Damping and Polarization Rates in Near Equilibrium Spin Transport

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Z.Wang, X.Guo and P.Zhuang, [arXiv:2009.10930 [hep-th]].
Z.Wang and P.Zhuang, [arXiv:2105.00915 [hep-ph]].

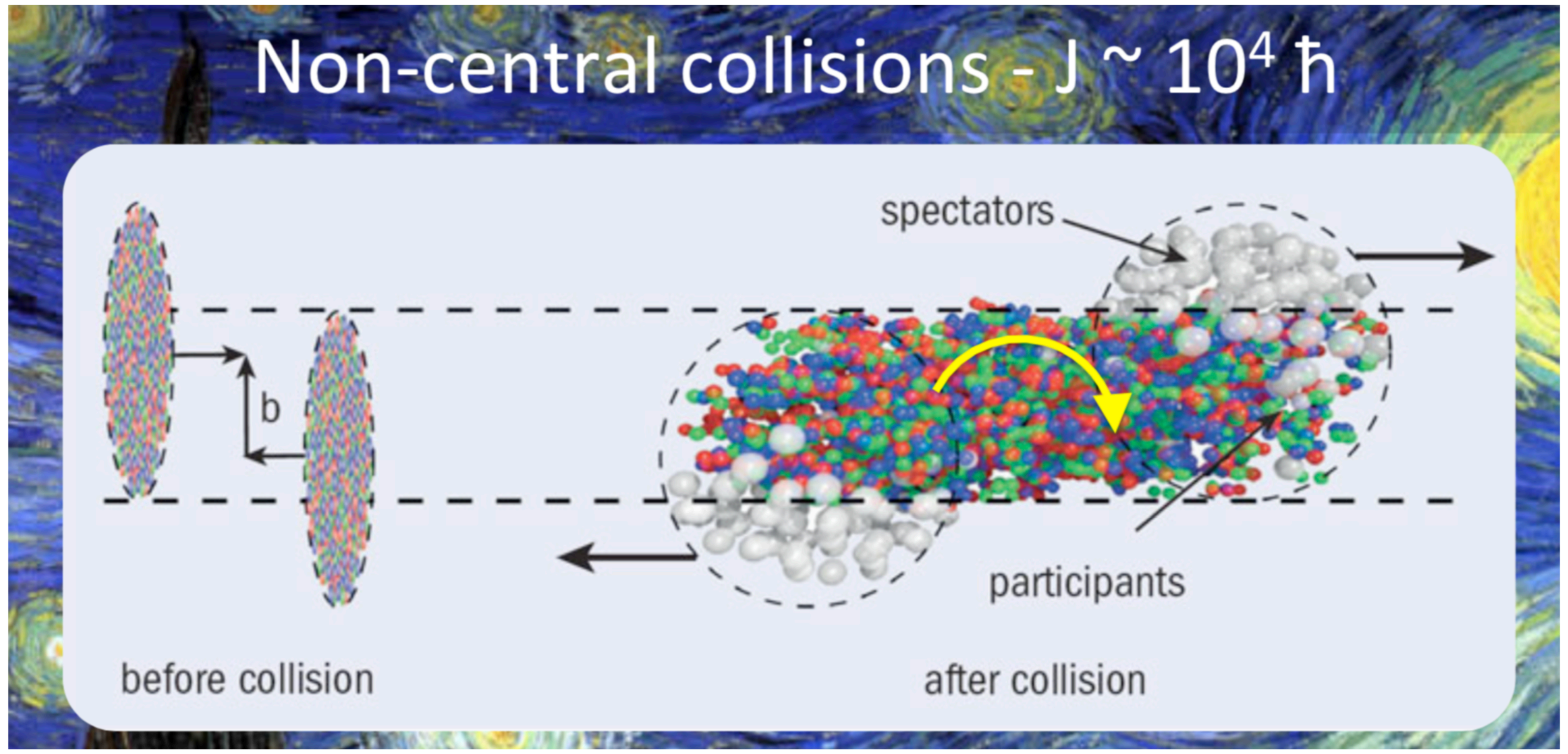
Outline

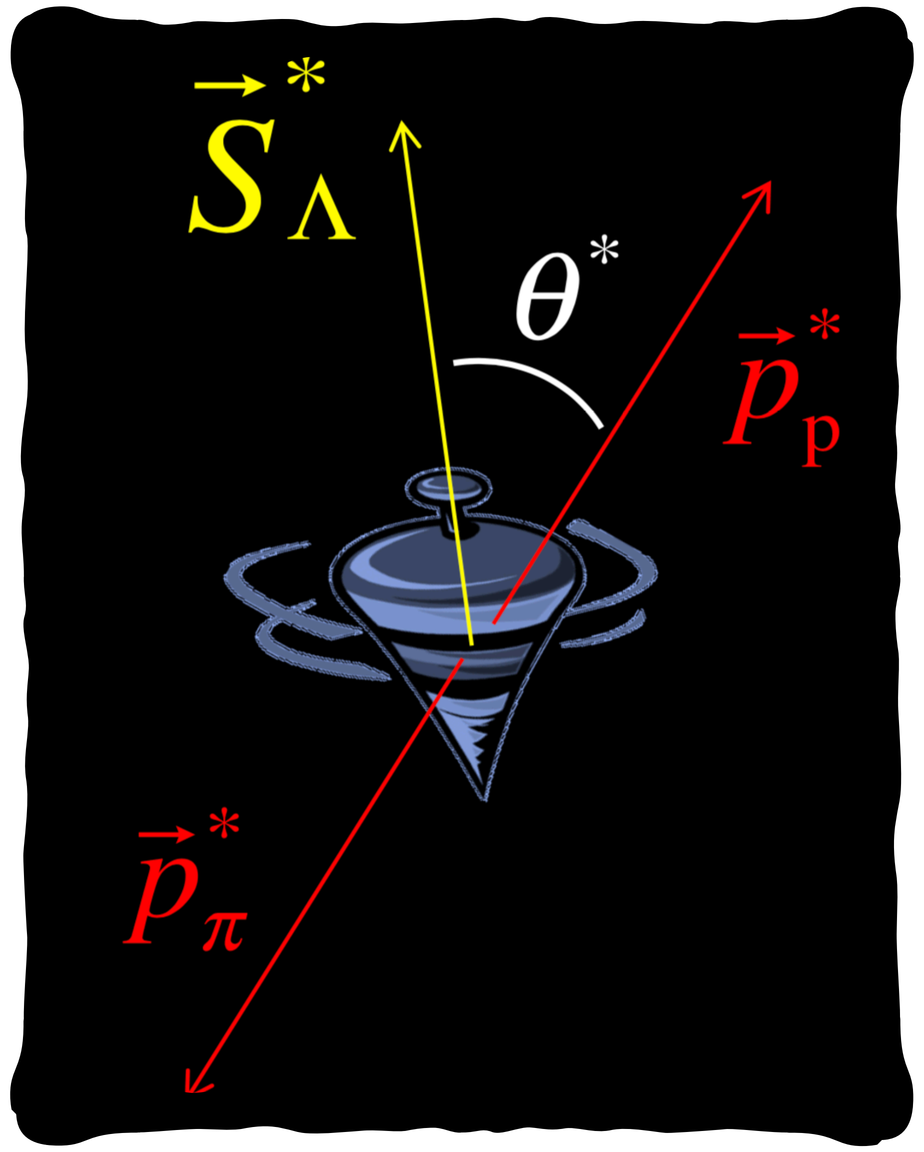
- **Background** Global OAM, Global polarization, Local Polarization, Spin Sign Puzzle, Collisions
- **Framework** Kadanoff-Baym equation
- **Global equilibrium — Detailed balance**
- **Close to equilibrium — Relaxation time approximation**

  RTA from Kadanoff-Baym equation; Interaction rate: NJL
- **Outlook**

Global Orbital Angular Momentum (OAM)

- Non central collision creates fireball with large OAM
- Some part can be transferred from orbital to spin
- Conservation $J_{\text{ini}} = L_{\text{ini}} = L_{\text{final}} + S_{\text{final}} \sim 10^6 \hbar$





