The 145th HENPIC Seminar

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Reconstruction of Heavy-Flavor Potential from Bottomonium Spectrum using DNN

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with:

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Ref: (arXiv)2105.07862

Outline

- Methodology inverting the Schroedinger equation
  - what is Deep Neural Network
  - new algorithm using DNN to obtain $V(r)$ from $\{E_n\}$

- Application
  - heavy flavor potential from Bottomonium mass & thermal width
  - phenomenological consequences

Schroedinger Equation

$$\hat{H}\psi_n = -\frac{\nabla^2}{2m}\psi_n + V(r)\,\psi_n = E_n\,\psi_n$$

- $V(r)$ known $\Longrightarrow \{E_n, \psi_n(r)\}$: numerical methods established.
- $\psi_n(r)$ known $\Longrightarrow V(r)$: $\dfrac{\nabla^2\psi_n}{2m\,\psi_n} = V(r) - E_n$
- $\{E_n\}$ known $\Longrightarrow V(r)$: ???

How to learn $V(r)$ from $\{E_n\}$?

- parameterize the potential $V(r\,|\,\boldsymbol{\theta})$, ***scan*** the whole $\boldsymbol{\theta}$-space, minimize $\chi^2 \equiv \sum_i \left(\frac{E_{\boldsymbol{\theta},i} - E_i}{\delta E_i}\right)^2$

- a ***gradient-descent*** based method:
  - goal -- find the $\boldsymbol{\theta}$-point that $\nabla_{\boldsymbol{\theta}}\chi^2 = 0$
  - update $\boldsymbol{\theta}$ iteratively according to $\Delta\boldsymbol{\theta} \propto \nabla_{\boldsymbol{\theta}}\chi^2$

$$\nabla_{\boldsymbol{\theta}}\chi^2 = 2\sum_i \frac{E_{\boldsymbol{\theta},i} - E_i}{(\delta E_i)^2}\nabla_{\boldsymbol{\theta}}E_{\boldsymbol{\theta},i}$$

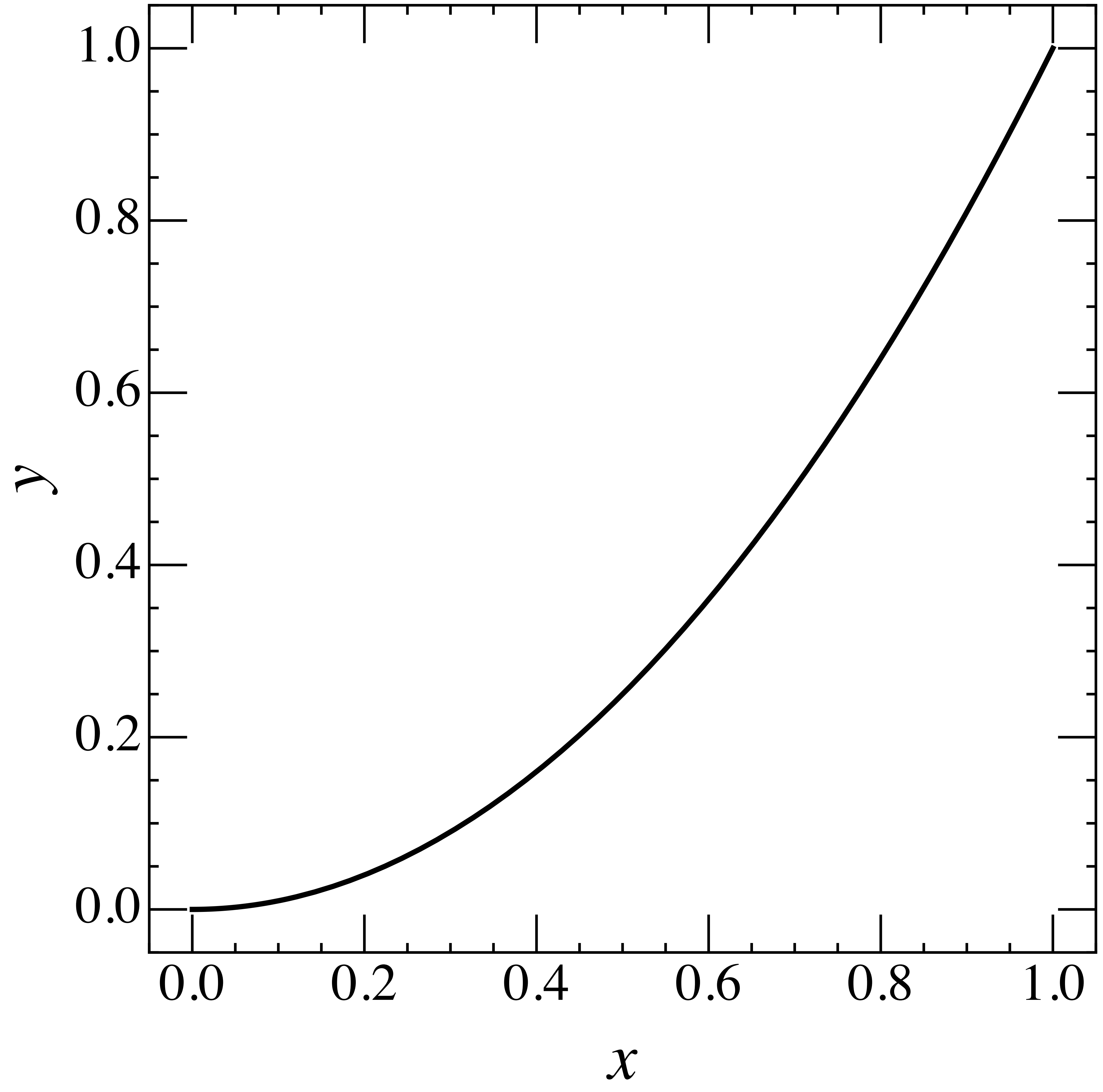
$$\nabla_{\boldsymbol{\theta}}E_{\boldsymbol{\theta},i} = \langle\psi_i\,|\,\nabla_{\boldsymbol{\theta}}V(\boldsymbol{\theta})\,|\,\psi_i\rangle$$

- general unbiased parameterization scheme? Deep Neural Network!

What are Deep Neural Networks?

--- a general parameterization scheme to approximate continuous functions.

example: approximate $y = x^2$ for $x \in [0,1]$



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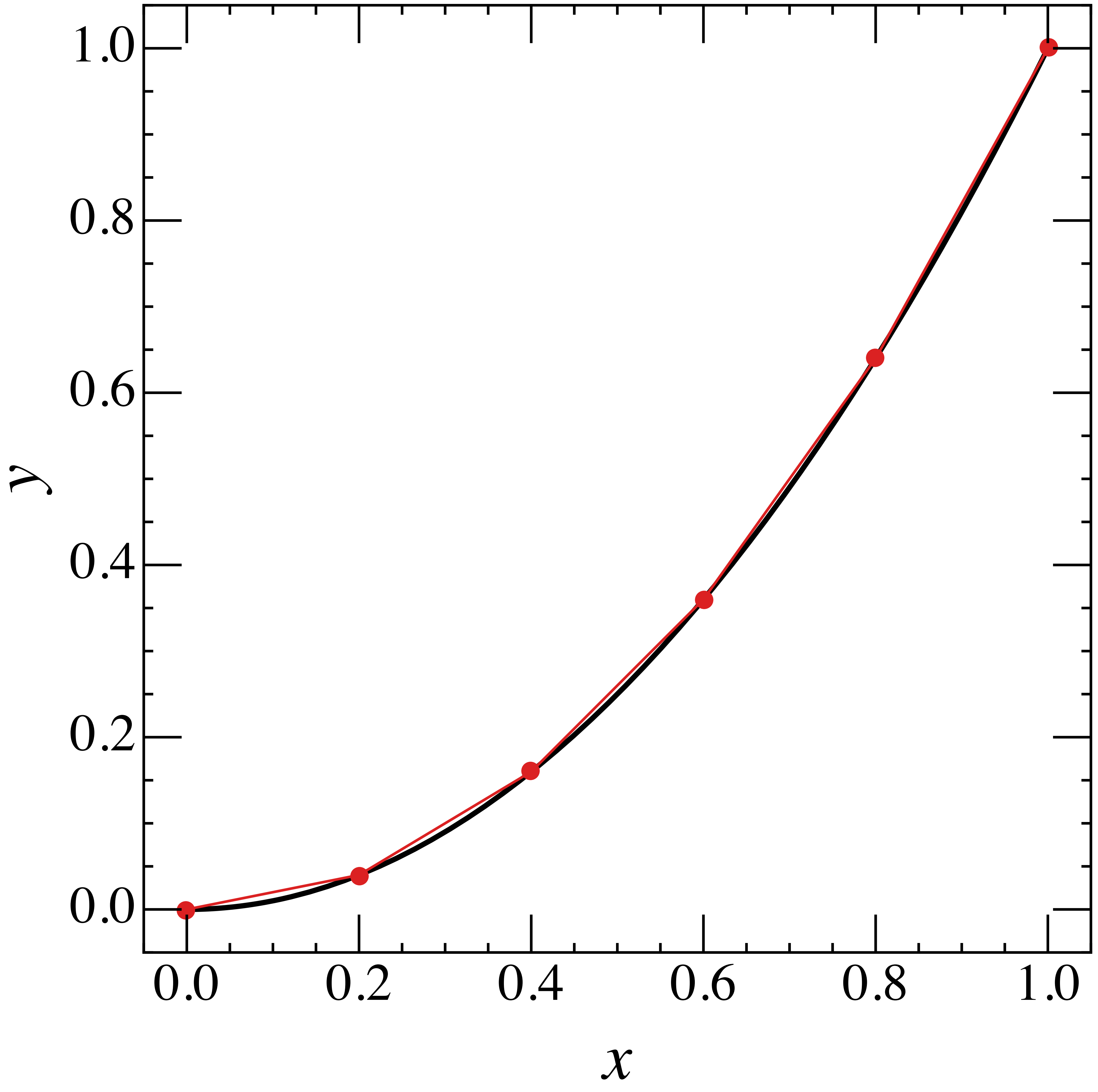

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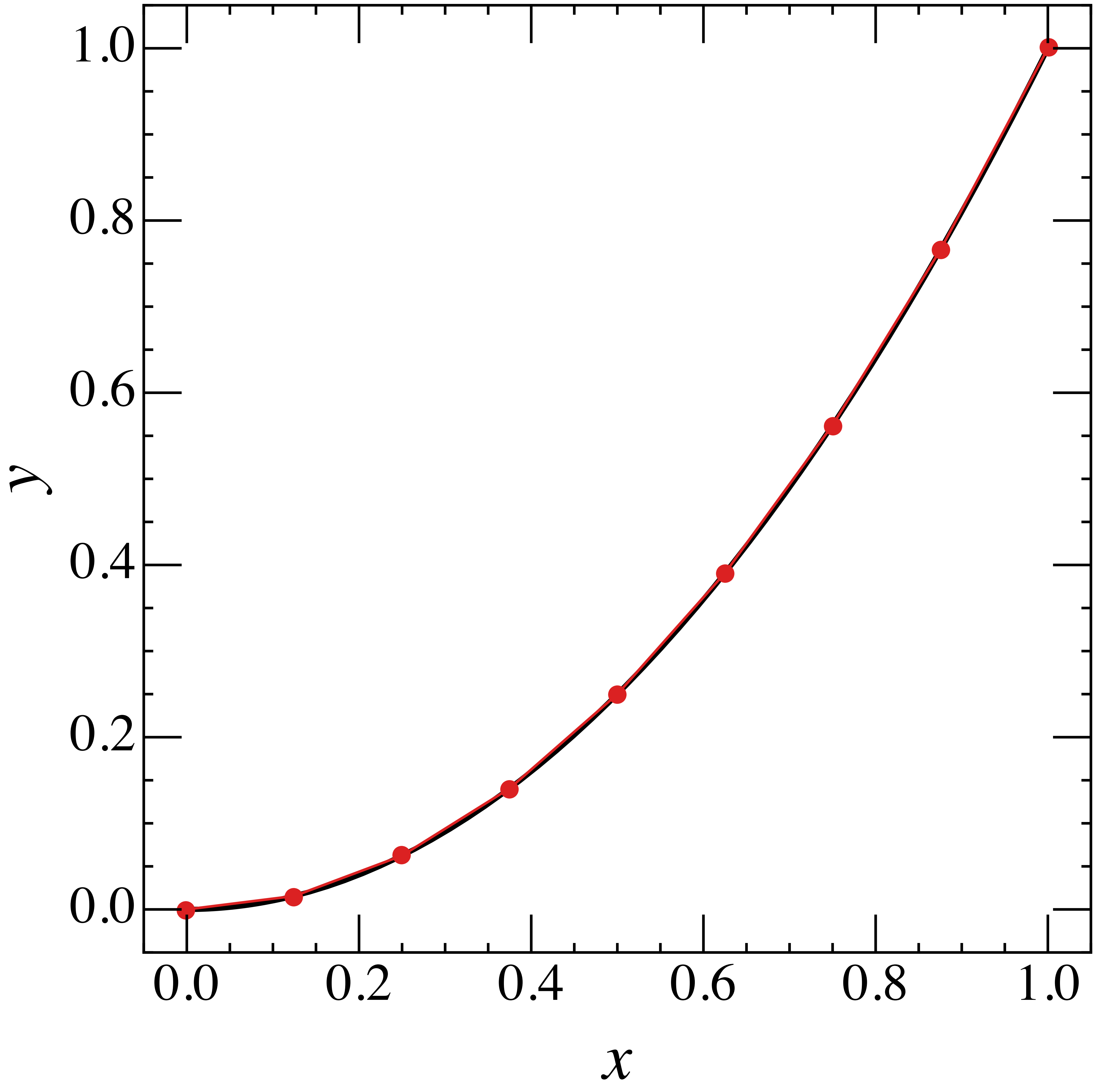

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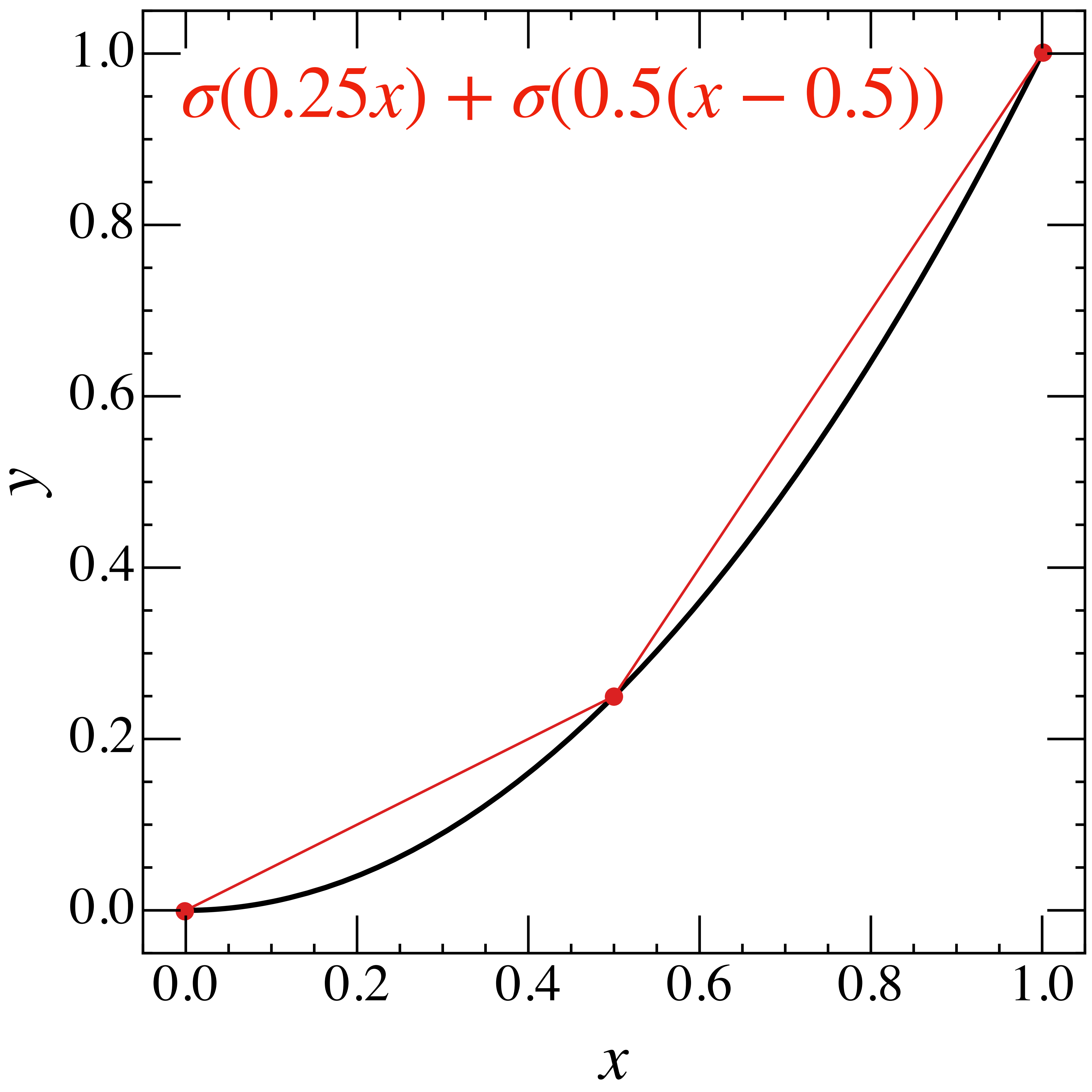
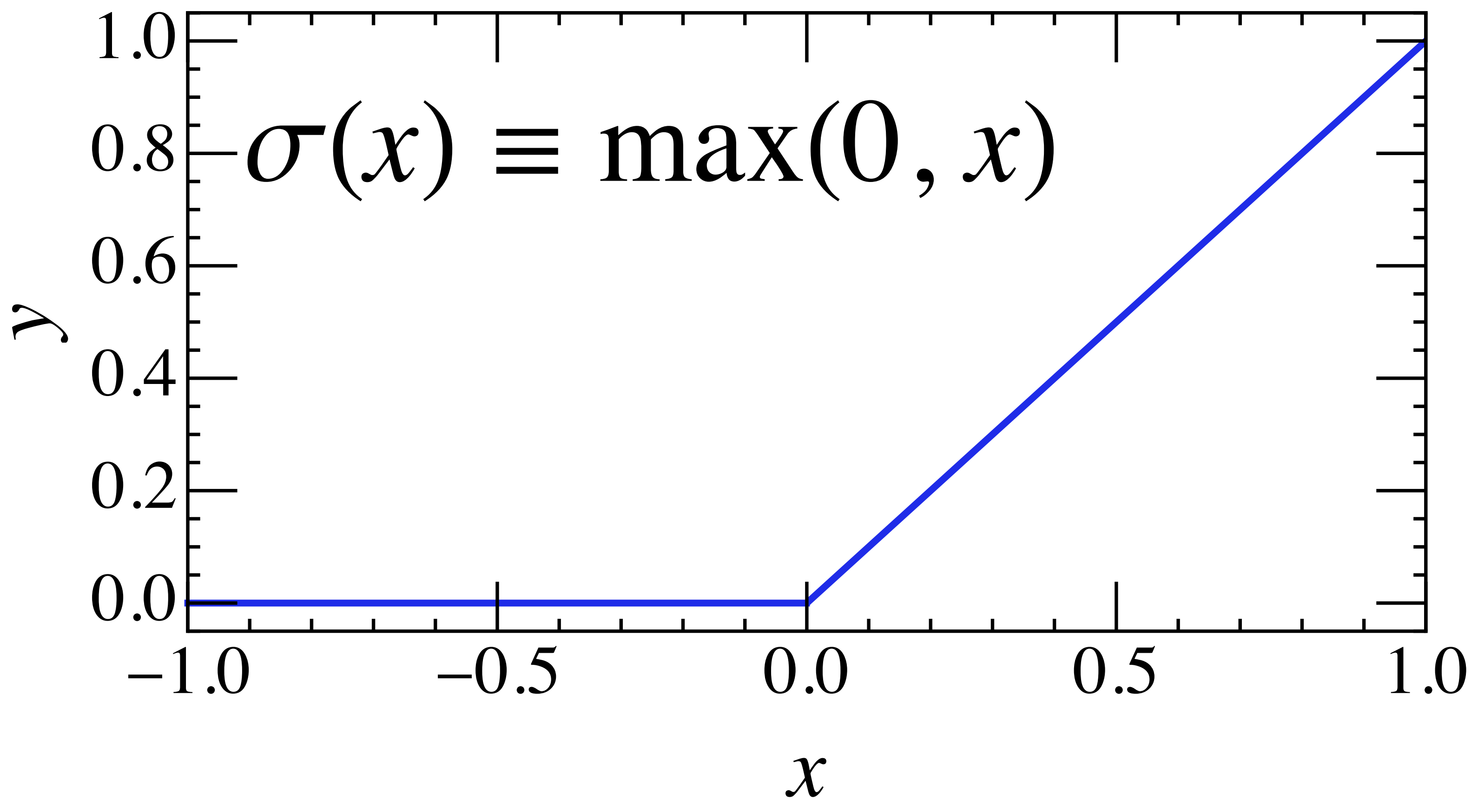


