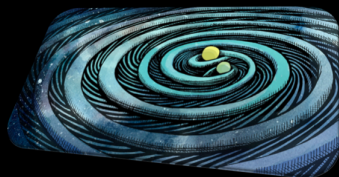


*That would be one of the most fascinating things
man could do, because it would tell you very much
how the universe started.*

— Rainer Weiss



Testing Gravity with GWs

Kavli Institute for Astronomy and Astrophysics

Speaker: Lijing Shao (邵立晶)

CCNU: June 10, 2020

Outline

- 1 Gravity Tests
- 2 Introduction to GWs
- 3 Binary Black Holes
- 4 Binary Neutron Stars
- 5 Summary

1. Gravity Tests

Absence of Quantum Gravity

- On one hand, we have **Quantum Field Theory** to describe the electromagnetic, strong, and weak interactions
- On the other hand, we have **General Relativity** to describe the gravity, as the dynamics of curved spacetime
- **However, QFT and GR are not compatible at their face values!**

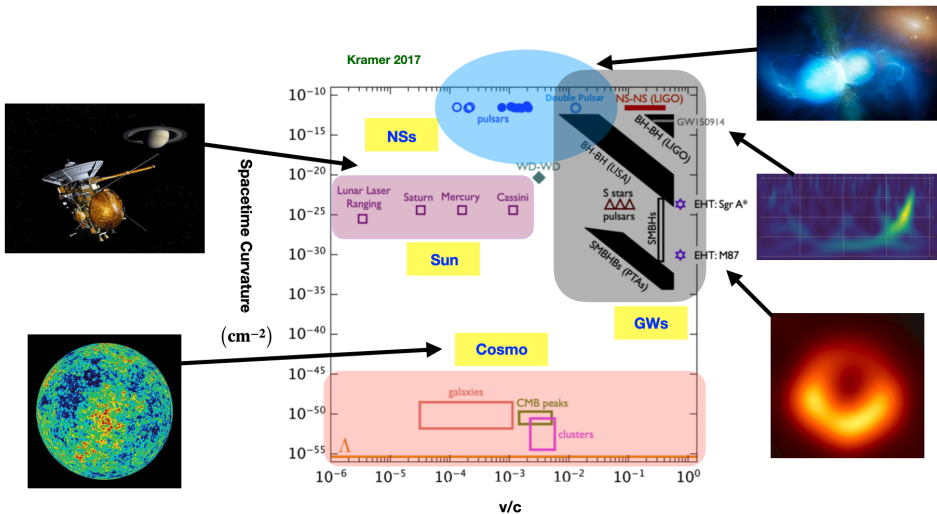


*No one wanted to talk
with Albert?*

**Theoretical physics is beautiful,
but not yet complete**

**Gravity may be holding
the key**

Parameter Space in Gravity Tests



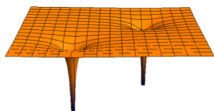
Parameter Space in Gravity Tests

- **G1**: Quasi-stationary weak-field regime
- **G2**: Quasi-stationary **strong-field** regime
- **G3**: **Highly dynamical strong-field** regime
- **GW**: Radiation regime



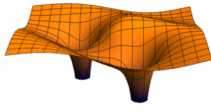
Solar System

G1



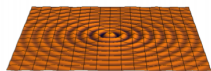
Binary Pulsar

G2



BH Merger

G3



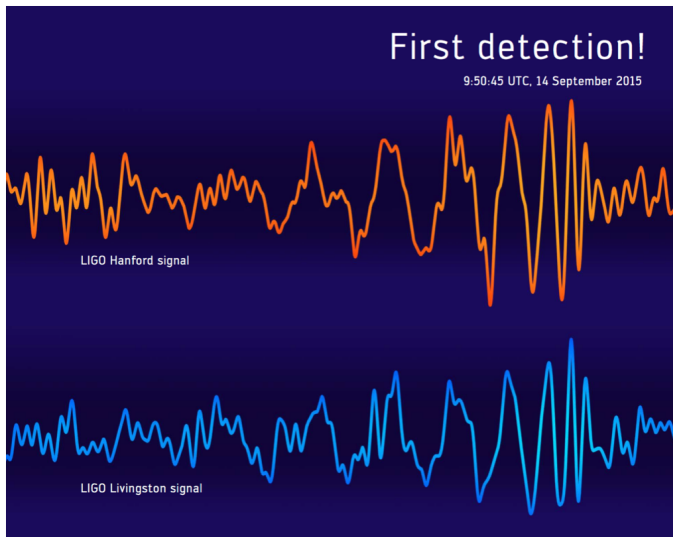
LIGO/Virgo Sites

GW

Wex 2014 (arXiv:1402.5594)

2. Introduction to GWs

Gravitational-wave Data



Gravitational Waveform (Time Domain)

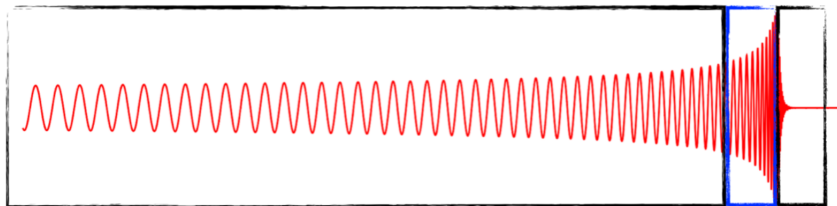
- **Inspiral**: post-Newtonian expansion
- **Merger**: numerical relativity
- **Ringdown**: black hole perturbation