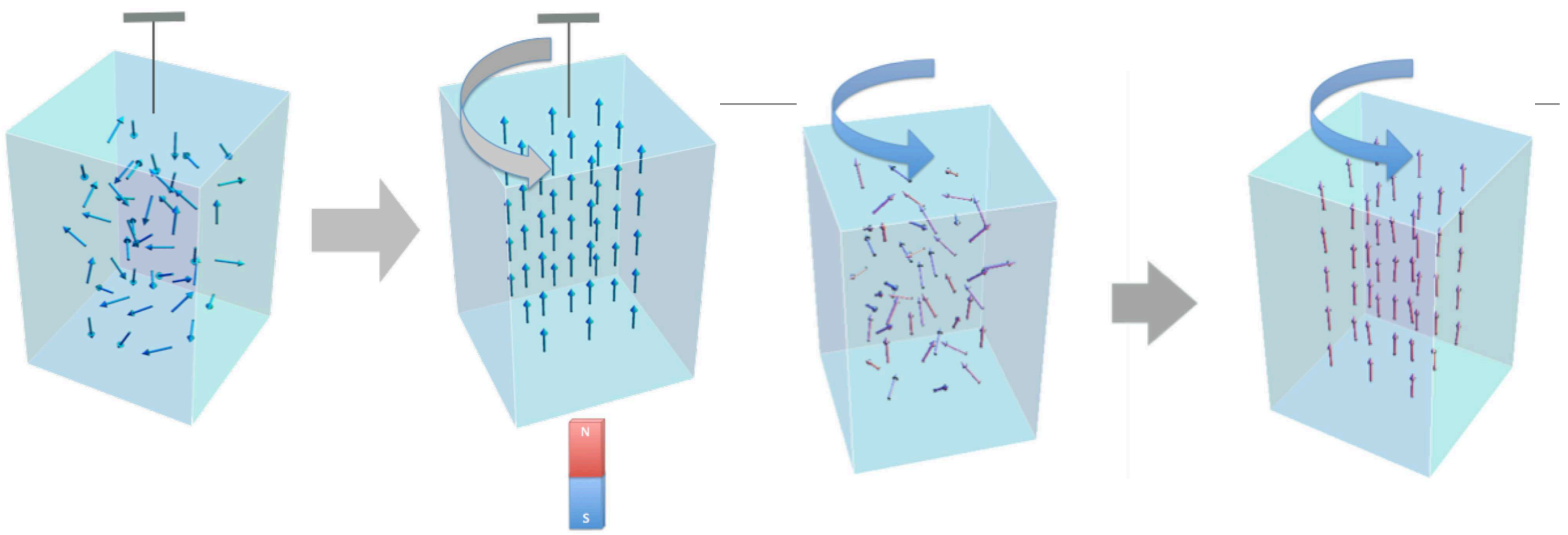
- Spin alignment with angular momentum of the emitting system
- Measurable through weak decay of Λ

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}\left(1 + \alpha \vec{P} \cdot \hat{p}^*_p\right) = \frac{1}{4\pi}(1 + \alpha P \cos\theta^*)$$

- Experimentally observed global Λ polarization.

Global Polarization

Einstein-de Haas effect   Barnett effect.



- Polarization of Λ through **spin-orbital coupling**

$$\frac{dN}{d\mathbf{p}} \sim e^{-(H_0 - \omega\cdot\mathbf{J})/T}$$

PRC 77, 044902(2008)

- Spin enslaved by thermal vorticity

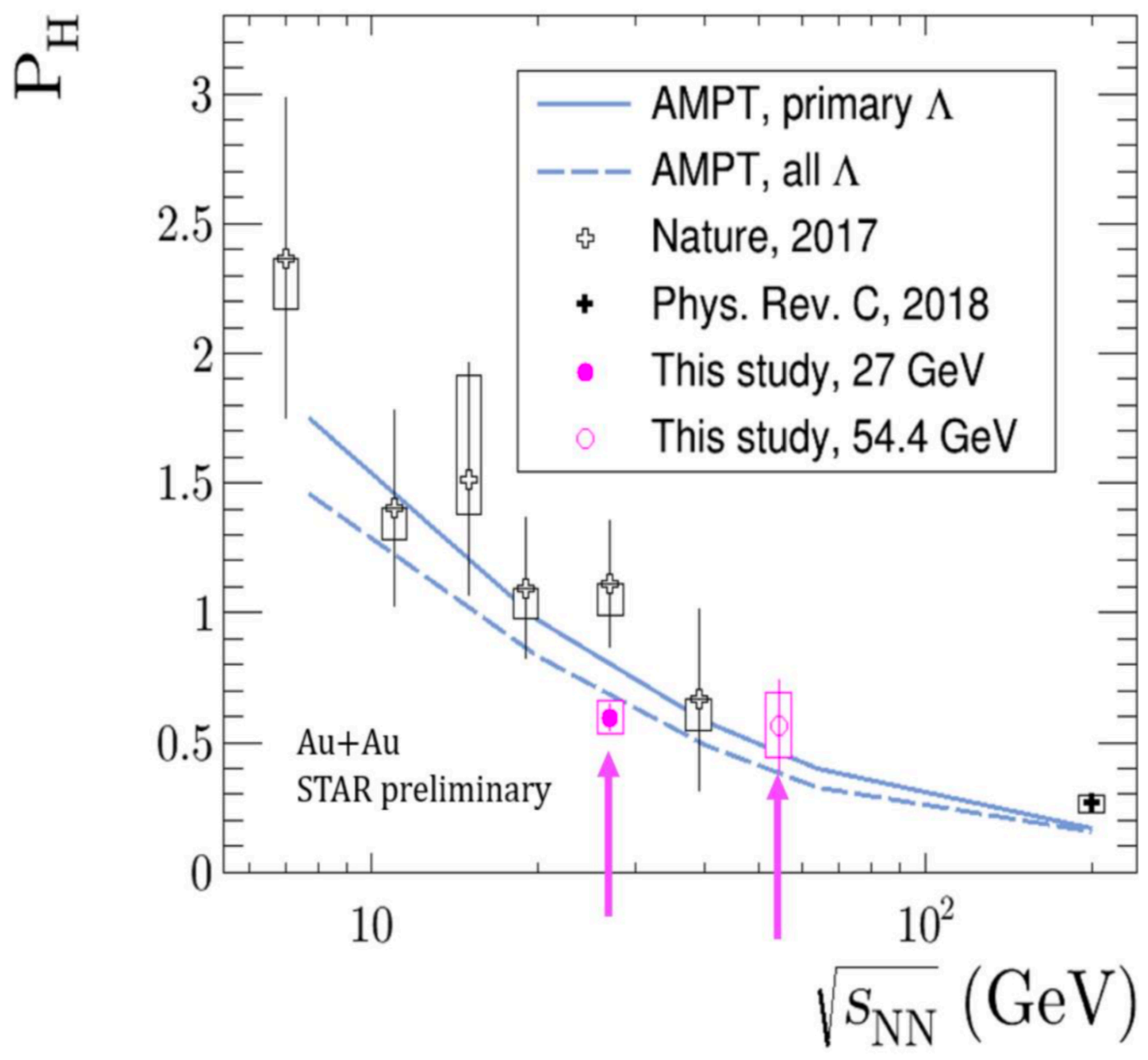
AP. 338 (2013) 32-49
PRC94 (2016), 024904

