Pre-equilibrium QCD plasma in heavy-ion collisions

Seminar @ High Energy Nuclear Physics in China (Online)

Sept. 23, 2021

Xiaojian Du | Bielefeld University

UNIVERSITÄT BIELEFELD

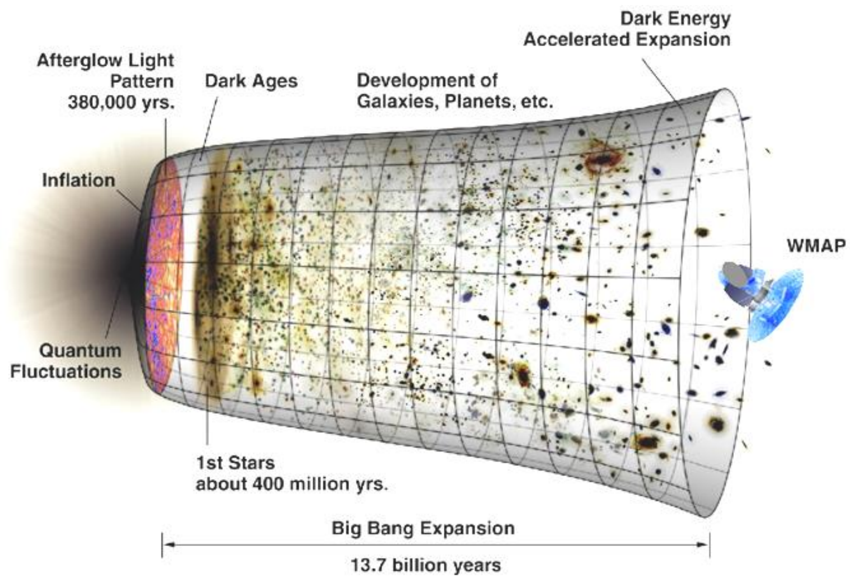
CRC–TR 211
Strong-interaction matter under extreme conditions

Outline

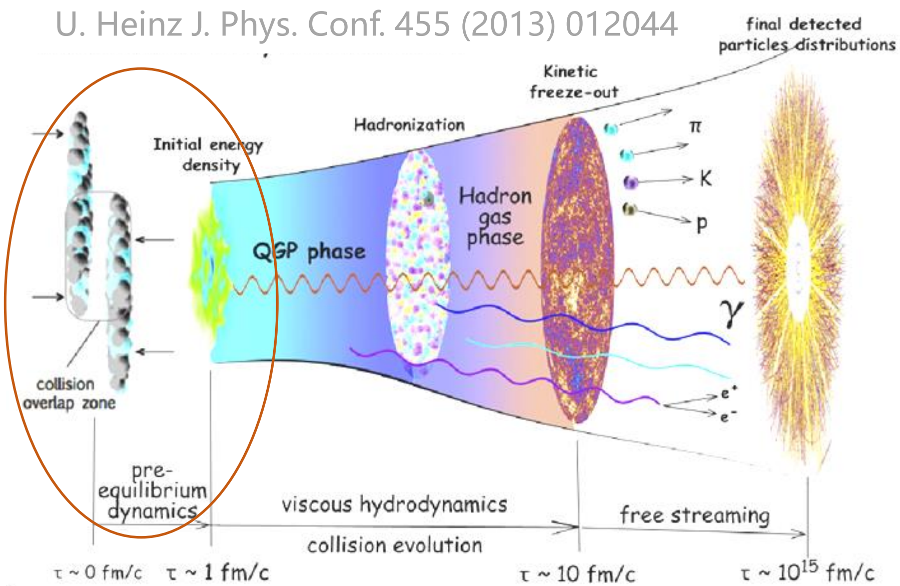
- **Pre-equilibrium QCD plasmas**
  - The motivation and the theoretical tool
- **Turbulence in pre-equilibrium QCD plasmas**
  - The turbulent nature of quark-gluon plasma
- **Pre-equilibrium QCD plasmas in early stage of HICs**
  - Attractor theory of quark-gluon plasma and its applications
- **Including space-time fluctuation**
  - Towards a complete picture of pre-equilibrium stage in HICs

[1] XD, Schlichting, PRL127(2021)122301.
[2] XD, Schlichting, PRD104(2021)054011
[3] Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626
[4] Others, in progress...

Equilibration after bangs



U. Heinz J. Phys. Conf. 455 (2013) 012044



Early universe (Big Bang)

Standard Model matter produced and equilibrated between inflation and Big Bang nucleosynthesis (BBN)

Heavy-ion collision (little bang)

Off-thermal plasma produced in initial collision and equilibrated into thermal hydrodynamic states:

Kinetic equilibration

Yang-Mills plasma (gluon saturated) equilibrated into quark-gluon plasma (quarks + gluon):

Chemical equilibration

QCD Effective Kinetic Theory

Simulation with QCD Effective Kinetic Theory (EKT)

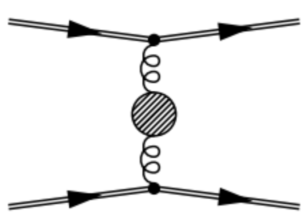
$$\left(\frac{\partial}{\partial\tau} - \frac{p_\parallel}{\tau}\frac{\partial}{\partial p_\parallel}\right) f_a(\tau, p_T, p_\parallel) = -C_a^{2\leftrightarrow 2}[f](\tau, p_T, p_\parallel) - C_a^{1\leftrightarrow 2}[f](\tau, p_T, p_\parallel)$$

Arnold, Moore, Yaffe, JHEP01 (2003) 030
Arnold, Moore, Yaffe, JHEP0206 (2002) 030
Kurkela, Mazeliauskas, PRD99 (2019) 054018

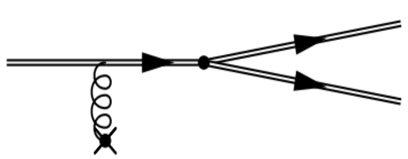
Solving a set of coupled Boltzmann equations

Including all light quarks/antiquarks and gluon $a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$

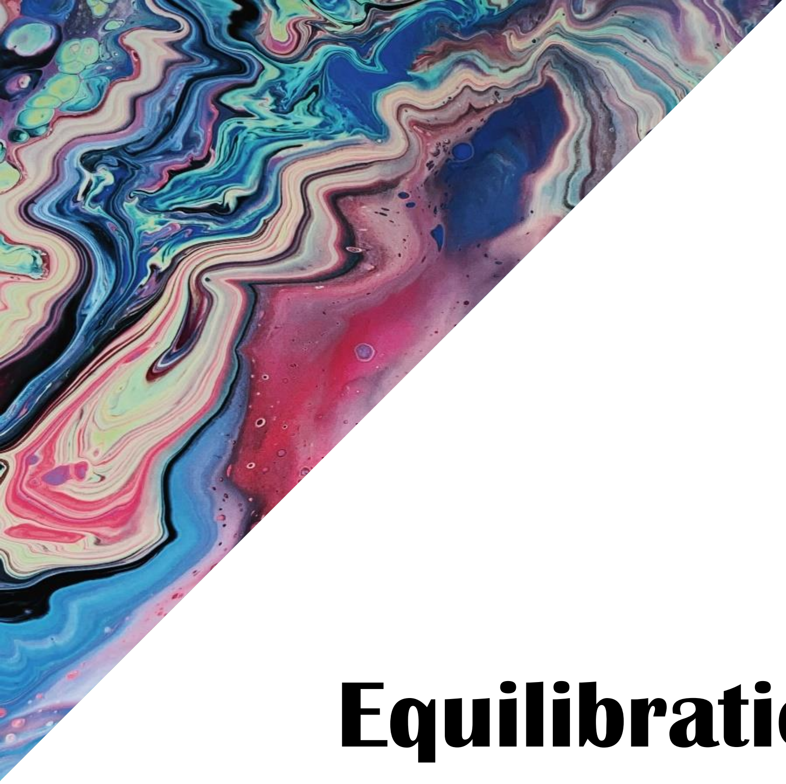
Including LO 2↔2 elastic scatterings and 1↔2 inelastic scatterings with back reaction



