Probing gluon spin correlation with jet substructure

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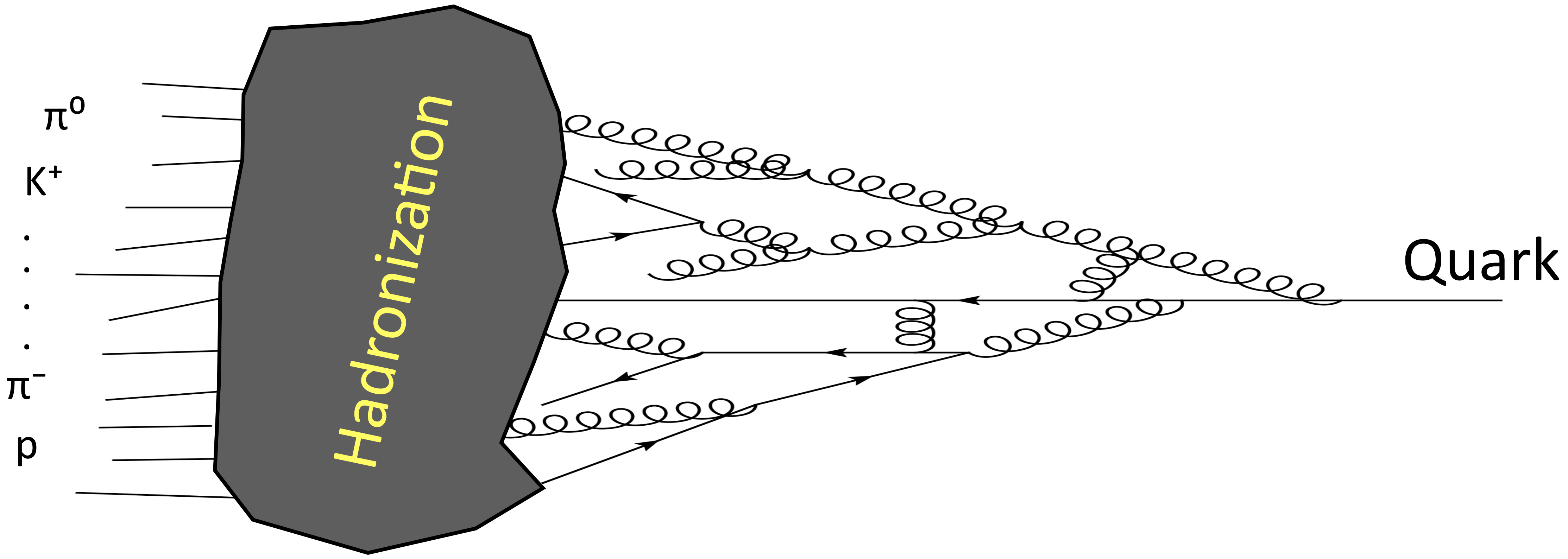
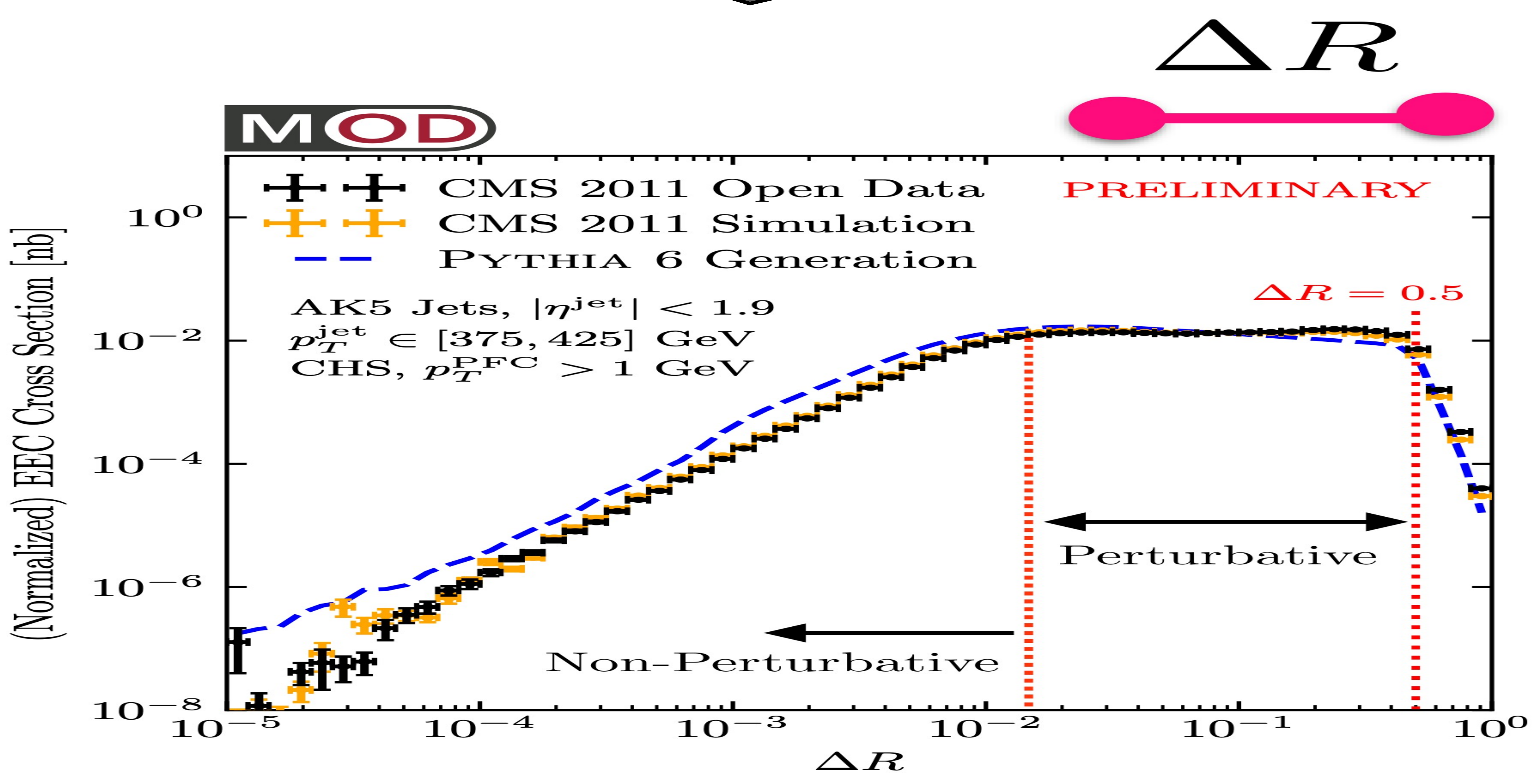
The 143rd HENPIC seminar

2011.02492 (PRL), 2104.00009, with Hao Chen, Ian Moult
with Hao Chen, Ian Moult, Joshua Sanders, work in progress

Jet substructure

Jets are collimated sprays of hadrons from evolution and fragmentation of high energy quark and gluon





The processes of fragmentation and evolution can be visualized by two-point energy correlator!

Komiske, Moult, Thaler, HXZ, in preparation

- Numerous substructure observable proposed in the last decades. These observables are designed to achieve different goals: maximizing BSM signal, reducing soft hadron contamination, ……

- For deciphering the dynamics of QCD, guiding principles are (subject to theorists own tastes):

  - better perturbative behavior

  - nice factorization properties

  - simple operator definition

Energy Correlations in Electron-Positron Annihilation: Testing Quantum Chromodynamics

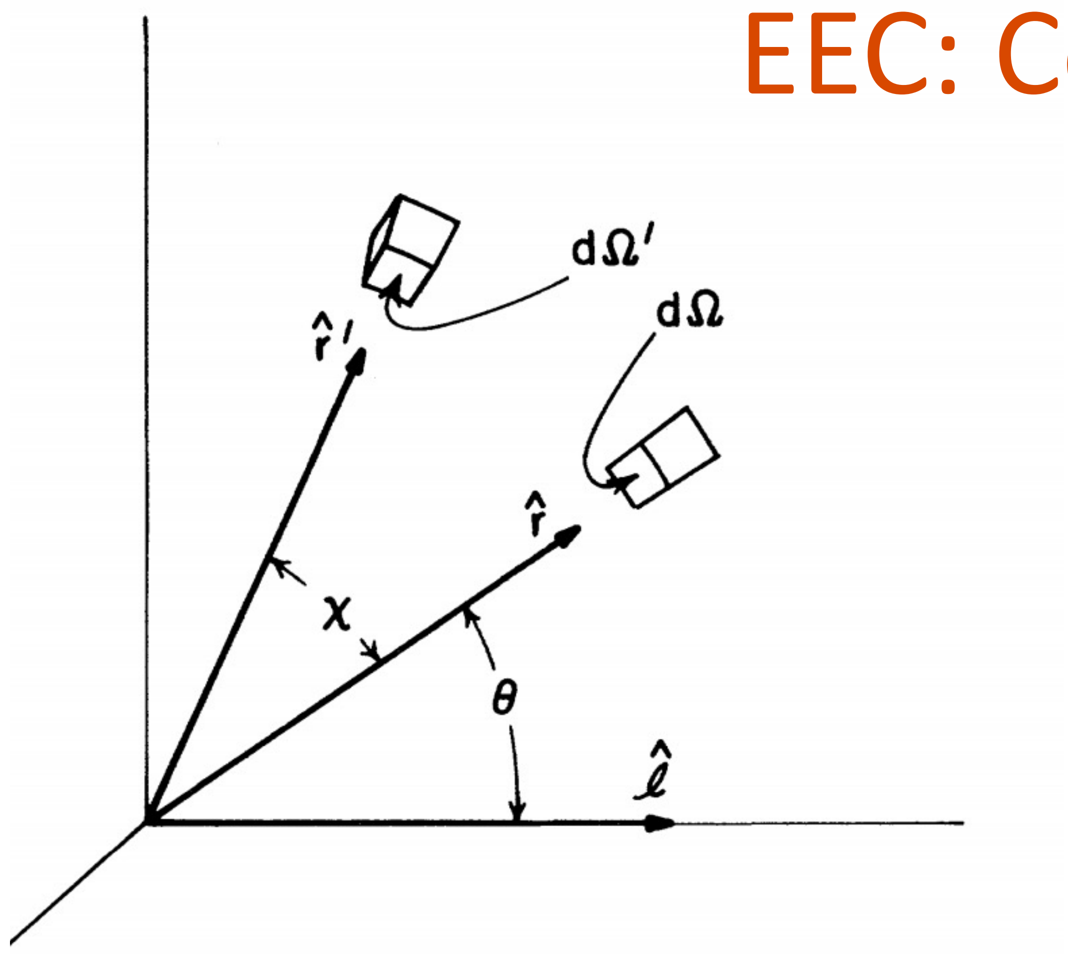
C. Louis Basham, Lowell S. Brown, Stephen D. Ellis, and Sherwin T. Love
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 21 August 1978)

An experimental measure is presented for a precise test of quantum chromodynamics. This measure involves the asymmetry in the energy-weighted opening angles of the jets of hadrons produced in the process $e^+e^- \to$ hadrons at energy W. It is special for several reasons: It is reliably calculable in asymptotically free perturbation theory; it has rapidly vanishing (order $1/W^2$) corrections due to nonperturbative confinement effects; and it is straightforward to determine experimentally.

Energy Correlations

Basham, Brown, Ellis, Love, 1978

EEC: Correlation of energy deposition between two detector at an angle χ

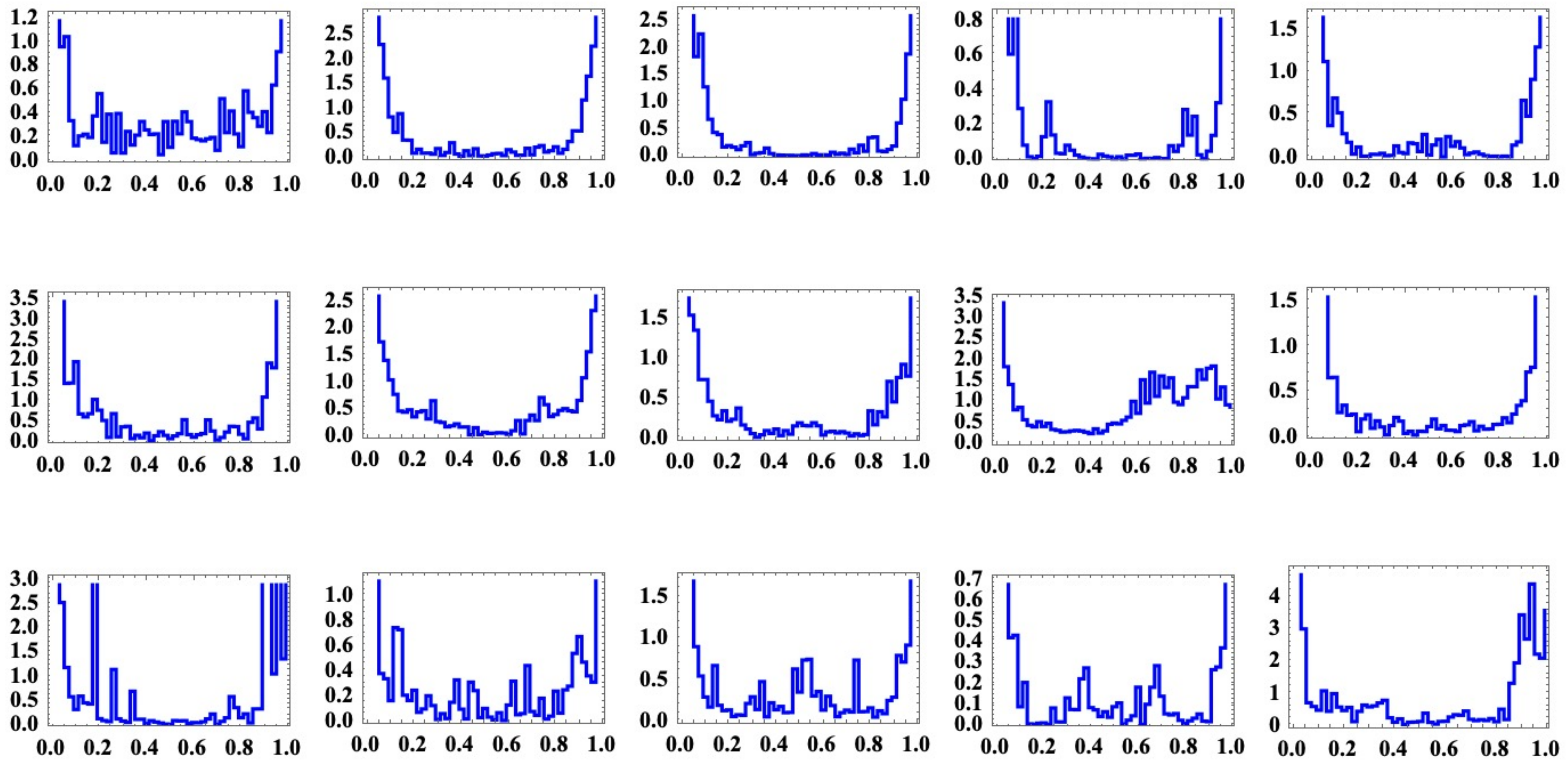


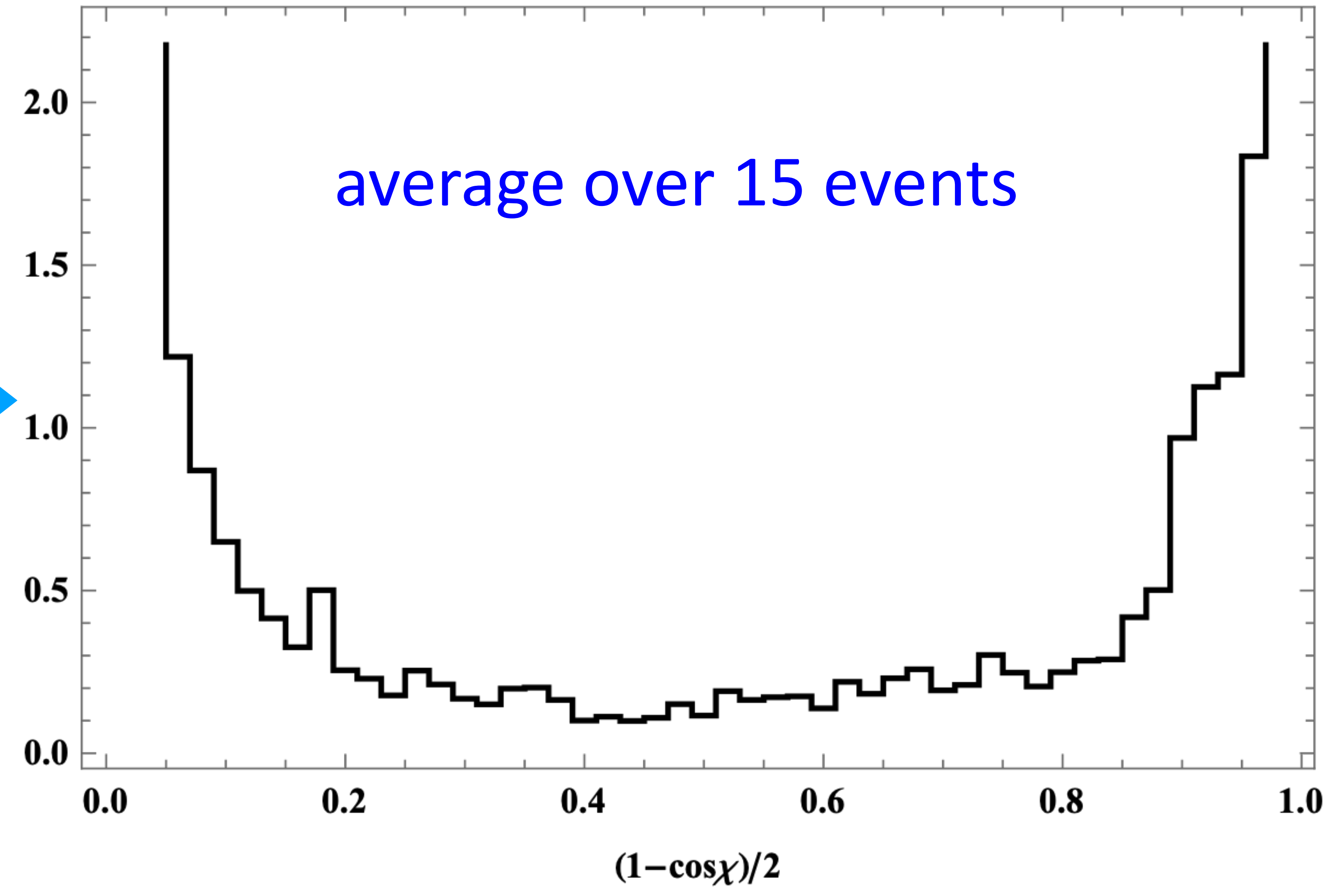
$$\mathrm{EEC}(\chi) = \frac{1}{\sigma_{\rm tot}} \sum_{i,j} \int d\sigma_{e^+e^- \to i,j+X} \frac{E_i E_j}{E_{\rm tot}^2} \delta(\chi - \chi_{ij})$$

Weighted cross section

Collinear safety requires weighting with energy.
Also no soft divergence due to energy weighting.