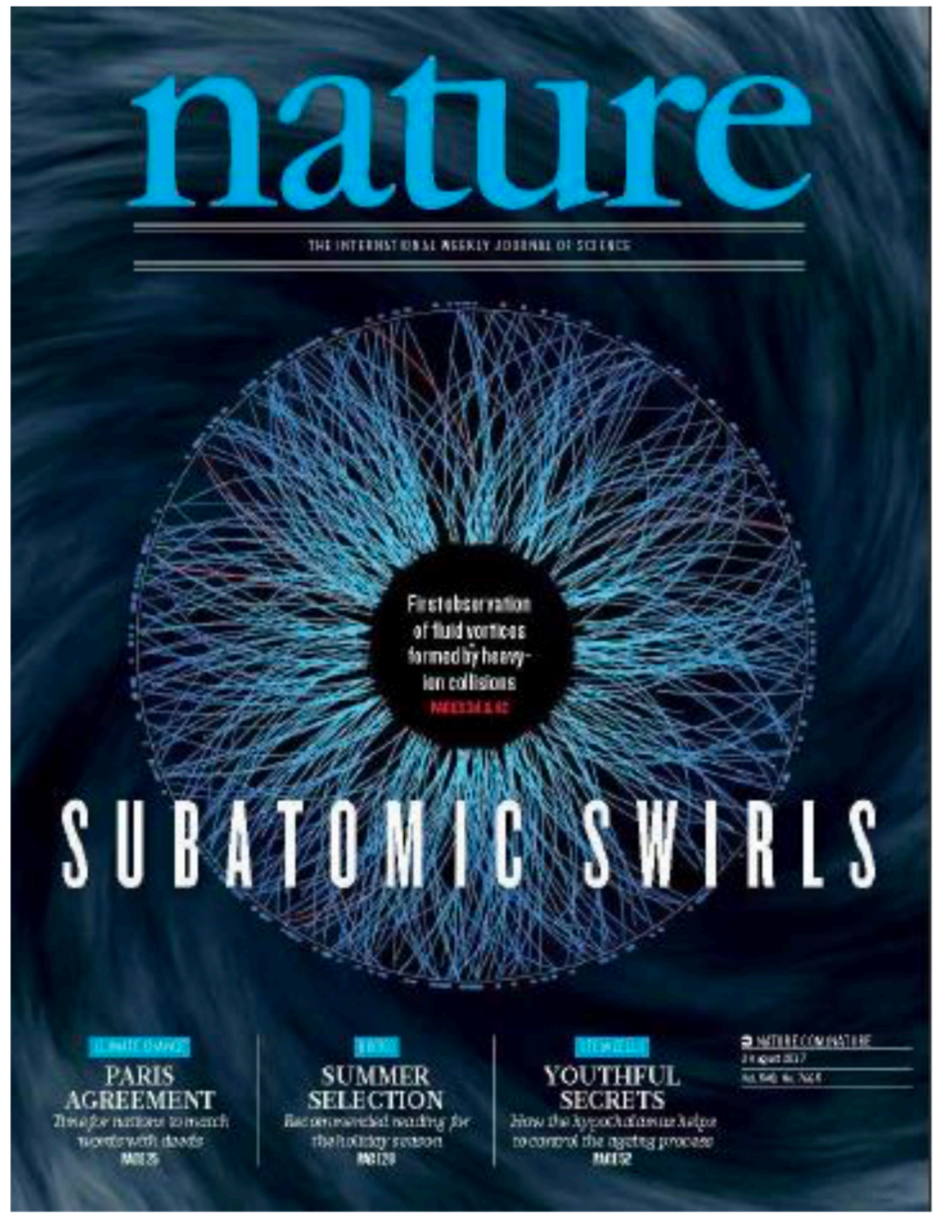
$$P^\mu(p) = \frac{1}{8m}\epsilon^{\mu\nu\rho\sigma} p_\nu \frac{\int d\Sigma_\lambda p^\lambda f'(x,p)\varpi_{\rho\sigma}(x)}{\int d\Sigma_\lambda p^\lambda f(x,p)} + \mathcal{O}(\varpi^2)$$

Polarization at freeze out

Global equilibrium

$$\varpi_{\rho\sigma} = \frac{1}{2}\left(\partial_\sigma\beta_\rho - \partial_\rho\beta_\sigma\right)$$





AMPT Li, Pang, Wang, Xia, PRC96, 054908(2017)

UrQMD Karpenko, Becattini, EPJC 77, 213(2017)

PICR hydro Xie, Wang, Csernai, PRC 95,031901(2017)

Chiral Kinetic + Collisions Sun, Ko, PRC96, 024906 (2017)

Experiment = Theory

- **How does initial OAM transfer to the matter created? (thermal vorticity)**

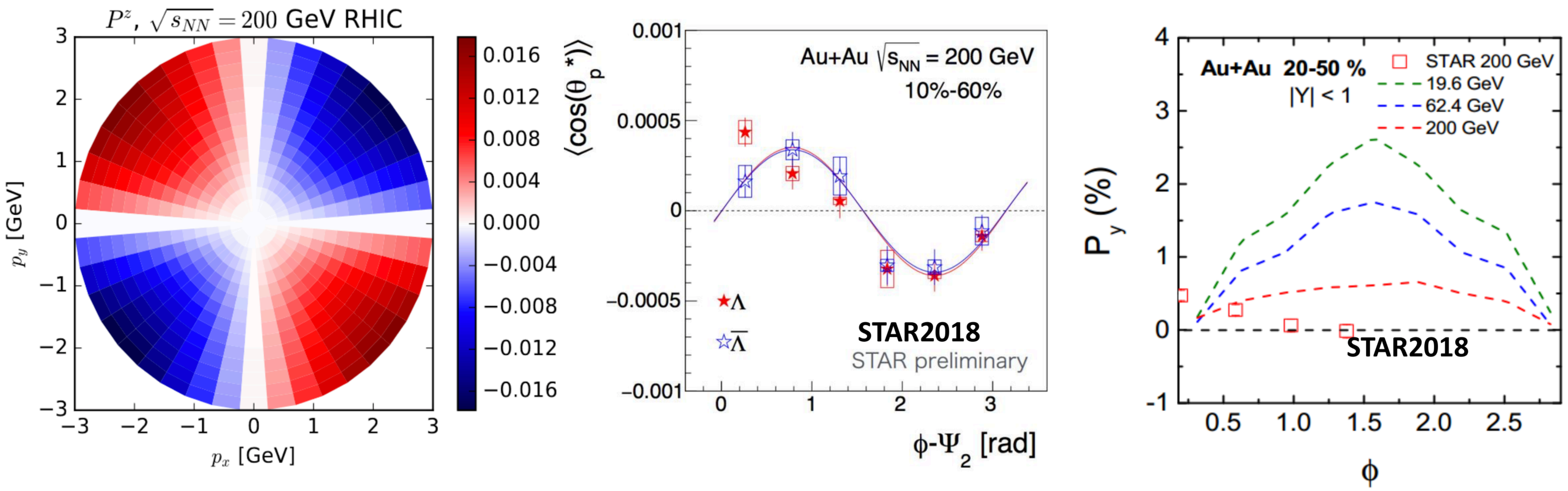
Local Polarization

Global polarization — total amount of angular momentum retains in the mid-rapidity region

Local polarization — its distribution in different ϕ

- Spin harmonic flow $$\frac{1}{2\pi}\frac{dP_{y,z}}{d\phi} = P_{y,z} + 2f_{2y,z}\sin(2\phi) + 2g_{2y,z}\cos(2\phi) + \cdots$$

Experiment ≠ Theory "Spin sign puzzle"



Longitudinal $f_{2z}^{\text{ther}} < 0, f_{2z}^{\text{exp}} > 0$

Transverse $g_{2y}^{\text{ther}} < 0, g_{2y}^{\text{exp}} > 0$

both thermal vorticity & thermal shear tensor — correct sign

S.Y.F.Liu, Y.Yin, arXiv:2103.09200
B.Fu, S.Y.F.Liu, L.Pang, H.Song and Y.Yin, arXiv:2103.10403
F. Becattini, M. Buzzegoli, A. Palermo ,arXiv:2103.10917
F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, I. Karpenko, arXiv:2103.14621
C. Yi, S. Pu and D.L.Yang, arXiv:2106.00238

Collisions

essential, not just for polarization

- Boltzmann eq — with collision but no spin
- Spin transport, free streaming— with spin but no collision

Mueller, Venugopalan, PRD 99 (2019), 056003
Hattori, Hidaka, Yang, PRD100 (2019), 096011
Weickgenannt, Sheng, Speranza, Wang, Rischke, PRD100 (2019), 056018
Gao, Liang, PRD100 (2019), 056021
Wang, Guo, Shi, Zhuang, PRD100 (2019), 014015

- **How does global OAM transfer to spin polarization?**
- **Does spin d.o.f reach local or global equilibrium? How? When?**
- **The contribution of shear indicates non-equilibrium effects.**

Li, Yee, PRD100 (2019), 056022 — Quantum kinetic theory, large quark mass, perturbative QCD, leading log, spin density matrix, collision in Γ

Ayala, PLB.801 (2020) 135169 — Relaxation time, effective coupling of spin to thermal vorticity $\lambda_a^\mu = g\frac{\sigma^{\alpha\beta}}{2}\varpi_{\alpha\beta}\gamma^\mu t_a$

Kapusta, Rrapaj, Rudaz, PRC101 (2020), 024907 — vorticity fluctuations, helicity flip in scatterings ,equilibrium time too large to be relevant

Chen, Son, Stephanov, PRL115 (2015) 021601 Liu, Sun, Ko, ArXiv:1910.06774 — Anomalous Lorentz transformation $X'^\mu = \Lambda^\mu{}_\alpha x^\alpha + \Delta^\mu_{\tilde{n}n'}$ enables angular momentum conservation in collisions.