“Inspiral”

post-Newtonian method

“Ringdown”

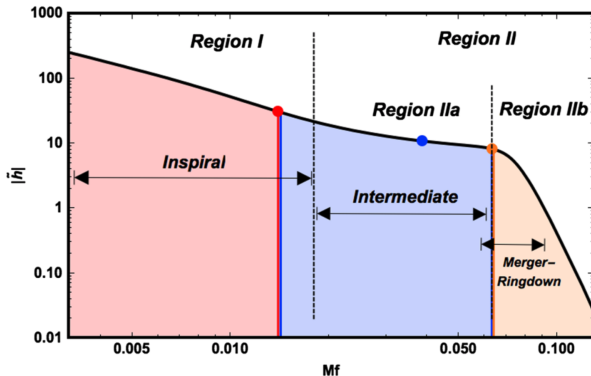
BH perturbation



Effective-one body (EOB): Bohé, Shao, Taracchini et al. 2017

“Merge”
Numerical relativity

Gravitational Waveform (Frequency Domain)

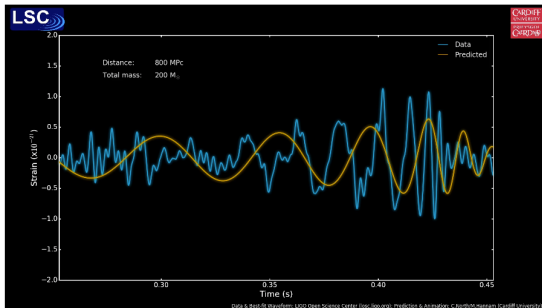


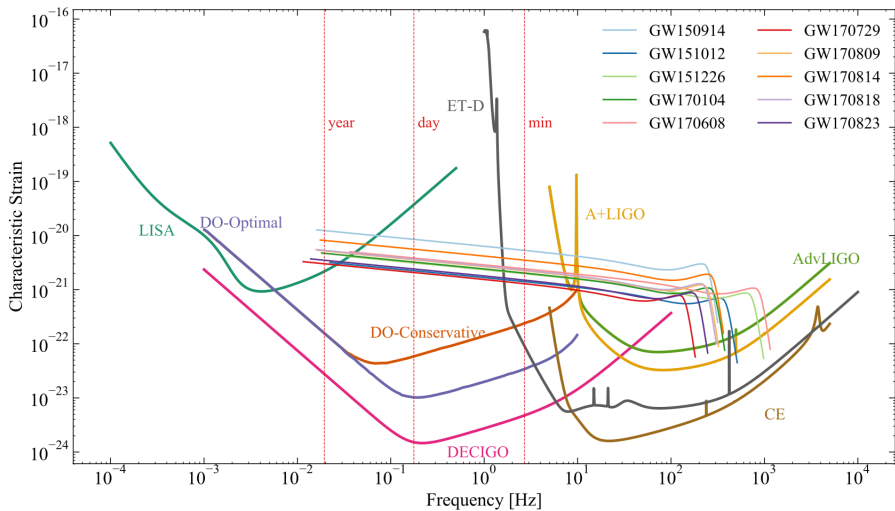
Phenomenological waveform: Husa et al. 2016, PRD; Khan et al. 2016, PRD

Matched Filter

- **Matched filtering** is a standard analysis method for **wideband** time series data

$$(\mathbf{g}|\mathbf{k}) \equiv 2 \int_0^\infty \frac{\tilde{g}^*(f)\tilde{k}(f) + \tilde{g}(f)\tilde{k}^*(f)}{S_n(f)} df$$

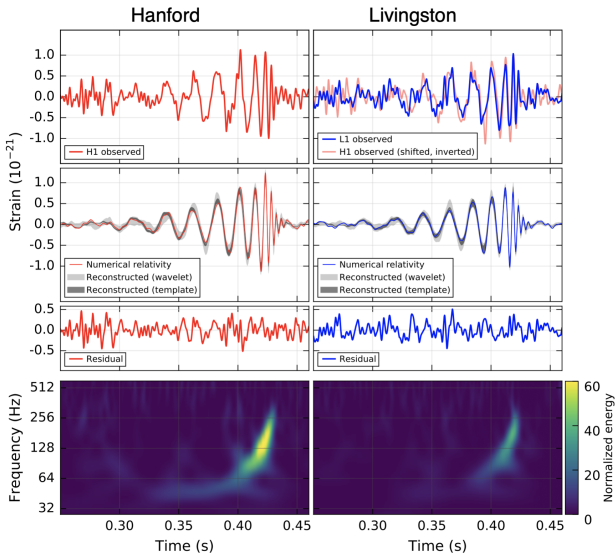




Liu, Shao, et al. 2020, MNRAS (arXiv:2004.12096)

GW150914 (LIGO/Virgo 2016)

