# Quantum kinetic theory for QED

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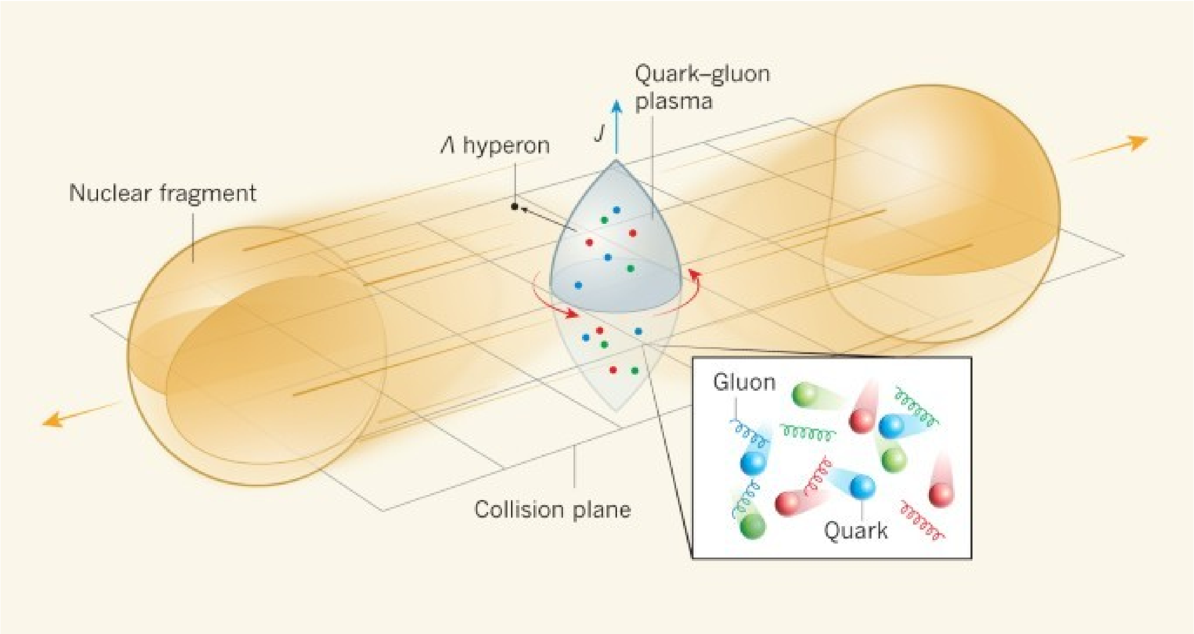
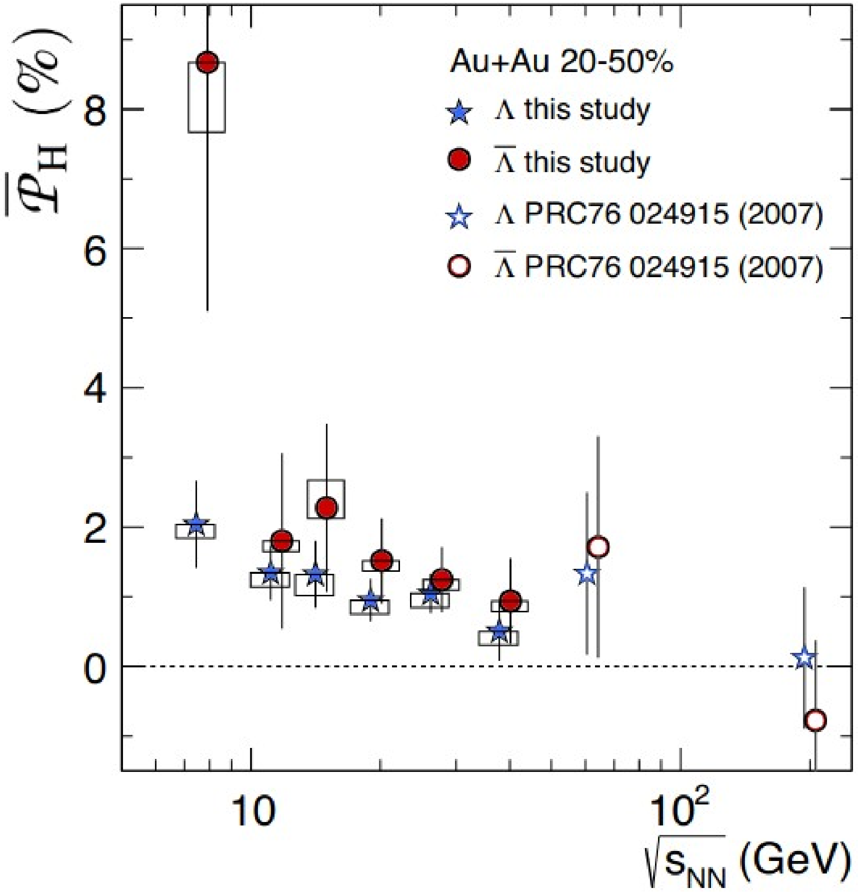
Seminar@CCNU, 2021.9.14

