Hadron structure from Large Momentum Effective Theory

Wei Wang
Shanghai Jiao Tong University

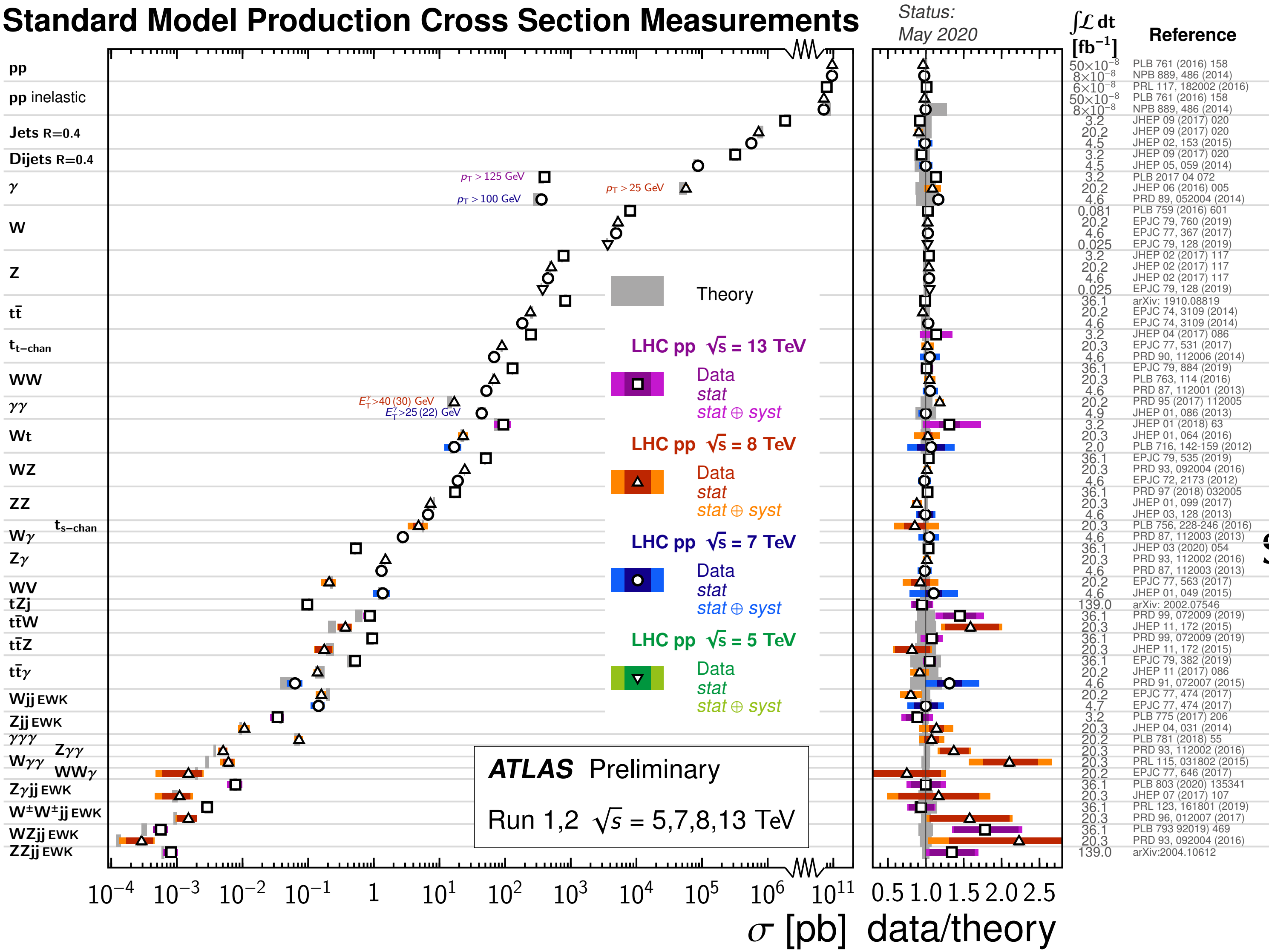
23.09. 2020
CCNU IOPP Forum

Outline

- **Parton Distribution Functions**
- **Quasi PDF and LaMET**
- **Brief Results for quark PDFs**
- **Recent Progress:**
  - ✓ **2-loop Perturbative Matching**
  - ✓ **Gluon quasi PDF**
  - ✓ **TMDWF**
- **Summary**

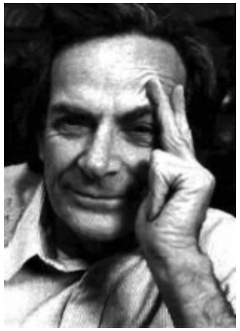
Disclaimer: There are many excellent works, but can not covered in this talk.

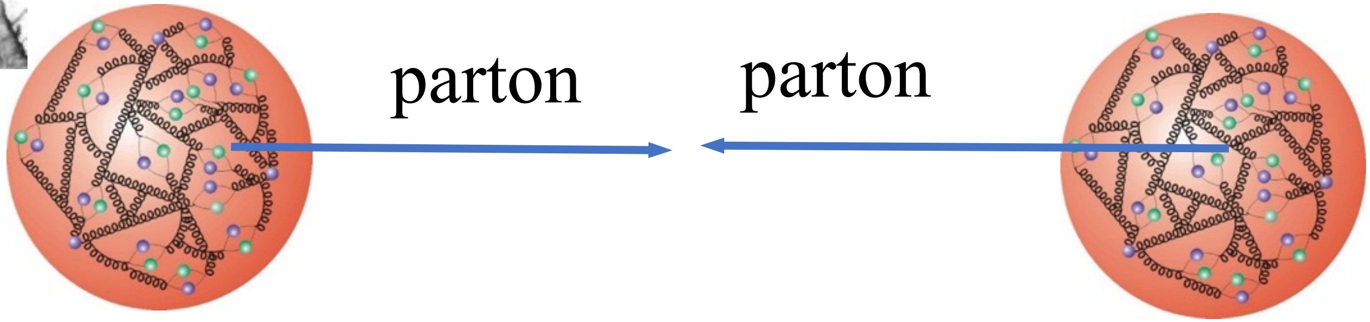
Success of the Standard Model(SM)