$36 + 29 M_{\odot}$: 0.2 sec, SNR=23



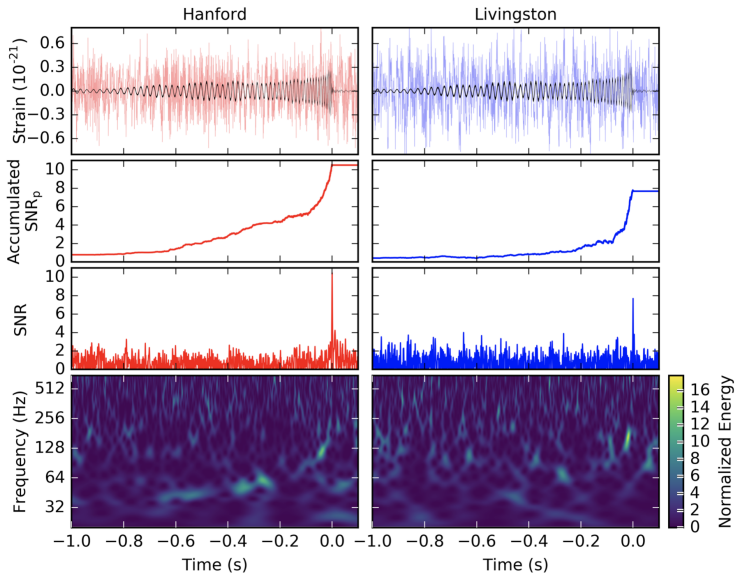
Parameter Estimation: GW150914

Primary black hole mass	$36^{+5}_{-4} M_{\odot}$
Secondary black hole mass	$29^{+4}_{-4} M_{\odot}$
Final black hole mass	$62^{+4}_{-4} M_{\odot}$
Final black hole spin	$0.67^{+0.05}_{-0.07}$
Luminosity distance	410^{+160}_{-180} Mpc
Source redshift z	$0.09^{+0.03}_{-0.04}$

LIGO/Virgo 2016, PRL

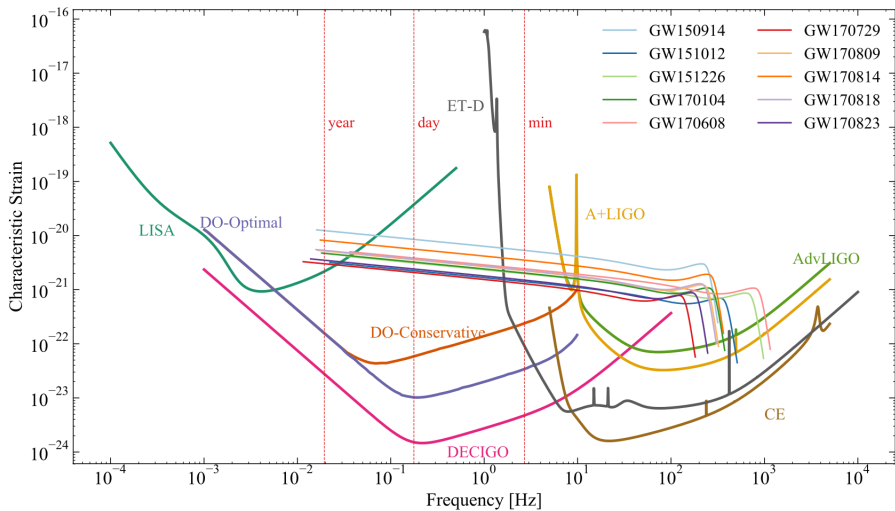
GW151226 (LIGO/Virgo 2016)

$14 + 8 M_{\odot}$: 1 sec, SNR=13



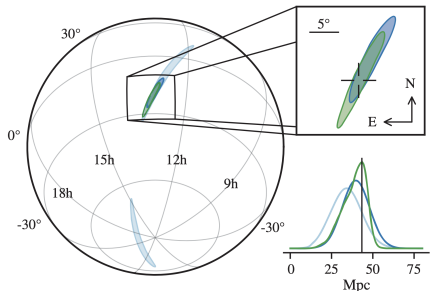
GW Transient Catalog GWTC-1 (LIGO/Virgo 2019)

	Type	$m_1 [M_\odot]$	$m_2 [M_\odot]$	d_L [Mpc]	Redshift z
GW150914	BBH	$35.6^{+4.8}_{-3.0}$	$30.6^{+3.0}_{-4.4}$	430^{+150}_{-170}	$0.09^{+0.03}_{-0.03}$
GW151012	BBH	$23.3^{+14.0}_{-5.5}$	$13.6^{+4.1}_{-4.8}$	1060^{+540}_{-480}	$0.21^{+0.09}_{-0.09}$
GW151226	BBH	$13.7^{+8.8}_{-3.2}$	$7.7^{+2.2}_{-2.6}$	440^{+180}_{-190}	$0.09^{+0.04}_{-0.04}$
GW170104	BBH	$31.0^{+7.2}_{-5.6}$	$20.1^{+4.9}_{-4.5}$	960^{+430}_{-410}	$0.19^{+0.07}_{-0.08}$
GW170608	BBH	$10.9^{+5.3}_{-1.7}$	$7.6^{+1.3}_{-2.1}$	320^{+120}_{-110}	$0.07^{+0.02}_{-0.02}$
GW170729	BBH	$50.6^{+16.6}_{-10.2}$	$34.3^{+9.1}_{-10.1}$	2750^{+1350}_{-1320}	$0.48^{+0.19}_{-0.20}$
GW170809	BBH	$35.2^{+8.3}_{-6.0}$	$23.8^{+5.2}_{-5.1}$	990^{+320}_{-380}	$0.20^{+0.05}_{-0.07}$
GW170814	BBH	$30.7^{+5.7}_{-3.0}$	$25.3^{+2.9}_{-4.1}$	580^{+160}_{-210}	$0.12^{+0.03}_{-0.04}$
GW170817	BNS	$1.46^{+0.12}_{-0.10}$	$1.27^{+0.09}_{-0.09}$	40^{+10}_{-10}	$0.01^{+0.00}_{-0.00}$
GW170818	BBH	$35.5^{+7.5}_{-4.7}$	$26.8^{+4.3}_{-5.2}$	1020^{+430}_{-360}	$0.20^{+0.07}_{-0.07}$
GW170823	BBH	$39.6^{+10.0}_{-6.6}$	$29.4^{+6.3}_{-7.1}$	1850^{+840}_{-840}	$0.34^{+0.13}_{-0.14}$



