Quantum Kinetic Theory

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I will discuss the following nontrivial questions:

- Spin distribution
- Beyond quasi-particles
- Covariance & equal-time
- Local & global equilibrium
- Relaxation time approximation

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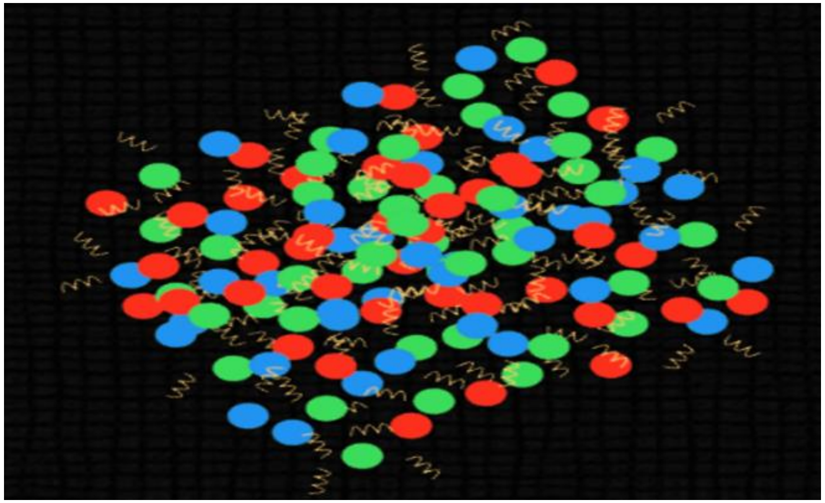
Introduction

Classical transport



1) Boltzmann equation for particle number f:

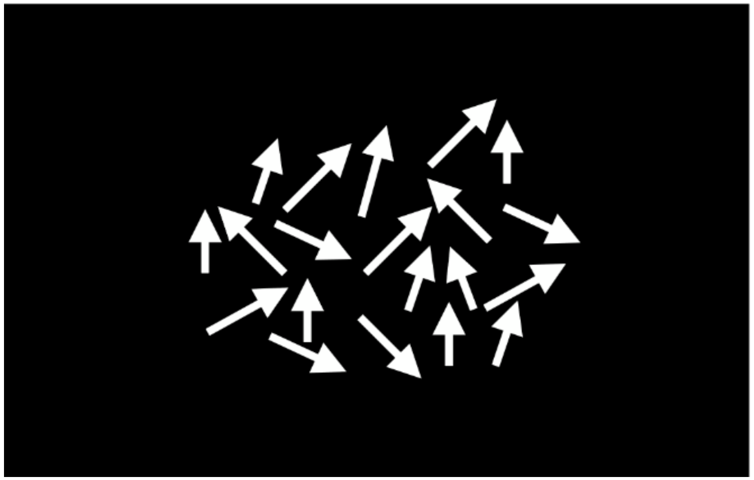
$$\left(p^\mu \partial_\mu + F^\mu \partial_\mu^p\right) f(x,p) = C$$

2) Quasi-particles (on-shell):

$$\left(p^2 - m^2\right) f(x,p) = 0 \rightarrow \ f(x,\vec{p})$$

Spin

- *Spin is a typical quantity in quantum mechanics. Many quantum anomalies in science are induced by spin, for instance the CME and CVE in nuclear physics..*



number distribution f (scalar) → Wigner function W (matrix):

$$W(x,p) = \int d^4 y e^{ipy} \left\langle \hat{\psi}(x+\frac{y}{2})\hat{\bar{\psi}}(x-\frac{y}{2}) \right\rangle$$

Off-shell

- *Considering inelastic interaction in medium, particles are in general not quasi-particles (off-shell effect), especially in high energy physics.*

1) On-shell → Off-shell:

$$\left(p^2 - m^2\right) f(x,p) \to \left[\left(p^2 - m^2\right) + \hbar \mathcal{A}(p)\right] W(x,p)$$

2) Equal-time formalism

$$W(x,\vec{p}) = \int dp_0 W(x,p)$$

Task:
self-consistently and completely construct a quantum kinetic theory including spin and off-shell effect.

General formalism:

Covariant & equal-time kinetic equations

Wigner function

- *Covariant Wigner operator for fermions interacting with a gauge field:*

$$\widehat{W}(x,p)=\int d^4y e^{ipy}\hat{\psi}(x+\frac{y}{2})e^{iq\int_{-1/2}^{1/2}ds\hat{A}(x+sy)y}\hat{\bar{\psi}}(x-\frac{y}{2})$$

gauge link $e^{iq\int_{-1/2}^{1/2}ds\hat{A}(x+sy)y}$ *to guarantee gauge invariance*

Wigner function

$$W(x,p)=\langle\widehat{W}(x,p)\rangle \qquad (W=\widehat{W}\ in\ quantum\ mechanics), \qquad 4\times 4\ matrix$$

16 distribution functions in phase space

- *Dyson-Schwinger equation for quantum fields $\hat{\psi}$ or Dirac equation for wave function ψ*

→ kinetic equations for $W(x,p)$

QED: D.Vasak, M.Gyulassy and H.-Th.Elze, Ann. Phys. 173, 462(1987)
QCD: H.-Th.Elze and U.Heinz, Phys. Rep. 183, 81(1989)

- *Problem:*

Initial $W(x,p)$ is related to the fields $\hat{\psi}(x)$ and $\hat{A}(x)$ at all times (due to $\int_{-\infty}^{\infty}dy_0$)
→ In general the covariant kinetic equation cannot be solved as an initial value problem !