define



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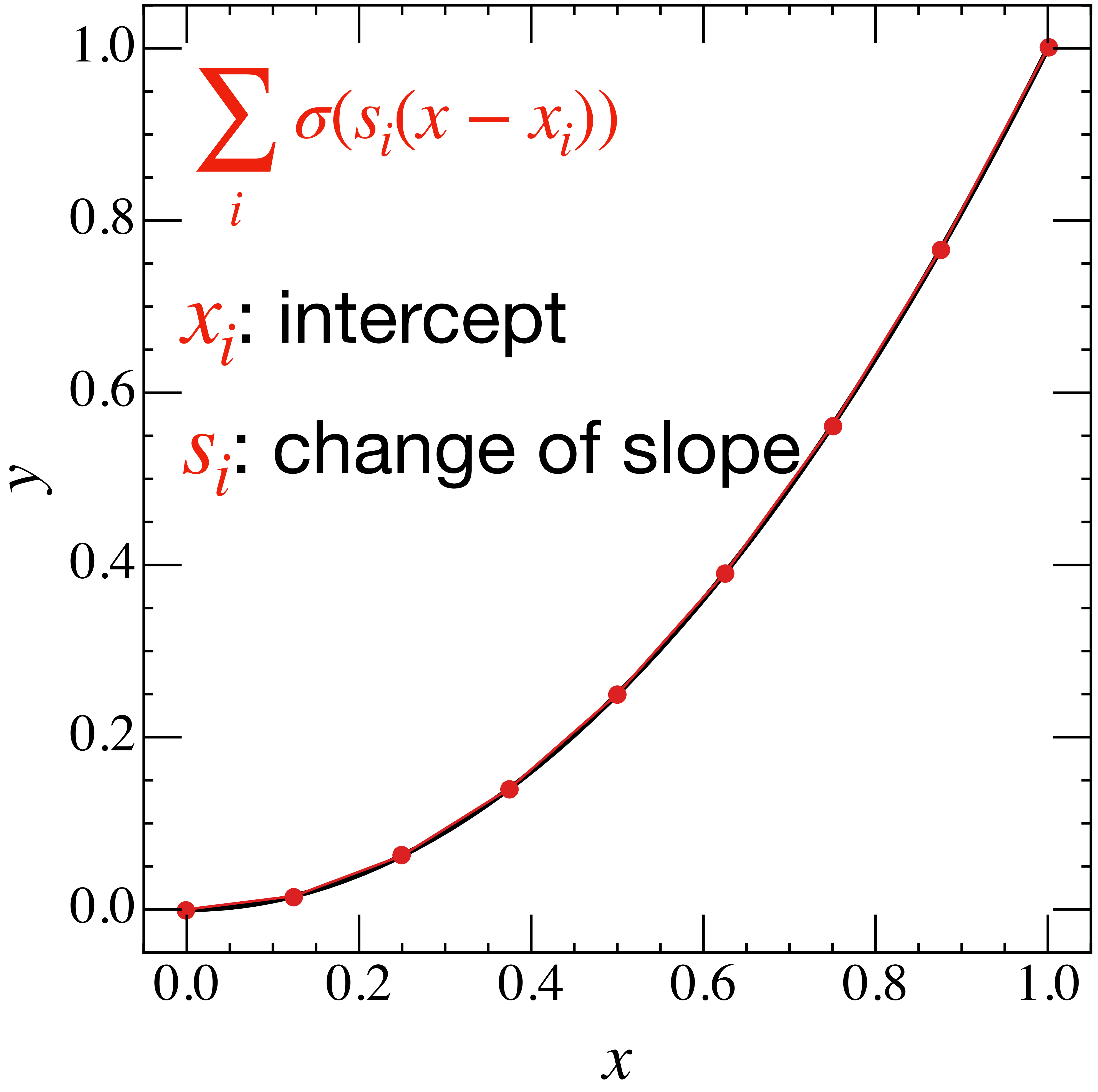

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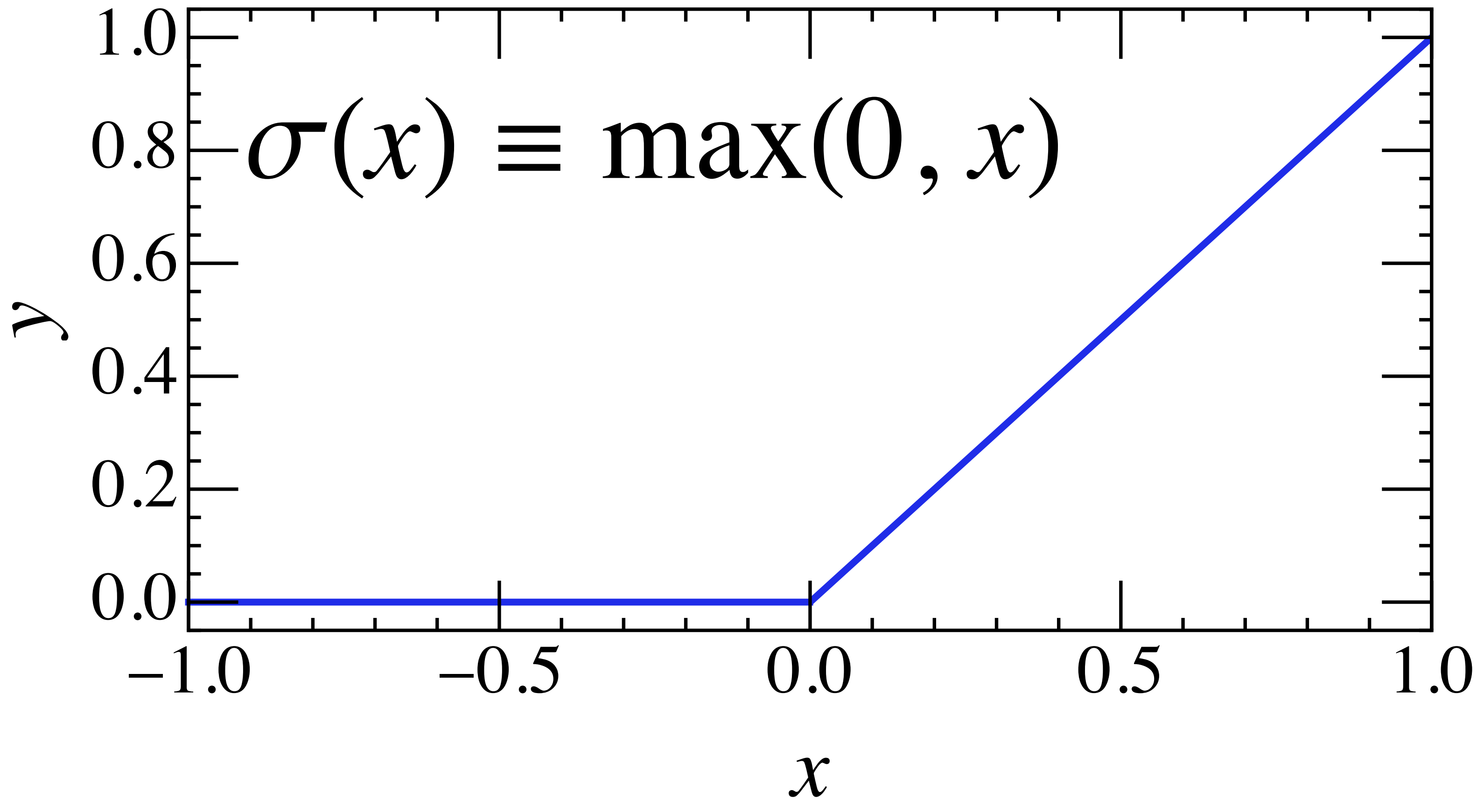

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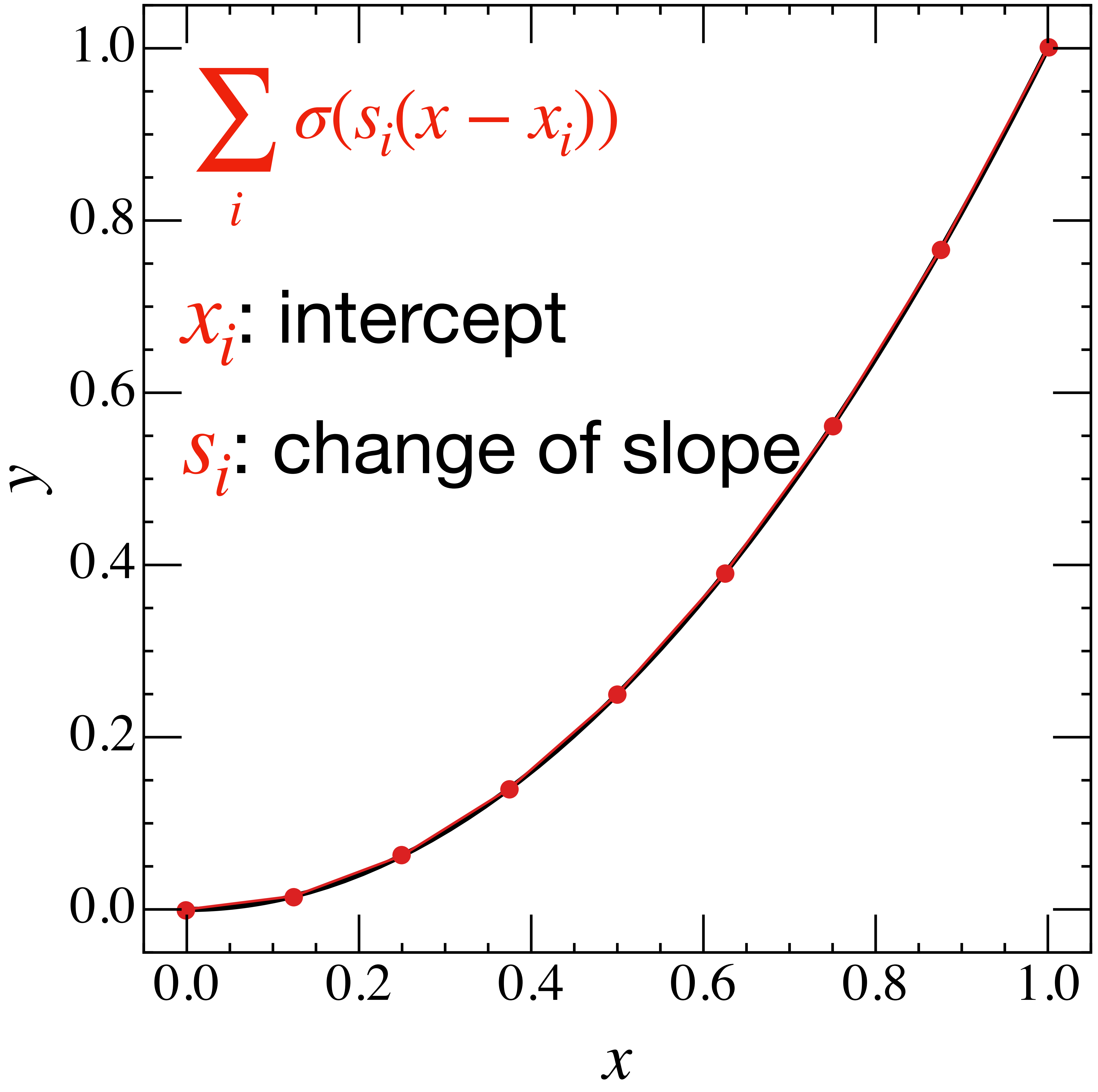
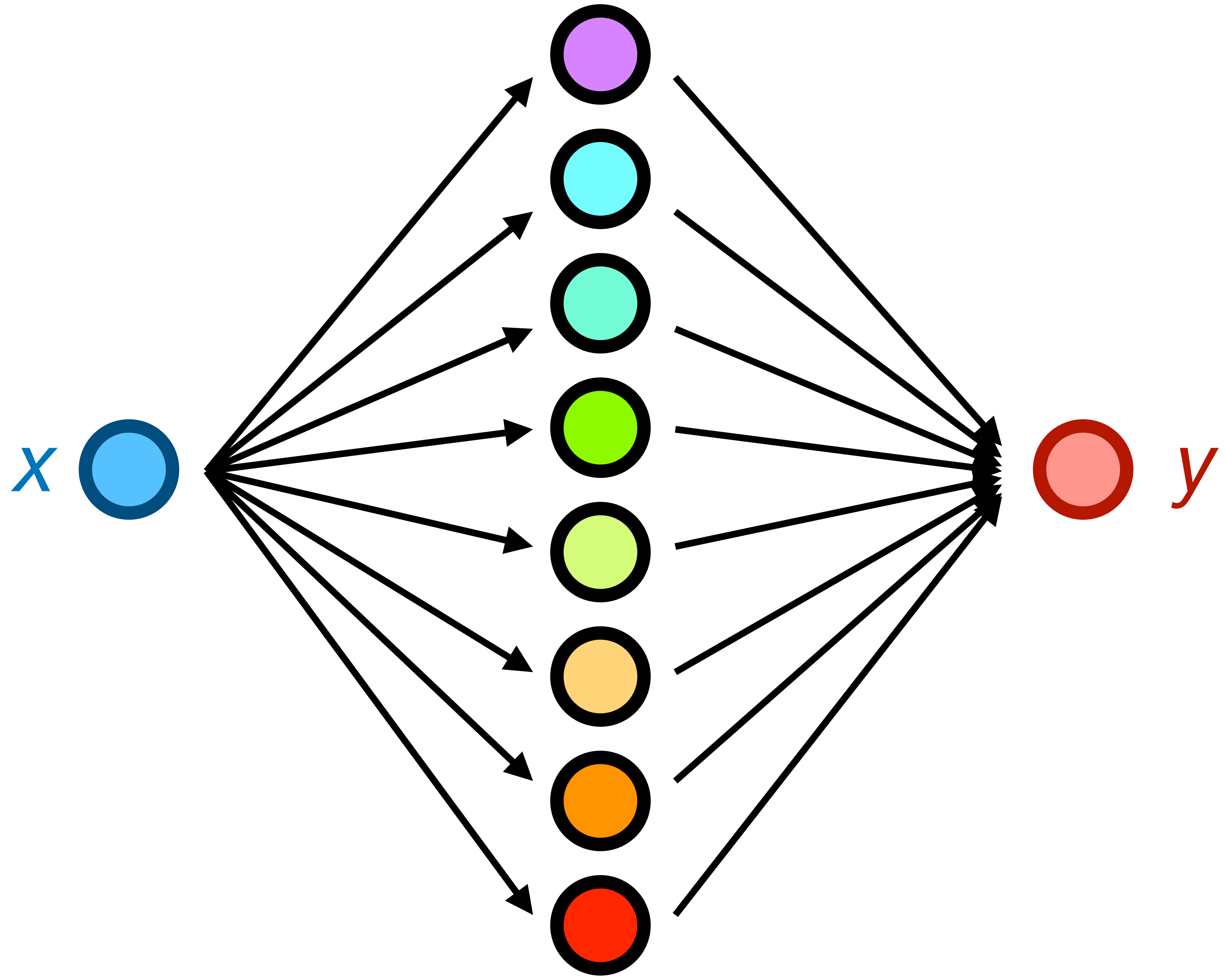