2↔2: Color screening by Debye mass fit to Hard Thermal Loop (HTL) calculation

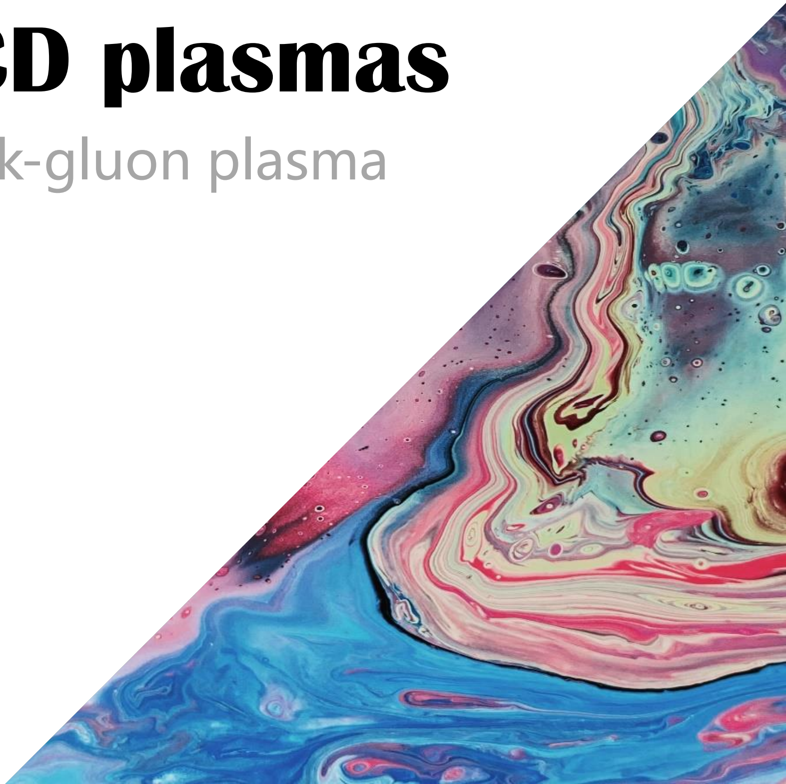


1↔2: Collinear radiation including Landau-Pomeranchuk-Migdal (LPM) effect via effective vertex resummation



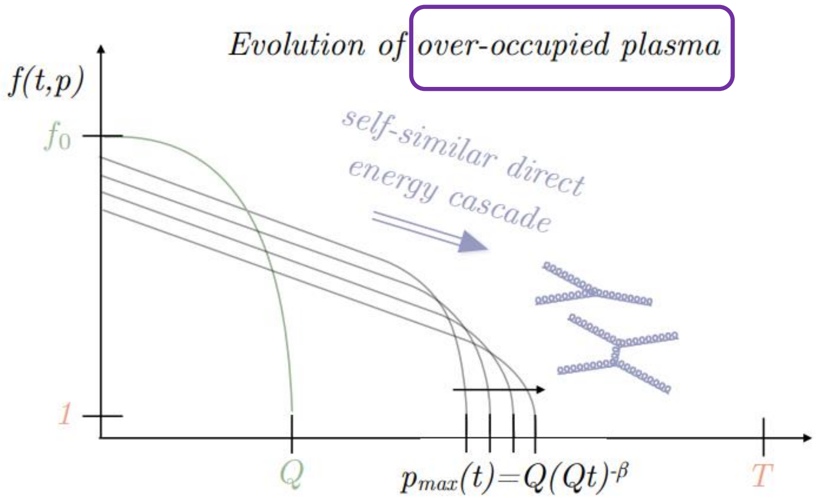
Equilibration of QCD plasmas

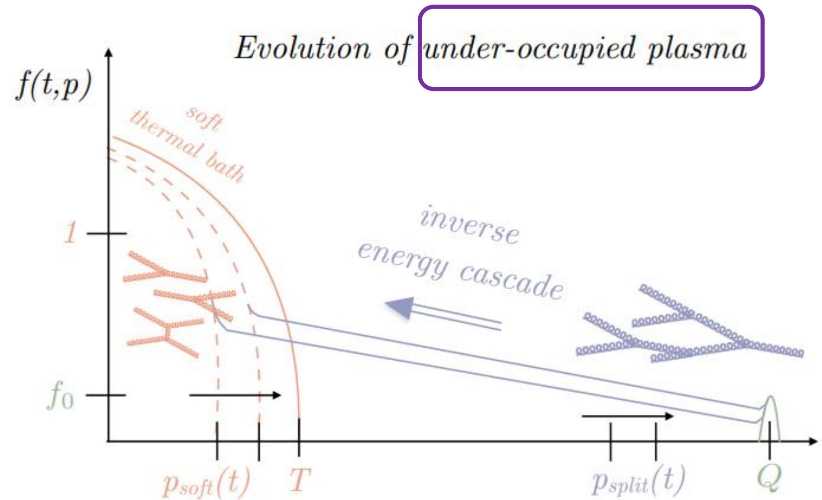
The turbulent nature of quark-gluon plasma



Turbulence in QCD equilibration

Two typical far-from-equilibrium systems





Over-occupied plasma

- Separation of scale

$$\langle p \rangle_0 \ll T$$

- Direct energy cascade

  low → high momentum

- Initial state in HICs

Under-occupied plasma

- Separation of scale

$$\langle p \rangle_0 \gg T$$

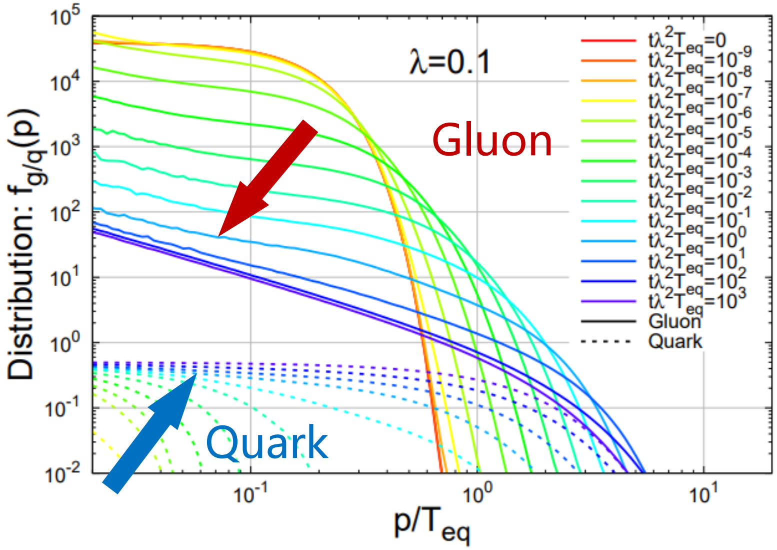
- Inverse energy cascade

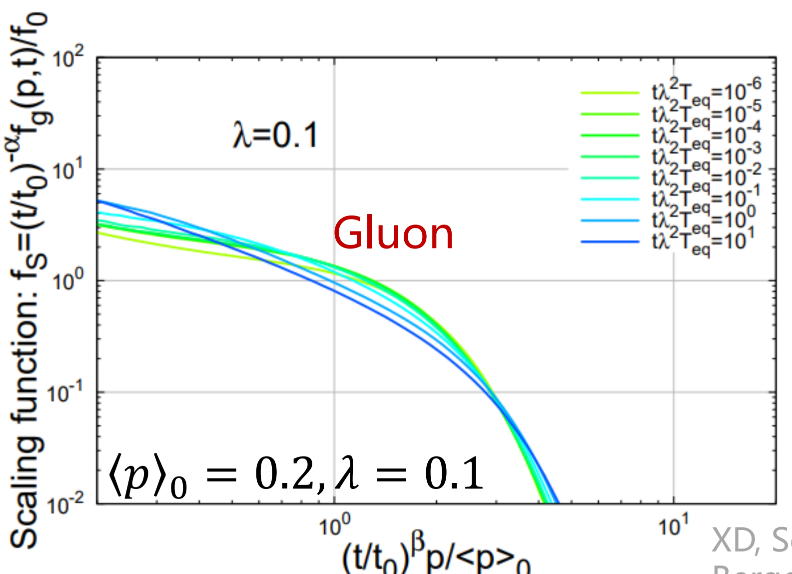
  high → low momentum

- Jets in HICs

Schlichting, Teaney, ARNPS 69 (2019) 447

Over-occupied plasma





Self-similar energy cascade

Self-similar scaling spectra

$$f_g(p,t) = (t/t_0)^{\alpha} f_0 f_S\left((t/t_0)^{\beta}\frac{p}{\langle p\rangle_0}\right)$$

