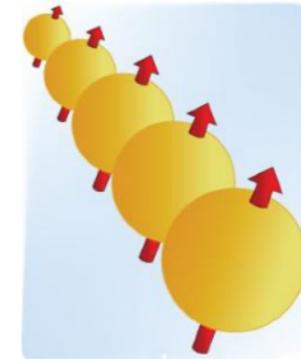
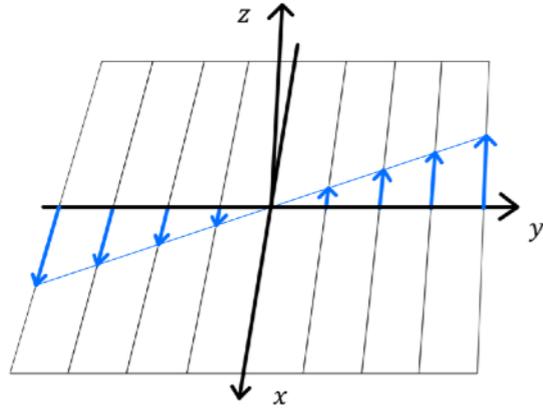


# Shear-induced polarization in heavy-ion collisions



$$\sigma^{ij} = \partial^i u^j + \partial^j u^i - \frac{2}{3} \delta^{ij} \nabla \cdot \vec{u}$$

shear strength

## Yi Yin



Institute of Modern Physics  
(IMP), Lanzhou, Chinese  
academy of sciences

Shuai Liu, YY, 2103.09200

Baochi Fu, Shuai Liu, Longgang Pang,  
Huichao Song, YY, 2103.10403;

CCNU, May. 6th, 2021



Shuai Liu,  
postdoc@IMP



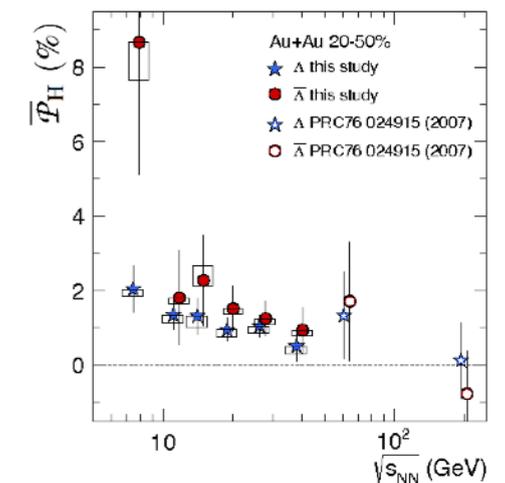
Baochi Fu,  
graduate@Peking  
U.

## Motivation

- Spin transport phenomena probe intriguing properties of quantum materials.

*Han et al, Nature Material, 19'*

- **New frontier:**  $\Lambda$  hyperon spin polarization and non-trivial  $\phi$  meson spin alignment have been observed in heavy-ion collisions.



*STAR, Nature 17'*

- other examples: CVE; magnetic-vorticity coupling ...

- Opportunities: exploring and understanding spin structure of QGP/nuclear matter.

- c.f. “proton spin puzzle”.

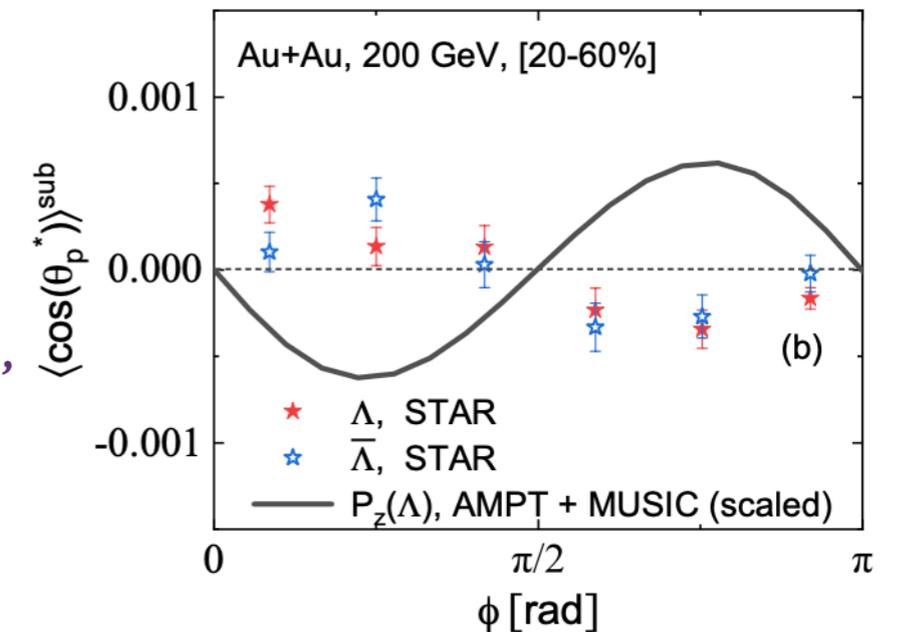
- Vorticity effects in heavy-ion collisions:
  - describe the trends of global (phase-space averaged)  $\Lambda$  polarization.

*Xin-Nian Wang, Zuo-Tang Liang, PRL 05'; Becattini et al, Annals Phys 13'*

- predict **qualitatively different behavior** in the differential measurements. (“sign puzzle”)

*STAR PRL 19'; hydro. simulations by many*

theory and phenomenology. Indeed, at this time, after having played the leading role, theory appears to have been surpassed by the experiments which have proved to be able to mea-



*Baochi Fu et. al, PRC21'*

— *Becattini, Lisa, Annals Phys. 2020*

## Outline

- Response theory
- **New!** Shear-induced spin polarization (SIP)
- The discovery of SIP?

# Response theory

## Spin polarization generation

- Rotation (independent of the direction of  $\vec{p}$ ):

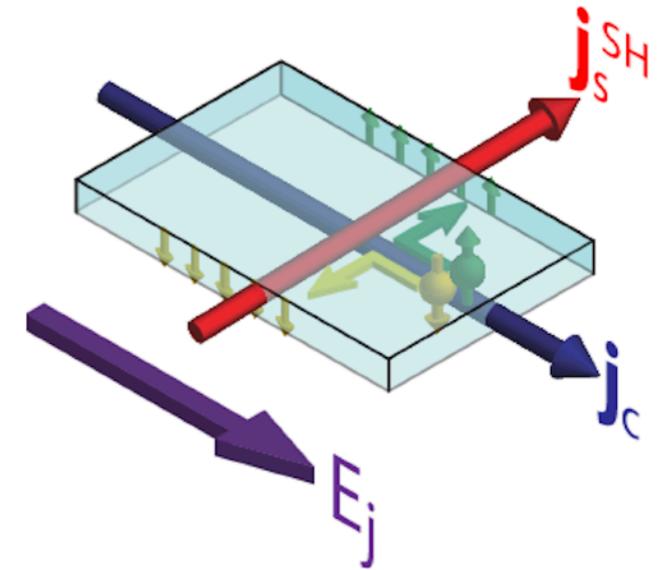
$$\Delta\epsilon = -\vec{s} \cdot \vec{\Omega} \rightarrow \vec{s} \parallel \vec{\Omega}$$