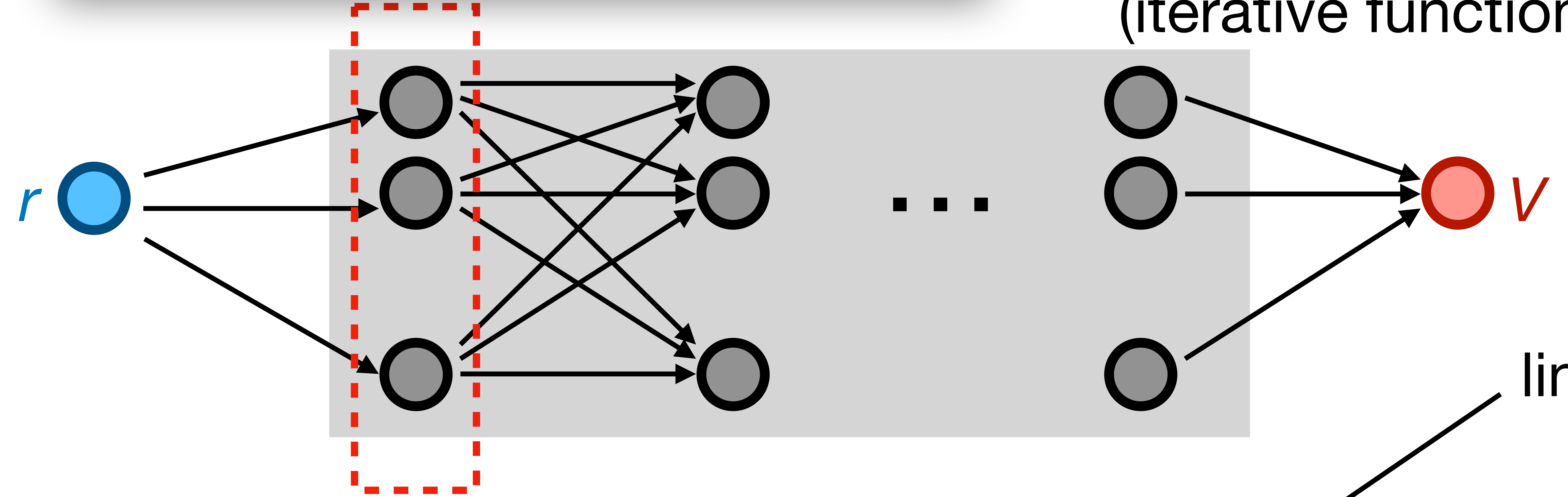
each ● represents one of $\sigma(s_i(x - x_i))$

What are Deep Neural Networks?

--- a general parameterization scheme to approximate continuous functions.

$$V(r) \approx V_{\text{DNN}}(r \,|\, \text{parameters})$$

(iterative function substitution)

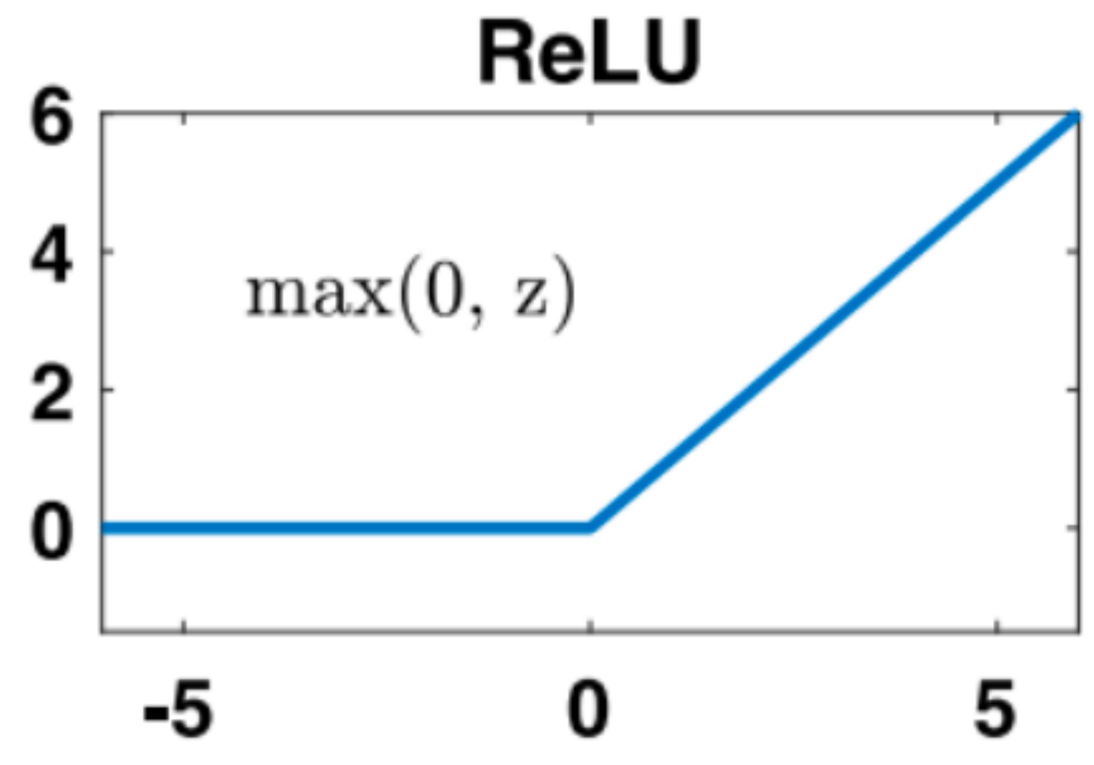


Each ● is an intermediate function ($a_i^{(l)}$):

- At the first layer: $z_i^{(1)} = b_i^{(1)} + W_{i,1}^{(1)}\, r,\qquad a_i^{(1)} = \sigma(z_i^{(1)})$

linear

nonlinear



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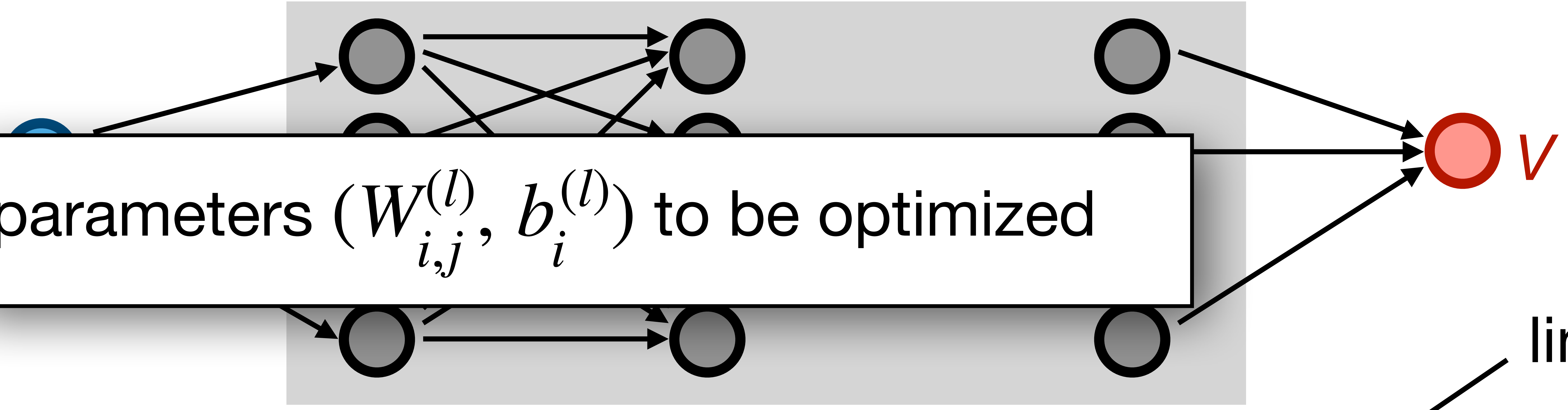


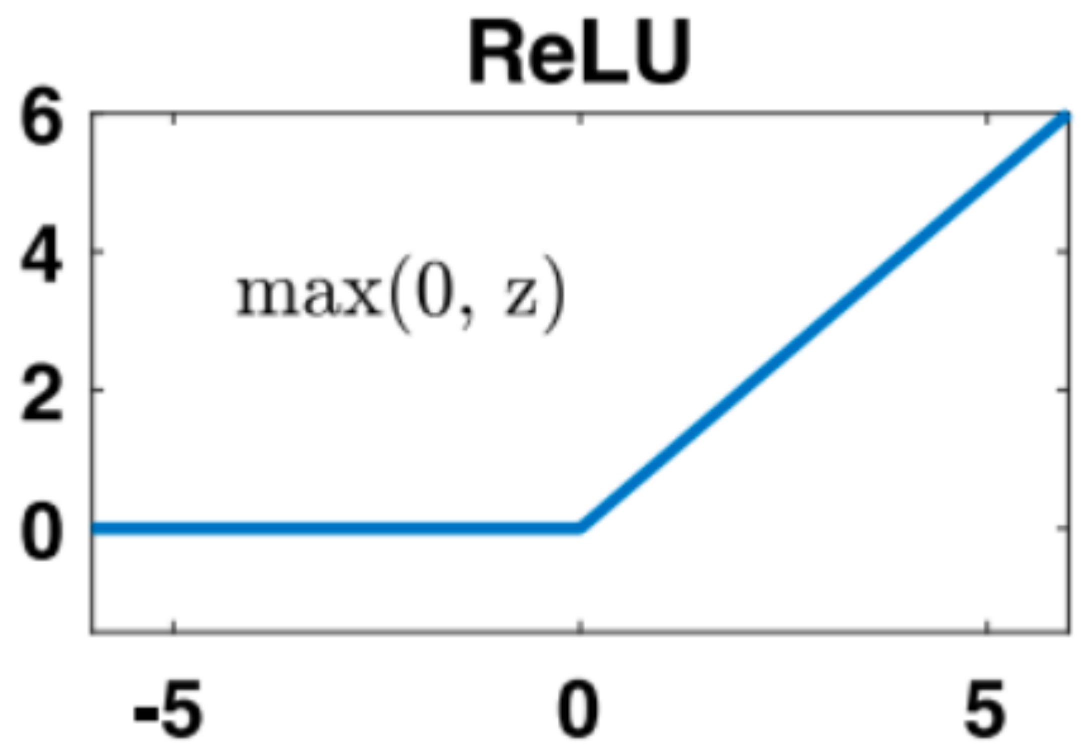
parameters $(W^{(l)}_{i,j},\, b^{(l)}_i)$ to be optimized

linear

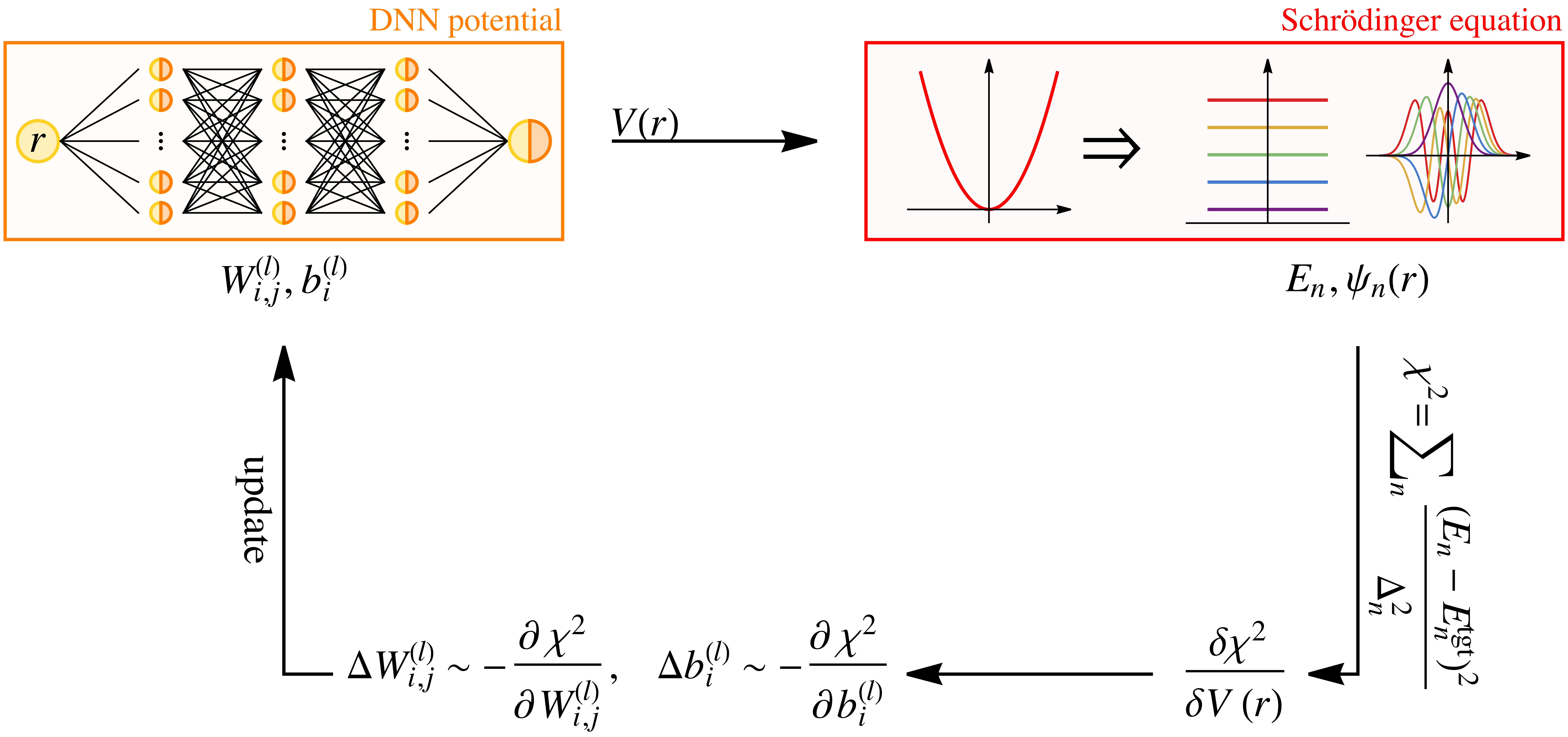
nonlinear

Each ● is an intermediate function $(a^{(l)}_i)$:

- At the first layer: $z^{(1)}_i = b^{(1)}_i + W^{(1)}_{i,1}\, r, \qquad a^{(1)}_i = \sigma(z^{(1)}_i)$
- At later layers: $z^{(l)}_i = b^{(l)}_i + \sum_j W^{(l)}_{i,j}\, a^{(l-1)}_j, \qquad a^{(l)}_i = \sigma(z^{(l)}_i)$



How to learn $V(r)$ from $\{E_n\}$ using DNN?