Measurement on a single event gives a function





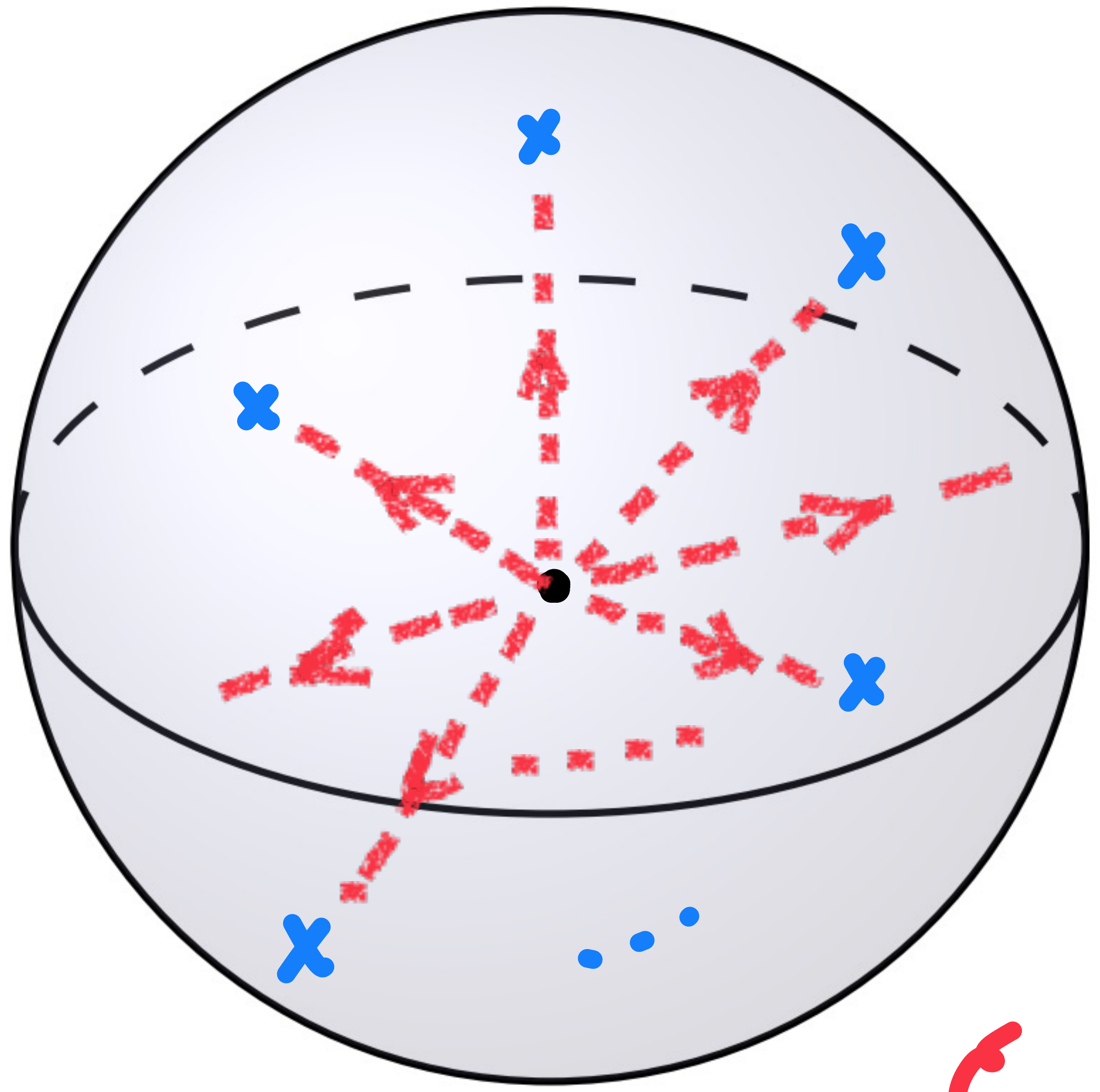
N-point energy correlator and celestial sphere

$$\mathrm{EEC}(\{\chi\}) = \frac{1}{\sigma_{\mathrm{tot}}} \sum_{i_1, i_2, \ldots, i_N} \int d\sigma_{e^+e^- \to i_1, \ldots, i_N + X} \frac{E_{i_1} E_{i_2} \cdots E_{i_N}}{E_{\mathrm{tot}}^N} \delta(\chi_{12} - \chi_{i_1 i_2}) \cdots \delta(\chi_{N-1,N} - \chi_{i_{N-1} i_N})$$

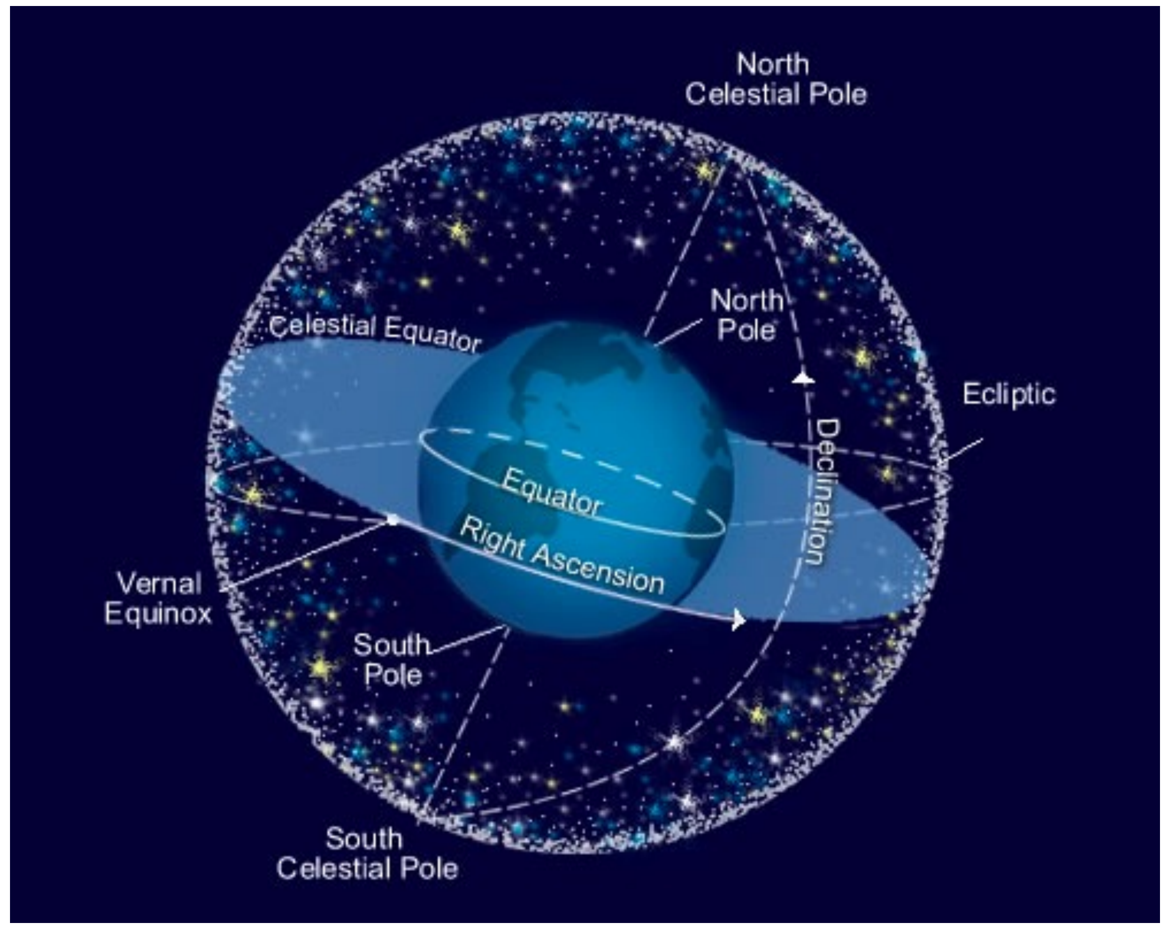
Place N energy detector at N marked point on the celestial sphere

Weighted cross section parameterized by N(N-1)/2 angles on the celestial sphere

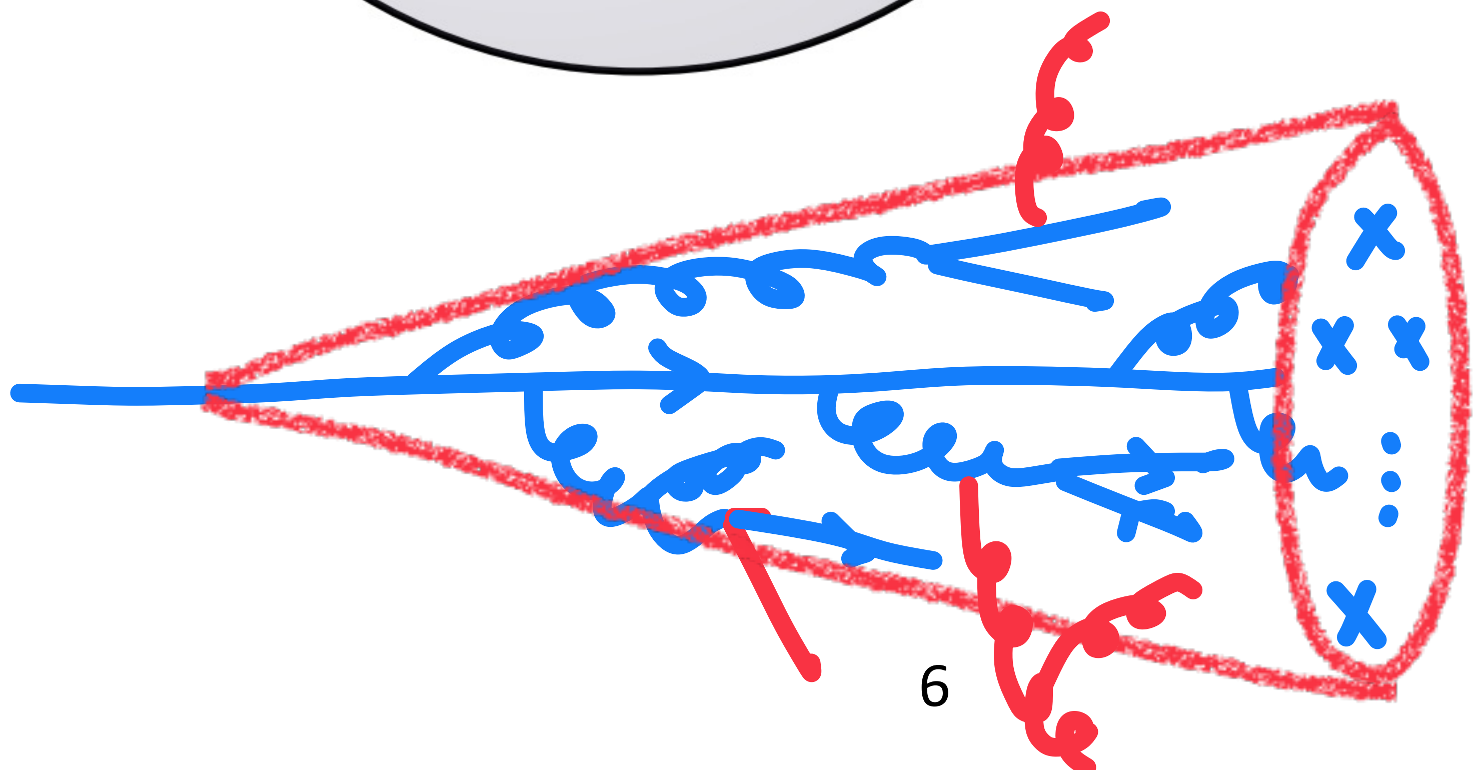
collider celestial sphere



astronomy celestial sphere



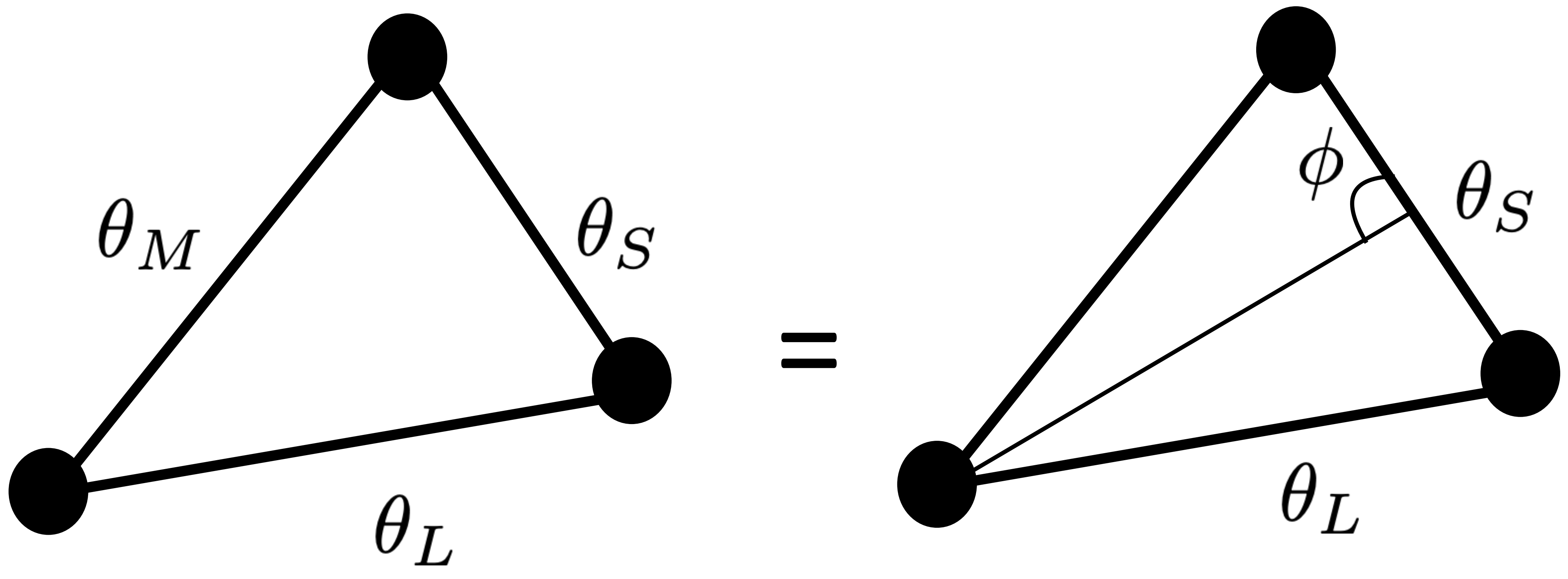
collinear limit
jet substructure

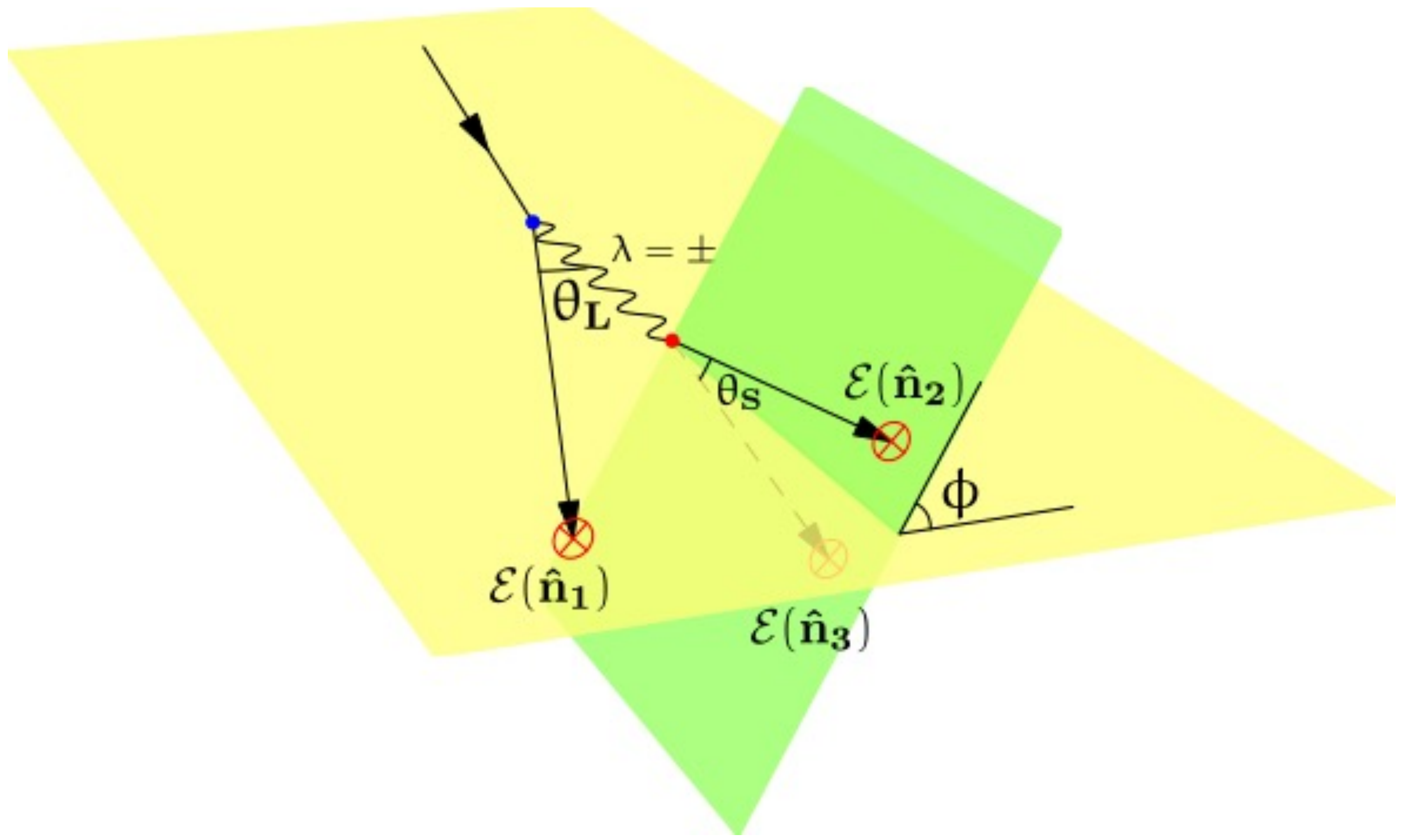


$\chi_{ab} \ll 1$

Non-global logarithms power suppressed

Three-point correlator





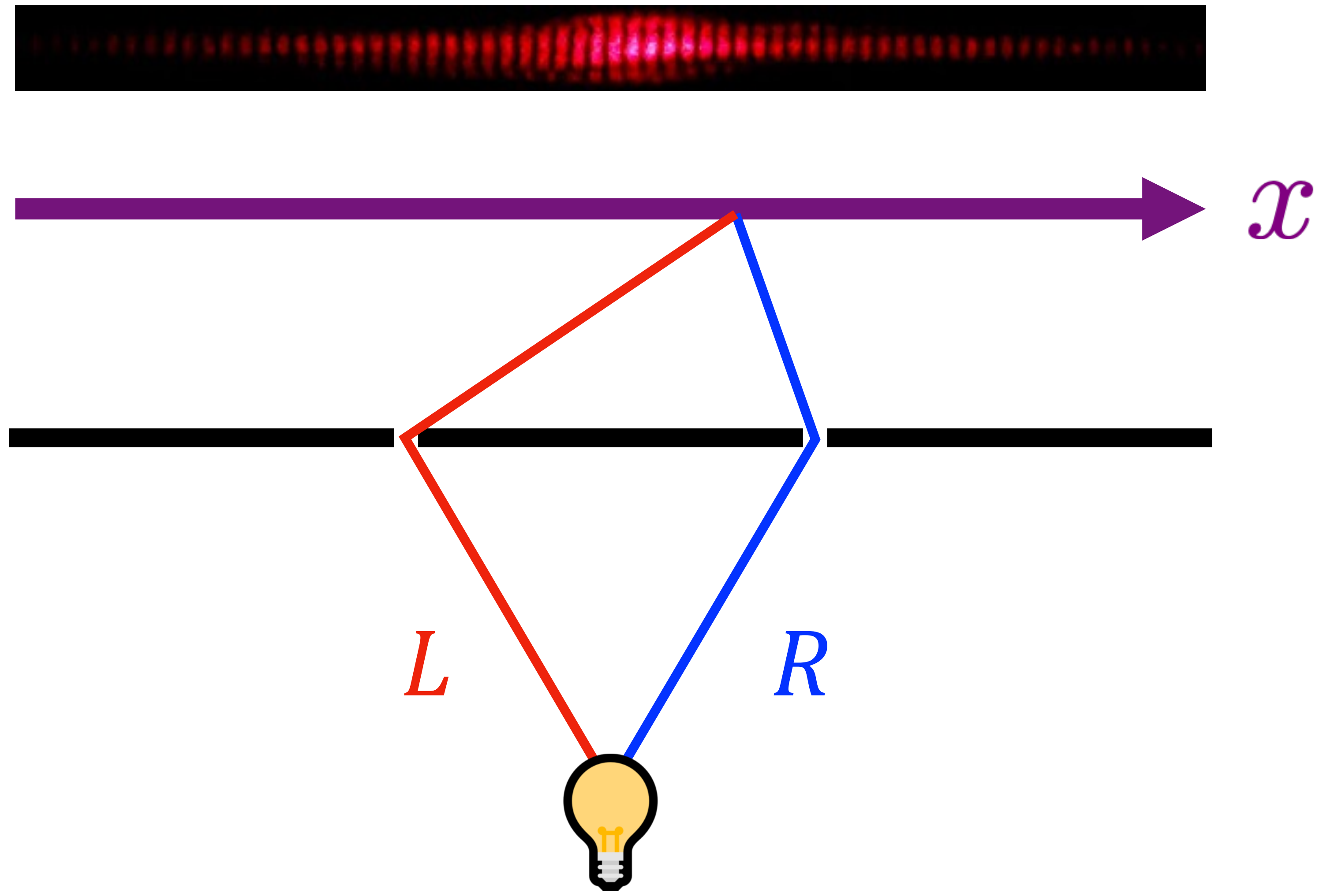
Analytic leading predictions available. First analytic calculation for three-prong jet substructure. [H. Chen, M.X. Luo, Moult, T.Z. Yang, X.Y. Zhang, HXZ, 2019, JHEP]