- Spin Hall effect:

$$\vec{s} \propto \vec{p} \times \vec{E}$$

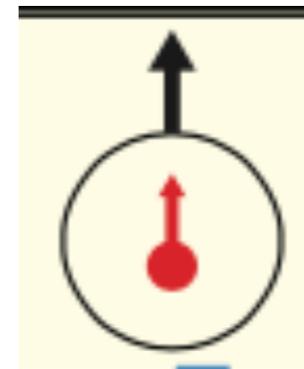
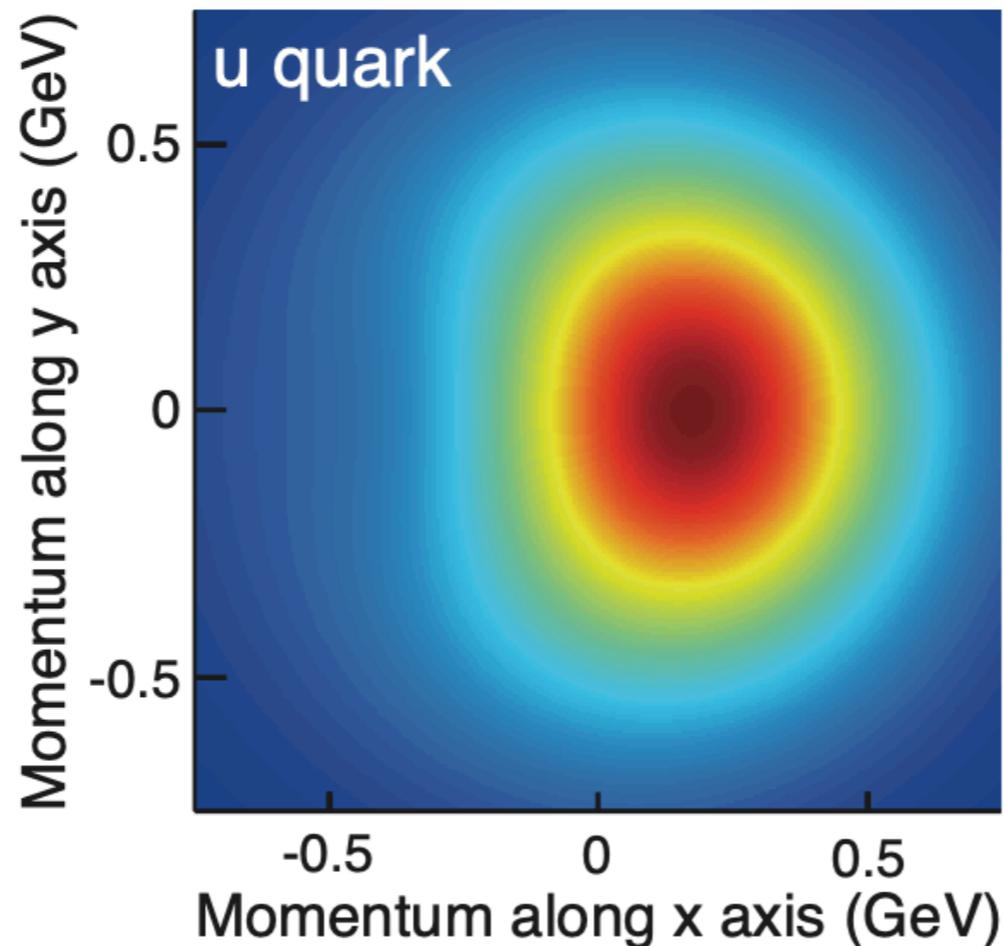
- More generally: **spin-momentum correlation**.

*Landau-Lifshitz volume 5*



*Illustration of spin Hall effect,  
Meyer et al, Nature material 17'*

## Spin-momentum correlation: proton c.f. QCD matter



*EIC white paper*

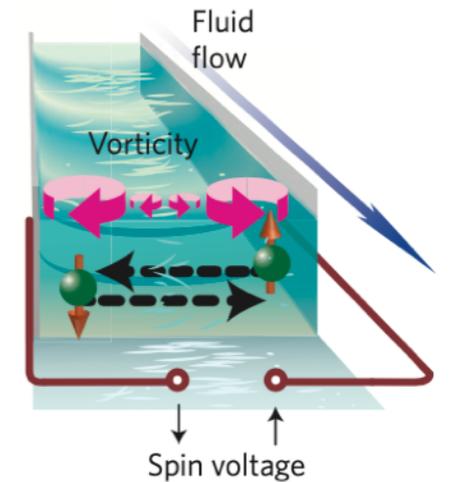
- e.g. transverse momentum dependent quark distribution in a proton.

*In heavy-ion collisions, differential spin polarizations probe the spin-momentum correlation in the medium. (NB: differential spin polarization  $\neq$  local vorticity)*

## Hydro. gradient generates spin polarization

- The gradient of hydro. field (e.g. flow and energy/charge density) leads to spin polarization of fermions in a fluid.

- A familiar example: vorticity-induced polarization.



*Nature phys.,  
Takahashi et al, 16'*

- Nevertheless, vorticity is just one example of hydro. gradients. **Can we systematically analyze all possible effects of hydro. gradients?**

*The answer is yes from response theory*

## Response theory

- Response to hydro. gradients:
  - expansion in gradient.
  - relating expansion coefficients to correlators  $\langle O(x)T^{\mu\nu}(x') \rangle$ .
- E.g.: viscous stress-tensor and viscosities.

$$(T^{\mu\nu})_{\text{vis}} \propto \eta \sigma^{\mu\nu} \quad q_{\text{heat}}^{\mu} \propto \kappa \partial_{\perp}^{\mu} T$$

- Applying similar procedure to spin polarization.

## Axial Wigner function

$$\mathcal{A}^\mu(t, \vec{x}, \vec{p}) = \int d^3\vec{y} e^{-i\vec{y}\cdot\vec{p}} \langle \bar{\psi}(t, \vec{x} - \frac{1}{2}\vec{y}) \gamma^\mu \gamma^5 \psi(t, \vec{x} + \frac{1}{2}\vec{y}) \rangle$$

- related to the phase space distribution of spin polarization vector.
- Building blocks for the gradient expansion:

$$\begin{aligned} \theta &= \partial_\perp \cdot u, \\ \omega^\mu &= \frac{1}{2} \epsilon^{\mu\nu\alpha\lambda} u_\nu \partial_\alpha^\perp u_\lambda, & \beta^{-1} \partial_\perp^\mu \beta, \\ \sigma^{\mu\nu} &= \frac{1}{2} (\partial_\perp^\mu u^\nu + \partial_\perp^\nu u^\mu) - \frac{1}{3} \Delta^{\mu\nu} \theta. \end{aligned}$$

*Tensors from gradient (focus on the neutral fluid)*

$$p^\mu = \epsilon u^\mu + p_\perp^\mu,$$

$$Q^{\mu\nu} = -\frac{p_\perp^\mu p_\perp^\nu}{p_\perp^2} - \frac{1}{3} \Delta^{\mu\nu}, \dots$$

*Tensors formed by single particle momentum*

$$\text{e.g.: } \mathcal{A}^\mu \sim \epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho_\lambda$$

- The most general expression consistent with symmetries (for the neutral fluid):

$$u \cdot \mathcal{A} = \tilde{c}_\omega p \cdot \omega,$$

$$\mathcal{A}_\perp^\mu = c_\omega \omega^\mu + c_T \epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \partial_\lambda \log \beta + g_\sigma \epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho_\lambda + g_\omega Q^{\mu\nu} \omega_\nu$$

vorticity effects

spin Nernst effect

shear-induced polarization

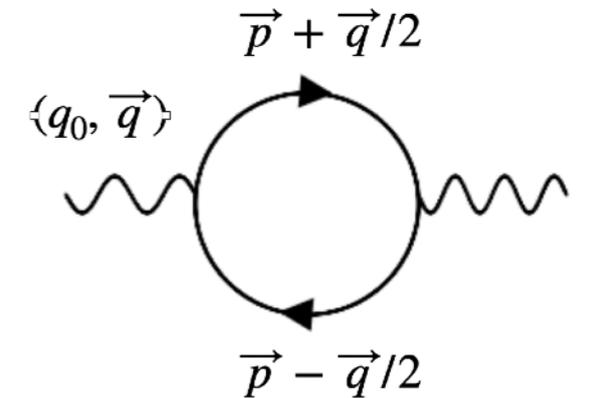
$$\vec{s} \propto \hat{p} \times \nabla \log T$$

Spin-momentum correlation

- Although allowed by symmetry, flow gradient and **momentum quadrupole coupling**, has never been discussed before.
- The expansion coefficients  $\tilde{c}_\omega, c_\omega, c_T, g_\sigma, g_\omega$ , i.e., energy-spin and **momentum-spin correlation functions**, depend on  $T, p \cdot u$  and can be determined from microscopic theories.
- c.f. Sivers function etc.