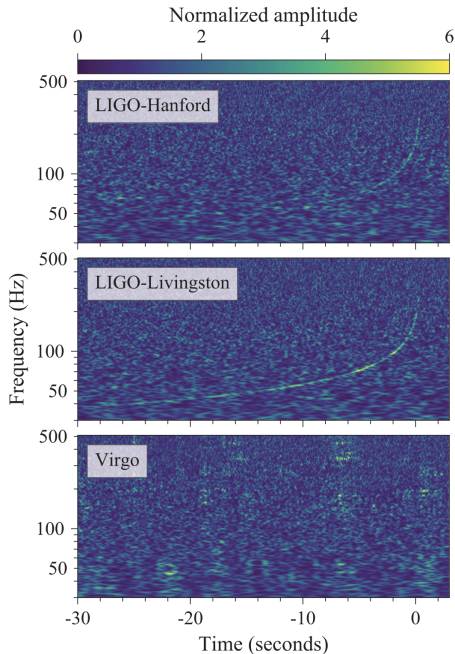
Liu, Shao, et al. 2020, MNRAS (arXiv:2004.12096)

GW170817 (LIGO/Virgo 2017)

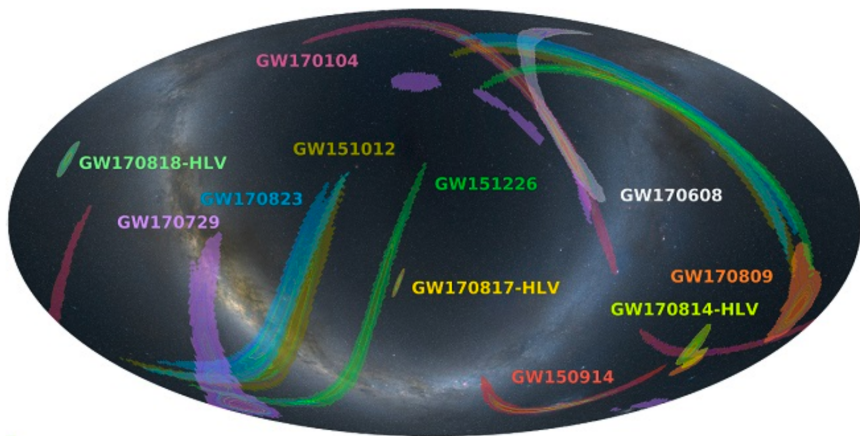


1 min, SNR=32

3000 cycles from 30 Hz



GWTC-1: sky position (LIGO/Virgo 2019)



3. Binary Black Holes

Testing Gravity with BBHs

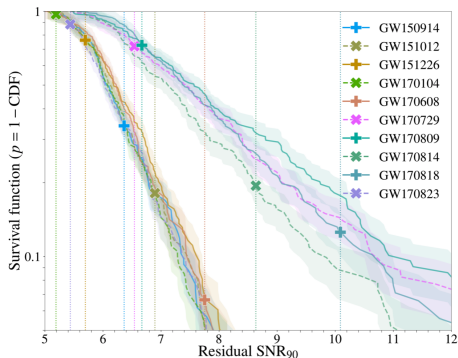
- Residual tests (RT)
- Inspiral-merger-ringdown consistency tests (IMR)
- Parameterized tests: inspiral & post-inspiral (PI & PPI)
- Modified dispersion relation (MDR)