- *Equal-time Wigner function*

$$W_0(x,\vec{p})=\int d^3\vec{y}e^{-i\vec{p}\cdot\vec{y}}\left\langle\hat{\psi}(x+\vec{y}/2)e^{-iq\int_{-1/2}^{1/2}ds\hat{\vec{A}}(x+s\vec{y})\cdot\vec{y}}\hat{\psi}^+(x-\vec{y}/2)\right\rangle$$

Dirac-Heisenberg-Wigner equation

I.Bialynicki-Birula, P.Gornicki and J.Rafelski, PRD44, 1825(1991)

- *Fermions in external electromagnetic field (quantum mechanics system)*

$$\left(i\gamma^\mu \mathcal{D}_\mu - m\right)\psi(x) = 0$$

→ *DHW transport equation:*

$$D_t W_0 = -\frac{1}{2}\vec{D}\cdot\{\rho_1\vec{\sigma}, W_0\} - \frac{i}{\hbar}\left[\rho_1\vec{\sigma}\cdot\vec{P} + \rho_3 m, W_0\right]$$

$$D_t = \partial_t + q\int_{-1/2}^{1/2} ds\vec{E}(\vec{x} + is\hbar\vec{\nabla}_p)\cdot\vec{\nabla}_p,$$

$$\vec{D} = \vec{\nabla} + q\int_{-1/2}^{1/2} ds\vec{B}(\vec{x} + is\hbar\vec{\nabla}_p)\times\vec{\nabla}_p$$

$$\vec{P} = \vec{p} - iq\hbar\int_{-1/2}^{1/2} dss\vec{B}(\vec{x} + is\hbar\vec{\nabla}_p)\times\vec{\nabla}_p$$

- *However, $W_0(x,\vec{p}) = \int dp_0 W(x,p)\gamma_0$ is not equivalent to $W(x,p)$!*

PZ and U.Heinz, Ann.Phys.245, 311(1996)

We must consider all the energy moments

$$W_n(x,\vec{p}) = \int dp_0 p_0^n W(x,p)\gamma_0 \qquad (n = 0,1,2,\dots).$$

- *Only when particles are quasi-particles (on-shell, $p^2 - m^2 = 0$),*

$$W_n(x,\vec{p}) = E_p^n W_0(x,\vec{p}),$$

$W_0(x,\vec{p})$ is enough to describe the system.

From covariant to equal-time kinetic equations I

PZ and U.Heinz, PRD57, 6525(1998)

以量子力学系统为例。

1) Covariant kinetic equations

$$\left(\gamma^\mu K_\mu - M\right)W = 0$$

$$K_\mu = \Pi_\mu + \frac{i\hbar}{2}D_\mu, \qquad M = M_1 + iM_2,$$

$$\Pi_\mu = p_\mu - iq\hbar\int_{-1/2}^{1/2} ds s F_{\mu\nu}(x - i\hbar s\partial_p)\partial_p^\nu, \qquad M_1 = m_0 - \cos\left(\frac{\hbar}{2}\partial_x\partial_p\right)(m(x) - m_0),$$

$$D_\mu = \partial_\mu - q\int_{-\frac{1}{2}}^{\frac{1}{2}} ds F_{\mu\nu}\left(x - i\hbar s\partial_p\right)\partial_p^\nu \qquad M_2 = \sin\left(\frac{\hbar}{2}\partial_x\partial_p\right)(m(x) - m_0)$$

Constraint (with p_μ, off-shell) **and transport** (with ∂_μ) **equations**

$$\begin{cases}\left[\gamma^\mu\left(K_\mu + K_\mu^+\right) - (M + M^+)\right]W(x,p) = 0\\ \left[\gamma^\mu\left(K_\mu - K_\mu^+\right) - (M - M^+)\right]W(x,p) = 0\end{cases}$$

2) Equal-time hierarchy for $W_n(x,\vec{p})$

p_0-integrating the covariant equations:

$$\begin{cases}Transport\ equations\ for\ W_0(x,\vec{p}) & \rightarrow\ DHW\ equation\\ Constraint\ equation\ for\ W_1(x,\vec{p})\end{cases}$$

p_0-integrating $p_0 \cdot$ (covariant equations):

$$\begin{cases}Transport\ equations\ for\ W_1(x,\vec{p})\\ Constraint\ equation\ for\ W_2(x,\vec{p})\end{cases}$$

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From covariant to equal-time kinetic equations II

PZ and U.Heinz, PRD57, 6525(1998)