Zhang, Fang, QWang, XNWang, PRC100 (2019), 064904 — Particle scattering with finite impact parameter -- finite OAM.In-state as wave packet

Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612 — Covariant transport for massive fermion with collision

Weickgenannt, Speranza, Sheng, Wang, Rischke, arXiv:2005.01506, arXiv: 2103.10636 — Non local collision, and Kadanoff-Baym equation

Defu Hou, Shu Lin, ArXiv: 2008.03862 — Polarization rotation of chiral fermion

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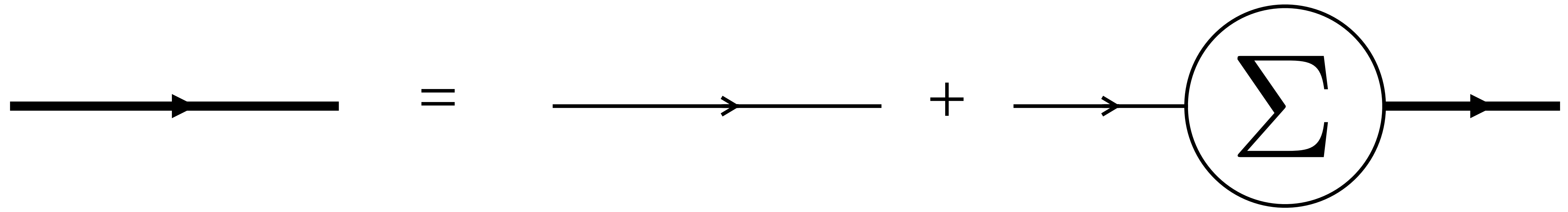
- **Outlook** Interaction rate: hot QED plasma, hot (dense) QCD plasma …
  Relate to field theory: Polarization rate from imaginary part
  Relate to hydro: Angular momentum conservation, entropy production

Dyson Equation

General Dyson equation

$$S(x,y) = S^0(x,y) + \int d^4z d^4w S^0(x,w)\Sigma(w,z)S(z,y)$$
$$= S^0(x,y) + \int d^4z d^4w S(x,w)\Sigma(w,z)S^0(z,y)$$



Dyson equation for contour Green's function

$$\begin{aligned}
\left(i\hbar\gamma^\mu\partial^x_\mu - m + \hbar\Sigma^\delta(x)\right)S^<(x,y) &= -\hbar\int_{-\infty}^{\infty} dz\left(\Sigma_R(x,z)S^<(z,y) + \Sigma^<(x,z)S_A(z,y)\right)\\
S^<(x,y)\left(i\hbar\gamma^\mu\overleftarrow{\partial}^y_\mu + m - \hbar\Sigma^\delta(y)\right) &= +\hbar\int_{-\infty}^{\infty} dz\left(S^<(x,z)\Sigma_A(z,y) + S_R(x,z)\Sigma^<(z,y)\right)
\end{aligned}$$

Wigner transformation

$$S(X,p) = \int d^4u e^{ip\cdot u/\hbar} S\left(X + \frac{u}{2}, X - \frac{u}{2}\right)$$

$$\begin{aligned}
\left(\gamma^\mu p_\mu - M(X)\right)S^< + \frac{i\hbar}{2}\gamma^\mu\nabla_\mu S^< + \frac{i\hbar}{2}(\nabla_\mu M)(\partial^p_\mu S^<) &= -\hbar\Sigma^<\hat{\Lambda}\mathrm{Re}S_R + \frac{i\hbar}{2}\left(\Sigma^<\hat{\Lambda}S^> - \Sigma^>\hat{\Lambda}S^<\right)\\
S^<\left(\gamma^\mu p_\mu - M(X)\right) - \frac{i\hbar}{2}S^<\gamma^\mu\overleftarrow{\nabla}_\mu - \frac{i\hbar}{2}(\partial^p_\mu S^<)(\nabla_\mu M) &= -\hbar\mathrm{Re}S_R\hat{\Lambda}\Sigma^< - \frac{i\hbar}{2}\left(S^>\hat{\Lambda}\Sigma^< - S^<\hat{\Lambda}\Sigma^>\right)
\end{aligned}$$

Expansion…

$$A \star B = AB + \frac{i\hbar}{2}[AB]_{\text{P.B.}} + \mathcal{O}(\hbar^2) \qquad [AB]_{\text{P.B.}} \equiv (\partial_q^\mu A)(\partial_\mu B) - (\partial_\mu A)(\partial_q^\mu B)$$

Transport and constraint

$$\left\{(\gamma^\mu p_\mu - M(X)), S^<\right\} + \frac{i\hbar}{2}\left[\gamma^\mu, \nabla_\mu S^<\right] = \frac{i\hbar}{2}\left([\Sigma^<, S^>]_\star - [\Sigma^>, S^<]_\star\right)$$

$$\left[(\gamma^\mu p_\mu - M(X)), S^<\right] + \frac{i\hbar}{2}\left\{\gamma^\mu, \nabla_\mu S^<\right\} + i\hbar(\nabla_\mu M)(\partial_\mu^p S^<) = \frac{i\hbar}{2}\left(\{\Sigma^<, S^>\}_\star - \{\Sigma^>, S^<\}_\star\right)$$

Wigner function and self-energy both have: spin structure; classical & quantum part; coordinate and momentum dependence; modified on-shell & off-shell;

Spin decomposition

$$S^< = \mathcal{S} + i\mathcal{P}\gamma^5 + \mathcal{V}_\mu\gamma^\mu + \mathcal{A}_\mu\gamma^5\gamma^\mu + \frac{1}{2}\mathcal{S}_{\mu\nu}\sigma^{\mu\nu},$$

$$S^> = \bar{\mathcal{S}} + i\bar{\mathcal{P}}\gamma^5 + \bar{\mathcal{V}}_\mu\gamma^\mu + \bar{\mathcal{A}}_\mu\gamma^5\gamma^\mu + \frac{1}{2}\bar{\mathcal{S}}_{\mu\nu}\sigma^{\mu\nu},$$

$$\Sigma^< = \Sigma_S + i\Sigma_P\gamma^5 + \Sigma_{V\mu}\gamma^\mu + \Sigma_{A\mu}\gamma^5\gamma^\mu + \frac{1}{2}\Sigma_{T\mu\nu}\sigma^{\mu\nu},$$

$$\Sigma^> = \bar{\Sigma}_S + i\bar{\Sigma}_P\gamma^5 + \bar{\Sigma}_{V\mu}\gamma^\mu + \bar{\Sigma}_{A\mu}\gamma^5\gamma^\mu + \frac{1}{2}\bar{\Sigma}_{T\mu\nu}\sigma^{\mu\nu}.$$

$\mathcal{S}$: scalar, related to mass distribution

$\mathcal{V}_\mu$: vector, number current

$\mathcal{A}_\mu$: axial-vector, related to spin density $\langle S^{\lambda,\mu\nu}\rangle = \frac{\hbar}{2}\int d^4p\,\epsilon^{\lambda\mu\nu\alpha}\mathcal{A}_\alpha$

$\mathcal{S}_{\mu\nu}$: tensor, magnetic moment tensor

Only 4 independent components, $\mathcal{V}_\mu, \mathcal{A}_\mu$

Semi-classical expansion

$$S^< = S^{<(0)} + \hbar S^{<(1)} + \cdots$$

$$\Sigma^< = \Sigma^{<(0)} + \hbar\Sigma^{<(1)} + \cdots$$

1) **mass density, number density — dominant by classical components**
2) **spin polarization generated by interaction with E, B, ω, all at $\mathcal{O}(\hbar)$**

Transport equation

- **Constraint & Transport**

$$\left\{(\gamma^\mu p_\mu - m), S^<\right\} + \frac{i\hbar}{2}\left[\gamma^\mu, \nabla_\mu S^<\right] = \frac{i\hbar}{2}\left([\Sigma^<, S^>]_\star - [\Sigma^>, S^<]_\star\right),$$
$$\left[(\gamma^\mu p_\mu - m), S^<\right] + \frac{i\hbar}{2}\left\{\gamma^\mu, \nabla_\mu S^<\right\} = \frac{i\hbar}{2}\left(\{\Sigma^<, S^>\}_\star - \{\Sigma^>, S^<\}_\star\right),$$

- **Spin Decomposition of both sides** Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612

Collision term

$$C = [\Sigma^<, S^>]_\star - [\Sigma^>, S^<]_\star = C_S + i\gamma^5 C_P + \gamma^\mu C_{V_\mu} + \gamma^5\gamma^\mu C_{A_\mu} + \frac{1}{2}\sigma^{\mu\nu} C_{T\mu\nu},$$
$$D = \{\Sigma^<, S^>\}_\star - \{\Sigma^>, S^<\}_\star = D_S + i\gamma^5 D_P + \gamma^\mu D_{V_\mu} + \gamma^5\gamma^\mu D_{A_\mu} + \frac{1}{2}\sigma^{\mu\nu} D_{T\mu\nu}.$$

- Take V and A as basis — vector charge & axial charge current
- Semi-classical expansion
  - V has one independent component

    $$p_{[\mu}\mathcal{V}^{(0)}_{\nu]} = 0 \qquad p_{[\mu}\mathcal{V}^{(1)}_{\nu]} = -\frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\nabla^\rho \mathcal{A}^{(0)\sigma} - \frac{1}{4}D^{(0)}_{T\mu\nu}$$