success of EW and flavor sectors but also QCD

Factorization: Parton Model & PDF



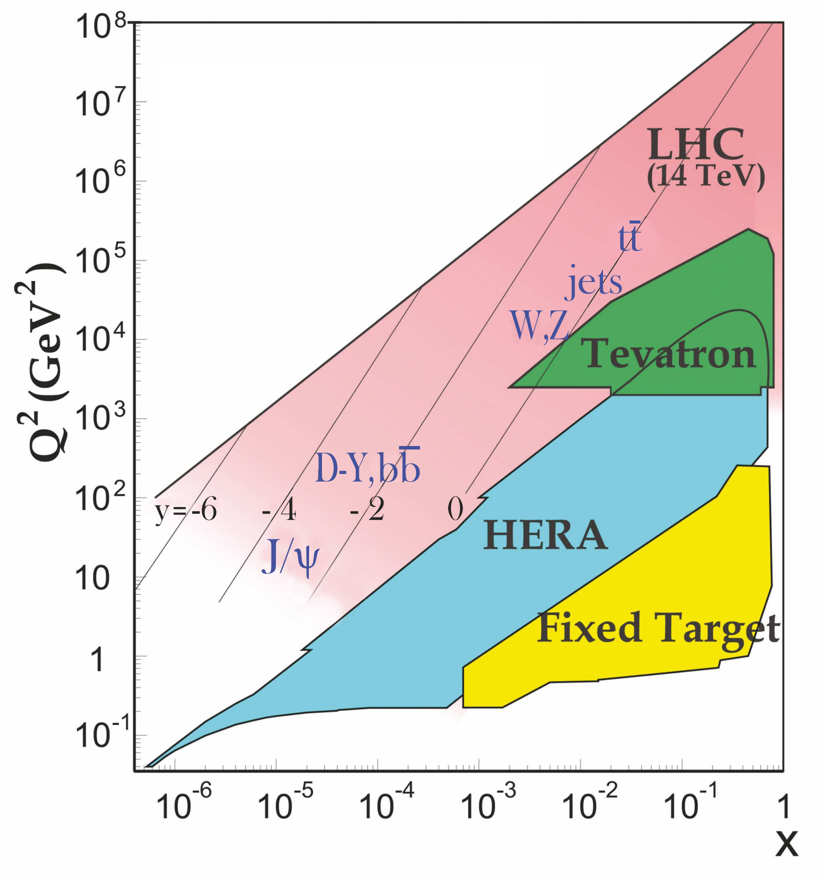


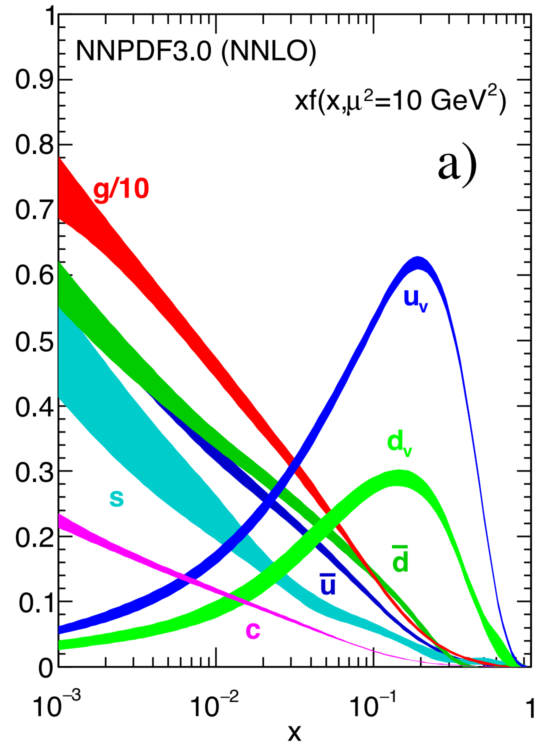
Factorization theorems:

$$d\sigma \sim \int dx_1 dx_2 * f(x_1) * f(x_2) * C(x_1, x_2, Q)$$

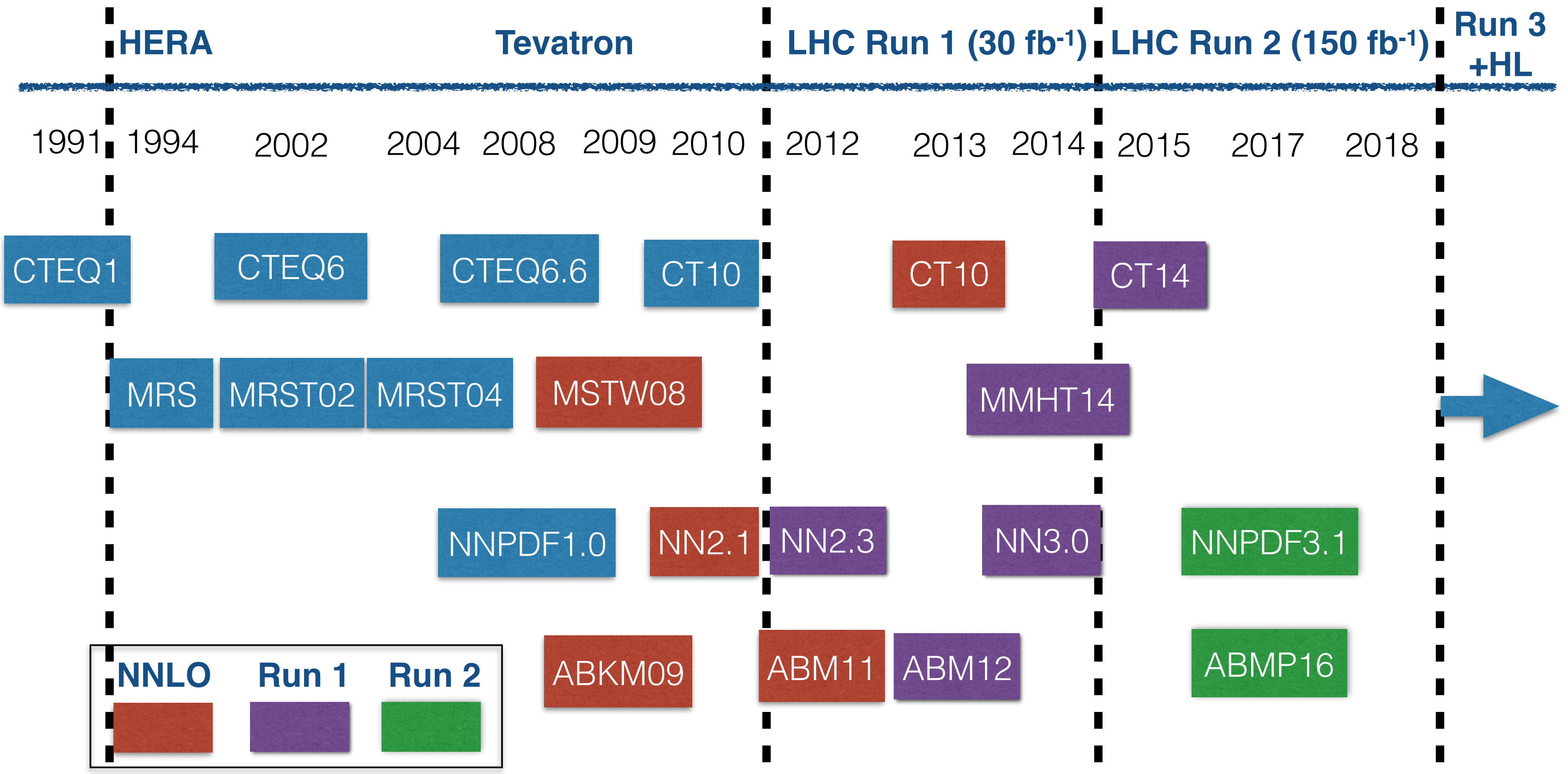
PDF: basic inputs for particle physics at hadron colliders.