- Computing retarded correlators:

$$\int_{\vec{y}} e^{i\vec{y}\cdot\vec{p}} \langle \bar{\psi}(t, \vec{x} - \frac{1}{2}\vec{y}) \gamma^\mu \gamma^5 \psi(t, \vec{x} + \frac{1}{2}\vec{y}) T^{\alpha\beta}(0,0) \rangle$$



- For general fermion mass at one-loop:

$$\mathcal{A}_\perp^\mu = (-n'_{FD}) \left[ \underbrace{\omega^\mu}_{\text{vorticity effects}} + \underbrace{\epsilon^{\mu\nu\alpha\lambda} u_\nu p_\alpha \partial_\lambda \log \beta}_{\text{spin Nernst effect}} + \frac{-p_\perp^2}{(p \cdot u)} \underbrace{\epsilon^{\mu\nu\alpha\lambda} u_\nu Q_{\alpha\rho} \sigma^\rho_\lambda}_{\text{shear-induced polarization}} \right] + 0 \times Q^{\mu\nu} \omega_\nu$$

(NB:  $p \cdot \mathcal{A} \sim \mathcal{O}(\text{higher loops})$ , see. Di-Lung Yang, Hattori, Yoshimasa, JHEP 20')

- Open question: dissipative or not?

## Chiral kinetic theory

- (analogous) magnetization current term contributes to axial Wigner function.

$$\vec{\mathcal{A}} = \sum_{\lambda=R,L} \left[ s_{\lambda} \hat{p} f_{\lambda} - \frac{\hat{p}}{2p} \times \nabla f_{\lambda} \right]$$

*Son, Yamamoto, PRD 12;*

*Chen, Son, Stephanov, Yee, YY, PRL 14;*

*Chen, Son, Stephanov, PRL 15;*

- Consider near equilibrium expansion:

$$\sum_{\lambda=R,L} \left[ -\frac{\hat{p}}{2p} \times \nabla n(\epsilon_p - \vec{p} \cdot \vec{u} + \Delta\epsilon_p) \right] \rightarrow \beta n(1-n) \epsilon^{ikj} Q_{jl} \sigma_k^l + (\sim \omega)$$

- Agrees with one loop calculations in the same settings.

*see also Hayata, Mameda, Yoshimasa 2021*

## Comparison

- Summary of one-loop results:

see also Becattini et al,  
2103.10917

Spin polarization=[Vorticity]+[T-gradient]+[Shear]

- Popular approach: spin distribution in a specific hydro. configuration (no entropy production)

*Becattini et al, Annals Phys. 323:2452 (08)  
Annals Phys. 338:32 (13) and follow-ups;*

$$\partial_{\mu}(\beta u_{\nu}) + \partial_{\nu}(\beta u_{\mu}) = 0 \leftrightarrow \partial_{\mu} s^{\mu} = 0$$

$$\rightarrow \mathcal{A}^{\mu} \propto \epsilon^{\mu\nu\alpha\beta} \left[ \partial_{\alpha}(\beta u_{\beta}) - \partial_{\beta}(\beta u_{\alpha}) \right] p_{\nu}$$

- Without shear, agrees with one-loop calculations.
- Response theory applies to general hydro. profile and can be improved systematically through higher-loop/non-perturbative calculations.

# Shear-induced polarization

## Interpretation

- In spatial components

$$(s^i)_{\text{SIP}} \propto \epsilon^{ikj} Q_{jl} \sigma_k^l = \epsilon^{ikj} \hat{p}_j \hat{p}_l \sigma_k^l, \quad Q_{ij} = \hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij}$$

- c.f. Spin Hall effect:

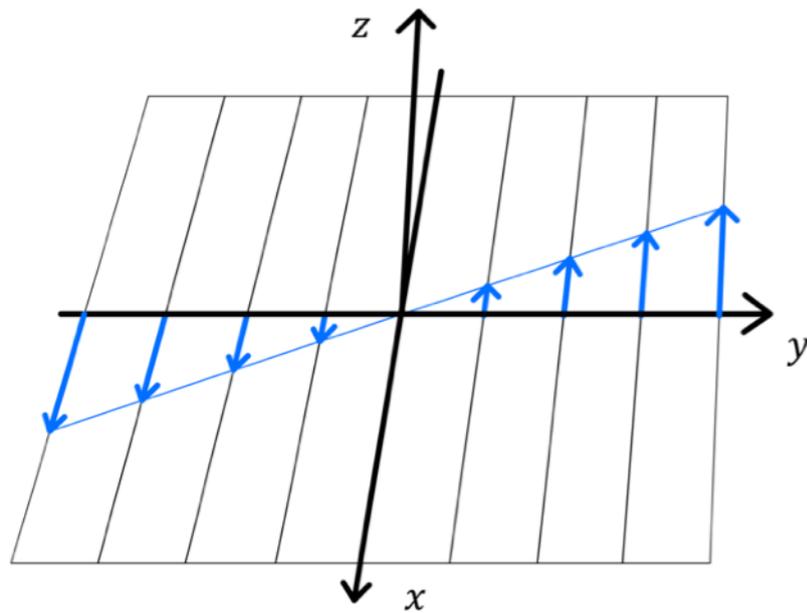
$$\vec{s} \propto \hat{p} \times \left( \overrightarrow{\sigma \odot p} \right)$$

*hydro. force*

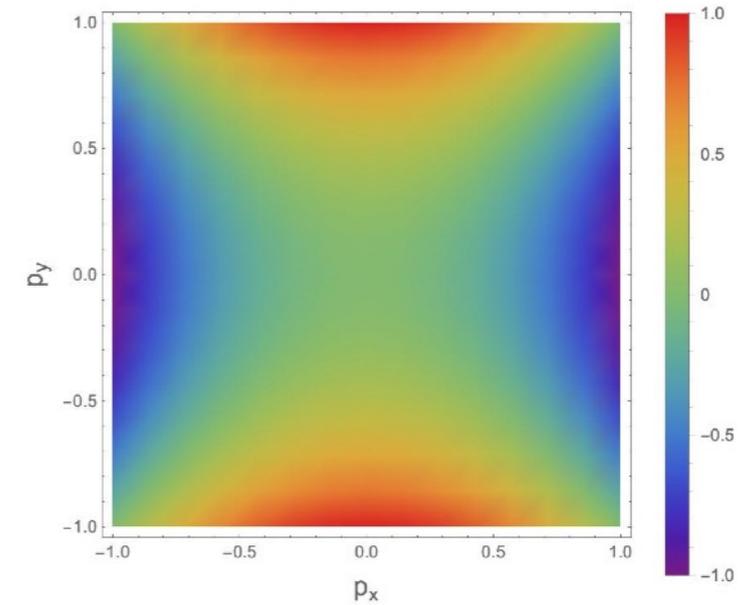
- $\sigma^{\mu\nu} \rightarrow T^{\mu\nu} \rightarrow M^{\mu\nu\alpha} \rightarrow$  spin polarization

$$(M^{\mu\nu\alpha} = x^\nu T^{\mu\alpha} - x^\mu T^{\nu\alpha}) \quad \text{Belinfante, 1940}$$

# Illustration



A standard shear flow profile:  
 $\omega^z \neq 0, \sigma^{xy} \neq 0$

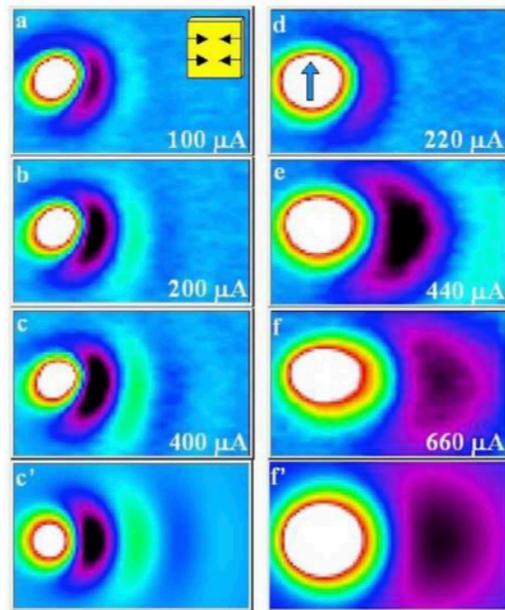


Spin polarization along z-direction in phase space from SIP.

