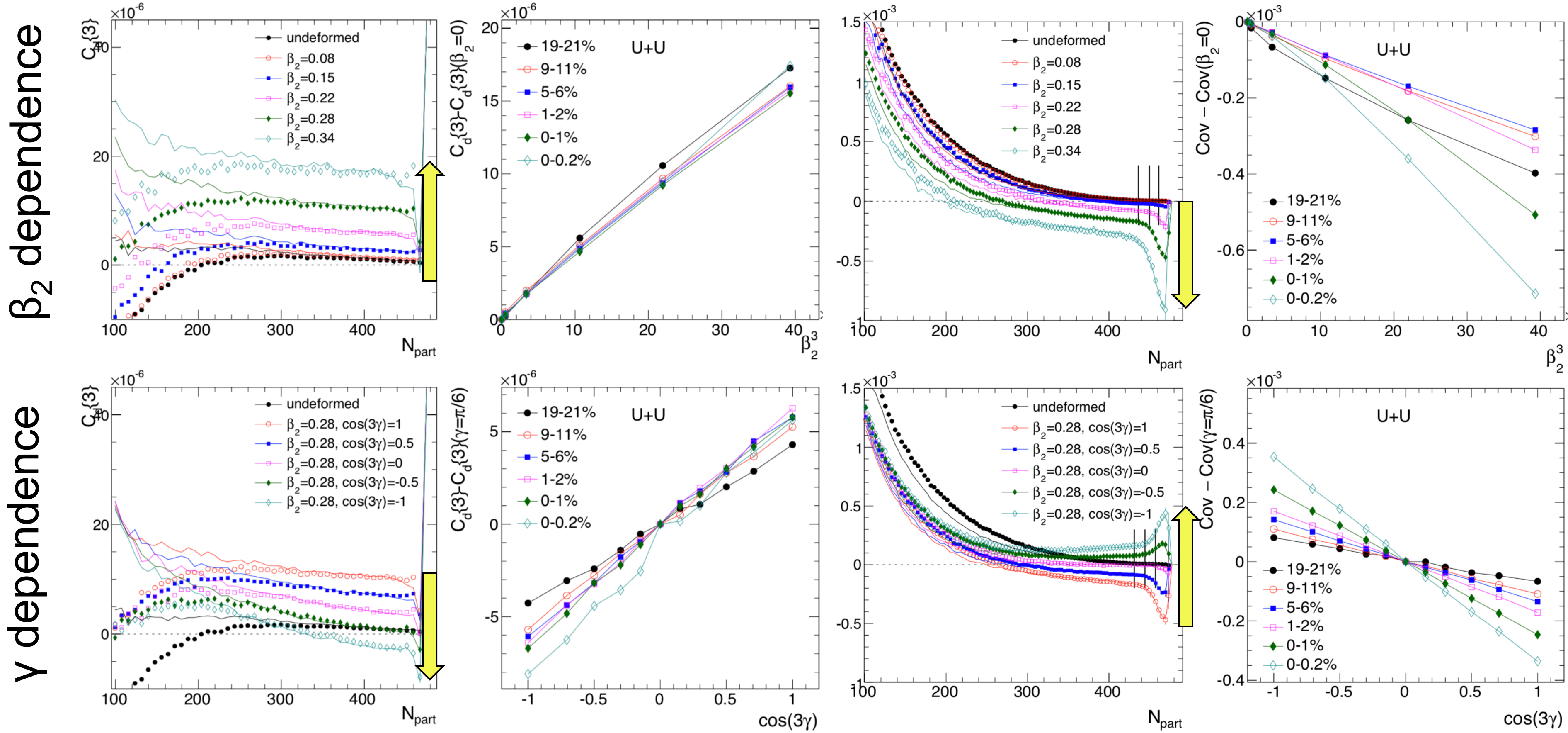
- Confirms β_2^3 dependence

β_2 and γ dependence for skewness

Glauber model

See 2109.00604

$$\left\langle \left(\frac{\delta d_\perp}{d_\perp}\right)^3 \right\rangle \qquad \left\langle \varepsilon_2^2 \frac{\delta d_\perp}{d_\perp} \right\rangle$$



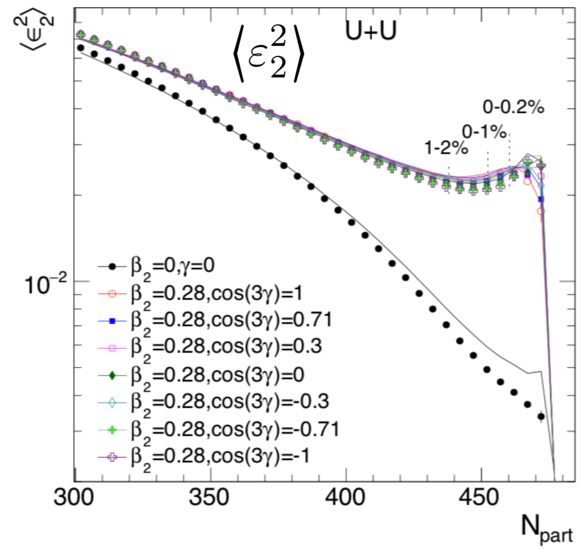
- Confirms $a' + (b' + c'\cos(3\gamma))\beta_2^3$