3) Spin decomposition

$W^{+} = \gamma_0 W \gamma_0 \neq W$, *the Wigner function itself is not a physics quantity.*

$$W_0(x,\vec{p}) = \frac{1}{4}\left[f_0 + \gamma^5 f_1 - i\gamma^0\gamma^5 f_2 + \gamma^0 f_3 + \gamma^5\gamma^0\vec{\gamma}\cdot\vec{g}_0 + \gamma^0\vec{\gamma}\cdot\vec{g}_1 - i\vec{\gamma}\cdot\vec{g}_2 - \gamma^5\vec{\gamma}\cdot\vec{g}_3\right]$$

D.Vasak, M.Gyulassy and H.-Th.Elze, Ann. Phys. 173, 462(1987)

Conservation laws → Physics of the spin components

f_0 : *number density*, $\vec{g}_0$: *spin density*

f_1 : *helicity density*, f_2 : *topologic charge density*, f_3 : *mass density*

$\vec{g}_1$: *number current*, $\vec{g}_3$: *magnetic moment*

16 constraint equations + 16 transport equations

4) Truncating the hierarchy

spin 1/2 particles: W_0 and W_1 form a closed subgroup

spin 0 particles: W_0, W_1 and W_2 form a closed subgroup

Quantum mechanics systems:
Spin interaction with external fields

Spin interaction with electromagnetic field: $\hbar^0$

PZ and U.Heinz, PRD53, 2096(1996)

$$W_0(x,\vec{p}) = W_0^{(0)}(x,\vec{p}) + \hbar W_0^{(1)}(x,\vec{p}) + \cdots$$

- Constraint equations → only 4 independent components:
  number density f_0 and spin density $\vec{g}_0$

- Boltzmann equation for number density f_0:

$$\left(D_t + \frac{\vec{p}}{E_p}\cdot\vec{D}\right) f_0 - \frac{1}{2E_p}\left(\vec{\nabla} m^2 \cdot \vec{\nabla}_p\right) f_0 = 0$$

$$D_t = \partial_t + q\vec{E}\cdot\vec{\nabla}_p, \qquad \vec{D} = \vec{\nabla} + q\vec{B}\times\vec{\nabla}_p$$

- Bargmann-Michel-Telegdi equation for spin density $\vec{g}_0$:

$$\left(D_t + \frac{\vec{p}}{E_p}\cdot\vec{D}\right)\vec{g}_0 - \frac{1}{2E_p}\left(\vec{\nabla} m^2\cdot\vec{\nabla}_p\right)\vec{g}_0 = \frac{q}{E_p^2}\left[\vec{p}\times\left(\vec{E}\times\vec{g}_0\right) - E_p\vec{B}\times\vec{g}_0\right]$$

the phase-space version of the Bargmann-Michel-Telegdi equation to describe spin precession in electromagnetic fields

V.Bargmann, L.Michel and V.Telegdi, PRL2, 435(1959)

Chiral fermions in electromagnetic field: $\hbar^1$

A.Huang, S.Shi, Y.Jiang, J.Liao and PZ, PRD98, 036010(2018)

CME: a chirality imbalance induced electric current in external magnetic field, a probe of nontrivial topology of QCD.

M.Stephanov and Y.Yin, PRL109, 162001 (2012)
D.Son and N.Yamamoto. PRD, 87, 85016(2013)
J.Chen, S.Pu, Q.Wang and X.Wang, PRL110, 26230
Y.Hidaka, S.Pu and D.Yang. PRD95, 091901(2017)
Wu, Hou, Ren, PRD 96 (2017)096015
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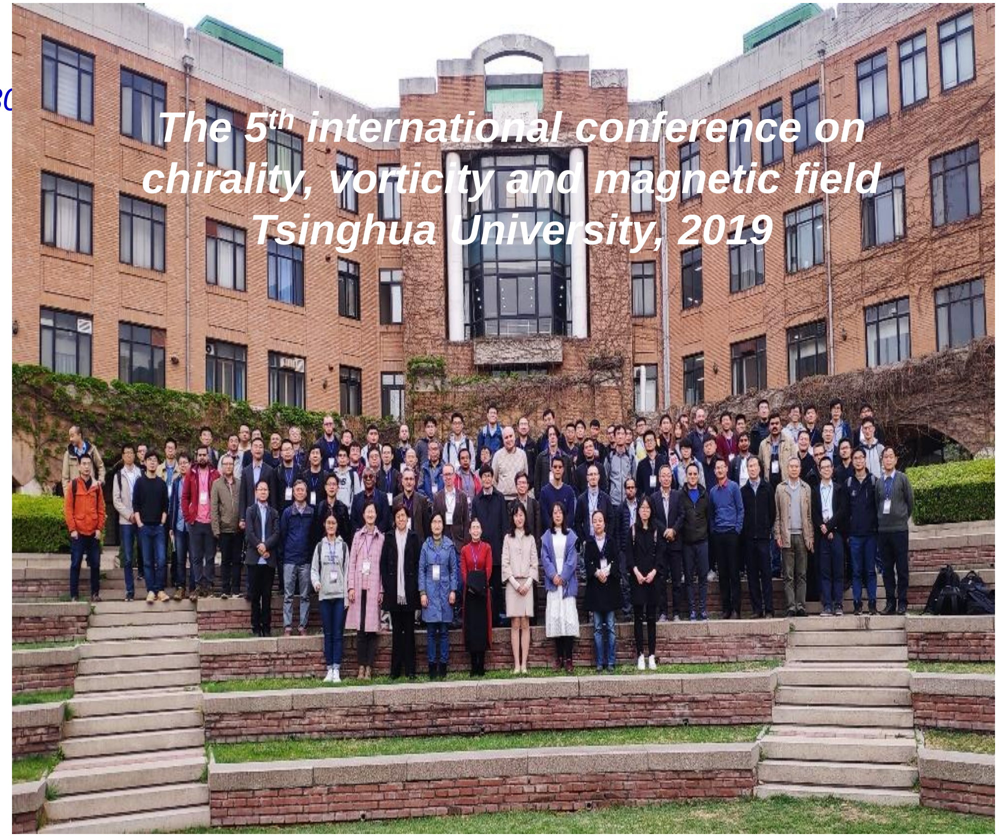