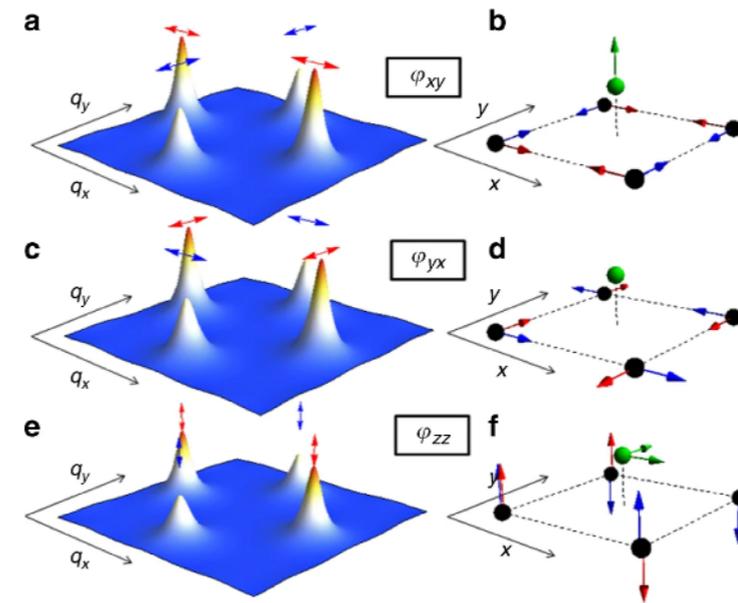
$$\mathcal{A}_{SIP}^i \propto \epsilon^{ikj} Q_{jl} \sigma_k^l, \quad Q_{ij} = \hat{p}_i \hat{p}_j - \frac{1}{3} \delta_{ij}$$

Shear-induced polarization (SIP): imaging anisotropy in a fluid into anisotropy in spin space.

## Observation?



*n-type GaAs, Crooker and Smith, PRL, 04'*



*BaFe<sub>2</sub>As<sub>2</sub>, Kissikov et al, Nature communication, 18'*

- The cousin effect, strain-induced polarization has been observed in crystals and liquid crystals.
- Shear-induced polarization (SIP): generic in fluids.
  - Can we/did we see SIP in heavy-ion collisions?
  - What can we learn ?

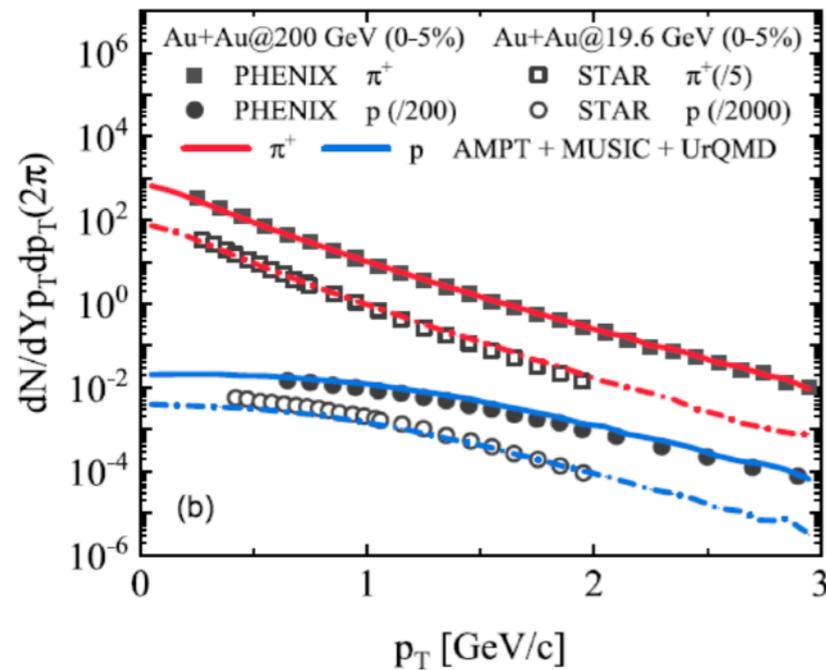
# Heavy-ion collisions

# Hydro. Model

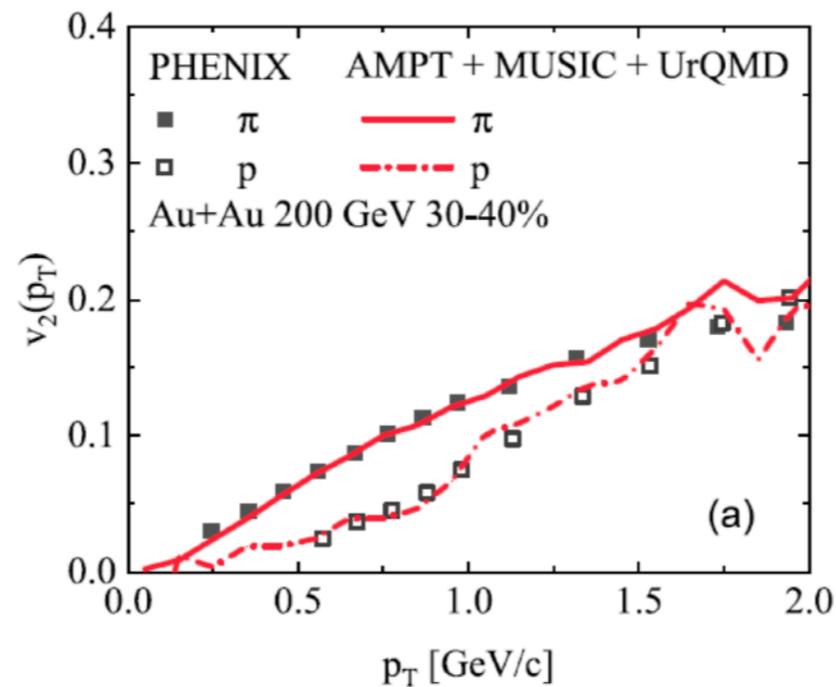


Baochi Fu, graduate@Peking U.