  - A has three independent components

    $$p_\mu \mathcal{A}^{(0)\mu} = 0 \qquad p_\mu \mathcal{A}^{(1)\mu} = \frac{1}{4}D^{(0)}_P$$

- On-shell condition & transport equations of V&A at each order

Transport equation

0th order transport

$$p\cdot\nabla\mathcal{V}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(0)}_\mu} - p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(0)}_\nu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{A}^{(0)\lambda}} - \frac{p_\beta}{m}\epsilon_{\mu\nu\alpha\lambda}p^\nu\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{A}^{(0)\lambda}},$$

$$p\cdot\nabla\mathcal{A}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(0)}_\mu} - p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(0)\nu}} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} + \widehat{\Sigma^{(0)}_{A\mu}p^\nu\mathcal{V}^{(0)}_\nu},$$

$\widehat{FG} = \bar{F}G - F\bar{G}$

Local collision term
Dynamical effect ,
e.g. diffusion effect

1st order transport

$$\begin{aligned}(p\cdot\nabla)\mathcal{V}^{(1)}_\mu =& - m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(1)}_\mu} - p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(1)}_\mu} + p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(1)}_\nu} + \frac{p_\nu}{m}\epsilon_{\rho\sigma\alpha\mu}p^\rho\widehat{\Sigma^{(0)\alpha\nu}_T\mathcal{A}^{(1)\sigma}} - \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma^{(0)\sigma\nu}_T\mathcal{A}^{(1)\lambda}}\\
&- m\widehat{\Sigma^{(1)}_S\mathcal{V}^{(0)}_\mu} - p^\nu\widehat{\Sigma^{(1)}_{V\nu}\mathcal{V}^{(0)}_\mu} + p_\mu\widehat{\Sigma^{(1)\nu}_A\mathcal{A}^{(0)}_\nu} + \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma^{(1)\alpha\nu}_T\mathcal{A}^{(0)\lambda}} - \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma^{(1)\sigma\nu}_T\mathcal{A}^{(0)\lambda}}\\
&- \frac{1}{2m}p^\nu[\Sigma^{(0)}_{T\mu\nu}\widehat{,(p^\alpha\mathcal{V}^{(0)}_\alpha)}]_{\text{P.B.}} + \frac{m}{2}[\Sigma^{(0)}_{T\mu\nu}\widehat{,\mathcal{V}^{(0)\nu}}]_{\text{P.B.}} - \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu[\Sigma^{(0)\alpha}_A\widehat{,\mathcal{V}^{(0)\beta}}]_{\text{P.B.}}\\
&- \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}\widehat{(\nabla^\alpha\Sigma^{(0)\nu}_V)\mathcal{A}^{(0)\beta}} + \frac{1}{2m}p_\mu\widehat{(\nabla^\nu\Sigma^{(0)}_P)\mathcal{A}^{(0)}_\nu} + \frac{1}{2m}\widehat{(p^\nu\nabla_\nu\Sigma^{(0)}_P)\mathcal{A}^{(0)}_\mu} - \frac{1}{2m}p^\nu\epsilon_{\mu\nu\alpha\beta}\widehat{(\nabla^\alpha\Sigma^{(0)}_S)\mathcal{A}^{(0)}}\\
&+ \frac{1}{2m}p_\nu\Sigma^{(0)}_{T\alpha\mu}\widehat{\nabla^{[\alpha}\mathcal{V}^{(0)\nu]}} - \frac{1}{2m}p_\nu\Sigma^{\alpha\nu(0)}_T\widehat{\nabla_{[\alpha}\mathcal{V}^{(0)}_{\mu]}} - \frac{1}{2}\epsilon_{\beta\nu\lambda\mu}\Sigma^{(0)\beta}_A\widehat{\nabla^\nu\mathcal{V}^{(0)\lambda}}\end{aligned}$$

- **Nonlocal collision term**
- **Related to spatial derivatives**
- **Correlated transport of V&A**
- **Polarization can be generated in a initially unpolarized system**

$$\begin{aligned}(p\cdot\nabla)\mathcal{A}^{(1)}_\mu =& - m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(1)}_\mu} - p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(1)}_\mu} - p^\nu\widehat{\Sigma^{(0)}_{A\mu}\mathcal{V}^{(1)}_\nu} - \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(1)\lambda}} + p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(1)\nu}}\\
&- m\widehat{\Sigma^{(1)}_S\mathcal{A}^{(0)}_\mu} - p^\nu\widehat{\Sigma^{(1)}_{V\nu}\mathcal{A}^{(0)}_\mu} - p^\nu\widehat{\Sigma^{(1)}_{A\mu}\mathcal{V}^{(0)}_\nu} - \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(1)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} + p_\mu\widehat{\Sigma^{(1)}_{A\nu}\mathcal{V}^{(0)\nu}}\\
&+ \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\widehat{(\nabla^\sigma\Sigma^{(0)\nu}_V)\mathcal{V}^{(0)\rho}} + \frac{m}{2}[\Sigma^{(0)}_P\widehat{,\mathcal{V}^{(0)}_\mu}]_{\text{P.B.}} - \frac{1}{2m}p_\mu[\Sigma^{(0)}_P\widehat{,(p^\nu\mathcal{V}^{(0)}_\nu)}]_{\text{P.B.}} - \frac{1}{2}\epsilon_{\mu\sigma\nu\rho}\widehat{\nabla^\sigma\Sigma^{(0)\nu}_A\mathcal{A}^{(0)\rho}}\\
&- \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}[\Sigma^{(0)\nu}_A\widehat{,(p^\alpha\mathcal{A}^{(0)\beta})}]_{\text{P.B.}} + \frac{m}{2}[\Sigma^{(0)}_{T\mu\nu}\widehat{,\mathcal{A}^{(0)\nu}}]_{\text{P.B.}} - \frac{1}{2m}p_\mu[\Sigma^{(0)}_{T\rho\nu}\widehat{,(p^\rho\mathcal{A}^{(0)\nu})}]_{\text{P.B.}}\\
&+ \frac{1}{2m}p_\sigma\nabla^\sigma\widehat{(\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)\nu})} - \frac{1}{2m}p^\nu\nabla^\sigma\widehat{(\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)}_\sigma)} - \frac{1}{2m}p_\mu\nabla^\sigma\widehat{(\Sigma^{(0)}_{T\sigma\nu}\mathcal{A}^{(0)\nu})} + \frac{1}{2m}p^\nu\nabla^\sigma\widehat{(\Sigma^{(0)}_{T\sigma\nu}\mathcal{A}^{(0)}_\mu)}.\end{aligned}$$

the interaction needs to be specified to calculate the off-diagonal self-energy $\Sigma^{>}$ & $\Sigma^{<}$

Angular momentum conservation

Total angular momentum

$$M_{\rho,\mu\nu} = x_\mu T_{\rho\nu} - x_\nu T_{\rho\mu} + \hbar S_{\rho,\mu\nu}$$

Relate to Wigner function

$$T^{\mu\nu} = \int d^4p \mathcal{V}^\mu p^\nu \qquad S^{\mu\nu\rho} = -\frac{1}{2}\int d^4p \epsilon^{\mu\nu\rho\lambda}\mathcal{A}_\lambda$$

Angular momentum conservation

$$\partial^\rho M_{\rho,\mu\nu} = T_{[\mu\nu]} + \hbar\nabla^\rho S_{\rho,\mu\nu} = 0$$

$$2p_{[\mu}\mathcal{V}_{\nu]} + \hbar\epsilon_{\mu\nu\rho\sigma}\nabla^\rho\mathcal{A}^\sigma = -\frac{\hbar}{2}D_{T\mu\nu}$$

$$\Rightarrow \quad \partial^\rho M_{\rho,\mu\nu} = T_{[\mu\nu]} + \hbar\nabla^\rho S_{\rho,\mu\nu} = \frac{\hbar}{4}\int d^4p\, D_{T\mu\nu}$$

$\mathcal{A}^{(1)}_\mu$ is non-trivial, verify conservation up to $\mathcal{O}(\hbar^2)$, requires $\frac{\hbar}{4}\int d^4p\, D^{(0),(1)}_{T\mu\nu} = 0$

$D^{(0)}_{T\mu\nu} = 0$, by power counting

$$D^{(1)}_{T\mu\nu} = -2\Big(\widehat{\Sigma_S\mathcal{S}_{\mu\nu}} + \widehat{\Sigma_{T\mu\nu}\mathcal{S}} + \epsilon_{\mu\nu\alpha\beta}\big(\widehat{\Sigma^\alpha_A\mathcal{V}^\beta} - \widehat{\Sigma^\alpha_V\mathcal{A}^\beta}\big)\Big)^{(1)} + \Big[\widehat{\Sigma_{V[\mu}\mathcal{V}_{\nu]}}\Big]^{(0)}_{\text{P.B.}}$$