based on 2109.00184

# $\Lambda$ Global Polarization at RHIC

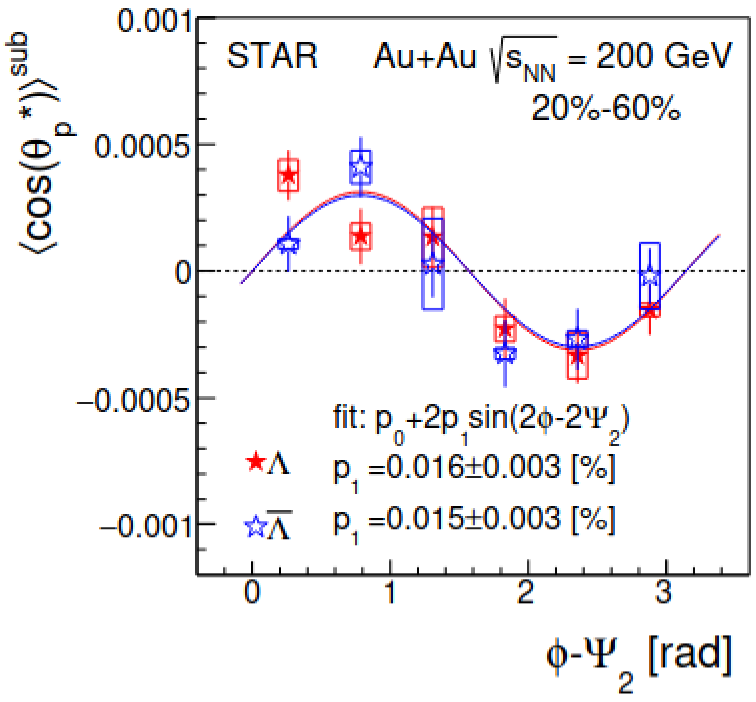


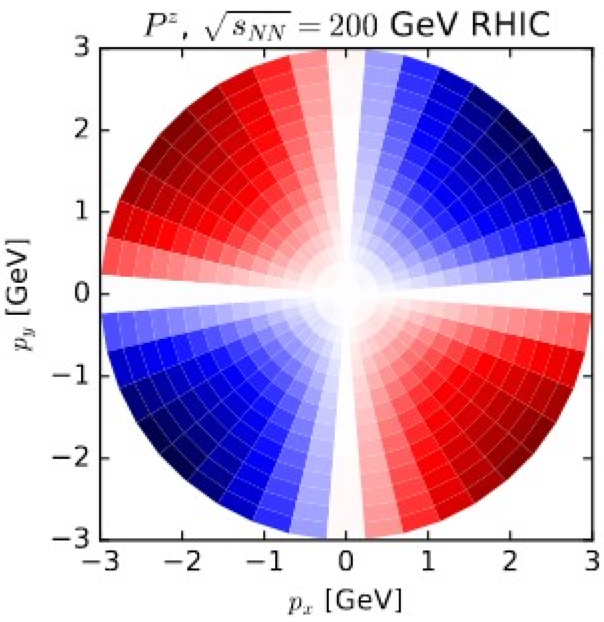

Liang, Wang, PRL 2005




STAR collaboration, Nature 2017

# $\Lambda$ Local polarization: sign puzzle




STAR collaboration, PRL 2019

Becattini, Karpenko, PRL 2018
Wei, Deng, Huang, PRC 2019
Wu, Pang, Huang, Wang, PRR 2019
Liu, Yin 2103.09200. Fu, Liu, Pang, Song, Yin, 2103.10403

thermal vorticity induced

Becattini, et al, PLB 2021, 2103.14621
Yi, Pu, Yang, 2106.00238

+thermal shear induced etc

# A microscopic model: QKT

aim to addressing the following questions

- Conversion from orbital angular momentum to spin angular momentum
- Transports of spin degree of freedom
- Systematic study of spin polarization with collision effect

# A Boltzmann equation for QCD

$$(\partial_t + \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}})\, f_s(\boldsymbol{x}, \boldsymbol{p}, t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$$

Spin-averaged(classical) kinetic theory

$f_s(\boldsymbol{x}, \boldsymbol{p}, t)$ distributions of quarks and transverse gluons