- chiral fermions to the first order in $\hbar$:

$$f_\pm = f_0 \pm f_1$$

$$\begin{cases} \left(\partial_t + \dot{\vec{x}} \cdot \vec{\nabla} + \dot{\vec{p}} \cdot \vec{\nabla}_p\right) f_\pm = -\dfrac{f_\pm - f_\pm^{th}}{\tau} \\ \dot{\vec{x}} = \dfrac{1}{1 + q\vec{B} \cdot \vec{b}_\pm} \dfrac{\vec{p}}{|\vec{p}|} \left(1 + 2q\vec{B} \cdot \vec{b}_\pm\right) \\ \dot{\vec{p}} = \dfrac{1}{1 + q\vec{B} \cdot \vec{b}_\pm} q \dfrac{\vec{p}}{|\vec{p}|} \times \vec{B}, \qquad \vec{b}_\pm = \pm \dfrac{\vec{p}}{|\vec{p}|^3} \end{cases}$$

- Mass correction

$$\partial_t f_\chi + \dot{\vec{x}} \cdot \vec{\nabla} f_\chi + \dot{\vec{p}} \cdot \vec{\nabla}_p f_\chi = \frac{\hbar m}{\sqrt{G}} A[\vec{g}_0]$$

Z.Wang and PZ, PRD100, 014015(2019)

Mass correction is from the residual spin effect, it is small.

Charge separation

A.Huang, Y.Jiang, S.Shi, J.Liao and PZ, PLB777, 177(2018)

■ *Analytic solution of the chiral transport equation*

$$f_{\pm}(x,\vec{p}) = f_{\pm}(x_0,\vec{p}_0)e^{-\frac{t-t_0}{\tau}} + \frac{1}{\tau}\int_{t_0}^{t} f_{\pm}^{eq}(x',\vec{p}')e^{-\frac{t-t\prime}{\tau}}dt'$$

$$\vec{B} = B(t)\vec{e}_y$$

initial imbalance $f_{\pm} \sim 1 \pm \lambda$

■ *Charge separation in nuclear collisions*

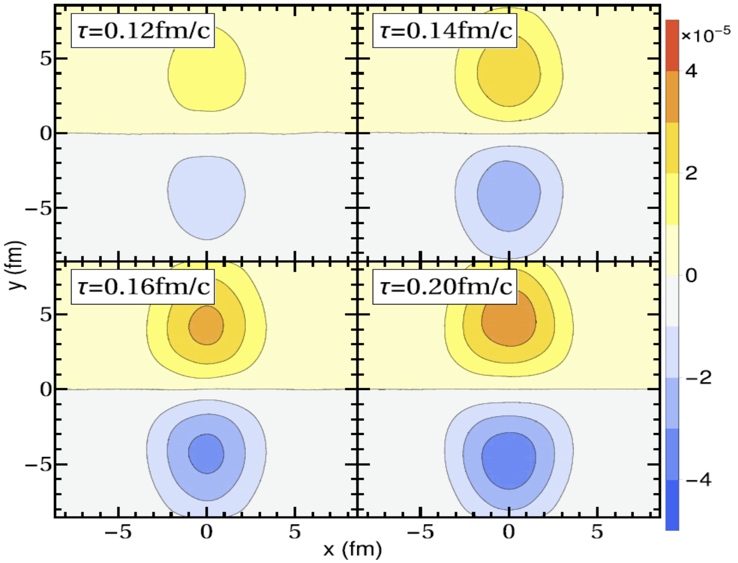


Fig. 3. Net charge density n^Q (normalized by ξQ_s^3) on the $x-y$ plane at different time (computed with FD initial distribution, ECHO magnetic field, $\tau_R = 0.1$ fm/c and $\lambda_5 = 0.2$).