Expand in the $\theta_S \to 0$ limit (**squeeze limit**), an interesting cos(2φ) arises

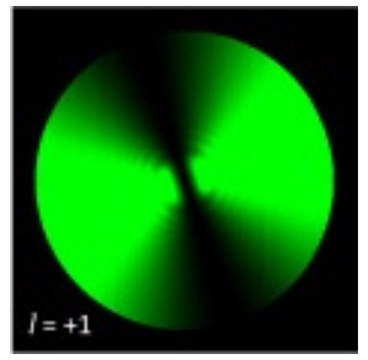
$$\mathrm{Sq}_q^{(0)}(\phi) = C_F n_f T_F \left(\frac{39 - 20\cos(2\phi)}{225} \right) + C_F C_A \left(\frac{273 + 10\cos(2\phi)}{225} \right) + C_F^2 \frac{16}{5}$$

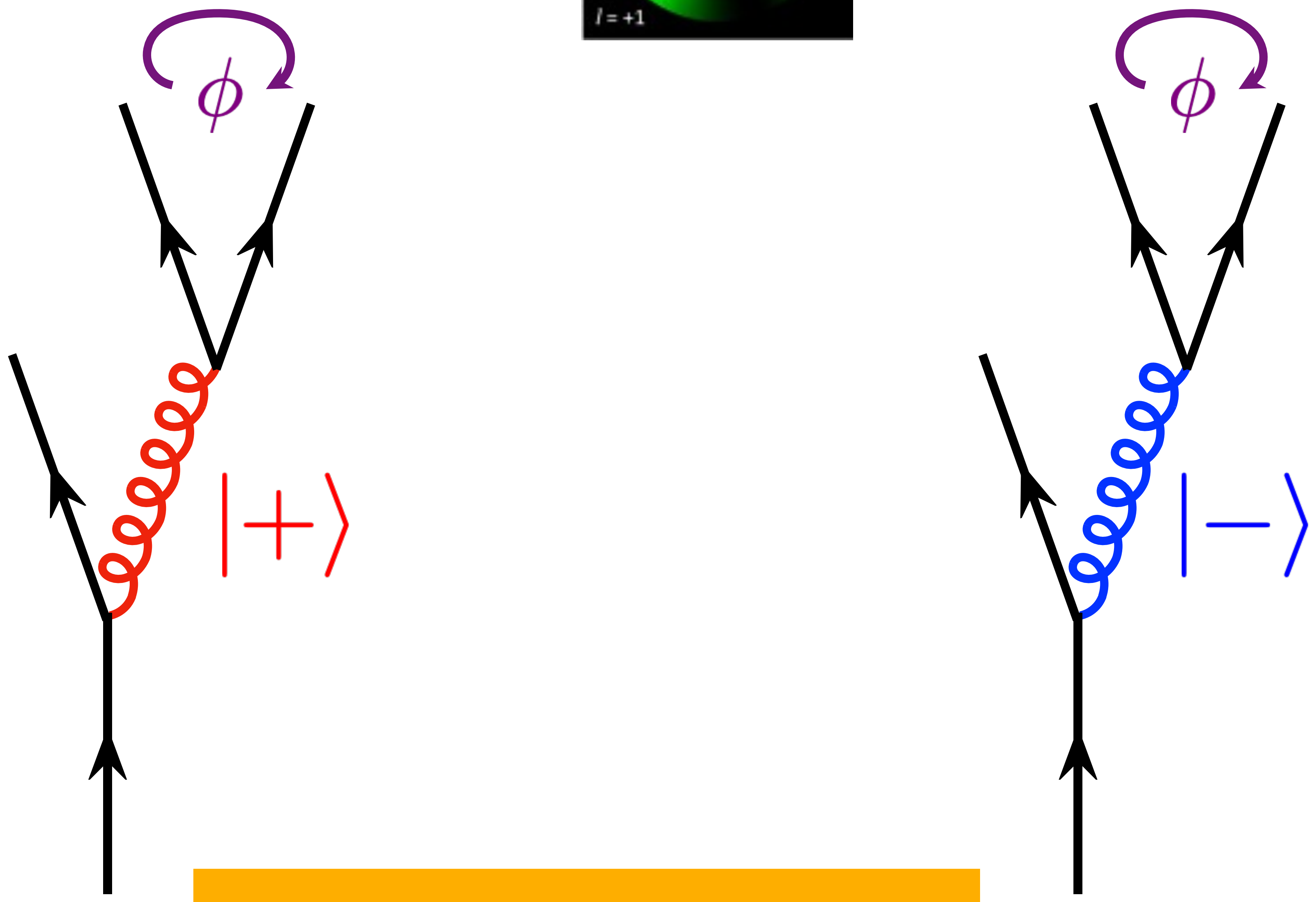
$$\mathrm{Sq}_g^{(0)}(\phi) = C_A n_f T_F \left(\frac{126 - 20\cos(2\phi)}{225} \right) + C_A^2 \left(\frac{882 + 10\cos(2\phi)}{225} \right) + C_F n_f T_F \frac{3}{5}$$

A double-slit experiment in spin space



Coherent source





incoherent source

$$\boldsymbol{E}_{\pm} = (\boldsymbol{\epsilon}_1 \pm i\boldsymbol{\epsilon}_2)\exp(i\boldsymbol{k}\cdot\boldsymbol{r} - i\omega t \pm i\phi)$$

Spin Space Interference leads to $\cos 2\phi$ pattern

H. Chen, I. Moult, HXZ, 2020, PRL

- Gluon spin is a relatively new topic in jet substructure

  See also A. Karlberg, G. Salam, L. Scyboz, R. Verheyen, 2021 using Lund plane
- Must be probed by angulation of jet constituents
- Superb angular resolution in the collinear limit make it promising to measure at the LHC
- Potentially could also be useful for Higgs discovery (H->bb v.s. g->bb), work in progress

- Gluon spin is a relatively new topic in jet substructure
  See also A. Karlberg, G. Salam, L. Scyboz, R. Verheyen, 2021 using Lund plane
- Must be probed by angular correlation of jet constituents
- Superb angular resolution in the collinear limit make it promising to measure at the LHC, in particular for tracks
- Potentially could also be useful for Higgs discovery (H->bb v.s. g->bb), work in progress

- Elliptic gluon Wigner distribution
  Y. Hatta, B.W. Xiao, F. Yuan, 2016; Hagiwara, C. Zhang, J. Zhou, Y.J. Zhou, 2021
- Linearly polarized photon in HIC
  C. Li, J. Zhou, Y.J. Zhou, 2019; B.W. Xiao, F. Yuan, J. Zhou 2020; H. Xing, C. Zhang, J. Zhou, Y.J. Zhou 2020; W. Zha, J. Brandenburg, L. Ruan, Z. Tang, Z. Xu, 2020; …
- Linearly polarized gluon in TMD PDFs
  P. Naldosky, C. Balazs, E. Berger, C.P. Yuan, 2007; Catai, Grazzini, 2011; D. Boer, W. Dunnen, C. Pisano, M. Schlegel, W. Vogelsang, 2011; …
- Polarized fragmentation function
  Z.B. Kang, K. Lee, F.Y. Zhao, 2020
- Dijet/Z+Jet azimuthal decorrelation
  Y. T. Chien, R. Rahn, S. Van Velzen, D.Y. Shao, W. Waalewijn; Y. Hatta, B.W. Xiao, F. Yuan, J. Zhou, 2020
- …… (appologize for missing references to your important works!)

There are intimate connection between these topics. How unified QCD is!

Plan for the remaining talk

- Light-ray operator and its OPE in QCD
- Squeeze limit of three-point energy correlator and gluon spin interference
- Partial wave expansion on celestial sphere

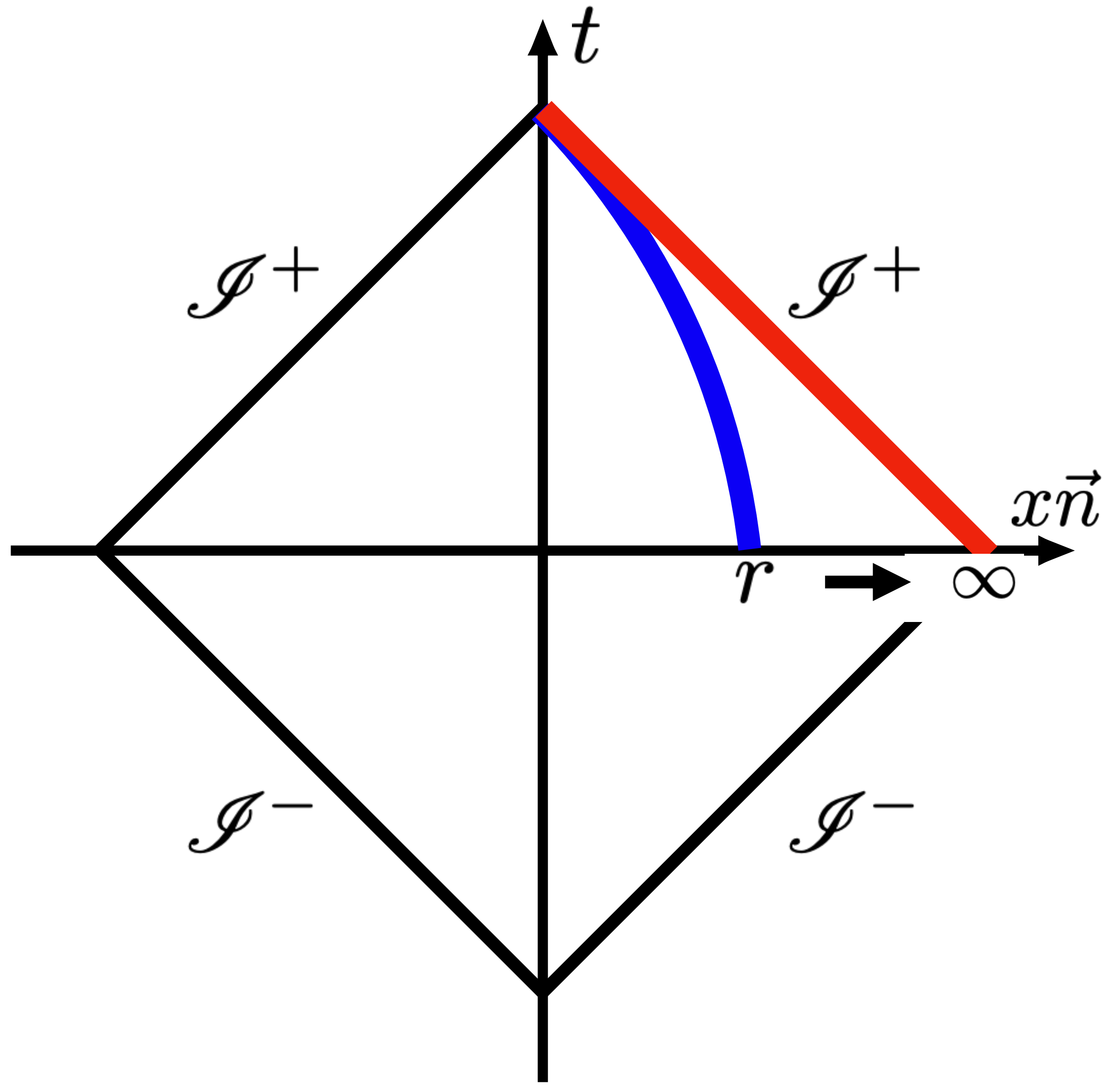
Rethinking Jet Substructure with Energy Correlators

Study jet substructure with weighted particle angular correlation.

Collinear safety requires weighting with energy.
Also no soft divergence due to energy weighting.

Operator definition for energy calorimeter

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\ \vec{n}_i T^{0i}(t, r\vec{n})$$



Energy flow operator(ANEC) Sveshnikov, Tkachov, 95; Tkachov, 95

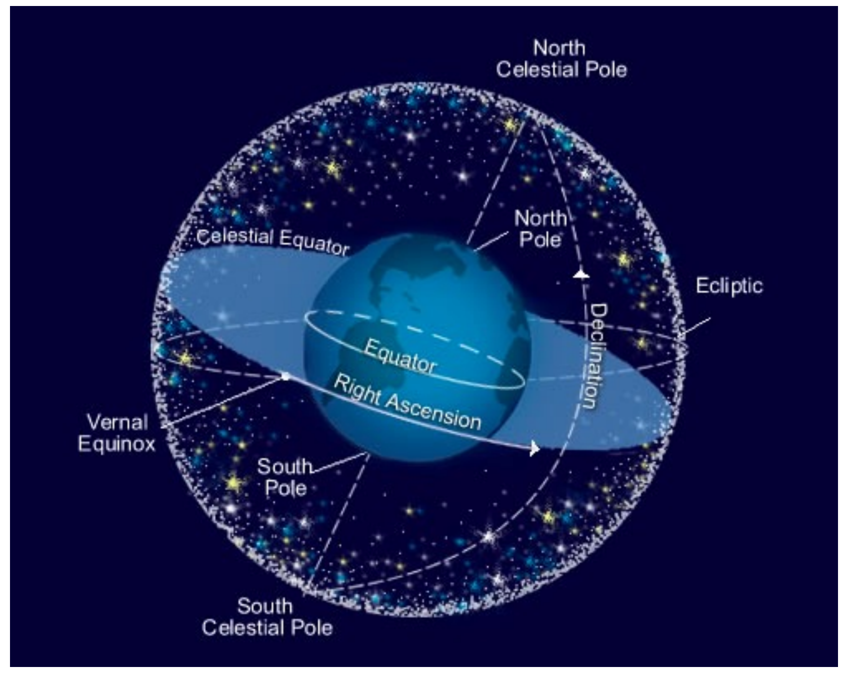
Korchemsky, Oderda, Sterman, 97; Korchemsky, 98; Belitsky, Korchemsky, Sterman, 01; Bauer, Fleming, Lee, Sterman, 08

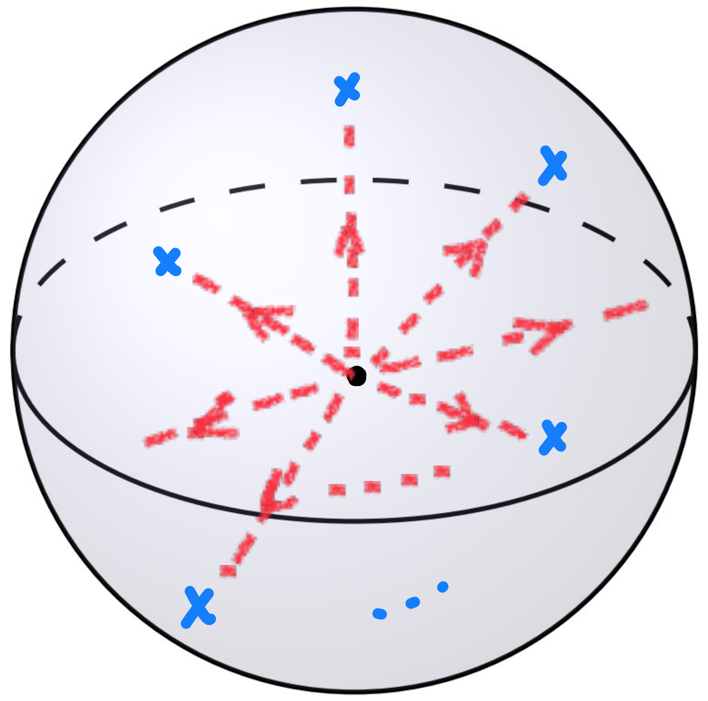
Energy flow operator can constraint CFT: conformal collider physics

Hofman, Maldacena, 2008
Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 19

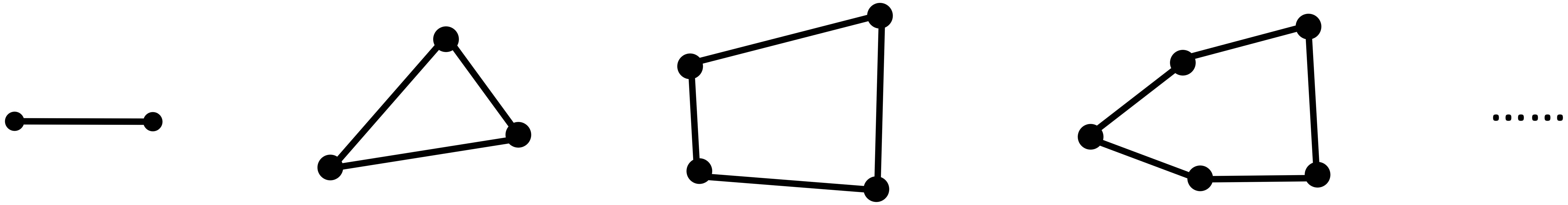
Adopted for QCD jet substructure study for the first time.

H. Chen, Moult, X.Y. Zhang, HXZ, 2020, PRD





N-point energy correlator can be regarded as (N+1)-point function of a fictitious 2D Euclidean **conformal field theory** on the sphere at future null infinity.



Form a complete basis for IRC safe jet substructure observable.

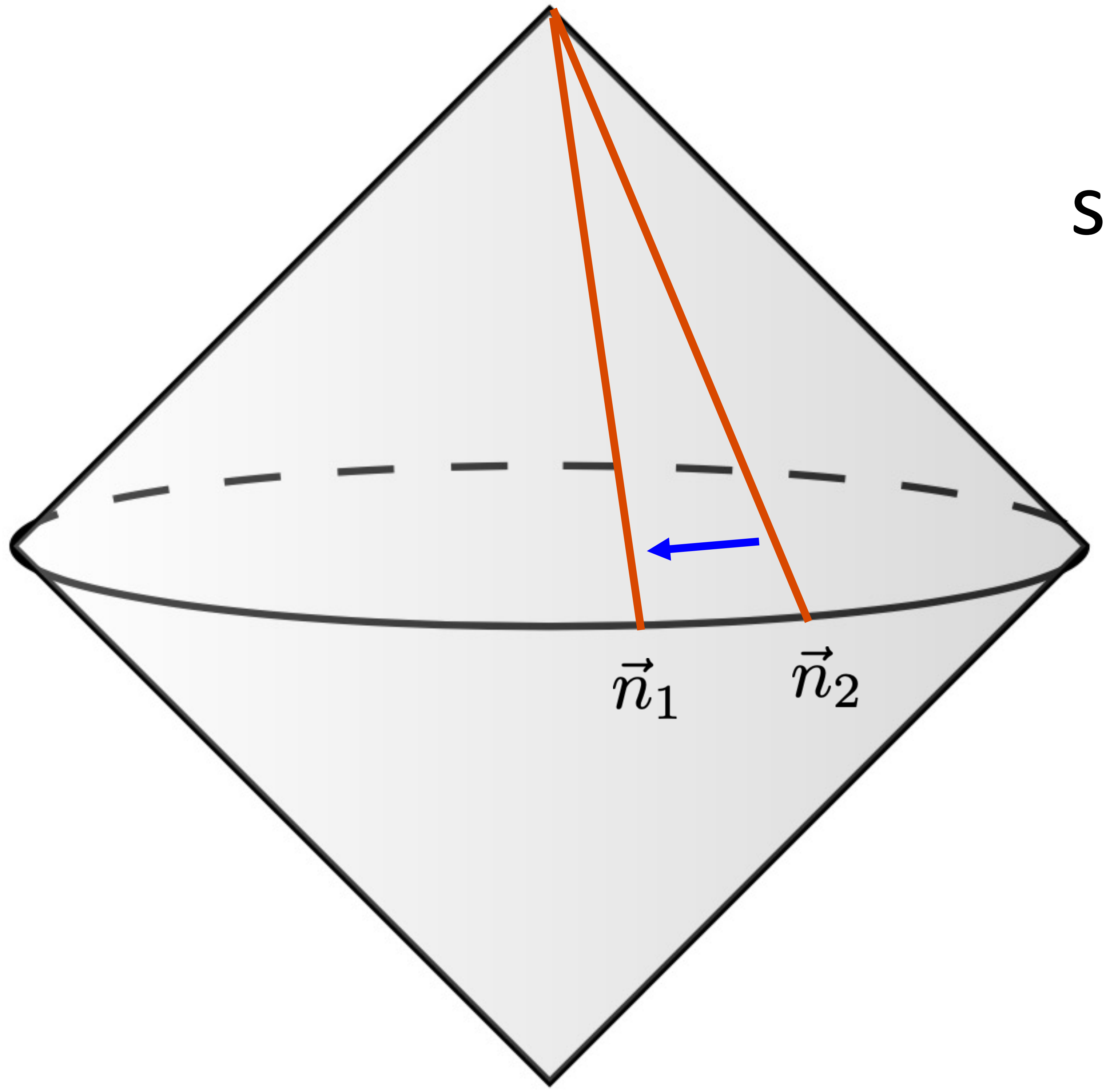
analogy: $$f(x) \leftrightarrow \tilde{f}(N) \equiv \int dx\, x^{N-1} f(x)$$

Observable := distribution

Energy correlator := Mellin moment

EEC as correlator of energy flow operator

$$\int d^4x e^{-iq\cdot x}\langle\Omega|j^{\dagger}_{\mathrm{em},\mu}(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)j^{\mu}_{\mathrm{em}}(0)|\Omega\rangle \qquad j^{\mu}_{\mathrm{em}}(x)=e_q\bar{q}\gamma^{\mu}q(x)$$



small angle limit ⟷ $\vec{n}_2\to\vec{n}_1$ ⟷ OPE of $\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)$

$$\lim_{\vec{n}_2\to\vec{n}_1}\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)=\sum_i\theta_{12}^{\gamma_i}\mathbb{O}_i(\vec{n}_1)$$

To make sense of the OPE, the two energy flow operator better be space-like separated. ✔

Then one can try to do light-transform on both side, and then establish the light-ray OPE. Cautious: not all light-ray operators can be written as light-transform of local operator.