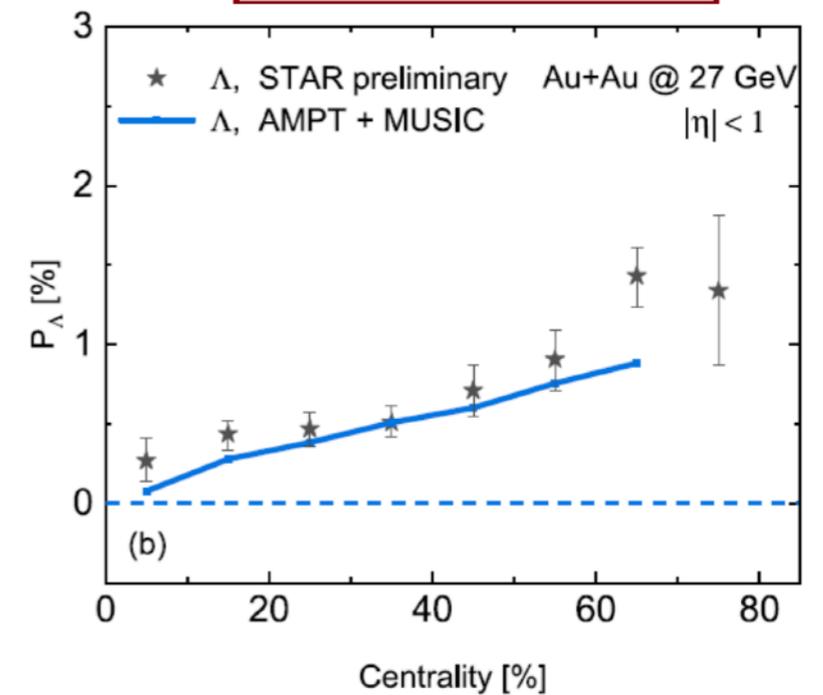
Transverse momentum spectra



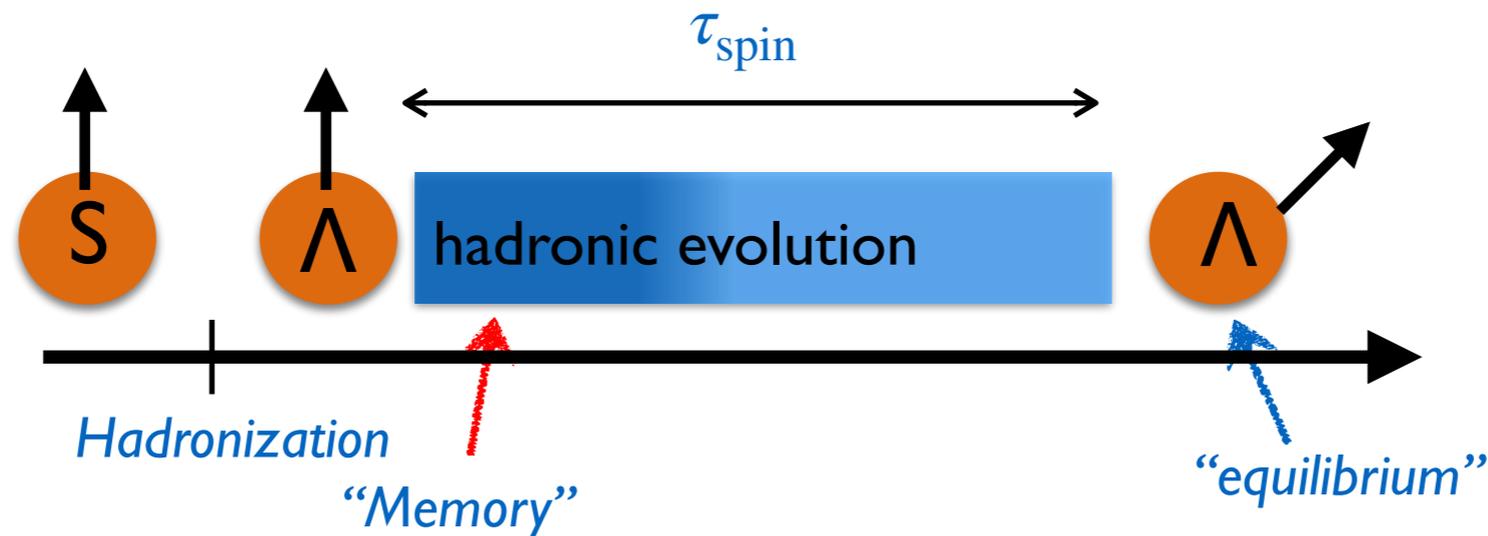
$v_2(p_T)$



Global Polarization from thermal vorticity

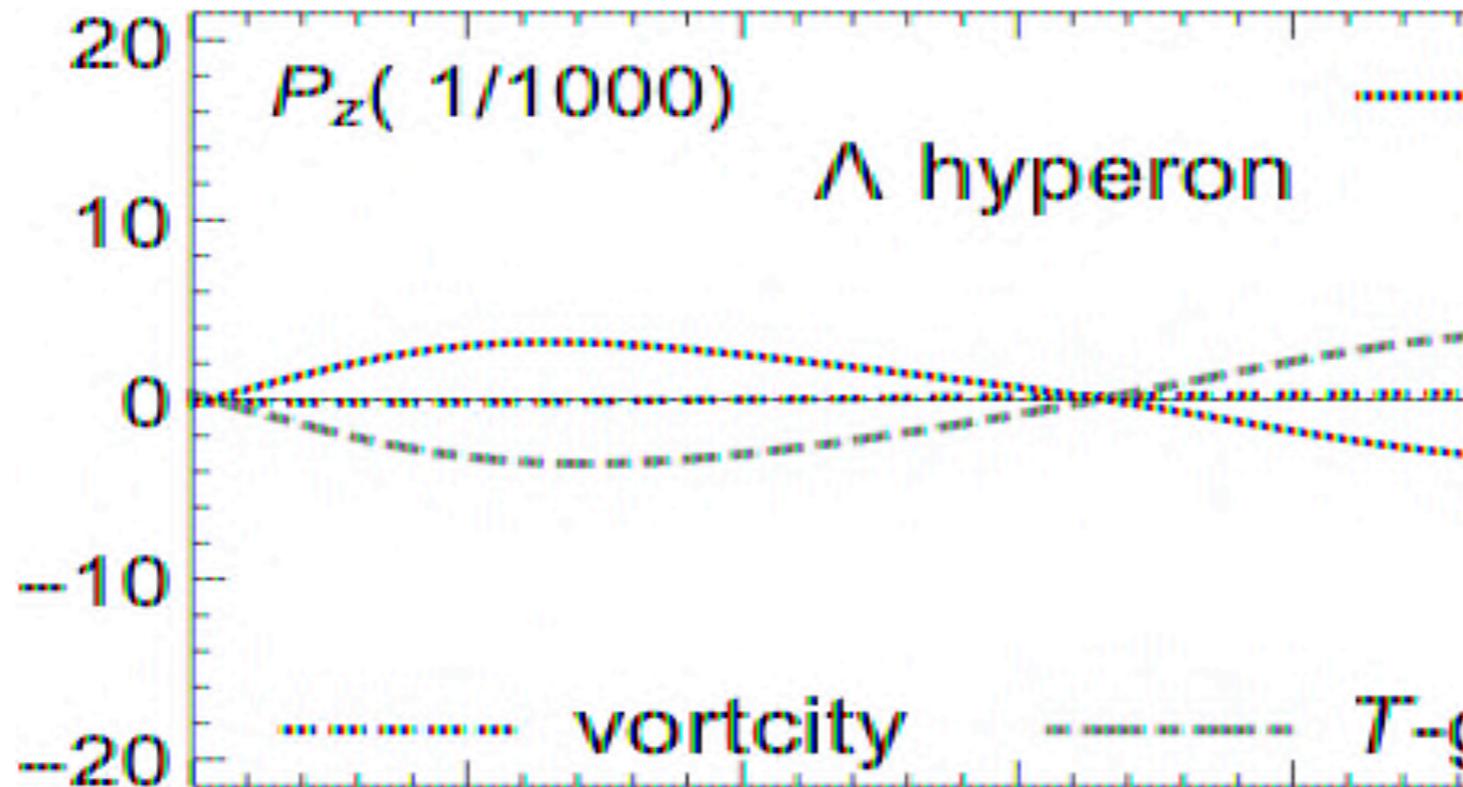


- Hydro. profile from the data-calibrated hydro. modeling (AMPT+MUSIC).

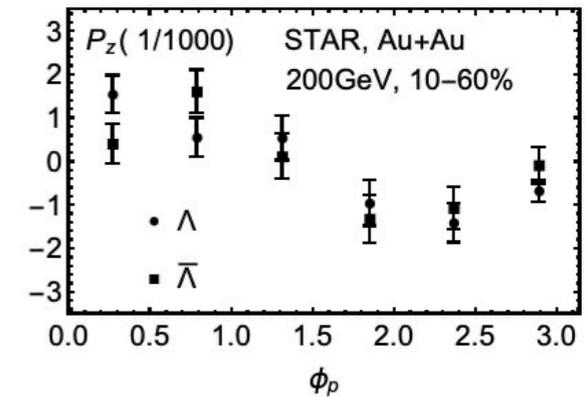


- “Local equilibrium”: expressible in terms of hydrodynamic field.
- Two benchmark scenarios:
  - “Lambda equilibrium”:  $\Lambda$  is born (shortly after hadronization) in equilibrium.
  - “**strange memory**”:  $\Lambda$  memorizes the polarization of strange quarks
- Focus on the qualitative feature.