Can we really learn $V(r)$ from $\{E_n\}$?

continuous → $V(r)$

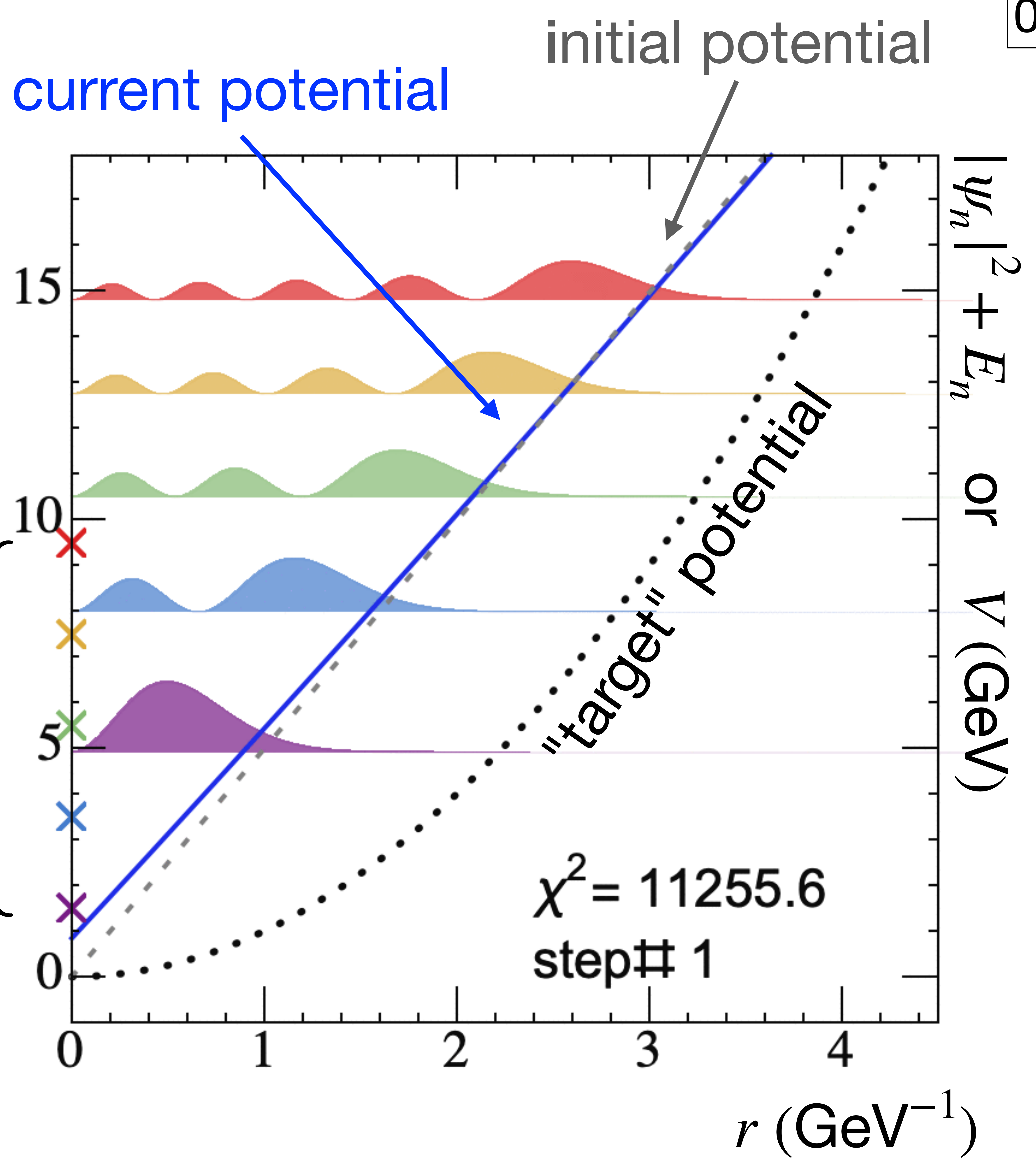
discrete, finite → $\{E_n\}$

Can we really learn $V(r)$ from $\{E_n\}$?

learn $V(r)$ according to

$$\{E_n\} = \left\{\frac{3}{2}, \frac{7}{2}, \frac{11}{2}, \frac{15}{2}, \frac{19}{2}\right\} \text{ GeV}$$

target spectrum

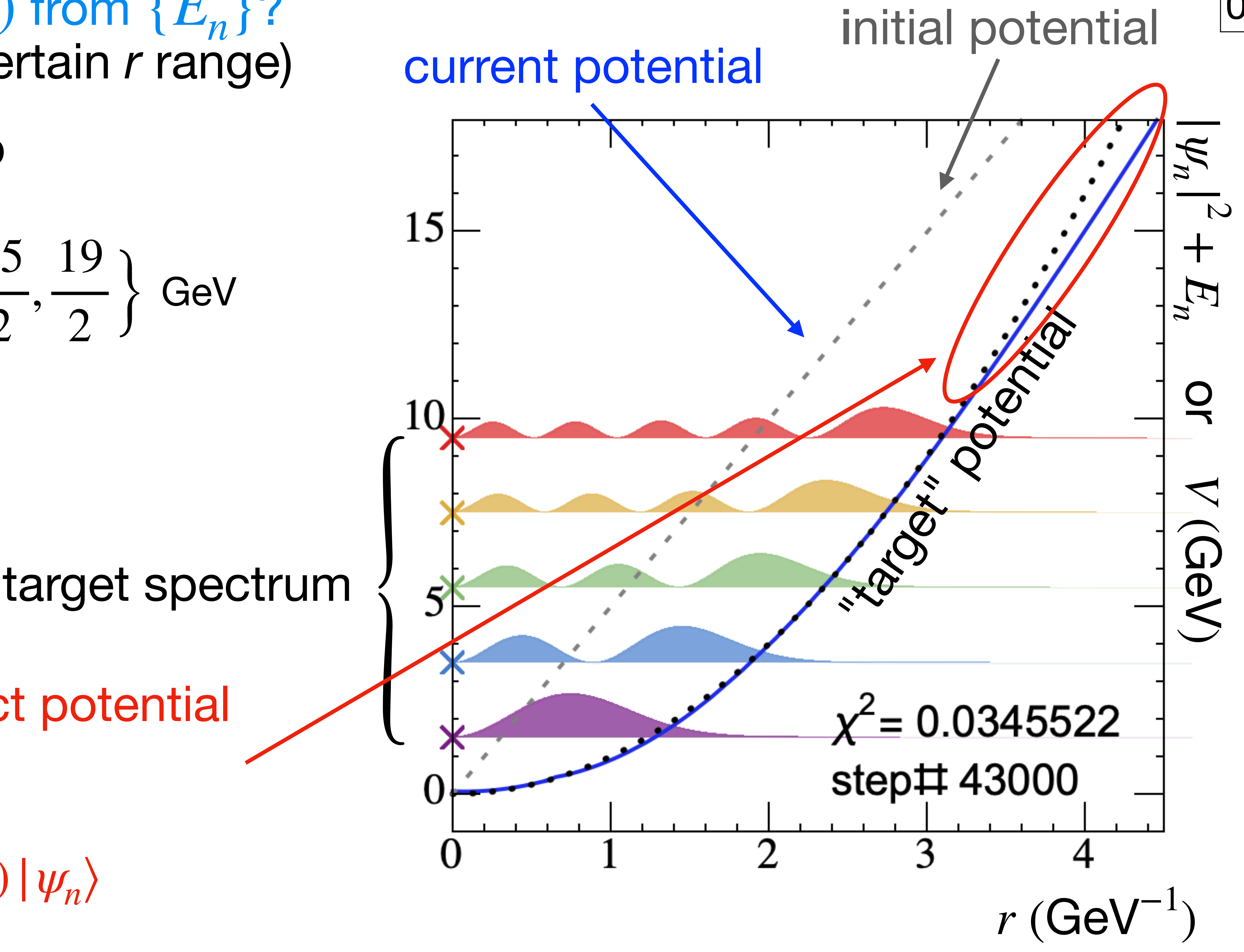


Can we really learn $V(r)$ from $\{E_n\}$?

-- Yes! (for a certain *r* range)

learn $V(r)$ according to

$$\{E_n\} = \left\{\frac{3}{2}, \frac{7}{2}, \frac{11}{2}, \frac{15}{2}, \frac{19}{2}\right\} \text{ GeV}$$



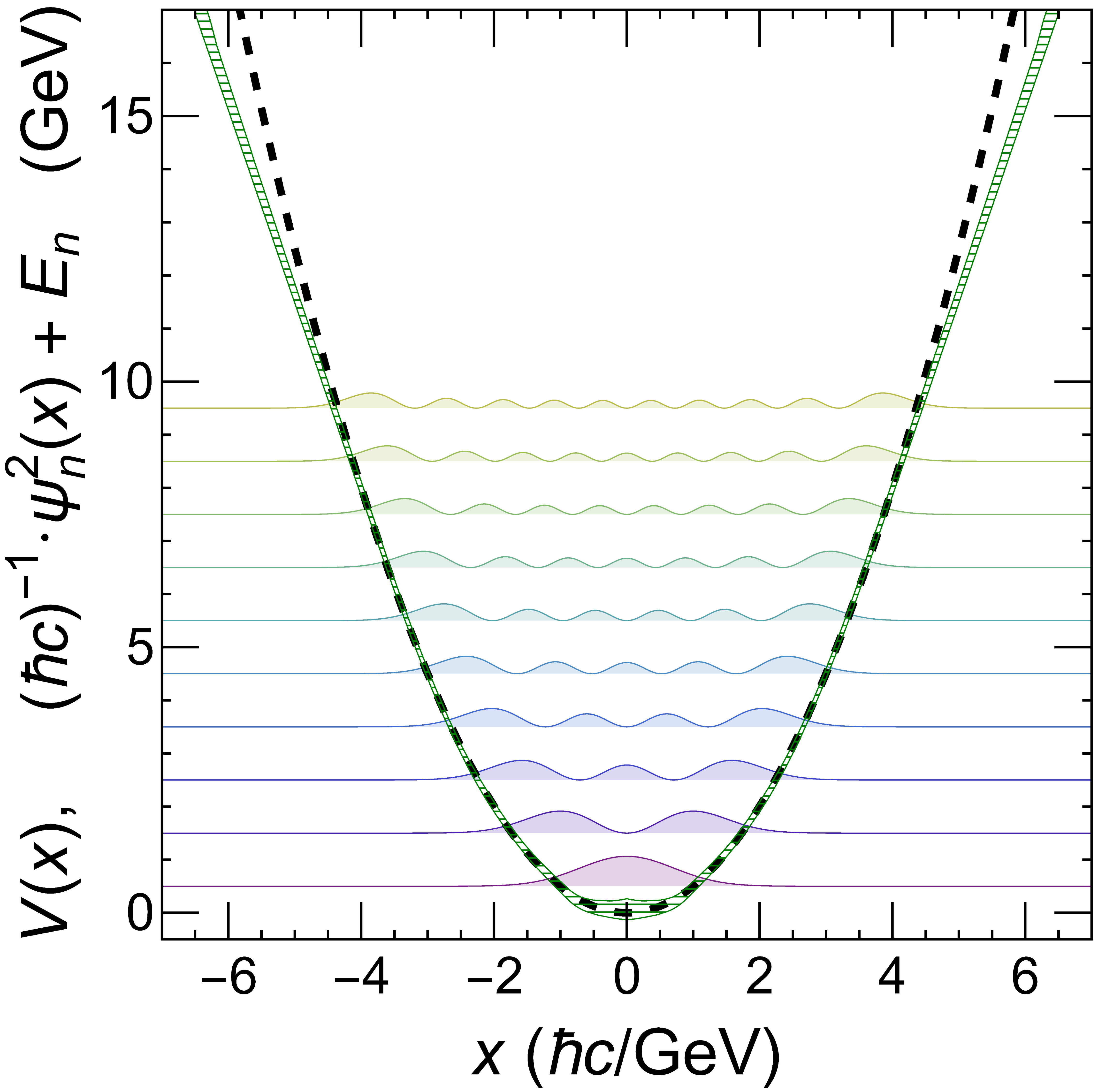
Deviate from the exact potential

where all $\psi_n \to 0$,

$$\delta E_n = \langle \psi_n | \delta V(r) | \psi_n \rangle$$

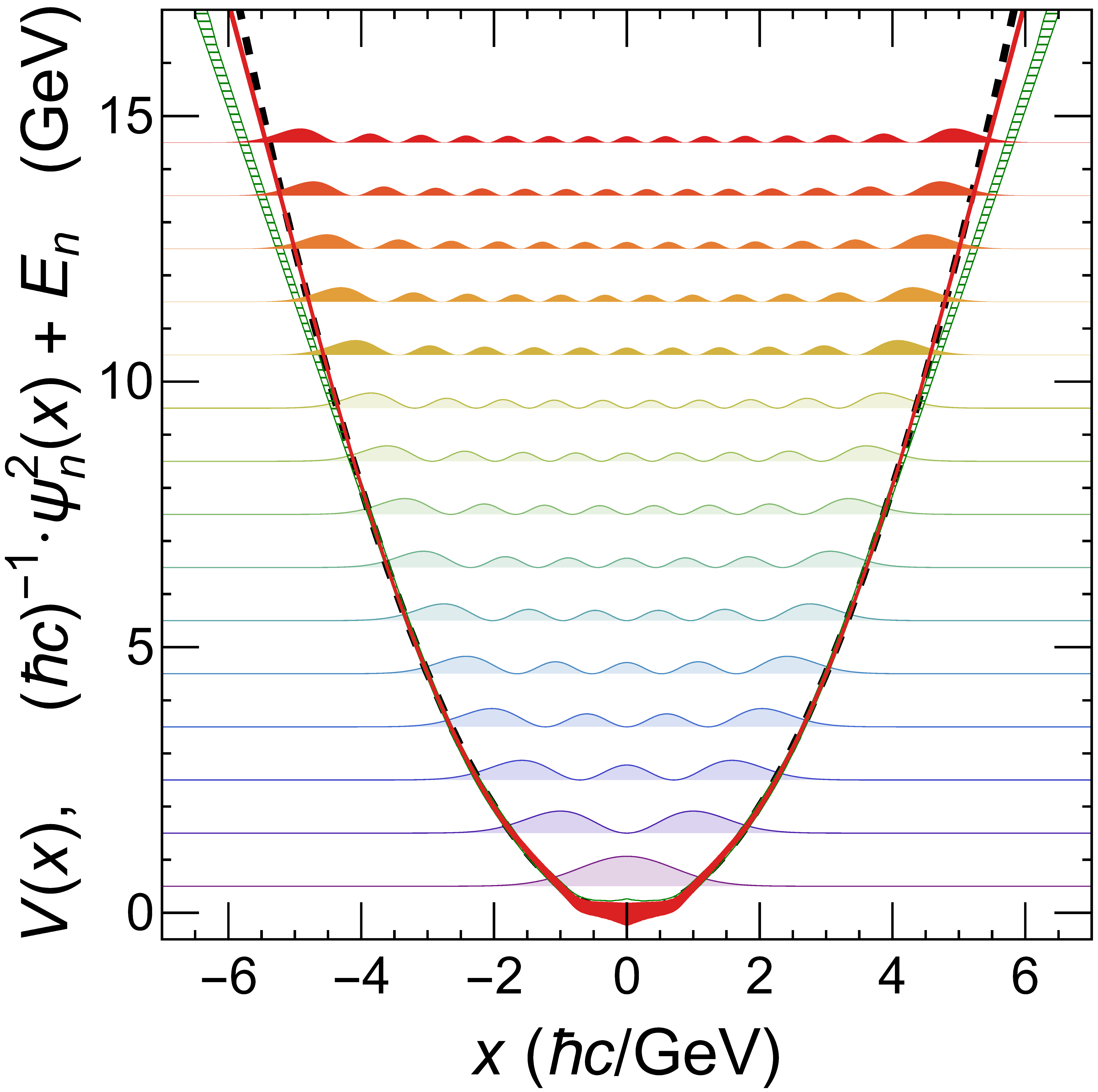
Can we really learn $V(r)$ from $\{E_n\}$?

$$\{E_n\} = \{\ 0.5,\ 1.5,\ 2.5,\ 3.5,\ 4.5,\ 5.5,\ 6.5,\ 7.5,\ 8.5,\ 9.5\}\ \text{GeV}$$



Can we really learn $V(r)$ from $\{E_n\}$?

$$\{E_n\} = \{\ 0.5,\ 1.5,\ 2.5,\ 3.5,\ 4.5,\ 5.5,\ 6.5,\ 7.5,\ 8.5,\ 9.5,\ 10.5,\ 11.5,\ 12.5,\ 13.5,\ 14.5\}\ \text{GeV}$$



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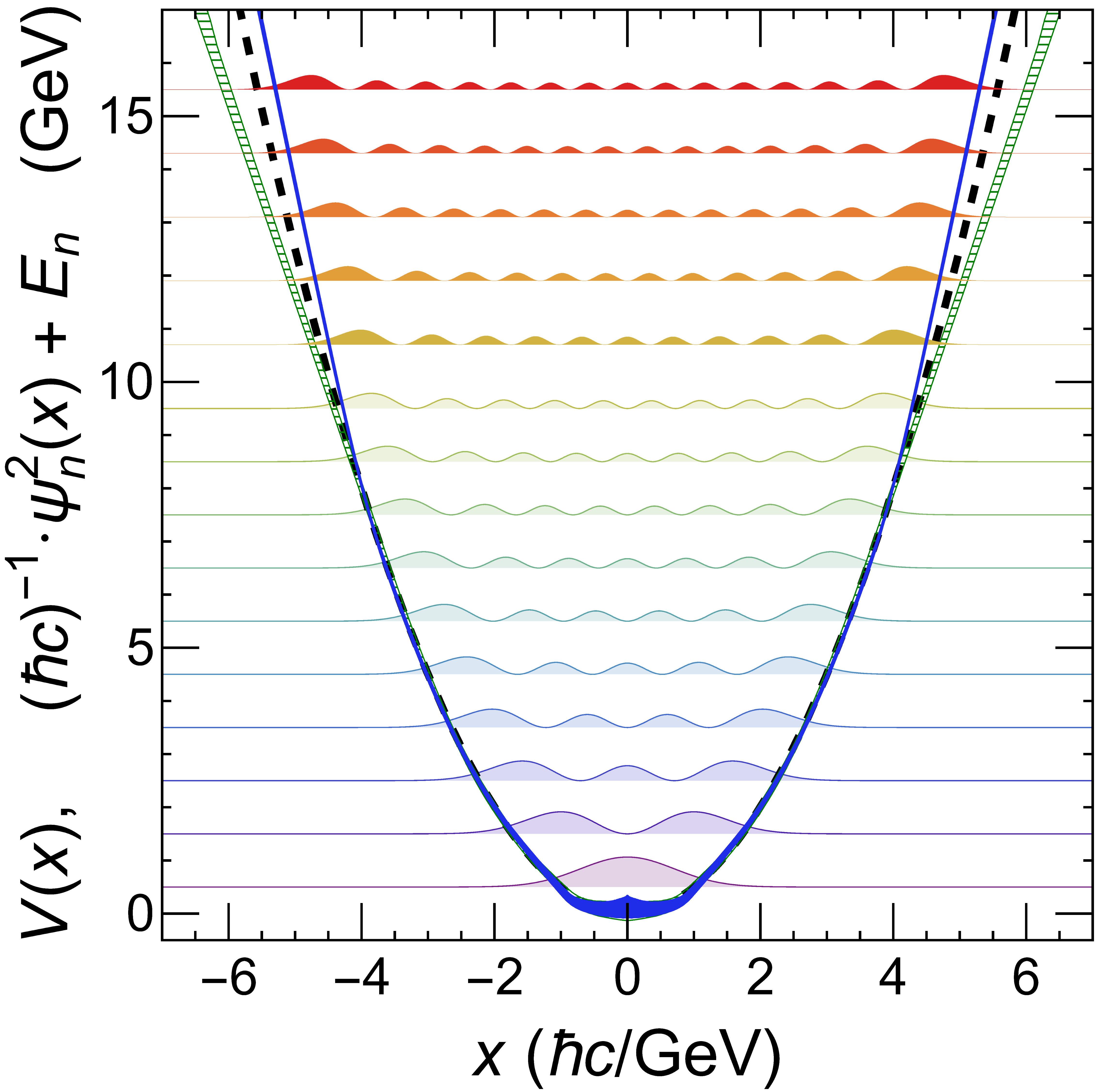
$$\{E_n\} = \{\ 0.5,\ 1.5,\ 2.5,\ 3.5,\ 4.5,\ 5.5,\ 6.5,\ 7.5,\ 8.5,\ 9.5,\ 10.6,\ 11.7,\ 12.8,\ 13.9,\ 15.0\}\ \text{GeV}$$



Application: Heavy Flavor Potential

Quarkonium in the QGP

- In heavy-ion collisions, quarkonium production serves as a probe of the QGP.
- Accurate understanding of the in-medium heavy-quark interaction?
    - Real potential modified by color-screening
    - Imaginary potential arises due to $(Q\bar{Q})_1 \to (Q\bar{Q})_8$, Landau damping, ...