$C_s^{2\leftrightarrow 2}[f]$ : elastic collisions

$C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$ : inelastic collisions

Expect to be the lowest order of QKT in hbar expansion

Arnold, Moore and Yaffe, early 00s

# A derivation of the Boltzmann equation not available!

1. One may begin with the full hierarchy of Schwinger-Dyson equations for (gauge-invariant) correlation functions in a weakly non-equilibrium state in the underlying quantum field theory. For weak coupling, one may systematically justify, and then in-

2. One may consider the diagrammatic expansion for the equilibrium correlator appearing in the Kubo relation (1.8) for some particular transport coefficient. After carefully

Gagnon, Jeon, PRD 2007

3. One may directly argue (by examining equilibrium finite temperature correlators) that, for sufficiently weak coupling, the underlying high temperature quantum field theory has well-defined quasi-particles, that these quasi-particles are weakly interacting with

Given the complexities of real-time, finite-temperature diagrammatic analysis in gauge theories (especially non-Abelian theories), we find the last approach to be the most physically transparent and compelling. But this is clearly a matter of taste.

AMY, JHEP 2000

This talk: derive and go beyond Boltzmann for QED

# Outline

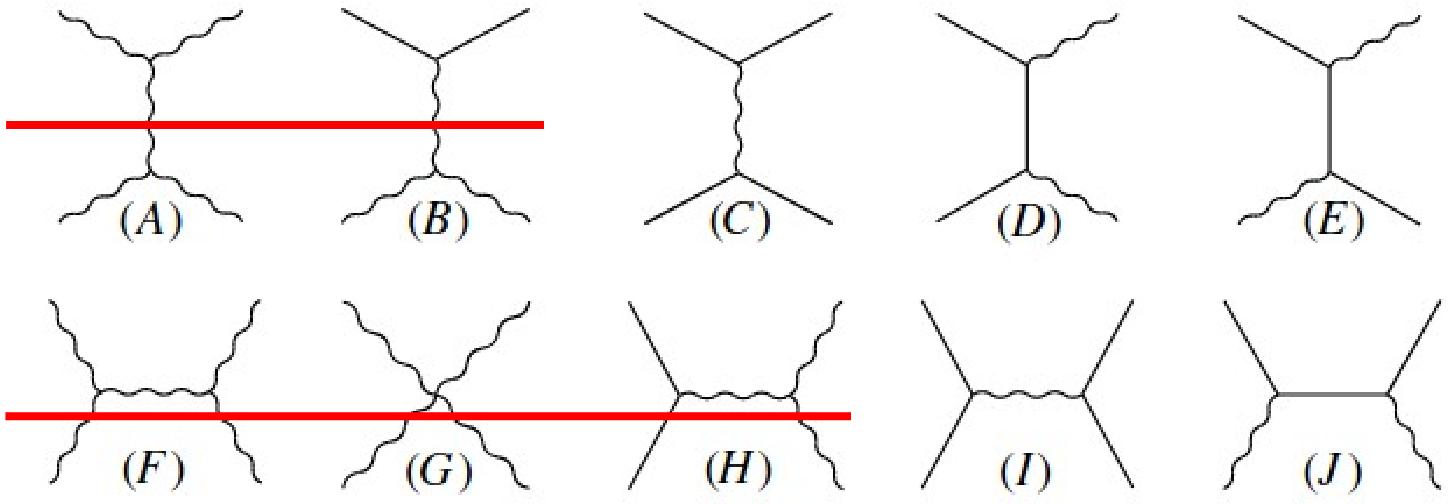
- Recent development on quantum kinetic theory with collisions
- Reduced DOFs by parity invariance
- Elastic/inelastic collisions from self-energies
- Spin polarizaion for fermion/photon
- Summary & Outlook

# Works on collision term

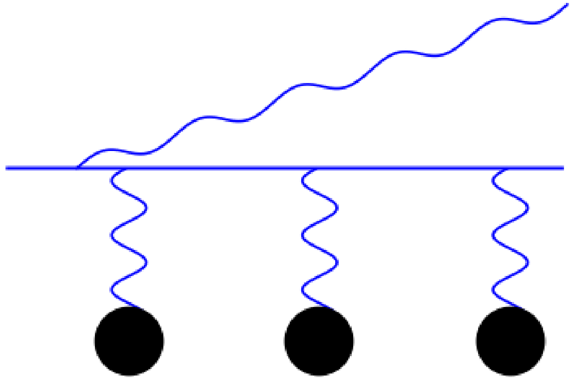
QKT with collisions
Yang, Hattori, Hidaka JHEP 2020 (general framework for fermion)
Hattori, Hidaka, Yamamoto, Yang JHEP 2021 (general framework for photon)
Hidaka, Pu, Yang, PRD 2017 (QED Coulomb scattering)
Li, Yee, PRD 2019 (QCD Coulomb scattering)
Carignano, Manuel, Torre-Rincon, PRD 2020 (QED Coulomb scattering)
Hou, SL, PLB 2021 (QED/QCD Coulomb scattering)
Weickgnnant, Speranza, Sheng, Q. Wang, Rischke, 2103.04896 ($2\rightarrow 2$ elastic scattering)
Sheng, Weickgnnant, Speranza, Rischke, Q. Wang 2103.10636 (Yukawa theory)
Z. Wang, Guo, Zhuang, 2009.10930, Z. Wang, Zhuang 2105.00915 ($2\rightarrow 2$ NJL)
Shi, Gale, Jeon, PRC 2021 ($2\rightarrow 2$ elastic scattering)