(Heavy) fermions in vortical field

S.Chen, Z.Wang and PZ, ArXiv: 2101.07596

- *In rotational frame,*

$$\left[i\gamma^\mu\partial_\mu + \gamma_0\boldsymbol{\omega}\cdot\boldsymbol{J} - m\right]\psi = 0.$$

- *Covariant kinetic equation*

$$\left[\gamma^\mu K_\mu + \frac{\hbar}{2}\gamma^5\gamma^\mu\omega_\mu - m\right]W(x,p) = 0,$$

- *Equal-time (on-shell) transport equations at order $\hbar^0$*

$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\boldsymbol{\nabla} - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\boldsymbol{\nabla}_p\right] f_0^{(0)\pm} = 0,$$

$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\boldsymbol{\nabla} - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\boldsymbol{\nabla}_p\right] \mathbf{g}_0^{(0)\pm} = -\boldsymbol{\omega}\times\mathbf{g}_0^{(0)\pm}.$$

- *Equal-time (off-shell) transport equations at order $\hbar^1$*

$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\boldsymbol{\nabla} - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\boldsymbol{\nabla}_p\right] f_0^{(1)\pm} = 0,$$

$$\left[\partial_t + \left(\pm\frac{\boldsymbol{p}}{\epsilon_p} + \boldsymbol{x}\times\boldsymbol{\omega}\right)\cdot\boldsymbol{\nabla} - (\boldsymbol{\omega}\times\boldsymbol{p})\cdot\boldsymbol{\nabla}_p\right] \mathbf{g}_0^{(1)\pm} = -\boldsymbol{\omega}\times\mathbf{g}_0^{(1)} - \frac{1}{2\epsilon_p^4}\boldsymbol{p}\times(\boldsymbol{p}\times\boldsymbol{\omega})(\boldsymbol{p}\cdot\boldsymbol{\nabla})f_0^{(0)\pm}.$$

Quantum field systems:

Spin interaction among particles (collision term)

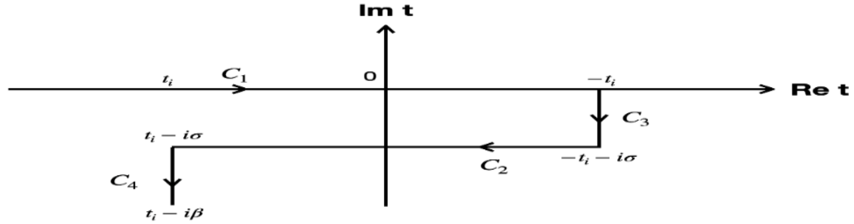
Li, Yee, PRD100 (2019), 056022
Ayala, PLB.801 (2020) 135169
Kapusta, Rrapaj, Rudaz, PRC101 (2020), 024907
Chen, Son, Stephanov, PRL115 (2015) 021601
Zhang, Fang, Q.Wang, X.Wang, PRC100 (2019), 064904
Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612
Weickgenannt, Speranza, Sheng, Wang, Rischke, arXiv:2005.01506, arXiv: 2103.10636
……

Kadanoff-Baym equations

Dyson-Schwinger equation

$$S(x,y) = S^0(x,y) + \int d^4z d^4w S^0(x,w)\Sigma(w,z)S(z,y)$$

Schwinger-Keldish formalism



$$\left(i\hbar\gamma^\mu\partial^x_\mu - m + \hbar\Sigma^\delta(x)\right)S^<(x,y) \;=\; -\hbar\int_{-\infty}^{\infty}dz\left(\Sigma_R(x,z)S^<(z,y) + \Sigma^<(x,z)S_A(z,y)\right)$$

$$S^<(x,y)\left(i\hbar\gamma^\mu\overleftarrow{\partial}^y_\mu + m - \hbar\Sigma^\delta(y)\right) \;=\; +\hbar\int_{-\infty}^{\infty}dz\left(S^<(x,z)\Sigma_A(z,y) + S_R(x,z)\Sigma^<(z,y)\right)$$

Constraint and transport equations with collision terms

$$\{(\gamma^\mu p_\mu - m), S^<\} + \frac{i\hbar}{2}[\gamma^\mu, \nabla_\mu S^<] = \frac{i\hbar}{2}([\Sigma^<, S^>]_* - [\Sigma^>, S^<]_*)$$

$$[(\gamma^\mu p_\mu - m), S^<] + \frac{i\hbar}{2}\{\gamma^\mu, \nabla_\mu S^<\} = \frac{i\hbar}{2}(\{\Sigma^<, S^>\}_* - \{\Sigma^>, S^<\}_*)$$

$$A * B = AB + \frac{i\hbar}{2}[AB]_{P.B.} + \mathcal{O}(\hbar^2)$$

Yang, Hattori, Hidaka, JHEP 2020 (2020) 070, arXiv:2002.02612

- *Spin decomposition for Wigner function, self-energies and Collision terms*

General collision terms

Z.Wang, X.Guo and P.Zhuang, [arXiv:2009.10930 [hep-th]]