Hard Thermal Loop potentials

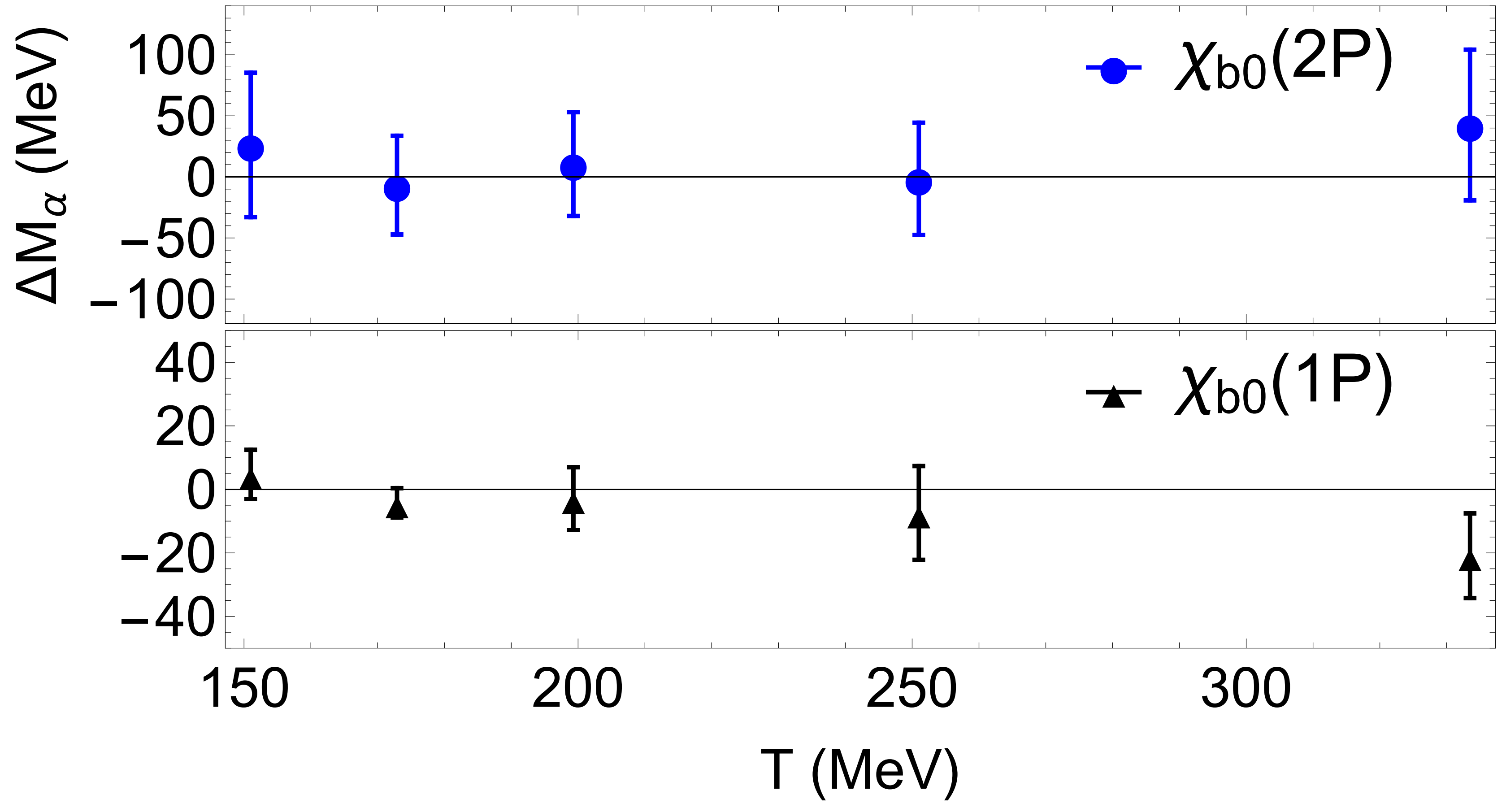
$$V_R(T,r) = \frac{\sigma}{\mu_D}\left(2 - (2+\mu_D r)e^{-\mu_D r}\right) - \alpha\left(\mu_D + \frac{e^{-\mu_D r}}{r}\right) + B\,,$$

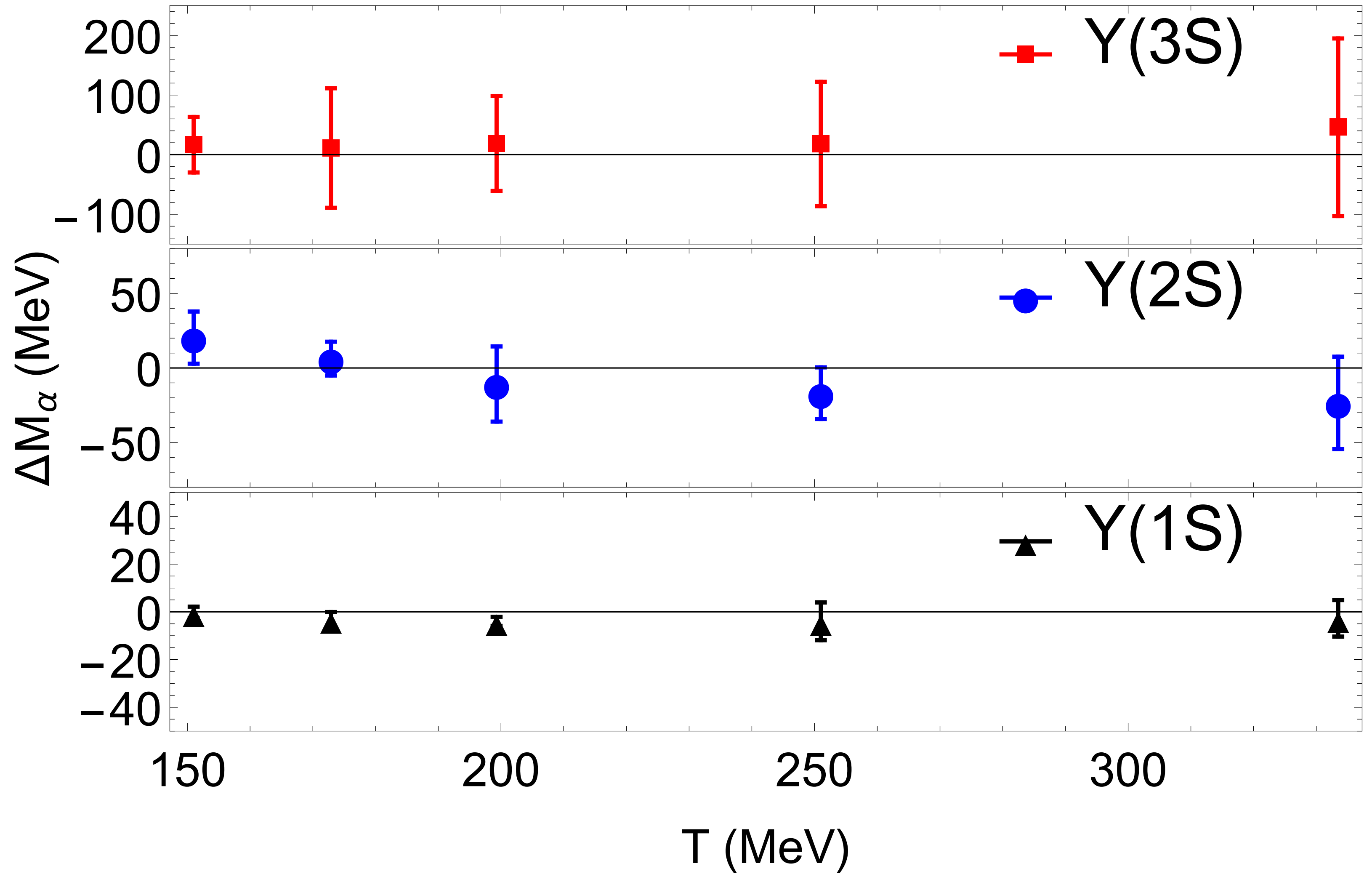
$$V_I(T,r) = -\frac{\sqrt{\pi}}{4}\mu_D T\sigma r^3 G^{2,2}_{2,4}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{2}\\ \frac{1}{2},\frac{1}{2},-\frac{3}{2},-1\end{matrix}\,\middle|\,\frac{\mu_D^2 r^2}{4}\right) - \alpha T\phi(\mu_D r)\,.$$

see e.g., Laine, Philipsen, Romatschke, and Tassler, JHEP 03, 054 (2007)

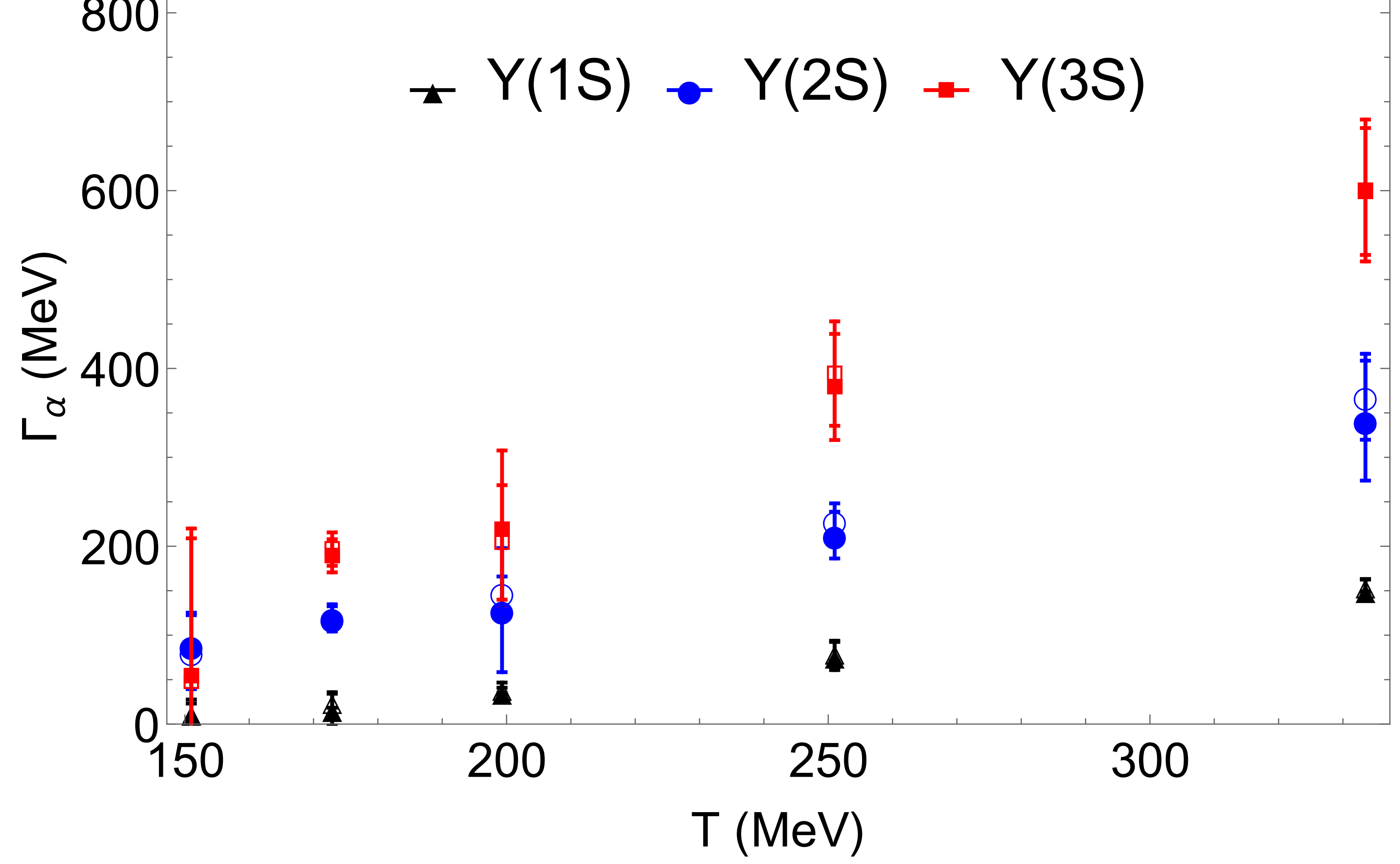
Bottomonium mass and thermal width, lattice QCD with finite m_Q