Event	Properties				SNR	GR tests performed				
	D_L [Mpc]	M_{tot} [M_\odot]	M_f [M_\odot]	a_f		RT	IMR	PI	PPI	MDR
GW150914^b	430 ⁺¹⁵⁰ ₋₁₇₀	66.2 ^{+3.7} _{-3.3}	63.1 ^{+3.3} _{-3.0}	0.69 ^{+0.05} _{-0.04}	25.3 ^{+0.1} _{-0.2}	✓	✓	✓	✓	✓
GW151012^b	1060 ⁺⁵⁵⁰ ₋₄₈₀	37.3 ^{+10.6} _{-3.9}	35.7 ^{+10.7} _{-3.8}	0.67 ^{+0.13} _{-0.11}	9.2 ^{+0.3} _{-0.4}	✓	-	-	✓	✓
GW151226^{b,c}	440 ⁺¹⁸⁰ ₋₁₉₀	21.5 ^{+6.2} _{-1.5}	20.5 ^{+6.4} _{-1.5}	0.74 ^{+0.07} _{-0.05}	12.4 ^{+0.2} _{-0.3}	✓	-	✓	-	✓
GW170104	960 ⁺⁴⁴⁰ ₋₄₂₀	51.3 ^{+5.3} _{-4.2}	49.1 ^{+5.2} _{-4.0}	0.66 ^{+0.08} _{-0.11}	14.0 ^{+0.2} _{-0.3}	✓	✓	✓	✓	✓
GW170608	320 ⁺¹²⁰ ₋₁₁₀	18.6 ^{+3.1} _{-0.7}	17.8 ^{+3.2} _{-0.7}	0.69 ^{+0.04} _{-0.04}	15.6 ^{+0.2} _{-0.3}	✓	-	✓	✓	✓
GW170729^d	2760 ⁺¹³⁸⁰ ₋₁₃₄₀	85.2 ^{+15.6} _{-11.1}	80.3 ^{+14.6} _{-10.2}	0.81 ^{+0.07} _{-0.13}	10.8 ^{+0.4} _{-0.5}	✓	✓	-	✓	✓
GW170809	990 ⁺³²⁰ ₋₃₈₀	59.2 ^{+5.4} _{-3.9}	56.4 ^{+5.2} _{-3.7}	0.70 ^{+0.08} _{-0.09}	12.7 ^{+0.2} _{-0.3}	✓	✓	-	✓	✓
GW170814	580 ⁺¹⁶⁰ ₋₂₁₀	56.1 ^{+3.4} _{-2.7}	53.4 ^{+3.2} _{-2.4}	0.72 ^{+0.07} _{-0.05}	17.8 ^{+0.3} _{-0.3}	✓	✓	✓	✓	✓
GW170818	1020 ⁺⁴³⁰ ₋₃₆₀	62.5 ^{+5.1} _{-4.0}	59.8 ^{+4.8} _{-3.8}	0.67 ^{+0.07} _{-0.08}	11.9 ^{+0.3} _{-0.4}	✓	✓	-	✓	✓
GW170823	1850 ⁺⁸⁴⁰ ₋₈₄₀	68.9 ^{+9.9} _{-7.1}	65.6 ^{+9.4} _{-6.6}	0.71 ^{+0.08} _{-0.10}	12.1 ^{+0.2} _{-0.3}	✓	✓	-	✓	✓

Residual Tests (LIGO/Virgo 2019)

- **Model**: best fitted model
- **Residual = Data – Model**
- **Residual tests**: consistent with noise distribution!

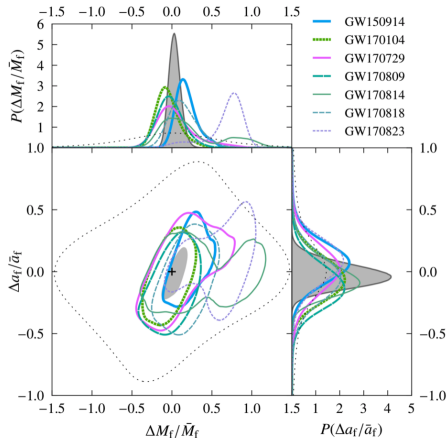
Event	IFOs	Residual SNR ₉₀	Fitting factor	<i>p</i> -value
GW150914	HL	6.4	≥ 0.97	0.34
GW151012	HL	6.9	≥ 0.81	0.18
GW151226	HL	5.7	≥ 0.91	0.76
GW170104	HL	5.2	≥ 0.94	0.97
GW170608	HL	7.8	≥ 0.90	0.07
GW170729	HLV	6.5	≥ 0.87	0.72
GW170809	HLV	6.7	≥ 0.91	0.73
GW170814	HLV	8.6	≥ 0.90	0.19
GW170818	HLV	10.1	≥ 0.78	0.13
GW170823	HL	5.4	≥ 0.92	0.89



IMR Consistency Tests (LIGO/Virgo 2019)

- Parameter estimation *separately* with **inpsiral** and **merger + ringdown**
- Check consistency!**

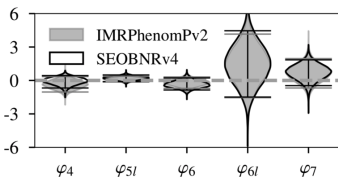
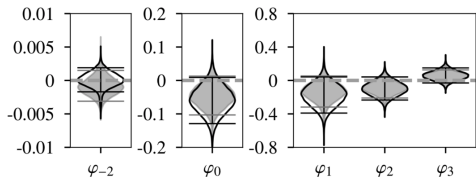
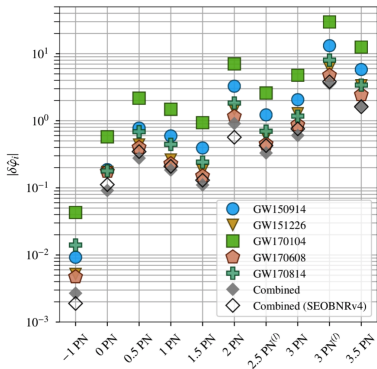
Event	f_c [Hz]	ρ_{IMR}	ρ_{insp}	$\rho_{\text{post-insp}}$	GR quantile [%]
GW150914	132	25.3	19.4	16.1	55.5
GW170104	143	13.7	10.9	8.5	24.4
GW170729	91	10.7	8.6	6.9	10.4
GW170809	136	12.7	10.6	7.1	14.7
GW170814	161	16.8	15.3	7.2	7.8
GW170818	128	12.0	9.3	7.2	25.5
GW170823	102	11.9	7.9	8.5	80.4