# Full classical collision term of QCD/QED

Elastic scattering
range: $1/eT$




Inelastic scattering
range: $1/e^2T$



Scattering with arbitrary medium particle coherently resummed: LPM effect

AMY, JHEP 2003

coarse-graining scale $1/e^4T$

$$(\partial_t + \hat{\boldsymbol{p}} \cdot \boldsymbol{\nabla}_{\boldsymbol{x}})\, f_s(\boldsymbol{x}, \boldsymbol{p}, t) = -C_s^{2\leftrightarrow 2}[f] - C_s^{\text{“}1\leftrightarrow 2\text{”}}[f]$$

Classical collision local!

However notion of locality depends on coarse-graining scale

# DOFs of full QKT

Massive fermion: $f_V^e$ $f_A^e$ $a_\mu$

Photon: $f_V^\gamma$ $f_A^\gamma$

Hattori, Hidaka, Yang, PRD 2019
Gao, Liang, PRD 2019
Weickgenannt et al, PRD 2019

Hattori, Hidaka, Yamamoto, Yang, JHEP 2021

# Assumptions & simplifications

Massive fermion: $f_V^e$ $\cancel{f_A^e \quad a_\mu}$

Hattori, Hidaka, Yang, PRD 2019
Gao, Liang, PRD 2019
Weickgenannt et al, PRD 2019

Photon: $f_V^\gamma$ $\cancel{f_A^\gamma}$

Hattori, Hidaka, Yamamoto, Yang, JHEP 2021

assume system parity invariant at lowest order

same DOF as in the classical kinetic theory by AMY

Arnold, Moore, Yaffe, JHEP 2003

# Kadanoff-Baym equations for fermions

$$S^{<}_{\alpha\beta}(X,P) = -\int d^4(x-y)e^{iP\cdot(x-y)/\hbar}\langle\bar{\psi}_\beta(y)\psi_\alpha(x)\rangle$$

$$\boxed{\frac{i}{2}\not{\partial}S^{<}} + \frac{\not{P}-m}{\hbar}S^{<} = \boxed{\frac{i}{2}\left(\Sigma^{>}S^{<} - \Sigma^{<}S^{>}\right)} - \frac{\hbar}{4}\left(\{\Sigma^{>}, S^{<}\}_{\mathrm{PB}} + \{\Sigma^{<}, S^{>}\}_{\mathrm{PB}}\right)$$

$$S^{<} = S^{<(0)} + \hbar S^{<(1)}$$

$S^{<(0)}$ Classical, describe momentum distribution

$S^{<(1)}$ Quantum correction, contains spin dynamics

$$S^{<(0)}(X,P) = -2\pi\epsilon(P\cdot u)\delta(P^2-m^2)(\not{P}+m)f_e(X,P)$$

u: frame vector

# Kadanoff-Baym equations for photons

$$D^{\mu\nu<}(X,P) = \int d^4(x-y) e^{iP\cdot(x-y)/\hbar} \langle A_\nu(y) A_\mu(x) \rangle$$