## Lambda spin polarization along longitudinal direction



vs transverse azimuthal angle  $\phi_p$

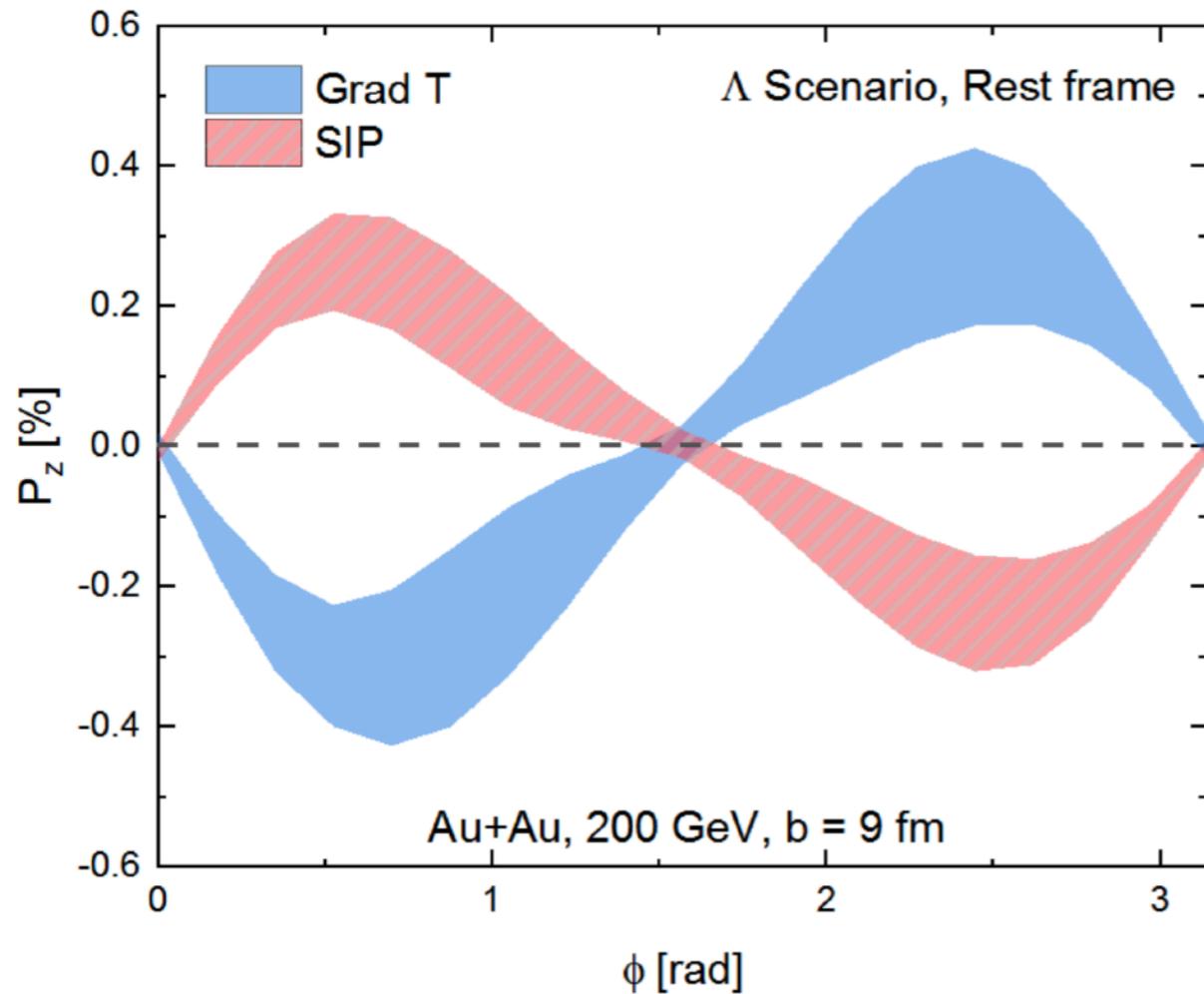


$$\text{Spin polarization} = [\text{vorticity}] + [\text{T-gradient}] + [\text{Shear}]$$

- SLP gives a “right sign” while the effect of T-gradient leads to “wrong sign”.

*also confirmed in 2103.14621 by Becattini et al from an independent hydro. simulation*

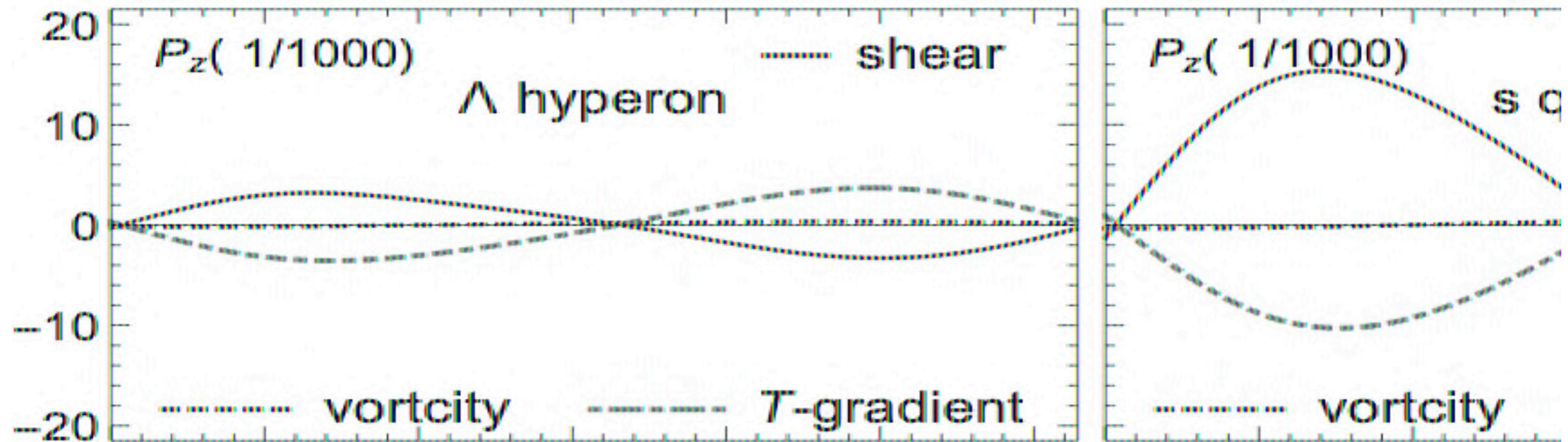
## Sensitivity to the inputs of hydro.



Band: possible flexibility of [Grad T] and [SIP]

- Initial flow: on  $\rightarrow$  off
- Initial condition: AMPT  $\rightarrow$  Glauber
- Shear viscosity: 0.08  $\rightarrow$  off
- Bulk viscosity:  $\zeta/s(T)$   $\rightarrow$  off
- Freeze-out temperature:  
167 MeV  $\rightarrow$  157 MeV