With the self-energy from NJL model,

$$\frac{\hbar}{4}\int d^4p D^{(1)}_{T\mu\nu} = 0$$

The collision terms conserve angular momentum, OAM and spin convert with each other

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- **Outlook**

Fermionic 2 by 2 collision by contact interaction

- derive the equilibrium distribution from the detailed balance principle
- different interaction determines only how fast the system reaches equilibrium state, but not the equilibrium distribution function

NJL-type model with scalar-channel of interaction

$$\mathcal{L} = \bar{\psi}(i\hbar\partial\!\!\!/ - m)\psi + G(\bar{\psi}\psi)^2$$

(fermionic 2 by 2 scattering)

chiral restored phase — consider only the current mass

strong coupling nature — 1/Nc expansion & semiclassical expansion

$\mathcal{O}(1/N_c)$ $$\Sigma^{>}_{\rm LO}(X,p) = G^2 \int dP\; S^{>}(X,p_1)\mathrm{Tr}\Big[S^{<}(X,p_2)S^{>}(X,p_3)\Big],$$

$\mathcal{O}(1/N_c^2)$ $$\Sigma^{>}_{\rm NL}(X,p) = -G^2 \int dP\; S^{>}(X,p_1)S^{<}(X,p_2)S^{>}(X,p_3),$$

$$\int dP = \int \frac{d^4p_1 d^4p_2 d^4p_3}{(2\pi)^4(2\pi)^4(2\pi)^4}(2\pi)^4\delta(p-p_1+p_2-p_3)$$

Simplification:

- the detailed balance — gain term and the loss term cancel with each other in arbitrary collision channel
- consider only self-energy at 1/Nc order

Detailed balance at 0th and 1st order

- Detailed balance at 0th order

$$f^{(0)}_{V\mathrm{eq}}(X,p) = n_F(\beta\cdot p)$$
$$f^{(0)}_{A\mathrm{eq}}(X,p) = 0$$

Comes from $f_V(X,\mathbf{p}_1)\bar{f}_V(X,\mathbf{p})\bar{f}_V(X,\mathbf{p}_2)f_V(X,\mathbf{p}_3) - f_V(X,\mathbf{p})\bar{f}_V(X,\mathbf{p}_1)f_V(X,\mathbf{p}_2)\bar{f}_V(X,\mathbf{p}_3)$

- Simplified 1st transport, considering $\mathcal{A}^{(0)\mathrm{eq}}_\mu = 0$ (or the power counting)

$$p\cdot\nabla\mathcal{A}^{(1)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(1)}_\mu} + p_\nu\widehat{\Sigma^{(0)\nu}_V\mathcal{A}^{(1)}_\mu} - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\widehat{(\nabla^\sigma\Sigma^{(0)\nu}_V)\mathcal{V}^{(0)\rho}} + p_\nu\widehat{\Sigma^{(1)}_{A\mu}\mathcal{V}^{(0)\nu}} - p_\mu\widehat{\Sigma^{(1)}_{A\nu}\mathcal{V}^{(0)\nu}} + \frac{m}{2}\epsilon_{\rho\nu\lambda\mu}\widehat{\Sigma^{(1)\rho\nu}_T\mathcal{V}^{(0)\lambda}}$$

- Detailed balance — vanishing collision term

$$\mathcal{A}^{(1)global}_\mu(p) = -\frac{1}{(2\pi)^3 4E_p}\epsilon_{\mu\nu\sigma\lambda}p^\nu\nabla^\sigma\beta^\lambda f'_V(p)$$

$$\nabla^\mu\beta^\nu + \nabla^\nu\beta^\mu = 0$$

Killing condition -- global equilibrium

- Spin polarization generated from coupling between vector and axial-vector charge.
- In equilibrium spin polarization generated by thermal vorticity.
- Gradient expansion of Wigner function — Non local effect — orbital angular momentum

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Relaxation time approximation

- RTA is a simplification to the collision kernel of the Boltzmann equation, very useful and has been employed extensively to derive the form of kinetic coefficients and dissipative hydrodynamic equations as well as find their exact solutions and attractors
- comes at the expense of ignoring the interaction mechanism of the microscopic constituent
- RTA assumes a single timescale for thermalization of all types of microscopic interactions.

$$p \cdot \partial f(x,p) = -\frac{f(x,p) - f_{eq}(x,p)}{\tau(p)}$$

Anderson-Witting $\tau(p) = \frac{\tau}{u \cdot p}$

Conseravtion by Landau matching condition

$$\nabla_\mu J^\mu = -\frac{1}{\tau} u_\mu \delta J^\mu = 0,$$

$$\nabla_\mu T^{\mu\nu} = -\frac{1}{\tau} u_\mu \delta T^{\mu\nu} = 0.$$

For spin-1/2 particles, the RTA of the collision kernel in Anderson-Witting form

$$\Big(\gamma^\mu p_\mu - m\Big)\mathcal{W}(x,p) + \frac{i\hbar}{2}\gamma^\mu \nabla_\mu \mathcal{W}(x,p) \;=\; -\frac{i\hbar}{2}\gamma^\mu u_\mu \frac{\delta\mathcal{W}(x,p)}{\tau(x,p)}$$

Relaxation time approximation

$$\left(\gamma^\mu p_\mu - m\right)\mathcal{W}(x,p) + \frac{i\hbar}{2}\gamma^\mu\nabla_\mu\mathcal{W}(x,p) = -\frac{i\hbar}{2}\gamma^\mu u_\mu \frac{\delta\mathcal{W}(x,p)}{\tau(x,p)}$$

Spin decomposition

$$p_\mu\mathcal{V}^\mu - m\mathcal{S} = 0$$

$$\hbar\nabla_\mu\mathcal{V}^\mu = -\frac{\hbar}{\tau}u_\mu\,\delta\mathcal{V}^\mu$$

$$2m\mathcal{P} + \hbar\nabla_\mu\mathcal{A}^\mu = -\frac{\hbar}{\tau}u_\mu\,\delta\mathcal{A}^\mu$$

$$p_\mu\mathcal{A}^\mu = 0$$

$$p_\mu\mathcal{S} - m\mathcal{V}_\mu + \frac{\hbar}{2}\nabla^\nu\mathcal{S}_{\mu\nu} = -\frac{\hbar}{2\tau}u^\nu\delta\mathcal{S}_{\mu\nu}$$

$$-p^\nu\mathcal{S}_{\mu\nu} + \frac{\hbar}{2}\nabla_\mu\mathcal{S} = -\frac{\hbar}{2\tau}u_\mu\delta\mathcal{S}$$

$$-\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu\mathcal{S}^{\alpha\beta} - m\mathcal{A}_\mu + \frac{\hbar}{2}\nabla_\mu\mathcal{P} = -\frac{\hbar}{2\tau}u_\mu\delta\mathcal{P}$$

$$2p_\mu\mathcal{P} + \frac{\hbar}{2}\epsilon_{\mu\nu\alpha\beta}\nabla^\nu\mathcal{S}^{\alpha\beta} = -\frac{\hbar}{2\tau}\epsilon_{\mu\nu\alpha\beta}u^\nu\delta\mathcal{S}^{\alpha\beta}$$

$$-2\epsilon_{\mu\nu\alpha\beta}p^\alpha\mathcal{A}^\beta - 2m\mathcal{S}_{\mu\nu} + \hbar\nabla_{[\mu}\mathcal{V}_{\nu]} = -\frac{\hbar}{\tau}u_{[\mu}\delta\mathcal{V}_{\nu]}$$

$$p_{[\mu}\mathcal{V}_{\nu]} + \frac{\hbar}{2}\epsilon_{\mu\nu\alpha\beta}\nabla^\alpha\mathcal{A}^\beta = -\frac{\hbar}{2\tau}\epsilon_{\mu\nu\alpha\beta}u^\alpha\delta\mathcal{A}^\beta$$

Conservation can easily be obtained from integrating over above equations, giving

$$\partial_\mu J^\mu = -\frac{1}{\tau}u_\mu\delta J^\mu,$$

$$\partial_\mu T^{\mu\nu} = -\frac{1}{\tau}u_\mu\delta T^{\mu\nu},$$

$$\partial_\alpha J^{\alpha,\mu\nu} = T^{[\mu\nu]} + \partial_\alpha S^{\alpha,\mu\nu} = -\frac{1}{\tau}u_\alpha\delta S^{\alpha,\mu\nu}.$$

Difference from Kadanoff-Baym equation

1) relations among components

2) does not modifies on-shell relation $(p^2 - m^2)\mathcal{V}_\mu^{(0),(1)} = 0$ $(p^2 - m^2)\mathcal{A}_\mu^{(0),(1)} = 0$