- **0th order transport**

$$p\cdot\nabla\mathcal{V}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(0)}_\mu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{A}^{(0)\lambda}} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma^{(0)\alpha\nu}_T\mathcal{A}^{(0)\lambda}} - p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(0)}_\nu},$$

$$p\cdot\nabla\mathcal{A}^{(0)}_\mu = m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(0)}_\mu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} + \widehat{\Sigma^{(0)}_{A\mu}p^\nu\mathcal{V}^{(0)}_\nu} - p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(0)\nu}},$$

$\widehat{FG} = \bar{F}G - F\bar{G}$

Local collision term
Dynamical effect ,
e.g. diffusion effect

- **1st order transport**

$$\begin{aligned}p\cdot\nabla\mathcal{V}^{(1)}_\mu =& + m\widehat{\Sigma^{(0)}_S\mathcal{V}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{V}^{(1)}_\mu} - p_\mu\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(1)}_\nu} - \frac{p_\nu}{m}\epsilon_{\rho\sigma\alpha\mu}p^\rho\widehat{\Sigma^{(0)\alpha\nu}_T\mathcal{A}^{(1)\sigma}} + \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma^{(0)\sigma\nu}_T\mathcal{A}^{(1)\lambda}}\\
&+ m\widehat{\Sigma^{(1)}_S\mathcal{V}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{V\nu}\mathcal{V}^{(0)}_\mu} - p_\mu\widehat{\Sigma^{(1)\nu}_A\mathcal{A}^{(0)}_\nu} - \frac{p_\nu}{m}\epsilon_{\alpha\mu\beta\lambda}p^\beta\widehat{\Sigma^{(1)\alpha\nu}_T\mathcal{A}^{(0)\lambda}} + \frac{m}{2}\epsilon_{\sigma\nu\lambda\mu}\widehat{\Sigma^{(1)\sigma\nu}_T\mathcal{A}^{(0)\lambda}}\\
&+ \frac{1}{2m}p^\nu[\widehat{\Sigma^{(0)}_{T\mu\nu},(p^\alpha\mathcal{V}^{(0)}_\alpha)}]_{\rm P.B.} - \frac{m}{2}[\widehat{\Sigma^{(0)}_{T\mu\nu},\mathcal{V}^{(0)\nu}}]_{\rm P.B.} + \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}p^\nu[\widehat{\Sigma^{(0)\alpha}_A,\mathcal{V}^{(0)\beta}}]_{\rm P.B.}\\
&- \frac{1}{2m}p_\nu\Sigma^{(0)}_{T\alpha\mu}\widehat{\nabla^{[\alpha}\mathcal{V}^{(0)\nu]}} + \frac{1}{2m}p_\nu\widehat{\Sigma^{\alpha\nu(0)}_T\nabla_{[\alpha}\mathcal{V}^{(0)}_{\mu]}} + \frac{1}{2}\epsilon_{\beta\nu\lambda\mu}\Sigma^{(0)\beta}_A\widehat{\nabla^\nu\mathcal{V}^{(0)\lambda}}\\
&+ \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}(\widehat{\nabla^\alpha\Sigma^{(0)\nu}_V)\mathcal{A}^{(0)\beta}} - \frac{1}{2m}p_\mu(\widehat{\nabla^\nu\Sigma^{(0)}_P)\mathcal{A}^{(0)}_\nu} - \frac{1}{2m}(\widehat{p^\nu\nabla_\nu\Sigma^{(0)}_P)\mathcal{A}^{(0)}_\mu} + \frac{p^\nu}{2m}\epsilon_{\mu\nu\alpha\beta}(\widehat{\nabla^\alpha\Sigma^{(0)}_S)\mathcal{A}^{(0)\beta}},\end{aligned}$$