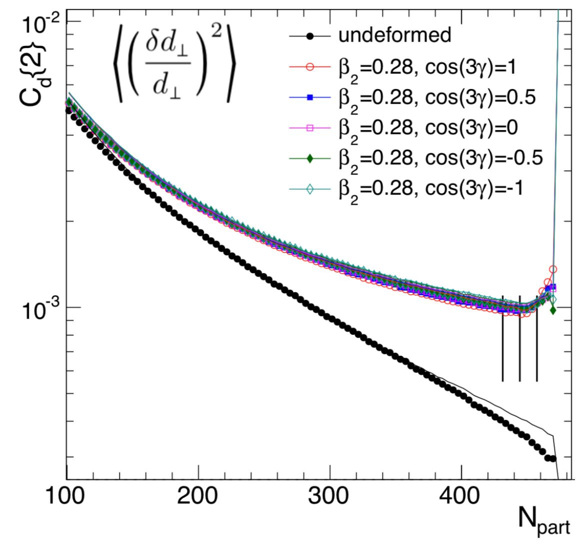
Influence of triaxiality γ

variances insensitive to γ

Only a function of β_2.

$$a' + b'\beta_2^2$$



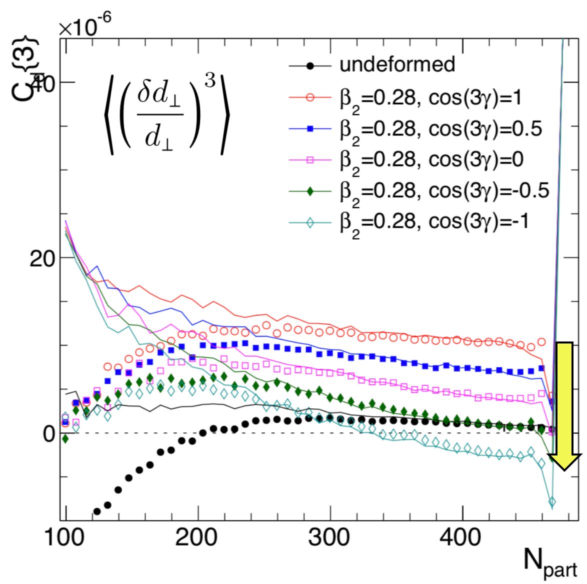


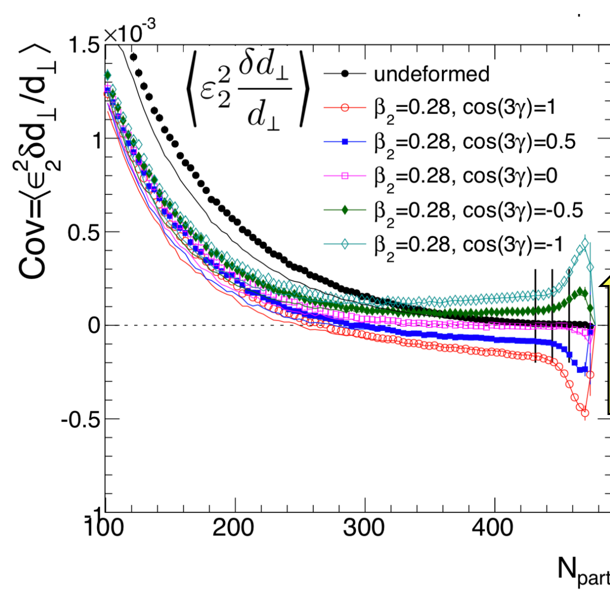
Skewness super sensitive

Opposite trends for the two observables.