3) transport equations

Relaxation time approximation

$$\left(\gamma^\mu p_\mu - m\right)\mathcal{W}(x,p) + \frac{i\hbar}{2}\gamma^\mu \nabla_\mu \mathcal{W}(x,p) = -\frac{i\hbar}{2}\gamma^\mu u_\mu \frac{\delta\mathcal{W}(x,p)}{\tau(x,p)}$$

transport equations obtained from spin decomposition, without hbar expansion

$$(p\cdot\partial)\mathcal{S} = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{S}$$

$$(p\cdot\partial)\mathcal{A}_\mu = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{A}_\mu - \frac{\hbar}{2m}\epsilon_{\mu\nu\alpha\beta}p^\beta\left(\partial^\alpha \frac{u^\nu}{\tau}\right)\delta\mathcal{S}$$

Damping **Polarization** $\tau = 5\bar{\eta}/T, \quad \bar{\eta} = \eta/s$

Relaxation time has spatial dependence through T

Polarization effect related to T-vorticity $p^\nu\partial_\nu A_\mu = -\frac{u^\nu p_\nu}{\tau}\delta A_\mu + \frac{1}{\tau}\frac{\hbar}{T}p^\nu \widetilde{\omega}^T_{\mu\nu}\delta f_V$

Magnitude of vorticity affects the magnitude of polarization, not the time-scale of polarization effect.

T-vorticity $\omega^T_{\mu\nu} = 1/2[\partial_\mu(Tu_\nu) - \partial_\nu(Tu_\mu)]$

correct quadrupole distribution in local-polarization

$$\widetilde{\omega}^T_{\mu\nu} = 1/2\epsilon_{\mu\nu\rho\sigma}\omega_T^{\rho\sigma}$$

Time-scale: consider a system with homogeneous temperature, in local rest frame of the fluid

$$\widetilde{\omega}^K_{\mu\nu} = 1/2\epsilon_{\mu\nu\rho\sigma}\partial^\rho u^\sigma = \omega_\mu u_\nu - \omega_\nu u_\mu + \epsilon_{\mu\nu\rho\sigma}\varepsilon^\rho u^\sigma$$

$$(p\cdot\partial)\mathcal{A}_i = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{A}_i + \frac{1}{\tau}(p\cdot u)\left[\hbar\omega_i\delta f_V\right].$$

Same time scale for various spin components

RTA from Kadanoff-Baym equation

- **close to equilibrium** add a fluctuation with momentum p to an equilibrium distribution, while all other modes are still at equilibrium

$$\mathcal{S}^{(0)} = \mathcal{S}^{(0)}_{eq} + \delta\mathcal{S}^{(0)}, \qquad \mathcal{A}^{(0)}_\mu = \mathcal{A}^{(0)eq}_\mu + \delta\mathcal{A}^{(0)}_\mu$$

The equilibrium part eliminates the collision terms

The self-energy does not contain modes with momentum p, thus is evaluated in equilibrium $\int \Pi_i \frac{d^4 p_i}{(2\pi)^4}\delta^4(p - \sum p_i)$

- **Summing 0th and 1st order** $\mathcal{S} = \mathcal{S}^{(0)} + \hbar\mathcal{S}^{(1)}, \quad \mathcal{A}_\mu = \mathcal{A}^{(0)}_\mu + \hbar\mathcal{A}^{(1)}_\mu, \quad \Sigma_X = \Sigma^{(0)}_X + \hbar\Sigma^{(1)}_X$

$$\mathcal{S} = \mathcal{S}_{eq} + \delta\mathcal{S}, \quad \mathcal{A}_\mu = \mathcal{A}^{eq}_\mu + \delta\mathcal{A}_\mu$$

- **Relaxation rate & polarization rate**

$$p^\nu\partial_\nu \begin{pmatrix} \delta\mathcal{S} \\ \delta\mathcal{A}_\mu \end{pmatrix} = -\begin{pmatrix} \omega_0 & \hbar\omega^\mu_s \\ -\hbar\omega_{s\mu} & \omega_0 \end{pmatrix}\begin{pmatrix} \delta\mathcal{S} \\ \delta\mathcal{A}_\mu \end{pmatrix},$$

$$\omega_0 = m\widehat{\Sigma}_F + p_\nu\widehat{\Sigma}^\nu_V,$$

$$\omega_{s\mu} = -m\widehat{\Sigma}_{A\mu} + \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}p^\nu\widehat{\Sigma}^{\sigma\rho}_S + \frac{p_\mu p_\nu}{m}\widehat{\Sigma}^\nu_A + \frac{1}{2m}\epsilon_{\mu\nu\rho\sigma}p^\rho\partial^\sigma\widehat{\Sigma}^\nu_V - \frac{1}{2}\partial_\mu\widehat{\Sigma}_P$$

relation between AW-RTA and KB-RTA

AW-RTA

$$(p\cdot\partial)\mathcal{S} = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{S}$$

$$(p\cdot\partial)\mathcal{A}_i = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{A}_i + \frac{1}{\tau}(p\cdot u)\left[\hbar\omega_i\delta f_V\right].$$

KB - RTA

$$p^\nu\partial_\nu \begin{pmatrix}\delta\mathcal{S}\\ \delta\mathcal{A}_\mu\end{pmatrix} = -\begin{pmatrix}\omega_0 & \hbar\omega_s^\mu \\ -\hbar\omega_{s\mu} & \omega_0\end{pmatrix}\begin{pmatrix}\delta\mathcal{S}\\ \delta\mathcal{A}_\mu\end{pmatrix},$$

$$\omega_0 = m\widehat{\Sigma}_F + p_\nu\widehat{\Sigma}_V^\nu,$$

$$\omega_{s\mu} = -m\widehat{\Sigma}_{A\mu} + \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}p^\nu\widehat{\Sigma}_S^{\sigma\rho} + \frac{p_\mu p_\nu}{m}\widehat{\Sigma}_A^\nu + \frac{1}{2m}\epsilon_{\mu\nu\rho\sigma}p^\rho\partial^\sigma\widehat{\Sigma}_V^\nu - \frac{1}{2}\partial_\mu\widehat{\Sigma}_P$$

Same damping rate for charge and spin

Polarization effect related to T-vorticity

Same relaxation time for different effects

Same damping rate for charge and spin

Polarization effect related to thermal vorticity, due to global equilibrium

Different relaxation time for different effect

Compare the both: specify the interaction

In local rest frame $\tau_0 = -E_p/\omega_0$

NJL contact interaction

$$p^\nu \partial_\nu \begin{pmatrix} \delta \mathcal{S} \\ \delta \mathcal{A}_\mu \end{pmatrix} = - \begin{pmatrix} \omega_0 & \hbar \omega_s^\mu \\ -\hbar \omega_{s\mu} & \omega_0 \end{pmatrix} \begin{pmatrix} \delta \mathcal{S} \\ \delta \mathcal{A}_\mu \end{pmatrix},$$

$$\omega_0 = m\widehat{\Sigma}_F + p_\nu \widehat{\Sigma}_V^\nu,$$

$$\omega_{s\mu} = -m\widehat{\Sigma}_{A\mu} + \frac{1}{2}\epsilon_{\mu\nu\sigma\rho} p^\nu \widehat{\Sigma}_S^{\sigma\rho} + \frac{p_\mu p_\nu}{m}\widehat{\Sigma}_A^\nu + \frac{1}{2m}\epsilon_{\mu\nu\rho\sigma} p^\rho \partial^\sigma \widehat{\Sigma}_V^\nu - \frac{1}{2}\partial_\mu \widehat{\Sigma}_P$$

NJL-type model with scalar-channel

$$\mathcal{L} = \bar{\psi}(i\hbar\partial\!\!\!/ - m)\psi + G(\bar{\psi}\psi)^2$$

(fermionic 2 by 2 scattering)

chiral restored phase, 1/Nc expansion

$$\Sigma^>(x,p) = G^2 \int dP S^>(x,p_1) \mathrm{Tr}\left[S^<(x,p_2) S^>(x,p_3)\right]$$

$$\int dP = \int \frac{d^4p_1 d^4p_2 d^4p_3}{(2\pi)^4(2\pi)^4(2\pi)^4}(2\pi)^4 \delta(p - p_1 + p_2 - p_3)$$