Global Fit of Data





Global Fit of Data



From Jun Gao

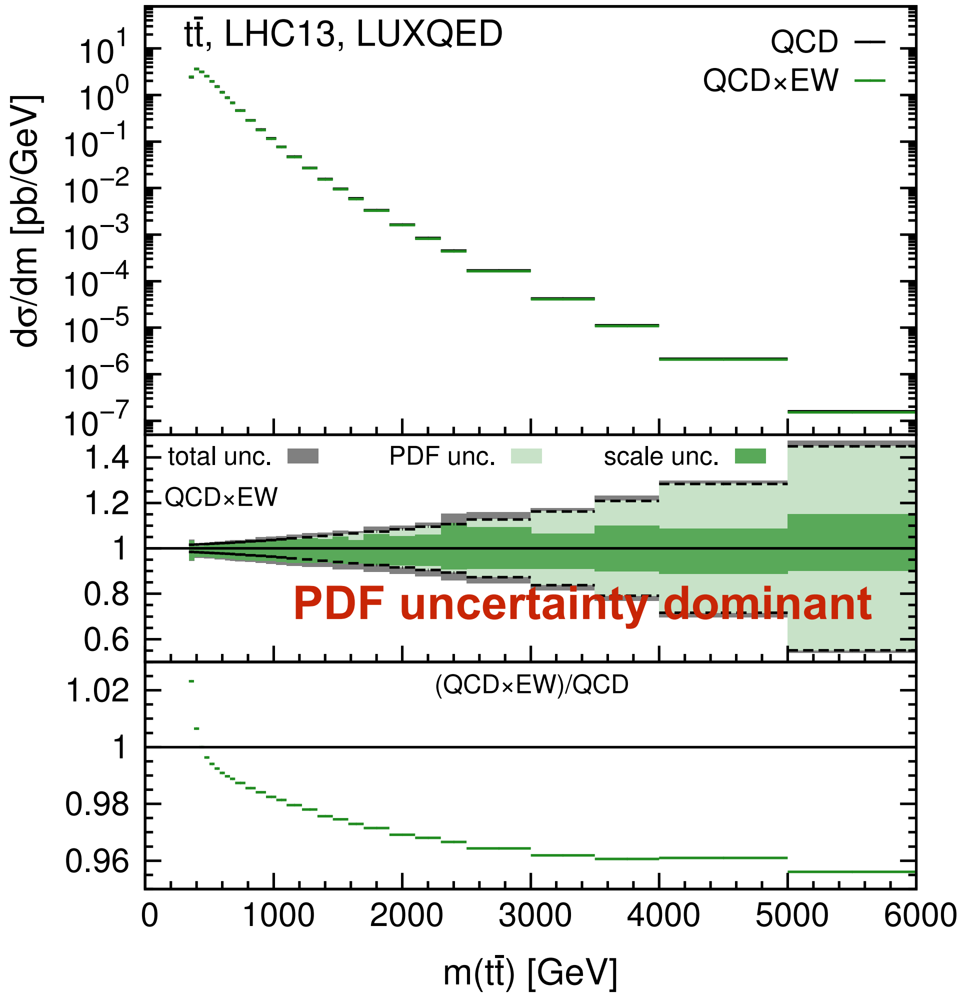
PDF From First Principle?

- Fitting Results rely on data

- First-principle calculation can cover regions where experiments cannot constrain so well

- The cost of improving calculations could be much lower than building large experiments.

Gluon PDF

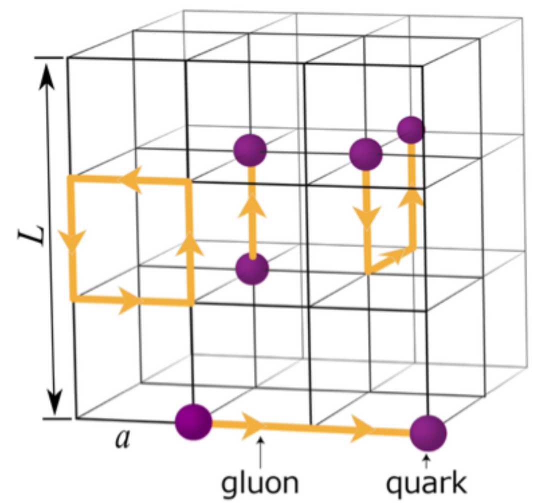


1705.04105v2

PDF at large x gives dominant errors: important to study heavy particles.

Lattice QCD(K.G.Wilson,1974)

- Numerical simulation in discretized Euclidean space-time
- Finite volume (L should be large)
- Finite lattice spacing (a should be small)



Tremendous successes in hadron spectroscopy, decay constants, strong coupling, form factors, etc.

Lattice QCD: PDF?

PDF (or more general parton physics):
Minkowski space, real time
infinite momentum frame, on the light-cone

Lattice QCD:

Euclidean space, imaginary time (t=i*tau)
Difficulty in time

$$x_E^{\mu}\, x_E^{\mu} = 0,\ x_E^{\mu} = (0,0,0,0)$$

Unable to distinguish local operator and light-cone operator
Sign problem in simulating real-time dynamics.

Lattice QCD: PDF?

One can form local moments to get rid of the time-dependence

- $\langle x^n \rangle = \int f(x) x^n dx$: matrix elements of local operators
- However, one can only calculate lowest few moments in practice.
- Higher moments quickly become noisy.

$$\int_0^1 dx\, x^n q(x,\mu) dx = a_n(\mu) \propto \left\langle P \middle| \bar{\psi}(0) \gamma^+ \overbrace{i\overleftrightarrow{D}^+ \cdots i\overleftrightarrow{D}^+}^{n} \psi(0) \middle| P \right\rangle$$

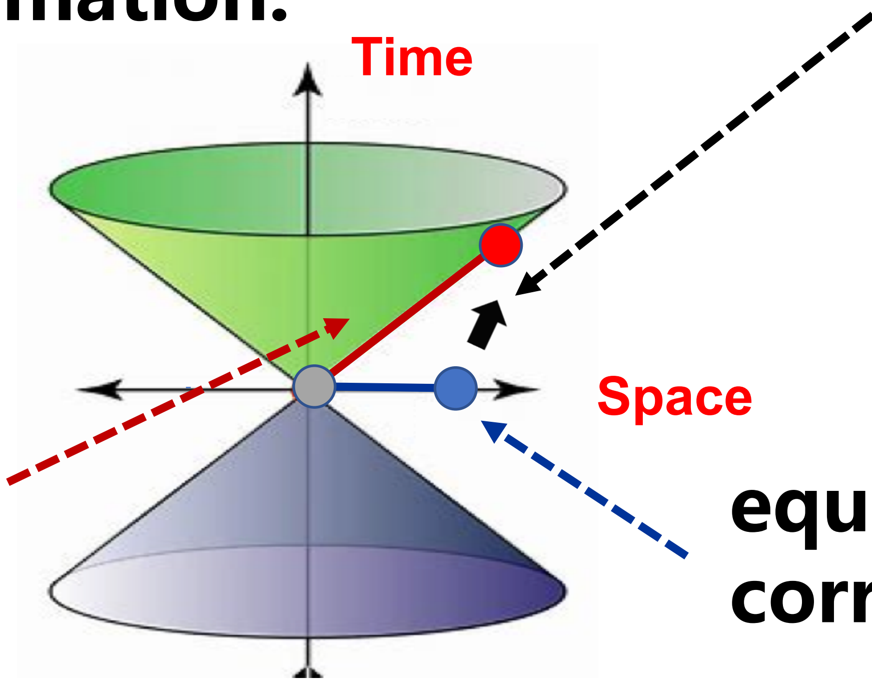
Quasi Parton Distribution Functions
and
Large Momentum Effective Theory
(LaMET)

X. Ji, Phys. Rev. Lett. 110 (2013) 262002
X. Ji, Sci.China Phys.Mech.Astron. 57 (2014) 1407-1412

Quasi-PDFs

$$\tilde{q}(x,\mu^2,P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \times \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$

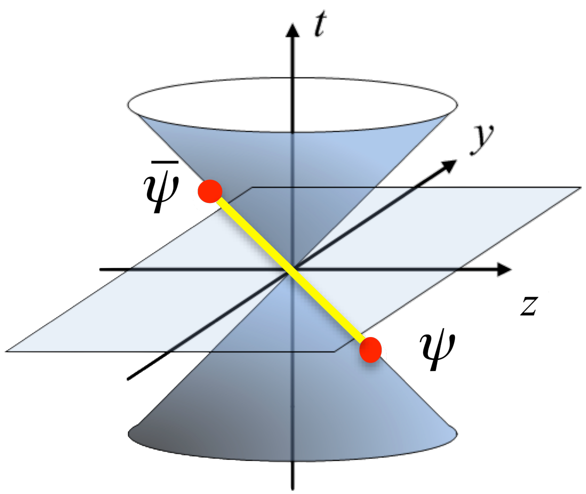
Frame transformation:



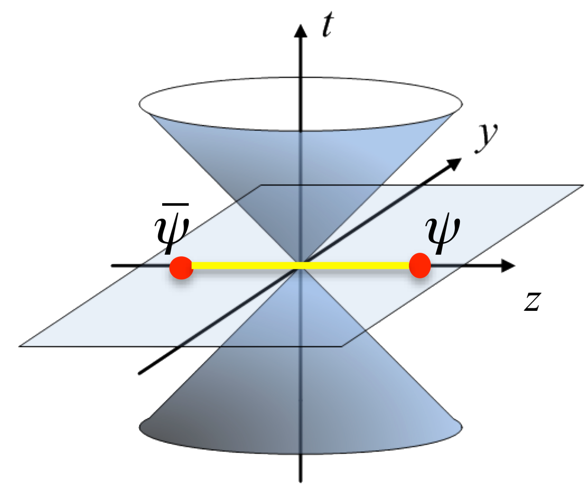
PDF:
light-cone correlation

equal time correlation

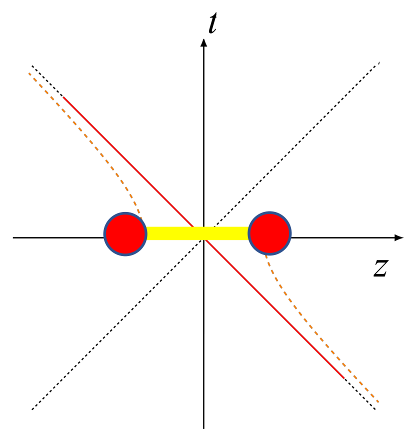
Quasi-PDFs



Lorentz boost



PDF:
light-cone separation;
Cannot be calculated on the lattice



Quasi-PDF :
Equal-time correlation;
Directly calculable on the lattice

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \; ,$$

Quasi-PDFs:**Finite but large Pz**

- The distribution at a finite but large Pz shall be calculable in lattice QCD.

- Since it differs from the standard PDF by simply an infinite Pz limit, it shall have the same infrared (collinear) physics.

- It shall be related to the standard PDF by a matching factor $Z(\frac{\mu}{Pz})$ which is perturbatively calculable.

$$Z(x, \mu/P^z) = \delta(x-1) + \frac{\alpha_s}{2\pi} Z^{(1)}(x, \mu/P^z) + \dots$$

quasi PDF in LaMET

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \quad ,$$

✓ Formalism: factorization, renormalization, power corrections

✓ Matching: perturbative corrections to Z

✓ Lattice QCD calculations

Lattice Simulations

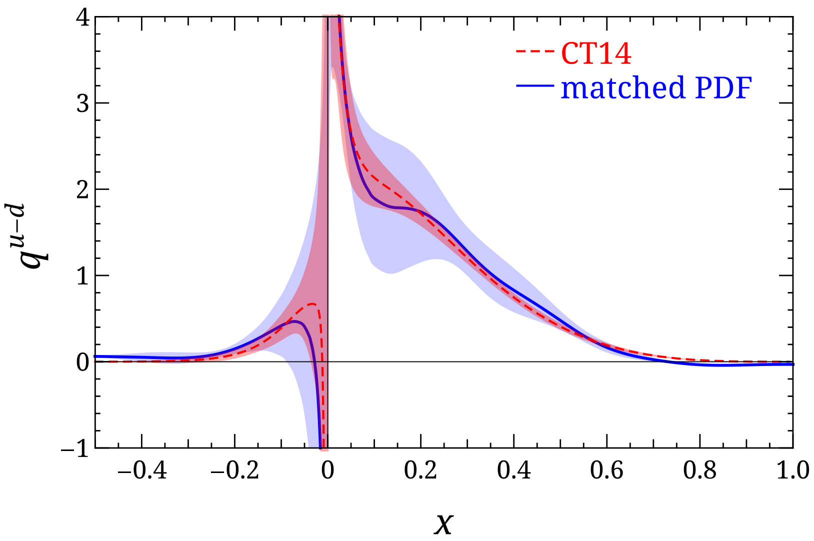
Lattice Collaboration working on quasi-PDFs:

- **Lattice Parton Physics Project (LP3) Collaboration**
  J.W. Chen, T. Ishikawa, L. Jin, R.-Z. Li, H.-W. Lin, Y.-S. Liu, A. Schaefer, Y.-B. Yang, J.-H. Zhang, R. Zhang, and Y. Zhao, et al
- **European Twisted Mass Collaboration (ETMC)**
  C. Alexandrou (U. Cyprus) , M. Constantinou (Temple U.), K.Cichy (Adam Mickiewicz U.), K. Jansen (NIC, DESY), F. Steffens (Bonn U.), et al.
- **DESY, Zeuthen** J. Green, et al.
- **Brookhaven group**
  T. Izubuchi, L. Jin, K. Kallidonis, N. Karthik, S. Mukherje, P. Petreczky, C. Schugert, S. Syritsyn.
- **MSU group**
  H.-W. Lin
- **Lattice Parton Collaboration (LPC)**
  X.Ji, P.Sun, A.Schafer, W.Wang, Y.Yang, J.Zhang, et al
- …