$$\begin{aligned}p\cdot\nabla\mathcal{A}^{(1)}_\mu =& + m\widehat{\Sigma^{(0)}_S\mathcal{A}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{V\nu}\mathcal{A}^{(1)}_\mu} + p^\nu\widehat{\Sigma^{(0)}_{A\mu}\mathcal{V}^{(1)}_\nu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(0)\alpha\beta}_T\mathcal{V}^{(1)\lambda}} - p_\mu\widehat{\Sigma^{(0)}_{A\nu}\mathcal{V}^{(1)\nu}}\\
&+ m\widehat{\Sigma^{(1)}_S\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{V\nu}\mathcal{A}^{(0)}_\mu} + p^\nu\widehat{\Sigma^{(1)}_{A\mu}\mathcal{V}^{(0)}_\nu} + \frac{m}{2}\epsilon_{\alpha\beta\lambda\mu}\widehat{\Sigma^{(1)\alpha\beta}_T\mathcal{V}^{(0)\lambda}} - p_\mu\widehat{\Sigma^{(1)}_{A\nu}\mathcal{V}^{(0)\nu}}\\
&- \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}(\widehat{\nabla^\sigma\Sigma^{(0)\nu}_V)\mathcal{V}^{(0)\rho}} - \frac{m}{2}[\widehat{\Sigma^{(0)}_P,\mathcal{V}^{(0)}_\mu}]_{\rm P.B.} + \frac{1}{2m}p_\mu[\widehat{\Sigma^{(0)}_P,(p^\nu\mathcal{V}^{(0)}_\nu)}]_{\rm P.B.}\\
&+ \frac{1}{2}\epsilon_{\mu\sigma\nu\rho}\nabla^\sigma\widehat{\Sigma^{(0)\nu}_A\mathcal{A}^{(0)\rho}} + \frac{1}{2}\epsilon_{\nu\mu\alpha\beta}[\widehat{\Sigma^{(0)\nu}_A,(p^\alpha\mathcal{A}^{(0)\beta})}]_{\rm P.B.} - \frac{m}{2}[\widehat{\Sigma^{(0)}_{T\mu\nu},\mathcal{A}^{(0)\nu}}]_{\rm P.B.} + \frac{p_\mu}{2m}[\widehat{\Sigma^{(0)}_{T\rho\nu},(p^\rho\mathcal{A}^{(0)\nu})}]_{\rm P.B}\\
&- \frac{1}{2m}p_\sigma\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)\nu}}) + \frac{1}{2m}p^\nu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\mu\nu}\mathcal{A}^{(0)}_\sigma}) + \frac{1}{2m}p_\mu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\sigma\nu}\mathcal{A}^{(0)\nu}}) - \frac{1}{2m}p^\nu\nabla^\sigma(\widehat{\Sigma^{(0)}_{T\sigma\nu}\mathcal{A}^{(0)}_\mu}).\end{aligned}$$

- **Nonlocal collision term**
- **Related to spatial derivatives**
- **Correlated transport of V&A**
- **Polarization can be generated in a initially unpolarized syste**

the interaction needs to be specified to calculate the off-diagonal self-energy $\Sigma^>$ & $\Sigma^<$

NJL model

Fermionic 2 by 2 collisions by contact interaction

$$\mathcal{L} = \bar{\psi}(i\hbar\partial\!\!\!/ - m)\psi + G(\bar{\psi}\psi)^2.$$

- *Collision term (non-equilibrium state) is model dependent.*
- *Local equilibrium state is determined by detailed balance principle (loss term and gain term cancel to each other), and therefore model independent.*
- *Global equilibrium state is determined by detailed balance + Killing condition*

$$\nabla^\sigma \beta^\lambda + \nabla^\lambda \beta^\sigma = 0.$$

$$\beta_\mu = u_\mu / T$$

and therefore model independent.

- *The model dependence of near-equilibrium state is weaker.*

Spin distribution in equilibrium state

Z.Wang, X.Guo and P.Zhuang, [arXiv:2009.10930 [hep-th]]

Detailed balance → Zeroth-order spin distribution:

$$A_\mu^{(0)}(x,p) = 0$$

Detailed balance → First-order spin distribution:

$$\mathcal{A}_\mu^{(1)} = -\frac{1}{(2\pi)^3 4E_p}\epsilon_{\mu\nu\sigma\lambda}p^\nu\nabla^\sigma\beta^\lambda f_V'(p)$$
$$-\frac{1}{(2\pi)^3[m^2+p\cdot(p+q)]4E_p}\epsilon_{\mu\nu\rho\lambda}(p+q)^\nu p^\rho p_\sigma(\nabla^\sigma\beta^\lambda+\nabla^\lambda\beta^\sigma)f_V'(p).$$

Single-particle Wigner function should have only one momentum scale. The second term depends on two momenta p and q, it should disappear ! This is guaranteed by the Killing condition!

$$\mathcal{A}_\mu^{(1)} = -\frac{1}{(2\pi)^3 4E_p}\epsilon_{\mu\nu\sigma\lambda}p^\nu\nabla^\sigma\beta^\lambda f_V'(p)$$

- Spin polarization generated from coupling between vector and axial-vector charge.
- The equilibrium spin polarization is created by a thermal vorticity and is orthogonal to the momentum.
- *Spin can reach only global equilibrium, very different from the number distribution!*

Anderson-Witting relaxation time approximation (RTA)

$$p \cdot \partial f(x,p) = -\frac{f(x,p) - f_{eq}(x,p)}{\tau(p)}$$

For spin-1/2 particles, the RTA of the collision kernel in Anderson-Witting form

$$\Big(\gamma^\mu p_\mu - m\Big)\mathcal{W}(x,p) + \frac{i\hbar}{2}\gamma^\mu \nabla_\mu \mathcal{W}(x,p) \;=\; -\frac{i\hbar}{2}\gamma^\mu u_\mu \frac{\delta \mathcal{W}(x,p)}{\tau(x,p)}$$

Different degrees of freedom (spin components) have the same thermalization time.

Is it true?

RTA from Kadanoff-Baym equations

Z.Wang and P.Zhuang, [arXiv:2105.00915 [hep-ph]].

