Several aspects of fRG-QCD at finite temperature and density

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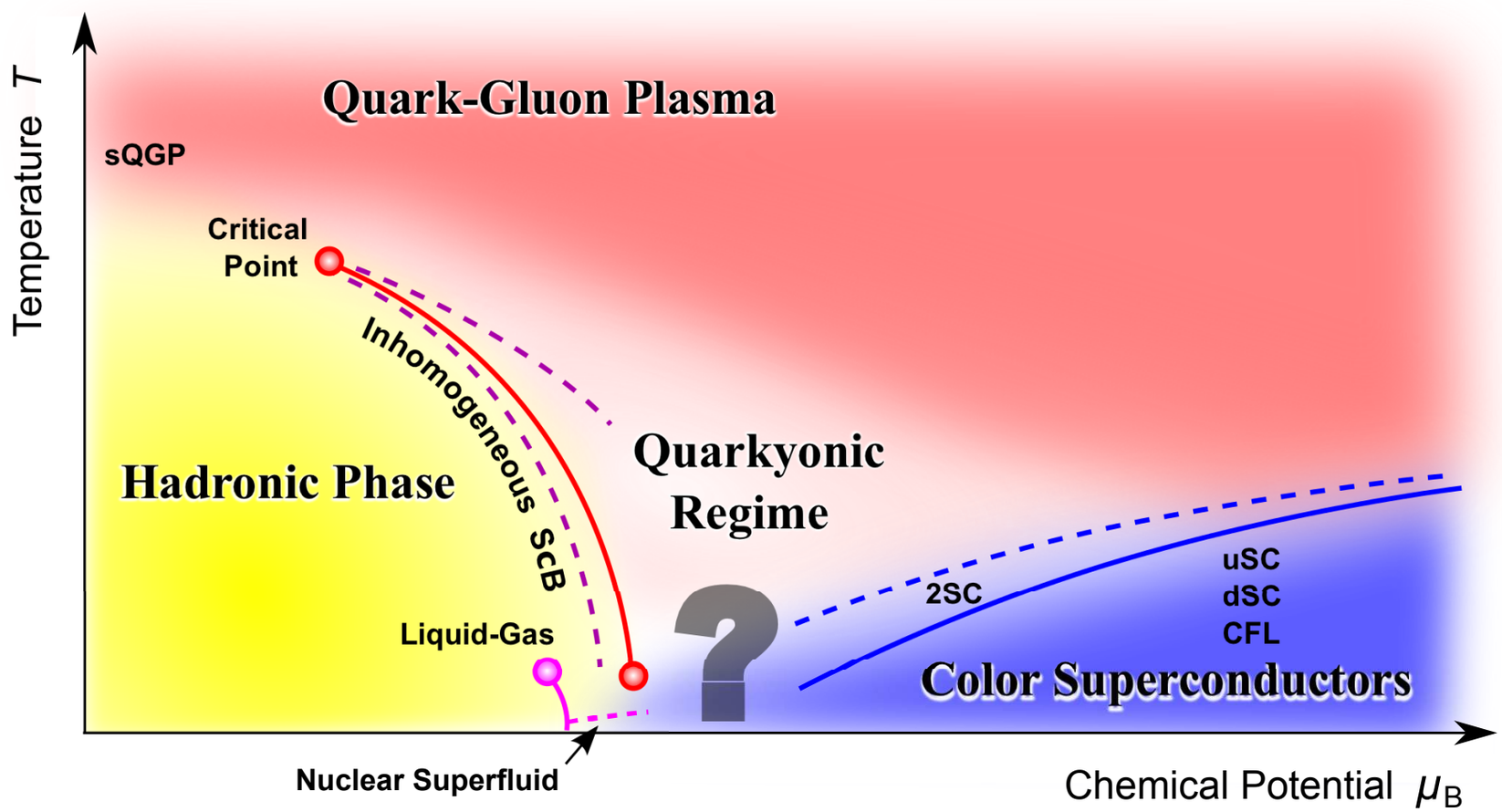
Central China Normal University, Wuhan, Jan 9, 2020

Based on :
WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991;
Rui Wen, WF, arXiv:1909.12564;
Shi Yin, Rui Wen, WF, PRD 100 (2019) 094029, arXiv:1907.10262.

Outline

- Introduction
- fRG-QCD formalism
- Phase structure in QCD
- QCD equation of state
- High-order baryon number fluctuations
- Strangeness neutrality and correlations
- Baryon number probability distribution
- Summary and outlook

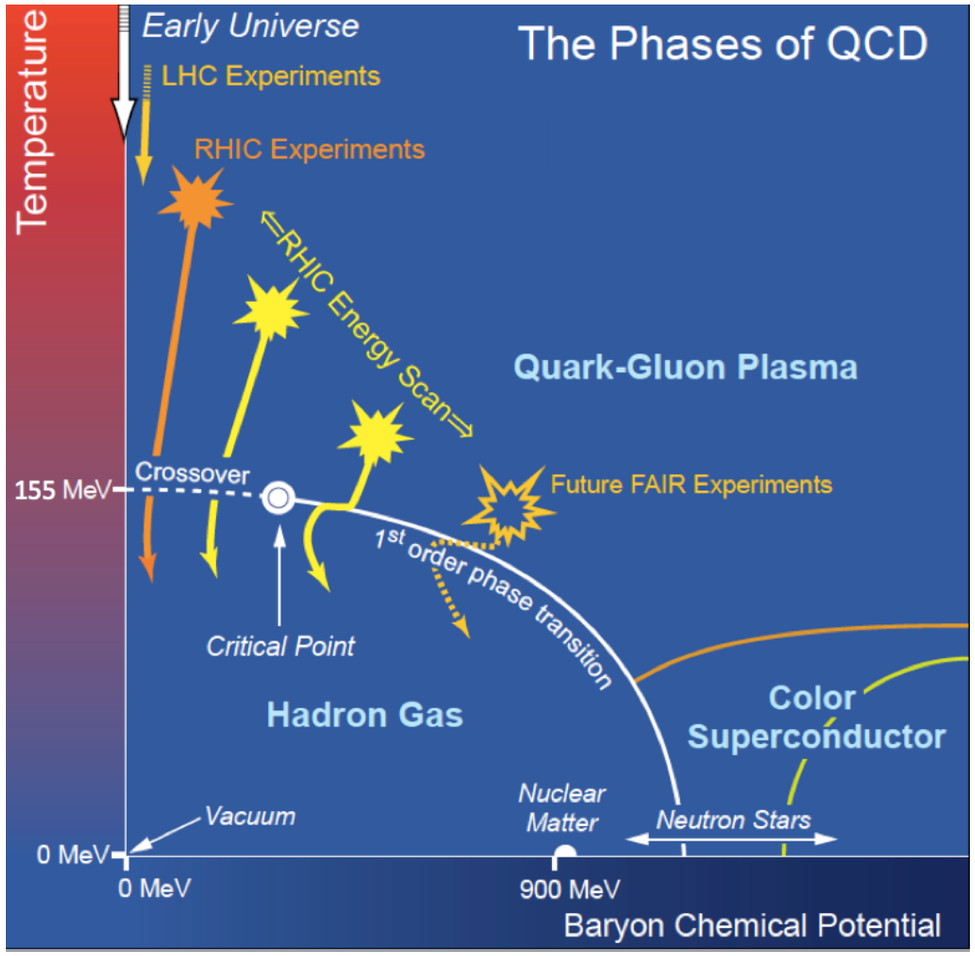
QCD phase diagram



K. Fukushima, C. Sasaki, arXiv:1301.6377.

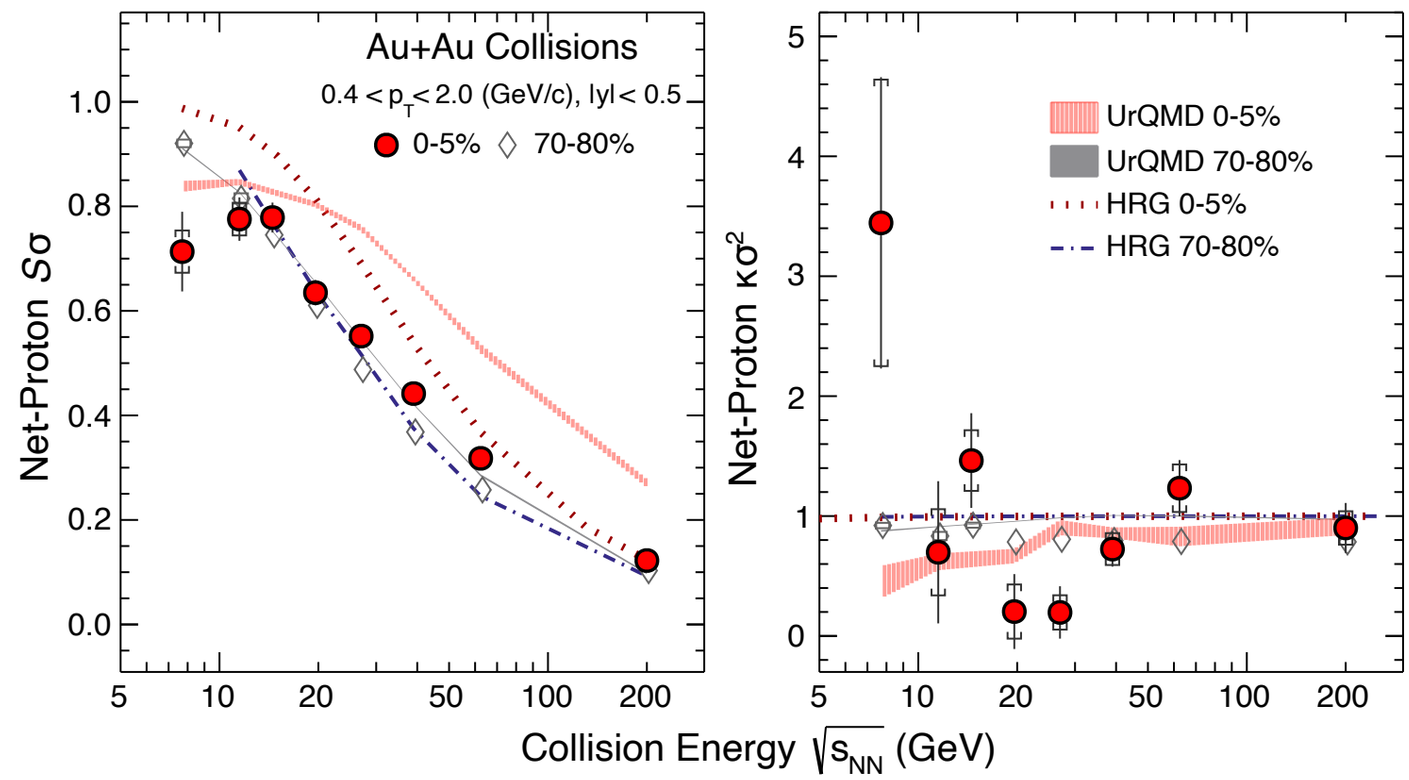
- QCD phase diagram in the T-muB plane.
- Critical end point (or critical point) is a key feature of QCD phase structure.
- Experimental programs: RHIC-BES, FAIR, NICA, HIAF.
- Some hints from RHIC-BES experiment: net-baryon (proton) cumulants, directed flow, HBT radii, light nuclei……

High-order net-proton cumulants



The Hot QCD White Paper (2015)

RHIC:

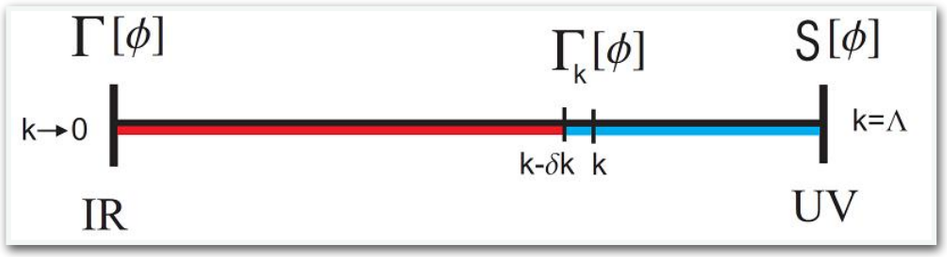


J. Adam *et al.* (STAR), arXiv: 2001.02852

- Non-monotonic energy dependence of the kurtosis => hint of entering critical region.

Quantum fluctuations with FRG

FRG



Rebosonized QCD Effective action:

$$\Gamma_k = \int_x \Big\{ \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + Z_c \left(\partial_\mu \bar{c}^a\right) D^{ab}_\mu c^b + \frac{1}{2\xi} (\partial_\mu A^a_\mu)^2$$

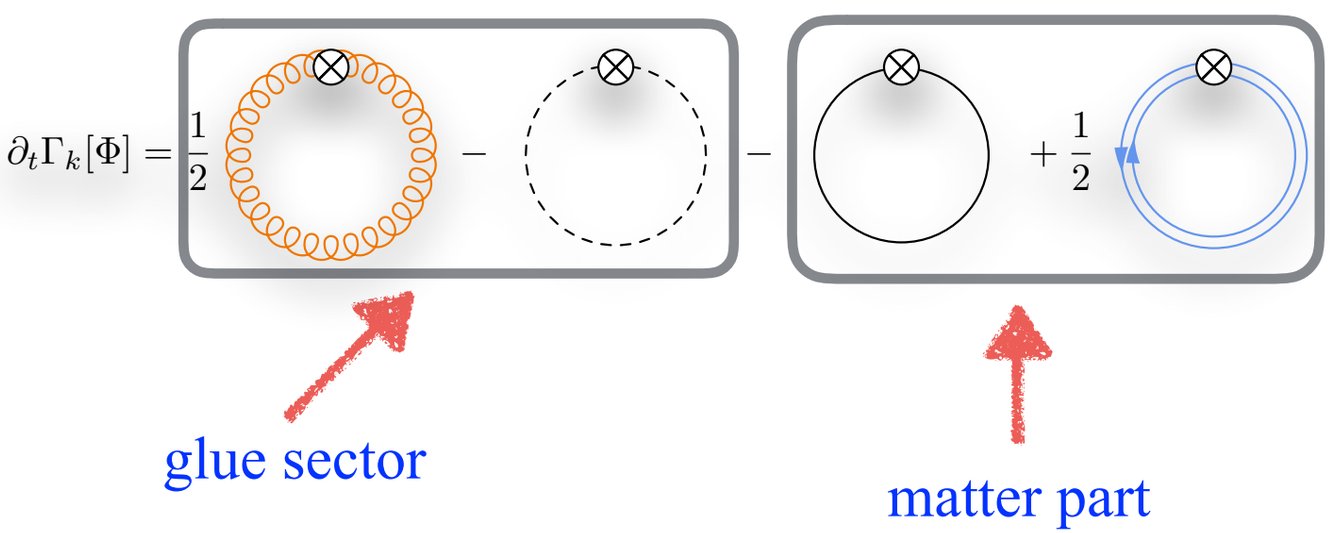
$$+ \frac{1}{2} \int_p A^a_\mu(-p) \left(\Gamma^{(2)\,ab}_{AA\mu\nu}(p) - Z_A \Pi^\perp_{\mu\nu} \delta^{ab} p^2 \right) A^b_\nu(p)$$

$$+ \bar{q} \left[Z_q \left(\gamma_\mu D_\mu - \gamma_0 \hat{\mu}\right) + m_s(\sigma_s) \right] q$$

$$- \lambda_q \left[(\bar{q}\,\tau^0 q)^2 + (\bar{q}\,\vec{\tau} q)^2 \right] + h\,\bar{q}\,\left(\tau^0 \sigma + \vec{\tau}\cdot\vec{\pi}\right)\, q$$

$$+ \frac{1}{2} Z_\phi \left(\partial_\mu \phi\right)^2 + V_k(\rho, A_0) - c_\sigma\, \sigma - \frac{1}{\sqrt{2}}\, c_{\sigma_s}\, \sigma_s \Big\},$$

Flow equation:

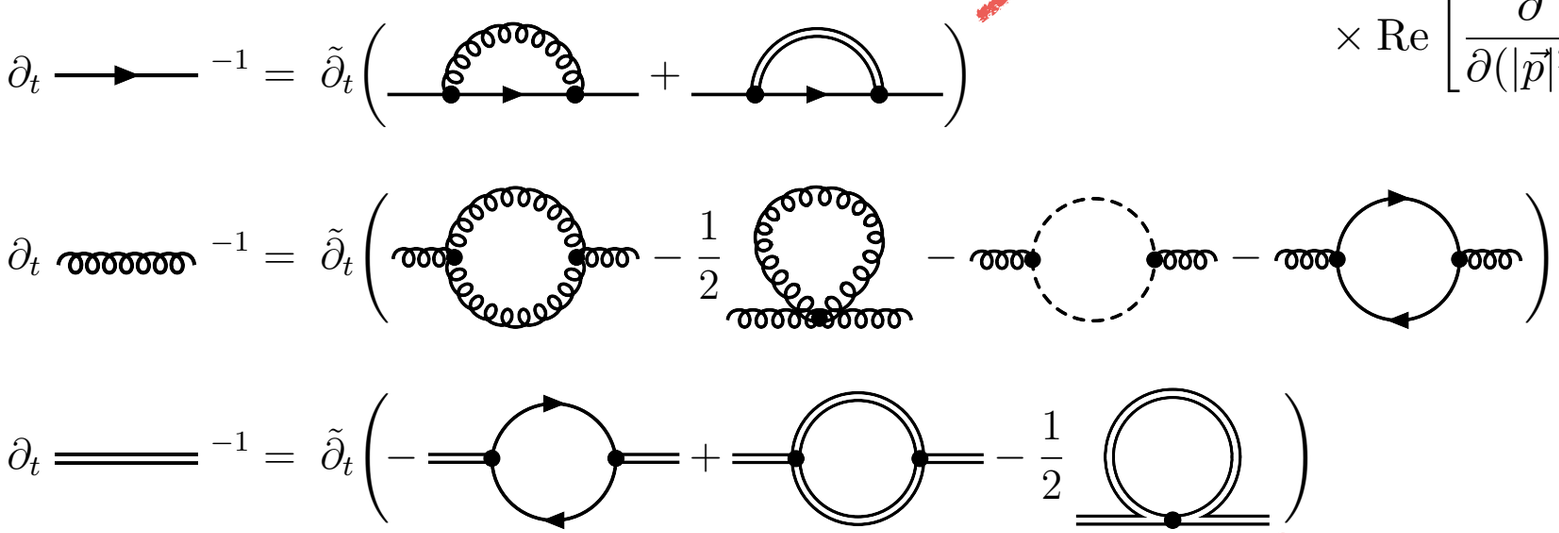


Propagators and anomalous dimensions

Quark anomalous dimension:

$$\eta_{q,k}(p_0,\vec{p}) = \frac{1}{4Z_{q,k}(p_0,\vec{p})} \times \mathrm{Re}\left[\frac{\partial}{\partial(|\vec{p}|^2)}\mathrm{tr}\left(i\vec{\gamma}\cdot\vec{p}\left(-\frac{\delta^2}{\delta\bar{q}(p)\delta q(p)}\partial_t\Gamma_k\right)\right)\right],$$

Flow equation:



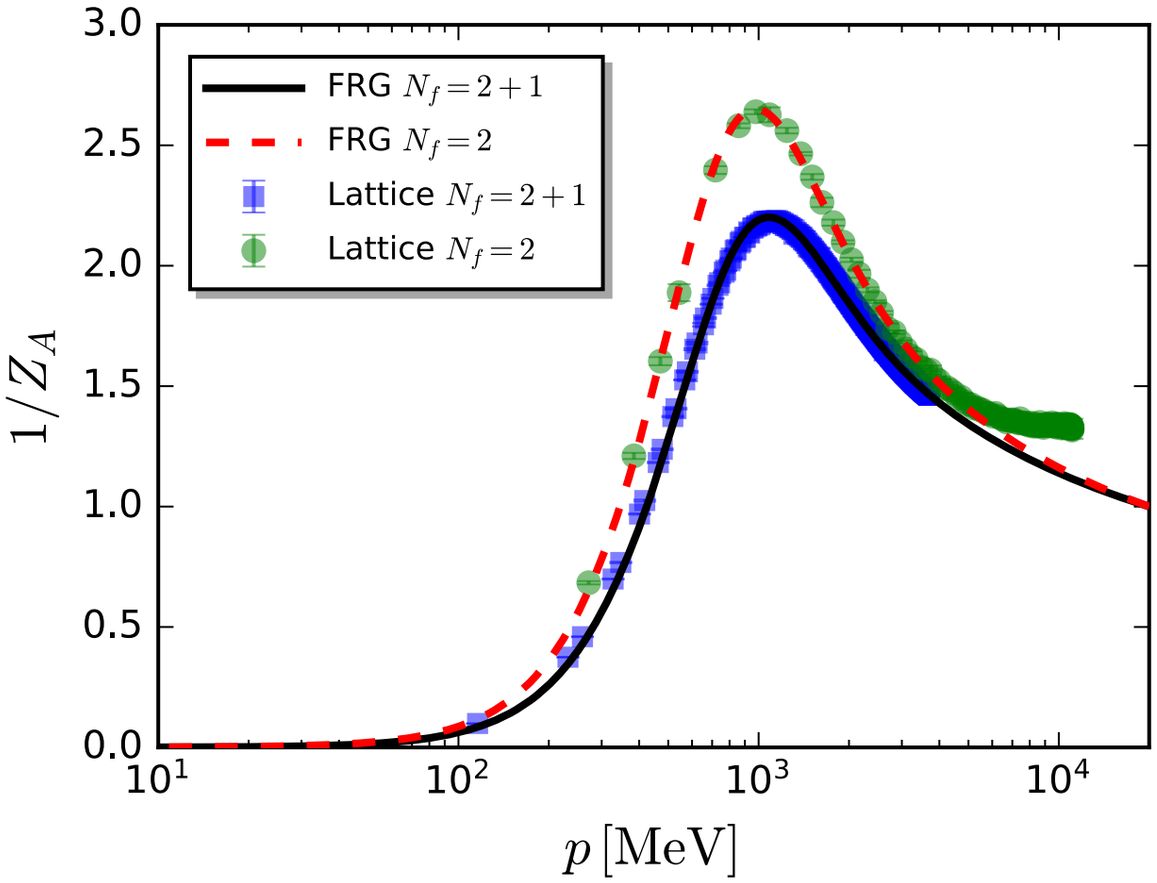
Glue sector:

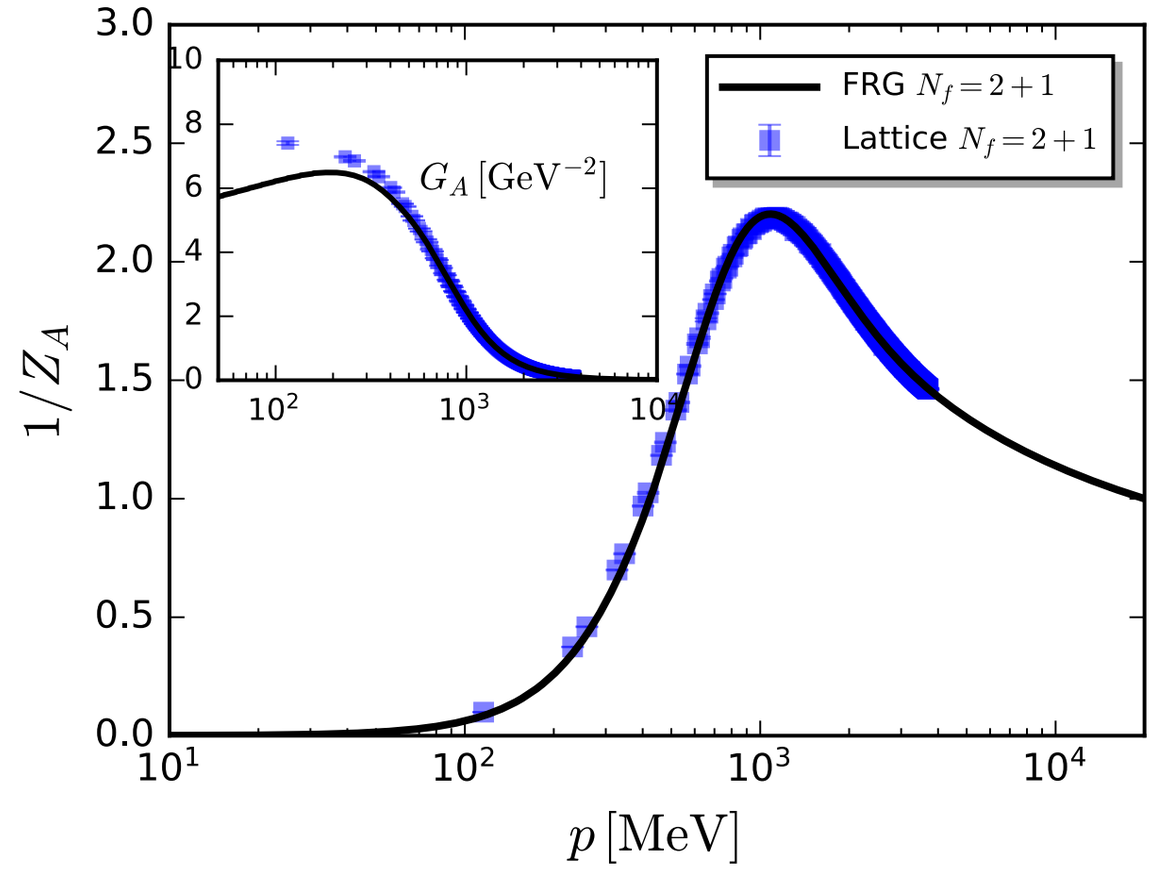
$$\eta_A = \eta_{A,\mathrm{vac}}^{\mathrm{QCD}} + \Delta\eta_A^{\mathrm{glue}} + \Delta\eta_A^{q}$$

Meson anomalous dimension:

$$\eta_{\phi,k}(p_0,\vec{p}) = -\frac{1}{Z_{\phi,k}}\frac{1}{\delta_{ij}}\frac{\partial}{\partial(|\vec{p}|^2)}\frac{\delta^2\partial_t\Gamma_k}{\delta\pi_i(-p)\delta\pi_j(p)},$$

Gluon dressing function





For the moment we adopt:

$$\eta_A = \eta_{A,\mathrm{vac}}^{\mathrm{QCD}} + \Delta\eta_A^{\mathrm{glue}} + \Delta\eta_A^{q}$$

WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

$$\eta_{A,\mathrm{vac}}^{\mathrm{QCD}} = \eta_{A,\mathrm{vac}}^{\mathrm{QCD}}\Big|_{N_f=2} + \eta_{A,\mathrm{vac}}^{s}\,,$$

Thermal quark loop:

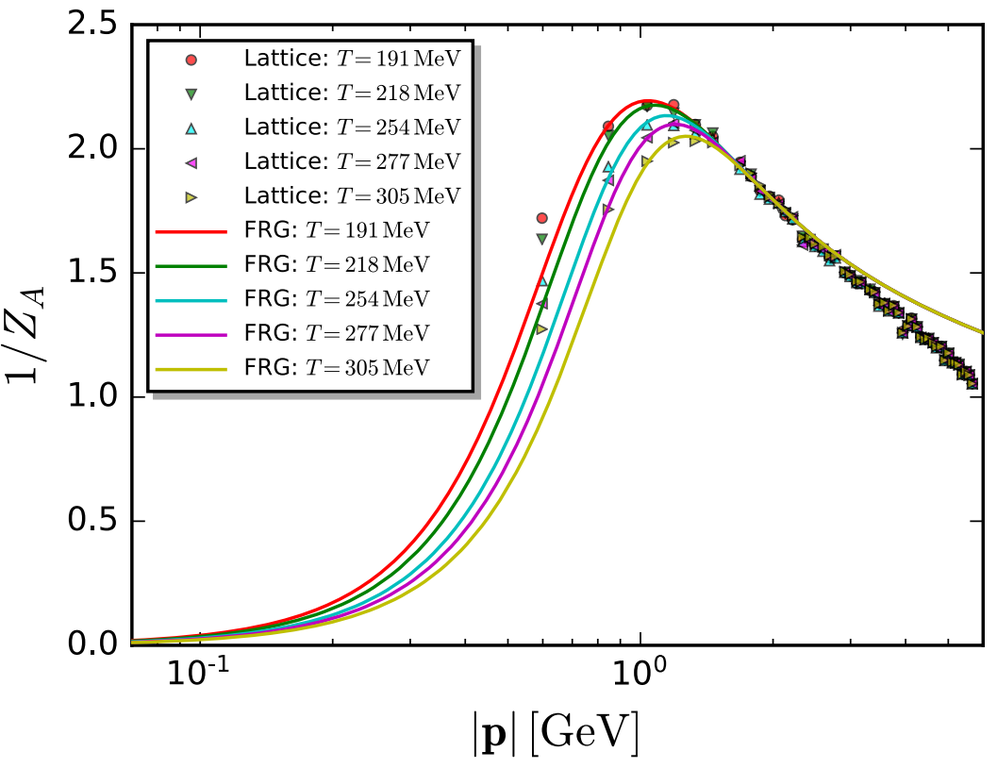
$$\Delta\eta_{A,T}^{q} = -\frac{1}{2(N_c^2-1)}\delta_{ab}\Pi_{\mu\nu}^{\mathrm{M}}(p)\left(\left[\overline{\mathrm{Flow}}_{AA\,\mu\nu}^{(2)\,ab}(p)\right]_T^{(q)} - \left[\overline{\mathrm{Flow}}_{AA\,\mu\nu}^{(2)\,ab}(p)\right]_{T=0}^{(q)}\right)_{\substack{p_0=0\\|\vec{p}|=k}}\,,$$

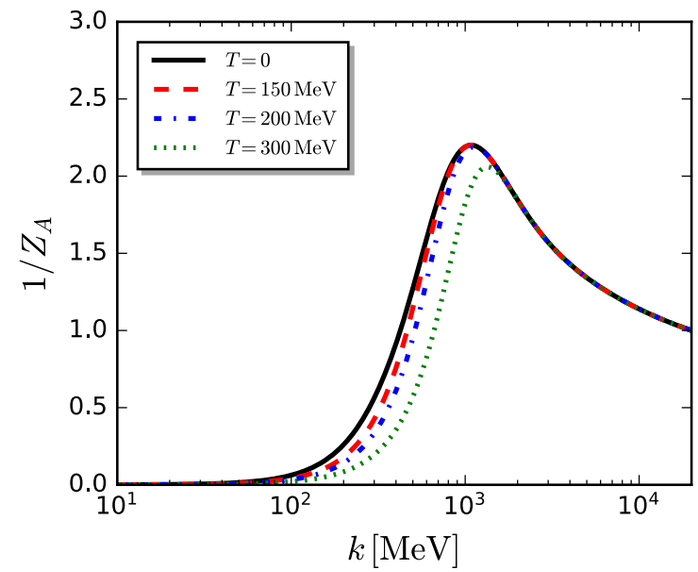
Debye screening mass for the gluon:

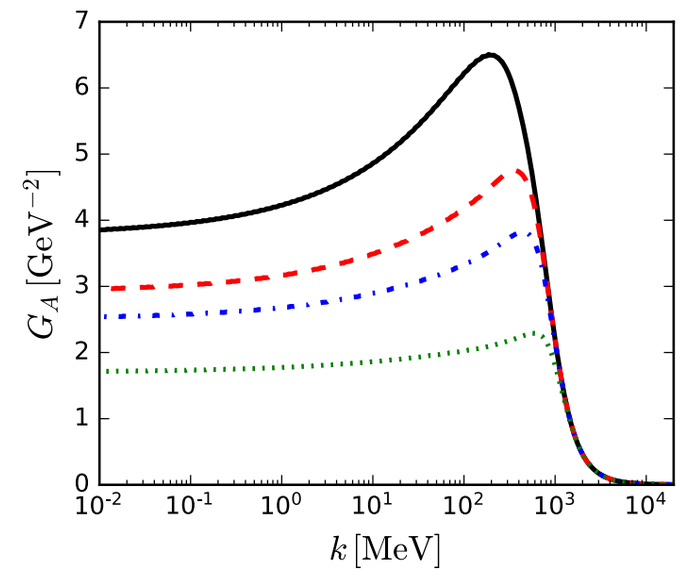
$$\bar{G}_A^{-1}(k) = \bar{Z}_{A,k}k^2 = Z_{A,k}k^2 + \Delta m_s^2(k,T)\,.$$

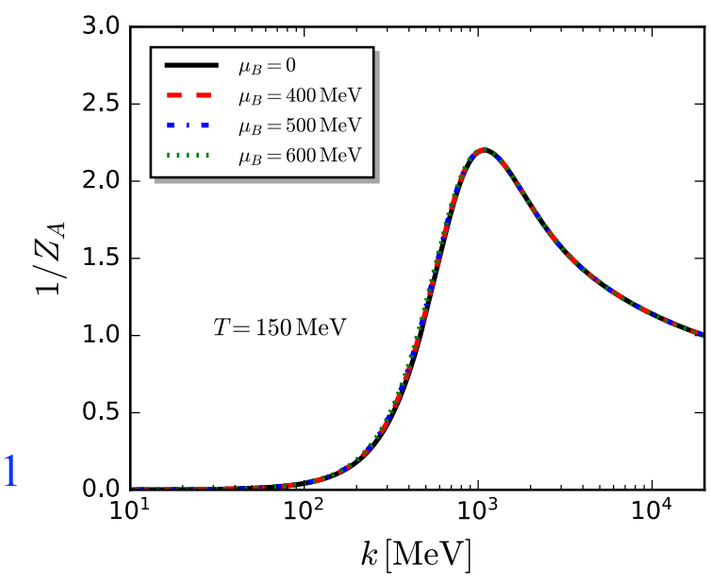
$$\Delta m_s^2(k,T) = (cT)^2\exp\left[-\left(\frac{k}{\pi T}\right)^n\right].$$

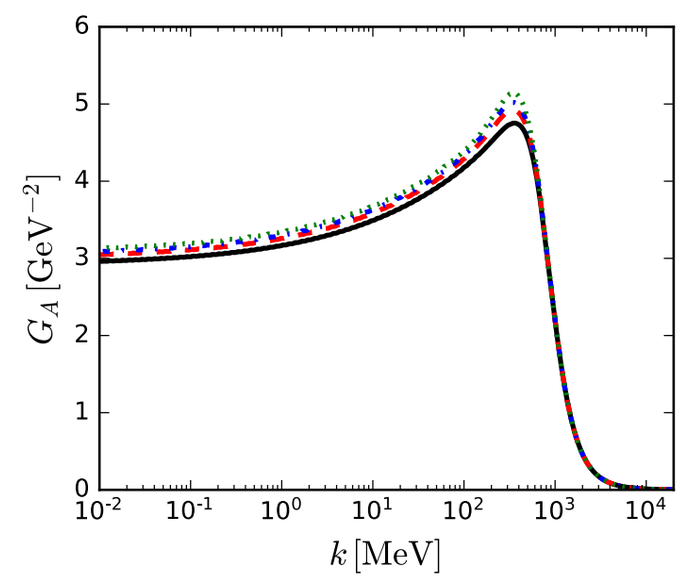
Gluon propagator at finite T and muB











WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

Dynamical hadronization

Introduce a scale-dependent meson field:

$$\langle \partial_t \hat{\phi}_k \rangle = \dot{A}_k \, \bar{q} \tau q + \dot{B}_k \, \phi + \dot{C}_k \, \hat{e}_\sigma \, ,$$

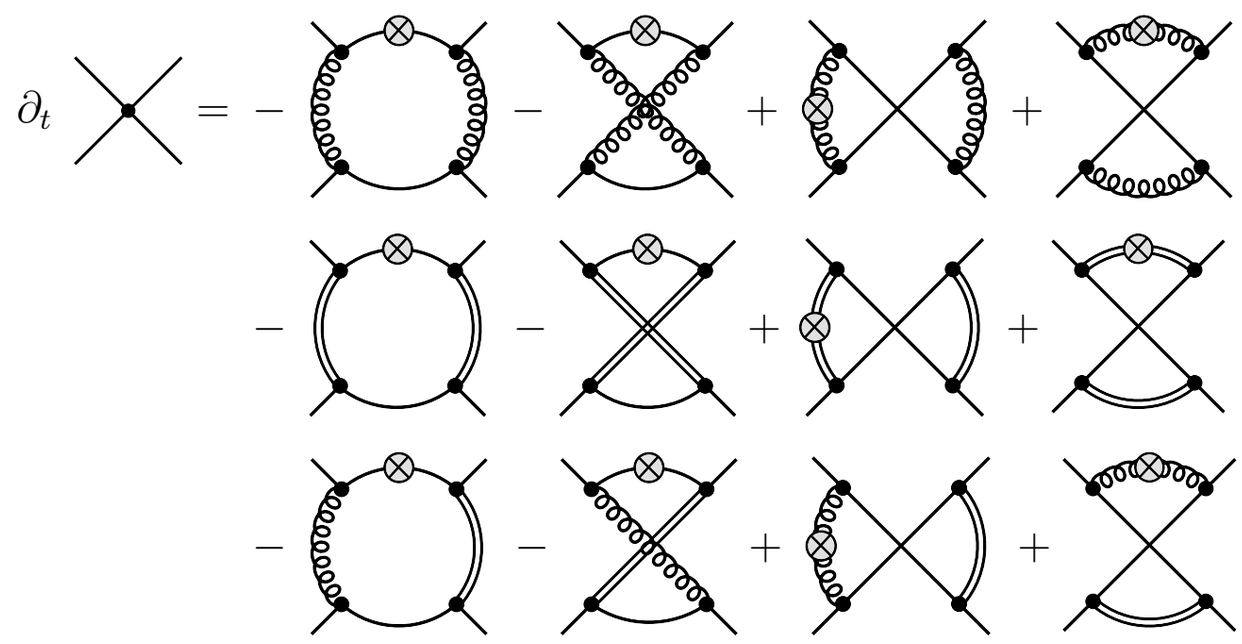
The Wetterich equation is modified:

$$\partial_t \Gamma_k[\Phi] + \int \langle \partial_t \hat{\phi}_{k,i} \rangle \left(\frac{\delta \Gamma_k[\Phi]}{\delta \phi_i} + c_\sigma \delta_{i\,\sigma} \right)$$

$$= \frac{1}{2} \mathrm{Tr}\big(G_k[\Phi] \, \partial_t R_k \big) + \frac{1}{2} \mathrm{Tr}\left(G_{\phi \Phi_j}[\Phi] \frac{\delta \langle \partial_t \hat{\phi}_k \rangle}{\delta \Phi_j} \, R_\phi \right) ,$$

The four-fermion couplings:

$$\partial_t \bar{\lambda}_q - 2\,(1+\eta_q)\,\bar{\lambda}_q - \bar{h}\,\dot{\bar{A}} = \frac{1}{4} \overline{\mathrm{Flow}}^{(4)}_{(\bar{q}q)(\bar{q}q)} \, ,$$



$+$ (gluons $\leftrightarrow$ mesons)

$+$ (permutations of the regulator insertions)

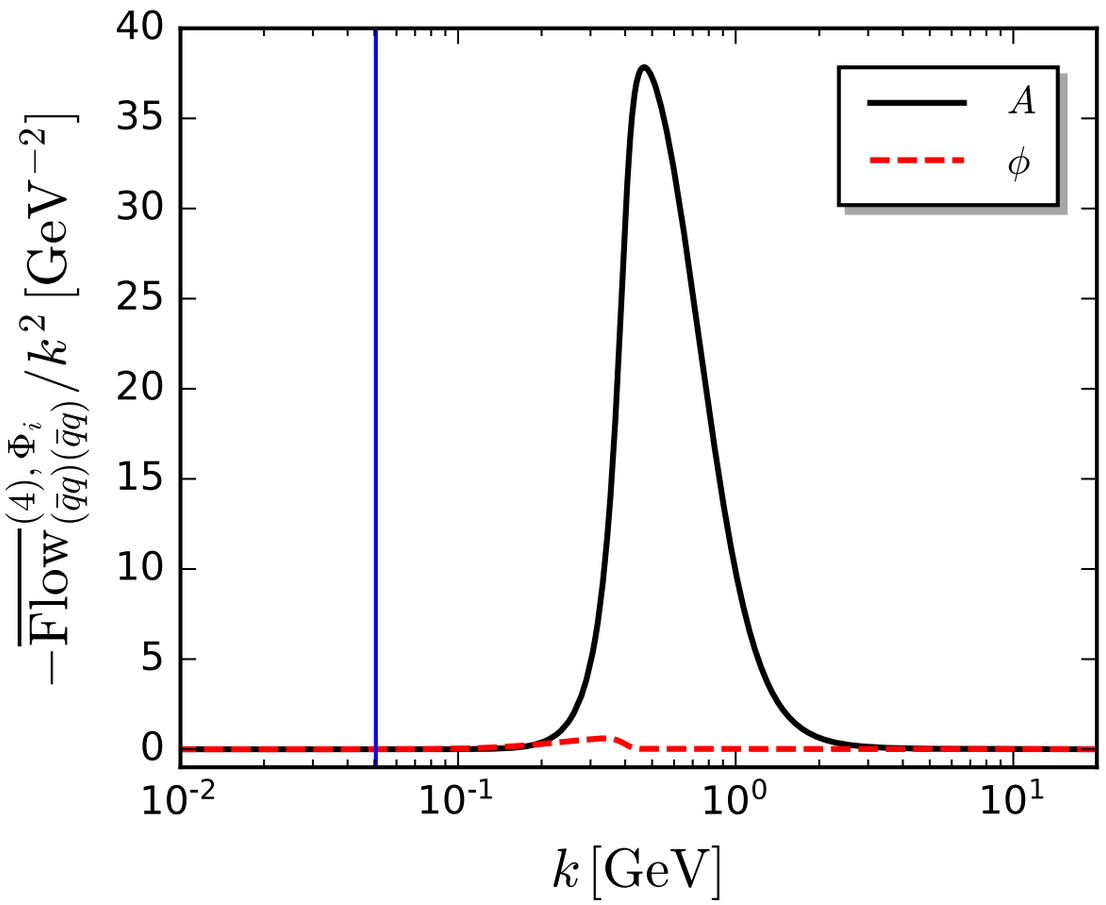
Demanding

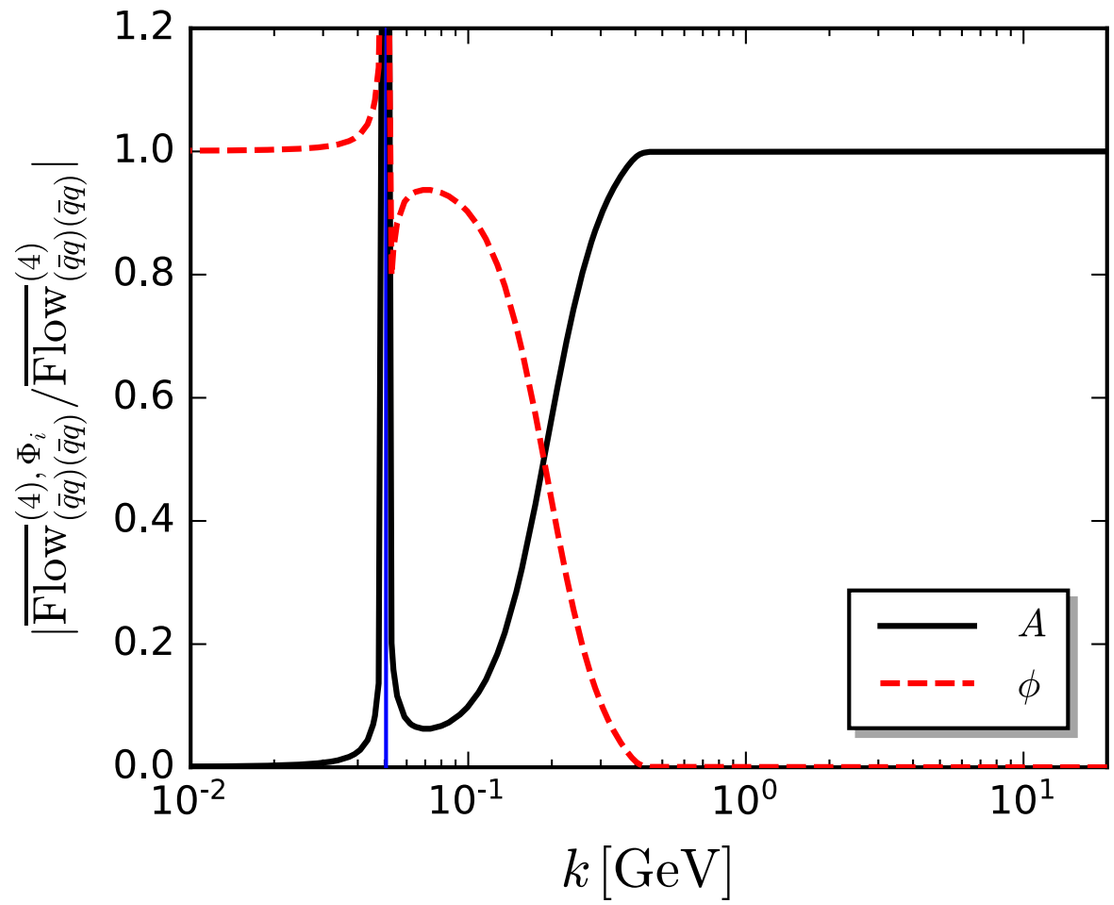
$$\bar{\lambda}_q \equiv 0 \, , \qquad \forall k \, .$$