Described by

$$a' + (b' + c'\cos(3\gamma))\beta_2^3$$





Use variance to constrain β_2, use skewness to constrain γ

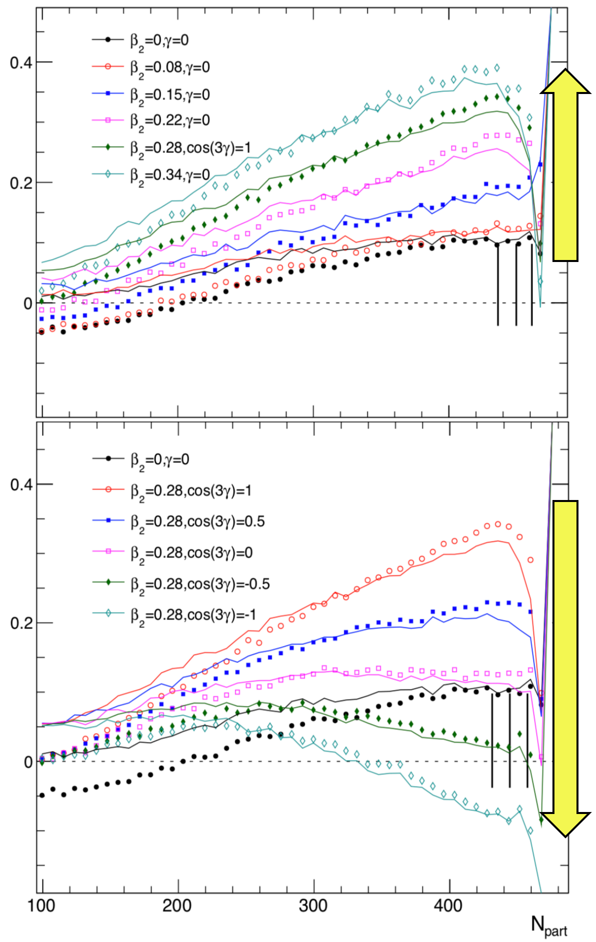
Skewness normalized by variances

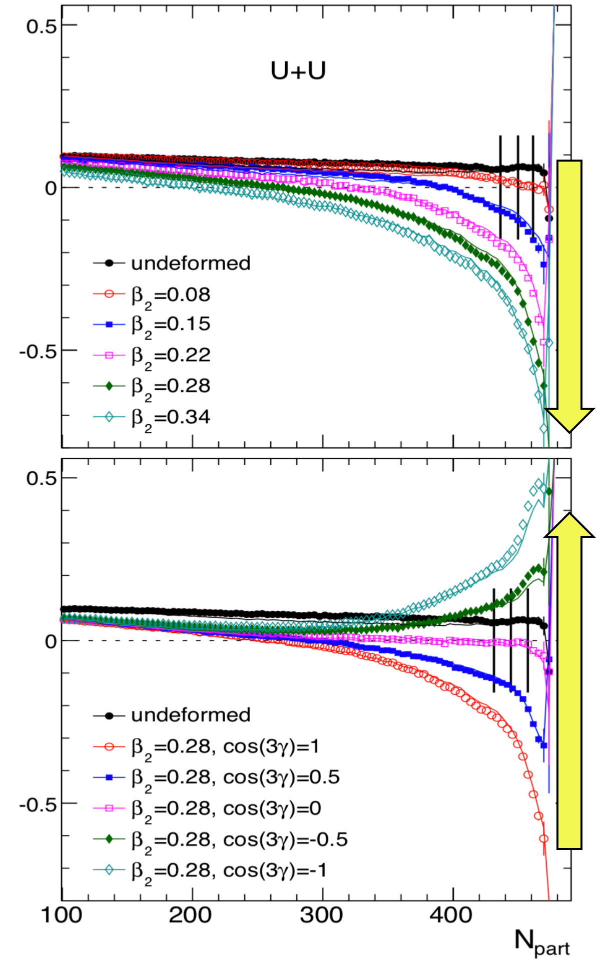
$$S_{\mathrm{d}} = \frac{\langle(\delta d_\perp)^3\rangle}{(\langle(\delta d_\perp)^2\rangle^{3/2}} \sim \frac{a_3 + (b_3 + c_3\cos(3\gamma))\beta_2^3}{(a_2 + b_2\beta_2^2)^{3/2}}$$

$$\rho = \frac{\langle \varepsilon_2^2 \delta d_\perp \rangle}{\langle \varepsilon_2^2 \rangle \sqrt{\langle(\delta d_\perp)^2\rangle}}$$

Different β_2

Different γ





Centrality dependence due to “a” terms

Simplified by subtracting the “a” terms

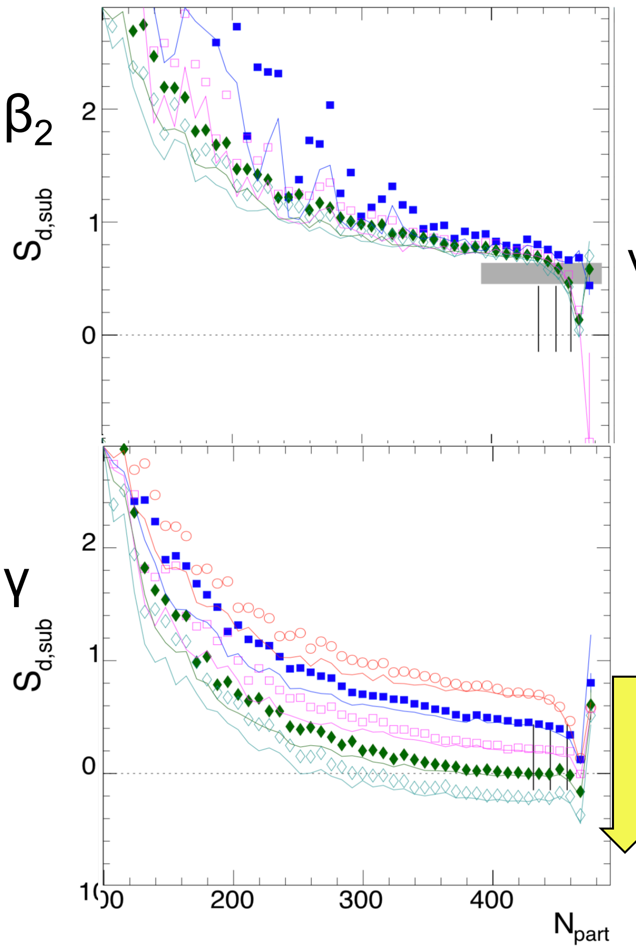
Skewness normalized by variances

$$S_{\mathrm{d,sub}} = \frac{\langle(\delta d_\perp)^3\rangle - \langle(\delta d_\perp)^3\rangle_{|\beta_2=0}}{(\langle(\delta d_\perp)^2\rangle - \langle(\delta d_\perp)^2\rangle_{|\beta_2=0})^{3/2}} \sim \frac{(b_3 + c_3\cos(3\gamma))\beta_2^3}{(b_2\beta_2^2)^{3/2}} = \frac{b_3}{b_2^{3/2}} + \frac{c_3}{b_2^{3/2}}\cos(3\gamma)$$

$$\rho_{\mathrm{sub}} = \frac{\langle\varepsilon_2^2\delta d_\perp\rangle - \langle\varepsilon_2^2\delta d_\perp\rangle_{|\beta_2=0}}{(\langle\varepsilon_2^2\rangle - \langle\varepsilon_2^2\rangle_{|\beta_2=0})\sqrt{\langle(\delta d_\perp)^2\rangle - \langle(\delta d_\perp)^2\rangle_{|\beta_2=0}}}$$