Parameterized Tests (LIGO/Virgo 2019)

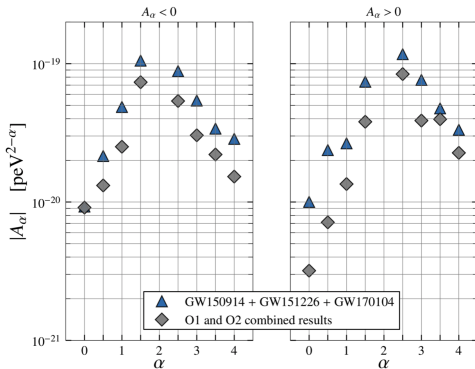
$$\psi \sim \frac{3}{128\eta} (\pi fM)^{-5/3} \sum_{i=0}^n \varphi_i^{\text{GR}} (\pi fM)^{i/3}$$

$$\varphi_i \rightarrow (1 + \delta\hat{\varphi}_i) \varphi_i^{\text{GR}}$$



Modified Dispersion Relation (LIGO/Virgo 2019)

- **Lorentz violation:** $E^2 = p^2 c^2 + A_\alpha p^\alpha c^\alpha$
 - except $\alpha = 0$: **massive gravity**
 - LIGO/Virgo GWTC-1 $\Rightarrow m_g \leq 5.0 \times 10^{-23} \text{eV}/c^2$



Modified Dispersion Relation (massive graviton)

- **Binary pulsars** are also used to constrain the graviton mass via bounding the **gravitational backreaction**

- **Linearized gravity** with a massive graviton [Miao, Shao, et al. 2019]

$$S = \frac{1}{64\pi} \int d^4x \left[\partial_\lambda h_{\mu\nu} \partial^\lambda h^{\mu\nu} - 2\partial^\nu h_{\mu\nu} \partial_\lambda h^{\mu\lambda} + 2\partial^\nu h_{\mu\nu} \partial^\mu h - \partial^\mu h \partial_\mu h - 32\pi h_{\mu\nu} T^{\mu\nu} + m_g^2 \left(h_{\mu\nu} h^{\mu\nu} - \frac{1}{2} h^2 \right) \right]$$

- **Cubic Galileon** with screening mechanics [Shao, Wex, Zhou 2020]

$$S = \int d^4x \left[-\frac{1}{4} h^{\mu\nu} (\mathcal{E}h)_{\mu\nu} + \frac{h^{\mu\nu} T_{\mu\nu}}{2M_{\text{Pl}}} - \frac{3}{4} (\partial\pi_s)^2 \left(1 + \frac{1}{3\Lambda^3} \square\pi_s \right) + \frac{\pi_s T}{2M_{\text{Pl}}} \right]$$

Modified Dispersion Relation

- The most generic **linearized gravity** has the Lagrangian

[Kostelecký & Mewes 2018]

$$\mathcal{L}_{\mathcal{K}^{(d)}} = \frac{1}{4} h_{\mu\nu} \hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} h_{\rho\sigma}$$

where $\hat{\mathcal{K}}^{(d)\mu\nu\rho\sigma} = \mathcal{K}^{(d)\mu\nu\rho\sigma} i_1 i_2 \cdots i_{d-2} \partial_{i_1} \partial_{i_2} \cdots \partial_{i_{d-2}}$

- It predicts a modified dispersion relation for GWs

$$\omega = \left(1 - \zeta^0 \pm \sqrt{(\zeta^1)^2 + (\zeta^2)^2 + (\zeta^3)^2} \right) p$$

Modified Dispersion Relation

$$\omega = \left(1 - \zeta^0 \pm \sqrt{(\zeta^1)^2 + (\zeta^2)^2 + (\zeta^3)^2} \right) p$$

$$\zeta^0 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\mathbf{n}}) k_{(I)jm}^{(d)}$$

$$\zeta^1 \mp i\zeta^2 = \sum_{djm} \omega_{\pm 4}^{d-4} Y_{jm}(\hat{\mathbf{n}}) \left[k_{(E)jm}^{(d)} \pm ik_{(B)jm}^{(d)} \right]$$

$$\zeta^3 = \sum_{djm} \omega^{d-4} Y_{jm}(\hat{\mathbf{n}}) k_{(V)jm}^{(d)}$$

- Therefore, gravitons of different **polarization** or **frequency** have different velocity

Polarization Tests (LIGO/Virgo 2019)

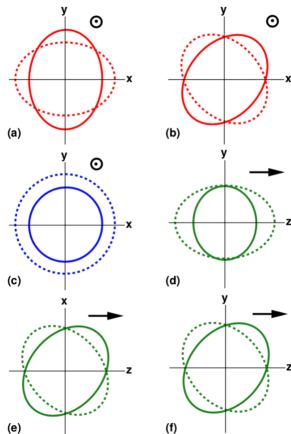
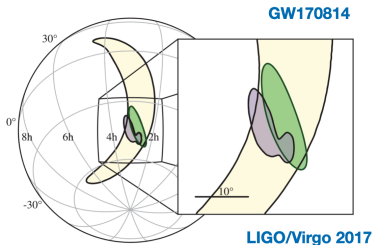
- **Triple** detections