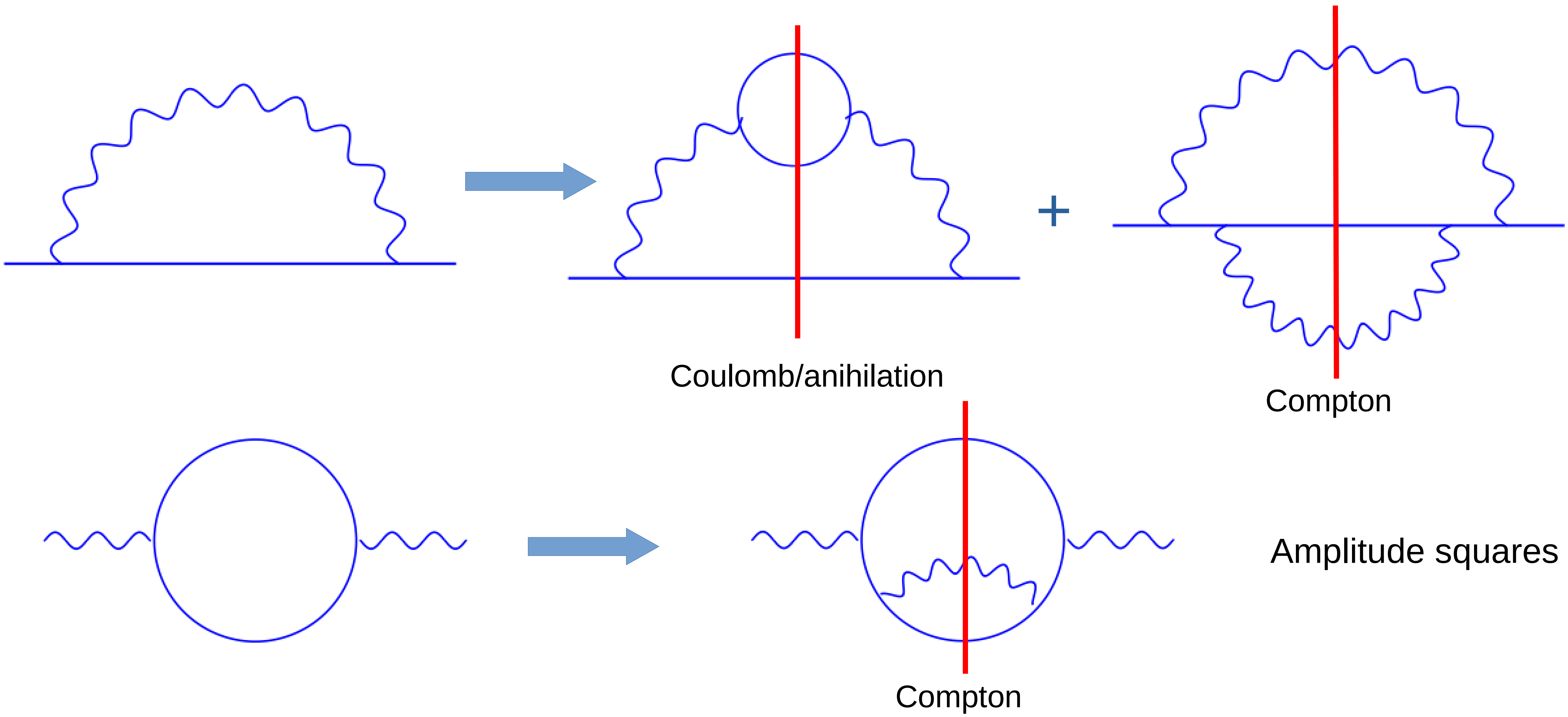
$$\Big[ \frac{1}{\hbar^2} \Big( -P^2 g^{\mu\nu} + P^\mu P^\nu - \frac{1}{\xi} P^{\mu\alpha} P^{\nu\beta} P_\alpha P_\beta \Big) + \frac{i}{2\hbar} \Big( -2P\cdot\partial g^{\mu\nu} + (\partial^\mu P^\nu + \partial^\nu P^\mu) - \frac{1}{\xi} P^{\mu\alpha} P^{\nu\beta} (\partial_\alpha P_\beta + \partial_\beta P_\alpha) \Big) + \frac{1}{4} \Big( \partial^2 g^{\mu\nu} - \partial^\mu \partial^\nu + \frac{1}{4\xi} P^{\mu\alpha} P^{\nu\beta} \partial_\alpha \partial_\beta \Big) \Big] D^<_{\nu\rho} =$$
$$\frac{i}{2} \left( \Pi^{\mu\nu>} D^<_{\nu\rho} - \Pi^{\mu\nu<} D^>_{\nu\rho} \right) + \frac{\hbar}{4} \{ \Pi^{\mu\nu>}, D^<_{\nu\rho} \} - \frac{\hbar}{4} \{ \Pi^{\mu\nu<}, D^>_{\nu\rho} \},$$