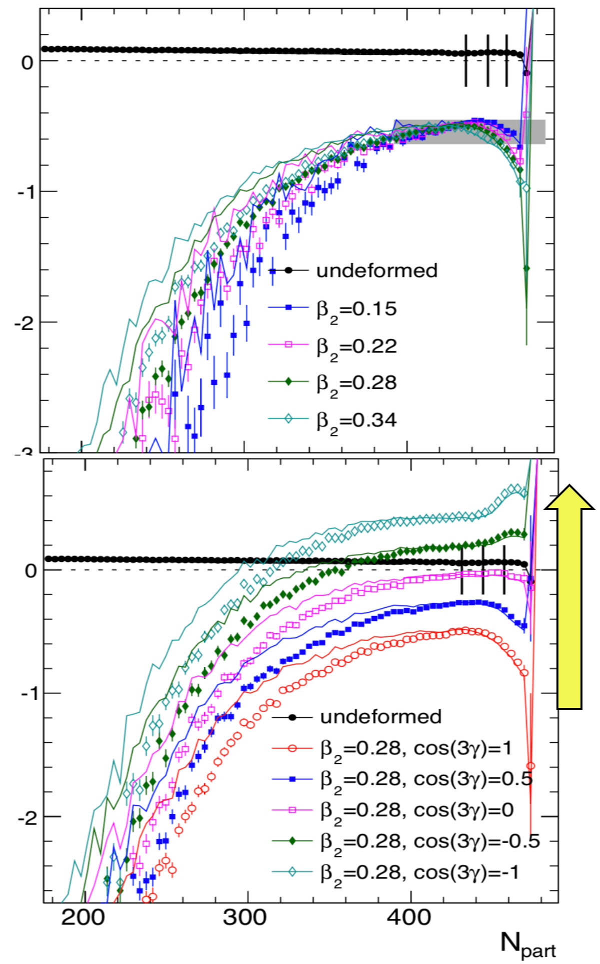
Different β_2

Different γ



β_2 dep. cancels

Values close to liquid-drop prediction (shaded band)