It’s convenient to write energy flow operator as light-transform of local twist 2 (collinear) spin 2 traceless symmetric tensor

$$\bar{n}_\mu = (1, -\vec{n}) \qquad A^\mu \bar{n}_\mu = A^+$$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{n}_i T^{0i}(t, r\vec{n}) \;\Rightarrow\; \mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, [O_q^{\mu_1\mu_2}(t, r\vec{n}) + O_g^{\mu_1\mu_2}(t, r\vec{n})]\bar{n}_{\mu_1}\bar{n}_{\mu_2}$$

twist τ = dimension Δ - (collinear) spin J

$$O_q^{\mu\nu} = \frac{1}{4}\bar{q}\gamma^\mu(iD^\nu)q \qquad O_g^{\mu\nu} = -\frac{1}{4}F^{\rho\mu}F_\rho{}^\nu$$

Therefore, for the local OPE, we need local Wilson operator label by various twist and spin. For twist 2 even spin, there are three different family in QCD

transverse spin-0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

$$\mathcal{O}_{\tilde{g}\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

helicity ±

transverse spin-2

$$\epsilon_\pm = (0, 1, \pm i, 0)$$

$$\frac{g_\perp^{\mu\nu}}{2} - \frac{b^\mu b^\nu}{b^2}$$

under light-transform, one can analytic continue even spin to odd spin.

Braun, Balitsky, 1989;

Kravchuk, Simmons-Duffin, 2018

ensure finite, non-vanishing light transform

Symmetries

Hofman, Maldacena 08; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 19

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} r^{\Delta-J} \int_0^\infty dt\, O^{\mu_1\ldots\mu_J}(t, r\vec{n})\bar{n}_{\mu_1}\ldots\bar{n}_{\mu_J}$$

Light-transform of $O_{(\Delta,J)}$

dimension	$J-\Delta-1$	$+$	Δ	$= J-1$
collinear spin	$-\Delta+J+1$	$+$	$-J$	$= 1-\Delta = 1-\tau+J$

for energy flow operator

$\Delta = 4\,, J = 2$

$$\lim_{\vec{n}_2\to\vec{n}_1} \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) = \sum_i \theta_{12}^{\gamma_i}\mathbb{O}_i(\vec{n}_1)$$

$\mathbb{O} = \mathbf{L}[O]$ $\quad \tau = \Delta - J$

dimension	$(2-1)$	$+$	$(2-1)$	$=$	0	$+$	$(3-1)$
collinear spin	$(1-4)$	$+$	$(1-4)$	$=$	γ_i	$+$	$(1-\tau_i-3)$

Only J=3 local operator appear in the OPE

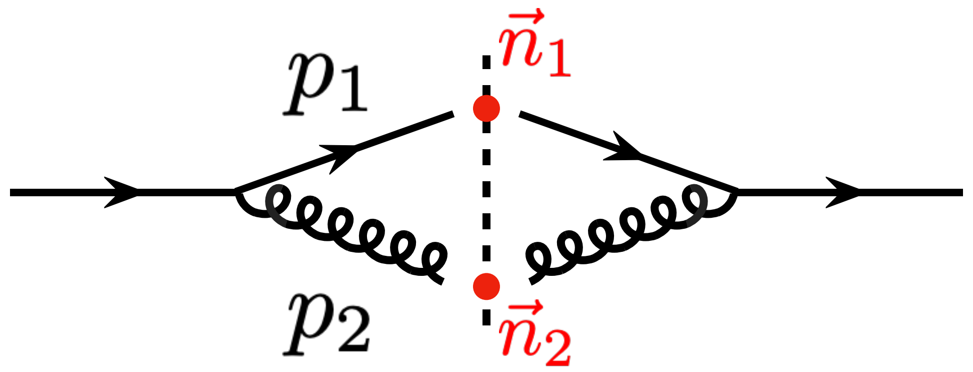
$\Rightarrow \gamma_i = \tau_i - 4$

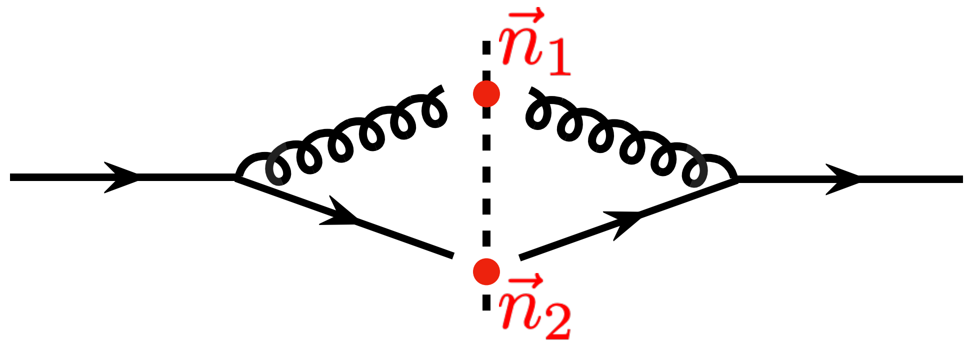
$$\mathcal{E}(\vec{n}_1)\,\mathcal{E}(\vec{n}_2) \sim \sum_i c_i\, \theta^{\tau_i-4}\mathbb{O}_i(\vec{n}_2)$$

Small angle expansion reduce to twist expansion of local operator

Matching for quark operator

$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) \approx C(\theta_{12})\mathbb{O}^{[3]}(\vec{n}_2)$$





omit the Wilson line for simplicity,
but does not affect the tree level matching

$$\langle 0|\psi(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\bar{\psi}(0)|0\rangle$$

$$= g^2 C_F \int \frac{E_1^2 dE_1}{(2\pi)^3 2E_1}\frac{E_2^2 dE_2}{(2\pi)^3 2E_2} e^{-i(p_1+p_2)\cdot x} E_1 E_2 \left(\sum_\lambda \frac{\not{p}_1 + \not{p}_2}{(p_1+p_2)^2} \not{\epsilon}_\lambda(p_2)\not{p}_1 \not{\epsilon}_{-\lambda}(p_2) \frac{\not{p}_1 + \not{p}_2}{(p_1+p_2)^2} + (2 \leftrightarrow 3) \right)$$

$$\overset{\theta\to 0}{=} \frac{1}{2\pi}\frac{4}{\theta_S^2}\frac{g^2}{(4\pi)^2} C_F \int_0^1 dz\, z(1-z) \left(\frac{1+z^2}{1-z} + \frac{1+(1-z)^2}{z} \right) \int \frac{E^2 dE}{(2\pi)^3 2E} \not{p} E^2 e^{-iEn\cdot x}$$

Pqq   Pgq

$$\Rightarrow \; -\frac{1}{2\pi}\frac{2}{\theta^2}\left[(\gamma_{qq}(2) - \gamma_{qq}(3)) + (\gamma_{gq}(2) - \gamma_{gq}(3)) \right] \langle \Omega | \psi(x)\, \mathbb{O}_q^{[3]}(\vec{n}_2) \bar{\psi}(0) | \Omega \rangle$$

Matching coefficient   momentum conservation: $\gamma_{qq}(2) + \gamma_{gq}(2) = 0$

Matching for gluon operator

$$\epsilon_\pm = (0, 1, \pm i, 0)$$

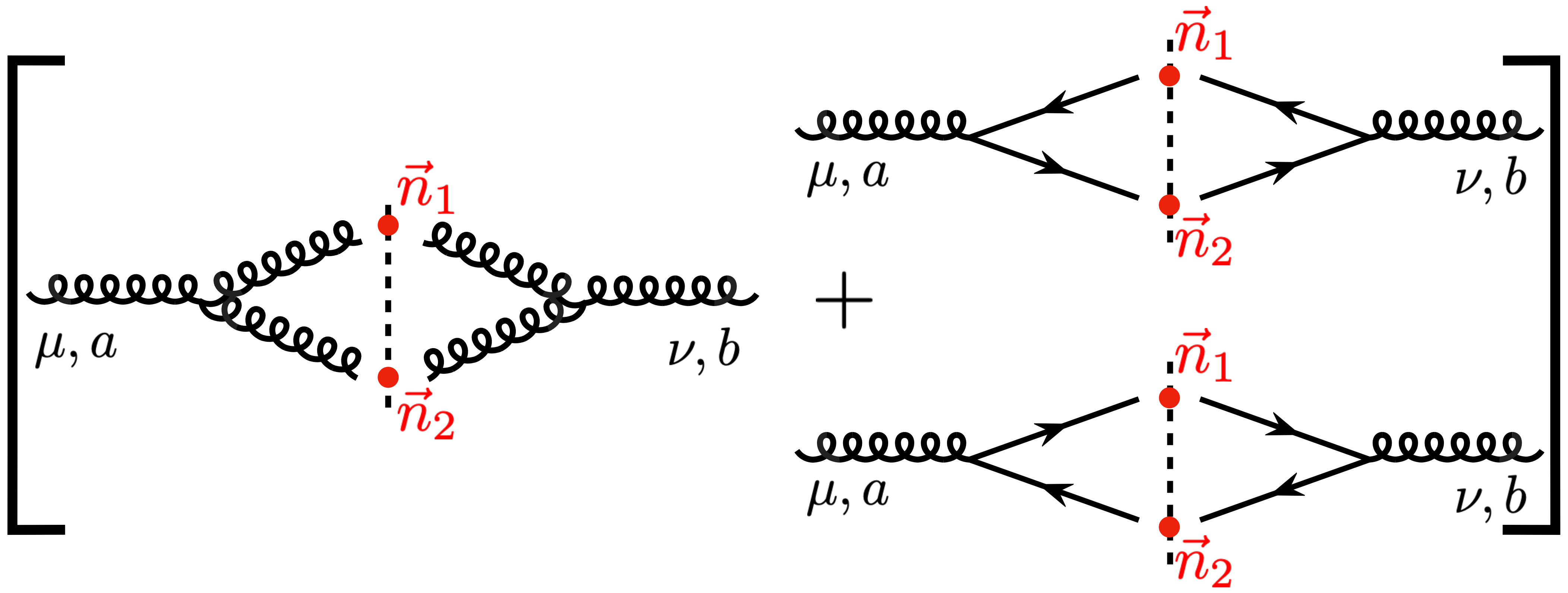
$$\langle \Omega | A_b^\nu(x)\ \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ A_a^\mu(0) | \Omega \rangle$$

$\epsilon_+^\mu \epsilon_-^\nu + \epsilon_-^\mu \epsilon_+ -^\nu$: $\mathbb{O}_g$ transverse spin 0

$\epsilon_+^\mu \epsilon_+^\nu$: $\mathbb{O}_{\tilde{g},+}$ transverse spin 2

$\epsilon_-^\mu \epsilon_-^\nu$: $\mathbb{O}_{\tilde{g},-}$ transverse spin 2

$$= \int \frac{E_1^2 dE_1}{(2\pi)^3 2E_1} \frac{E_2^2 dE_2}{(2\pi)^3 2E_2}\ E_1 E_2\ e^{-i(p_1+p_2)\cdot x}$$



$$\xrightarrow{\theta \to 0} \quad -\frac{1}{2\pi}\frac{2}{\theta^2}\Big[c_g\ \langle \Omega | A_b^\nu(x)\ \mathbb{O}_g^{[3]}\ A_a^\mu(0) | \Omega \rangle$$

$$+ c_{\tilde{g}} \left(e^{2i\phi} \langle \Omega | A_b^\nu(x)\ \mathbb{O}_{\tilde{g},-}^{[3]}\ A_a^\mu(0) | \Omega \rangle + e^{-2i\phi} \langle \Omega | A_b^\nu(x)\ \mathbb{O}_{\tilde{g},+}^{[3]}\ A_a^\mu(0) | \Omega \rangle \right) \Big]$$

$$\langle 1\, 2 \rangle^2 = s_{12} e^{2i\phi}$$

$$c_g = (\gamma_{gg}(2) - \gamma_{gg}(3)) + 2n_f\,(\gamma_{qg}(2) - \gamma_{qg}(3))$$

$$c_{\tilde{g}} = (\gamma_{g\tilde{g}}(2) - \gamma_{g\tilde{g}}(3)) + 2n_f\,(\gamma_{q\tilde{g}}(2) - \gamma_{q\tilde{g}}(3))$$

$$\vec{\mathbb{O}}^{[J]} = \begin{pmatrix} \mathbb{O}_q^{[J]} \\ \mathbb{O}_g^{[J]} \\ \mathbb{O}_{\tilde{g},+}^{[J]} \\ \mathbb{O}_{\tilde{g},-}^{[J]} \end{pmatrix}$$

$$\boxed{\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \text{higher twist}}$$

special case: $\mathcal{E} \sim \mathbb{O}_q^{[2]} + \mathbb{O}_g^{[2]}$

$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$

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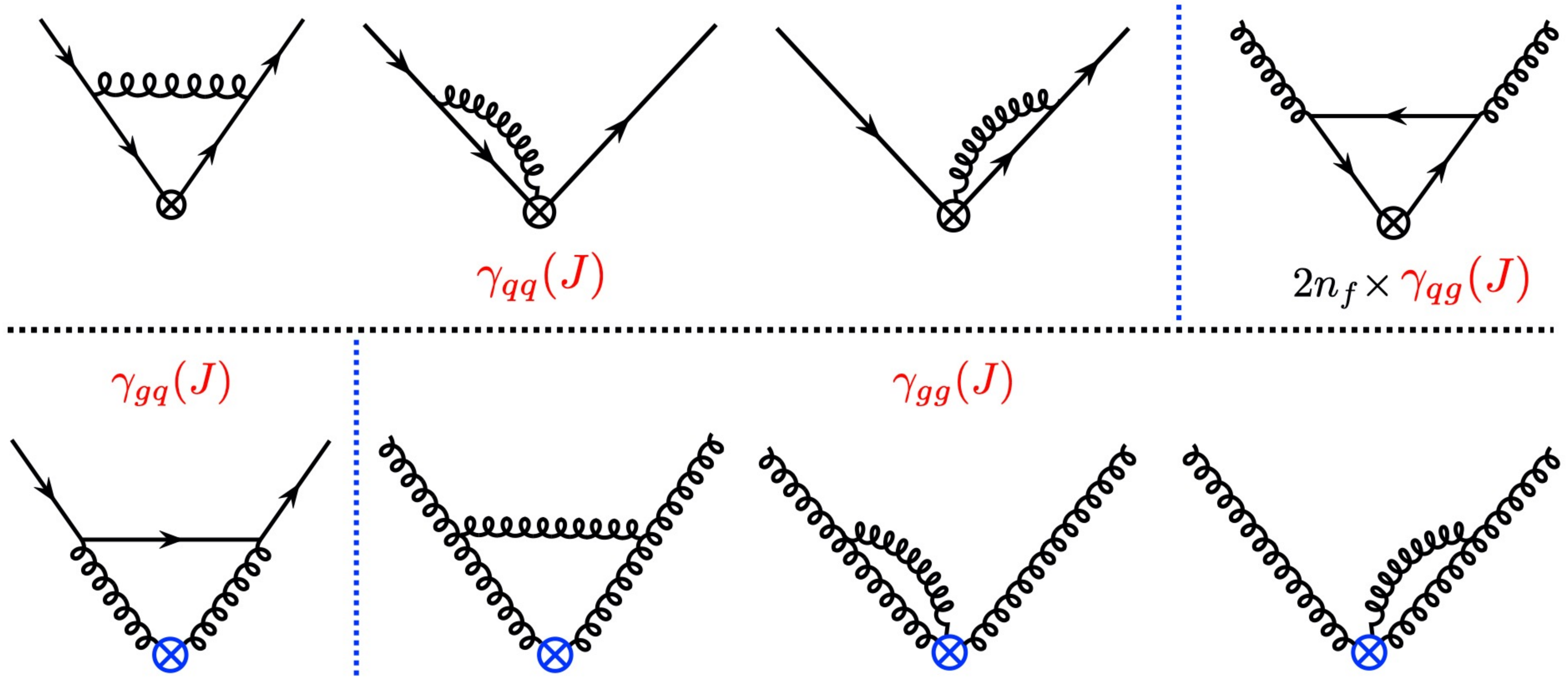

QCD 4 loops and 5 loops! Moch, Ruijl, Ueda, Vermeresen, 17; + Herzog, 18

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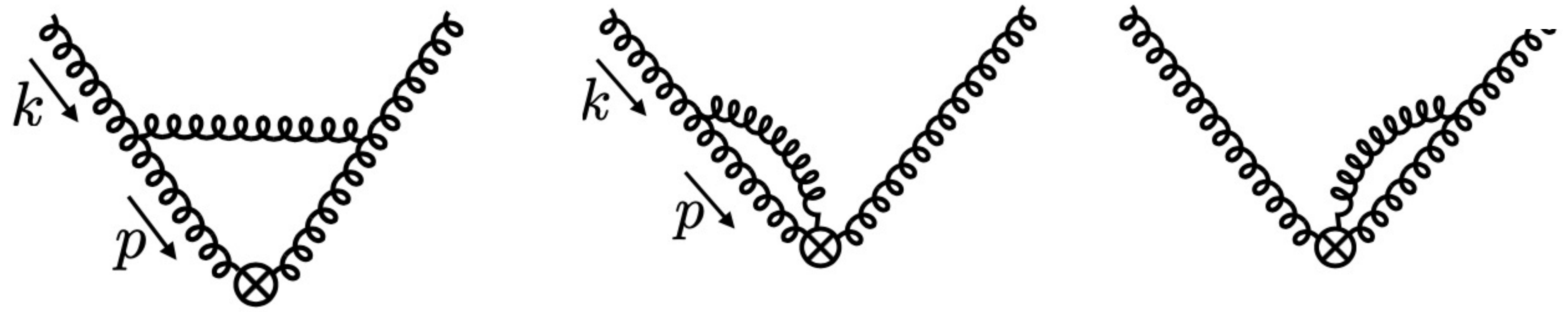
$$\boxed{\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \text{higher twist}}$$

special case: $\mathcal{E} \sim \mathbb{O}_q^{[2]} + \mathbb{O}_g^{[2]}$

$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$

Helicity-flip operator $\langle k,+|O_{J,\lambda}|k,-\rangle$ or $\langle k,-|O_{J,\lambda}|k,+\rangle$,



$$O_{J,\lambda} = \epsilon_{\lambda,i}\epsilon_{\lambda,j}O_J^{ij} = \epsilon_{\lambda,i}\epsilon_{\lambda,j}F^{+i}(iD^+)^{J-2}F^{+j}$$

$$\gamma_{\tilde{g}\tilde{g}}(J) = \frac{g^2}{(4\pi)^2}\left(4C_A\sum_{j=1}^{J}\frac{1}{j} - \beta_0\right) \Rightarrow P_{\tilde{g}\tilde{g}}(z) = 2C_A\left(\frac{1}{[1-z]_+} - 1\right) + \frac{\beta_0}{2}\delta(1-z)$$