Universal Scaling Function

$$f_S\left((t/t_0)^{\beta}\frac{p}{\langle p\rangle_0}\right)$$

Scaling Exponents from Yang-Mills plasma

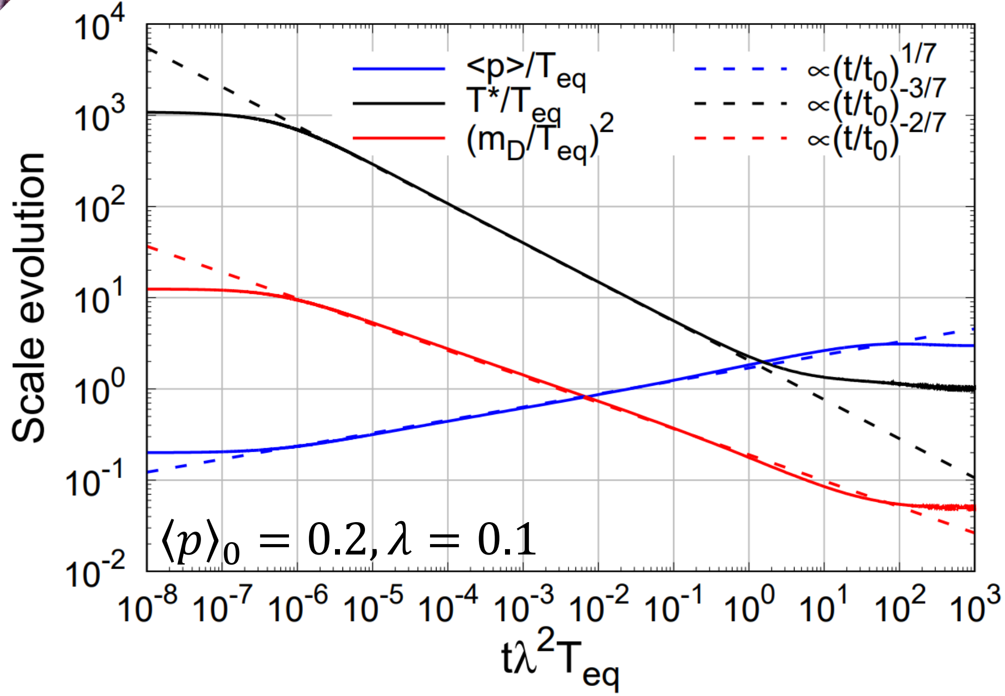
$$\alpha = -\frac{4}{7}, \beta = -\frac{1}{7}$$

Also work for quark-gluon plasma
gluon dominated

Quark spectra following gluon spectrum

XD, Schlichting, PRD104(2021)054011
Berges, Boguslavski, Schlichting, Venugopalan, PRD89 (2014) 114007
Abraao York, Kurkela, Lu, Moore, PRD89(2014)074036

Over-occupied plasma



Self-similar scaling

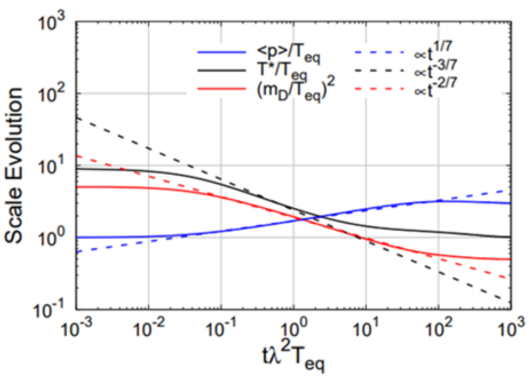
$$f \sim f_0 \left(\frac{t}{t_0}\right)^{-\frac{4}{7}}$$

Power-law evolution

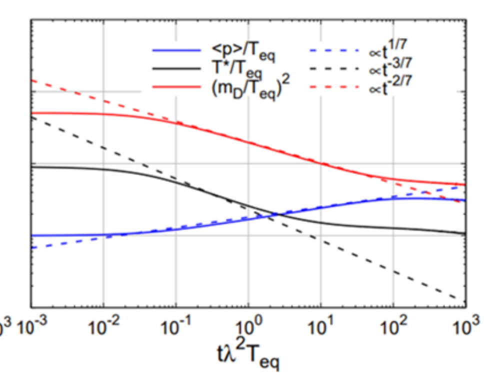
$$p \sim \langle p\rangle_0 \left(\frac{t}{t_0}\right)^{\frac{1}{7}} \quad T \sim g^2 f_0 \langle p\rangle_0 \left(\frac{t}{t_0}\right)^{-\frac{3}{7}}$$

$$m_D^2 \sim g^2 f_0 \langle p\rangle_0^2 \left(\frac{t}{t_0}\right)^{-\frac{2}{7}}$$

Not limited to Yang-Mills plasma
But also for quark-gluon plasma



$\langle p\rangle_0 = 1, \lambda = 1$



$\langle p\rangle_0 = 1, \lambda = 10$

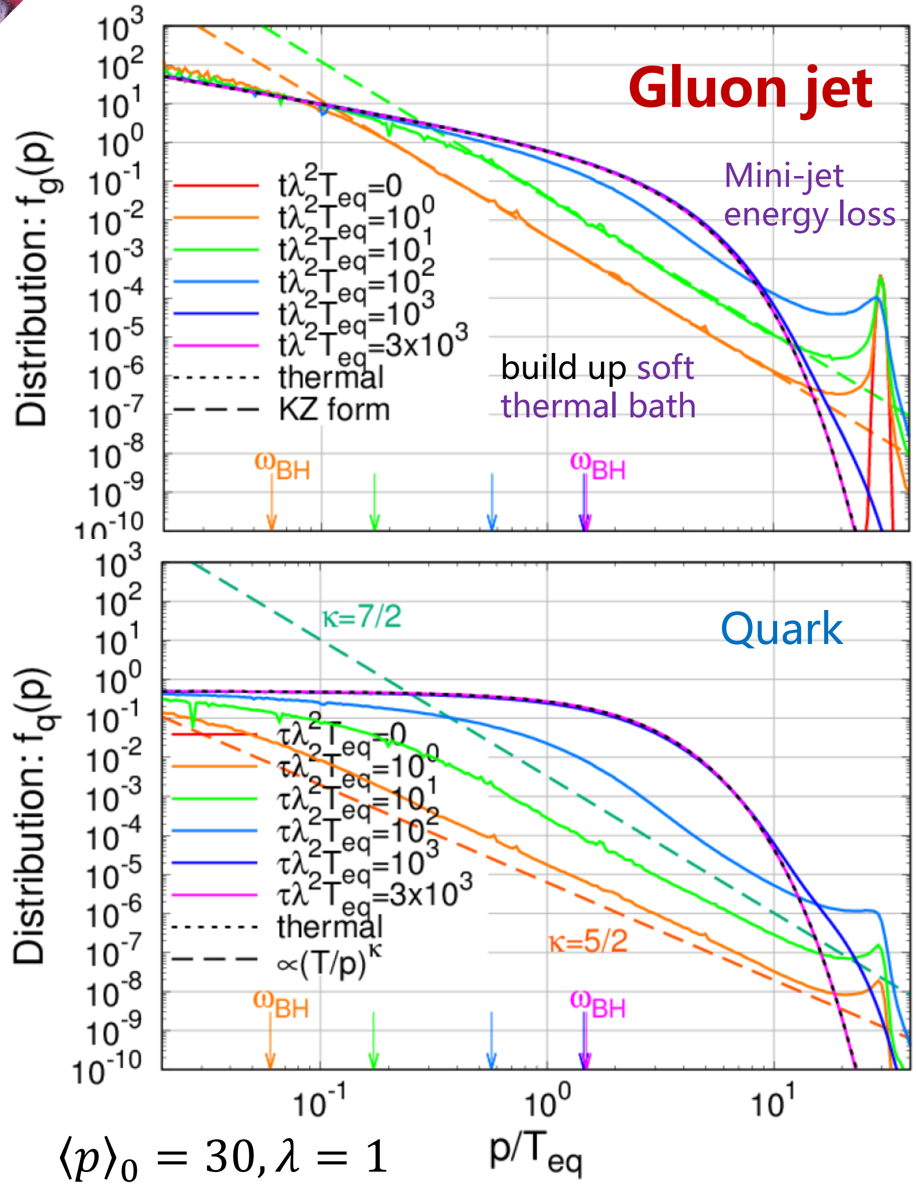
Even work for stronger coupling

t'Hooft coupling

$$\lambda = 4\pi\alpha_s N_c$$

XD, Schlichting, PRD104(2021)054011

Under-occupied plasma



$\langle p\rangle_0 = 30, \lambda = 1$

Wave turbulence

Kolmogorov-Zakharov spectrum (exponent κ=7/2 for gluon)