## “Lambda equilibrium” vs “strange memory”

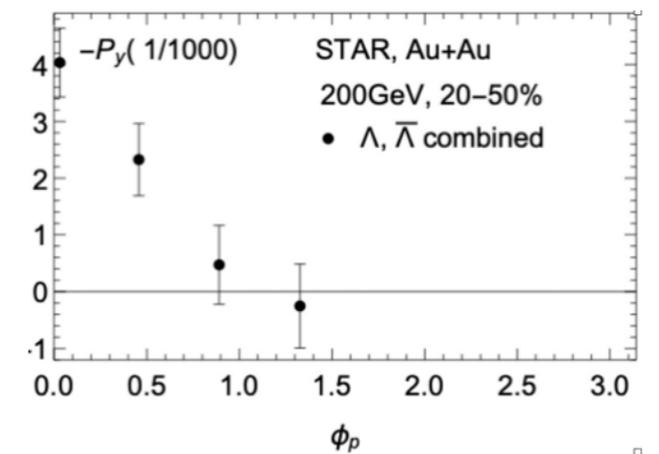
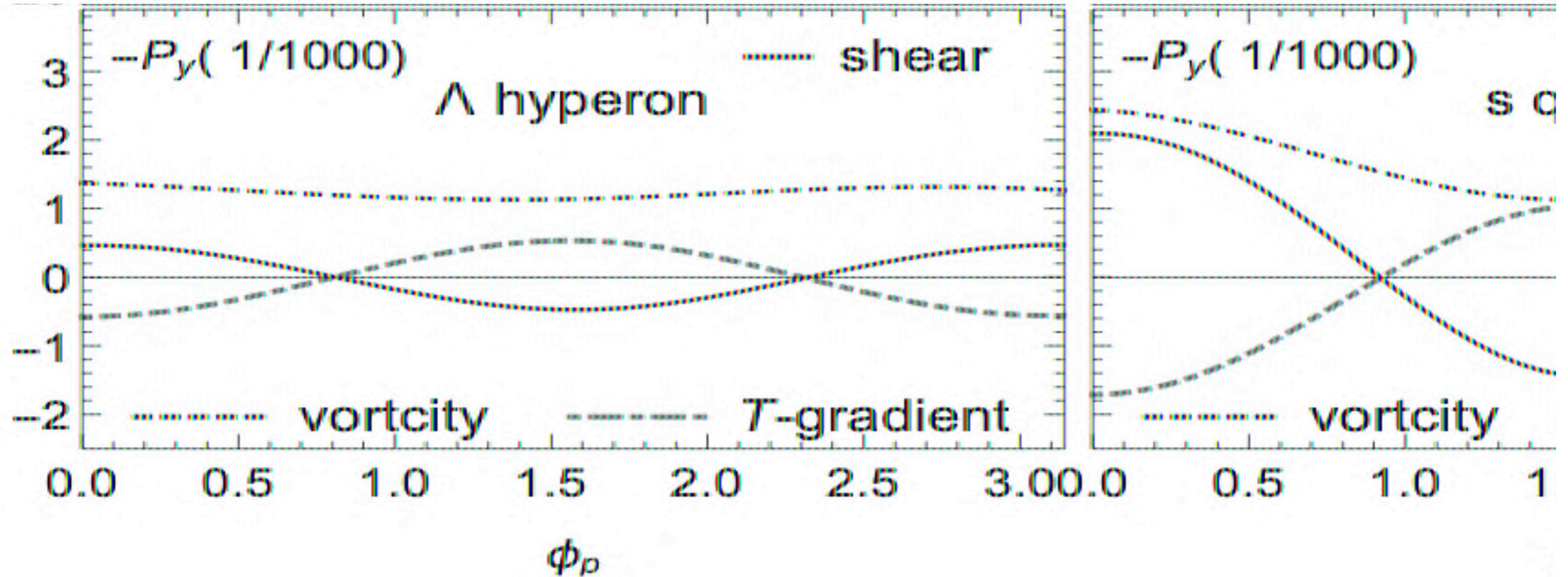


vs transverse azimuthal angle  $\phi_p$

$$\text{Spin polarization} = [\text{Vorticity}] + [\text{T-gradient}] + [\text{Shear}]$$

- SIP becomes more prominent when the mass of spin carrier becomes smaller.