NJL contact interaction

$$p^\nu \partial_\nu \begin{pmatrix} \delta\mathcal{S} \\ \delta\mathcal{A}_\mu \end{pmatrix} = -\begin{pmatrix} \omega_0 & \hbar\omega_s^\mu \\ -\hbar\omega_{s\mu} & \omega_0 \end{pmatrix} \begin{pmatrix} \delta\mathcal{S} \\ \delta\mathcal{A}_\mu \end{pmatrix},$$

$$\omega_0 = m\widehat{\Sigma}_F + p_\nu \widehat{\Sigma}_V^\nu,$$

$$\omega_{s\mu} = -m\widehat{\Sigma}_{A\mu} + \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}p^\nu \widehat{\Sigma}_S^{\sigma\rho} + \frac{p_\mu p_\nu}{m}\widehat{\Sigma}_A^\nu + \frac{1}{2m}\epsilon_{\mu\nu\rho\sigma}p^\rho \partial^\sigma \widehat{\Sigma}_V^\nu - \frac{1}{2}\partial_\mu \widehat{\Sigma}_P$$

$$\omega_0(p) = G^2 \int dP \frac{m^2 + p_2 \cdot p_3}{m^3}(m^2 + p\cdot p_1)\left(\overline{\mathcal{S}}^{(0)}(p_1)\mathcal{S}^{(0)}(p_2)\overline{\mathcal{S}}^{(0)}(p_3) + \mathcal{S}^{(0)}(p_1)\overline{\mathcal{S}}^{(0)}(p_2)\mathcal{S}^{(0)}(p_3)\right),$$

$$\begin{aligned}\omega_{s\mu}(p) = G^2 \int dP \frac{m^2 + p_2 \cdot p_3}{m^4}\Big[&- m(m^2 + p\cdot p_1)\left(\overline{\mathcal{A}}_\mu^{(1)}(p_1)\mathcal{S}^{(0)}(p_2)\overline{\mathcal{S}}^{(0)}(p_3) + \mathcal{A}_\mu^{(1)}(p_1)\overline{\mathcal{S}}^{(0)}(p_2)\mathcal{S}^{(0)}(p_3)\right) \\ &+ m(p_\mu + p_{1\mu})p^\nu \left(\overline{\mathcal{A}}_\nu^{(1)}(p_1)\mathcal{S}^{(0)}(p_2)\overline{\mathcal{S}}^{(0)}(p_3) + \mathcal{A}_\nu^{(1)}(p_1)\overline{\mathcal{S}}^{(0)}(p_2)\mathcal{S}^{(0)}(p_3)\right) \\ &+ \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu p_1^\beta \left([\partial^\alpha \overline{\mathcal{S}}^{(0)}(p_1)]\mathcal{S}^{(0)}(p_2)\overline{\mathcal{S}}^{(0)}(p_3) + [\partial^\alpha \mathcal{S}^{(0)}(p_1)]\overline{\mathcal{S}}^{(0)}(p_2)\mathcal{S}^{(0)}(p_3)\right) \\ &+ \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu p_1^\beta \partial^\alpha \left(\overline{\mathcal{S}}^{(0)}(p_1)\mathcal{S}^{(0)}(p_2)\overline{\mathcal{S}}^{(0)}(p_3) + \mathcal{S}^{(0)}(p_1)\overline{\mathcal{S}}^{(0)}(p_2)\mathcal{S}^{(0)}(p_3)\right)\Big].\end{aligned}$$

Schouten identity
Killing condition

$\propto$ **Thermal vorticity**

In general, collision terms could generate all kinds of vorticity structures in off-equilibrium systems.

However, for systems close to **global equilibrium**, only thermal vorticity contributes.

collision term vanishes only in global equilibrium

NJL contact interaction

AW-RTA

$$(p\cdot\partial)\mathcal{S} = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{S}$$

$$(p\cdot\partial)\mathcal{A}_i = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{A}_i + \frac{1}{\tau}(p\cdot u)\big[\hbar\omega_i\delta f_V\big].$$

KB-RTA

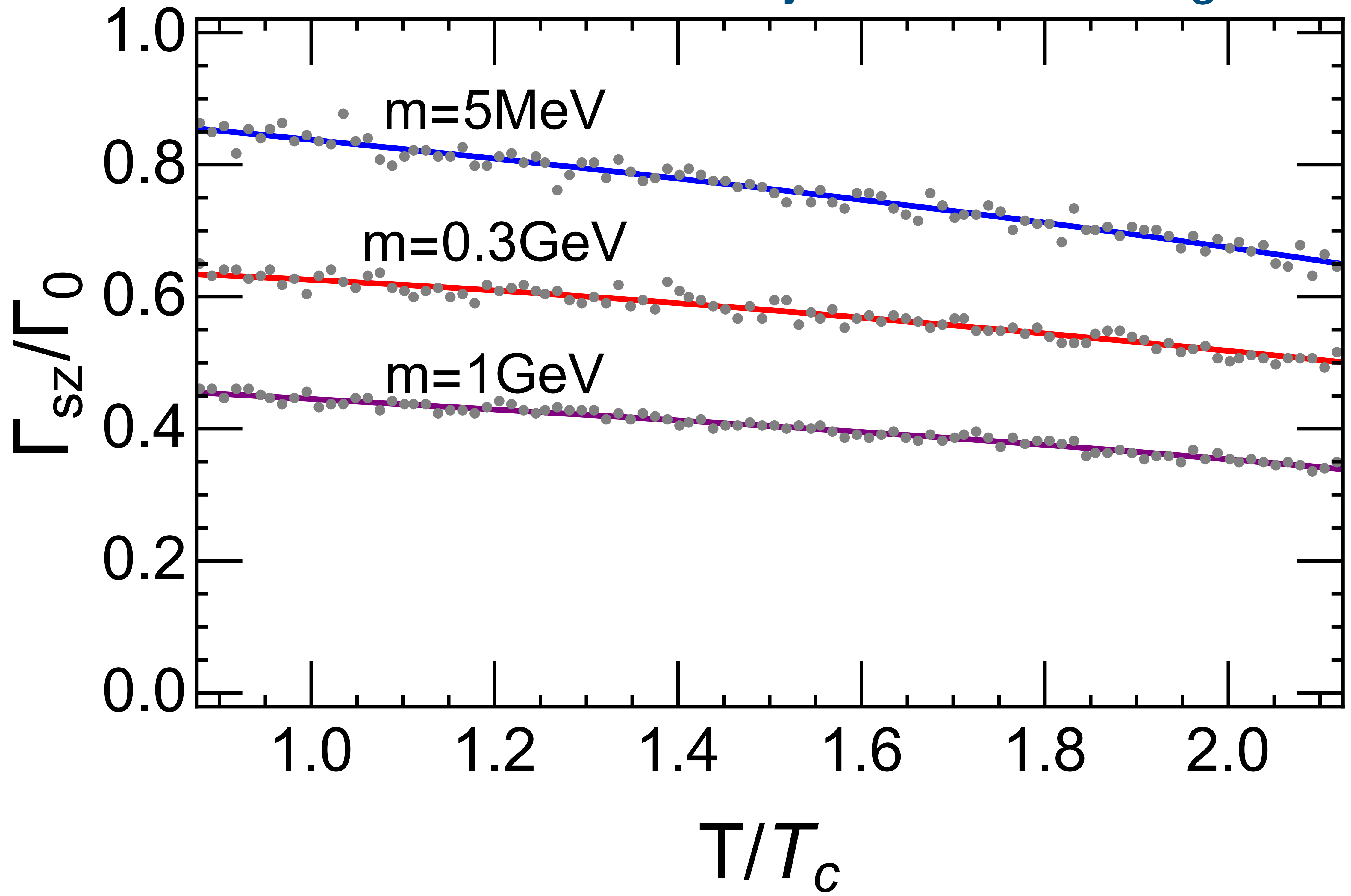
$$p^\mu\partial_\mu f_V = -\Gamma_0\delta f_V$$

$$p^\mu\partial_\mu \mathbf{A} = -\Gamma_0\delta\mathbf{A} + \mathbf{\Gamma}_s\left(\hbar\omega\delta f_V\right)$$

$$\frac{\Gamma_{si}}{\Gamma_0} = \frac{\tau_0}{\tau_{si}} = \frac{2m}{\omega}\frac{\omega_{si}}{\omega_0}$$

Evaluate the ratio between both time-scales

consider a system with homogeneous temperature, in local rest frame of the fluid



- High temperature: polarization slower
- Heavy spin carrier: polarization slower
- Massless & vacuum: same time scale

Outline

- **Background** Global OAM, Global polarization,
  Local Polarization, Spin Sign Puzzle, Collisions

- **Framework** Kadanoff-Baym equation

- **Global equilibrium — Detailed balance**

- **Close to equilibrium — Relaxation time approximation**

  RTA from Kadanoff-Baym equation; Interaction rate: NJL

- **Outlook** Interaction rate: hot QED plasma, hot (dense) QCD plasma …
  Relate to field theory: Polarization rate from imaginary part
  Relate to hydro: Entropy production