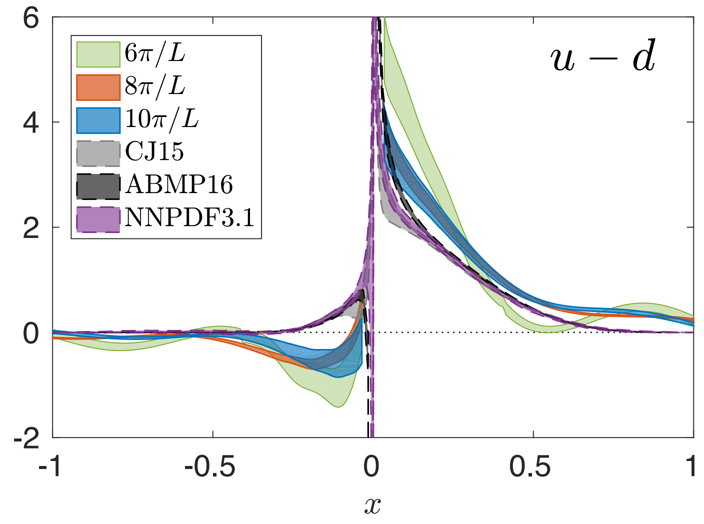
Lattice Progress on quasi PDF:quark

$$u(x) - d(x) - \bar{u}(-x) + \bar{d}(-x)$$

LP3: 1803.04393



ETMC:1803.02685

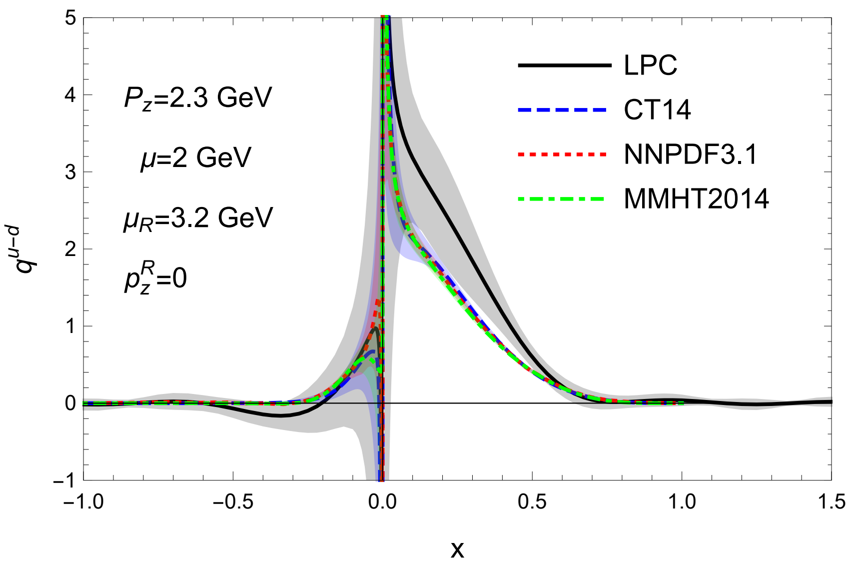


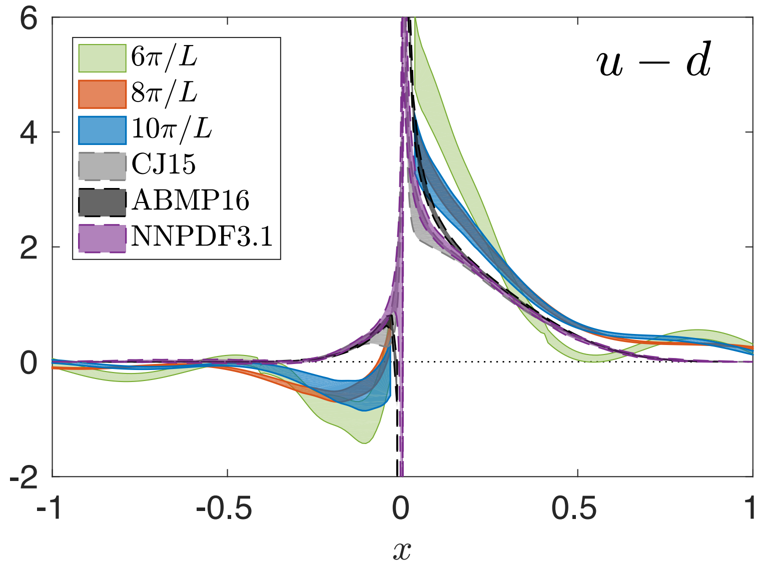
Lattice Progress on quasi PDF:quark

$$u(x) - d(x) - \bar{u}(-x) + \bar{d}(-x)$$

LPC:1807.06566

ETMC:1803.02685





Progress on quasi-PDF

- ✓ More Precision Calculations:
  - ✓ Lattice QCD Simulations
  - ✓ 2-loop Perturbative Matching
  - ✓ …
- ✓ New Distributions:
  - ✓ Gluons PDFs
  - ✓ Transverse Momentum Dependent PDF
  - ✓ …

Many Progress has been made on quasi PDFs, see Reviews:

Alexandrou et al., 1902.00587

Ji, et al. 2004.03543, Rev.Mod.Phy.

2-Loop Perturbative Corrections

$$\tilde{q}(x,\mu^2,P^z) = \int_{-1}^{1} \frac{dy}{|y|} Z\left(\frac{x}{y},\frac{\mu}{P^z}\right) q(y,\mu^2) + \mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right) \ ,$$

Higher-order corrections are important:

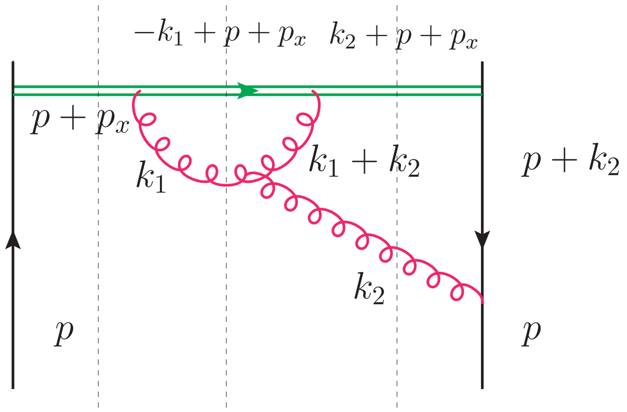
- ✓ If μ = 2*GeV*, α*s*(μ = 2*GeV*) ~ 0.3, α*s*^{2}-correction is needed for a precision prediction
- ✓ The factorization proof at NNLO is nontrivial

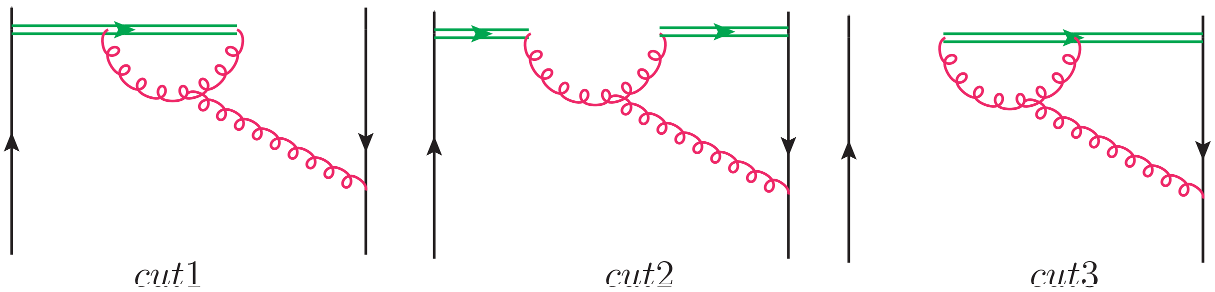
Chen, WW, Zhu, *2005.13757, PRD RC*
2006.10917, JHEP
2006.14825

See also Li,Ma,Qiu,2006.12370

2-Loop Perturbative Corrections

Higher order corrections bring about a large number (79+ at NNLO) of Feynman diagrams





2-Loop Perturbative Corrections

Check Master Integrals using the numerical integration package FIESTA

Analytic:

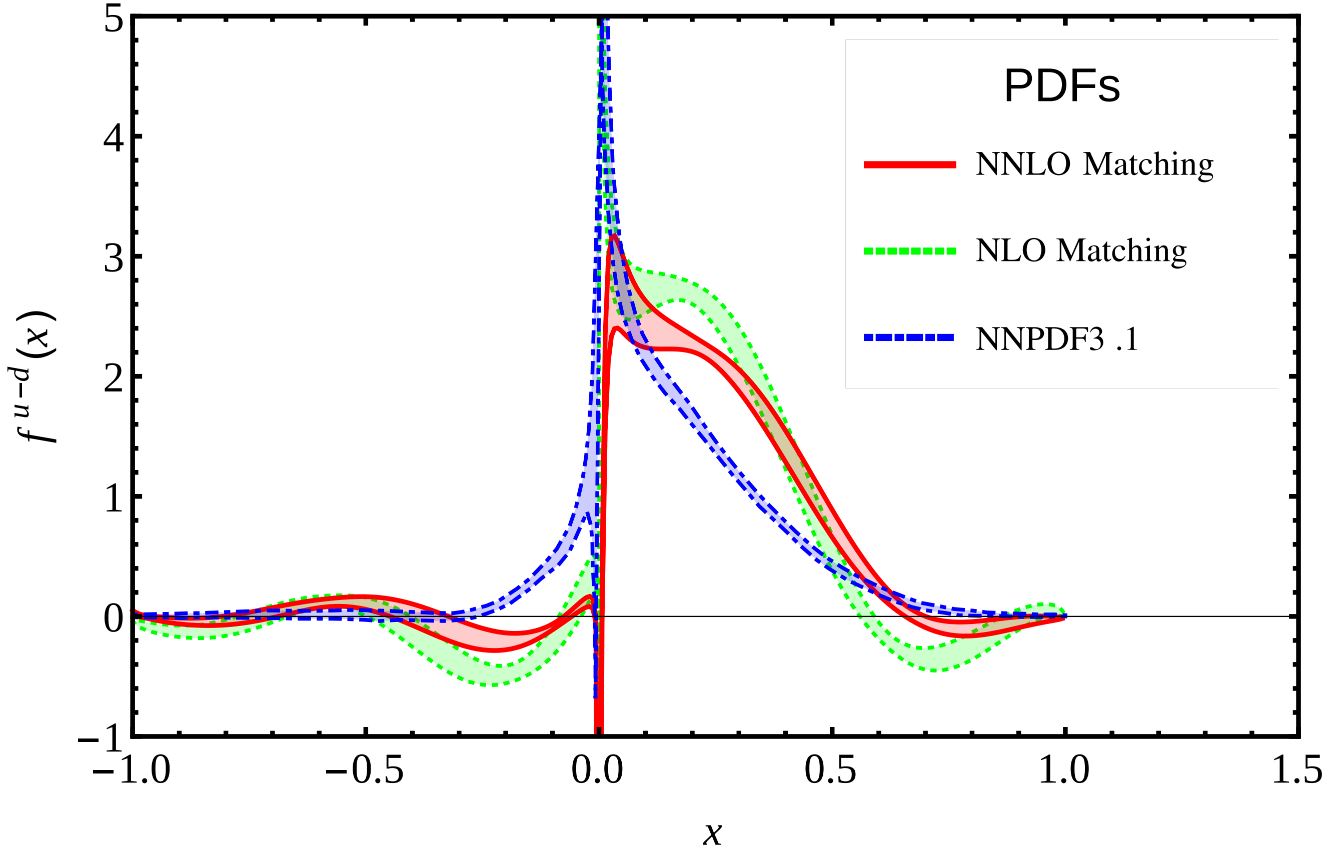
$$I^{1}_{1,1,0,0,2,1,0} = \frac{-2.492900960}{\epsilon} + 0.4498613241 + \epsilon(-21.287203876),$$