Applied to EEC

$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)$$
$$= -\frac{1}{2\pi}\frac{2}{\theta^2}\Big\{[(\gamma_{qq}(2)-\gamma_{qq}(3))+(\gamma_{gq}(2)-\gamma_{gq}(3))]\,\mathbb{O}_q^{[3]}+[(\gamma_{gg}(2)-\gamma_{gg}(3))+2n_f(\gamma_{qg}(2)-\gamma_{qg}(3))]\,\mathbb{O}_g^{[3]}$$
$$+\frac{1}{2}[(\gamma_{g\tilde{g}}(2)-\gamma_{g\tilde{g}}(3))+2n_f(\gamma_{q\tilde{g}}(2)-\gamma_{q\tilde{g}}(3))]\left(e^{2i\phi}\mathbb{O}_{\tilde{g},-}^{[3]}+e^{-2i\phi}\mathbb{O}_{\tilde{g},+}^{[3]}\right)\Big\}+\mathcal{O}(\theta^0)\,.$$

Leading log series: $\widehat{C}_\phi(J) \simeq \alpha_s(1+\alpha_s\ln\theta+\alpha_s^2\ln^2\theta+\cdots)$

transverse spin-0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

transverse spin-2

$$\left[\mathcal{O}_{\tilde{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}\right]$$

CANNOT MIX

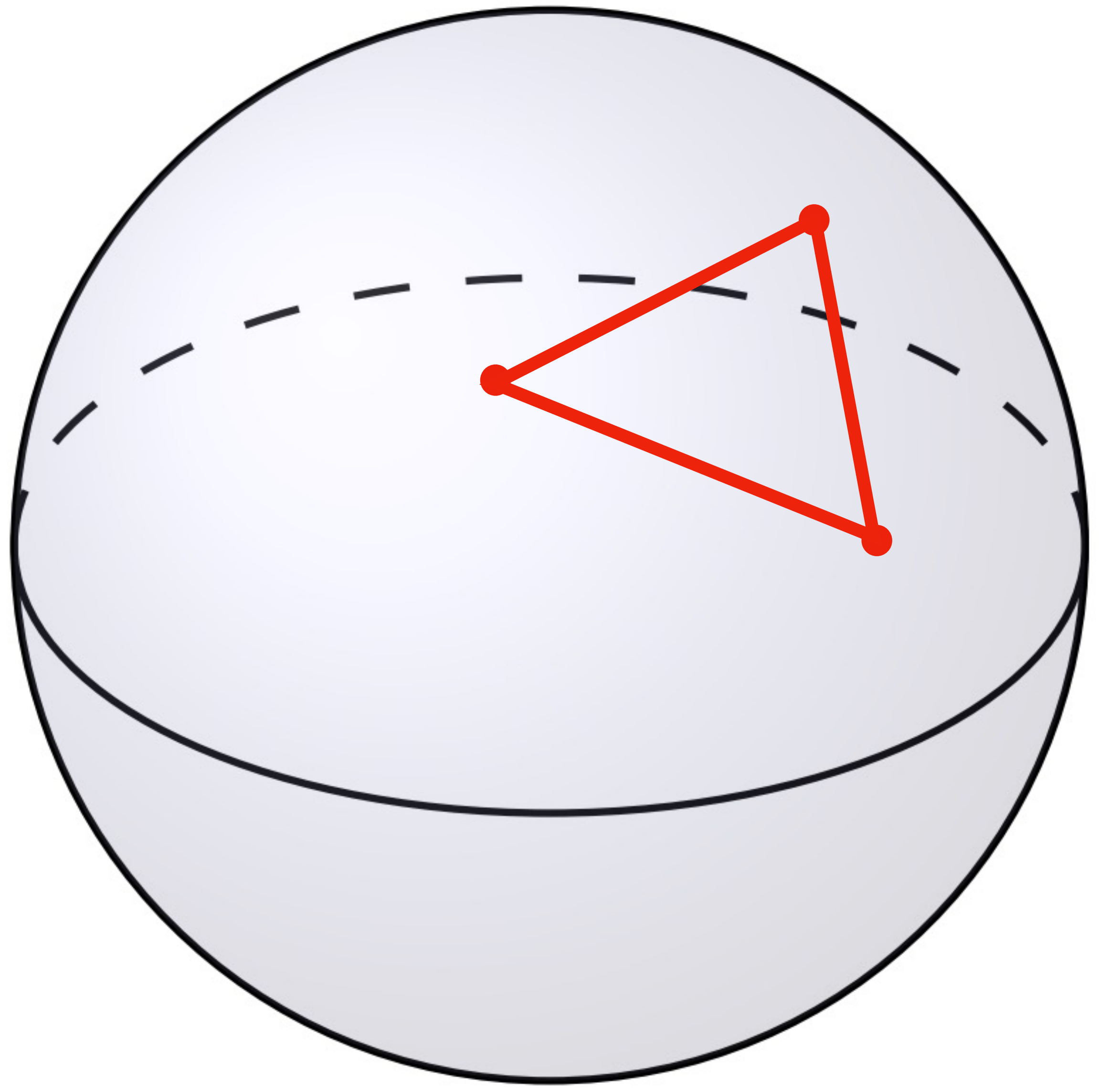
RG equation:

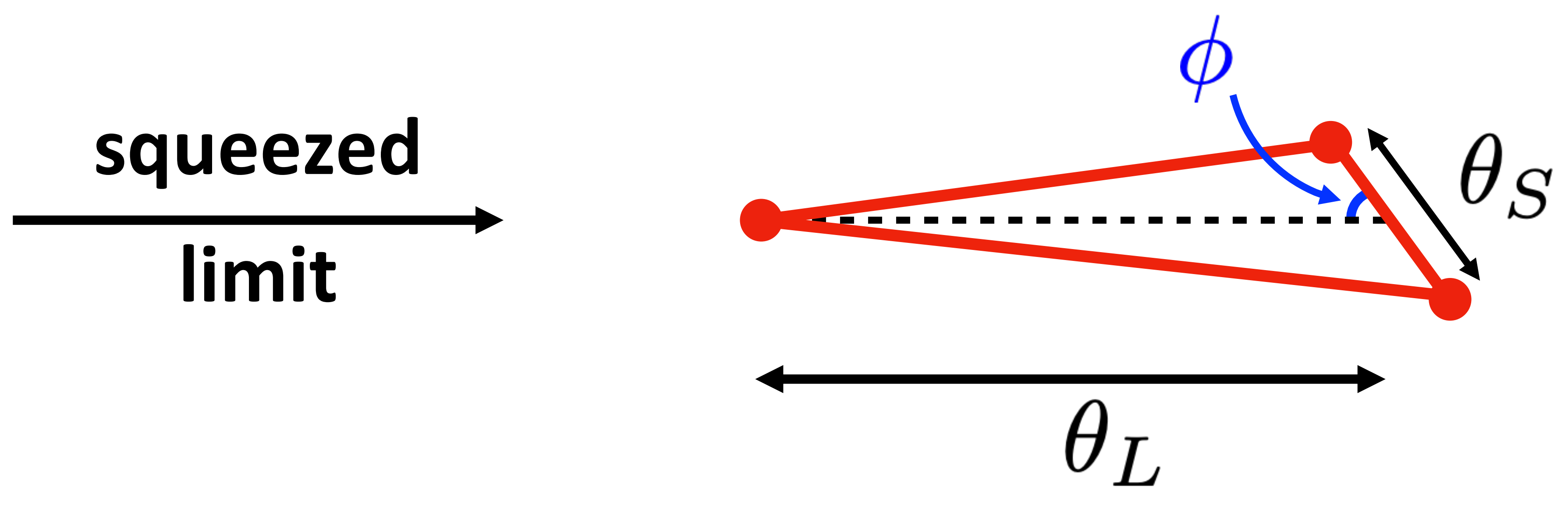
$$\frac{d}{d\ln\mu^2}\vec{\mathcal{O}}^{[J]} = -\widehat{\gamma}(J)\cdot\vec{\mathcal{O}}^{[J]}$$

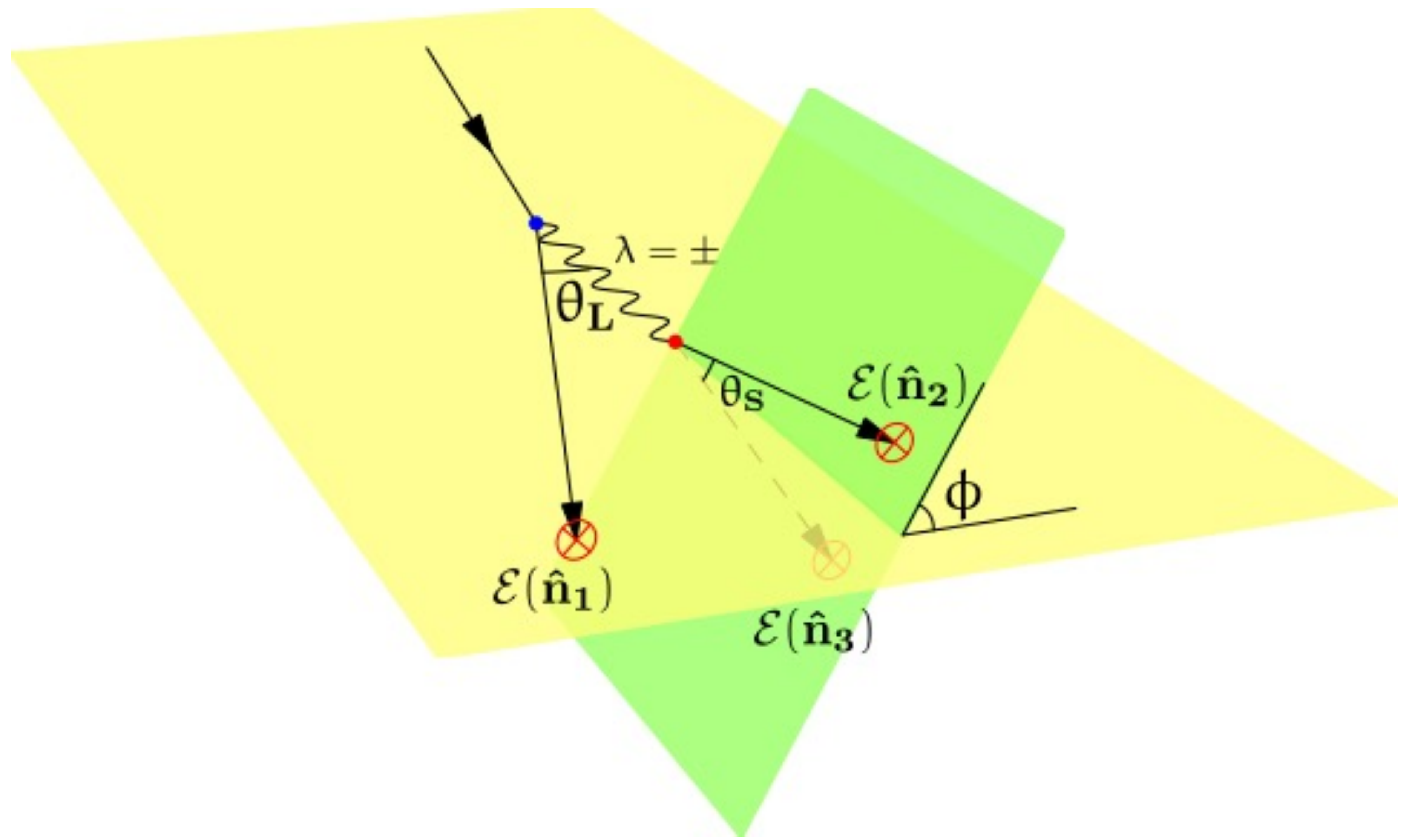
$$\widehat{\gamma}(J) = \begin{pmatrix}\gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 0\\ \gamma_{gq}(J) & \gamma_{gg}(J) & 0\\ 0 & 0 & \gamma_{\tilde{g}\tilde{g}}(J)\mathbf{1}\end{pmatrix}$$

$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\sim\frac{1}{\theta^{2-\gamma}}\mathbb{O}_{q,g}+\frac{1}{\theta^{2-\tilde{\gamma}}}\mathbb{O}_{\tilde{g}}+\text{twist corrections}$$

Squeeze three-point correlator







$$\frac{d^3\Sigma_i}{d\theta_L^2\, d\theta_S^2\, d\phi} \simeq \frac{1}{\pi}\left(\frac{\alpha_s}{4\pi}\right)^2 \frac{\mathrm{Sq}_i^{(0)}(\phi)}{\theta_L^2\theta_S^2} + \cdots$$

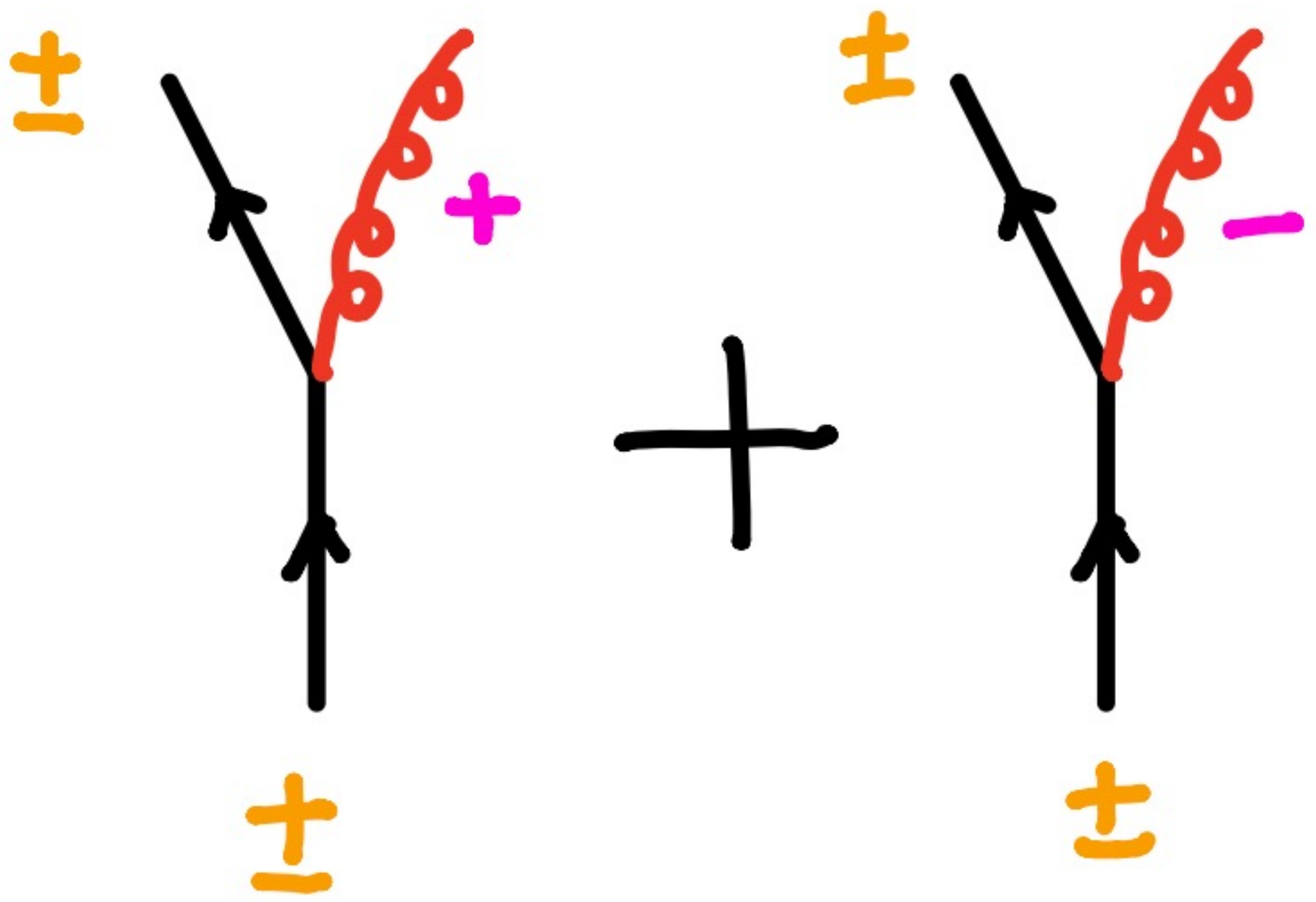
$$\mathrm{Sq}_q^{(0)}(\phi) = C_F n_f T_F \left(\frac{39 - 20\cos(2\phi)}{225}\right) + C_F C_A \left(\frac{273 + 10\cos(2\phi)}{225}\right) + C_F^2 \frac{16}{5}$$

$$\mathrm{Sq}_g^{(0)}(\phi) = C_A n_f T_F \left(\frac{126 - 20\cos(2\phi)}{225}\right) + C_A^2 \left(\frac{882 + 10\cos(2\phi)}{225}\right) + C_F n_f T_F \frac{3}{5}$$

Interference Effect

1. Cancellation between boson and fermion
2. The equal coefficient due to an effective N=1 supersymmetry

Squeeze three-point correlator



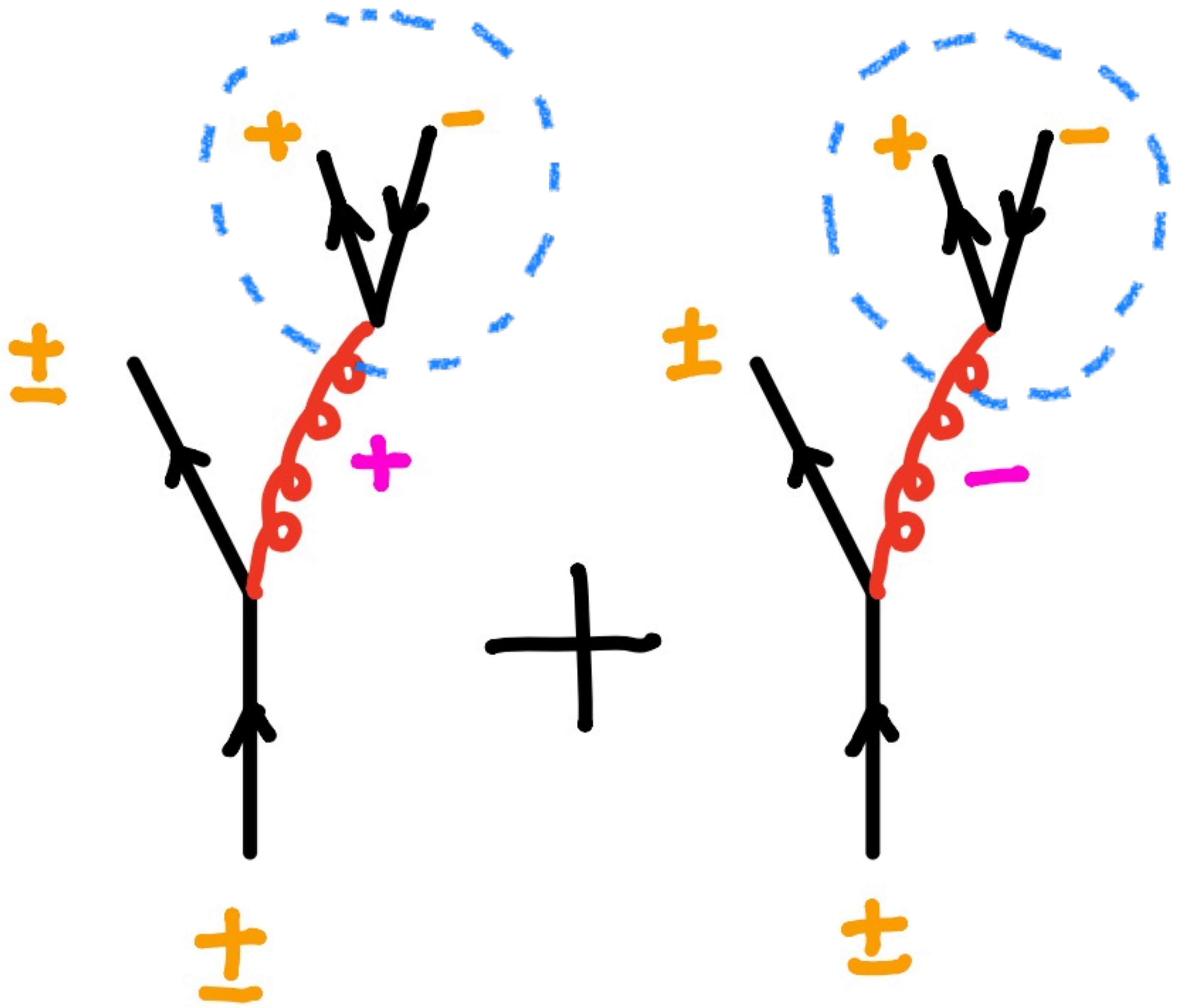
$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |-\rangle)$$

$$\hat{\rho} = \frac{1}{2}\Big(|+\rangle\langle+| + |-\rangle\langle-| + (|+\rangle\langle-| + |-\rangle\langle+|)\Big)$$