The hadronization function reads

$$\dot{\bar{A}} = -\frac{1}{\bar{h}} \, \overline{\mathrm{Flow}}^{(4)}_{(\bar{q}q)(\bar{q}q)/4} \, .$$

Flow of 4-quark coupling—gluon versus meson





WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

Yukawa coupling

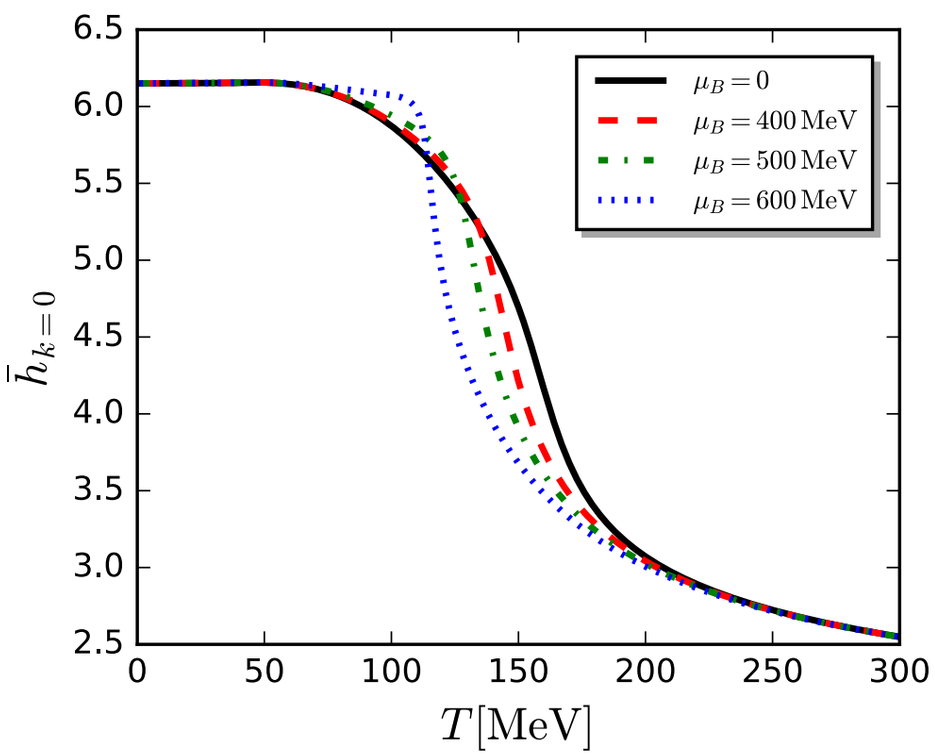
Two different Yukawa coupling:

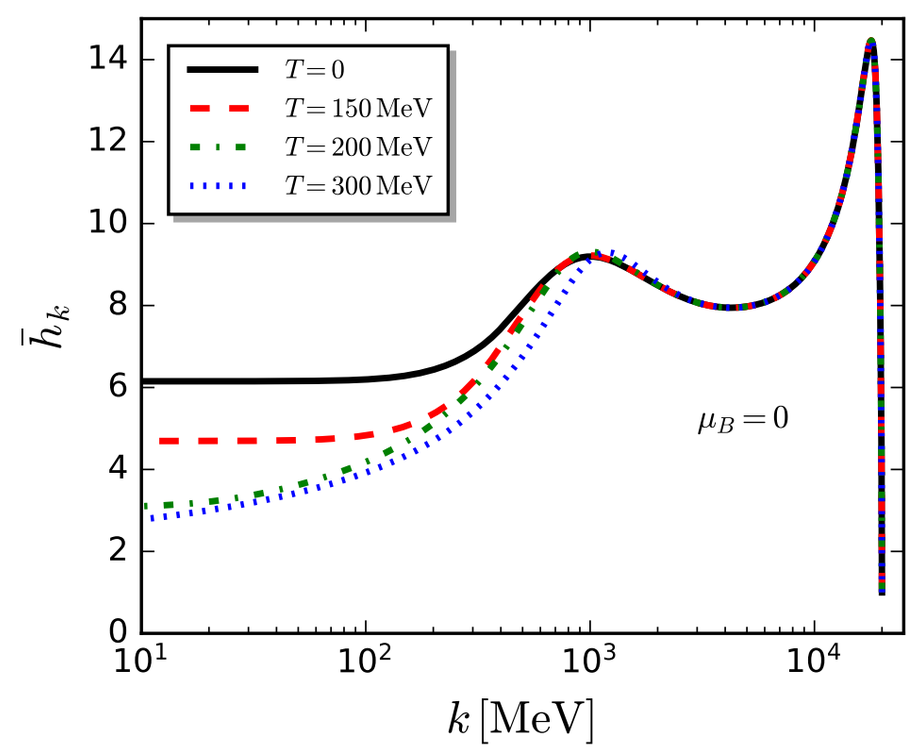
$$h_\pi = h(\rho_0) = \Gamma^{(3)}_{(\bar{q}\vec{\tau}q)\vec{\pi}}[\Phi_0]\,,$$

$$h_\sigma = h(\rho_0) + \rho\, h'(\rho_0) = \Gamma^{(3)}_{(\bar{q}\tau^0 q)\sigma}[\Phi_0]\,.$$

The flow equation:

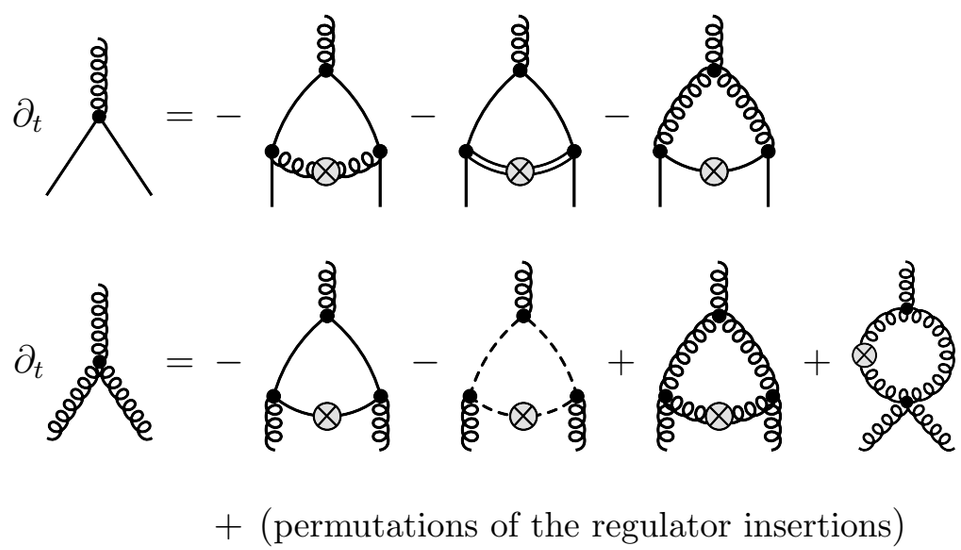
$$\partial_t \bar{h} = \left(\frac{1}{2}\eta_{\phi,k} + \eta_{q,k}\right)\bar{h} - \bar{m}_\pi^2\,\dot{\bar{A}} + \frac{1}{\bar{\sigma}}\mathrm{Re}\,\overline{\mathrm{Flow}}^{(2)}_{(\bar{q}\tau^0 q)}\,,$$

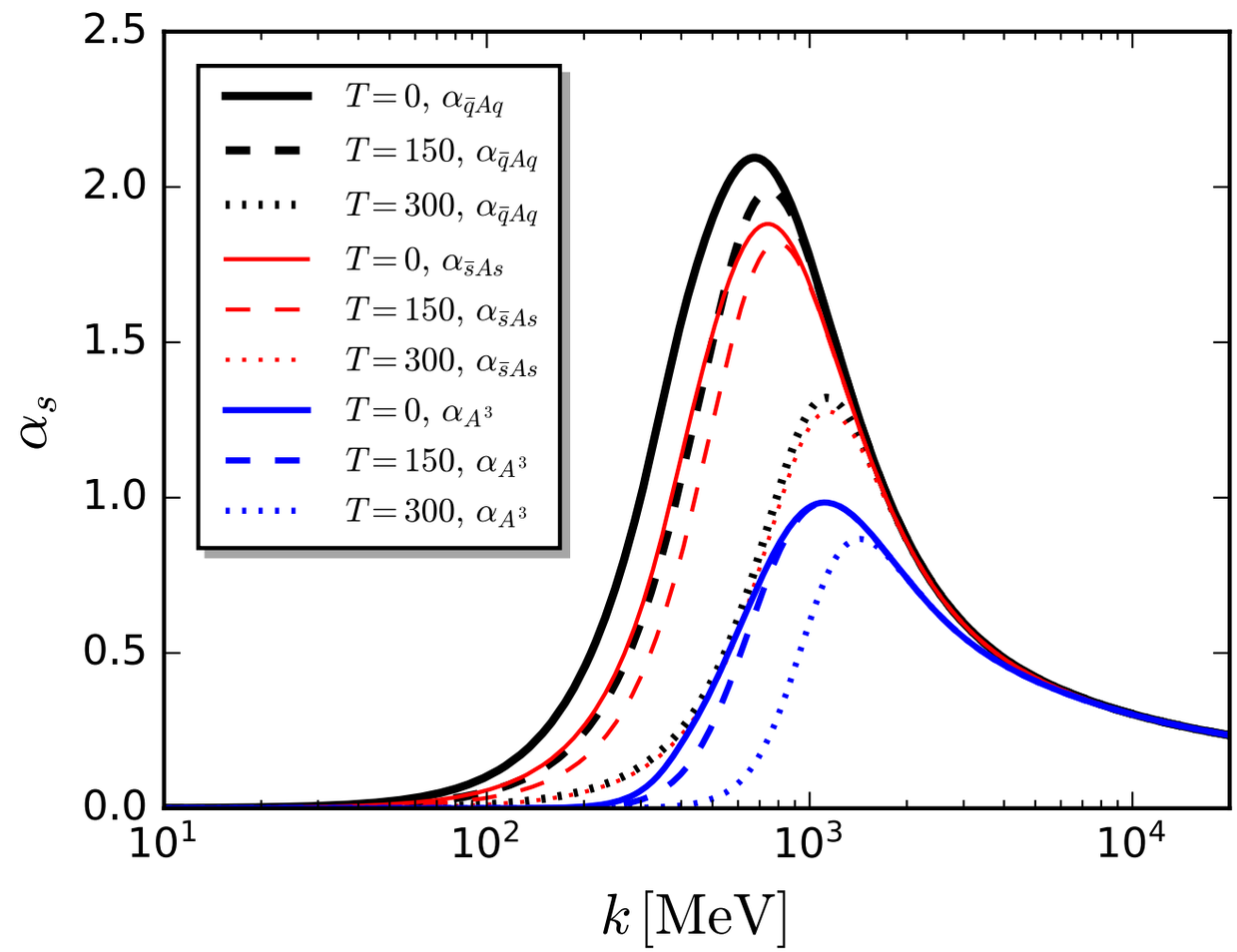




WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

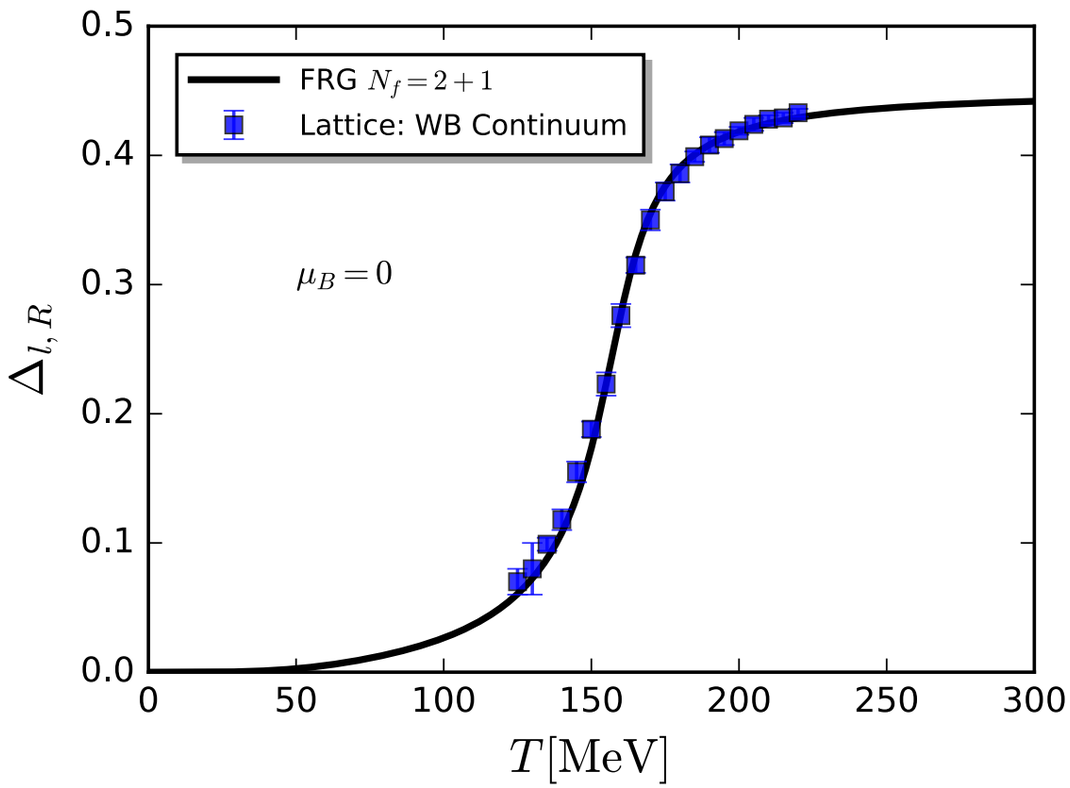
QCD strong couplings among quarks and gluons





WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

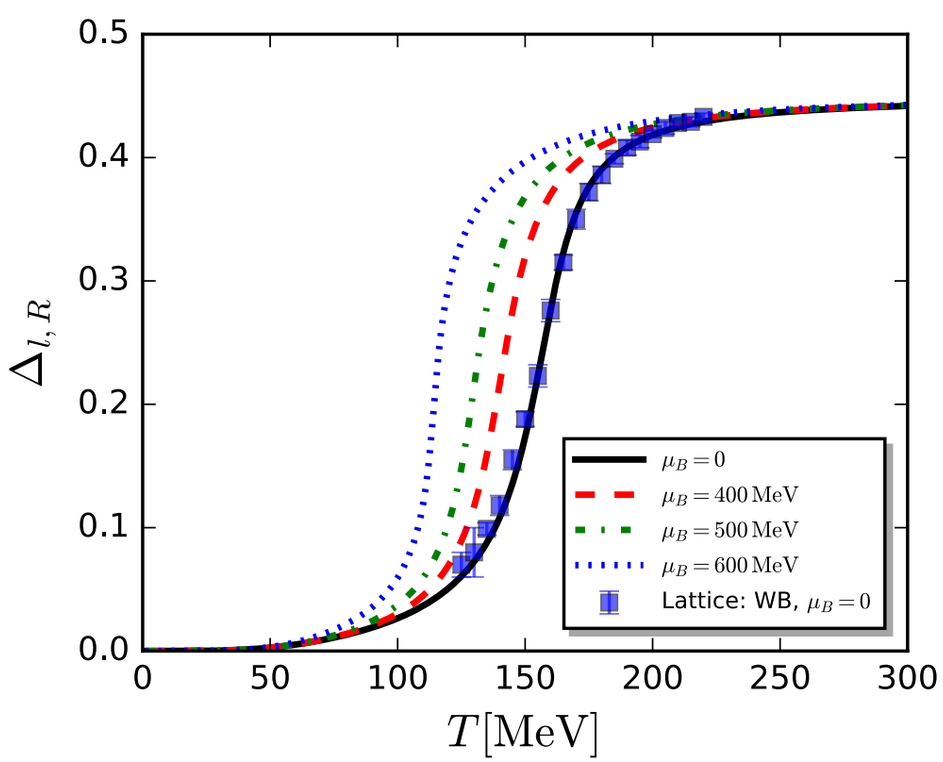
Renormalized light quark condensate

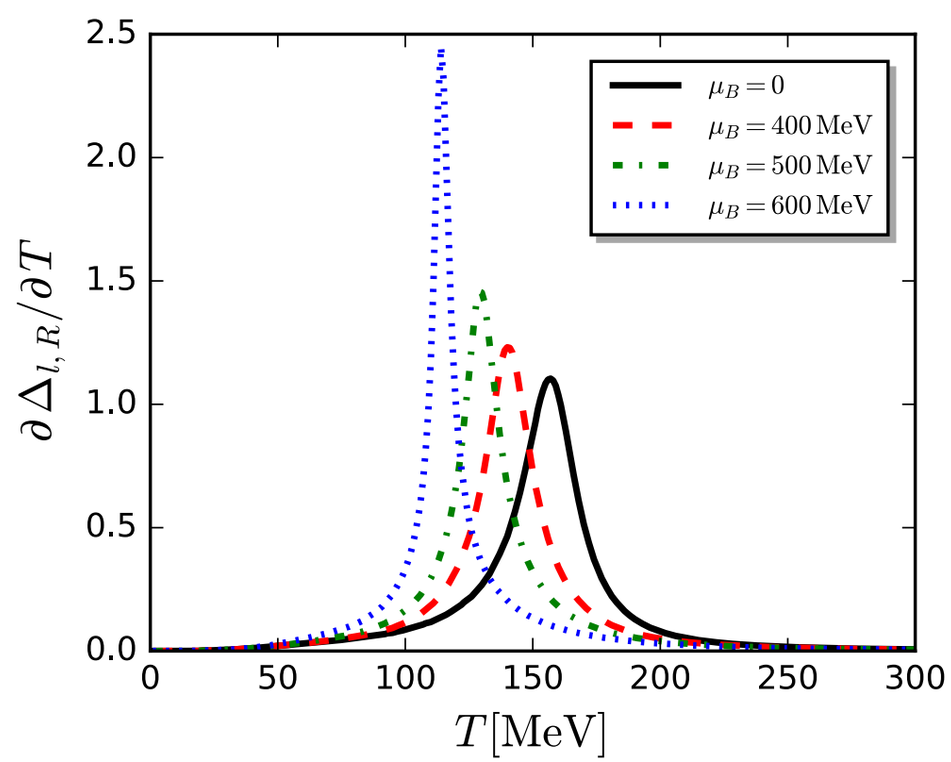


$$\Delta_{q_i} \simeq -m^0_{q_i} T \sum_{n\in\mathbb{Z}} \int \frac{d^3q}{(2\pi)^3} \mathrm{tr}\, G_{q_i\bar{q}_i}(q)\,.$$

$$\Delta_{q_i,R} = \frac{1}{\mathcal{N}_R}\left[\Delta_{q_i}(T,\mu_q) - \Delta_{q_i}(0,0)\right]\,.$$

WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

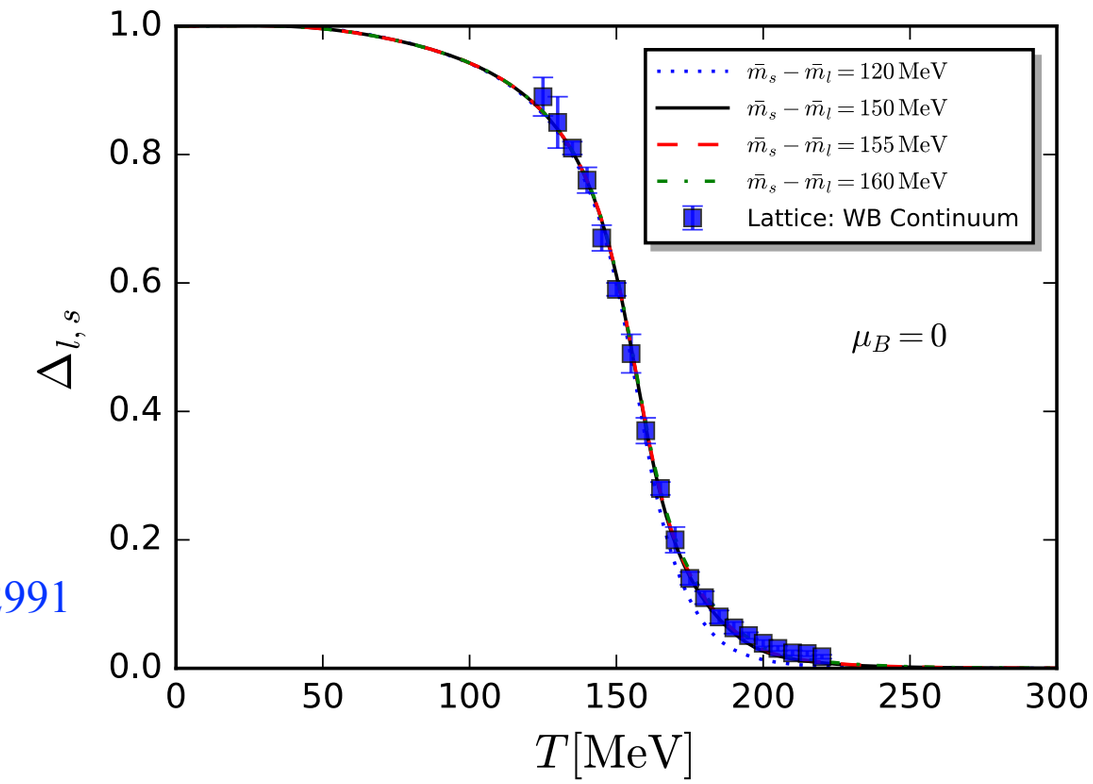




Other fermionic observables

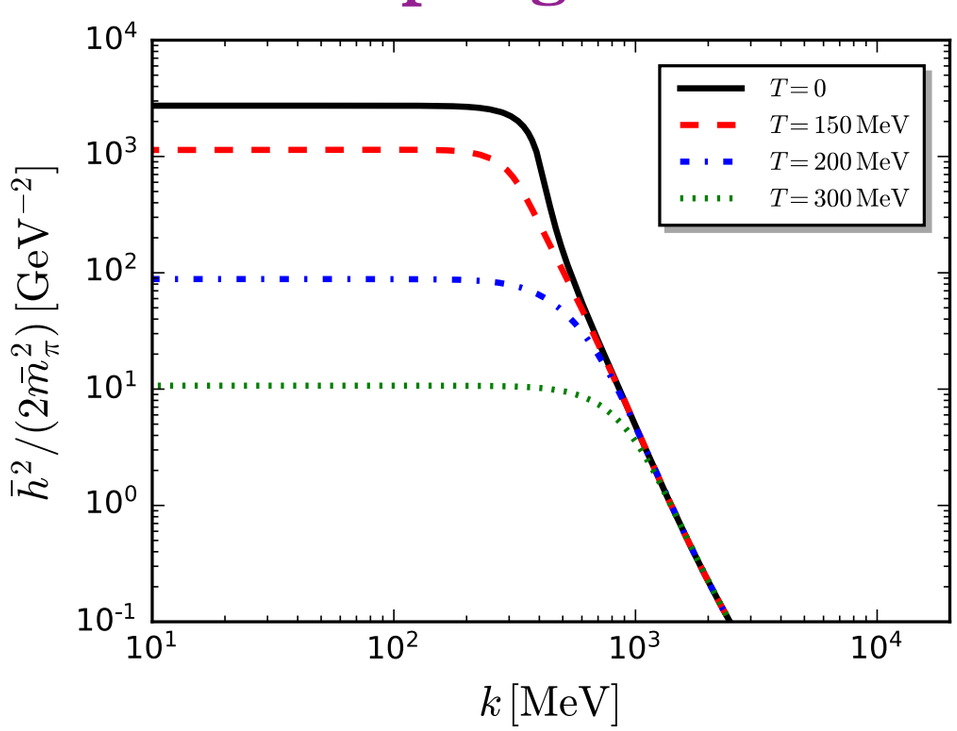
Reduced condensate:

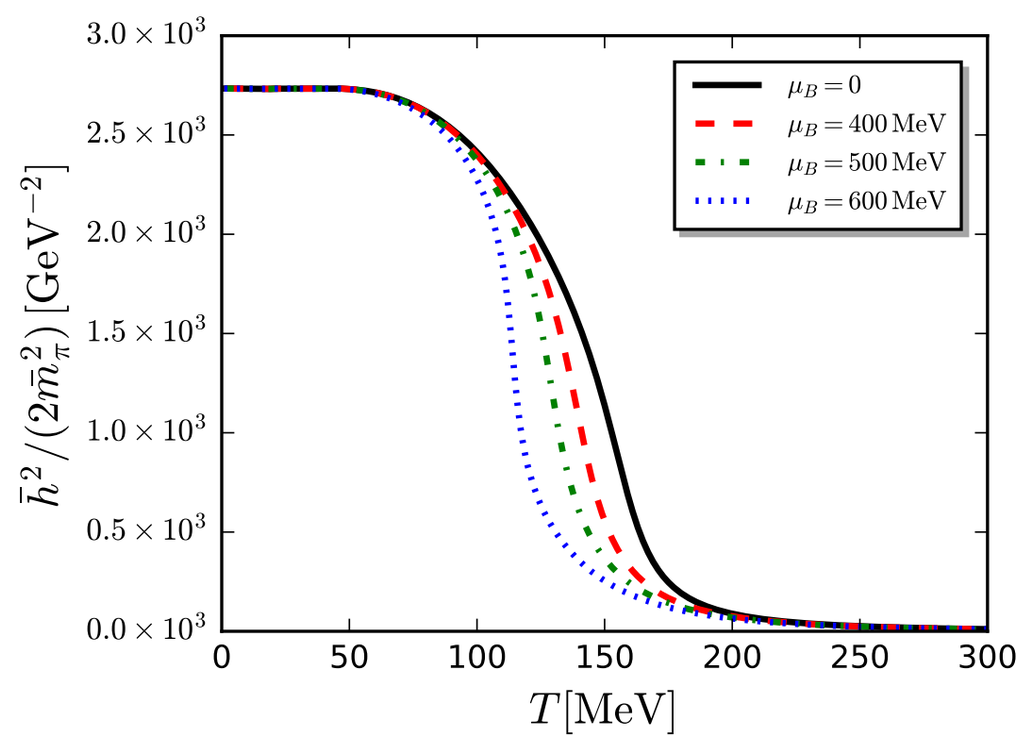
$$\Delta_{l,s} = \frac{\Delta_l(T,\mu_q) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T,\mu_Q)}{\Delta_l(0,0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0,0)} \, .$$



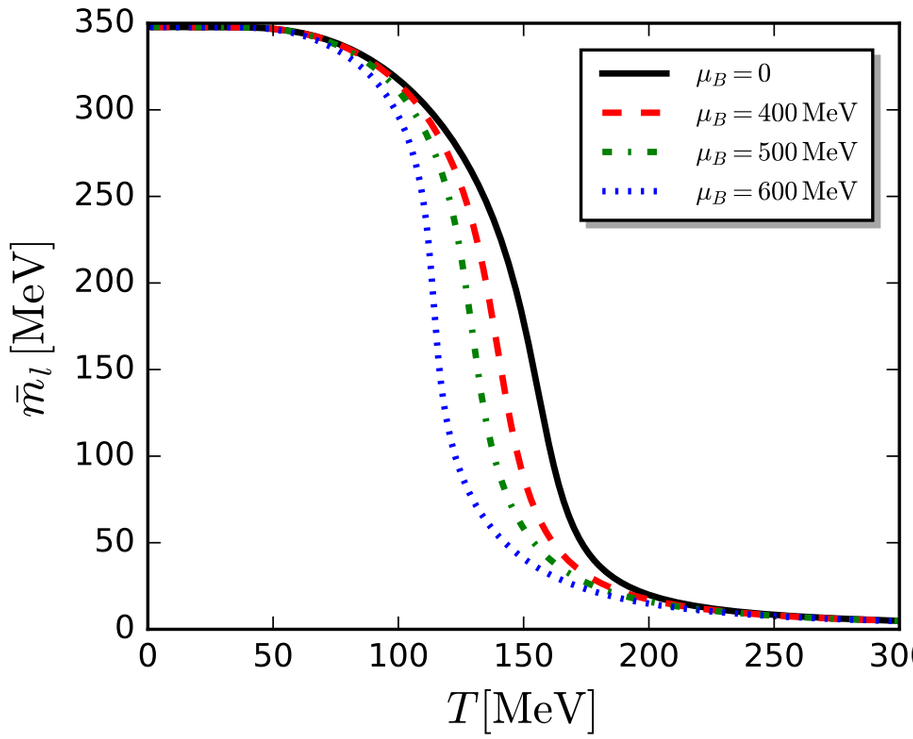
WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

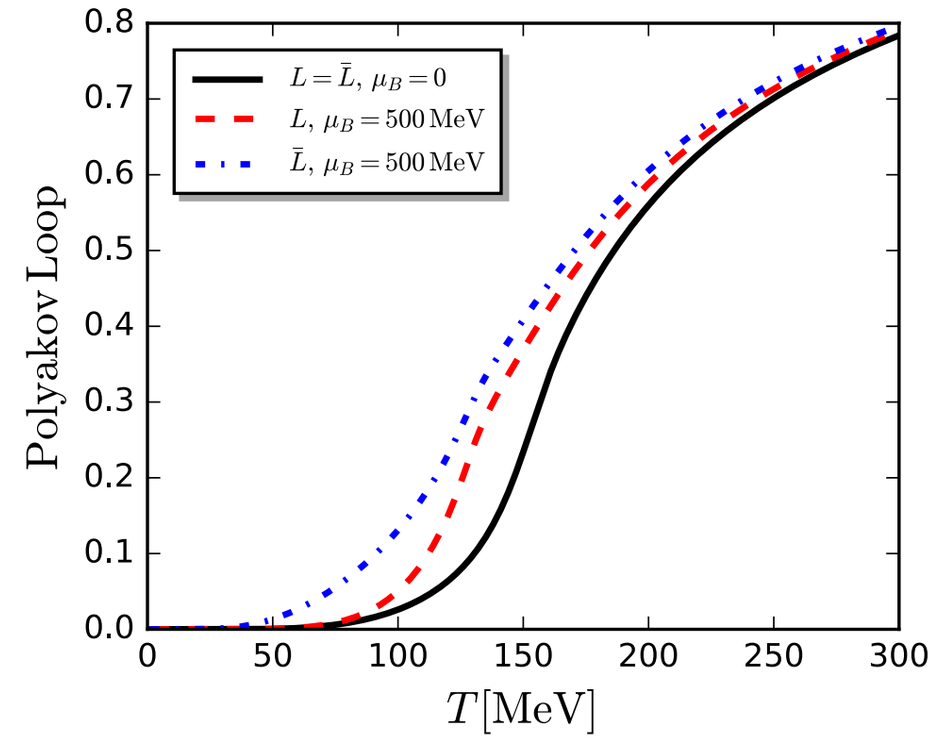
Effective four-quark coupling:

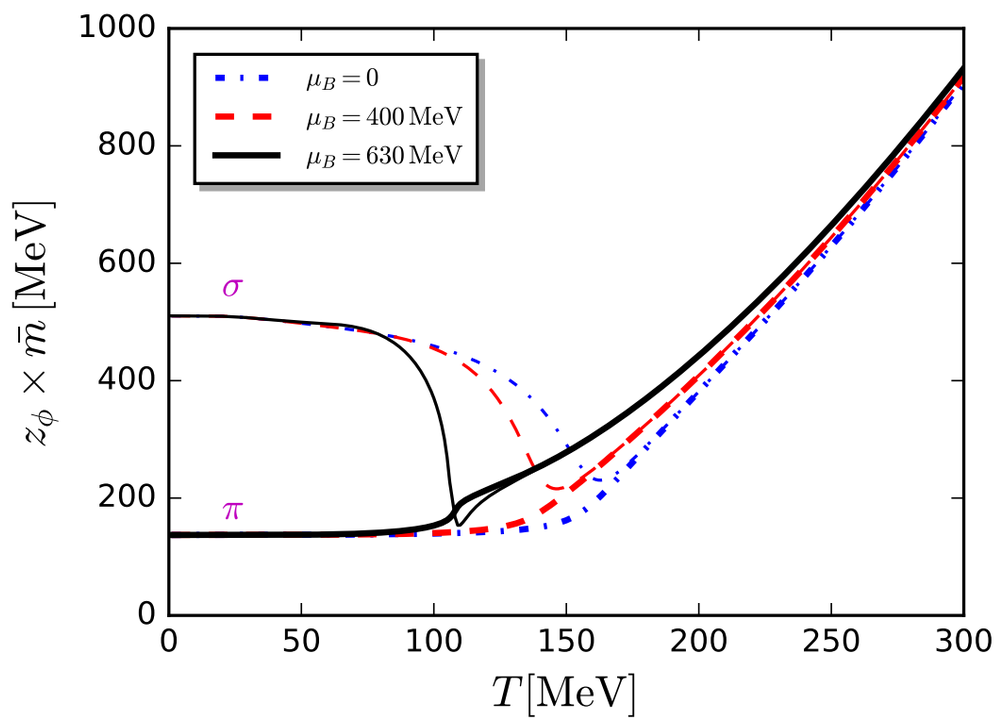




Quark, meson mass and Polyakov loop







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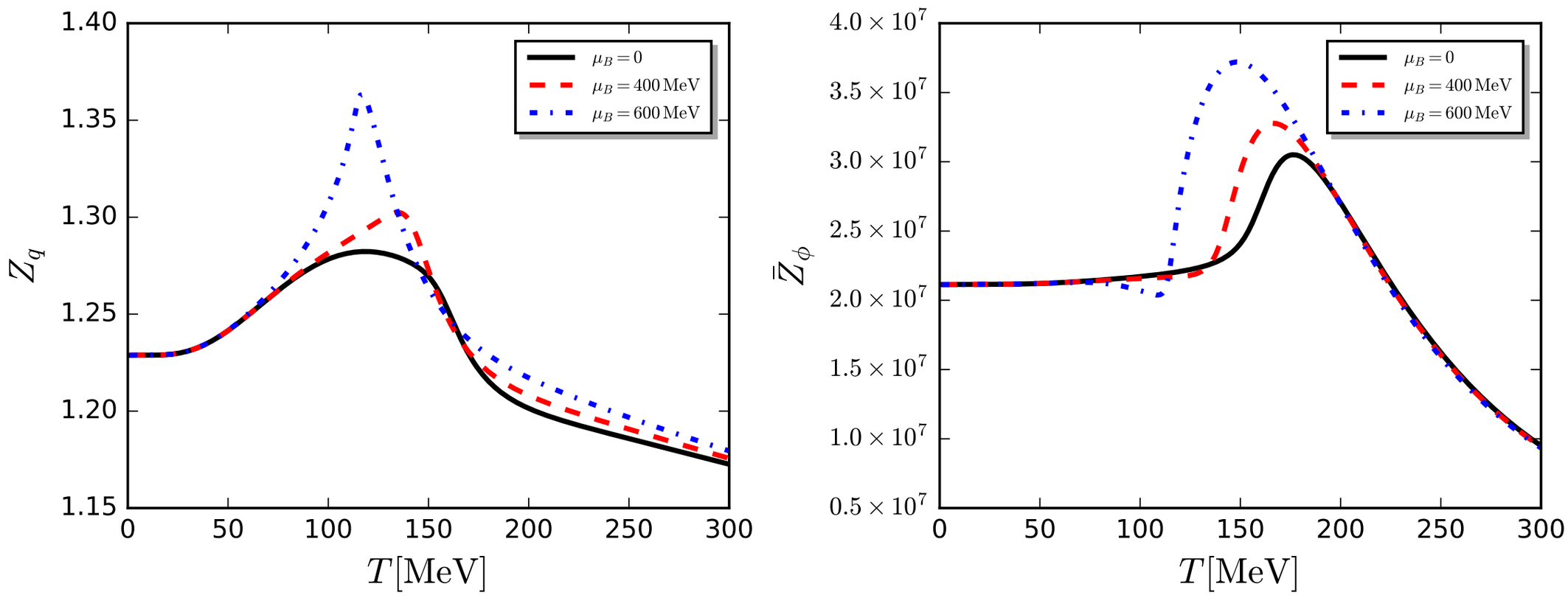
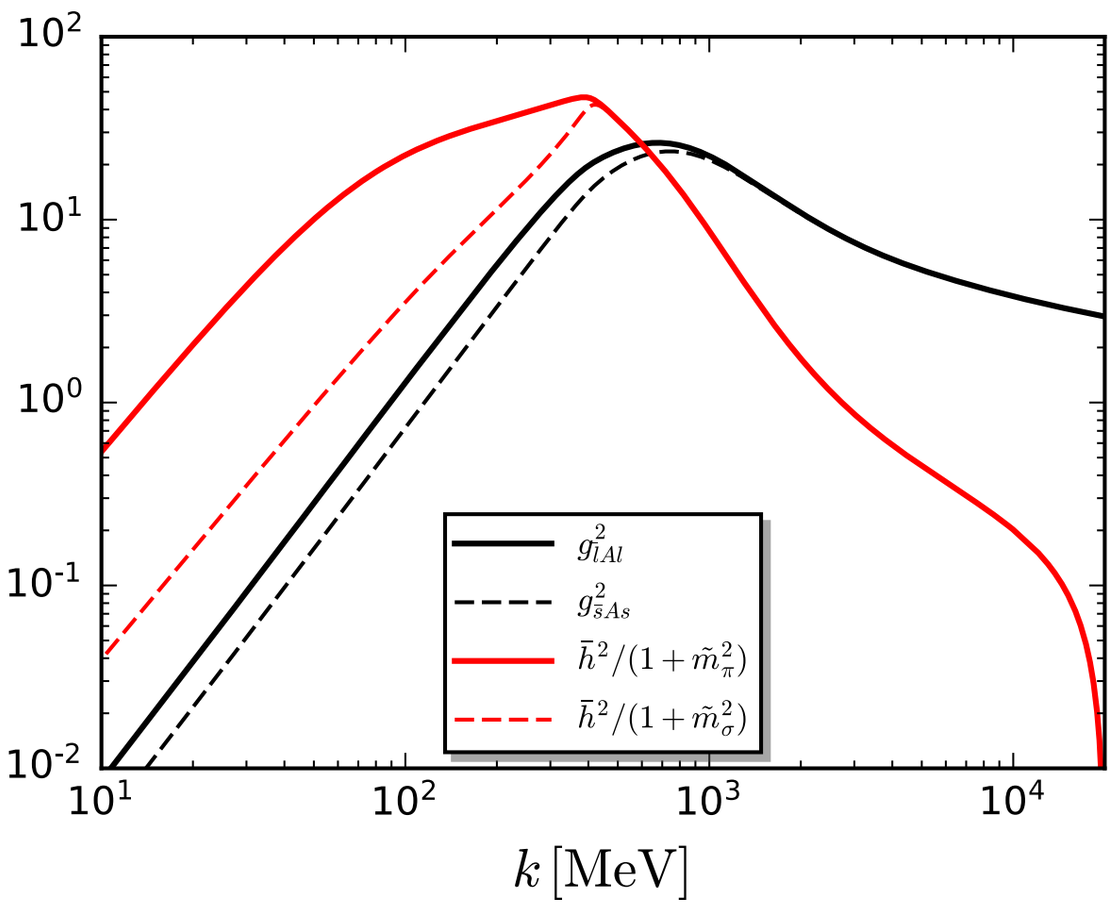
Quark and meson wave function renormalization



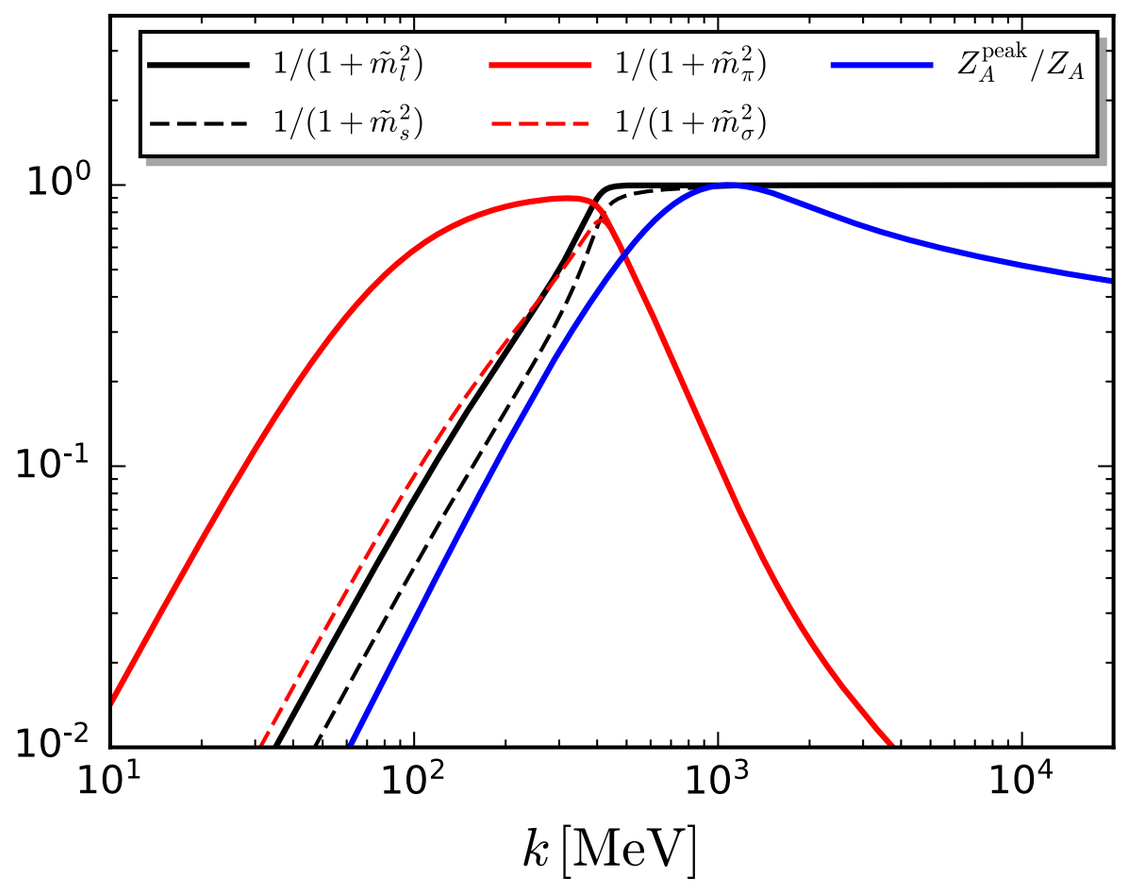
WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