- GW170729, GW170809, GW170814, GW170818

- Bayes factors: $10^1 - 10^2$

- tensor **vs** vector

- tensor **vs** scalar

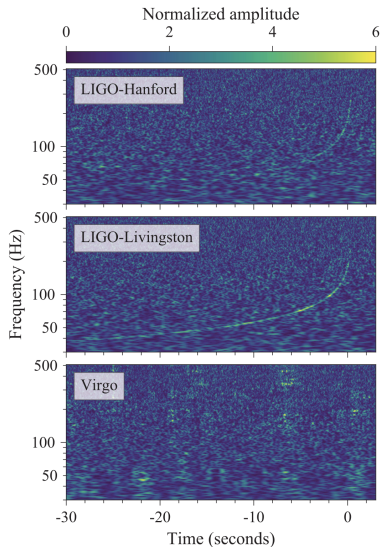
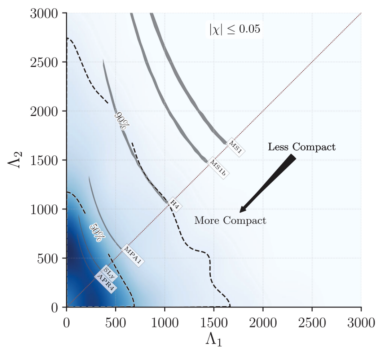


4. Binary Neutron Stars

Waveform: tidal deformability (LIGO/Virgo 2017)

■ SEOBNRv4T

- tidal deformability
- equation of state

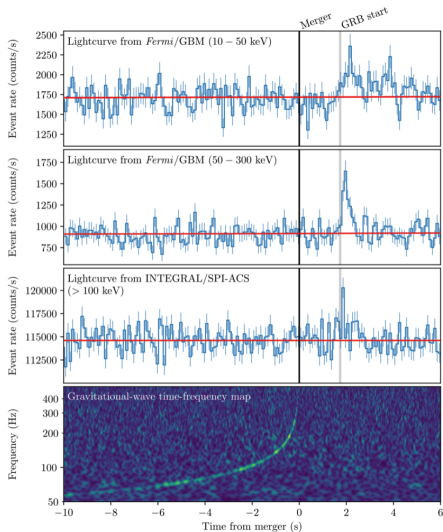


Speed of Gravity (LIGO/Virgo 2017)

- The famous 1.7 sec

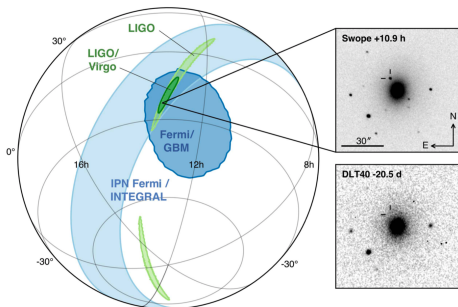
$$-3 \times 10^{-15} \leq \frac{\Delta v}{v_{EM}} \leq +7 \times 10^{-16}$$

- strong implications on cosmological models
 - ... tons of PRL papers



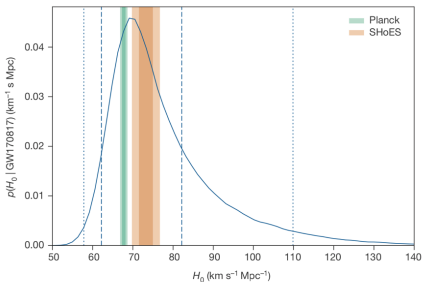
Polarization Tests (LIGO/Virgo 2019)

- Precise localization: NGC 4993
- Bayes factors
 - tensor vs vector: 10^{21}
 - tensor vs scalar: 10^{23}
- **much** tighter than BBHs

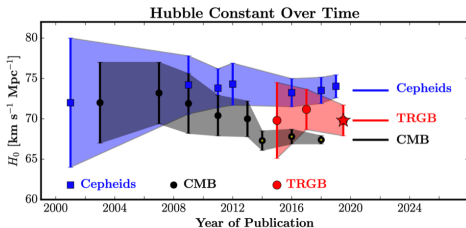


Hubble Constant (LIGO/Virgo 2017)

- By simultaneously measuring **redshift** and **luminosity distance**, GWs provide an independent way to probe cosmological parameters [Schutz 1986]



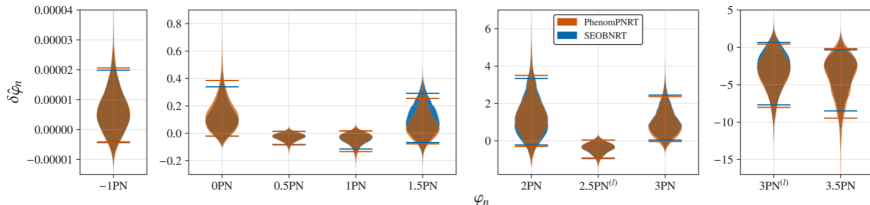
LIGO/Virgo 2017



[arXiv:1907.05922](https://arxiv.org/abs/1907.05922) (ApJ, in press)

The Carnegie-Chicago Hubble Program

Parameterized Tests (LIGO/Virgo 2019)