unpolarized — spin 2 linearly polarized

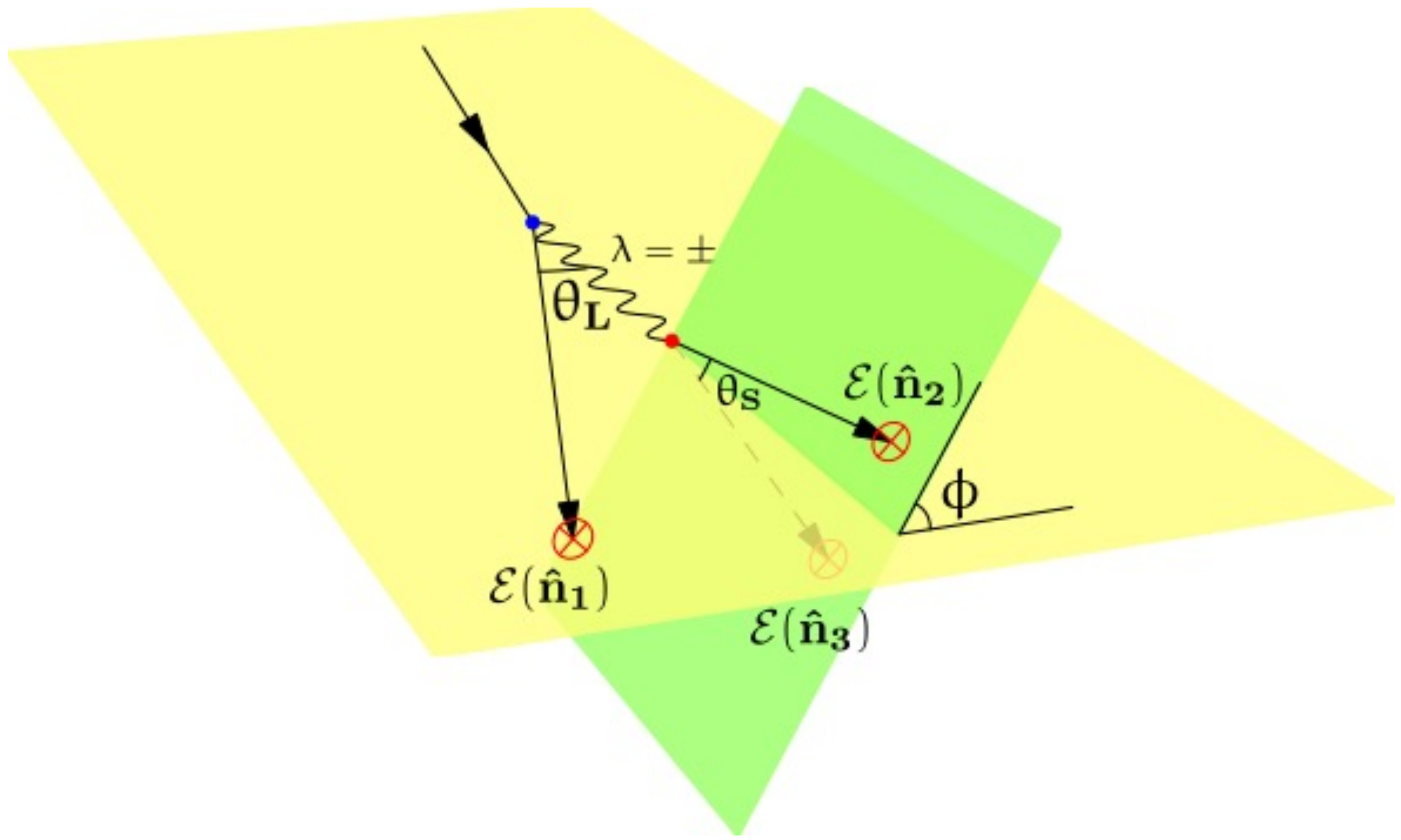
The off diagonal component of density matrix can not be observed directly

Measurement of off-diagonal component by splitting

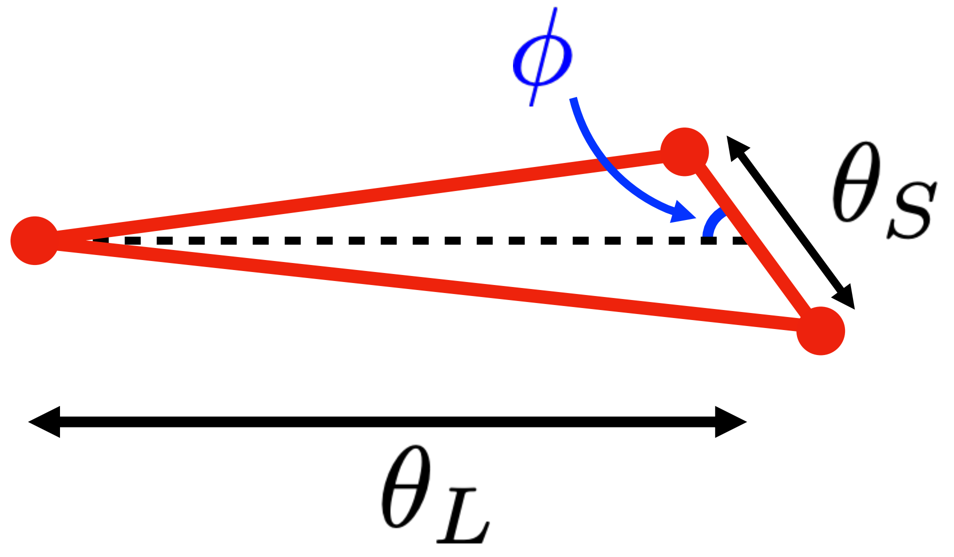


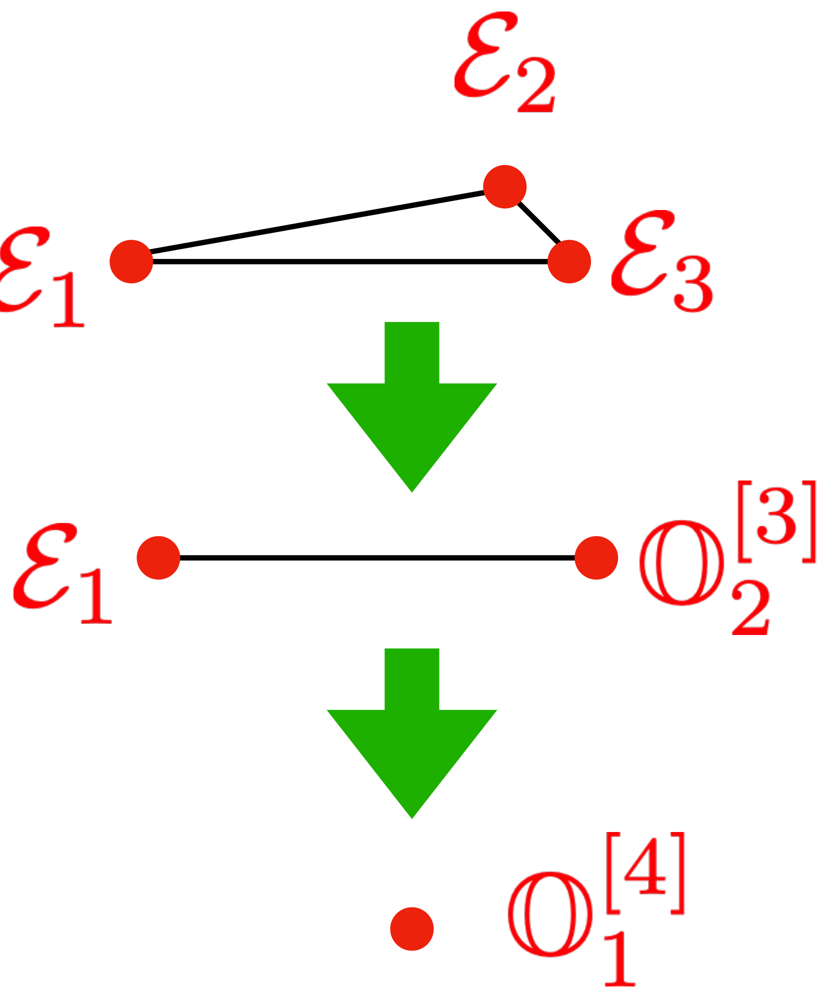
Density operator after second splitting

$$\hat{\rho} = C_0 | + - \rangle \langle + - | + C_1 \cos(2\phi) | + - \rangle \langle + - |$$



Sequential light-ray OPE





$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle$$

$e^{2i\phi_S}$

$$\longrightarrow -\frac{1}{2\pi}\frac{2}{\theta_S^2}\vec{\mathcal{J}}\left[\widehat{C}_{\phi_S}(2)-\widehat{C}_{\phi_S}(3)\right]\left\langle \mathcal{E}(\vec{n}_1)\vec{\mathbb{O}}^{[3]}(\vec{n}_2)\right\rangle$$

$$\longrightarrow \frac{1}{(2\pi)^2}\frac{2}{\theta_S^2}\frac{2}{\theta_L^2}\vec{\mathcal{J}}\left[\widehat{C}_{\phi_S}(2)-\widehat{C}_{\phi_S}(3)\right]\left[\widehat{C}_{\phi_L}(3)-\widehat{C}_{\phi_L}(4)\right]\boxed{\left\langle \vec{\mathbb{O}}^{[4]}(\vec{n}_1)\right\rangle}$$

$e^{2i\phi_S}$ $\quad e^{-2i\phi_L}$

$\cos(2\phi)$

Hierarchy $1 \gg \theta_L \gg \theta_S$ **fixed order result needs to be resummed**

$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle \sim C_1(\theta_S,\mu)\left\langle \mathcal{E}(\vec{n}_1)\mathbb{O}^{[3]}(\vec{n}_2)\right\rangle_\mu \sim C_1(\theta_S,\mu)C_2(\theta_L,\mu)\left\langle \mathbb{O}^{[4]}(\vec{n}_1)\right\rangle_\mu$$

$$\widehat{\gamma}(J)=\begin{pmatrix}\gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 0\\ \gamma_{gq}(J) & \gamma_{gg}(J) & 0\\ 0 & 0 & \gamma_{\tilde{g}\tilde{g}}(J)\mathbf{1}\end{pmatrix}$$

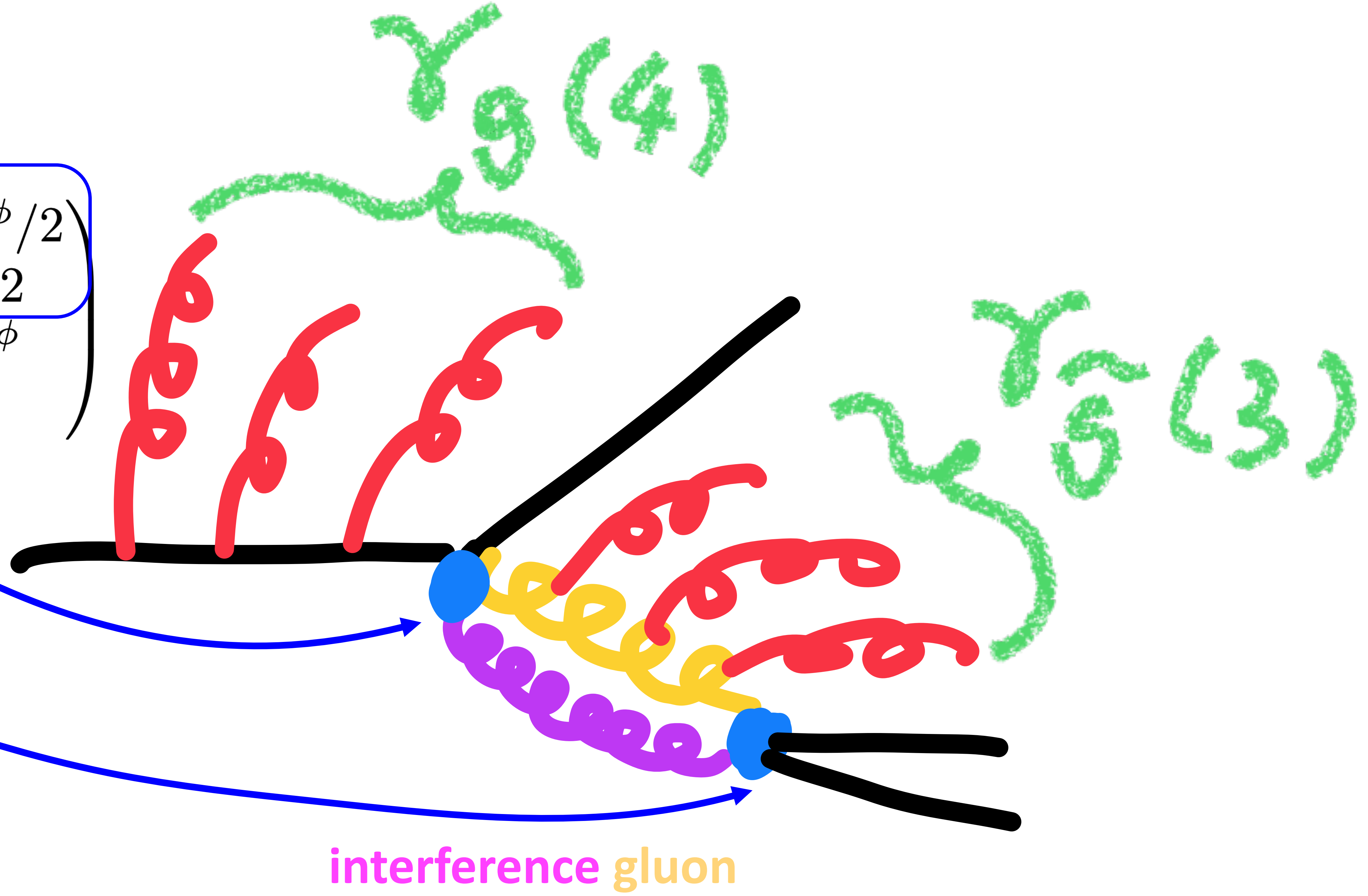
Leading log approximation

$$\widehat{\gamma}(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 0 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & 0 \\ 0 & 0 & \gamma_{\tilde{g}\tilde{g}}(J)\mathbf{1} \end{pmatrix}$$

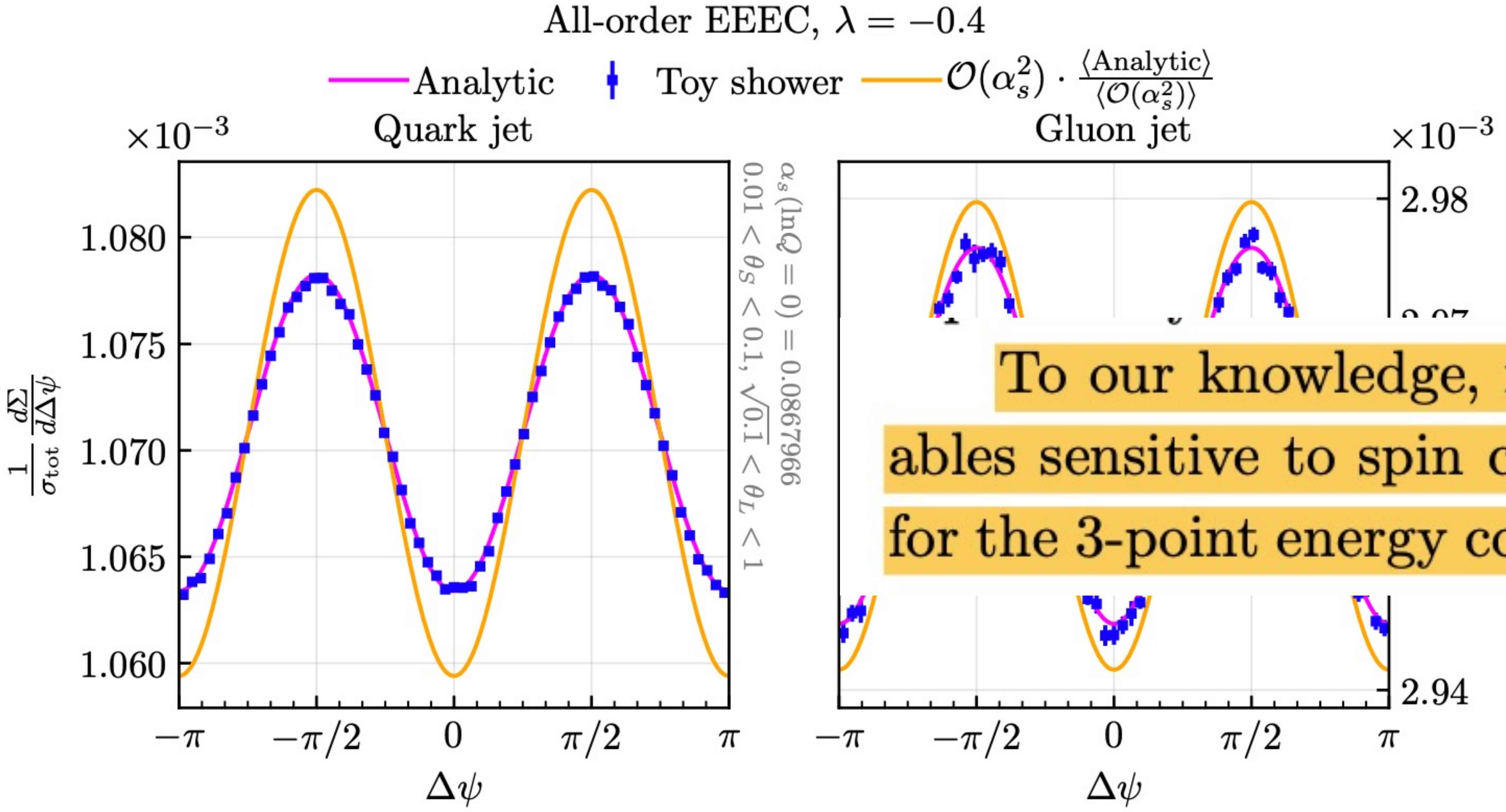
$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3)$$

$$= \frac{1}{(2\pi)^2}\frac{2}{\theta_S^2}\frac{2}{\theta_L^2}\vec{\mathcal{J}}\left[\widehat{C}_{\phi_S}(2) - \widehat{C}_{\phi_S}(3)\right]\left[\frac{\alpha_s(\theta_L Q)}{\alpha_s(\theta_S Q)}\right]^{\frac{\widehat{\gamma}(3)}{\beta_0}}\left[\widehat{C}_{\phi_L}(3) - \widehat{C}_{\phi_L}(4)\right]\left[\frac{\alpha_s(Q)}{\alpha_s(\theta_L Q)}\right]^{\frac{\widehat{\gamma}(4)}{\beta_0}}\vec{\mathbb{O}}^{[4]}(\hat{n}_1)$$

$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$



Our analytic resummed was confirmed shortly by a numerical Monte Carlo parton shower pro



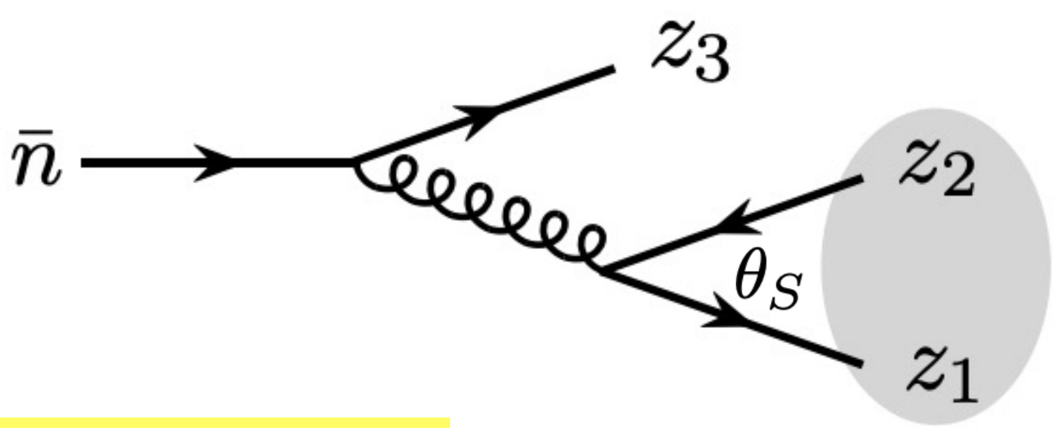
Spin correlations in final-state parton showers and jet observables Karlberg, Salam, Scyboz, Verheyen, 2021

To our knowledge, no analytical result exists for the logarithmic structure of observables sensitive to spin correlations, except for the recently computed all-order result [38] for the 3-point energy correlator, reproduced in section 3.3. In order to enable comparisons