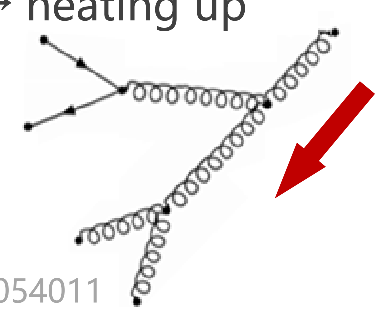
$$f_{KZ}(p,t) = \eta(t)\left(\frac{\langle p\rangle_0}{p}\right)^{\kappa}$$

Blaizot, Iancu, Mehtar-Tani, PRL 111, 052001 (2013)
Mehtar-Tani, Schlichting, JHEP 09, 144 (2018)

Bottom-up thermalization

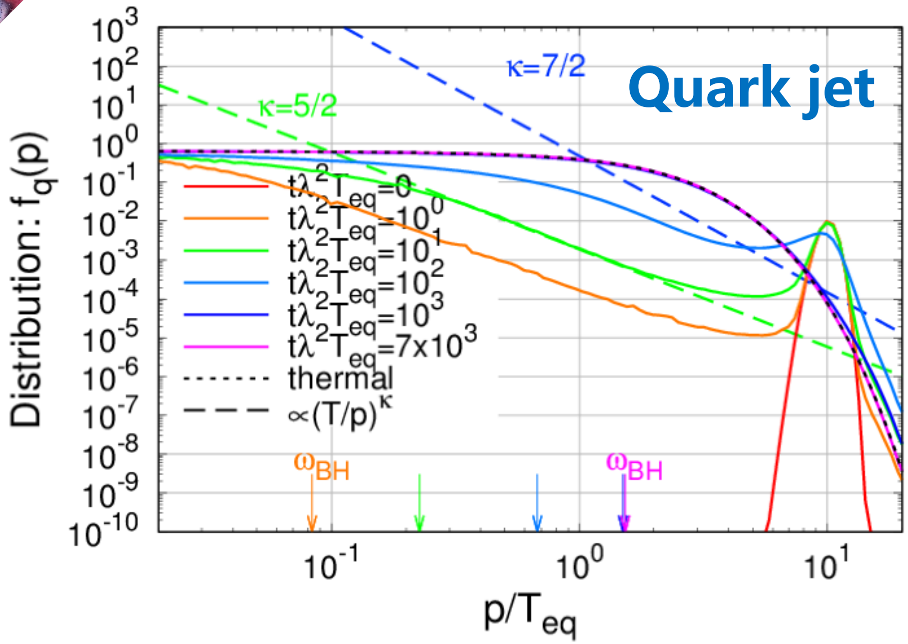
1. Emission of (soft) quarks and gluon
2. Radiative breakup by multiple branchings → build up soft thermal bath
3. Mini-Jet energy loss → heating up thermal bath

Baier, et al. PLB 502 (2001) 51



XD, Schlichting, PRD104(2021)054011

Under-occupied plasma



Wave turbulence

1. Quark follows κ=5/2 to κ=7/2
2. Gluon follows κ=7/2
3. Antiquark follows gluon (secondary production)

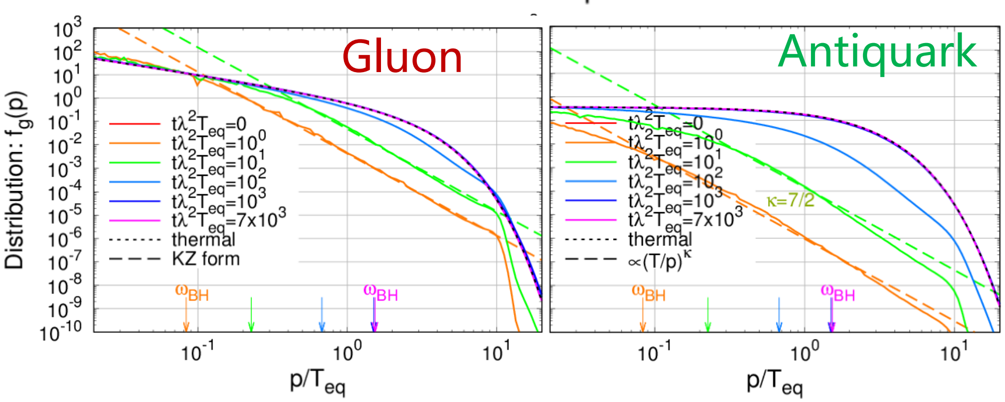
$$f_{KZ}(p,t) = \eta(t)\left(\frac{\langle p\rangle_0}{p}\right)^{\kappa}$$

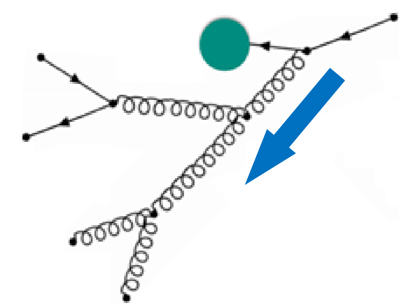
Bottom-up thermalization

Same pattern as for

in-medium jet energy loss and equilibration

with unified description of soft and hard sectors





$\langle p\rangle_0 = 10, \lambda = 1$

XD, Schlichting, PRD104(2021)054011

For turbulence from jet
Soudi, Schlichting, JHEP07(2021)077

Early stage of heavy-ion collisions

Attractor of quark-gluon plasma and its applications

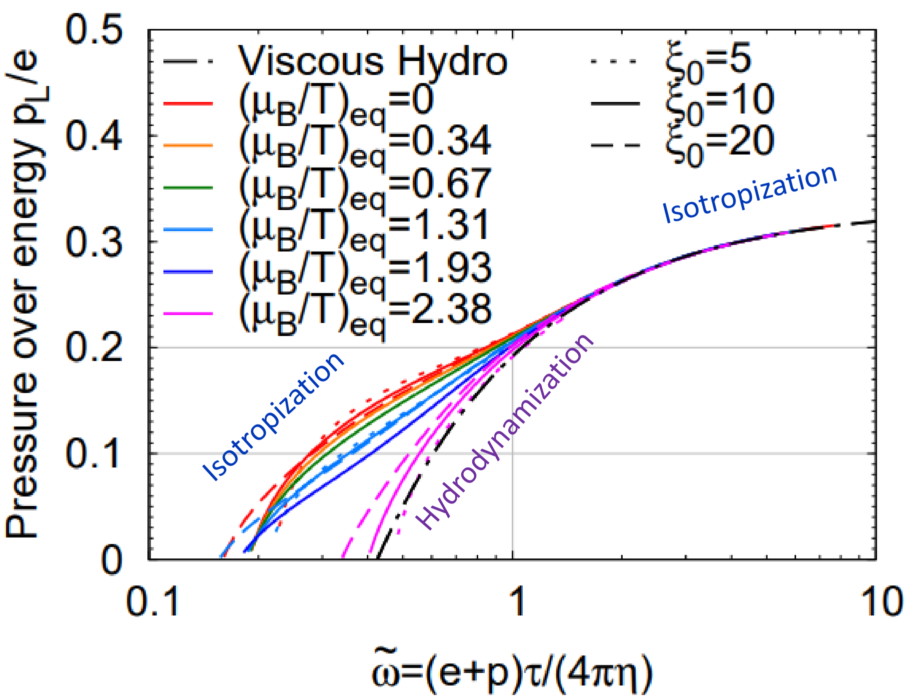
Hydrodynamization in HICs

Isotropization

longitudinal pressure/energy density
0 (initial) → 1/3 (final equilibrium)