FIESTA:

$$I^{1}_{1,1,0,0,2,1,0} = \frac{-2.49290 \pm 0.0000652}{\epsilon} + 0.449836 \pm 0.000847 + \epsilon(-21.2872 \pm 0.004169).$$

Divergences between quasi and lightcone PDFs cancel!

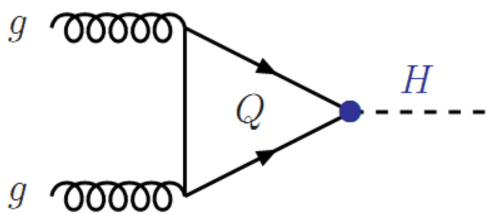
2-Loop Perturbative Corrections

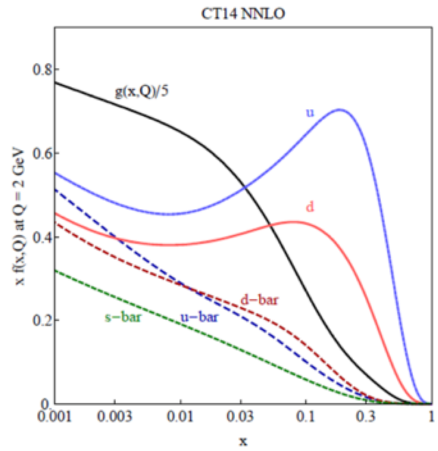


Chen,WW, Zhu, 2006.14825

using LPC data with $z_{cut} = 10a$, $\mu = 2GeV$ and in modified MS scheme; uncertainty is from lattice data

Gluon quasi PDF: Renormalization



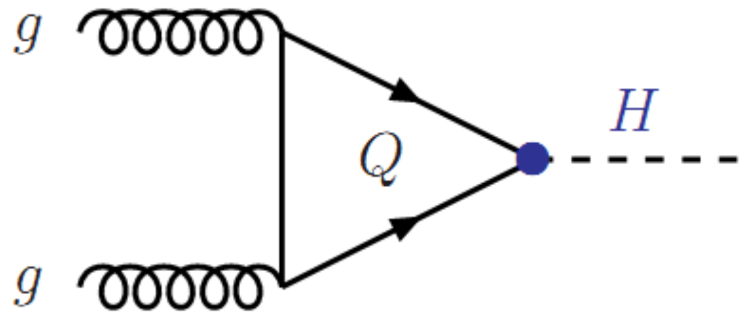


WW,Zhao,Zhu,1708.02458, EPJC
WW, Zhao, 1712.03830,JHEP
Zhang, Ji, Schafer, WW, Zhao,1808.10824,PRL
WW, Zhang, Zhao, Zhu, 1904.00978,PRD

See also Li, Ma, Qiu, 1809.01836

Gluon PDF

Higgs Production:
gluon-gluon fusion



Cross sections are calculated by Zürich group at N^3LO QCD and NLO EW accuracies [Anastasiou:2016cez]

mH=125.09 GeV, $\sqrt{s}$=13 TeV

σ=48.52pb

Total Uncertainty: 3.9% (Gaussian)

PDF: 1.9%

α_s: 2.6%

quasi PDF for gluon: definition?

Definition of quasi and light-cone gluon distribution

$$f_{g/H}(x,\mu) = \int \frac{d\xi^-}{2\pi x P^+} e^{-ix\xi^- P^+} \langle P|F^+{}_i(\xi^-)W(\xi^-,0,L_{n^+})F^{i+}(0)|P\rangle$$

$$\tilde{f}_{g/H}(x,\mu) = \int \frac{dz}{2\pi x P^z} e^{-ixzP^z} \langle P|F^z{}_i(z)W(z,0,L_{n^z})F^{iz}(0)|P\rangle$$

- Field Strength Tensor: F

WW,Zhao,Zhu,1708.02458

- i sums over transverse directions (i=1,2) or full directions
- $W(z_1,z_2, C)$ is a Wilson line along contour C.

Renormalization of gluon PDF: Auxiliary Field

Gervais and Neveu, 1980

Wilson line $W(z_1, z_2; C) = \langle \mathcal{Z}(\lambda_1)\bar{\mathcal{Z}}(\lambda_2)\rangle_z$

Gauge invariant non-local operators
pairs of gauge invariant composite local operators

$$F^a_{\mu\nu}(z_1)W_{ab}(z_1, z_2; C)F^b_{\rho\sigma}(z_2) = \left\langle (F^a_{\mu\nu}(z_1)\mathcal{Z}_a(\lambda_1)) \middle| \overline{(\mathcal{Z}_b}(\lambda_2)F^b_{\rho\sigma}(z_2)) \right\rangle$$
$$= \Omega^{(1)}_{\mu\nu}(z_1)\overline{\Omega^{(1)}_{\rho\sigma}}(z_2)$$

$$\Omega^{(1)}_{\mu\nu}(z_1) = F^a_{\mu\nu}(z_1)\mathcal{Z}_a(\lambda_1))$$

Renormalization of gluon quasi-PDF

Three operators with the same quantum number

$$\Omega^{(1)}_{\mu\nu} = F^a_{\mu\nu}\mathcal{Z}_a,$$
$$\Omega^{(2)}_{\mu\nu} = \Omega^{(1)}_{\mu\alpha}\frac{\dot{x}_\alpha\dot{x}_\nu}{\dot{x}^2} - \Omega^{(1)}_{\nu\alpha}\frac{\dot{x}_\alpha\dot{x}_\mu}{\dot{x}^2},$$
$$\Omega^{(3)}_{\mu\nu} = |\dot{x}|^{-2}(\dot{x}_\mu A^a_\nu - \dot{x}_\nu A^a_\mu)(D\mathcal{Z})_a,$$

$$\begin{pmatrix}\Omega^{z\mu}_{1,R}\\ \Omega^{z\mu}_{3,R}\end{pmatrix} = \begin{pmatrix}Z_{22} & Z_{13}\\ 0 & Z_{33}\end{pmatrix}\begin{pmatrix}\Omega^{z\mu}_{1}\\ \Omega^{z\mu}_{3}\end{pmatrix};$$

Different components are renormalized differently!

$$\Omega^{ti}_{1,R} = Z_{11}\Omega^{ti}_{1}$$

Renormalization of gluon PDF:

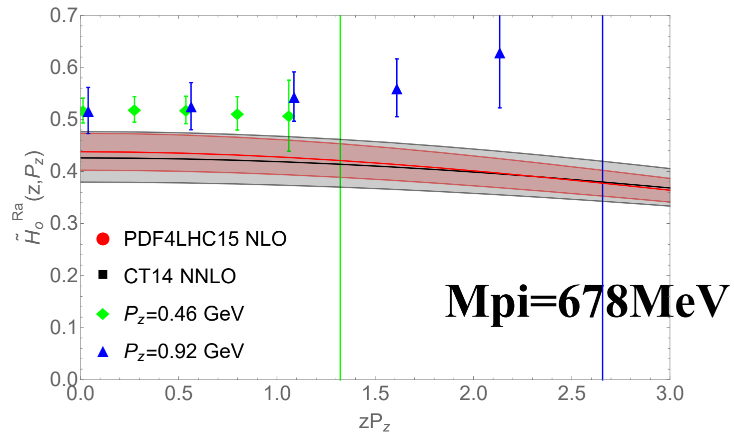
Multiplicatively Renormalizable Operators

$$O^{(1)}(z_1, z_2) \equiv F^{ti}(z_1)L(z_1, z_2)F_i^{\ t}(z_2),$$
$$O^{(2)}(z_1, z_2) \equiv F^{zi}(z_1)L(z_1, z_2)F_i^{\ z}(z_2),$$
$$O^{(3)}(z_1, z_2) \equiv F^{ti}(z_1)L(z_1, z_2)F_i^{\ z}(z_2),$$
$$O^{(4)}(z_1, z_2) \equiv F^{z\mu}(z_1)L(z_1, z_2)F_\mu^{\ z}(z_2),$$

Four multiplicative Renormalizable operators can be used to define gluon quasi-PDFs

Zhang, Ji, Schafer, WW, Zhao,1808.10824 (PRL 2019)

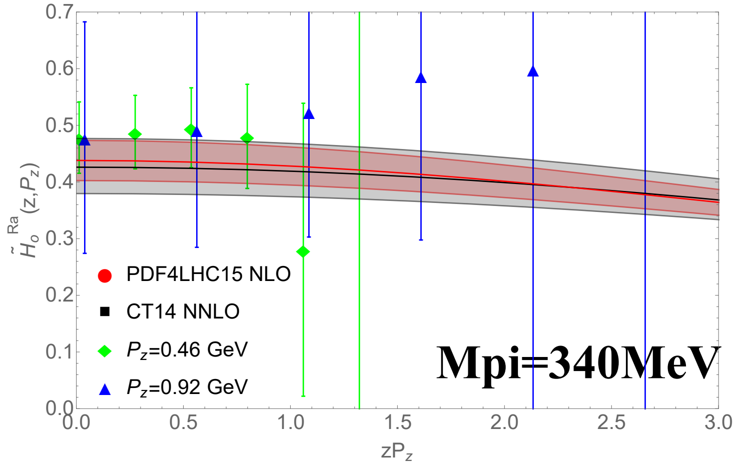
First Lattice Simulation



Fan, Yang, Anthony, Lin, Liu,1808.02077(PRL)

$$\tilde{H}_0(z, P_z) = \langle P|\mathcal{O}_0(z)|P\rangle,$$

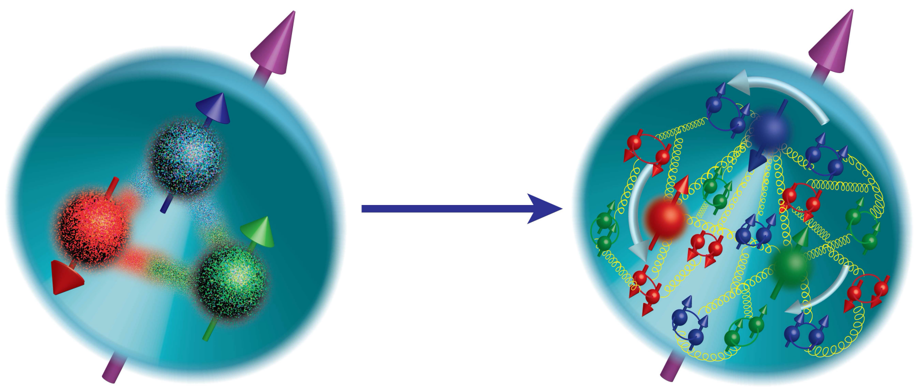
$$\mathcal{O}_0 \equiv \frac{P_0\left(\mathcal{O}(F^t_\mu, F^{\mu t}; z) - \frac{1}{4}g^{tt}\mathcal{O}(F^\mu_\nu, F^\nu_\mu; z)\right)}{\frac{3}{4}P_0^2 + \frac{1}{4}P_z^2}.$$



In future:

- More precise
- Physical Pion
- Large Momentum
- Quark-gluon Mixing

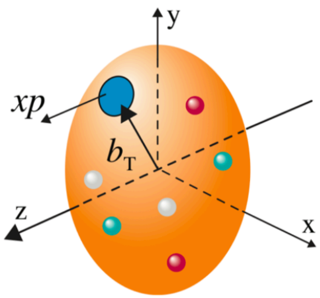
TMD PDF



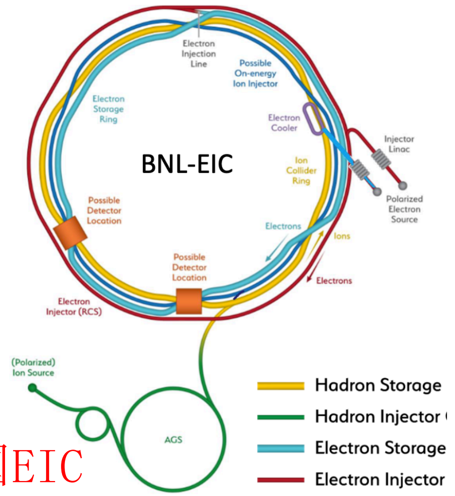
核子内部结构

TMD PDF

尽管人类对核子(质子)结构的研究取得了多次突破性进展，但核子结构的完整图像尚未建立，尤其是广泛研究的部分子分布函数[1969年诺贝尔奖]只描述了核子内部夸克和胶子一维结构。



随着美国能源部刚批准的电子离子对撞机(EIC)和我国正在筹建的EicC的不断推进，人们将首次有可能从实验上深入到核子三维结构。



美国EIC

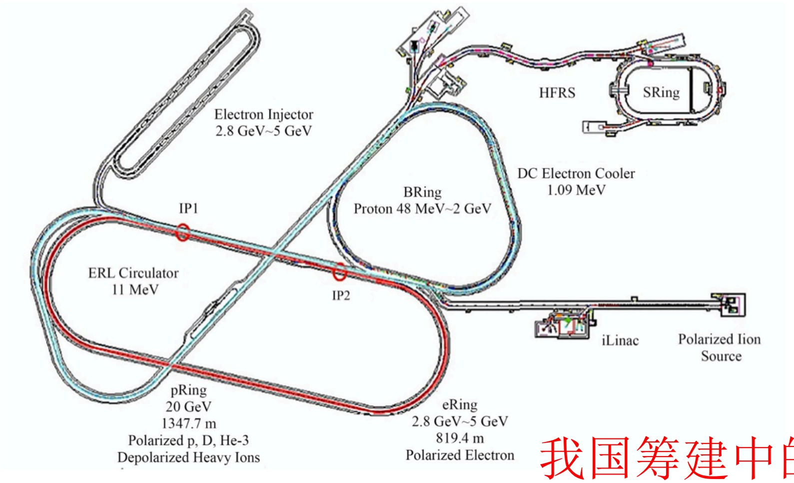


图31 EicC装置总体布局
Fig.31 The layout of EicC

我国筹建中的EicC

TMD PDF

How to obtain TMDPDF from first-principle?

$$f^{\mathrm{TMD}}(x,\zeta,b_\perp,\mu) = \frac{f(x,\tau,b_\perp,\mu)}{\sqrt{S(\tau,\tau',b_\perp,\mu)}}$$