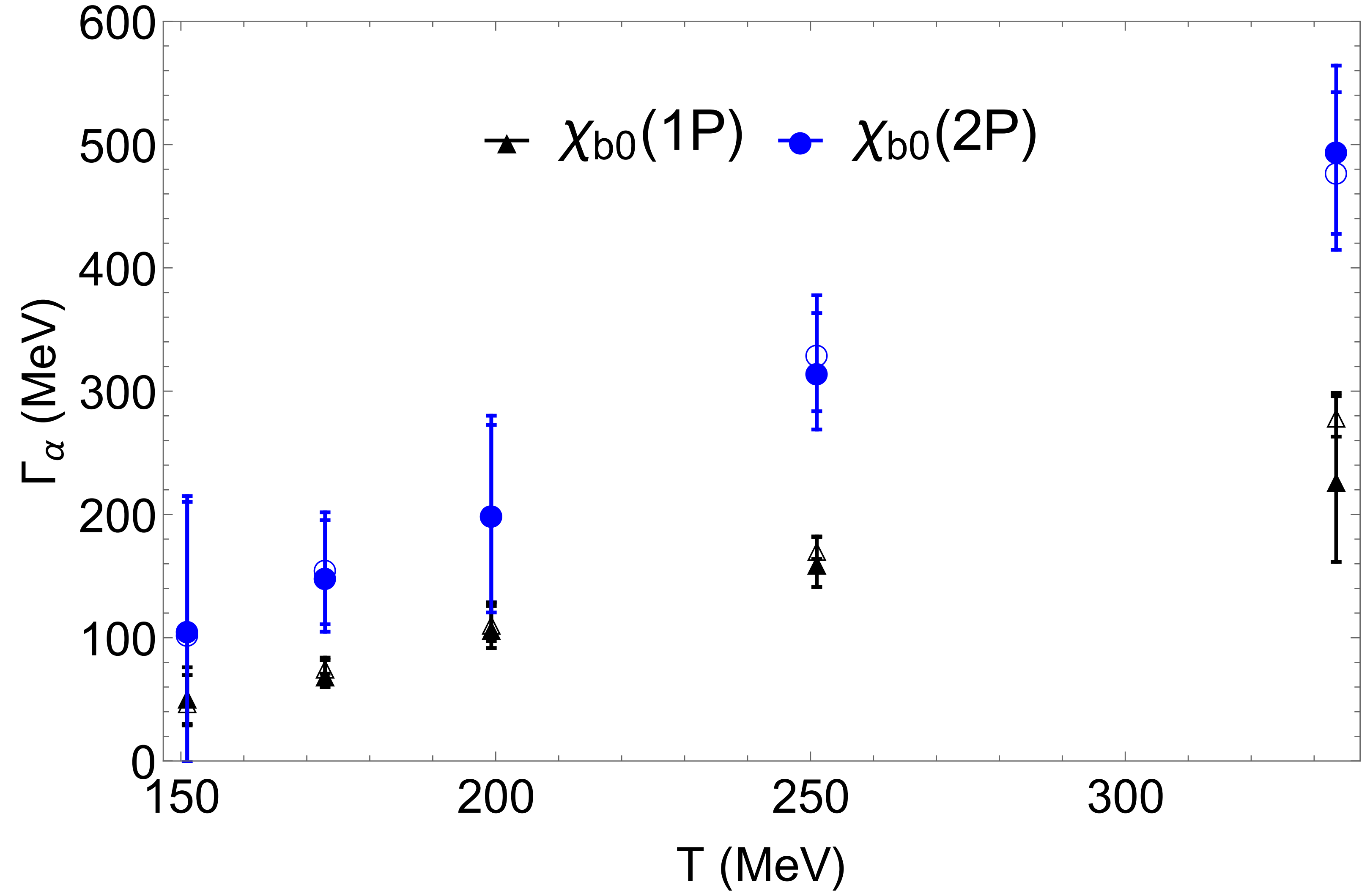
Mass





Thermal Width

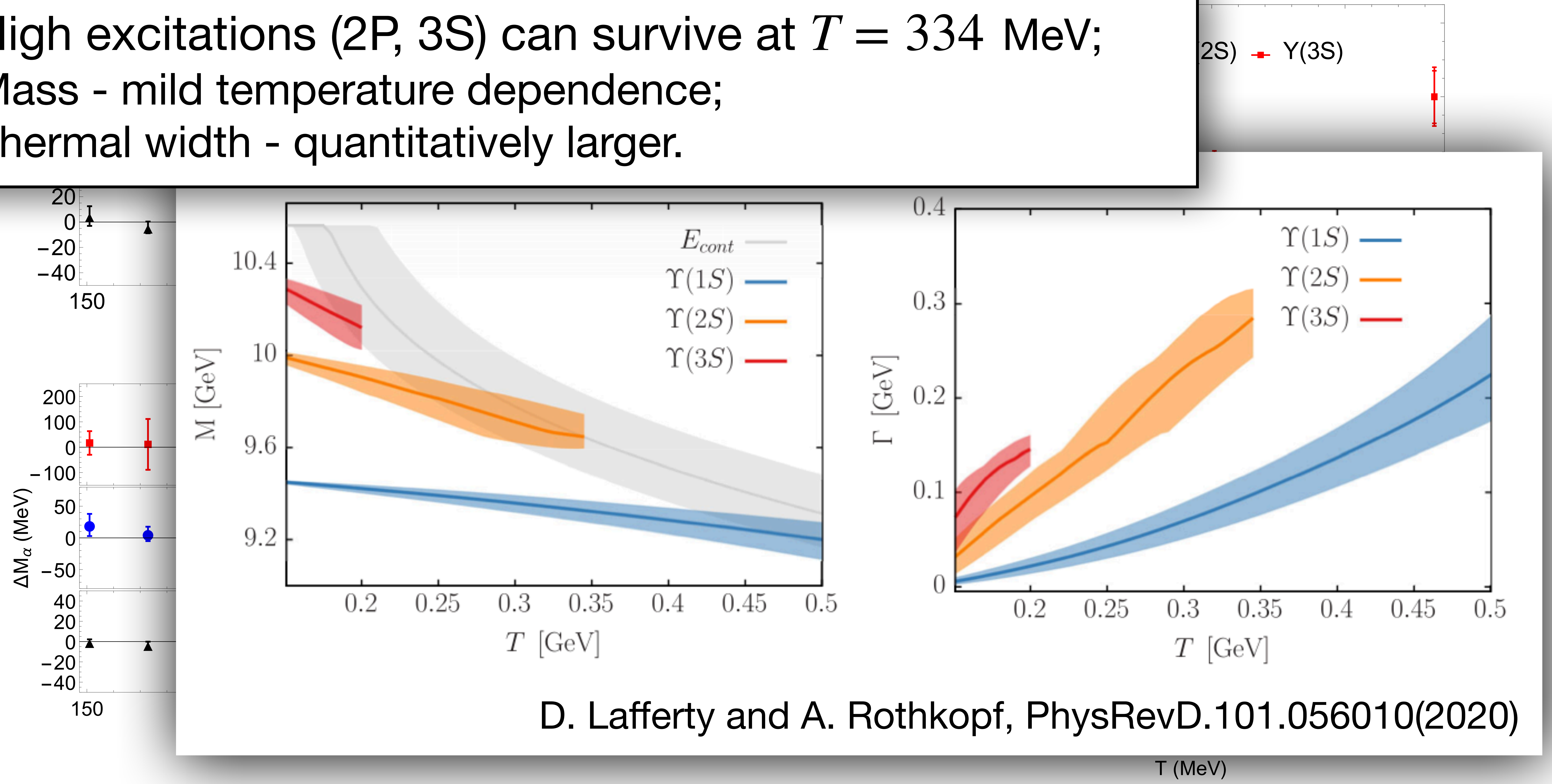




R. Larsen, S. Meinel, S. Mukherjee, and P. Petreczky:
Phys.Rev.D100,074506(2019), Phys.Lett.B800,135119(2020), Phys.Rev.D102,114508(2020)

Bottomonium mass and thermal width, lattice QCD with finite m_Q

High excitations (2P, 3S) can survive at $T = 334$ MeV;
Mass - mild temperature dependence;
Thermal width - quantitatively larger.



D. Lafferty and A. Rothkopf, PhysRevD.101.056010(2020)

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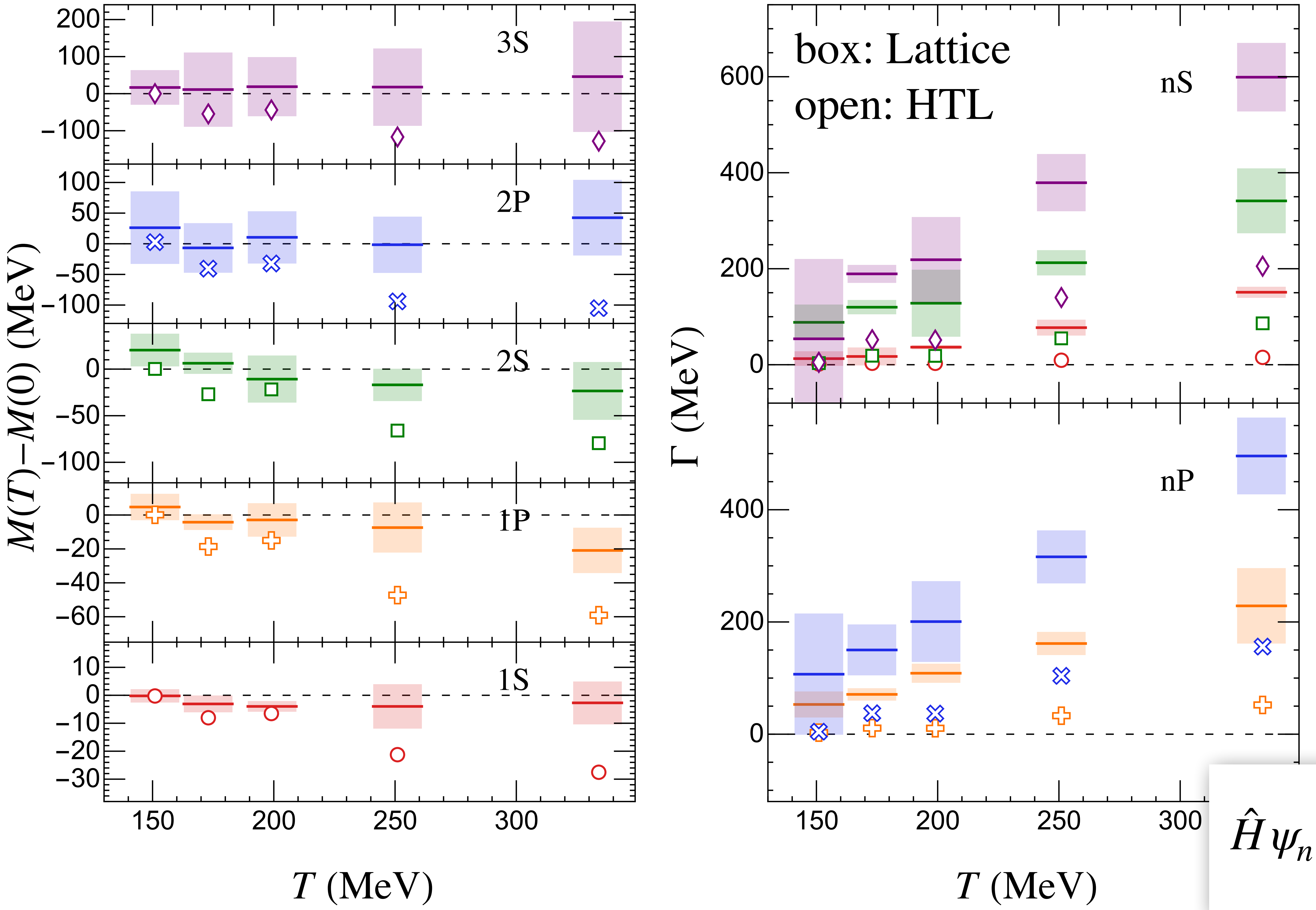
Can we understand the new lattice result using Hard Thermal Loop potential?

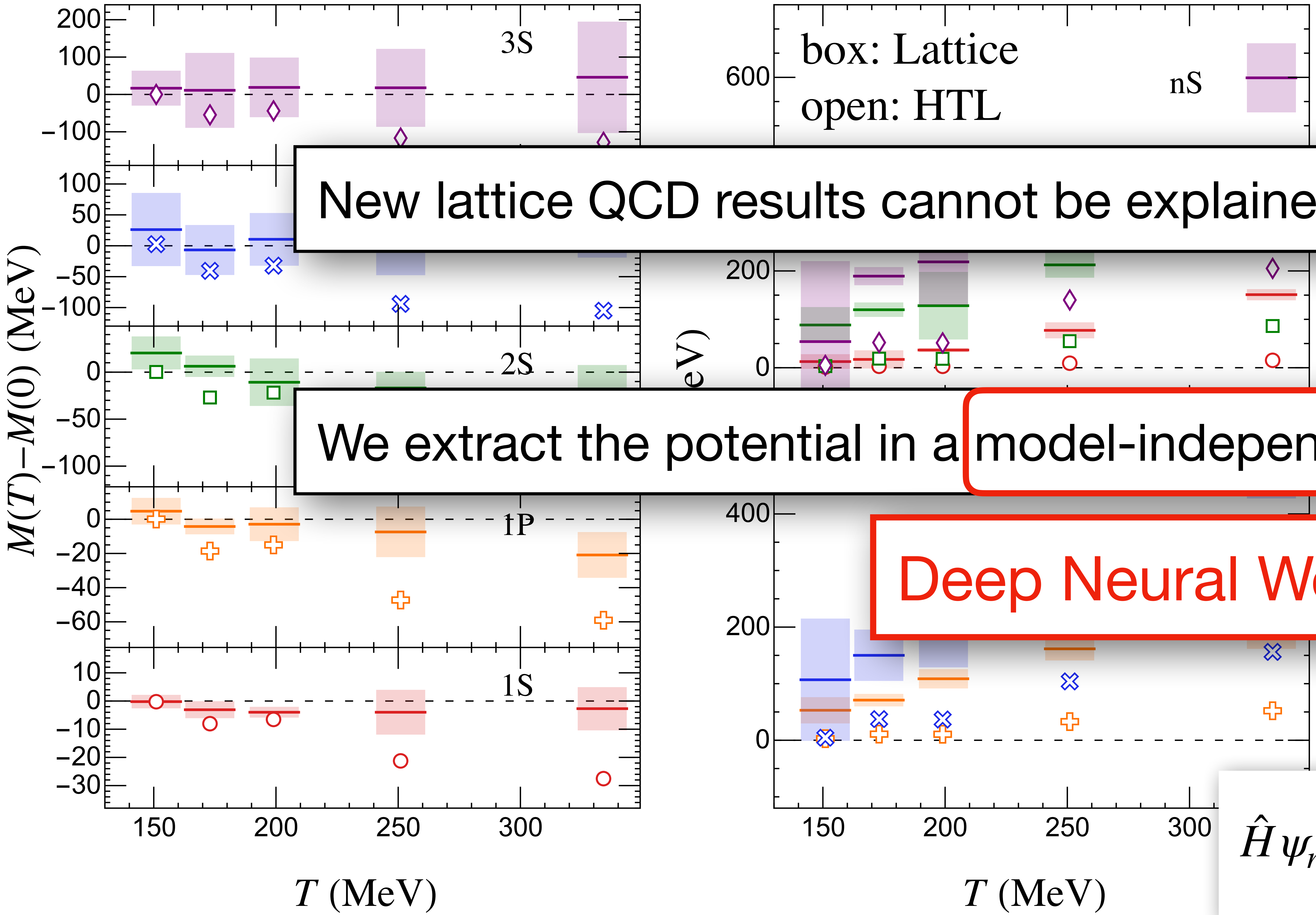


$m_D(T)$ fitted by LQCD

$$V(T,r) = V_R(T,r) + i\, V_I(T,r)$$

$$\hat{H}\psi_n = -\frac{\nabla^2}{2m_\mu}\psi_n + V(r)\,\psi_n = E_n\psi_n$$

Can we understand the new lattice result using Hard Thermal Loop potential?



New lattice QCD results cannot be explained by the HTL potential

We extract the potential in a model-independent way

Deep Neural Works (DNN)

$$V(T,r) = V_R(T,r) + i\, V_I(T,r)$$

$$\hat{H}\psi_n = -\frac{\nabla^2}{2m_\mu}\psi_n + V(r)\,\psi_n = E_n\psi_n$$

How to learn potential using DNN?

$$V(T,r) = V_R(T,r) + i\,V_I(T,r)$$

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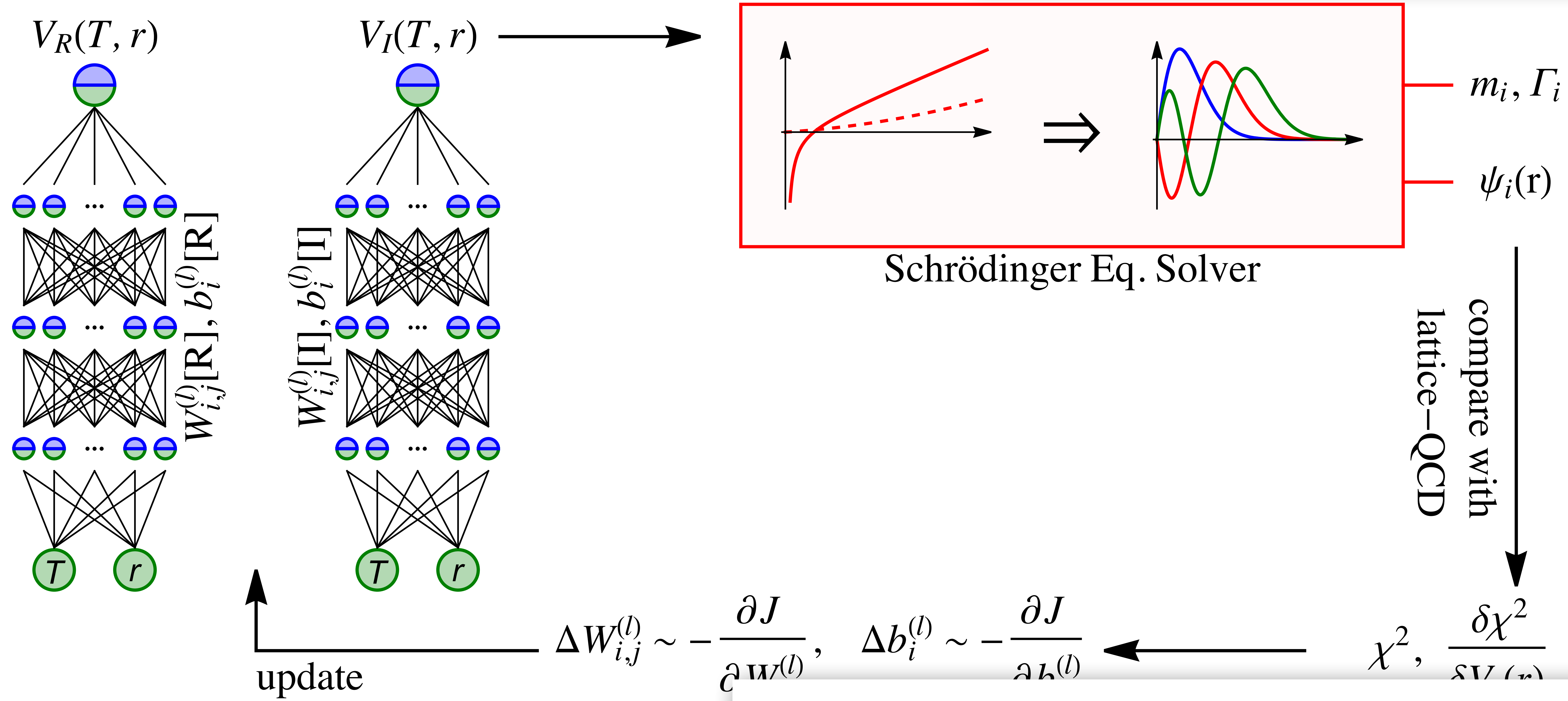

$$\chi^2 = \sum_{T,i}\left(\frac{m_{T,i} - m^{\text{lattice}}_{T,i}}{\delta m^{\text{lattice}}_{T,i}}\right)^2 + \left(\frac{\Gamma_{T,i} - \Gamma^{\text{lattice}}_{T,i}}{\delta\Gamma^{\text{lattice}}_{T,i}}\right)^2,$$

Test - Can we recover a known complex $V(T, r)$? -- Yes!