Coulomb gauge $\xi = 0$ singles out transverse components

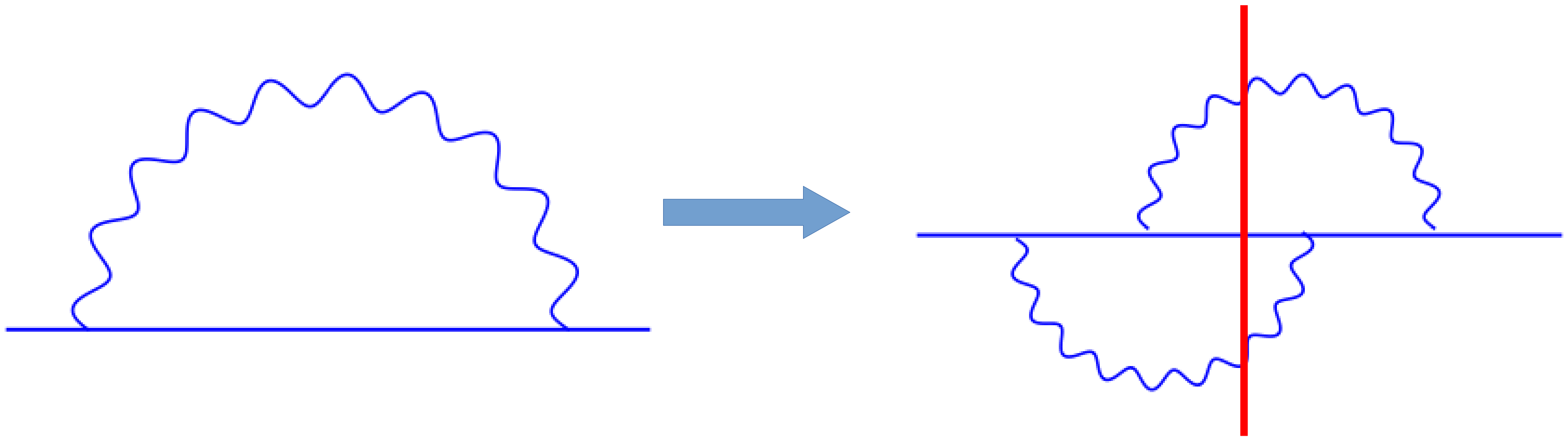
$$D^{\mu\nu<} = D^{\mu\nu<(0)} + \hbar D^{\mu\nu<(1)}$$

$$D^{<(0)}_{\mu\nu}(X,P) = 2\pi\epsilon(P\cdot u)\delta(P^2) P^T_{\mu\nu} f_\gamma(X,P).$$ u: frame vector

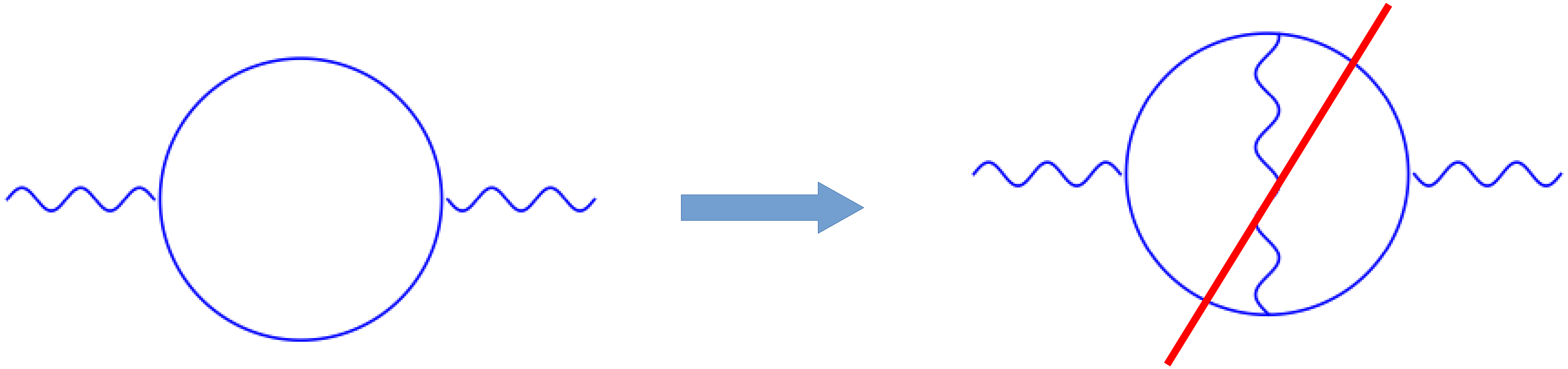
# Self-energies: correction to propagators



# Self-energies: correction to vertices (elastic $2 \rightarrow 2$)



Inteference terms



# Screening effect

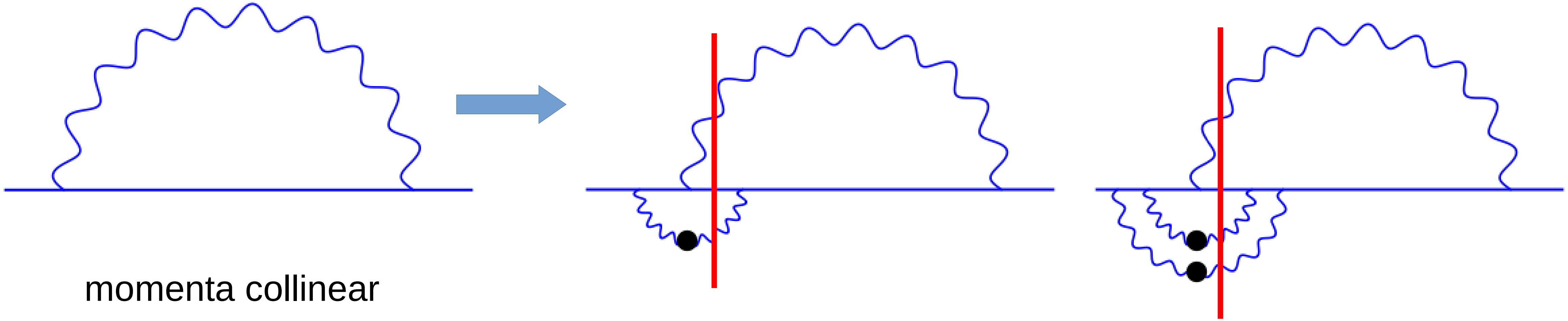
Potential IR divergence rendered finite by screening effect:
fermion/photon gain medium dependent thermal mass