Sequential decoupling and natural emergence of low energy effective theories

Exchange couplings



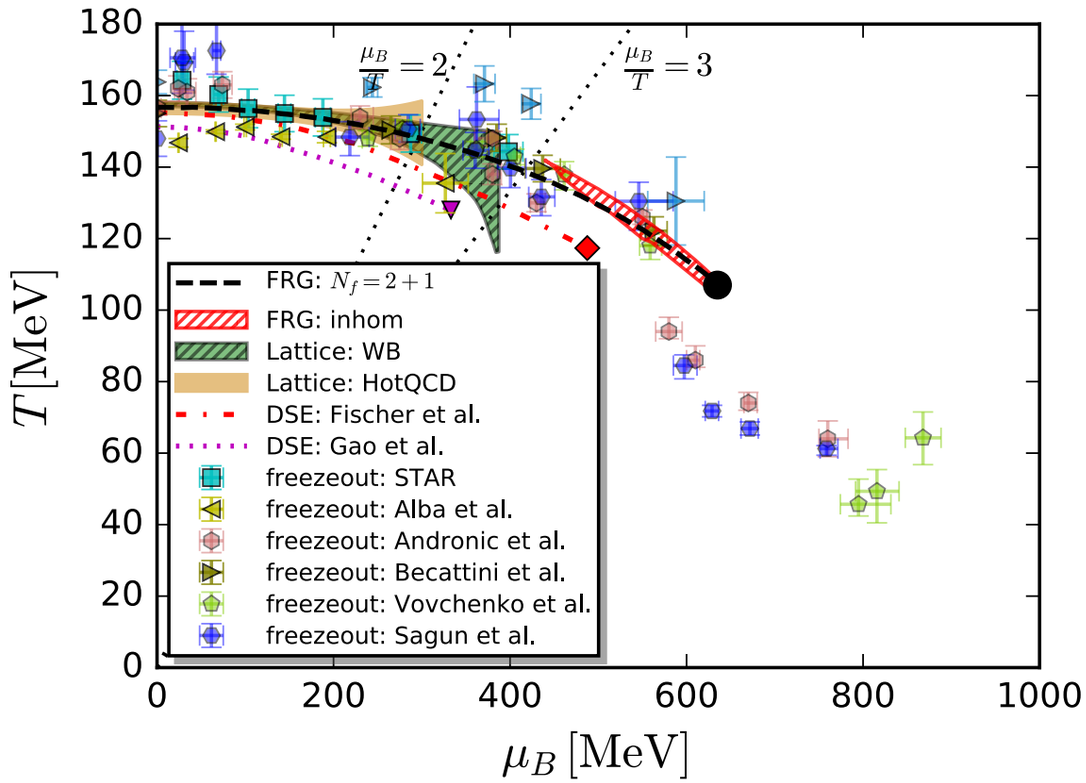
Propagator gapping

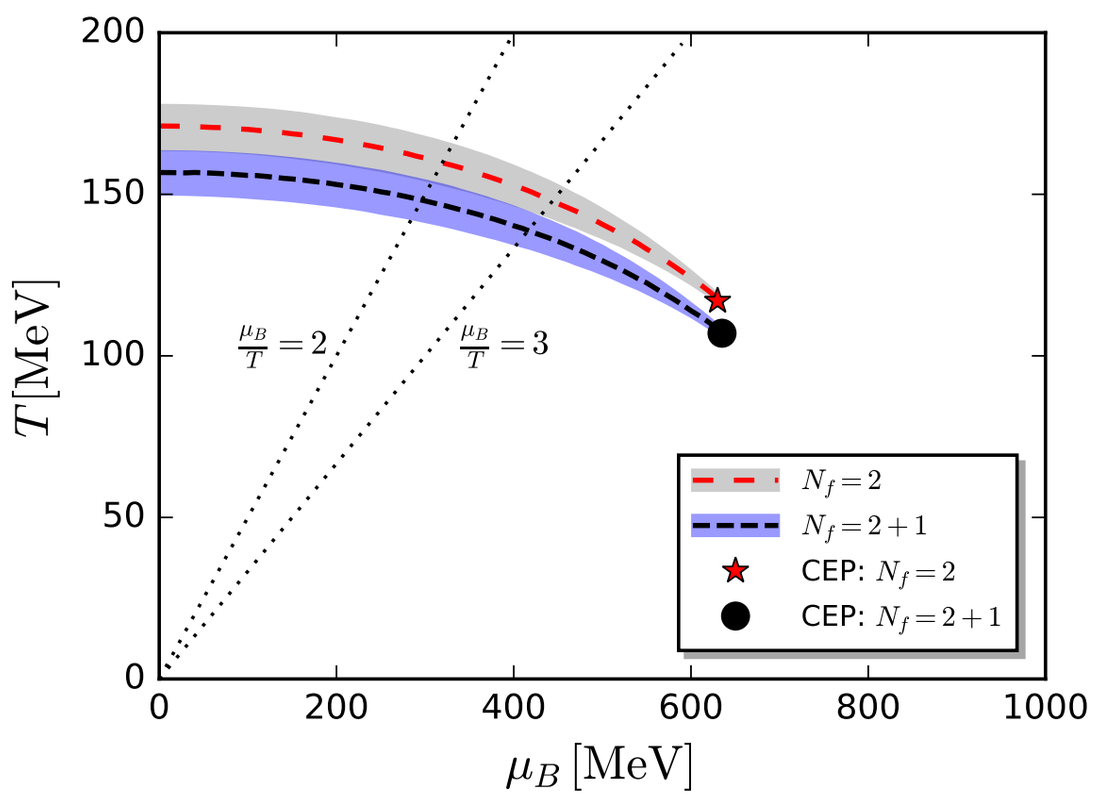


WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

Phase diagram and curvature

WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991





CEP:

$$(T_{\rm CEP}, \mu_{B_{\rm CEP}})_{N_f=2+1} = (107\,\text{MeV}, 635\,\text{MeV})\,,$$

$$(T_{\rm CEP}, \mu_{B_{\rm CEP}})_{N_f=2} = (117\,\text{MeV}, 630\,\text{MeV})\,,$$

FRG curvature of the phase boundary:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa\left(\frac{\mu_B}{T_c}\right)^2 + \lambda\left(\frac{\mu_B}{T_c}\right)^4 + \cdots\,,$$

$$\kappa_{N_f=2+1} = 0.0142(2)$$

$$\kappa_{N_f=2} = 0.0176(1)$$

Lattice result:

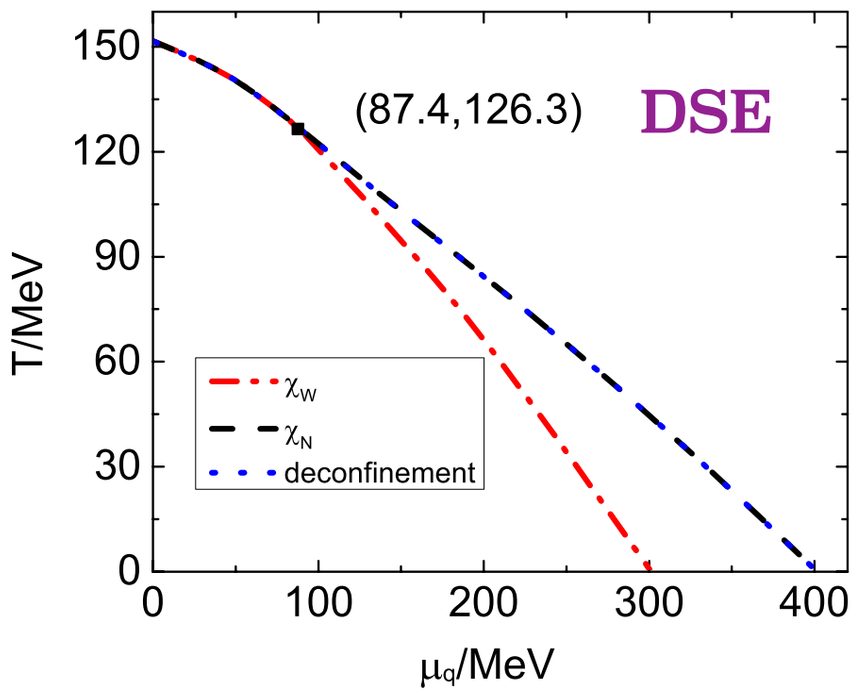
$$\kappa = 0.0149 \pm 0.0021$$

R. Bellwied *et al.* (WB), arXiv:1507.07510.

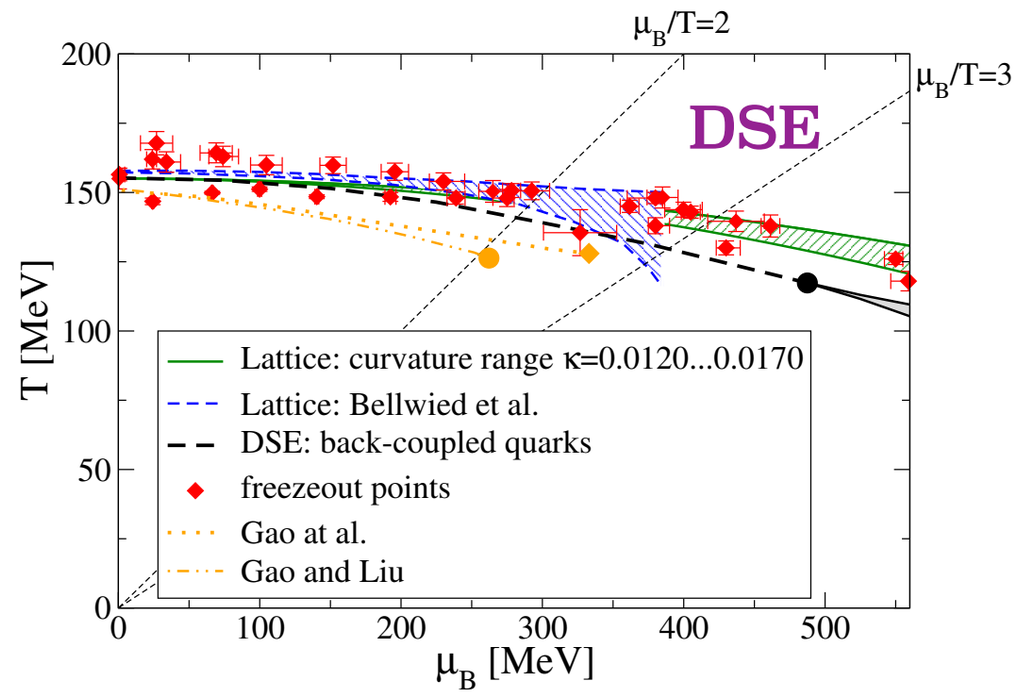
$$\kappa = 0.015 \pm 0.004$$

A. Bazavov *et al.* (HotQCD), arXiv:1812.08235.

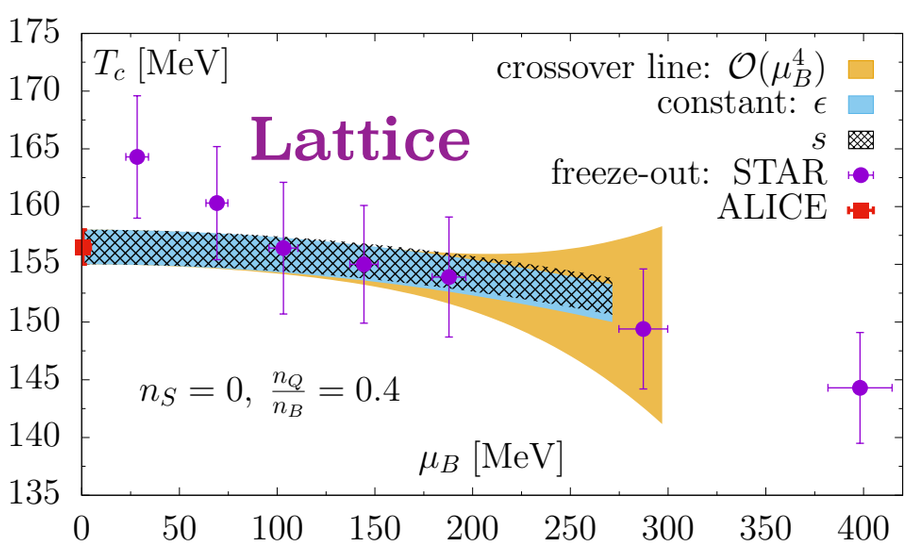
QCD Phase Structure: Lattice, DSE and FRG



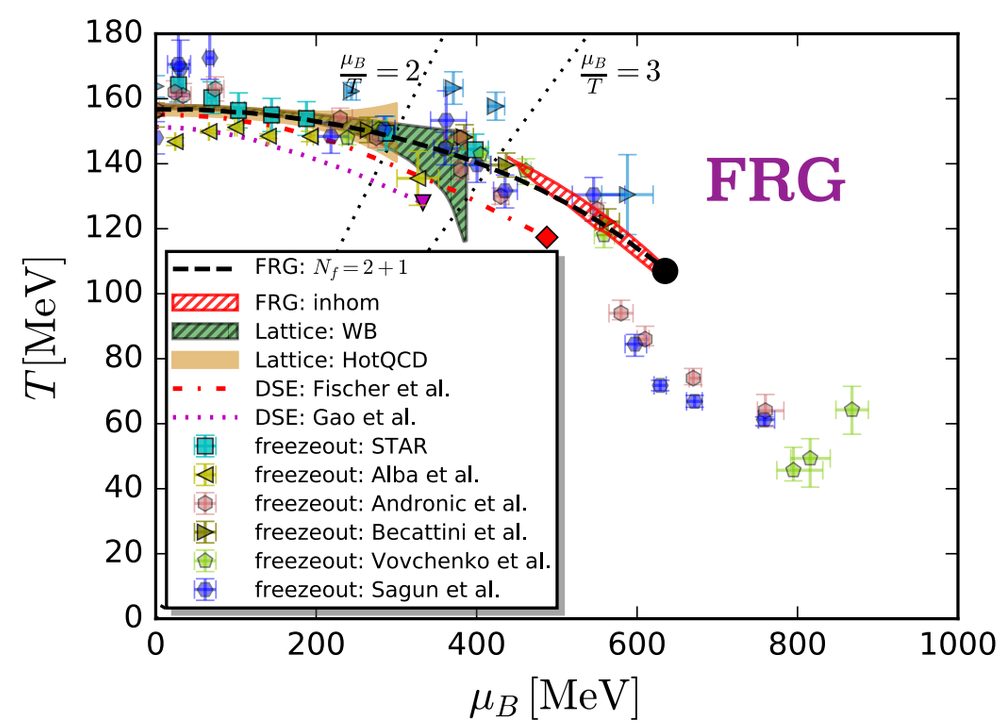
F. Gao and Y.-x. Liu , PRD 94 (2016) 076009



C.S. Fischer, Prog.Part.Nucl.Phys. 105 (2019) 1, arXiv:1810.12938.

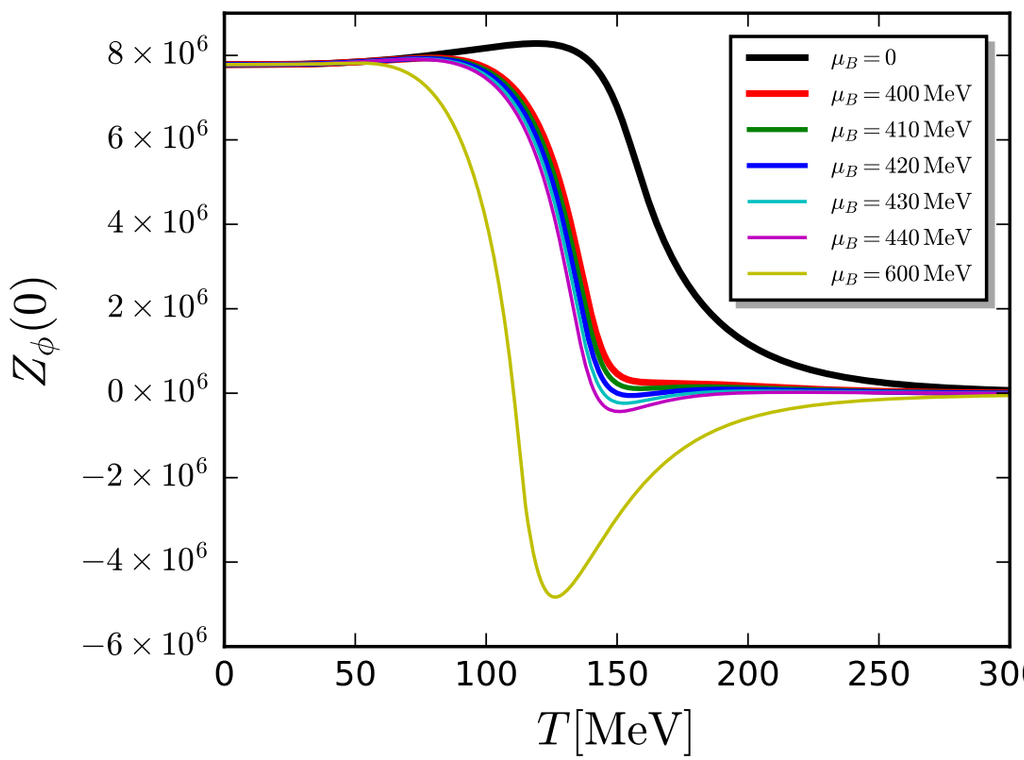


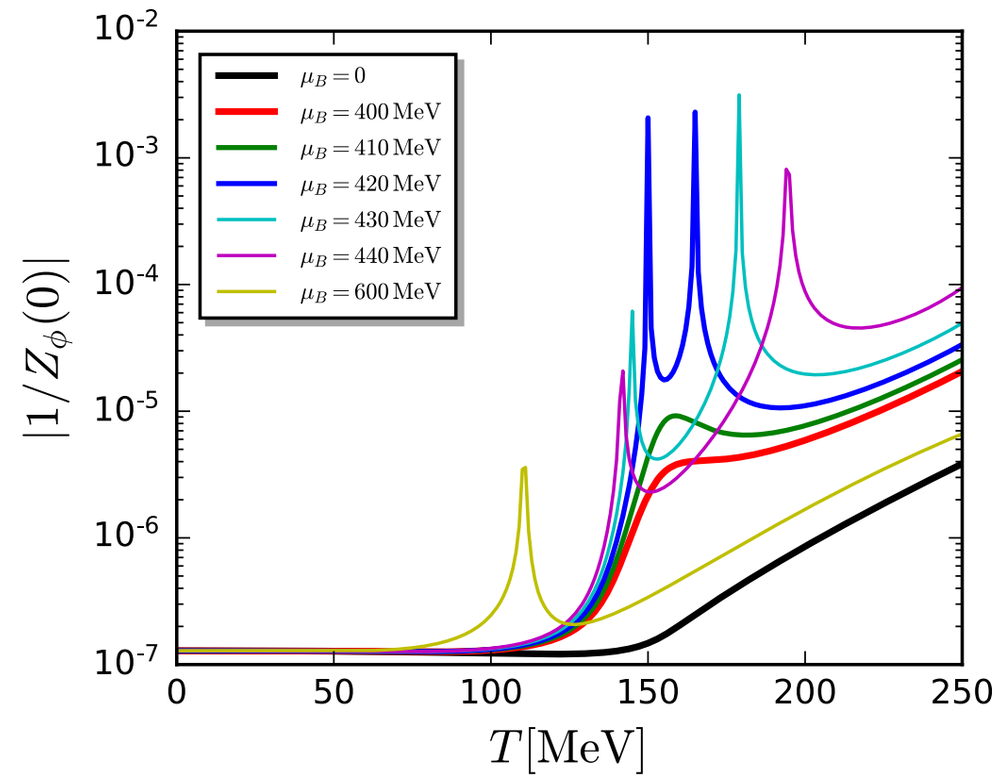
A. Bazavov *et al.* (HotQCD), PLB 795 (2019) 15, arXiv:1812.08235.

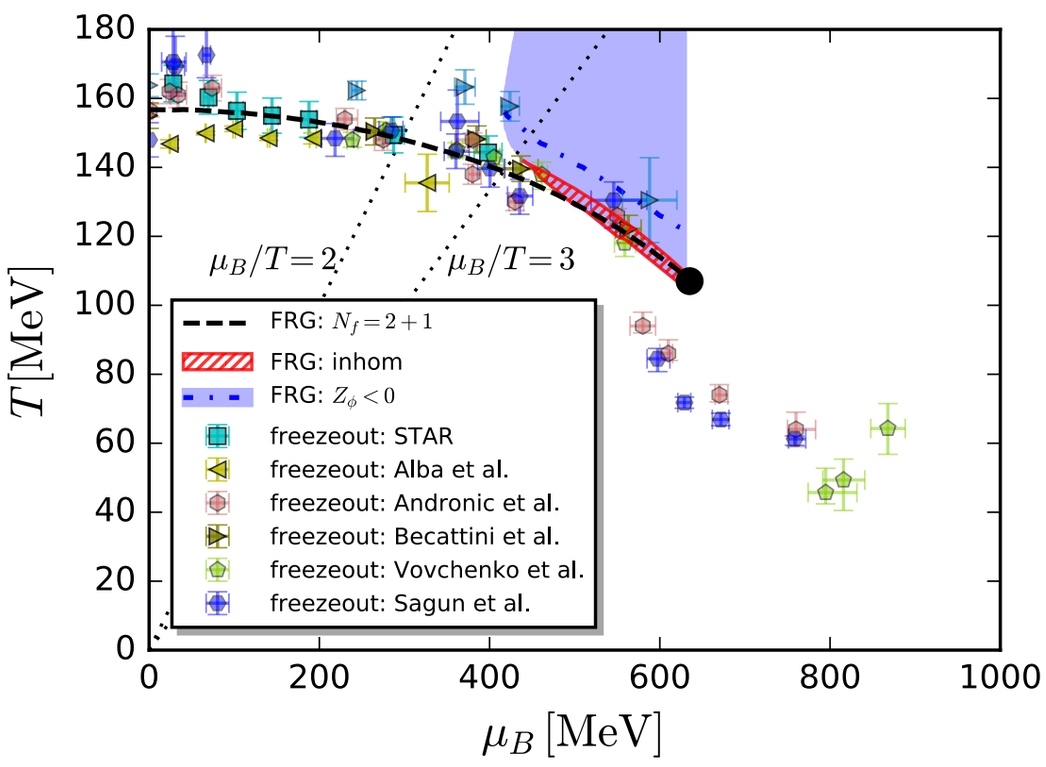


WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

Inhomogeneous phase?