Understand power corrections through Lorentz symmetry

Hidden analytic structure in three-point correlator

Consider the squeeze limit of (12)



highest transverse spin series

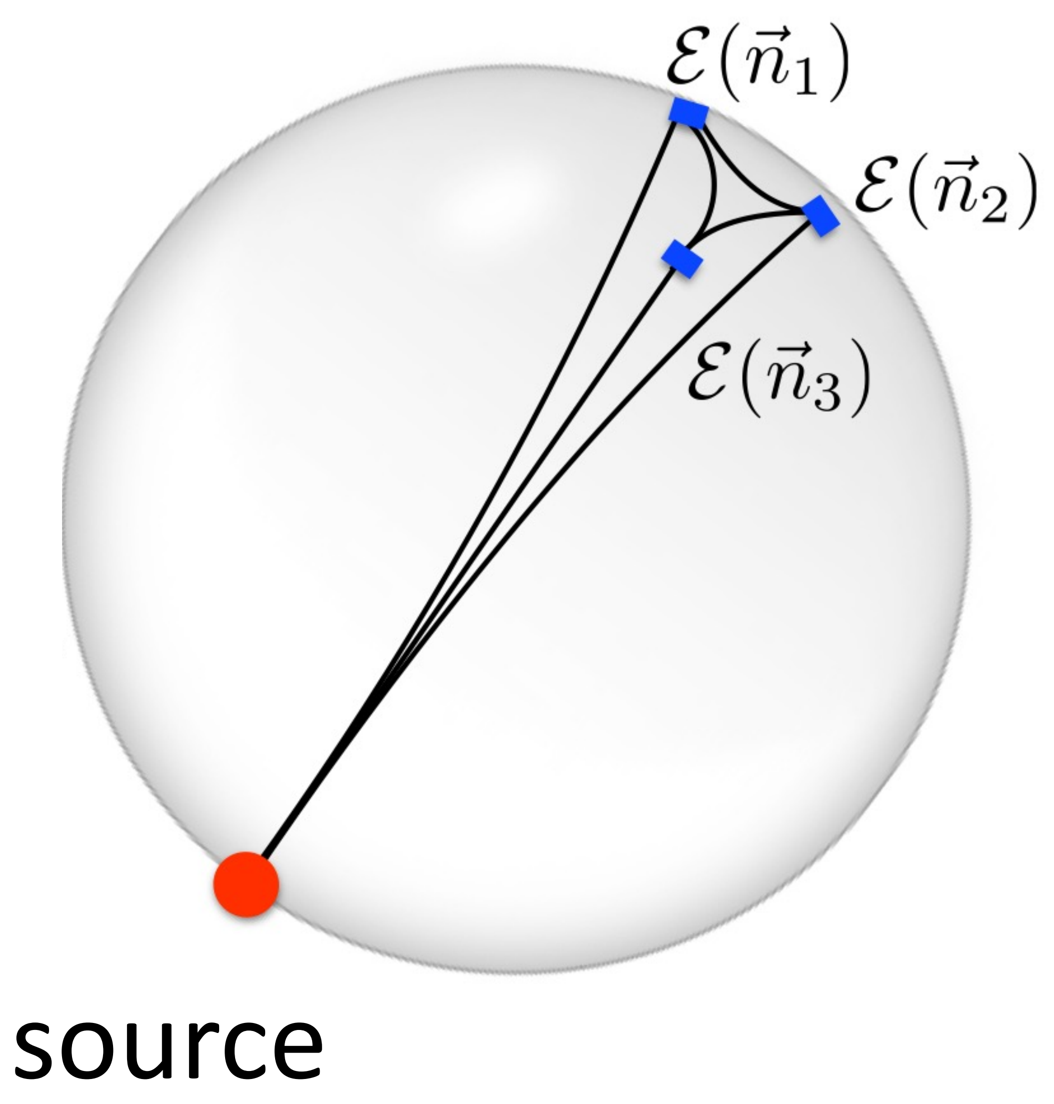
transverse twist = δ - j = 2

							δ
$-z^3\bar{z}$ (cos(2ϕ))	$+\frac{39}{10}z^2\bar{z}^2$	$-z\bar{z}^3$				θ_S^{-2}	4
$-z^4\bar{z}$ (cos(4ϕ))	$+\frac{39}{20}z^3\bar{z}^2$	$+\frac{39}{20}z^2\bar{z}^3$	$-z\bar{z}^4$			θ_S^{0}	6
$-\frac{6}{7}z^5\bar{z}$ (cos(6ϕ))	$+\frac{229}{140}z^4\bar{z}^2$	$-\frac{211}{140}z^3\bar{z}^3$	$+\frac{229}{140}z^2\bar{z}^4$	$-\frac{6}{7}z\bar{z}^5$		θ_S^{2}	8
$-\frac{5}{7}z^6\bar{z}$ (cos(8ϕ))	$+\frac{207}{140}z^5\bar{z}^2$	$-\frac{233}{140}z^4\bar{z}^3$	$-\frac{233}{140}z^3\bar{z}^4$	$+\frac{207}{140}z^2\bar{z}^5$	$-\frac{5}{7}z\bar{z}^6$	θ_S^{4}	10
$\cdots$		$\cdots$			$\cdots$		

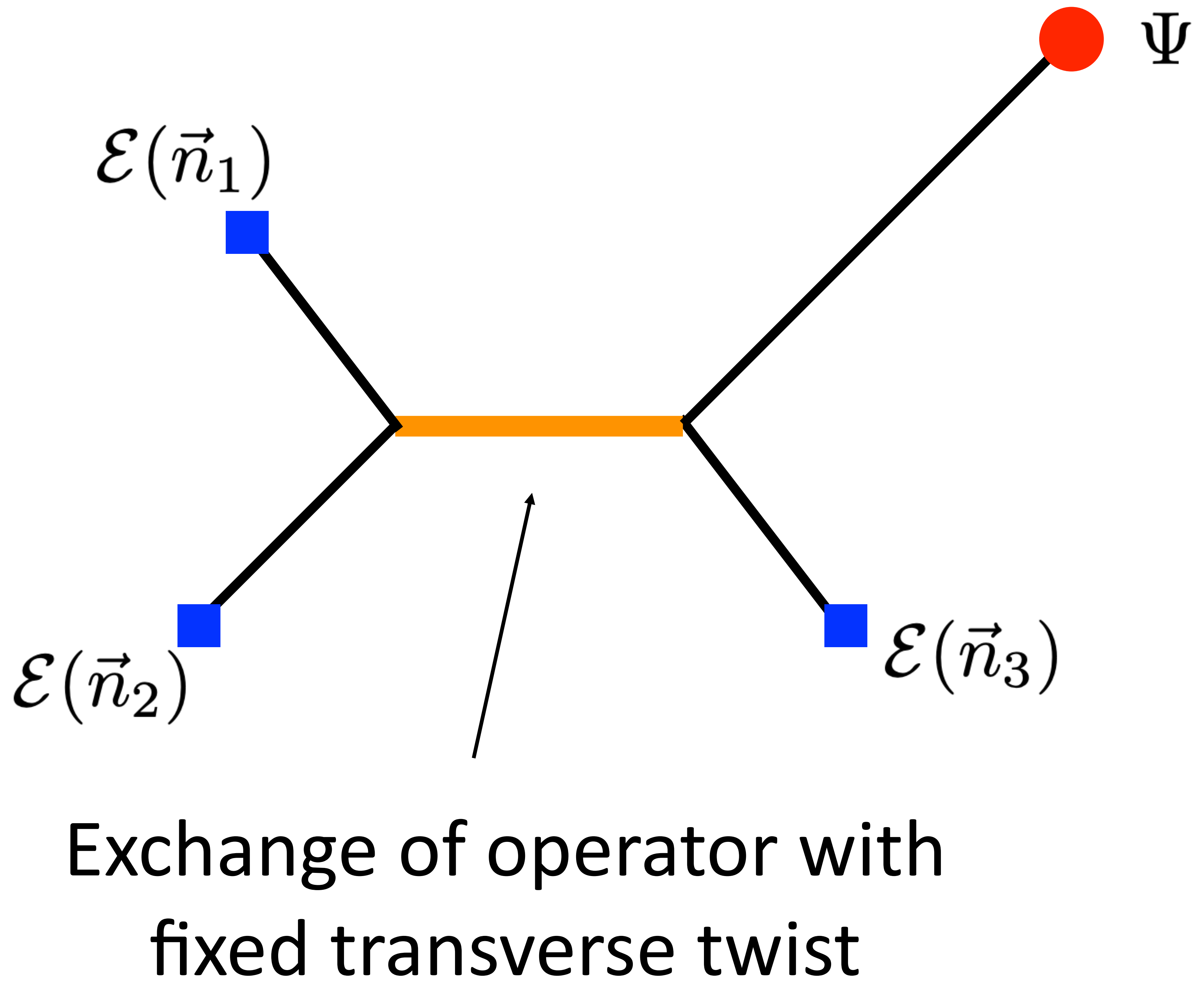
resummed to

$$-z^3\bar{z}\;{}_2F_1\left(3,2,6,z\right)$$

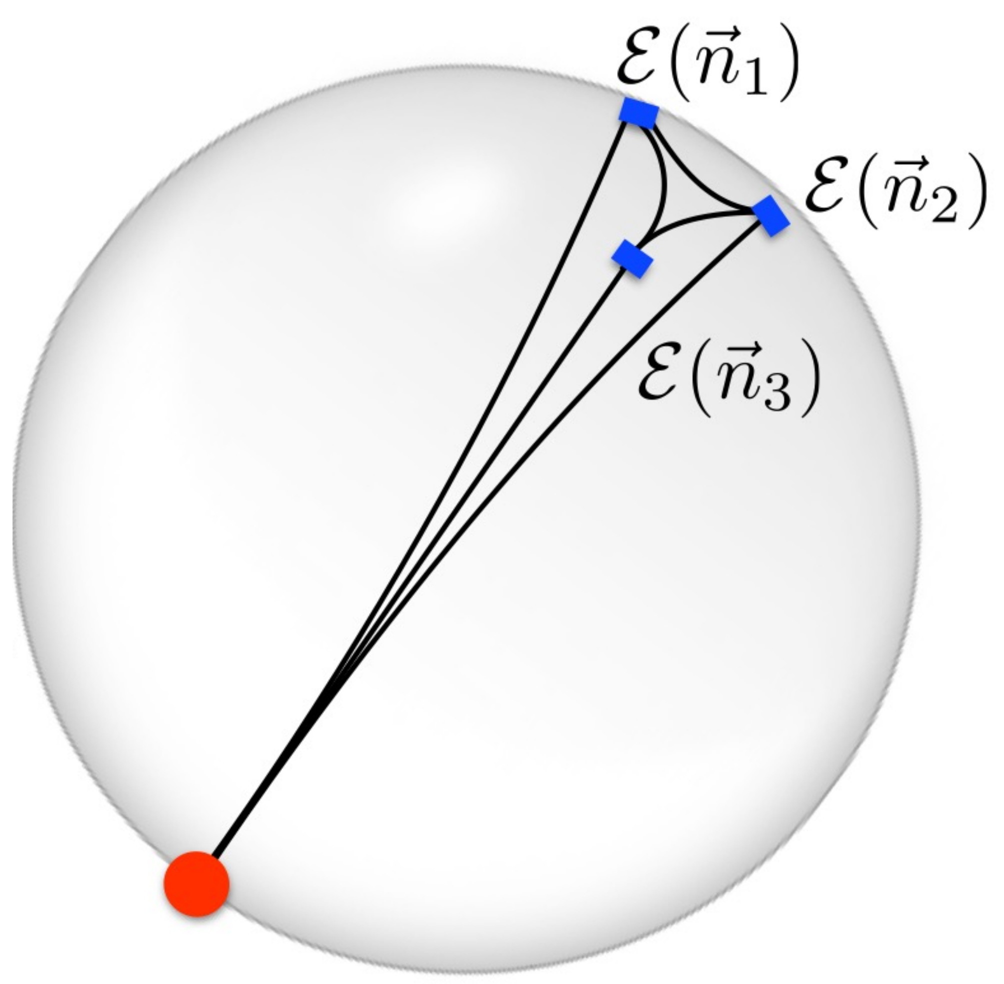
Every simplicity should has a reason. What is it?



$\mathcal{E}(\vec{n}_1)$
$\mathcal{E}(\vec{n}_2)$
$\mathcal{E}(\vec{n}_3)$
source

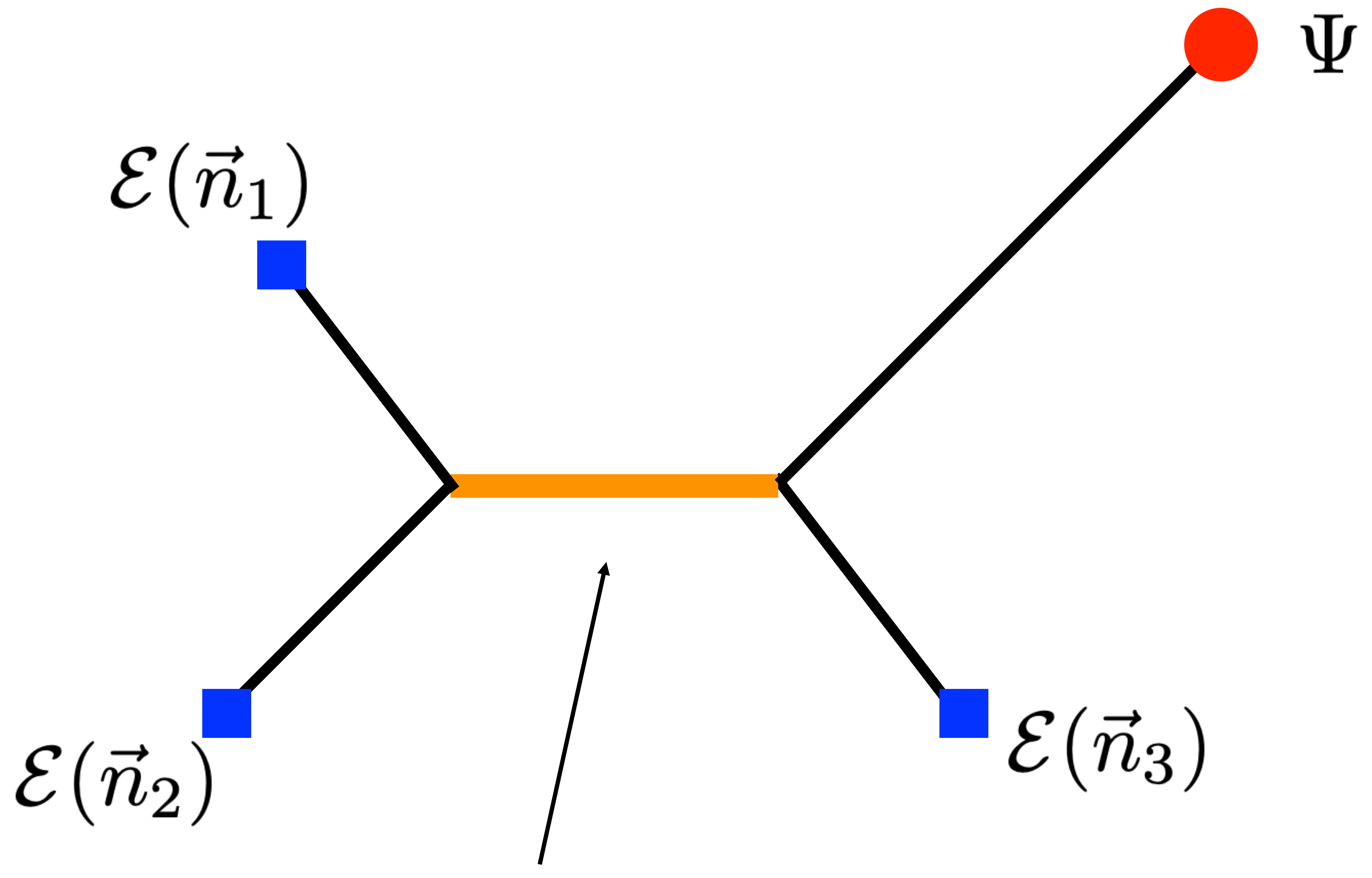


Ψ
$\mathcal{E}(\vec{n}_1)$
$\mathcal{E}(\vec{n}_2)$
$\mathcal{E}(\vec{n}_3)$
Exchange of operator with fixed transverse twist



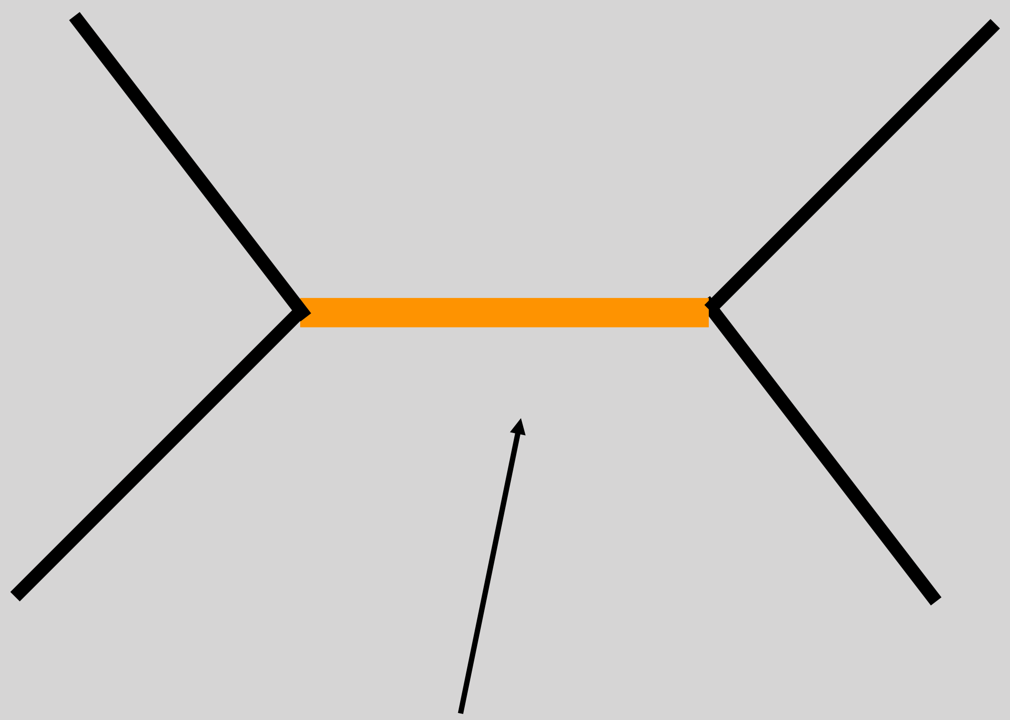
source

Partial wave expansion



Exchange of operator with fixed transverse twist

Recall: for 2->2 **Amplitude**



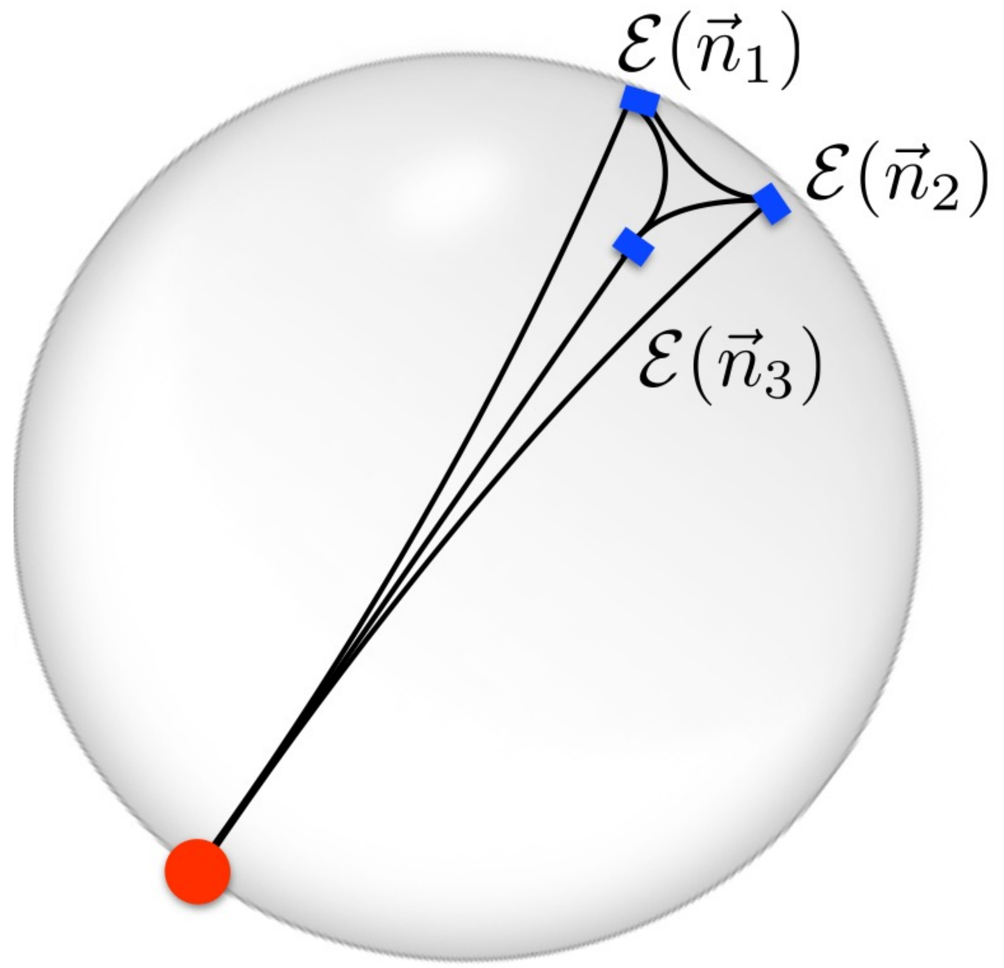
Exchange of particle with spin J

Partial wave amplitude, dynamics

$$A(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1)\, a_\ell(s)\, P_\ell(z), \quad z = \cos\theta$$