Quantum kinetic equations in states close to equilibrium:

$$W = W_{th} + \delta W$$

relation between AW-RTA and KB-RTA

AW-RTA

$$(p\cdot\partial)\mathcal{S} = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{S}$$

$$(p\cdot\partial)\mathcal{A}_i = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{A}_i + \frac{1}{\tau}(p\cdot u)\left[\hbar\omega_i\delta f_V\right].$$

KB - RTA

$$p\cdot\nabla\begin{pmatrix}\delta\mathcal{S}\\ \delta\mathcal{A}_\mu\end{pmatrix} = \begin{pmatrix}\omega_0(p) & +\hbar\omega_s^\mu(p)\\ -\hbar\omega_{s\mu}(p) & \omega_0(p)\end{pmatrix}\begin{pmatrix}\delta\mathcal{S}\\ \delta\mathcal{A}_\mu\end{pmatrix},$$

$$\omega_0(p) = m\widehat{\Sigma_S} + p^\nu\widehat{\Sigma_{V\nu}}$$

$$\omega_{s\mu}(p) = -m\widehat{\Sigma_{A\mu}} + \frac{1}{2}\epsilon_{\mu\nu\sigma\rho}p^\nu\widehat{\Sigma_T^{\sigma\rho}} + \frac{p_\mu}{m}p^\nu\widehat{\Sigma_{A\nu}} + \frac{1}{2m}\epsilon_{\mu\nu\rho\sigma}p^\rho(\nabla^\sigma\widehat{\Sigma_V^\nu}) - \frac{1}{2}(\partial_\mu\widehat{\Sigma_P})$$

Same damping rate for charge and spin

Polarization effect related to T-vorticity

Same relaxation time for different effects

Same damping rate for charge and spin

Polarization effect related to thermal vorticity, due to global equilibrium

Different relaxation time for different effect

Compare the both: specify the interaction

In local rest frame $\tau_0 = -E_p/\omega_0$

Different thermalization time scales

Z.Wang and P.Zhuang, [arXiv:2105.00915 [hep-ph]].

NJL contact interaction

AW-RTA

$$(p\cdot\partial)\mathcal{S} = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{S}$$

$$(p\cdot\partial)\mathcal{A}_i = -\frac{1}{\tau}(p\cdot u)\delta\mathcal{A}_i + \frac{1}{\tau}(p\cdot u)\left[\hbar\omega_i\delta f_V\right].$$

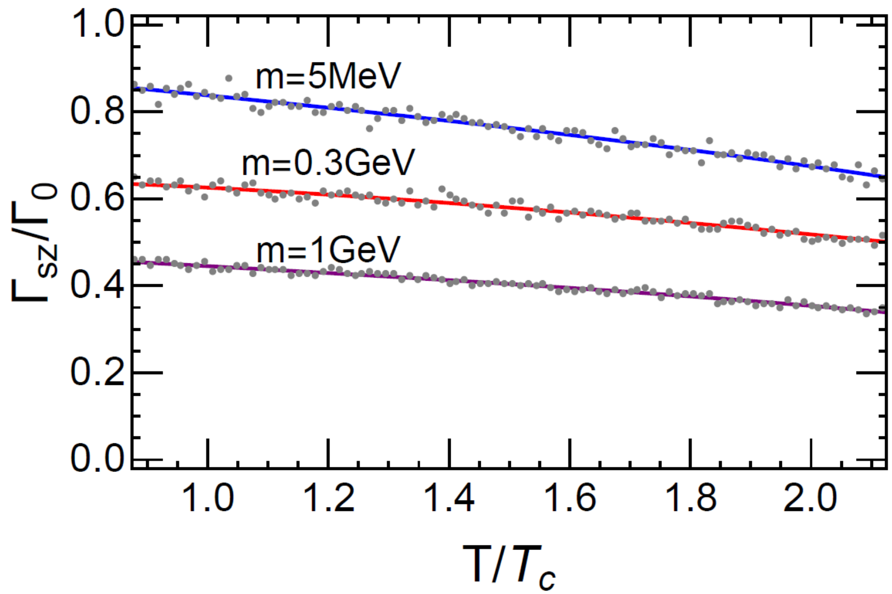
$$\frac{\Gamma_{si}}{\Gamma_0} = \frac{\tau_0}{\tau_{si}} = \frac{2m}{\omega}\frac{\omega_{si}}{\omega_0}$$

KB-RTA

$$p^\mu\partial_\mu f_V = -\Gamma_0\delta f_V$$

$$p^\mu\partial_\mu \mathbf{A} = -\Gamma_0\delta\mathbf{A} + \mathbf{\Gamma}_s\left(\hbar\omega\delta f_V\right)$$

Evaluate the ratio between both time-scales



charge is thermalized earlier and spin is thermalized later.

Summary

- *A self-consistent way to go from quantum field theory (quantum mechanics) to quantum kinetic theory in Wigner function formalism.*
- *Massive particles are in general not quasi-particles → equal-time kinetic hierarchy.*
- *Semi-classical expansion.*
- *CME & CVE at first order in $\hbar$.*
- *Spin polarization can be generated via thermal vorticity in global equilibrium state.*
- *Spin is thermalized later in comparison with charge.*
- *……*

谢谢大家！

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