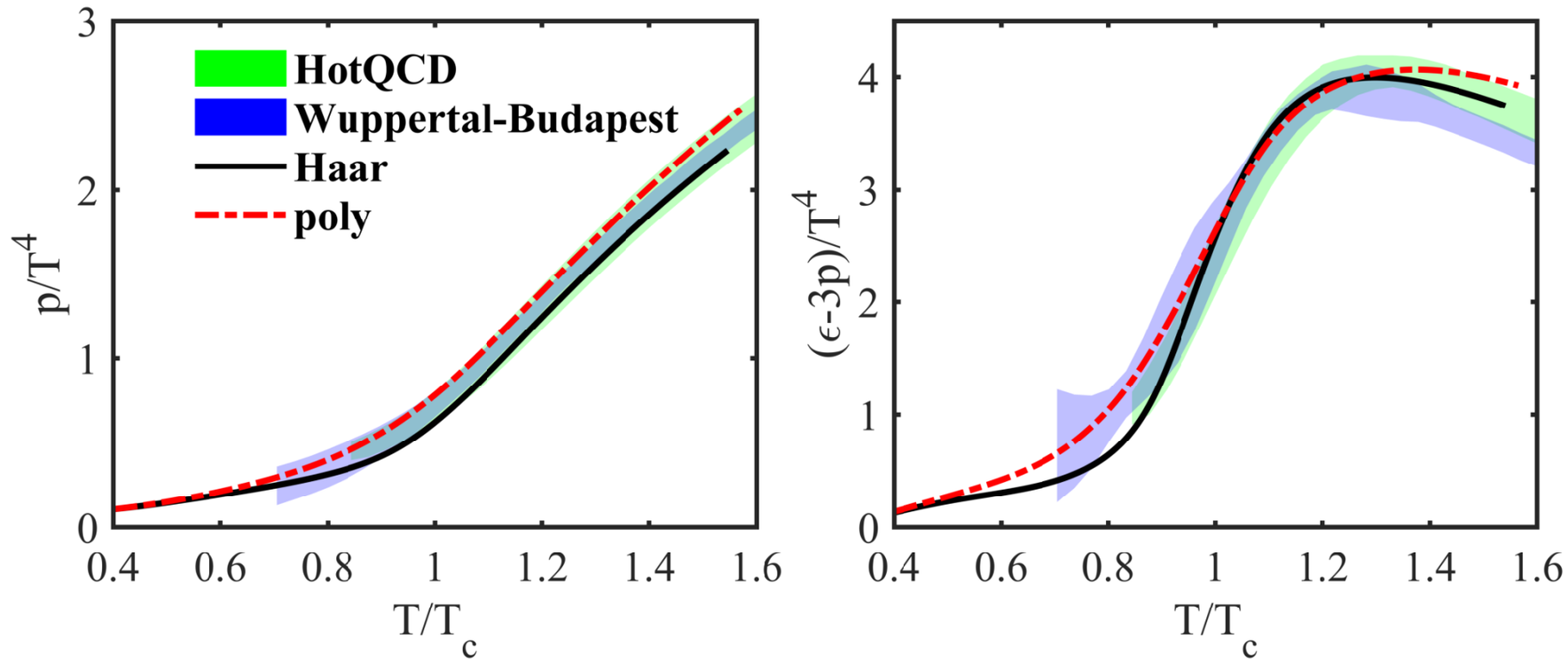


Two point function for the meson:

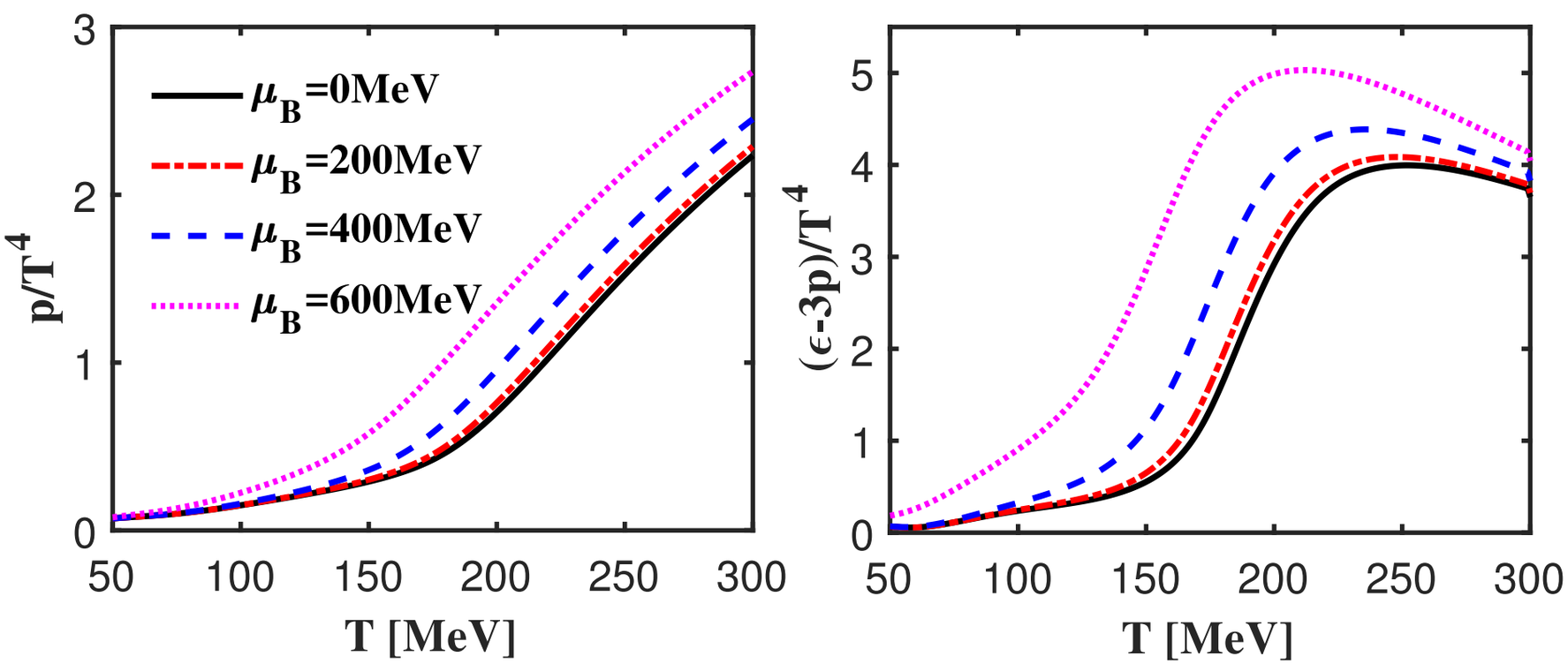
$$\Gamma^{(2)}_{\phi\phi}(p) = Z_\phi(p^2)\, p^2 + m_\phi^2 \,,$$

WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

QCD equation of state



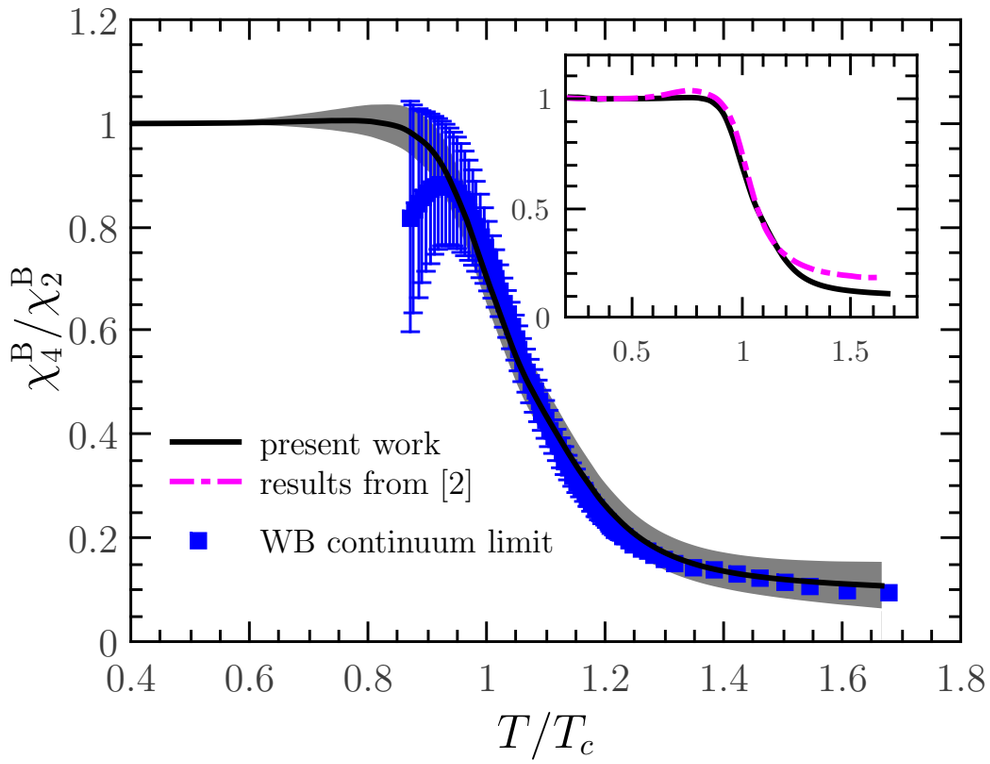
FRG results at muB=0



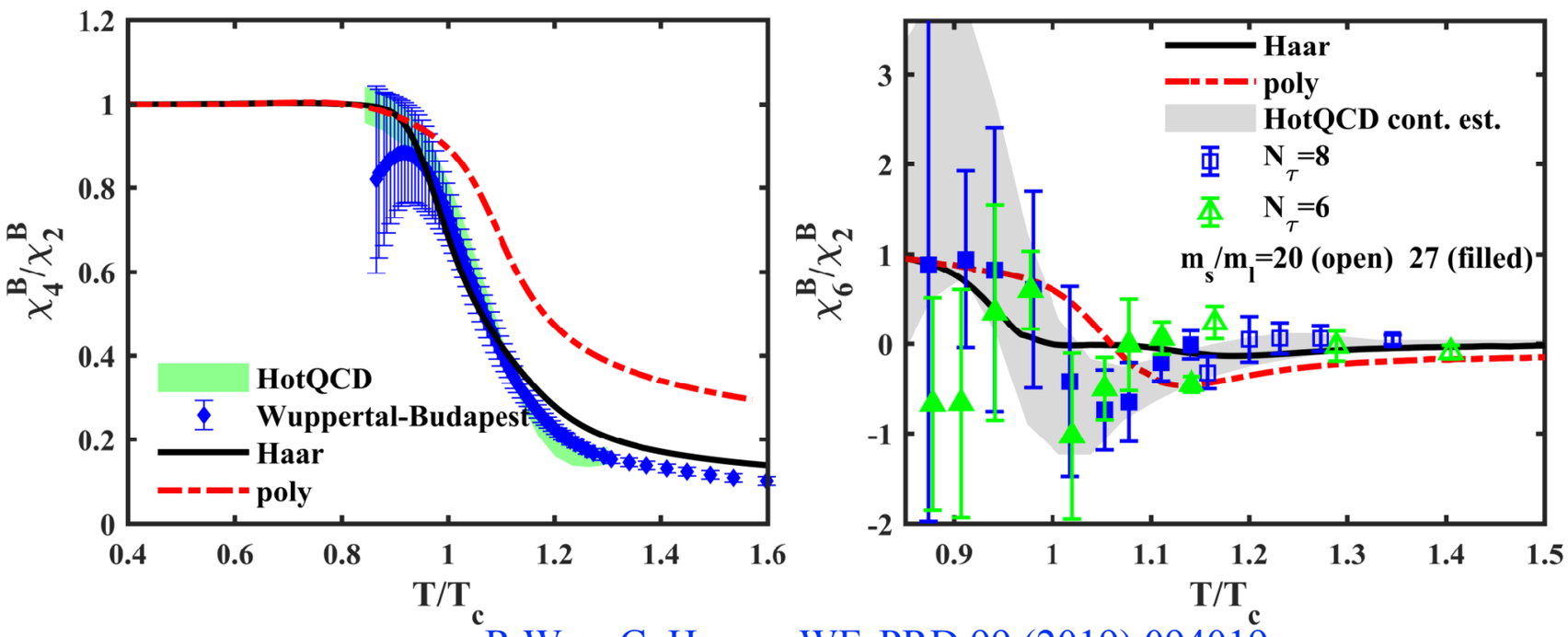
FRG results at high muB

R.Wen, C. Huang, WF, PRD 99 (2019) 094019, arXiv:1809.04233

Baryon number fluctuations from FRG



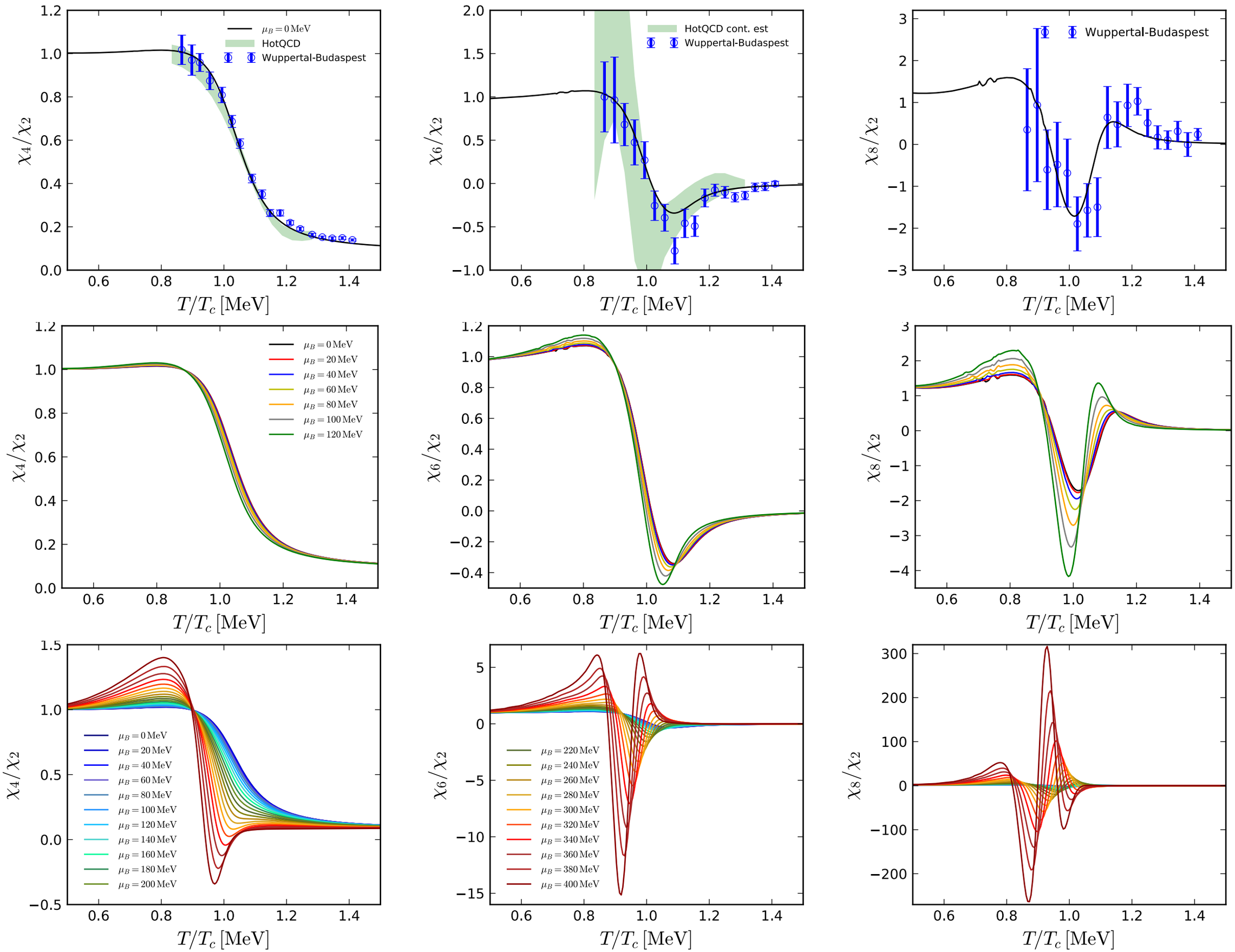
$N_f = 2$



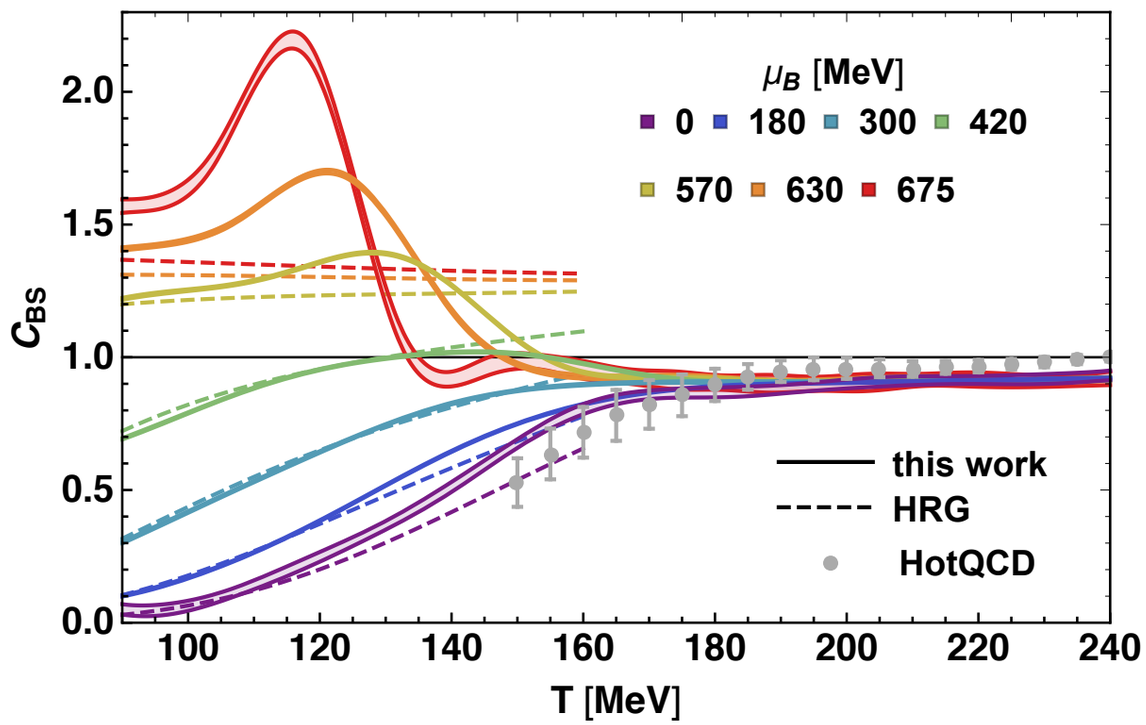
$N_f = 2 + 1$

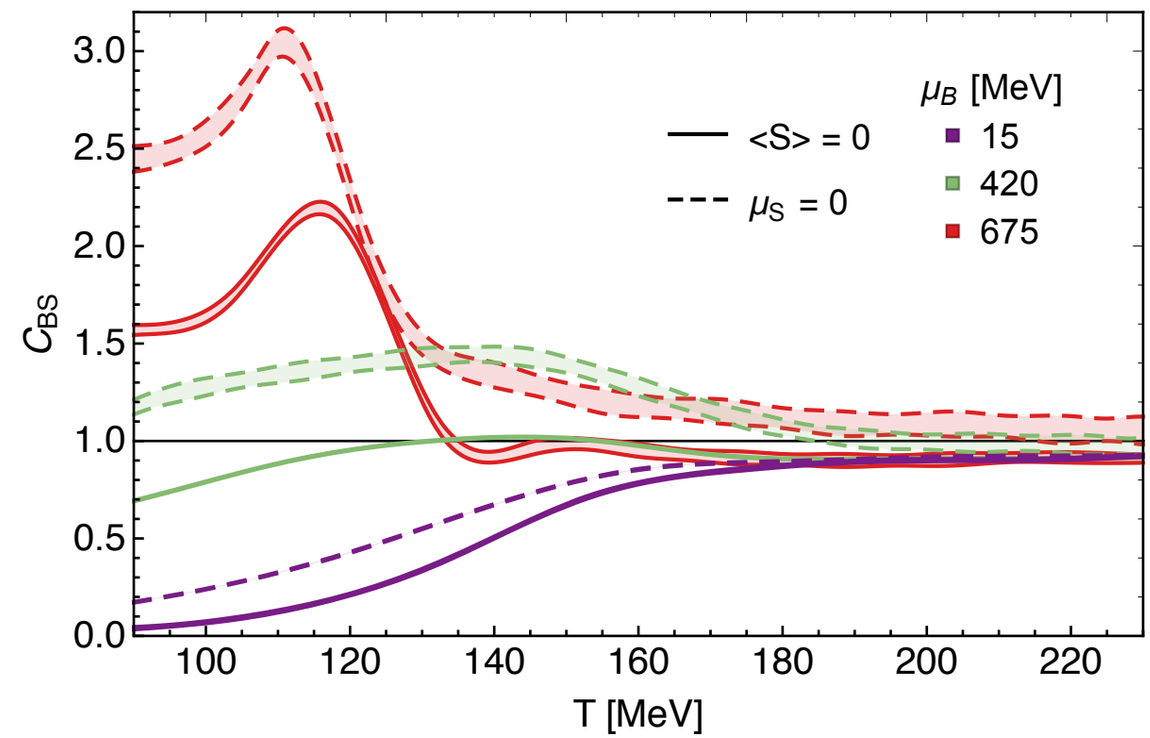
R.Wen, C. Huang, WF, PRD 99 (2019) 094019, arXiv:1809.04233

High-order baryon number fluctuations



Strangeness neutrality and baryon-strangeness correlations





Baryon-strangeness correlation:

$$C_{BS}(T,\mu_B,\mu_{S0}) = -3\frac{\langle BS\rangle}{\langle S^2\rangle} = -3\frac{\chi_{11}^{BS}(T,\mu_B,\mu_{S0})}{\chi_2^S(T,\mu_B,\mu_{S0})}$$