$$\delta m_e^2 = 2Q^\mu \Sigma_\mu = e^2 \int \frac{d^3p}{(2\pi)^3} \frac{2}{p} \left(2 f_\gamma(\vec{p}) + f_e(\vec{p}) + f_{\bar{e}}(\vec{p})\right)$$

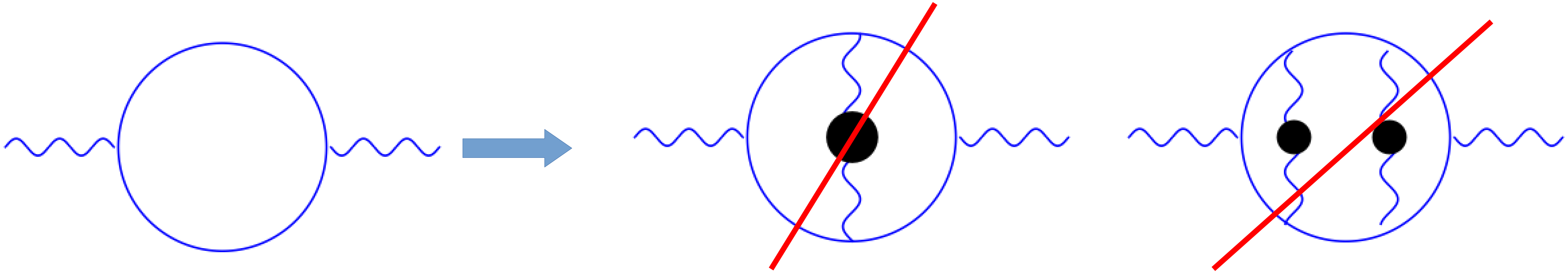
$$m_\gamma^2 = -\Pi_T^R = e^2 \int \frac{d^3p}{(2\pi)^3} \frac{2}{E_p} \left(f_e(\vec{p}) + f_{\bar{e}}(\vec{p})\right)$$

$\delta m_e, m_\gamma \sim eT$ provides cutoff for IR divergence

# Self-energies: correction to vertices (inelastic 1→2)



momenta collinear



# Two scenarios for fermion mass

| | 1→2 scatterings | 2→2 scatterings |
|---|---|---|
| $m \gg eT$ | suppressed, too heavy to radiate | screening needed for Coulomb only |
| $m \sim eT$ | modified rate | screening needed for all |

# Classical solution

$$S^{<(0)} = -2\pi\epsilon(P\cdot u)\delta(P^2-m^2)\left((\not{P}+m)f_e\right)$$

$$D^{<(0)}_{\mu\nu}(X,P) = 2\pi\epsilon(P\cdot u)\delta(P^2)P^T_{\mu\nu}f_\gamma(X,P).$$

$$\Rightarrow \quad \partial_t f_e - \mathbf{v}_e\cdot\nabla f_e = C^e[2\to 2] + C^e[1\to 2]$$

$$\partial_t f_\gamma - \mathbf{v}_\gamma\cdot\nabla f_\gamma = C^\gamma[2\to 2] + C^\gamma[1\to 2]$$

massive generalization of the Boltzmann equation by Arnold, Moore, Yaffe

# Quantum correction to fermion solution: Non-dynamical

$$\frac{i}{2}\partial\!\!\!/ S^{<(0)} + \frac{P\!\!\!/ - m}{\hbar} S^{<(1)} = \frac{i}{2}\left(\Sigma^{>(0)} S^{<(0)} - \Sigma^{<(0)} S^{>(0)}\right)$$

$$S^{<(1)}(P) = \gamma^5 \gamma_\mu \mathcal{A}^\mu + \frac{i[\gamma_\mu, \gamma_\nu]}{4} \mathcal{S}^{\mu\nu}$$