Hydrodynamic constitutive relation:

$$\frac{p_L}{e} = \frac{1}{3} - \frac{16\eta}{9(e+p)\tau}$$



Universal scaling

$$\tilde{\omega} = \frac{(e+p)\tau}{4\pi\eta}$$

Recast:

$$\frac{p_L}{e} = \frac{1}{3} - \frac{4}{9\pi\tilde{\omega}}$$

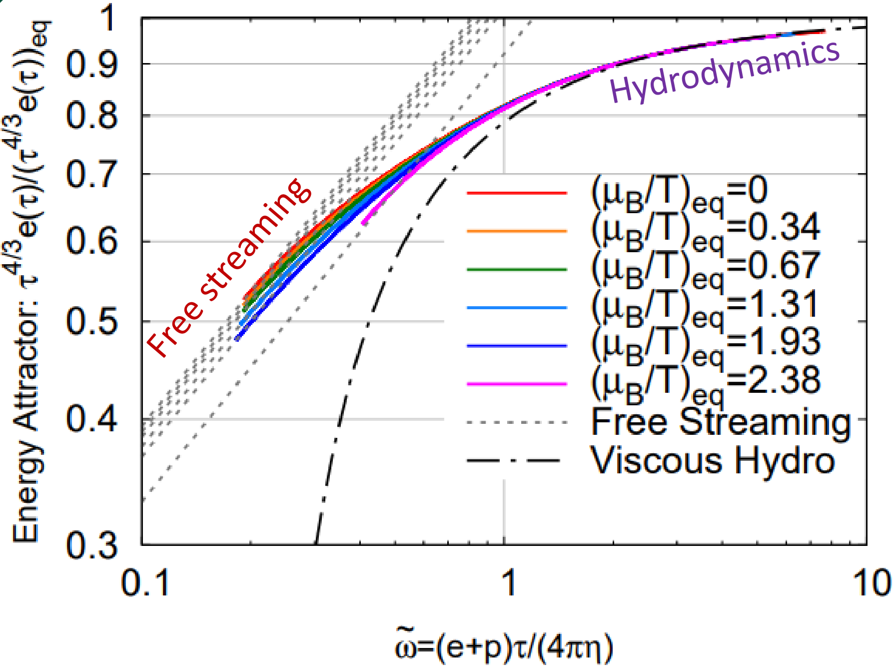
Pressure attractor

Effective constitutive relation from EKT

$$\frac{p_L}{e} = f(\tilde{\omega})$$

Kurkela, Mazeliauskas, PRD99(2019)054018
KOMPOST, PRC99(2019)034910
XD, Schlichting, PRL127(2021)122301

Attractor in Hydrodynamization



Energy attractor

$$\mathcal{E}\left(\tilde{\omega}=\frac{(e+p)\tau}{4\pi\eta}\right)=\frac{\tau^{4/3}e}{(\tau^{4/3}e)_{eq}}$$

Asymptotically

$$\mathcal{E}(\tilde{\omega}\gg 1)\approx 1-\frac{2}{3\pi\tilde{\omega}}$$ Hydrodynamics

$$\mathcal{E}(\tilde{\omega}\gg 1)\approx C_\infty^{-1}\tilde{\omega}^{4/9}$$ Free streaming

Universal non-equilibrium attractor

Pre-equilibrium description connects initial to hydro in HICs

$$(\tau^{4/3}e)_{\tilde{\omega}}=\left(4\pi\frac{\eta T_{eff}}{e+p}\right)^{\frac{4}{9}}\left(\frac{\pi^2}{30}\nu_{eff}\right)^{\frac{1}{9}}(\tau e)_0^{\frac{8}{9}}C_\infty\mathcal{E}(\tilde{\omega})$$

$$(\tau\Delta n_f)_{\tilde{\omega}}=(\tau\Delta n_f)_0$$

Two-way

Provide input for hydrodynamics

Learn the past !(pre-eq, initial)

Giacalone, Mazeliauskas, Schlichting PRL123(2019)262301
XD, Schlichting, PRL127(2021)122301

Pre-equilibrium QGP trajectory

Fix the final equilibrium quantities

From EKT: entropy

$$(\tau s)_{eq} = \frac{\tau\left(e + p - \sum_f \mu_f \Delta n_f\right)}{T}$$

From EKT: net baryon number

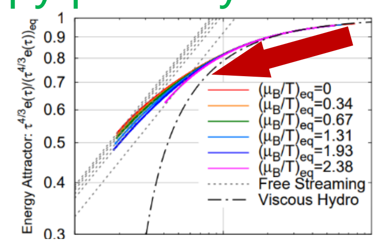
$$\Delta n_B = \frac{1}{3}\Delta n_u + \frac{1}{3}\Delta n_d$$

From data: charged particle multiplicity

$$\frac{dN_{ch}}{d\eta} = \frac{N_{ch}}{JS}(\tau s)_{eq} S_T \approx 0.12 (\tau s)_{eq} S_T$$

From data: entropy per baryon

$$\frac{S}{N_B} = \left(\frac{\tau s}{\tau \Delta n_B}\right)_{eq}$$



Learn the pre-equilibrium QGP

1. Apply non-equilibrium attractor

$$\left(\tau^{4/3} e\right)_{\tilde{\omega}} = \mathcal{E}(\tilde{\omega})\left(\tau^{4/3} e\right)_{eq}$$

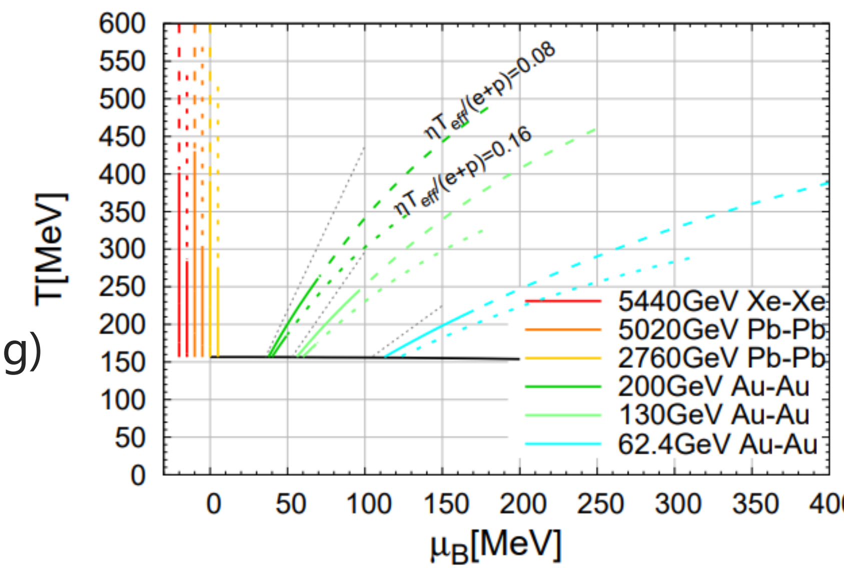
$$\left(\tau \Delta n_f\right)_{\tilde{\omega}} = \left(\tau \Delta n_f\right)_{eq}$$

2. Define effective T and μ_B (Landau matching)

Non-equilibrium QGP trajectory

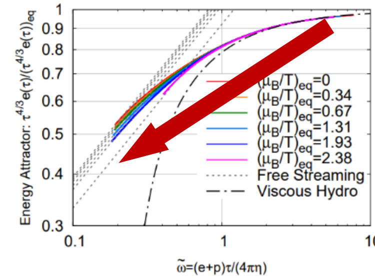
(at large baryon density)

XD, Schlichting, PRL127(2021)122301



Inferring the initial state

Push attractor further to initial state



$$\frac{dN_{ch}}{d\eta} = \frac{1}{J}\frac{4}{3}C_\infty^{3/4}\left(4\pi\frac{\eta}{s}\right)^{1/3}\left(\frac{\pi^2}{30}\nu_{\text{eff}}\right)^{1/3}\frac{N_{ch}}{S}\int d^2b\,(\tau e)_0^{2/3}$$

with equation of state e≈3p

Model the initial state distribution

An energy deposition model: k_T-factorized Color Glass Condensate (CGC) form with Golec-Biernat Wusthoff (GBW) gluon distribution

$$(\tau e)_0 = \frac{(N_c^2-1)}{4g^2N_c\sqrt{\pi}}\frac{Q_A^2Q_B^2}{(Q_A^2+Q_B^2)^{5/2}}(2Q_A^4+7Q_A^2Q_B^2+2Q_B^4) \qquad Q_{A/B}^2(x,b)\sim(Q_{s,0},\lambda,\eta/s)$$

parameters (x) ↔ (CGC+EKT) ↔ observables (y_{exp})