Strangeness neutrality:

$$0 = \frac{d}{d\hat{\mu}_B}\chi_1^S(T,\mu_B,\mu_{S0})$$

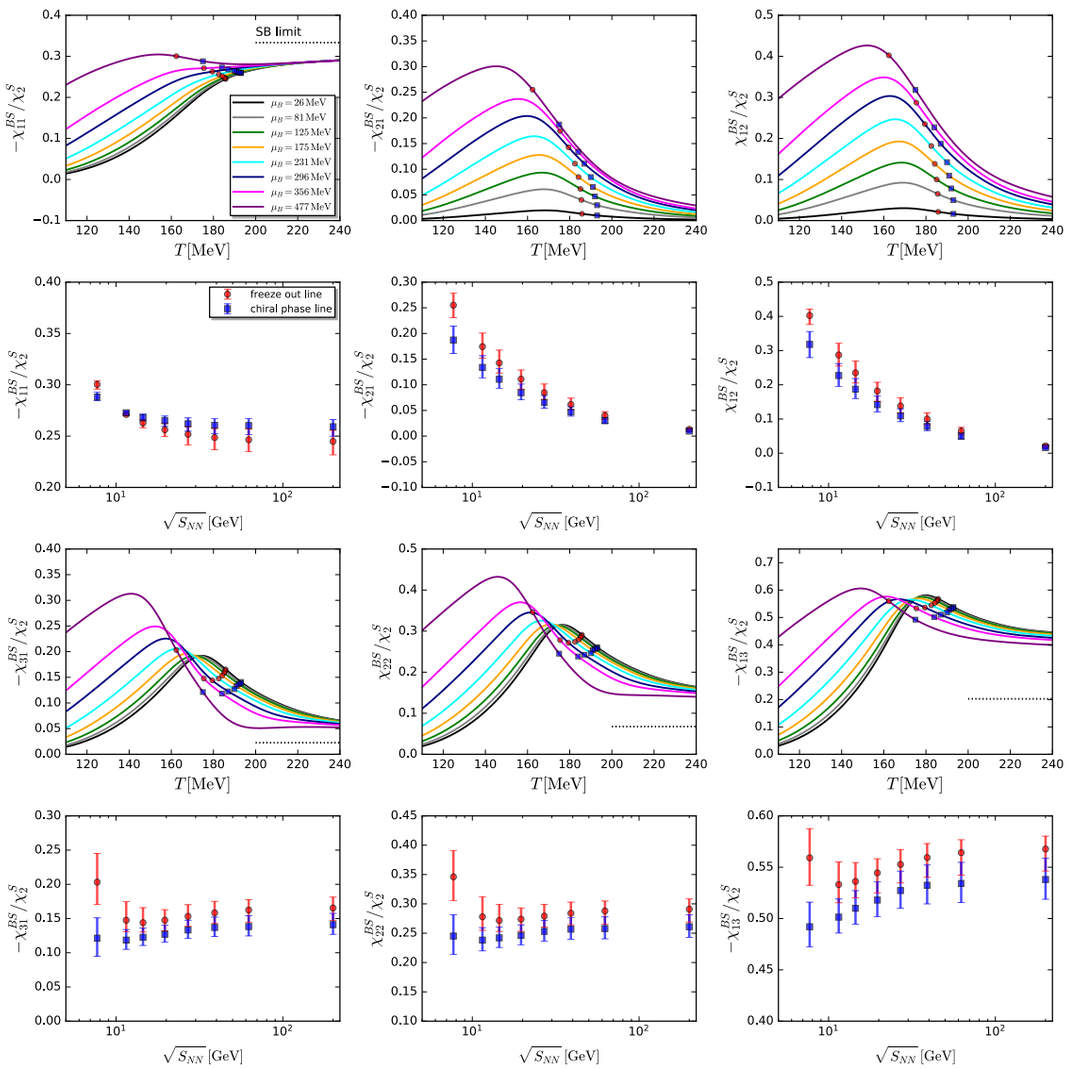
$$= \chi_{11}^{BS}(T,\mu_B,\mu_{S0}) + \chi_2^S(T,\mu_B,\mu_{S0})\frac{\partial\hat{\mu}_{S0}}{\partial\hat{\mu}_B}$$

$$\frac{\partial\mu_{S0}(T,\mu_B)}{\partial\mu_B} = \frac{1}{3}C_{BS}(T,\mu_B,\mu_{S0})\,.$$

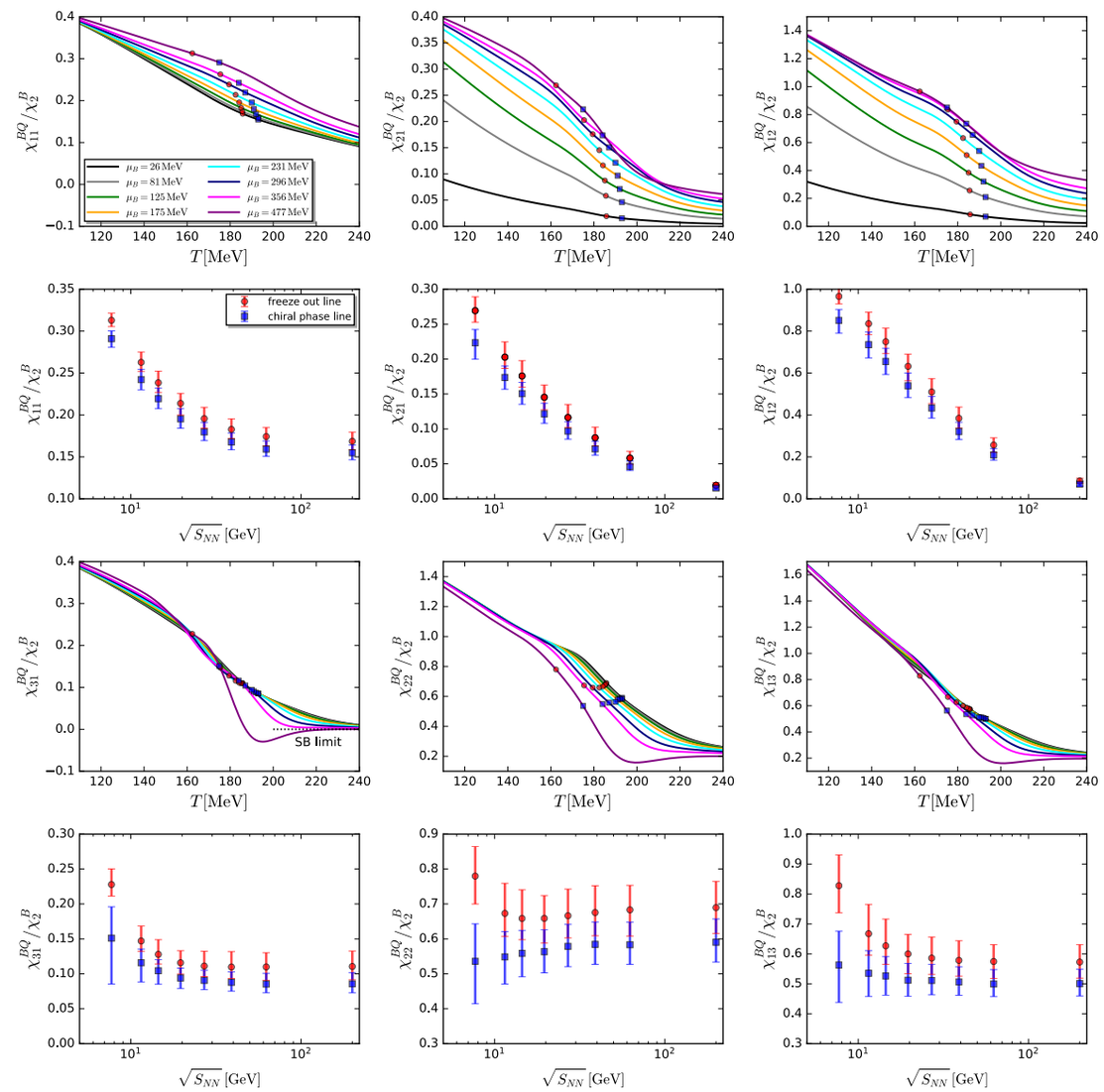
WF, J.M. Pawlowski, F. Rennecke, PRD 100 (2019) 111501(R), 1809.01594; 1808.00410.

Correlations of conserved charges

Baryon-strangeness correlations



Baryon-electric-charge correlations



Generalized susceptibilities:

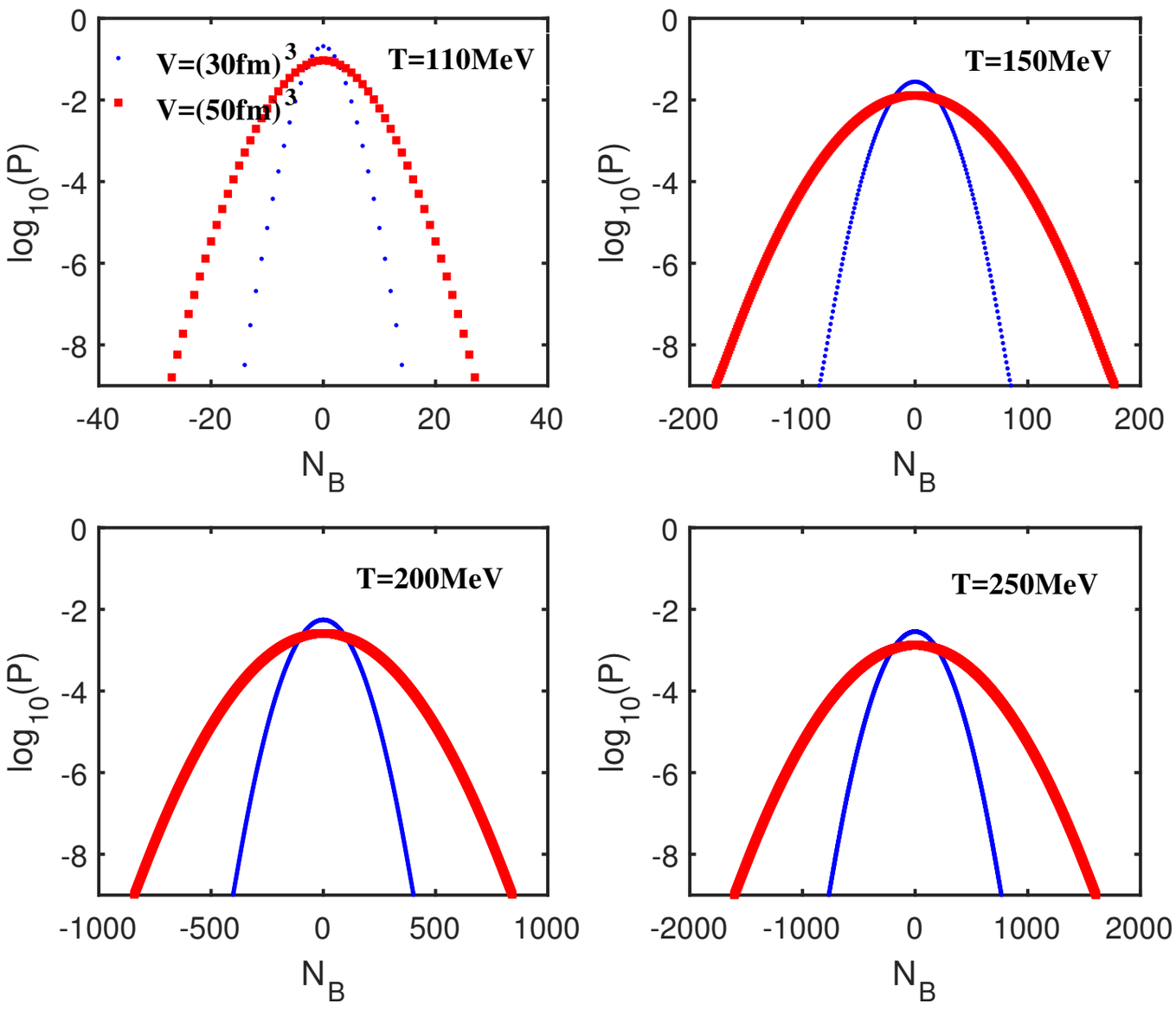
$$\chi_{ijk}^{BQS}(T,\mu_B,\mu_Q,\mu_S) = \frac{\partial^{i+j+k}(p/T^4)}{\partial\hat{\mu}_B^i\partial\hat{\mu}_Q^j\partial\hat{\mu}_S^k}$$

Rui Wen, WF, arXiv:1909.12564

Correlations:

$$\chi_{11}^{XY} = \frac{1}{VT^3}\langle(\delta N_X)(\delta N_Y)\rangle$$

Baryon number probability distribution



K.-x. Sun, R. Wen, WF, PRD 98 (2018) 074028, 1805.12025

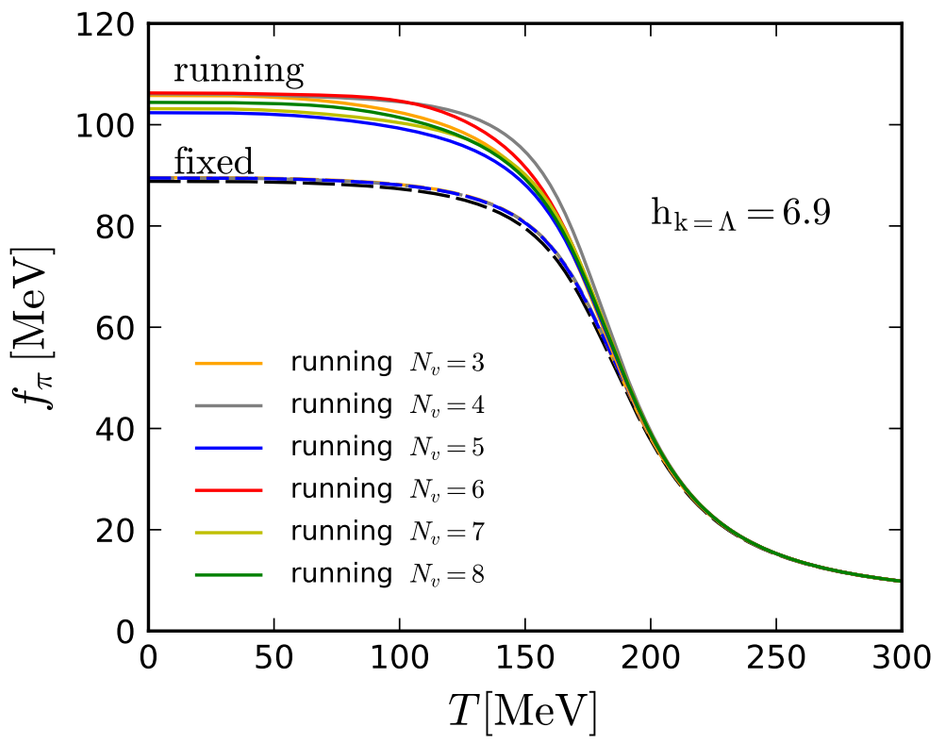
Probability distribution:

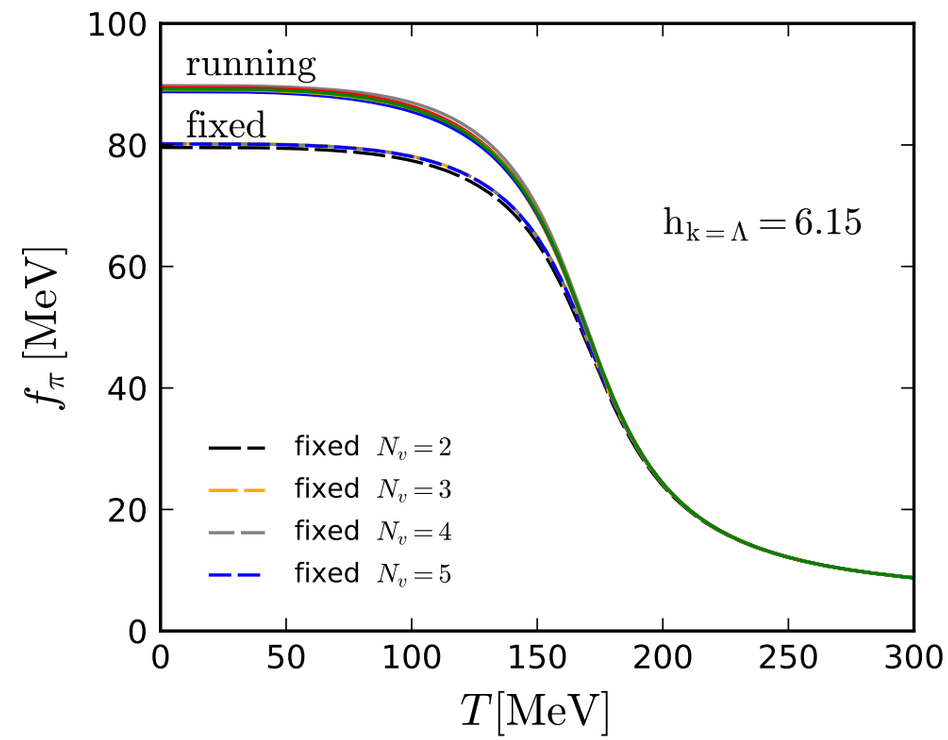
canonical partition function

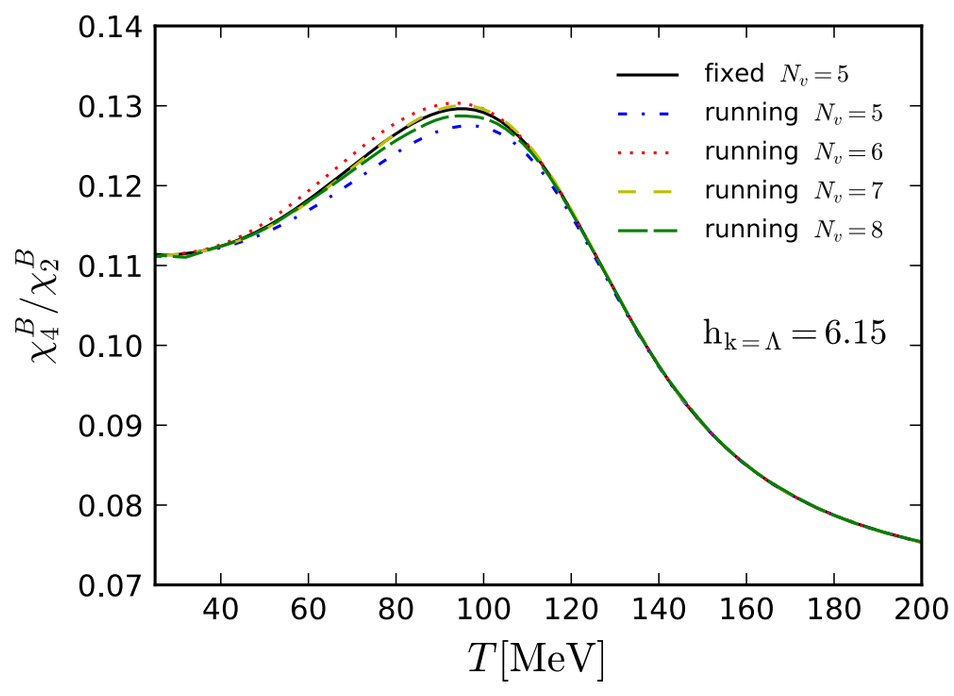
$$P(N_B;T,V,\mu_B) = \frac{Z(T,V,N_B)}{\mathcal{Z}(T,V,\mu_B)} \exp\left(\frac{\mu_B N_B}{T}\right) ,$$