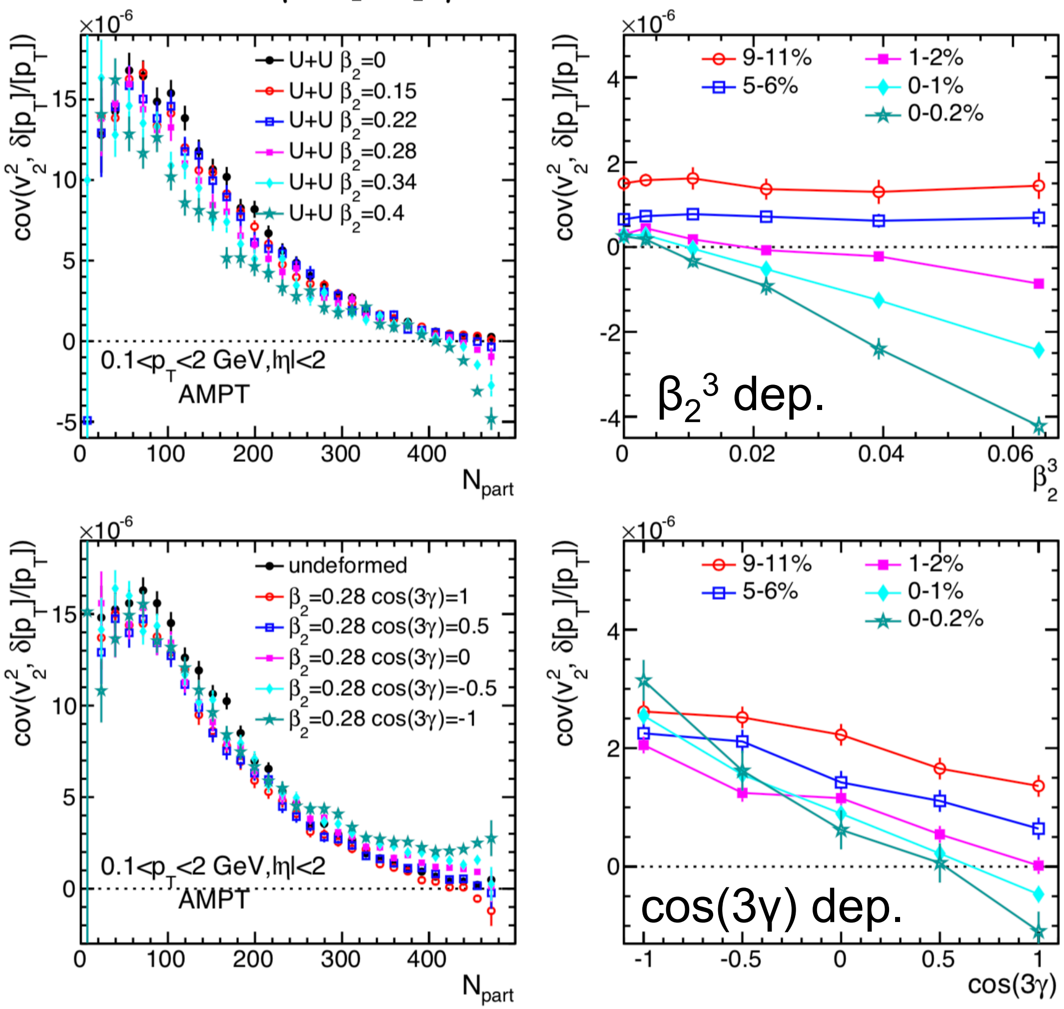


γ dependences after subtraction indep. of centrality ➔ $c_3/b_2^{3/2}$ indep. of centrality

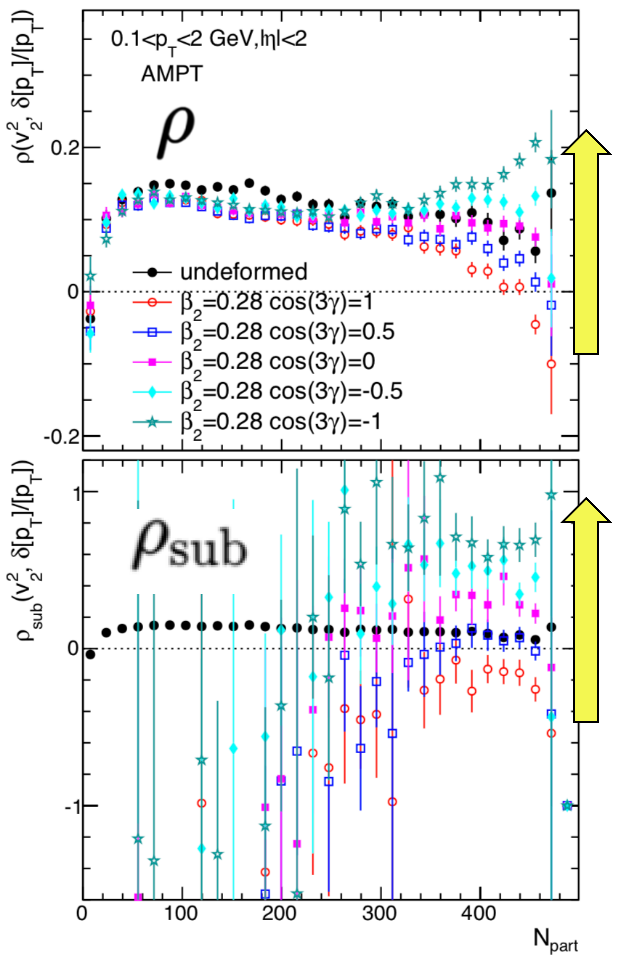
Unique and sensitive way to constrain the γ! contrast to NS

Do they survive to the final state: AMPT

$$\left\langle v_2^2 \frac{\delta[p_T]}{[p_T]} \right\rangle = a + (b + c\cos(3\gamma))\beta_2^3$$