Legendre function, kinematics

eigenfunction of quadratic Casimir operator of **SO(3)**



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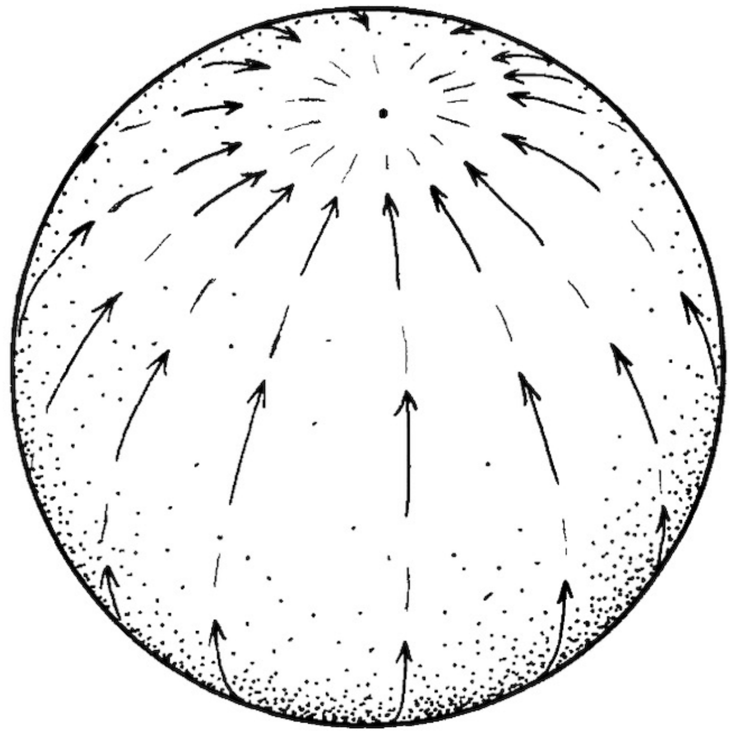
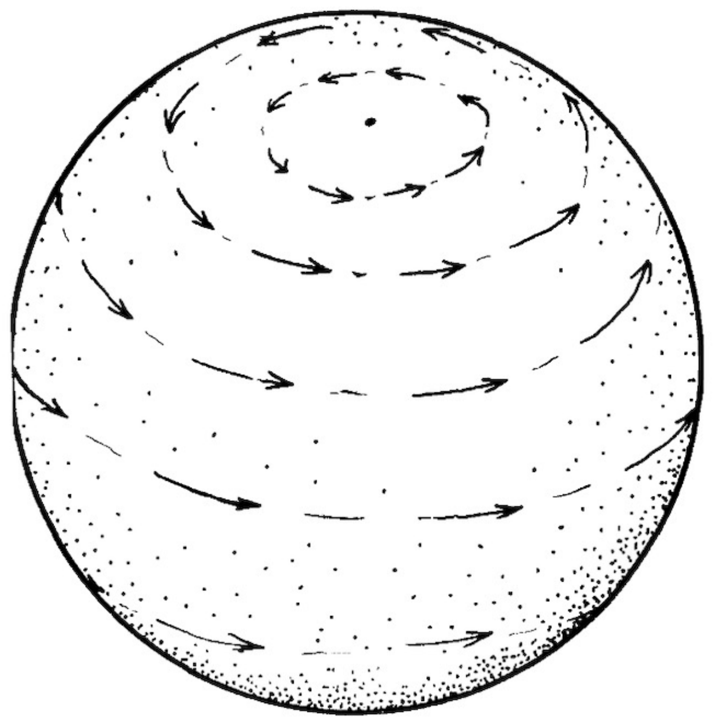
Three-point correlator transform non-trivially under Lorentz transformation



boost



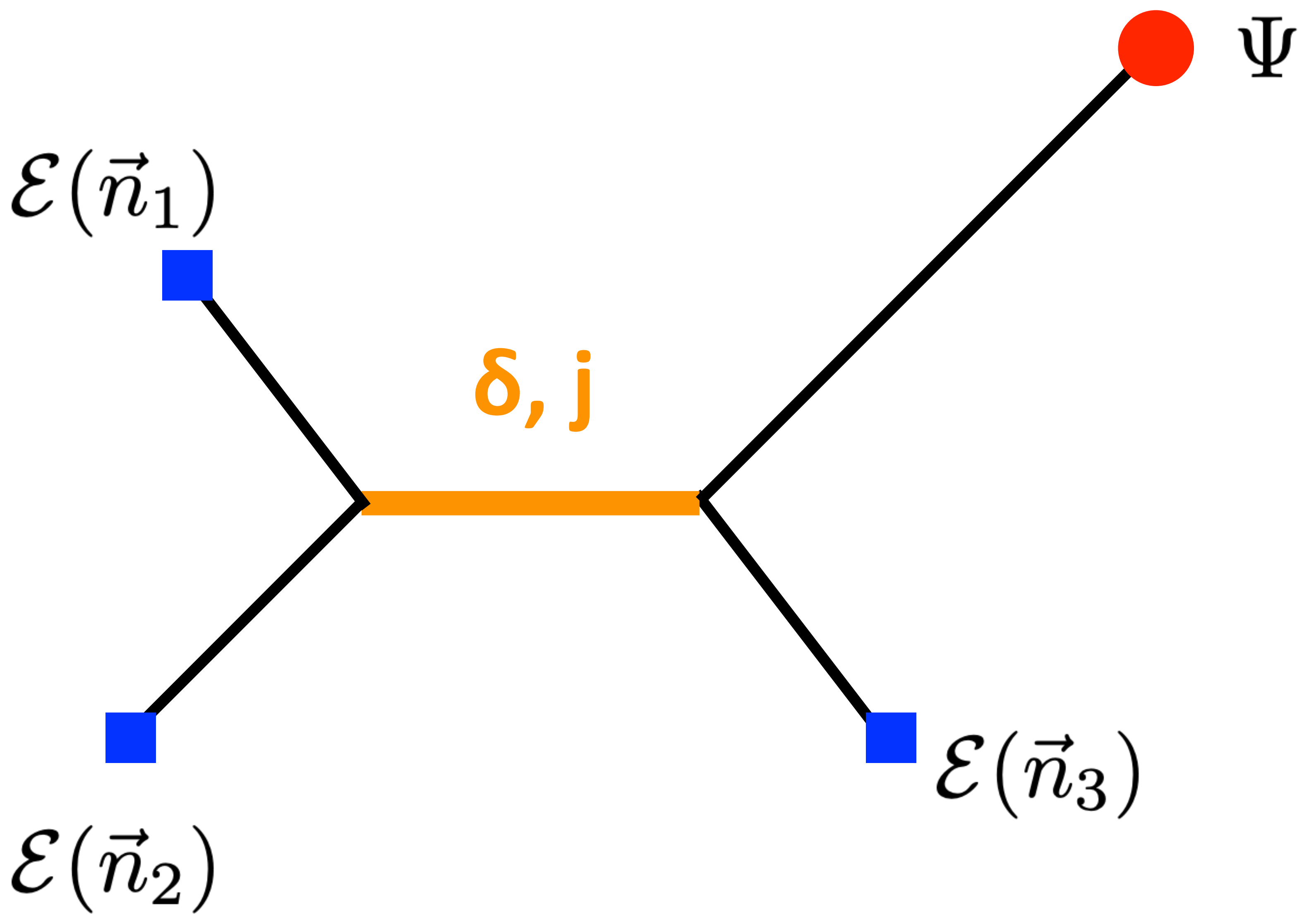
rotation

Art work of Penrose

Lorentz group SO(3,1) on celestial sphere = conformal group on Riemann sphere

Two representation labels: celestial dimension δ, transverse spin j

Control the power   Control cos(jφ)



$$g(z,\bar{z}) = \sum_{\delta,j} c_{\delta,j}\, g_{\delta,j}(z,\bar{z})$$

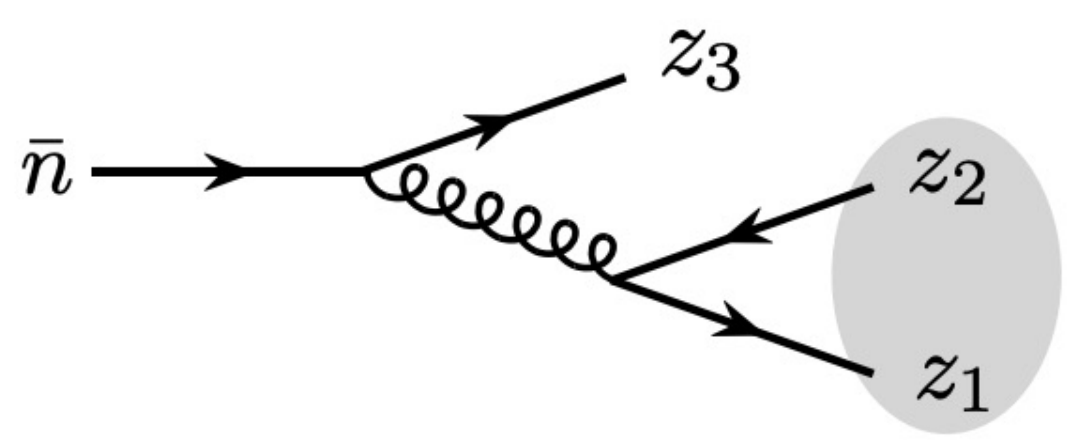
Partial wave expansion for 3-pt correlator

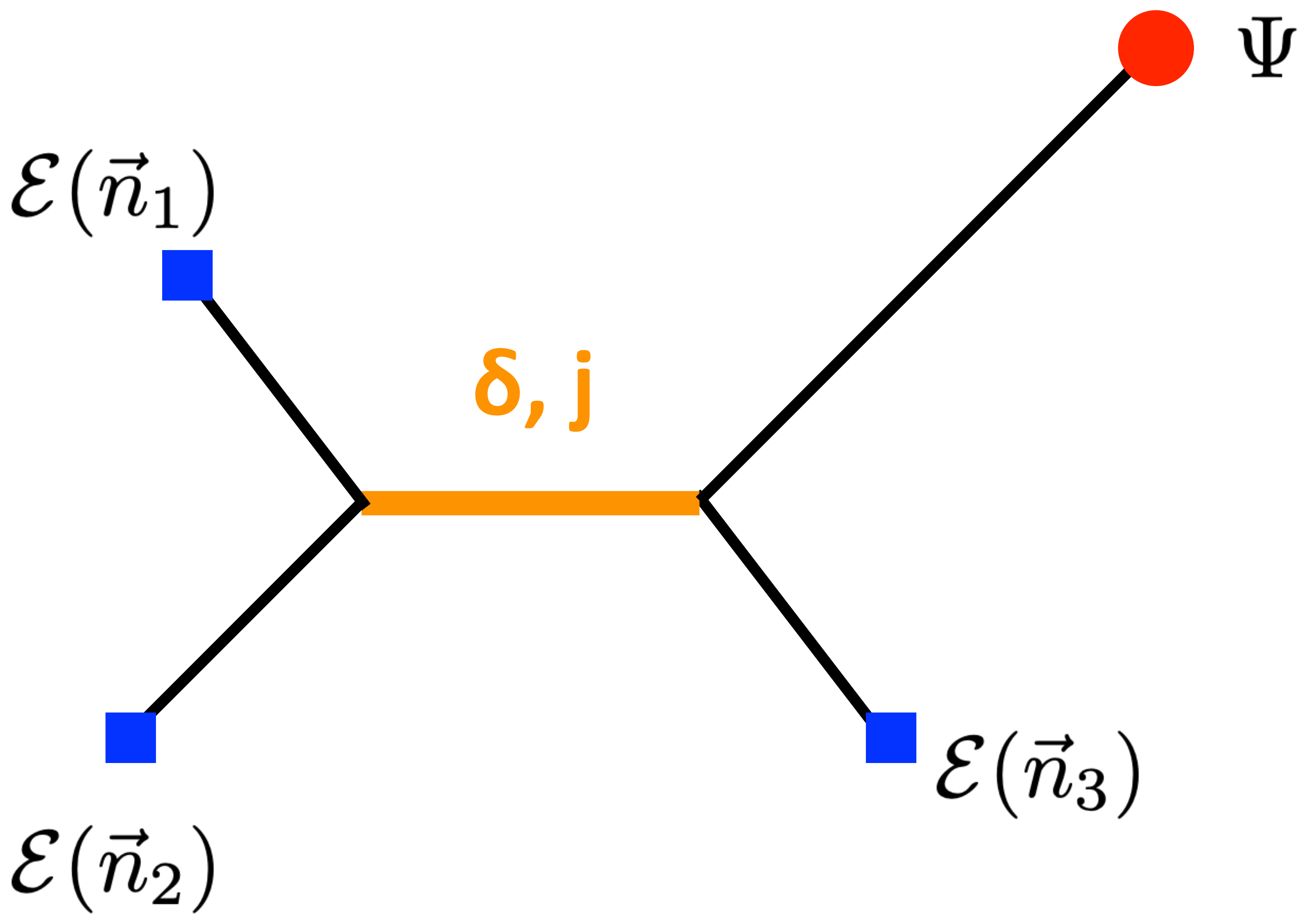
Casimir equation $\quad \mathcal{L}^{\mu\nu}(z_1,z_2)\mathcal{L}_{\mu\nu}(z_1,z_2) G_{\delta,j} = -(\delta(\delta-2)+j^2) G_{\delta,j}$

Eigenvalue

$$\mathcal{L}^{\mu\nu}(z_1,z_2) \equiv \sum_{i=1,2}\left(z_i^\mu \frac{\partial}{\partial z_{i\nu}} - z_i^\nu \frac{\partial}{\partial z_{i\mu}}\right)$$

Quadratic Casimir operator





$$g(z,\bar{z}) = \sum_{\delta,j} c_{\delta,j}\, g_{\delta,j}(z,\bar{z})$$

Partial wave expansion for 3-pt correlator

Solutions:

$$g_{\delta,j}(z,\bar{z}) = \frac{1}{1+\delta_{j,0}}\left(k_{\delta-j}(z)k_{\delta+j}(\bar{z}) + k_{\delta+j}(z)k_{\delta-j}(\bar{z})\right)$$

$$k_\beta(x) = x^{\beta/2}\,{}_2F_1\left(\frac{\beta}{2},\frac{\beta}{2}-1,\beta,x\right)$$

2D euclidean conformal block [Dolan, Osborn, 2003]

Previous example: $-z^3\bar{z}\;{}_2F_1(3,2,6,z)$

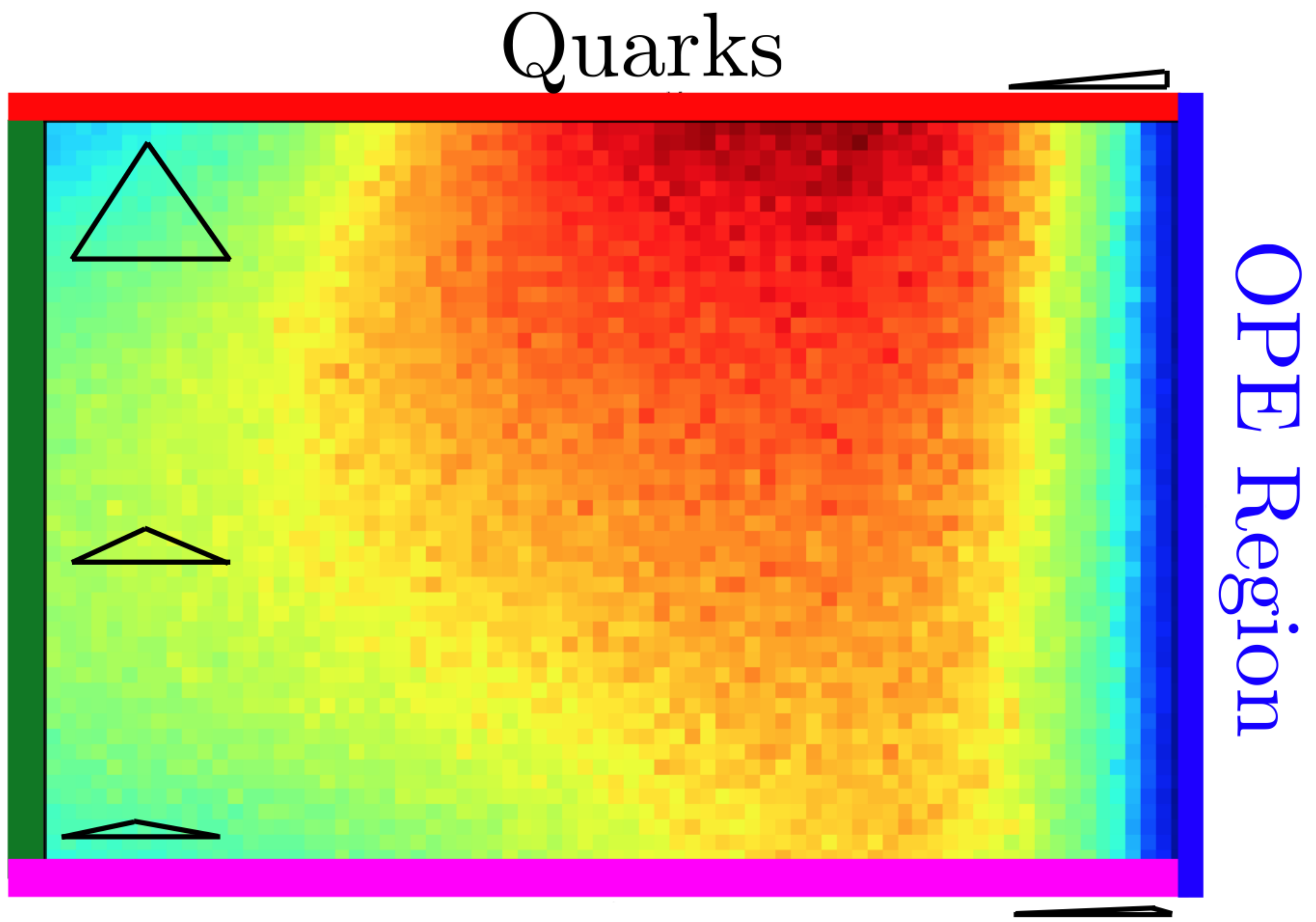
Corresponds to exchange of single primary and its descendent: $\delta = 4,\; j = 2$ $\quad \epsilon_\mu \epsilon^\nu G^{\mu\rho}(D^+)^2 G_{\nu\rho}$

$$k_6(z) = z^3\;{}_2F_1(3,2,6,z) \qquad k_2(\bar{z}) = \bar{z}$$

This establishes a surprising connection between conformal field theory techniques and QCD jet substructure.

2D euclidean conformal symmetry = 4D Lorentz symmetry.

More to be understood in the future.



You can see it in the real data!

Komiske, Moult, Thaler, HXZ, in preparation

Summary

- Jet substructure is an ideal tool to uncover the high energy dynamics of QCD.
- Light-ray operator product expansion, originally developed for N=4 SYM, now applied to QCD successfully at LO/LL.
- Symmetry allows transparent treatment of spin interference effects from light-ray OPE
- Partial wave expansion on celestial sphere. CFT <=> Jet substructure

Thank you very much!