grand canonical partition function

Running versus fixed expansion for the effective potential

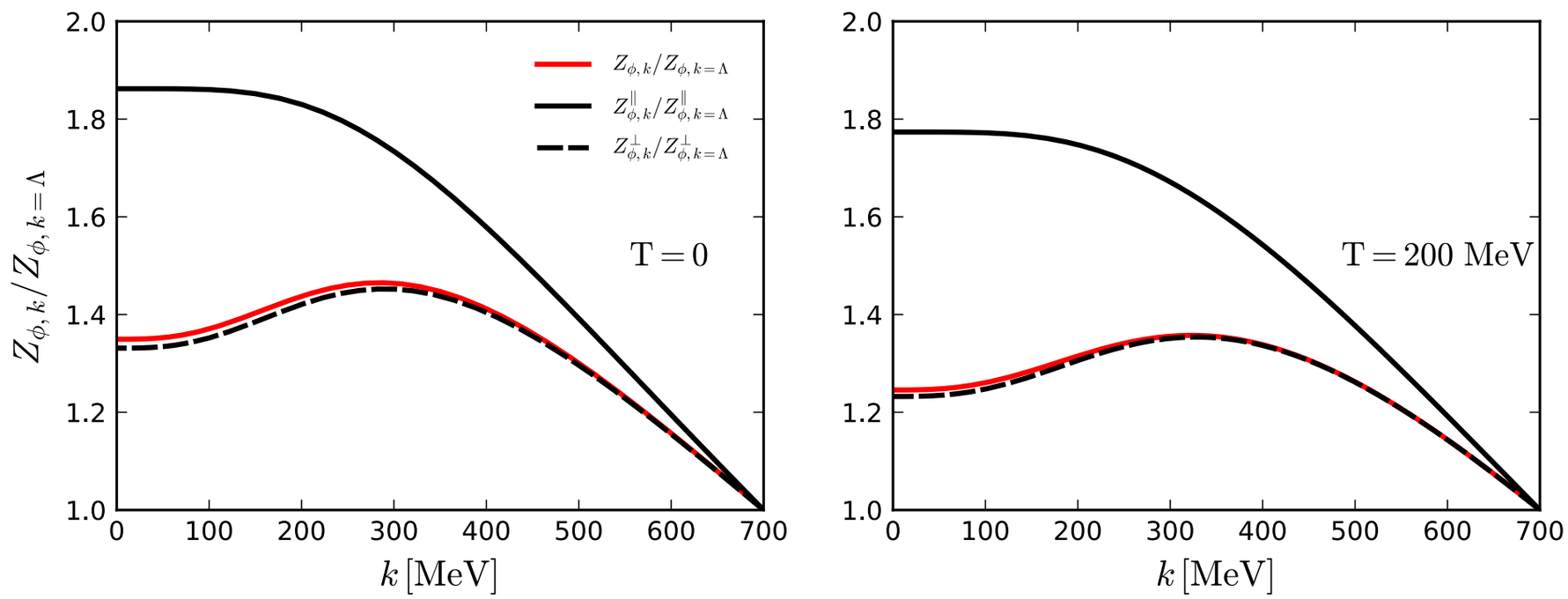




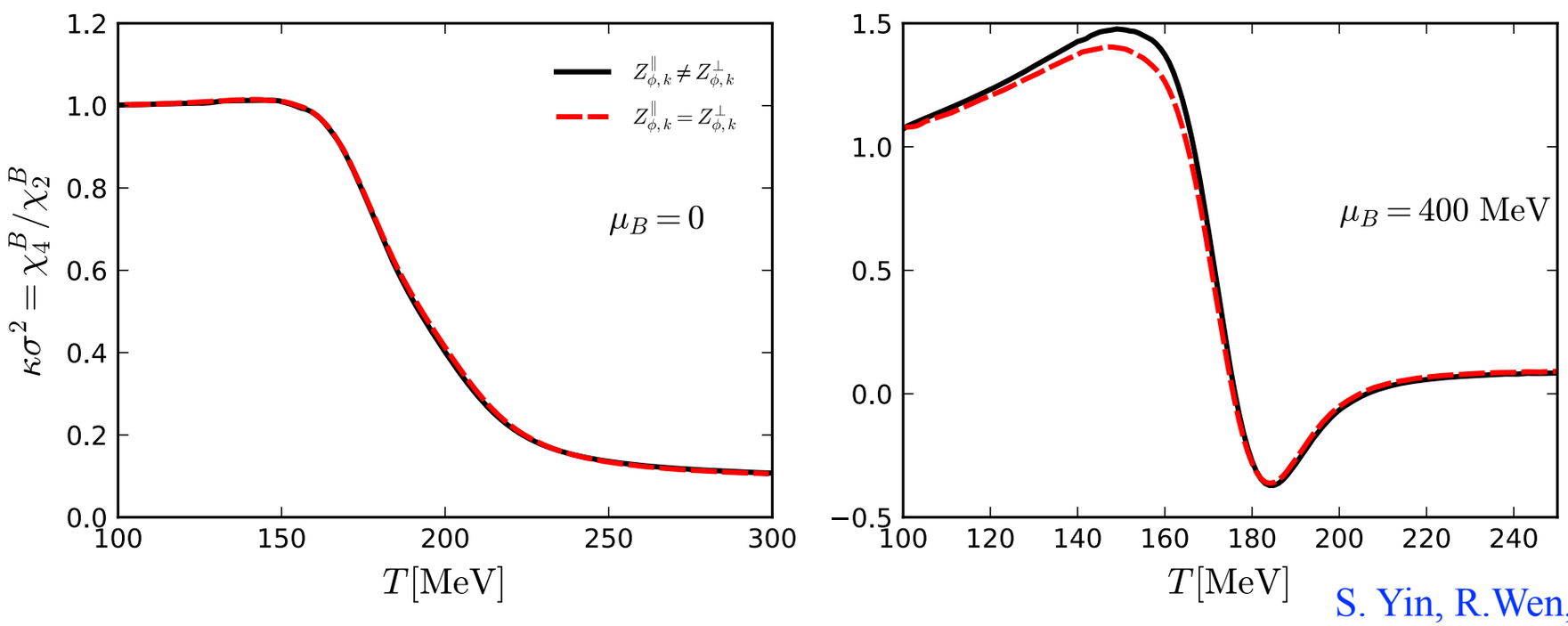


S. Yin, R.Wen, WF, PRD 100 (2019) 094029, arXiv:1907.10262.

Thermal splitting of the meson wave function renormalization



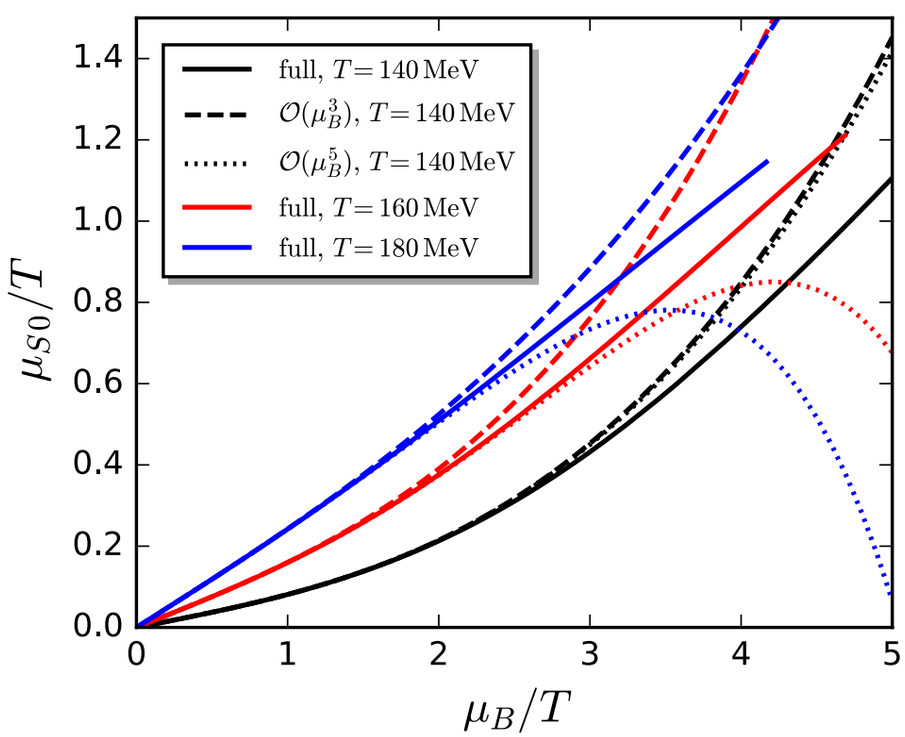
Zphi

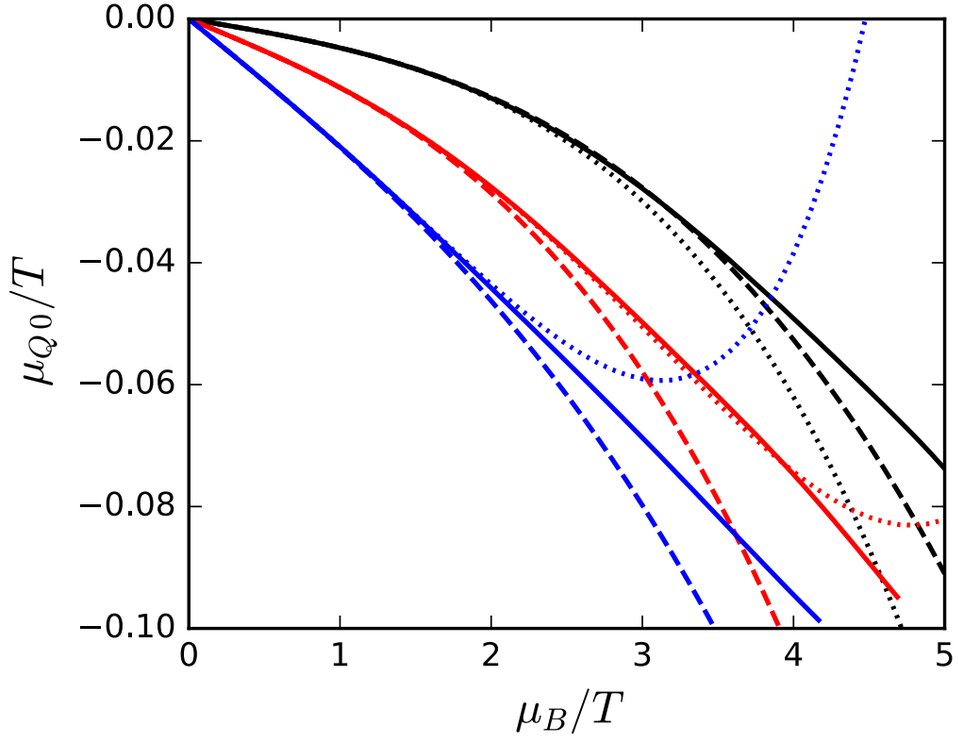


Kurtosis

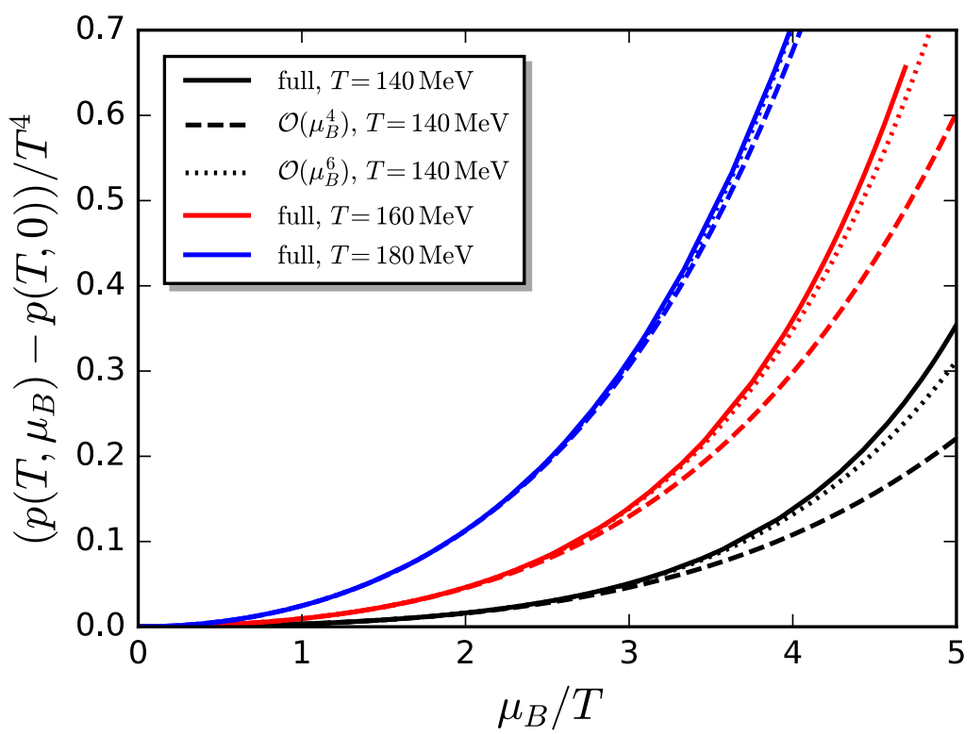
S. Yin, R.Wen, WF, PRD 100 (2019) 094029, arXiv:1907.10262.

Convergency of the Taylor expansion for muB





Pressure:



Rui Wen, WF, arXiv:1909.12564

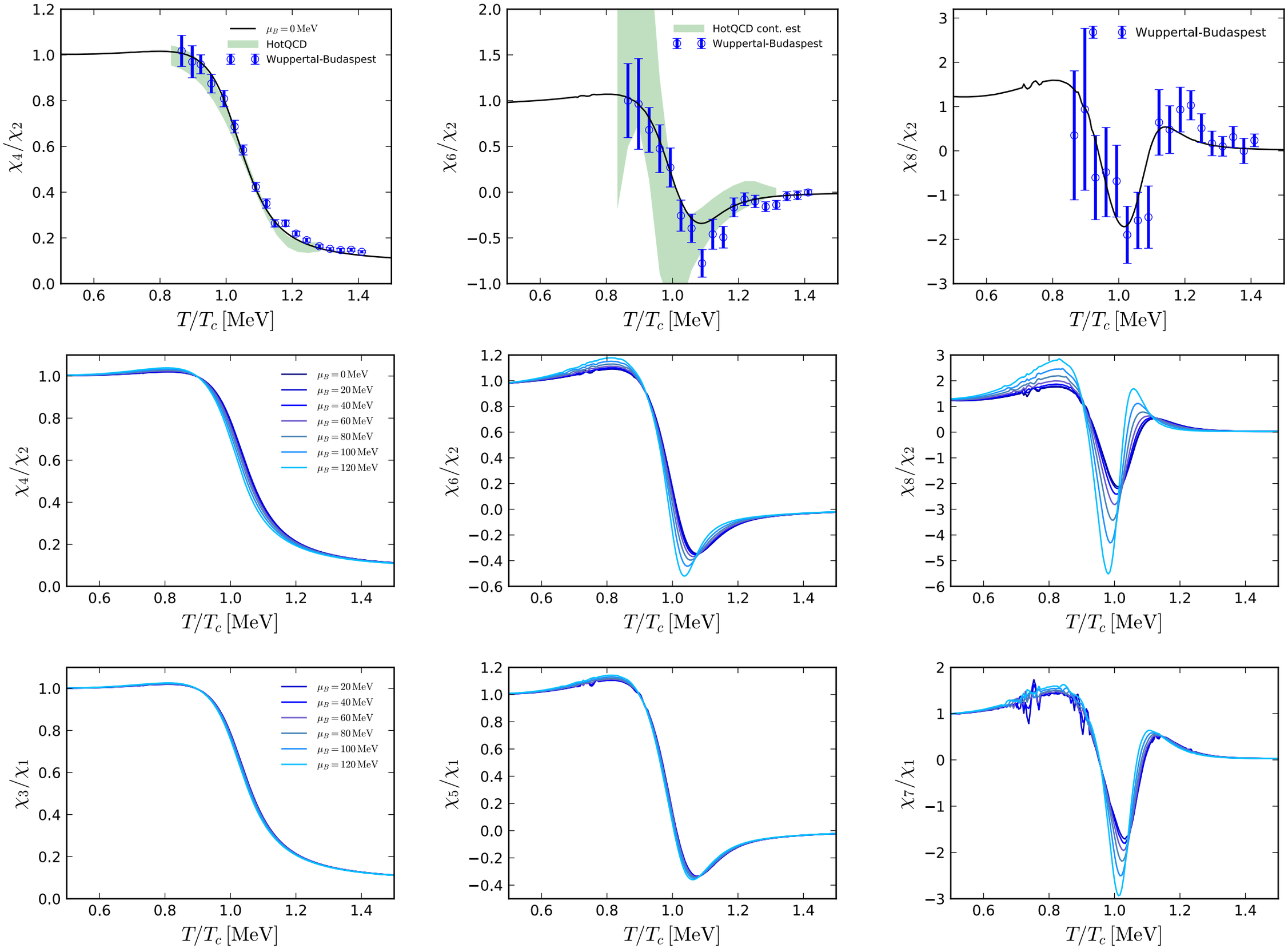
Summary and outlook

- ★ We have investigated the QCD phase structure within FRG.
- ★ A CEP at (T, muB)=(107,635) MeV, as well as indications for an inhomogeneous phase for muB >=420 MeV is founded.
- ★ The curvature of the phase boundary at small chemical potential is kappa=0.0142(2), in agreement with lattice results.
- ★ Improvement of our calculation and its applications on other observables, e.g., EoS, fluctuations, etc. are highly required.

Thank you very much for your attentions!

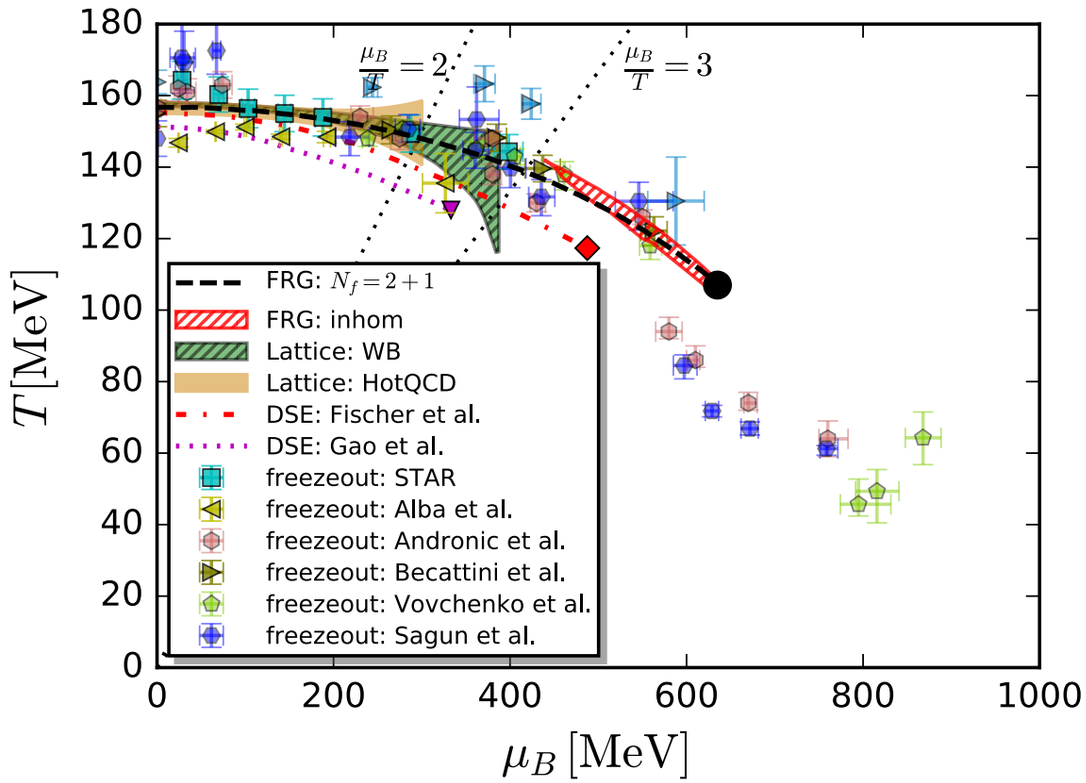
Backup

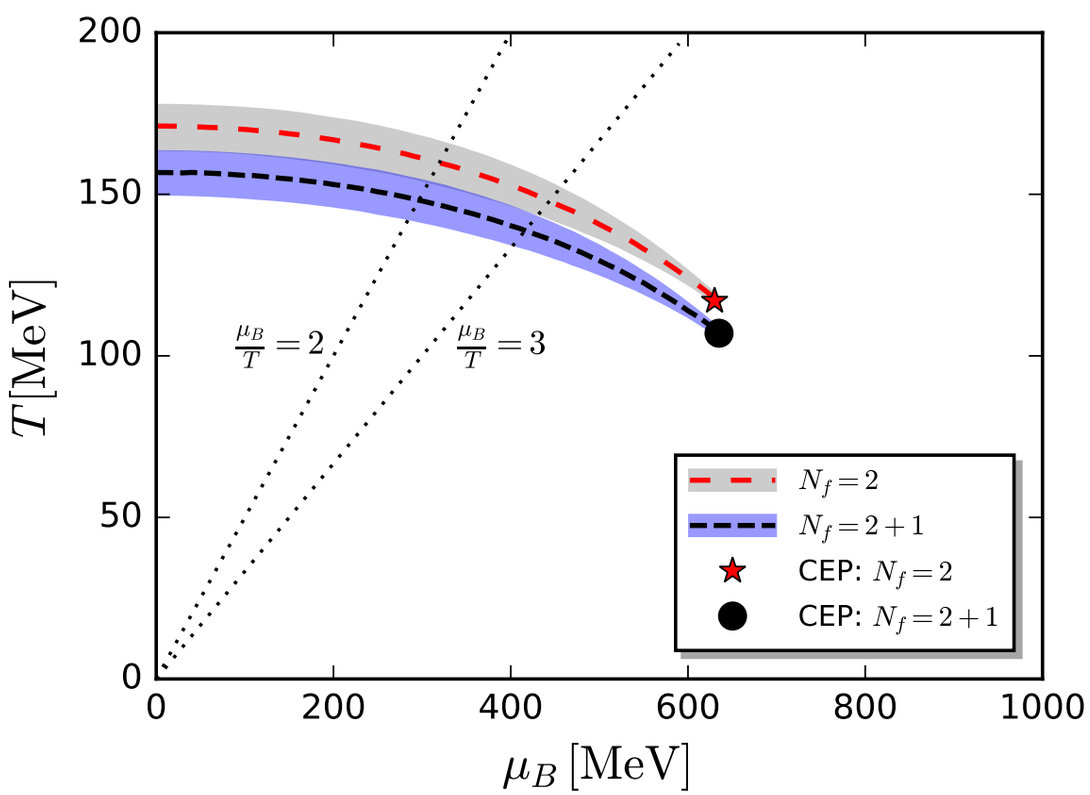
High-order baryon number fluctuations



Phase diagram and curvature

WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991





CEP:

$$(T_{\rm CEP}, \mu_{B\,\rm CEP})_{N_f=2+1} = (107\,\text{MeV}, 635\,\text{MeV})\,,$$

$$(T_{\rm CEP}, \mu_{B\,\rm CEP})_{N_f=2} = (117\,\text{MeV}, 630\,\text{MeV})\,,$$

FRG curvature of the phase boundary:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa\left(\frac{\mu_B}{T_c}\right)^2 + \lambda\left(\frac{\mu_B}{T_c}\right)^4 + \cdots\,,$$

$$\kappa_{N_f=2+1} = 0.0142(2)$$

$$\kappa_{N_f=2} = 0.0176(1)$$

Lattice result:

$$\kappa = 0.0149 \pm 0.0021$$

R. Bellwied *et al.* (WB), arXiv: 1507.07510.

$$\kappa = 0.015 \pm 0.004$$

A. Bazavov *et al.* (HotQCD), arXiv:1812.08235.

DSE result:

$$\kappa = 0.0238$$

C.S. Fischer *et al.*, arXiv: 1405.4762.

$$\kappa = 0.038$$

F. Gao *et al.*, arXiv: 1507.00875.

$$\kappa_{\rm DSE}\frac{\mu_{B,\rm DSE}^2}{T_c^2} = \kappa_{\rm FRG}\frac{\mu_B^2}{T_c^2}\,, \quad \text{where} \quad \mu_B = \sqrt{\frac{\kappa_{\rm DSE}}{\kappa_{\rm FRG}}}\mu_{B,\rm DSE}\,,$$

$$\mu_{B\,\rm CEP} = 631\,\text{MeV}\,.$$