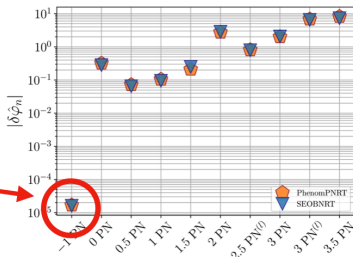


$$\phi \sim \frac{3}{128\eta} (\pi f M)^{-5/3} \sum_{i=0}^n \varphi_i^{\text{GR}} (\pi f M)^{i/3}$$

$$\varphi_i \rightarrow (1 + \delta \hat{\varphi}_i) \varphi_i^{\text{GR}}$$

A tight constraint on dipole radiation

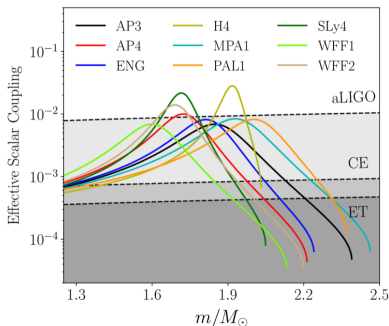
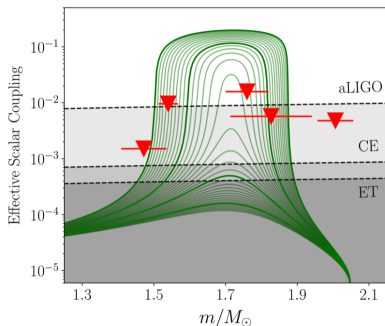
LIGO/Virgo 2019



Dipole Radiation strong-field scalarization

- Strong-field scalarization happens for NSs

$$S = \frac{c^4}{16\pi G_*} \int \frac{d^4x}{c} \sqrt{-g_*} [R_* - 2g_*^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - V(\varphi)] + S_m [\psi_m; A^2(\varphi) g_{\mu\nu}^*]$$

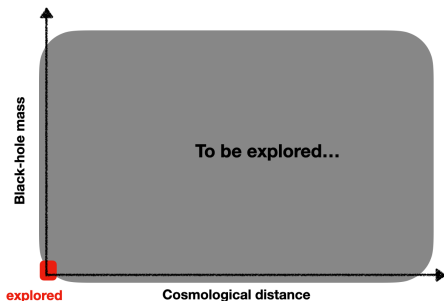


Damour & Esposito-Farèse 1993, PRL; Shao et al. 2017, PRX; Zhao, Shao, et al. 2019, PRD

5. Summary

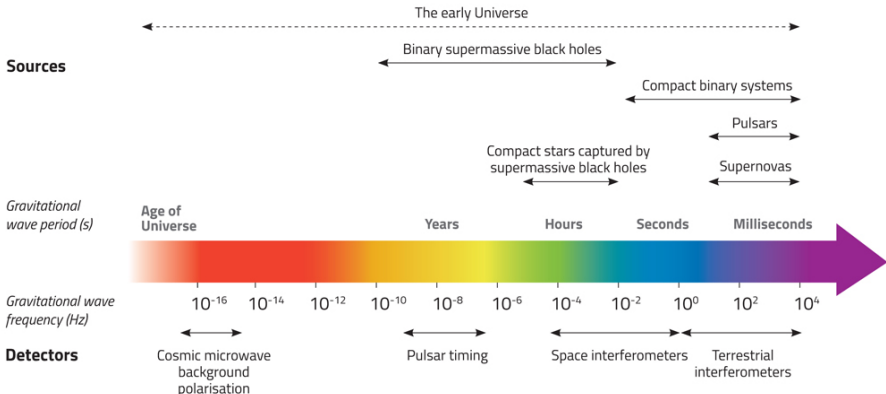
Summary

- **Einstein is still right**
- GWs launch **a new era** to test gravity
- Hope something new emerges **soon**

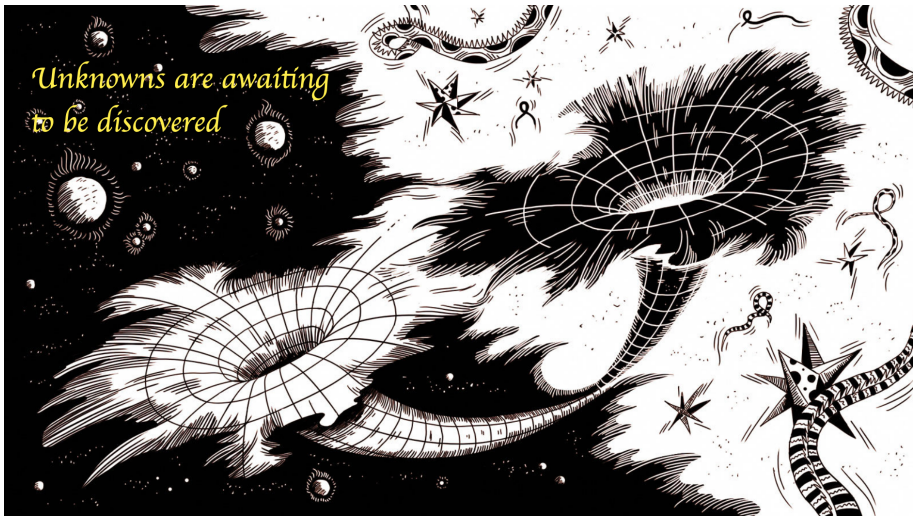


$$G_{\mu\nu} = 8\pi GT_{\mu\nu}$$

Albert Einstein (1915)



Only a tiny part of GW spectrum was revealed by now
Stay tuned!



*Unknowns are awaiting
to be discovered*

An exciting era for astronomers & physicists

THANK YOU