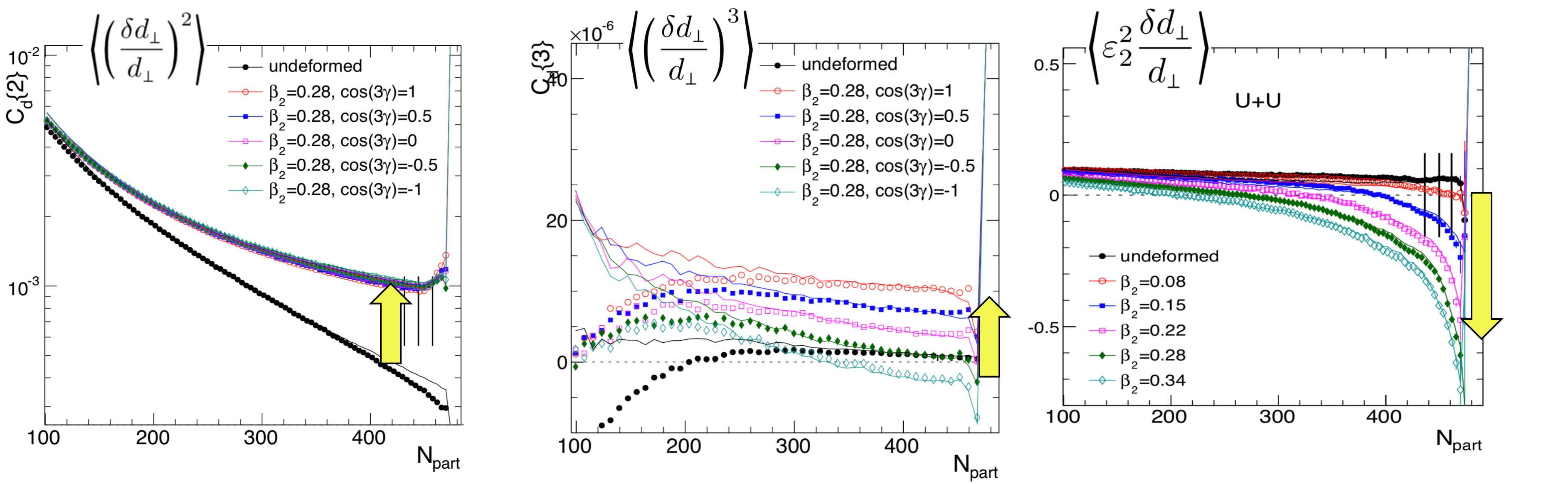


Normalized cumulant has similar behavior as Glauber

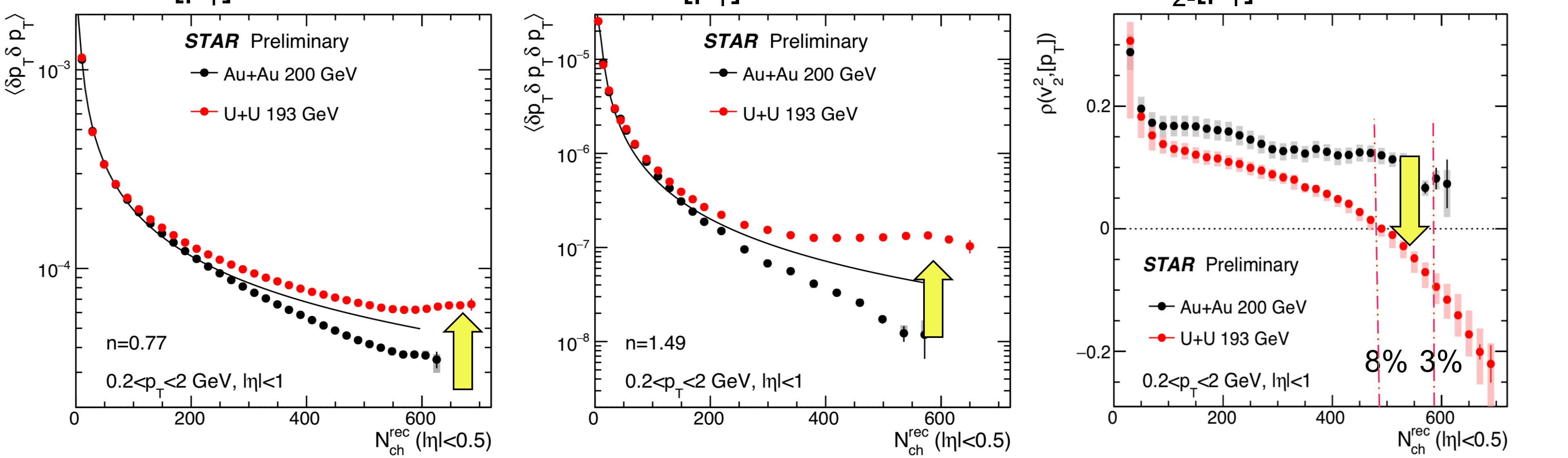


Initial shape/size fluctuations survive to the final state!

Contrast Glauber model with STAR data



[p_T] variance   [p_T] skewness v_2-[p_T] covariance



Require high-stat. hydro model simulation to quantify the response!

(β_2,γ) diagram in heavy-ion collisions

The (β_2,γ) dependence in 0-1% U+U Glauber model can be approximated by

$$\langle\varepsilon_2^2\rangle \approx [0.02 + \beta_2^2] \times 0.235$$

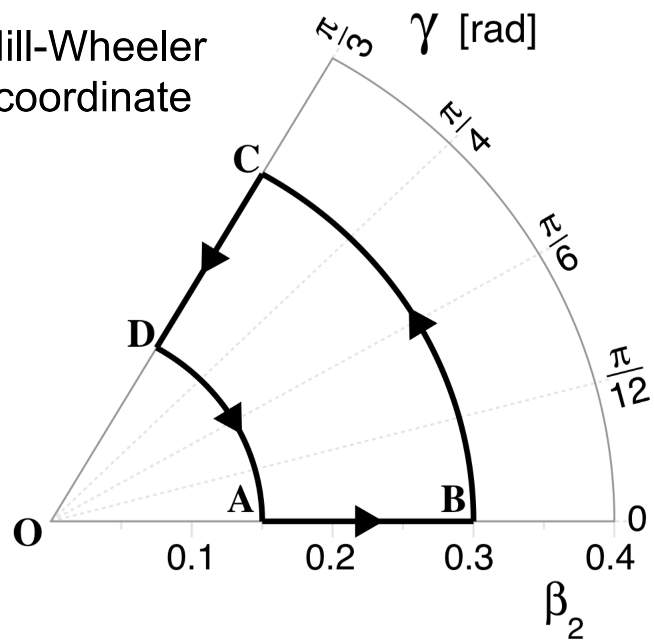
$$\langle(\delta d_\perp/d_\perp)^2\rangle \approx [0.035 + \beta_2^2] \times 0.0093$$

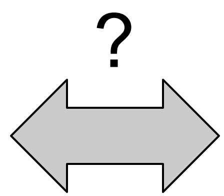
$$\langle\varepsilon_2^2 \delta d_\perp/d_\perp\rangle \approx [0.0005 - (0.07 + 1.36\cos(3\gamma))\beta_2^3] \times 10^{-2}$$