- Start with a known potential (solid line)

$$V_R(T,r) = \frac{\sigma}{\mu_D}\left(2 - (2+\mu_D r)e^{-\mu_D r}\right) - \alpha\left(\mu_D + \frac{e^{-\mu_D r}}{r}\right) + B\,,$$

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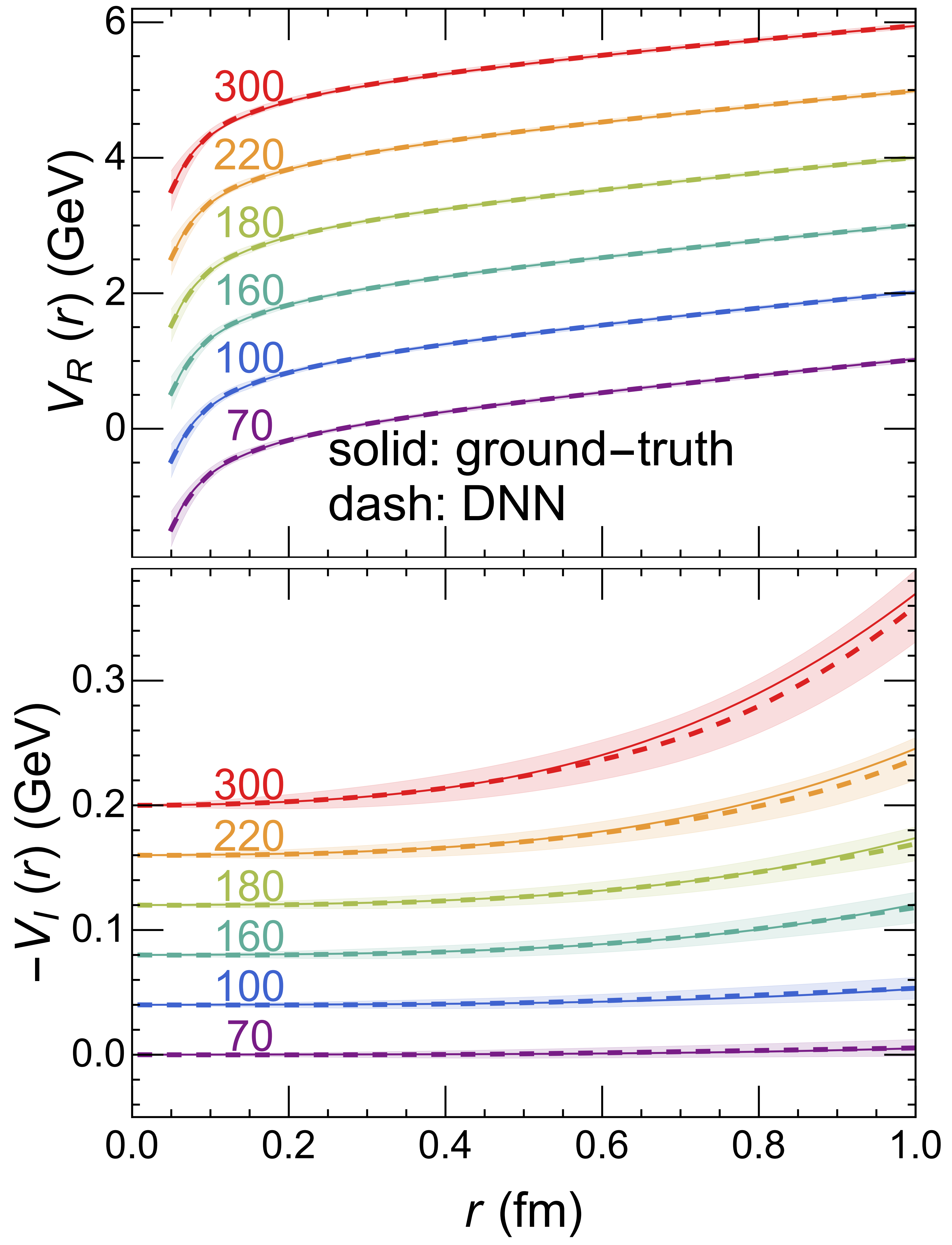
- Compute $\{m_n\,,\,\Gamma_n\}$ at

$$T = \{0, 151, 173\,, 199, 251, 334\}\ \text{MeV}$$

- Learn the potential using DNN (dash + band)



Test - Can we recover a known complex $V(T,r)$?   extrapolation is risky!

- Start with a known potential (solid line)

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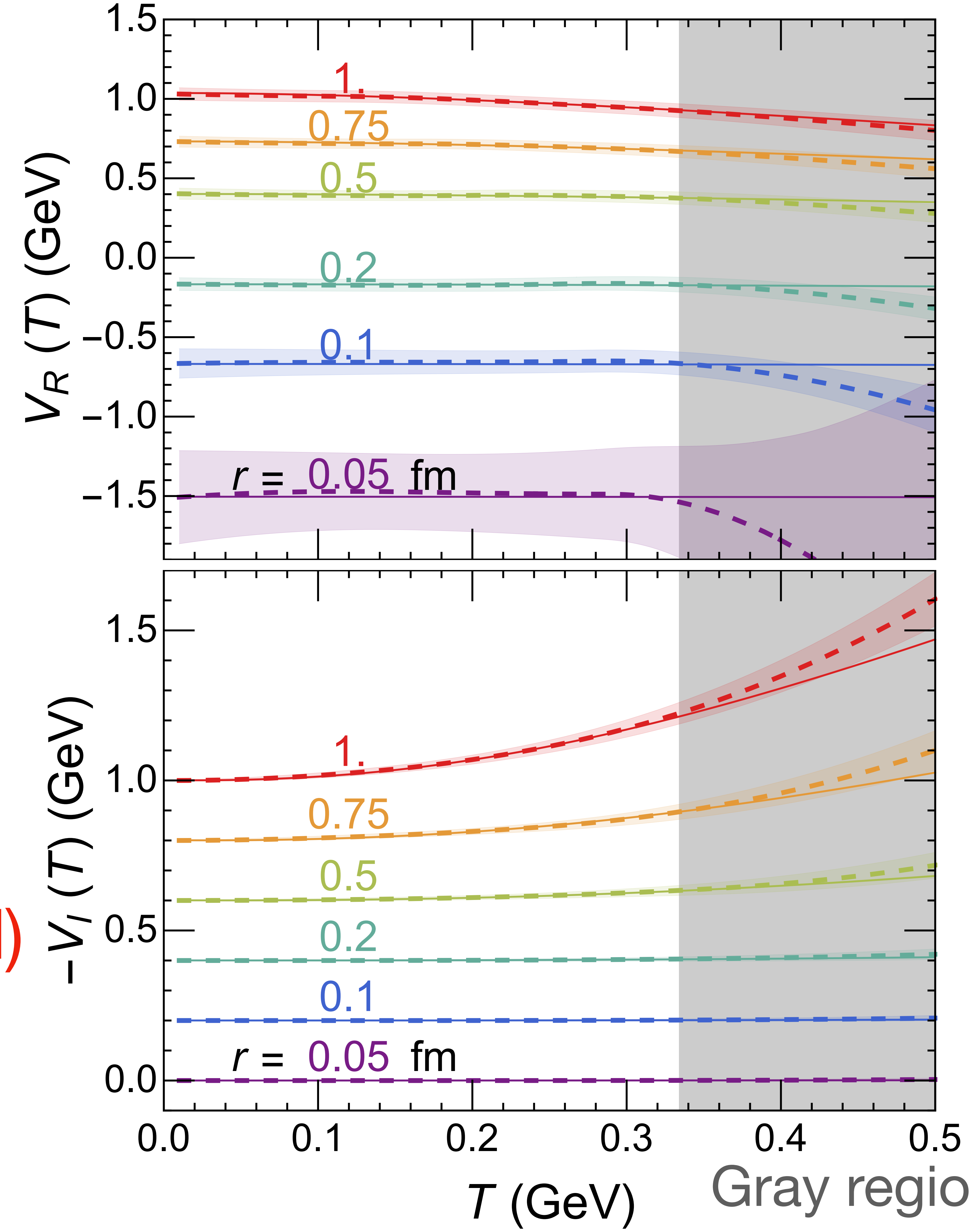
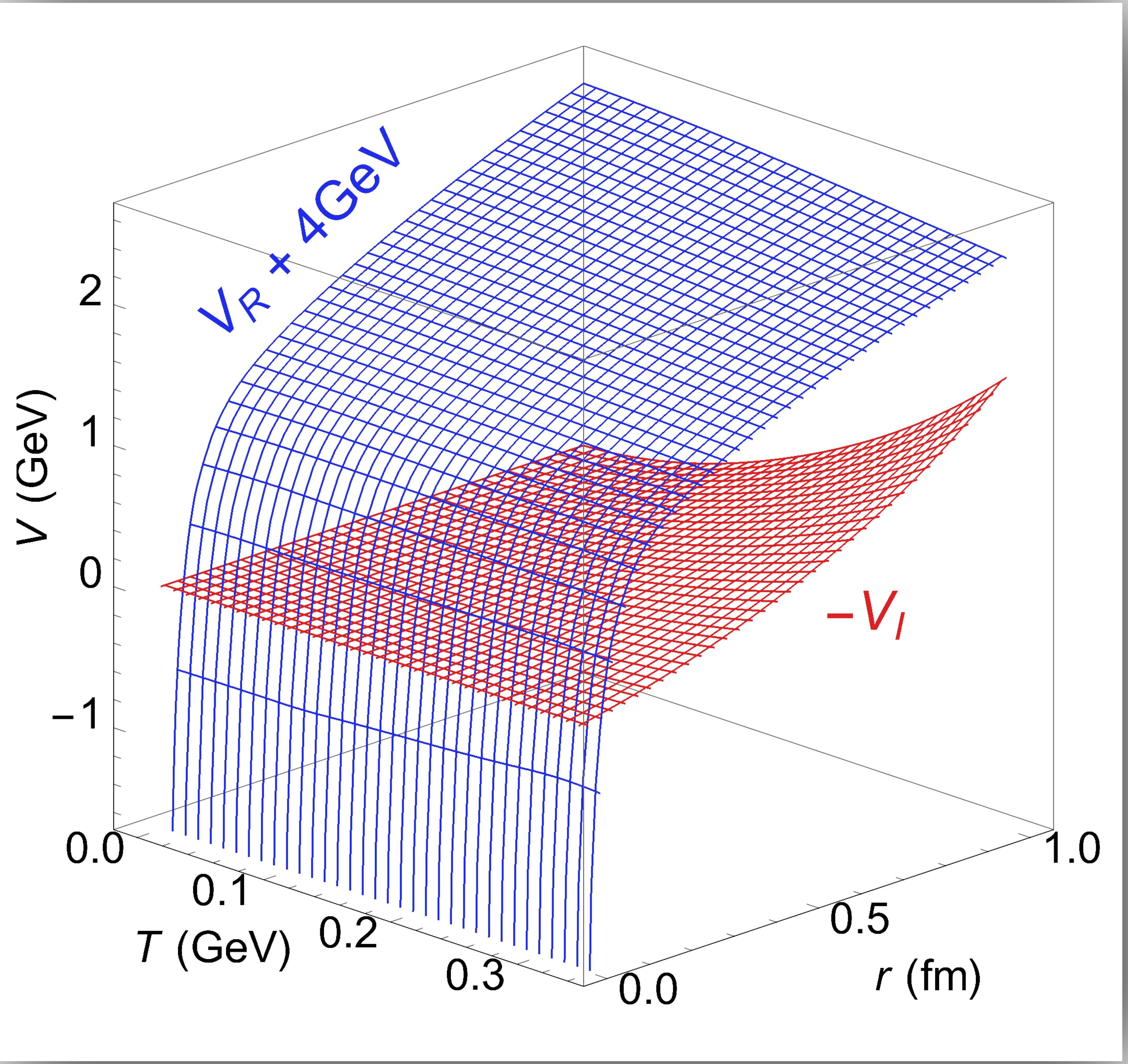