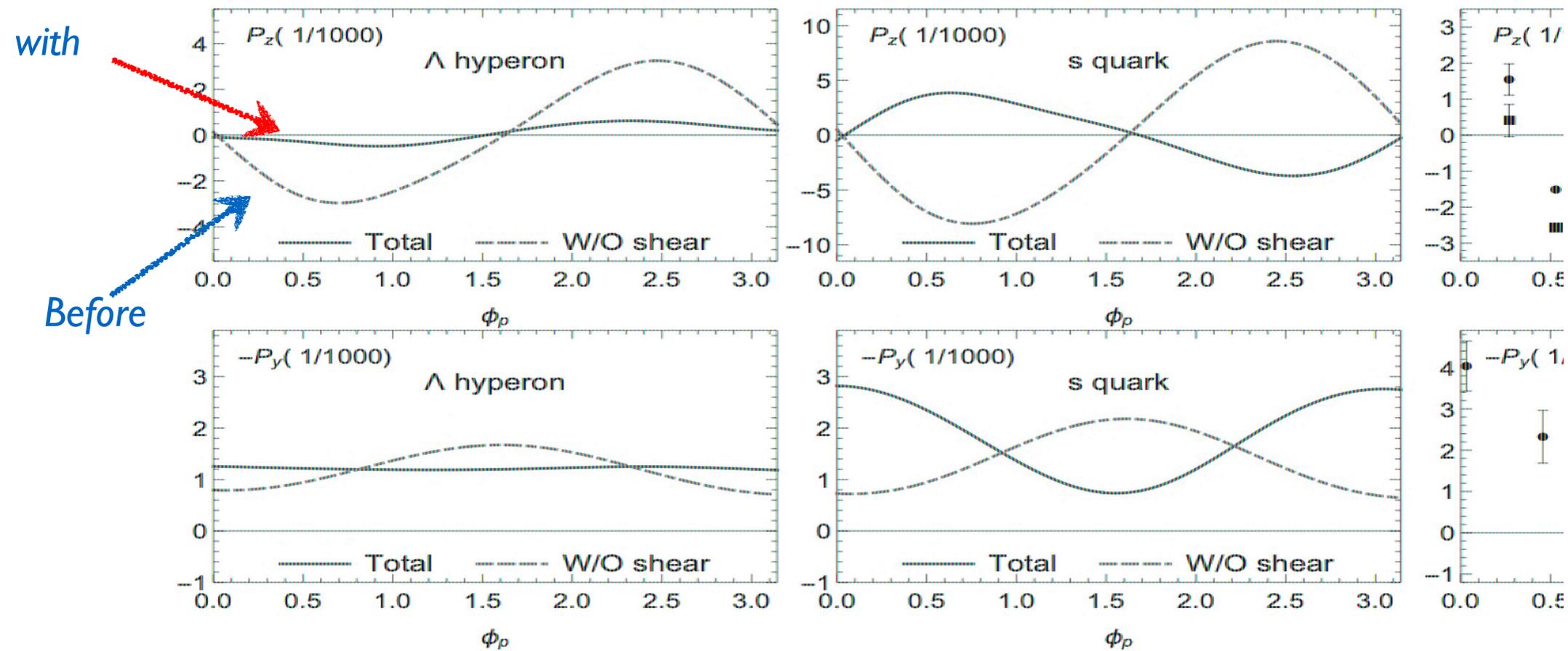
# Similar story for $P_y$



STAR preliminary results

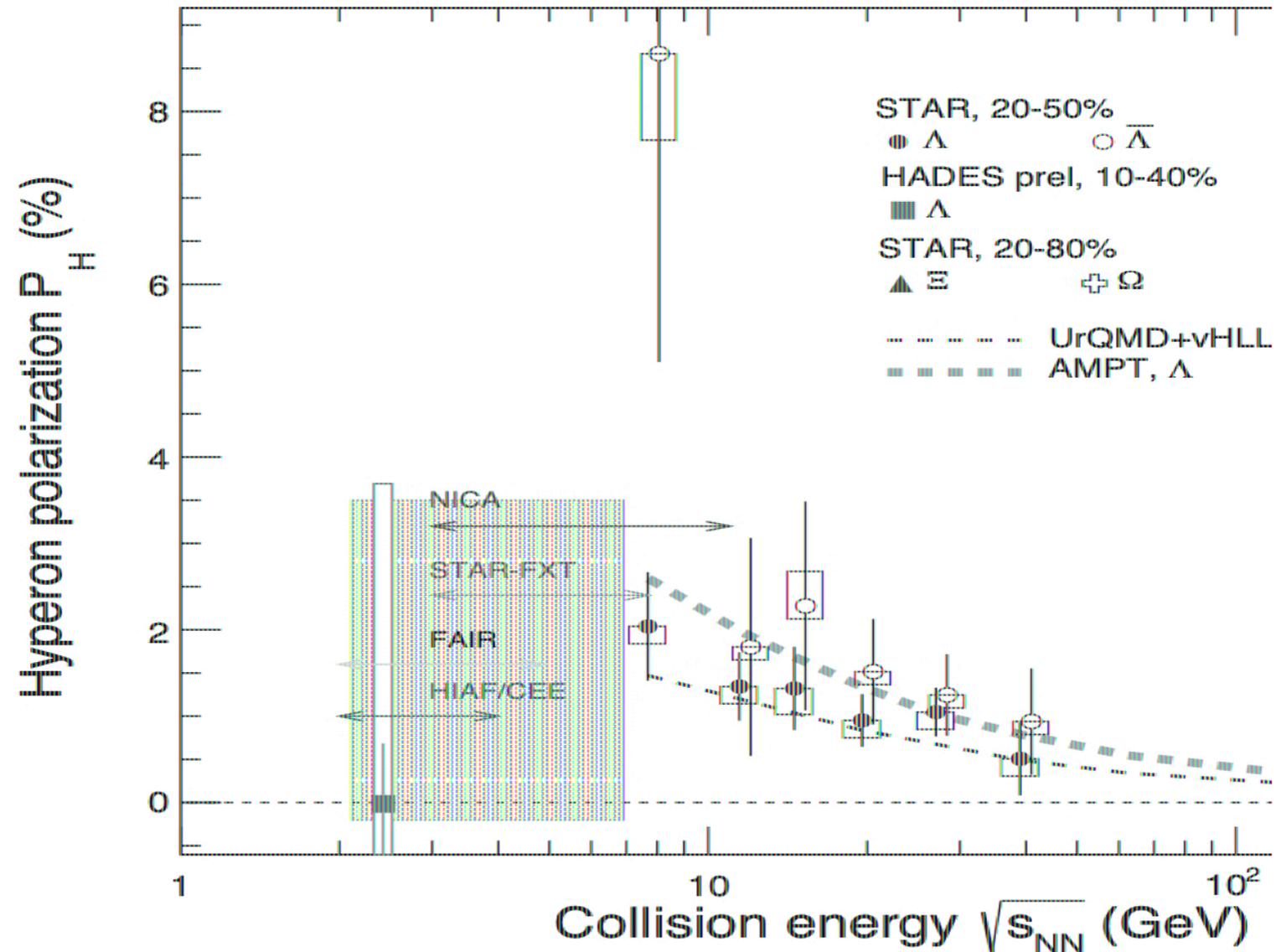
# Total spin polarization

Baochi Fu, Shuai Liu, Longgang Pang, Huichao Song and YY, 2103.10403



- Shear-induced polarization (SIP) effects are indispensable.
- SIP determines the qualitative feature of differential polarization in the “strange memory” scenario .
- $\Lambda$  polarization may probe the properties of QGP.

## The “switching off” of QGP signature



The suppression of global  $\Lambda$  polarization may tell us below which beam energy QGP “is switched” off.

# Summary and outlook

## Summary

- Differential spin polarization: probes the spin-momentum correlation of QCD matter
- Response theory analyses the effects of hydro. gradient on spin polarization systematically.
- **New!** Shear-induced (SIP): spin polarization generation through shear stress tensor.
- SIP and Lambda's memory of strange quark polarization: key to understand qualitative behavior of heavy-ion collisions data.
- For quantitative study, a comprehensive transport theory with spin is crucially needed.

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### Uniaxial strain control of spin-polarization in multicomponent nematic order of BaFe<sub>2</sub>As<sub>2</sub>

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### Observation of the spin Nernst effect

- To claim the discovery of shear-induced polarization and spin Nernst effect (effects of T-gradient), what is the road map?
- Baryonic Spin Hall effect (future): exploring QCD matter at finite baryon density.

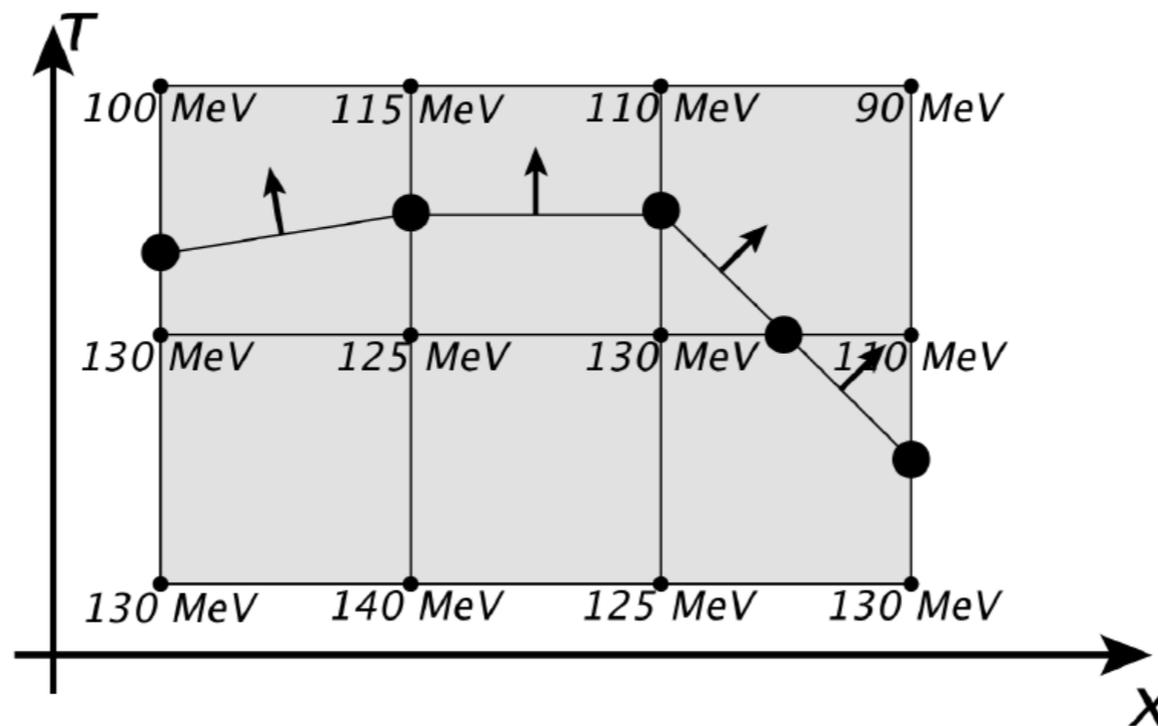
$$\vec{P}_{\pm} \propto \pm \hat{p} \times \nabla \mu_B$$

*Shuai Liu-YY, 2020. | 242 |. Also, Son, Yamamoto, PRD 12; Di-Lung Yang, Hattori, Yoshimasa, PRD 19'*

**Ultimate goal: spin structure of QCD matter.**

# Back-up

## Comments on 2103.14621 by Becattini et al



*T-gradient on equal-T surface from hydro. simulation (Fig. by Schenke, 16')*

- confirms that  $\Delta \Pi$  gives right sign contribution.
- Assuming that T-gradient effects can be ignored on the equal-T surface, they further argue that “Lambda equilibrium” scenario agrees with data with SIP.
- However, T-gradient is generically (and explicitly) non-zero on the equal-T surface.
- So, we stand for “strange memory” scenario.

## Can $\Lambda$ spin flipping rate be small?

Quark model+vector meson dominance  $\Rightarrow$  nucleon (N)-hyperon interaction is mediated by  $\omega$  meson which only couples with constituent u and d quark.

*Jennings, PLB 1990; Cohen-Weber PRC 1991*

However, spin of  $\Lambda$  is carried by s quark. So

(spin-dependent) N- $\Lambda$  interaction  $\ll$  (spin-dependent) N-N interaction.

This picture explains the puzzling experimental results

$$\text{N-}\Lambda \approx \frac{1}{40} \text{N-N}$$

*S.Ajimura et al. PRL 2001*

Under this picture,  $\Lambda$  spin flip rate could be (much) smaller than its equilibration rate  $\Rightarrow$  worthy checking in future.