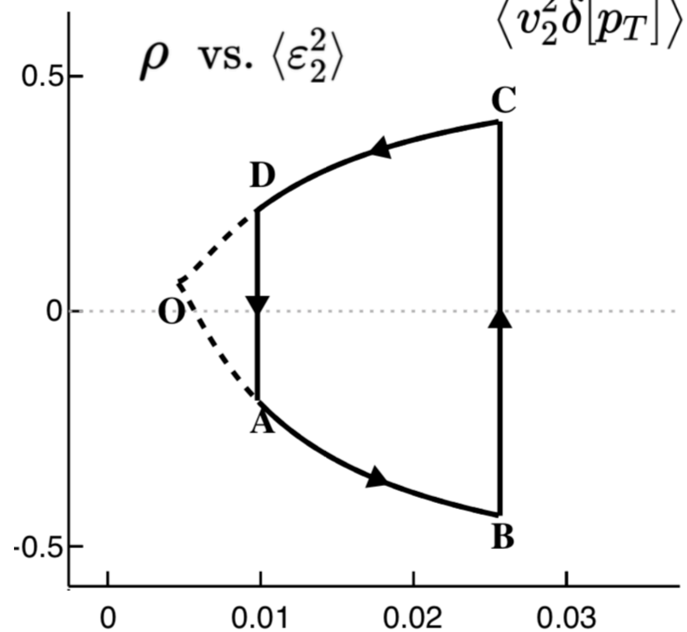
$$\rho = \frac{\langle\varepsilon_2^2 \delta d_\perp\rangle}{\langle\varepsilon_2^2\rangle\sqrt{\langle(\delta d_\perp)^2\rangle}}$$

Map from (β_2,γ) plane to HI observables

Hill-Wheeler coordinate







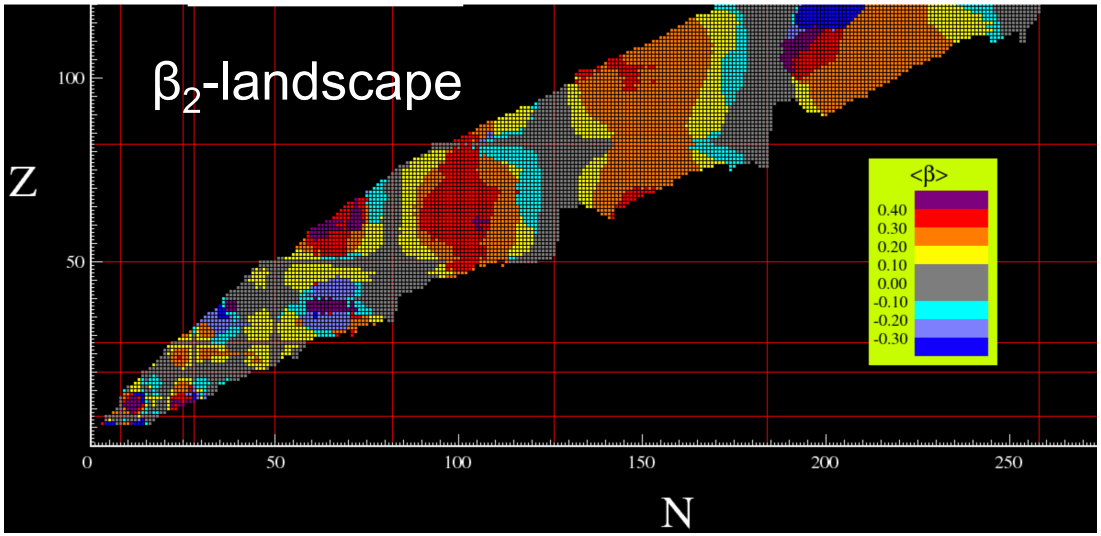
How about $\langle v_2^2 \delta[p_T]\rangle$ vs. $\langle v_2^2\rangle$

Collision system scan to map out this trajectory: calib. coefficients with species with known β,γ, then predict for species of interest.