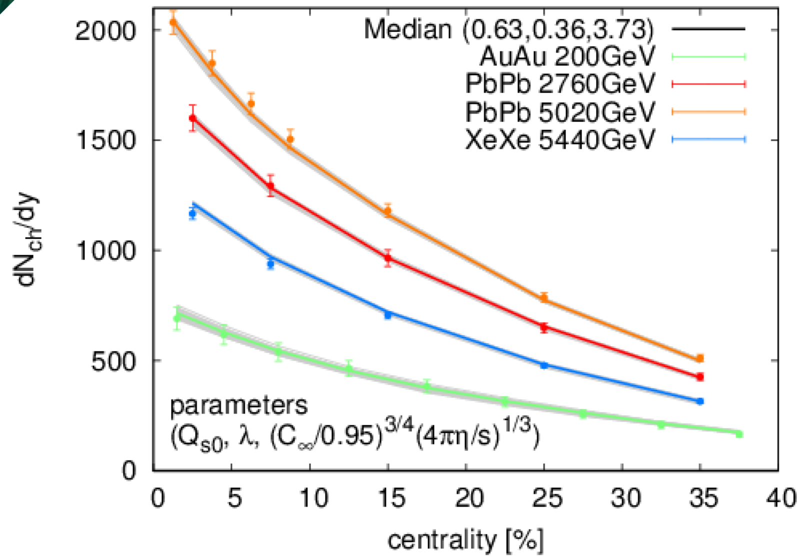
Bayes' theorem and Bayesian Inference

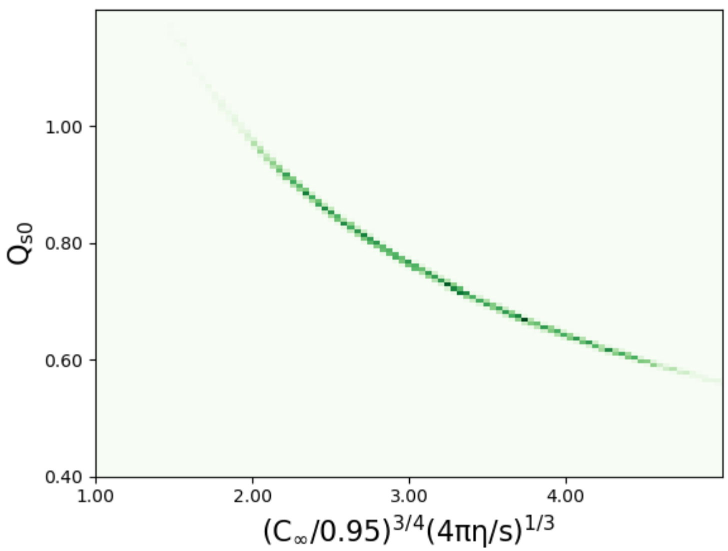
We want to construct the initial distribution by constraining parameters from observables. Find probability of parameters using Bayesian inference updating the likelihood function from model simulation

$$\boldsymbol{P(x|y_{exp})} = \frac{\boldsymbol{P(y_{exp}|x)}P(x)}{P(y_{exp})}$$

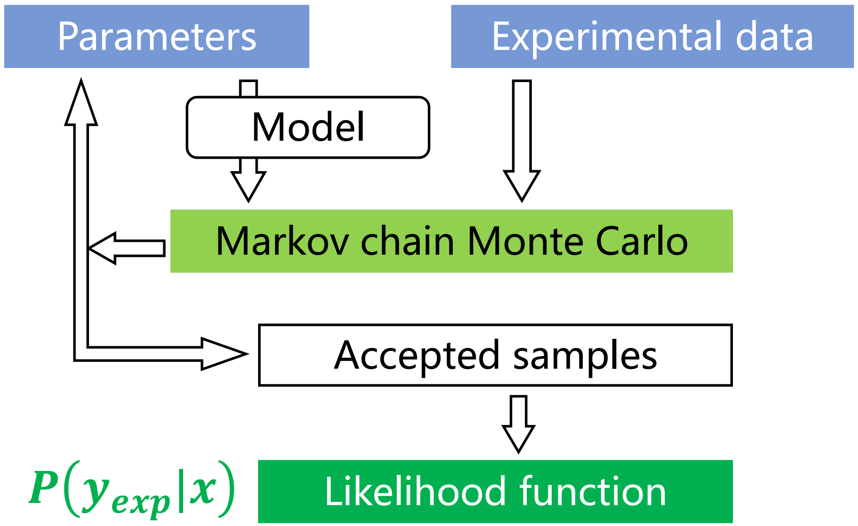
XD, Hoffmann, Schlichting, in progress

Inferring the initial state





Bayesian Inference



Fitting and correlations

Can find best fit parameters

Can find correlations between parameters

Further constrain initial distribution

XD, Hoffmann, Schlichting, in progress

Pre-equilibrium di-lepton production

Electromagnetic probes

Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626

Produced through-out HICs, not interacted with QGP, photon, di-lepton

Di-lepton production proportional to exp(-M/T), important at early stage of HICs

$$\frac{dN^{l+l-}}{d^4x d^4K} = \int \frac{d^3p_1}{(2\pi)^3}\frac{d^3p_2}{(2\pi)^3} 4N_c \sum_f f_q(x,p_1) f_{\bar{q}}(x,p_1) v_{q\bar{q}} \sigma_{q\bar{q}}^{l+l-} \delta^{(4)}(K-P_1-P_2)$$

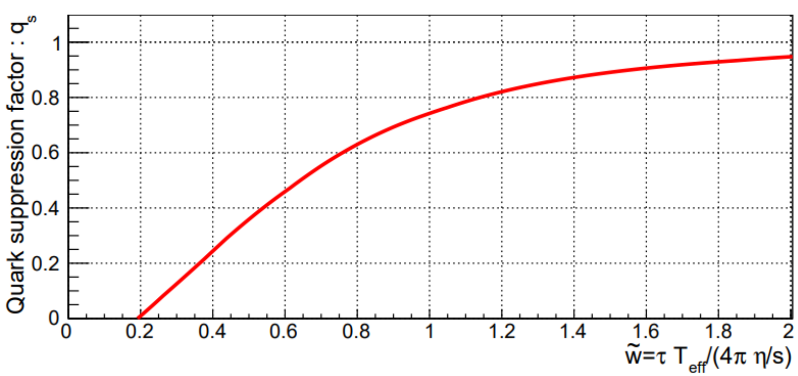
Pre-equilibrium quark suppression

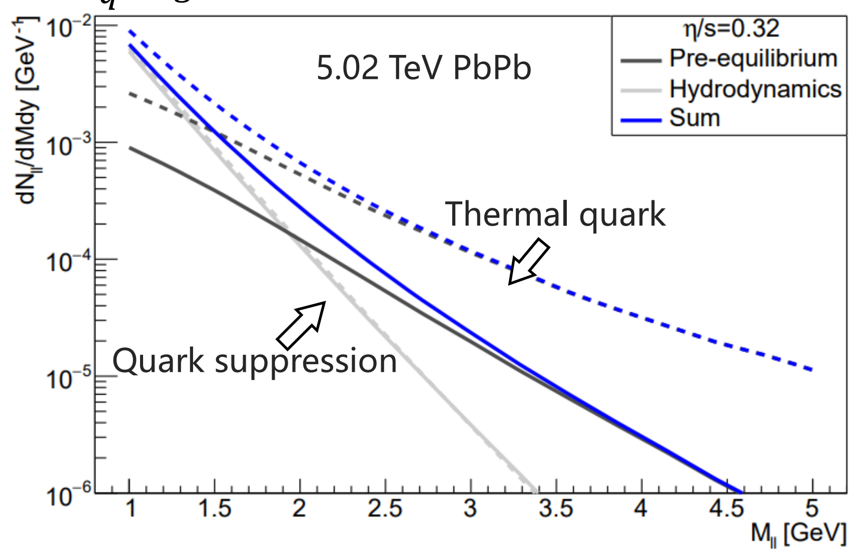
- Thermal di-lepton production: $f_{q/\bar{q}}(x,p)$ Fermi-Dirac distribution
- Pre-equilibrium di-lepton production: $q(\tau) = \frac{e_g^{eq}}{e_q^{eq}} \frac{e_q(\widetilde{\omega})}{e_g(\widetilde{\omega})}$