Un-subtracted TMDPDF

Soft-factor

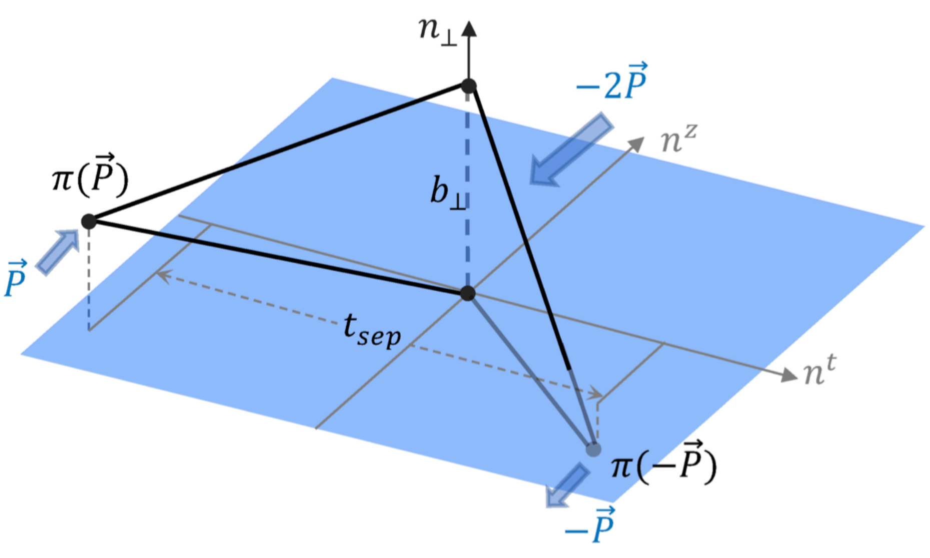
TMD PDF

Ji, Liu, Liu, arXiv: 1910.11415, 1911.03840

A four-quark form-factor:

$$\Pi(P, P', b_\perp) = \langle P'|\bar{\eta}(b_\perp)\Gamma'\eta(b_\perp)\bar{\psi}(0)\Gamma\psi(0)|P\rangle,$$

$$\Pi(P, P', b_\perp) = \int dx dx' H(x, x', P, P') \frac{\phi(x', \bar{Y}', P', b_\perp)}{S(\bar{Y}', Y', b_\perp)} \frac{\phi^\dagger(x, \bar{Y}, P, b_\perp)}{S(Y, \bar{Y}, b_\perp)} S(Y, Y', b_\perp).$$



TMD PDF

Ji, Liu, Liu, arXiv: 1910.11415, 1911.03840

Reduced Soft-factor can be calculated from form-factor and quasi-TMDWF:

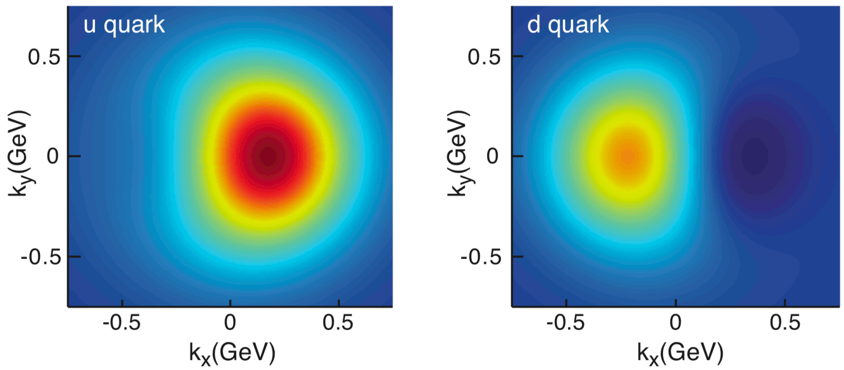
$$S_I(b_\perp) = \frac{S(Y, Y', b_\perp)}{S(Y, 0, b_\perp) S(0, Y', b_\perp)} \qquad (20)$$
$$= \frac{\Pi(P, P', b_\perp)}{\int dx dx' H(x, x', P, P') \widetilde{\phi}(x', P', b_\perp) \widetilde{\phi}^\dagger(x, P, b_\perp)}$$

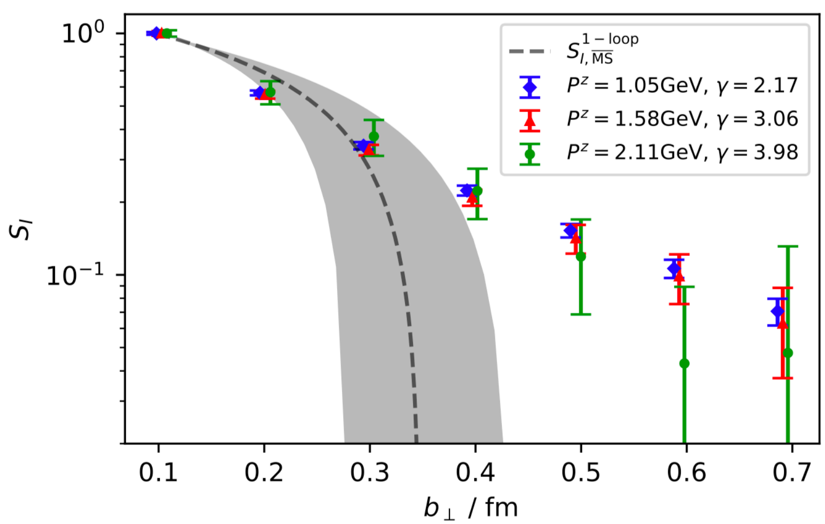
TMD PDF

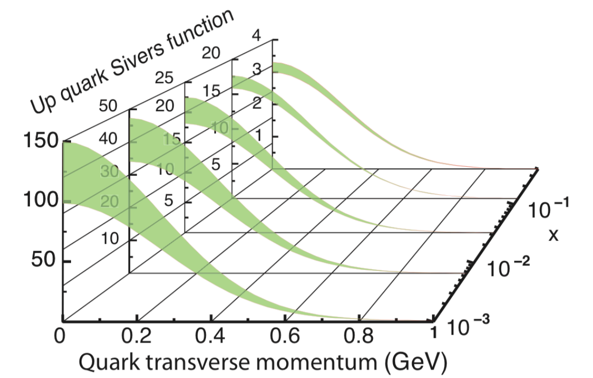
格点量子色动力学模拟

核子三维结构
EIC模拟图，非预言

$$f^{\mathrm{TMD}}(x,\zeta,b_\perp,\mu)=\frac{f(x,\tau,b_\perp,\mu)}{\sqrt{S(\tau,\tau',b_\perp,\mu)}}$$







初步完成工作：
软函数(LPC：2005.14572,submitted to PRL)

Summary

LaMET: Parton physics demands new ideas to solve non-perturbative QCD.

Recent Progress:

- ✓ Precision:
  - ✓ 2-loop Perturbative Matching
- ✓ New Distributions:
  - ✓ Gluons PDFs
  - ✓ Transverse Momentum Dependent PDF

In near future, we expect:

- ✓ Lattice calculation of quark PDFs with 2-loop: 10%
- ✓ New Distributions: gluon, TMDPDF, GPD, twist-3

Thank you very much!

BACKUP

TMD PDF

Ji, Liu, Liu, arXiv: 1910.11415, 1911.03840

TMDWF:

$$\tilde{\phi}(x, P, b_\perp) = \lim_{L\to\infty} \int \frac{P^z dz}{4\pi} e^{ixzP^z} \frac{\langle P|\bar{\psi}(0, z, \vec{b}_\perp)\tilde{\Gamma}\tilde{U}\psi(0)|0\rangle}{\sqrt{Z_E(0, 2L, b_\perp)}}.$$

$$\widetilde{\phi}(x, P, b_\perp) = H_\phi(x, P^z) \frac{\phi(x, \overline{Y}, P, b_\perp)}{S(Y, \overline{Y}, b_\perp)} S(Y, 0, b_\perp) \quad ($$