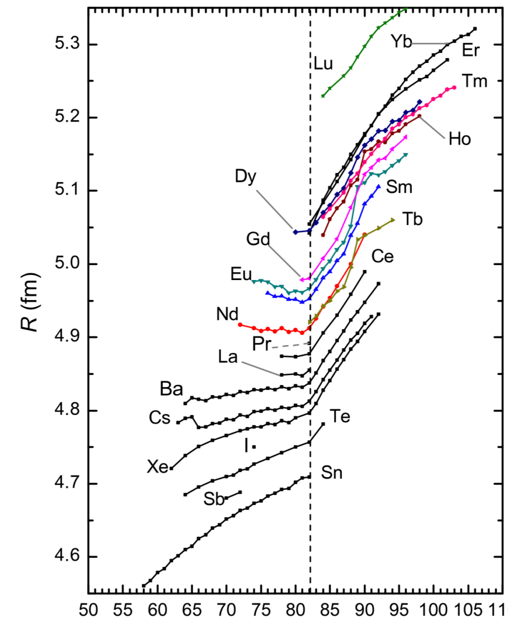
Outlook

shape/size landscapes from NS

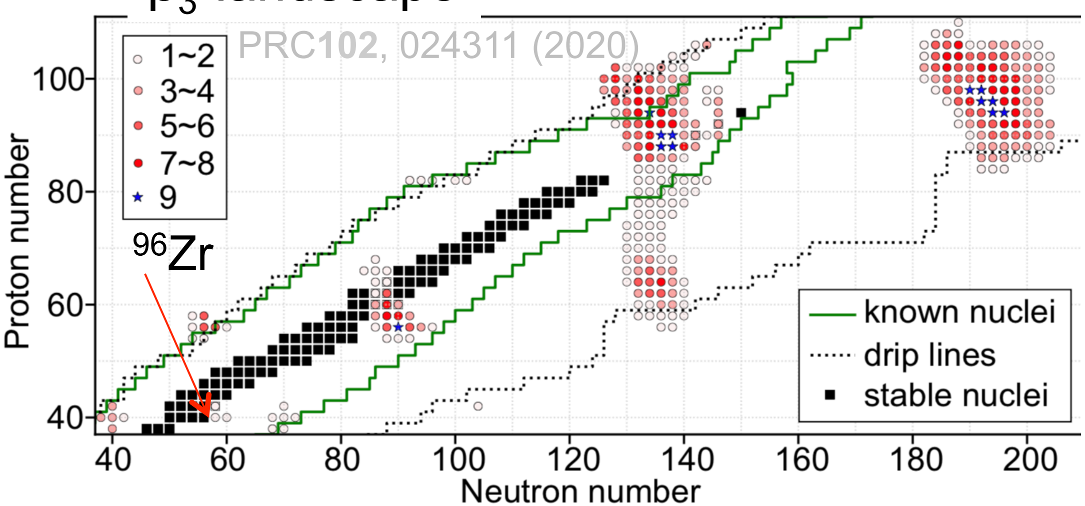
A lot of possibility for scan. But need to first establish the HI systematics using species with known deformations



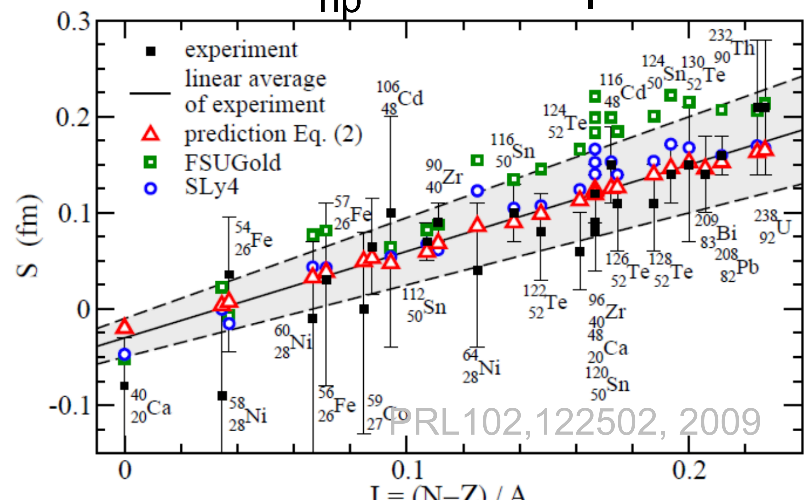
Radii-landscape



β3-landscape



Δr$_{np}$-landscape



Proposal in STAR BUR

STAR Beam Use Request for RUN 2022-2025
https://drupal.star.bnl.gov/STAR/system/files/STAR_Beam_Use_Request_Runs22_25.pdf

- β_n available mostly for n=2 and even-even, but we collided several odd-mass ones ☹

A list of large systems from RHIC and LHC

	β_2	β_3	β_4		β_2	β_3	β_4
^{238}U	0.286 [9]	0.078 [10]	0.094 [10]	^{208}Pb	0.06 [9]	0.04[11]	?
^{197}Au	-(0.13-0.16) [12, 13]	?	-0.03 [12]	^{129}Xe	0.16 [12]	?	?
^{96}Ru	0.16 [14]	?	?	^{96}Zr	0.06 [14]	0.20-0.27	0.06 [12]

- Step1: calibrate systematics with two species around ^{197}Au: ^{208}Pb & ^{198}Hg (β_2= -0.11)
  - ^{208}Pb √s=0.2 RHIC vs 5 TeV @LHC: Precision on IS and pre-equlibrium dynamics
  - ^{208}Pb √s=0.2 vs ^{197}Au √s=0.2 TeV: Quantify effects of Au deformation
  - ^{198}Hg √s=0.2 TeV: with known β_2 cross-check the consistency of β_{2Au}, γ in ^{197}Au.
- Step2: explore more exotic regions for triaxial and octupole deformations
  - Scan a isotopic chain: ^{144}Sm (β_2=0.08),^{148}Sm (β_2=0.14,triaxial),^{154}Sm (β_2=0.34)
    - These species are in region Z~56/N~88, where large octupole is expected/predicted.
  - Compare a pair with equal mass: ^{154}Sm (β_2 = 0.34) and ^{154}Gd (β_2 = 0.31)
- Due to constrain of sPHENIX program, can only do this opportunistically at RHIC, but how about LHC beyond 2030? What about NICA at √s=11GeV?

Open-questions and Opportunities

- How can we use hydrodynamic response to image the shape and radial profile of nuclei? and how are they related to properties measured in nuclear structure experiments?
- How does the uncertainty brought by nuclear structure impact the initial state of heavy-ion collisions and the extraction of QGP transport properties?
- What is the best nuclear structure knowledge of the species used so far in heavy-ion experiments? Conversely, what is the implication of heavy-ion data for the development of ab-initio methods of nuclear structure?
- What additional systems would be beneficial for both communities? What can be done at the LHC and at RHIC before EIC?

Planning an INT program to discuss connection between NS and HI in 2022-2023