Chemical equilibration of QGP

Quark abundance increases

(YM plasma → QCD plasma)





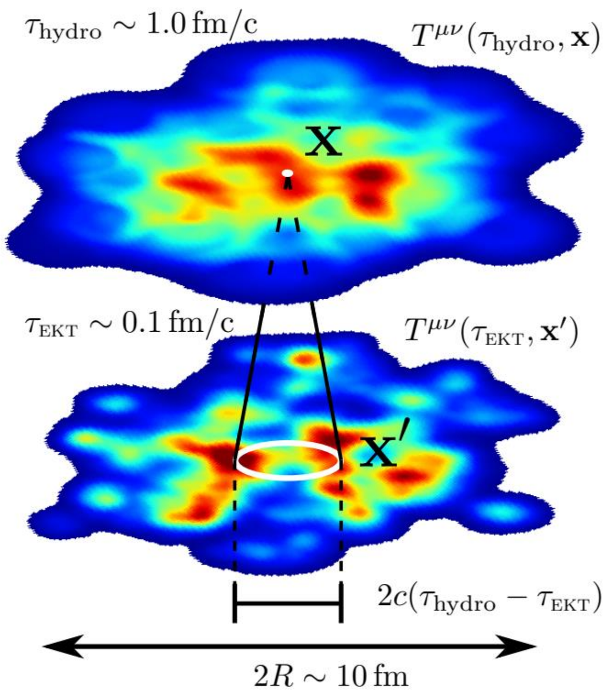
Space-time fluctuation

Towards a complete picture of pre-equilibrium stage in HICs

Linearized Effective Kinetic Theory

Linearized Effective Kinetic Theory

For spatial inhomogeneous and momentum anisotropic evolution



$$T^{\mu\nu}(\tau_{\rm EKT}, \mathbf{x}') = \overline{T}^{\mu\nu}_{\mathbf{x}}(\tau_{\rm EKT}) + \delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\rm EKT}, \mathbf{x}')$$

Background — Effective kinetic theory (EKT)

Perturbation — Linearized EKT

KøMPøST framework:
Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney,
PRL122 (2019) 12, 122302, PRC99 (2019) 3, 034910

Linear response/Green's function

Propagate energy-momentum fluctuation in both position and momentum space (with FFT)

$$\delta T^{\mu\nu}_{\mathbf{x}}(\tau_{\rm hydro}, \mathbf{x}) = \int d^2\mathbf{x}'\; G^{\mu\nu}_{\alpha\beta}(\mathbf{x}, \mathbf{x}', \tau_{\rm hydro}, \tau_{\rm EKT})\, \delta T^{\alpha\beta}_{\mathbf{x}}(\tau_{\rm EKT}, \mathbf{x}') \frac{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\rm hydro})}{\overline{T}^{\tau\tau}_{\mathbf{x}}(\tau_{\rm EKT})}$$

Extended from RTA: Kamata, Martinez, Plaschke, Ochsenfeld, Schlichting, PRD102(2020)056003
Extended from nonlinear Boltzmann: XD, Schlichting, in progress (both Yang-Mills and QCD)

Response functions

Response function in Fourier space

$$G^{\mu\nu}_{\alpha\beta}(\mathbf{x}-\mathbf{x}_0,\tau,\tau_0) = \int \frac{d^2\mathbf{k}}{(2\pi)^2}\tilde{G}^{\mu\nu}_{\alpha\beta}(\mathbf{k},\tau,\tau_0)e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}_0)}$$

Energy perturbation

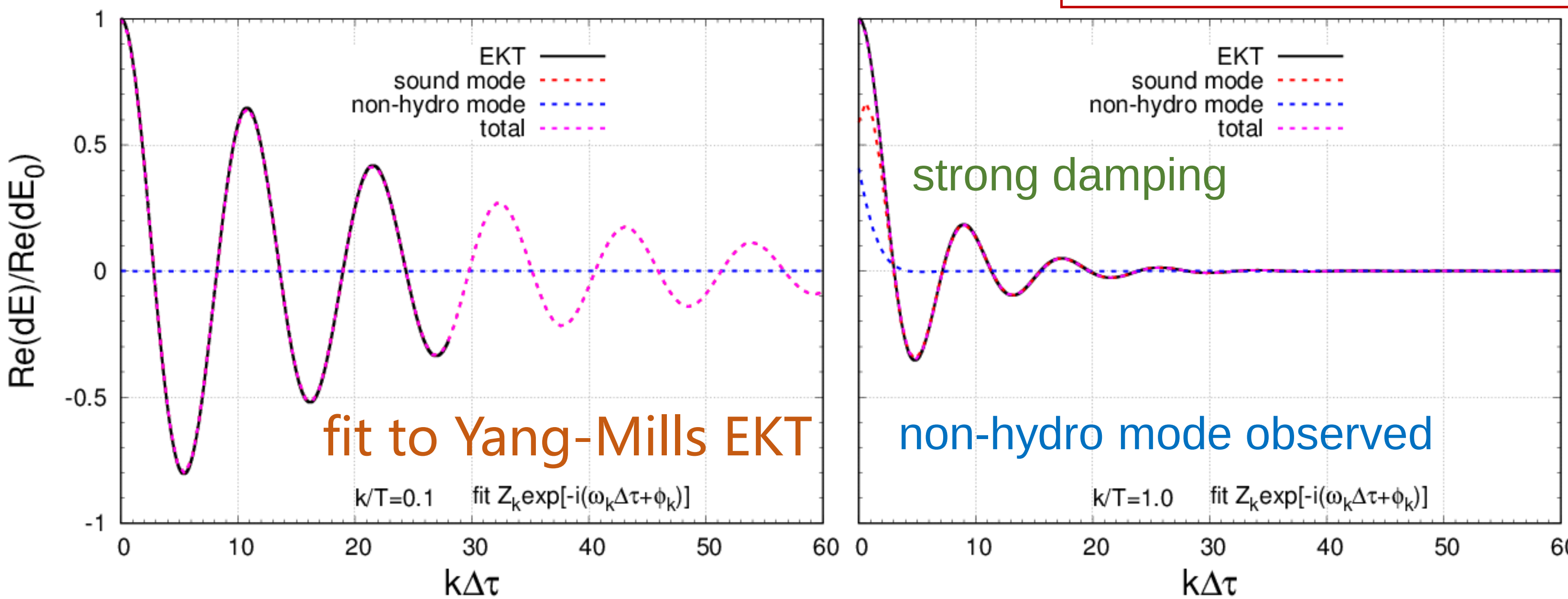
Simple relation to calculate response function

$$\delta T^{\mu\nu}_{\mathbf{k},(s)}(\tau,\mathbf{k}) = \frac{T^{\tau\tau}(\tau)}{T^{\tau\tau}(\tau_0)}\tilde{G}^{\mu\nu}_{\alpha\beta}(\mathbf{k},\tau,\tau_0)\delta T^{\mu\nu}_{\mathbf{k},(s)}(\tau_0,\mathbf{k})$$

Initial conditions

$$f_a(\tau_0,\vec{p}) = \frac{1}{e^{p/T}-1}$$

$$\delta f_{\mathbf{k},a}(\tau_0,\vec{p}) = -\frac{\delta T}{T}p\partial_p f_a(\tau_0,\vec{p})$$