Gray region: extrapolations

Results



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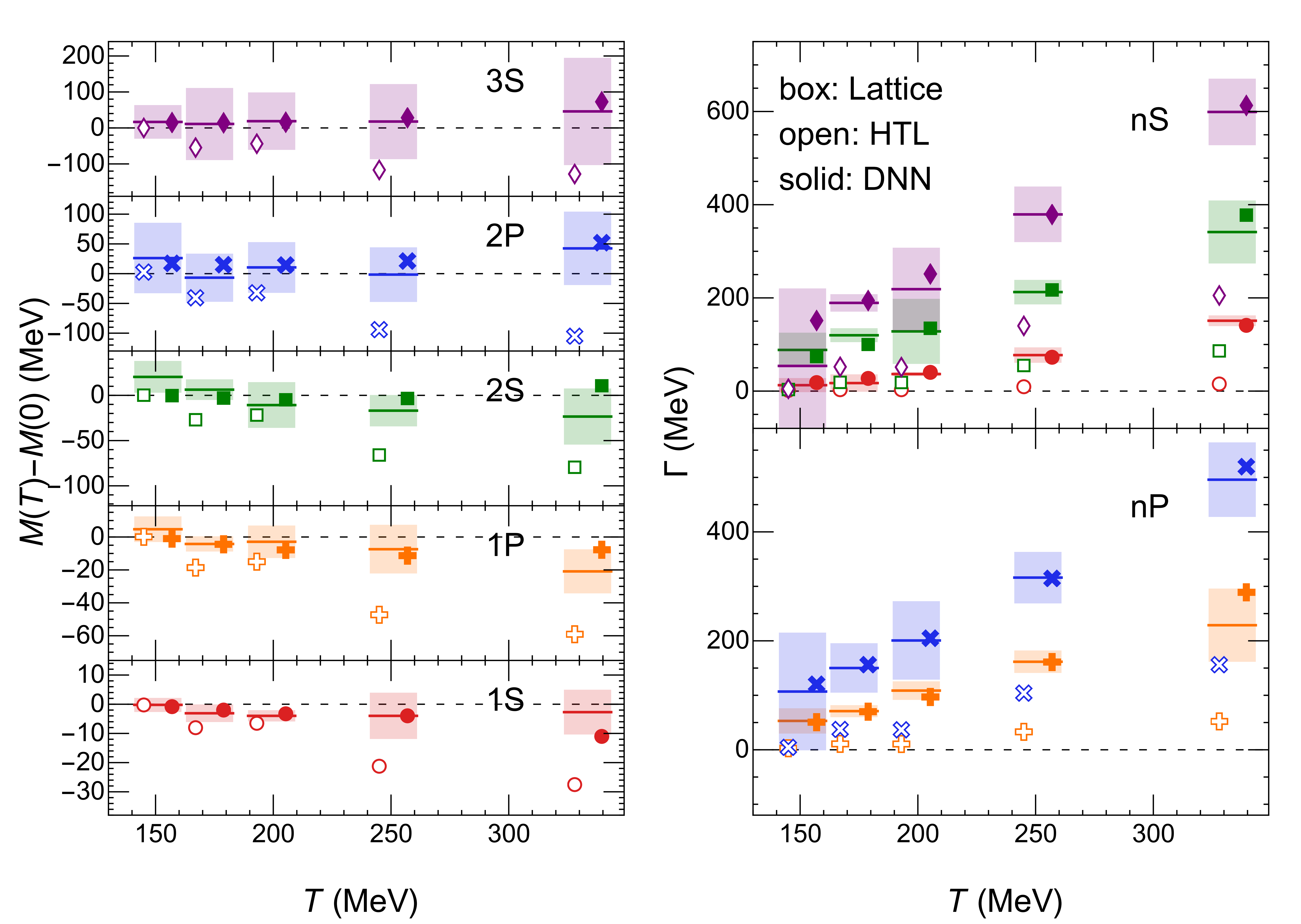
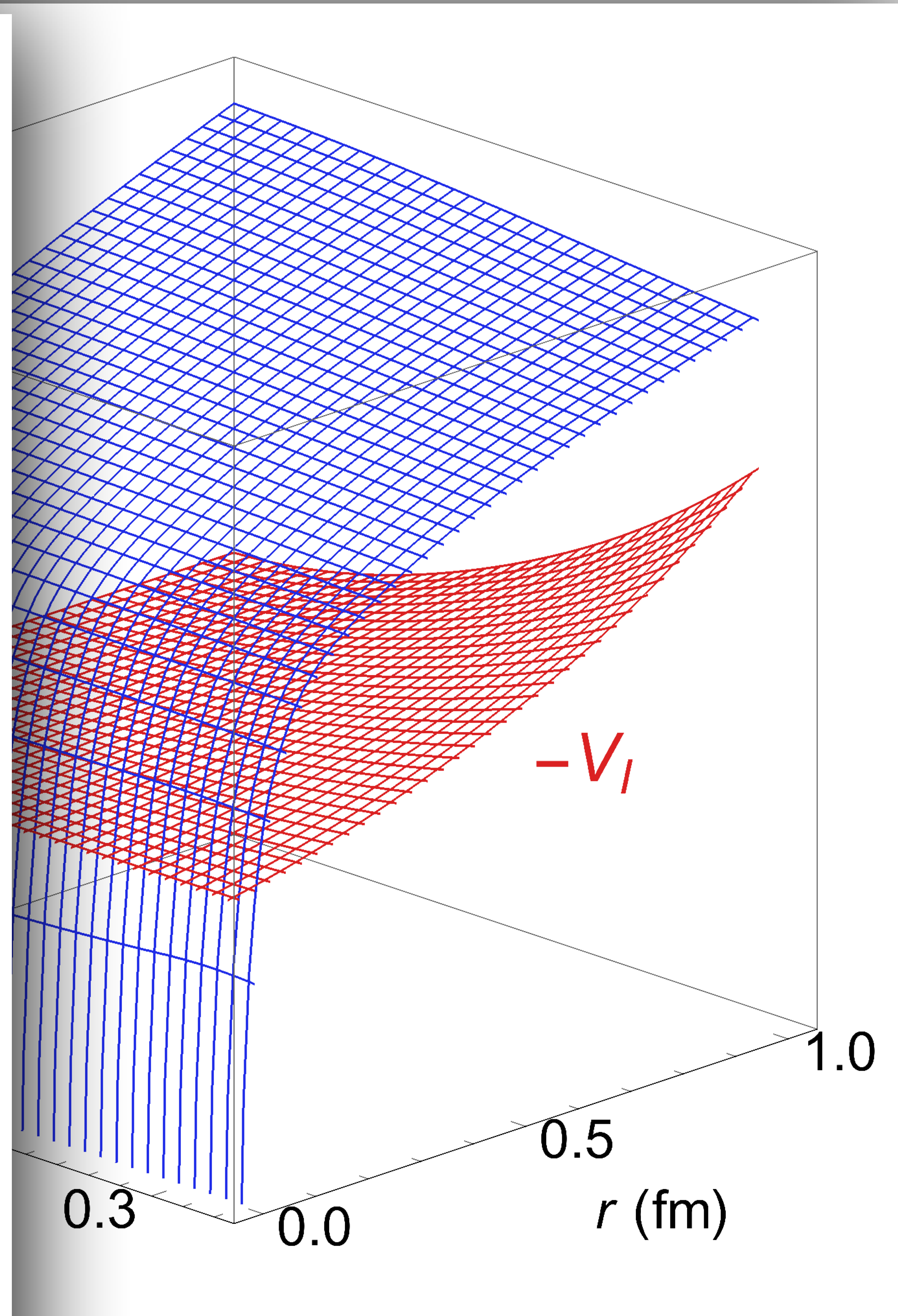
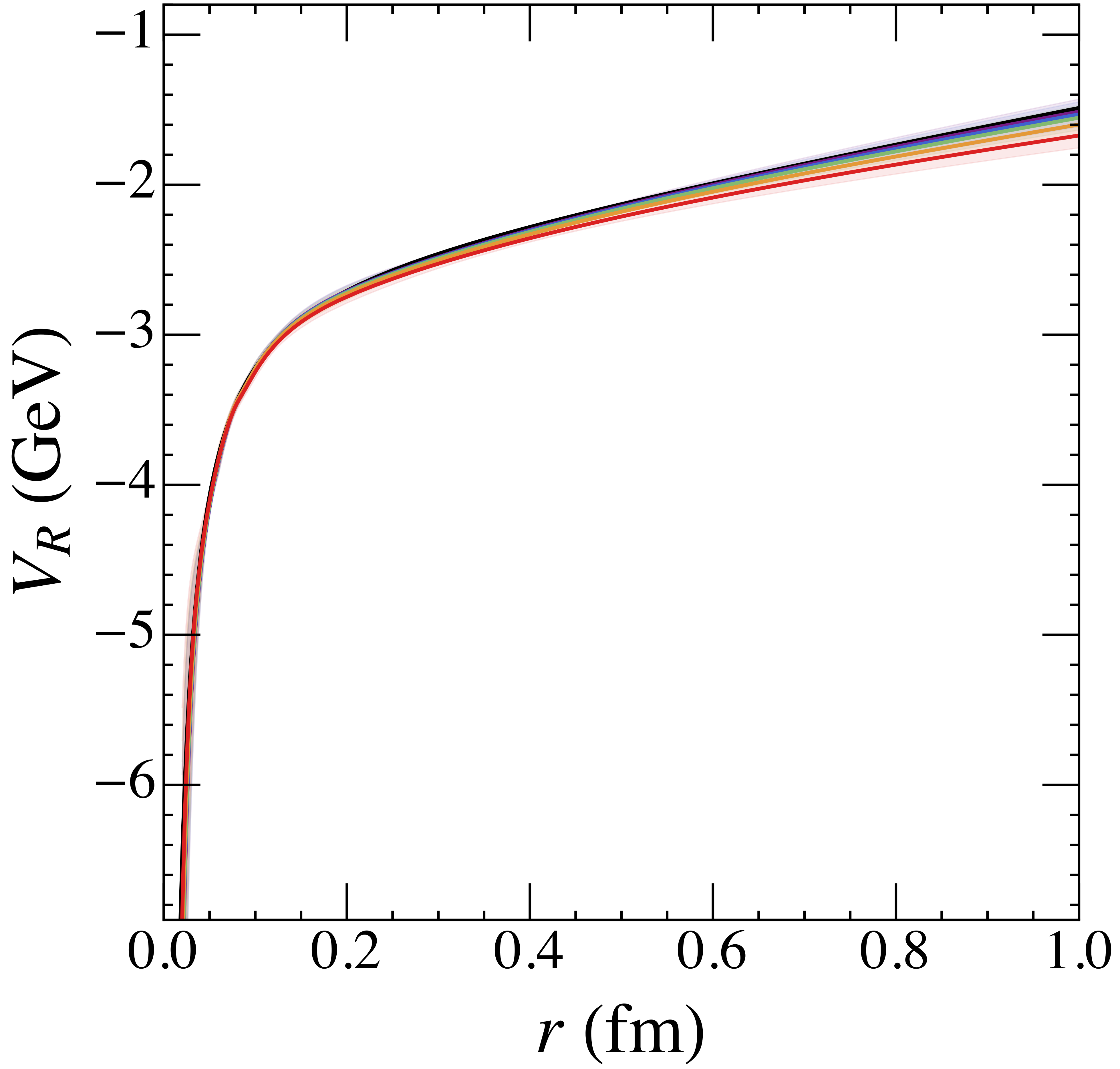
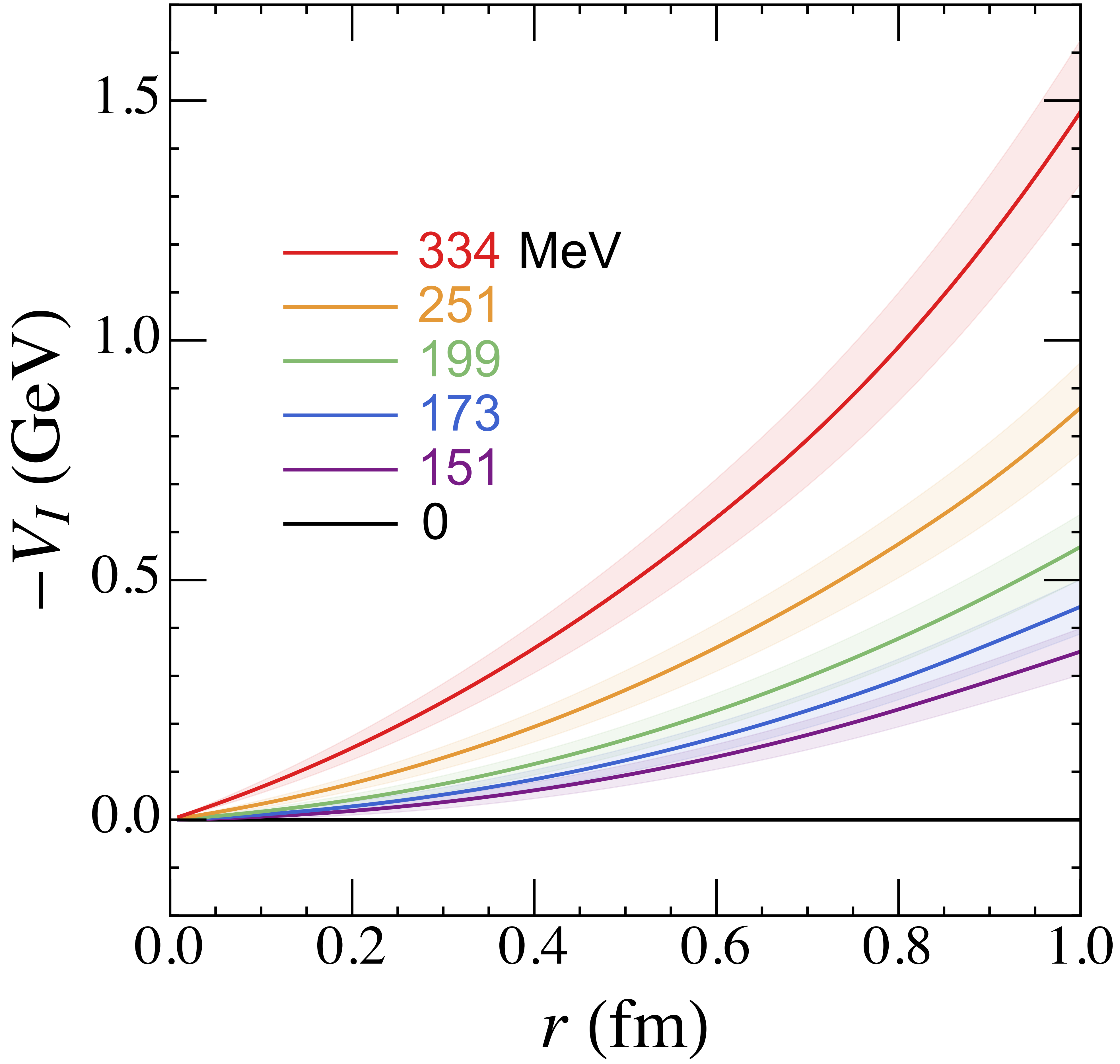

3S
2P
2S
1P
1S
M(T)−M(0) (MeV)
T (MeV)
box: Lattice
open: HTL
solid: DNN
nS
nP
Γ (MeV)
T (MeV)


−V_I
0.3
0.0
0.5
1.0
r (fm)

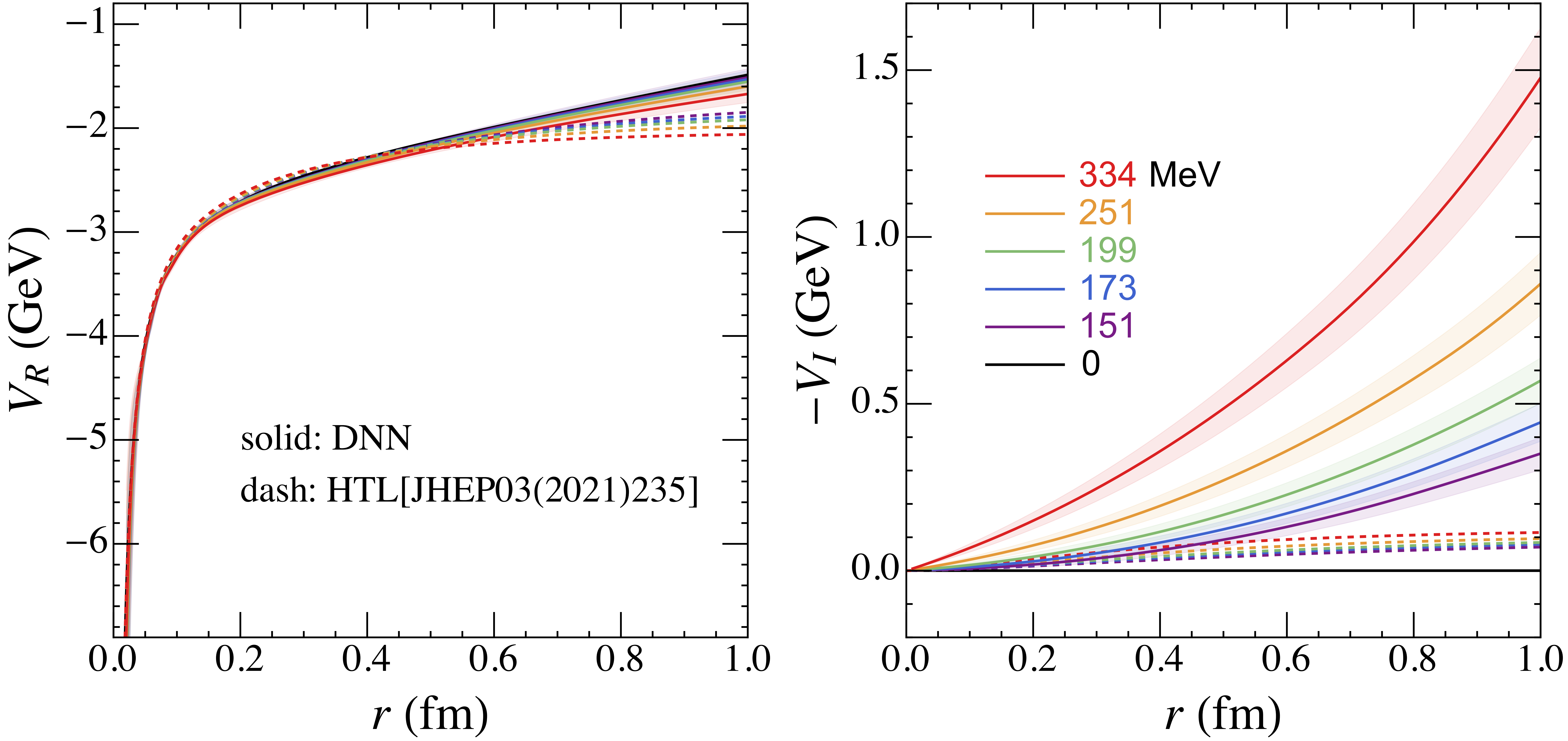
What physics we have learned from $V_{\text{DNN}}(T, r)$?





What physics we have learned from $V_{\text{DNN}}(T, r)$?

--- compare with HTL potential used in [1]



[1] A. Islam and M. Strickland, JHEP03(2021)235

What physics we have learned from $V_{\text{DNN}}(T, r)$?

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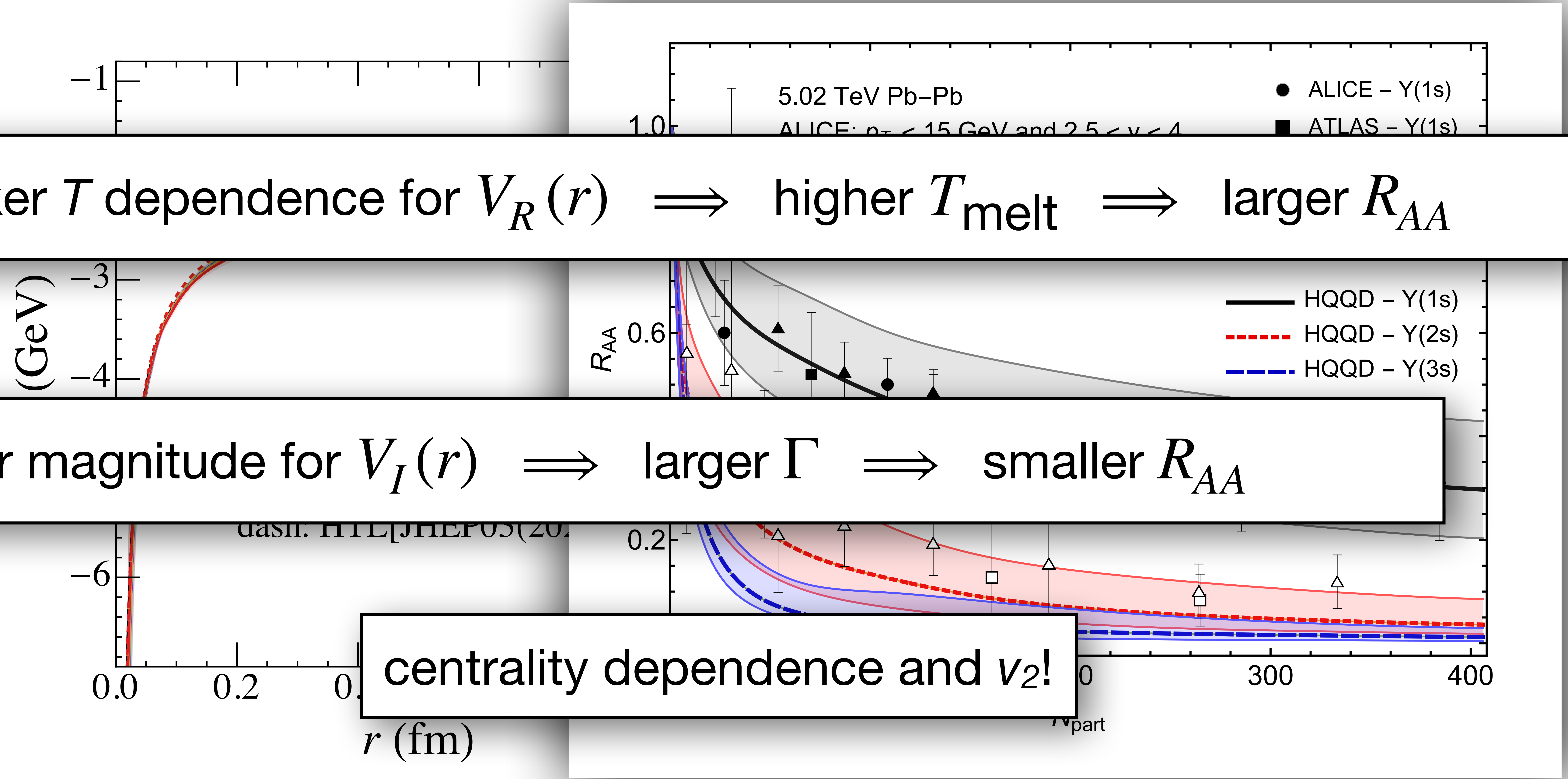

weaker T dependence for $V_R(r)$ $\Longrightarrow$ higher T_{melt} $\Longrightarrow$ larger R_{AA}

larger magnitude for $V_I(r)$ $\Longrightarrow$ larger Γ $\Longrightarrow$ smaller R_{AA}

centrality dependence and v_2!

[1] A. Islam and M. Strickland, JHEP03(2021)235

Summary and Outlook

- Develop new algorithm employing DNN to learn $V(r)$ from $\{E_n\}$.

- Extract HF complex $V(T, r)$ from LQCD results of bottomonium m and Γ.

- Phenomenological consequences in heavy-ion collisions?