Chen et al, PRL 2014, PRL 2015
Hidaka, Pu, Yang, PRD 2019, JHEP 2020

$$\mathcal{A}^\mu = -2\pi\epsilon(P \cdot u) \frac{\epsilon^{\mu\nu\rho\sigma} P_\rho u_\sigma \mathcal{D}_\nu f_e}{2(P \cdot u + m)} \delta(P^2 - m^2)$$

gives spin polarization, restores frame independence at O(hbar)

$$\mathcal{D}_\nu = \boxed{\partial_\nu} - \boxed{\Sigma_\nu^> - \Sigma_\nu^< \frac{1-f_e}{f_e}}$$

quantum correction non-local

green term: included in phenomenological studies
blue term: should also be included

# Quantum correction to photon solution: Non-dynamical

$$\left(-P^2 g^{\mu\nu} + P^\mu P^\nu - \frac{1}{\xi} P^{\mu\alpha} P^{\nu\beta} P_\alpha P_\beta\right) D^{<(1)}_{\nu\rho} + \frac{i}{2}\big( - 2P\cdot\partial g^{\mu\nu} + \partial^\mu P^\nu + \partial^\nu P^\mu$$
$$- \frac{1}{\xi} P^{\mu\alpha} P^{\nu\beta}(\partial_\alpha P_\beta + \partial_\beta P_\alpha)\big) D^{<(0)}_{\nu\rho} = \frac{i}{2}\left(\Pi^{\mu\nu>(0)} D^{<(0)}_{\nu\rho} - \Pi^{\mu\nu<(0)} D^{>(0)}_{\nu\rho}\right),$$

$$D^{<(1)}_{\lambda\rho} = -\frac{i P_{\lambda\alpha} P^{\nu\beta} P^\alpha \partial_\beta D^{<(0)}_{\nu\rho}}{2(-P^2 + (P\cdot u)^2)} + \frac{i u_\chi u_\mu \left(\Pi^{\mu\nu>(0)} D^{<(0)}_{\nu\rho} - \Pi^{\mu\nu<(0)} D^{>(0)}_{\nu\rho}\right)}{2(-P^2 + (P\cdot u)^2)} - (\lambda \leftrightarrow \rho)$$

photonic CVE

possible collisional correction

Huang, Mitkin, Sadofyev, Speranza, JHEP 2020

Hattori, Hidaka, Yamamoto, Yang, JHEP 2021

# Quantum correction: dynamical

$$\frac{i}{2}\partial\!\!\!/ S^< + \frac{p\!\!\!/ - m}{\hbar} S^< = \frac{i}{2}\left(\Sigma^> S^< - \Sigma^< S^>\right) - \frac{\hbar}{4}\left(\{\Sigma^>, S^<\}_{\mathrm{PB}} + \{\Sigma^<, S^>\}_{\mathrm{PB}}\right)$$

- quantum correction to classical collision term $S^{<(1)} \sim \partial_X f_e$
- Poisson bracket terms $\{A, B\}_{\mathrm{PB}} = \partial_P A \cdot \partial_X B - \partial_P B \cdot \partial_X A$

Both are non-local

# Summary

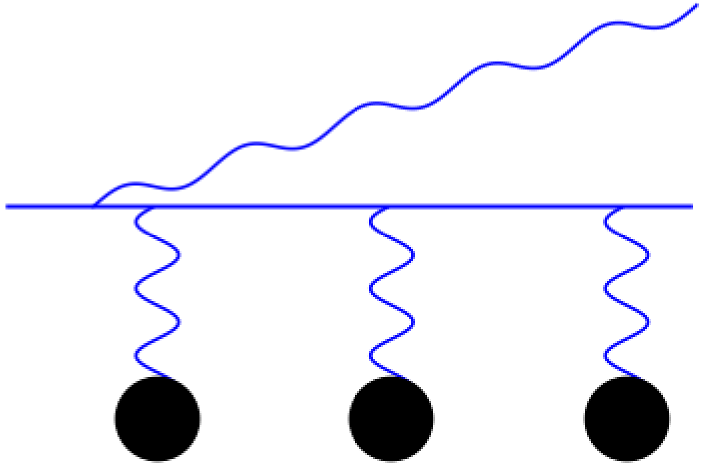
- Derived quantum kinetic equation for QED
- Classical solution generalizes Boltzmann equation to massive case
- (Non-dynamical) quantum correction gives fermion/photon spin polarizations

# Outlook

- (Dynamical) quantum correction needs quantum correction to collision term
- Parity non-invariant system

Thank you!

# Two scenarios for fermion mass



medium kicks: $\sim eT$

fermion mass: $\sqrt{m^2 + m_{\rm th}^2}$ $m_{\rm th} \sim eT$

1 → 2 processes irrelevant for mass regime

$m \gg eT$ heavy fermion barely radiates!

$m \sim eT$ Modified 1 → 2 collision term