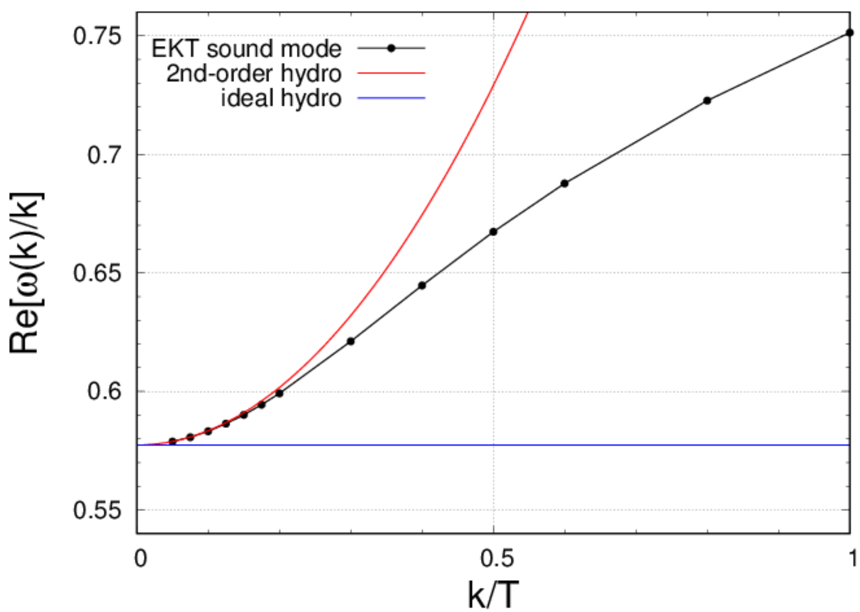


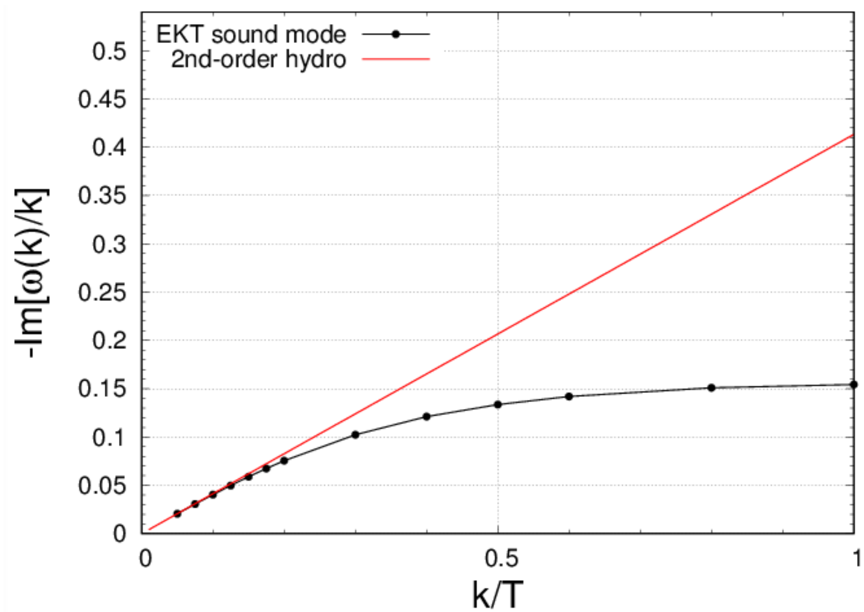
XD, Schlichting, in progress

Comparing to hydrodynamics

Dispersion relations in Yang-Mills EKT

For different k-wave modes





Compare to 2nd-order hydrodynamic

For different k-wave modes, 2nd-order hydro has

$$\omega_{1,2} = \pm c_s k - i\Gamma k^2 \pm \frac{\Gamma}{c_s}\left(c_s^2 \tau_\Pi - \frac{\Gamma}{2}\right) k^3 + \mathcal{O}(k^4)\,, \quad \Gamma = \frac{d-2}{d-1}\,\frac{\eta}{\varepsilon + P}$$

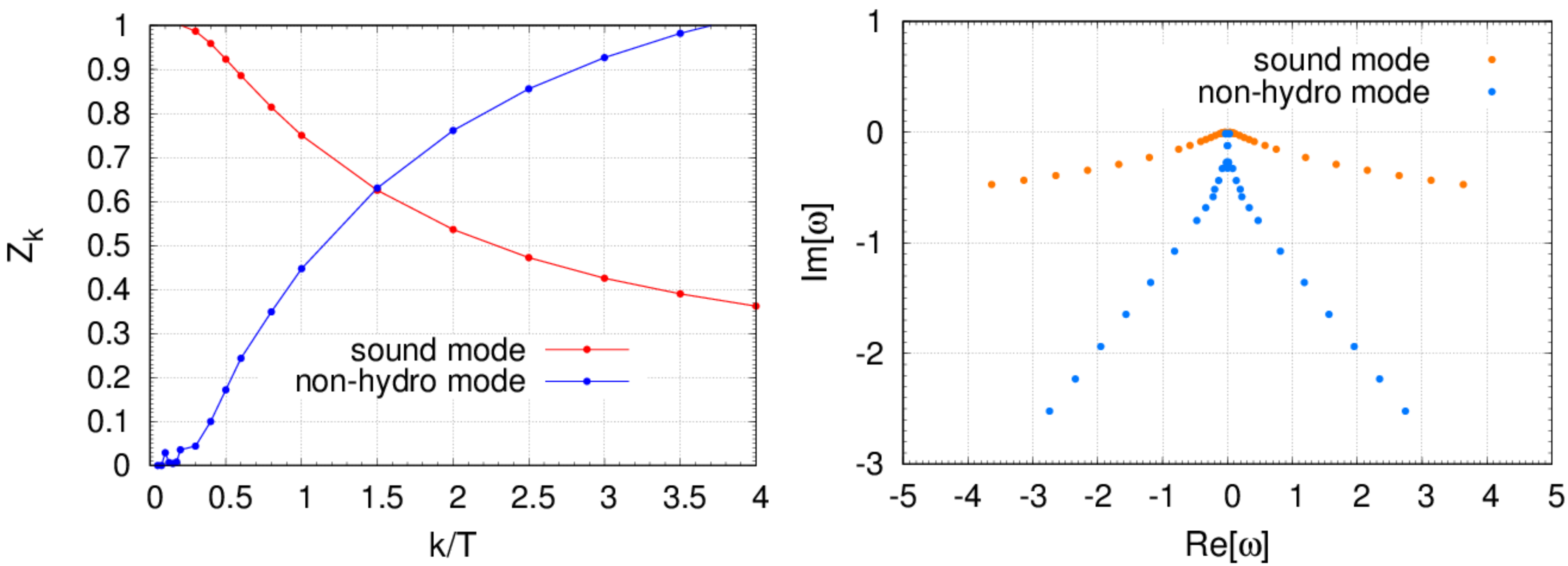
Baier, Romatschke, Son, Starinets, Stephanov, JHEP04(2008)100
Keegan, Kurkela, Mazeliauskas, Teaney, JHEP08(2016)171

XD, Schlichting, in progress

Non-hydrodynamic modes

Residue and poles in the complex plane

Pre-equilibrium Yang-Mills plasma described by a sound mode + a non-hydro mode



More discussion:
RTA: Romatschke, EPJC76(2016)352, Kurkela, Wiedeman, EPJC79(2019)776
AdS/CFT: Buchel, Heller, Noronha, PRD94(2016)106011

XD, Schlichting, in progress

Conclusions

From theory to practice

Conclusions

- **Pre-equilibrium QCD plasmas**
  - QCD effective kinetic theory numerical solver at finite density
- **Turbulence in pre-equilibrium QCD plasmas**
  - Self-similar scaling equilibration
  - Kolmogorov-Zakharov spectrum
  - Bottom-up thermalization
- **Pre-equilibrium QCD plasmas and early stage of HICs**
  - Universal attractor solution and its applications:
  - Pre-equilibrium QGP Trajectory in HICs
  - Inferring the initial state from observables via attractor
  - Pre-equilibrium di-lepton production in HICs
- **Including space-time fluctuation**
  - Extension with linearized EKT (now Yang-Mills)
  - Yang-Mills plasma described by a sound mode+a non-hydro mode
  - Towards a complete pre-eq picture in HIC: Extension to QCD in the future!

[1] XD, Schlichting, PRL127(2021)122301.
[2] XD, Schlichting, PRD104(2021)054011
[3] Coquet, XD, Ollitrault, Schlichting, Winn, PLB821(2021)136626
[4] Others, in progress…

Celebrate 100th Anniversary of Prof. C. N. Yang

Since we are discussing Yang-Mills theory today