discussion. focus on divergent part

$$\int \frac{d^n k}{(2\pi)^n} \frac{\gamma_\nu (\not p' + \not k)\gamma_\mu (\not p + \not k)\gamma^\nu}{(p'+k)^2 (p+k)^2 k^2} = D \int \frac{d^n k}{(2\pi)^n} \frac{\gamma_\nu \not k \gamma_\mu \not k \gamma^\nu}{k^2 k^2 k^2}.$$

note

$$\frac{1}{ABC} = \int d\alpha\, d\beta \frac{1}{(\alpha A + \beta B + (1-\alpha-\beta)C)^3} \quad (1)$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{1}{(l_E^2+\Delta)^m} = \frac{1}{(4\pi)^{\frac{d}{2}}} \frac{\Gamma(m-\frac{d}{2})}{\Gamma(m)} \left(\frac{1}{\Delta}\right)^{m-\frac{d}{2}} \quad (2)$$

$$\int \frac{d^d l}{(2\pi)^d} \frac{(l_E^2)^n}{(l_E^2+\Delta)^m} = \frac{\Gamma(m-\frac{d}{2}-n)\Gamma(n+\frac{d}{2})}{(4\pi)^{d/2}\Gamma(m)\Gamma(\frac{d}{2})} \left(\frac{1}{\Delta}\right)^{m-\frac{d}{2}-n}. \quad (3).$$

} Euclidean space integral!

$$\int \frac{d^4 l}{(2\pi)^4} \frac{l_E^2}{(l_E^2-\Delta)^3} = \frac{i}{16\pi^2} \frac{\Gamma(\varepsilon)}{(\Delta)^\varepsilon}.$$

$$\frac{1}{分母} = \frac{1}{(p'+k)^d (p+k)^2 k^2}.$$

$$= \int da\, db \frac{1}{(a(p'+k)^2 + b(p+k)^2 + k^2(1-a-b))^3}$$

$$= \int da\, db \frac{1}{((k + ap' + bp)^2 - \square)^3}$$

from Minkowski space to Euclidean space

$$\int \frac{d^d l}{(2\pi)^d} \frac{(l^2)^n}{(l^2-\Delta)^m} = \frac{i(-1)^n}{(-1)^m} \times \int \frac{d^d l_E}{(2\pi)^d} \frac{(l_E^2)^n}{(l_E^2+\Delta)^m}.$$

$l = k^2 + ap' + bp$

$\Delta = \square$

$$\frac{分子}{分母} = \int da\, db \frac{1}{(l^2-\Delta)^3} = \int da\, db$$

$k \to l$. 对 dl 积分.

分子: k 的 0次 1次 2次 对应 l 对 0次 1次. 2次

k 的 0次 (l 的 0次) 积分有限(对 l) from (2)

k 的 1次 (l 的 1次) 积分为0 奇偶性.

仅剩 k 的 2次项 $\gamma_\nu \not k \gamma_\mu \not k \gamma^\nu = \gamma^\mu k^2$!

D.

$$\int da\int db\int d^4l\,\frac{k^2}{(l^2-\Delta)^3}=\int da\,db\int d^4l_E\,\frac{k^2}{(l_E^2+\Delta)^3}$$

Δ is positive.
see it as an effective mass term. peskin p191.

$$=\int da\,db\int d^4l_E\,\frac{[l_E-(ap'+bp)]^2}{(l_E^2+\Delta)^3}\quad 分子$$

分子出现 l 的0次 1次 2次

仅保留2次（0次 from (2) 有限. 1次 此积分为0, 奇函数性）

$$=\int da\,db\int d^4l_E\,\frac{l_E^2}{(l_E^2+\Delta)^3}+\cdots$$

$$=\int da\,db\int d^4k'\,\frac{(k'^2\mp\Delta)^2}{(k'^2)^3}+\cdots$$

同样地 仅2次(分子)项发散.

$$=\int da\,db\int d^4k'\,\frac{k'^2}{(k')^6}$$

$$=\int da\,db\int d^4k\,\frac{k^2}{k^6}\sim\frac{1}{k^4}$$

$$=\int da\,db\,\frac{(-1)}{(4\pi)^{\frac{d}{2}}}\,i\Gamma(\varepsilon)\left(\frac{1}{q^2}\right)^{\varepsilon}.$$

question 2.

from δZ_A δZ_4. δZ_F $\longrightarrow$ Z_g.

$$Z_g = \frac{Z_F}{Z_4 Z_A^{\frac{1}{2}}}$$

$Z_4 = 1+\delta Z_4$ $\quad Z_A = 1+\delta Z_A$ $\quad Z_F = 1+\delta Z_F$.

here. all loops contribution. $\delta Z_4 \to$ finite, we can ~~ju~~ use taylor exposion.

$\therefore$ However, In final result, we just keep 1 loop contribution

$$\frac{1}{1+\delta Z_4(\text{all loops})} = \cancel{吉}\ 1-\delta Z_4(\text{all loop})$$

$$\overset{1\text{ loop}}{=} 1-\delta Z_4(1\text{ loop}) \Longrightarrow \text{keep 1 loop contribution}$$

$(\Delta')'$.

question 3. $\varepsilon \to 0$

$\Gamma(\varepsilon)\cdot\left(\frac{1}{q^2}\right)^{\varepsilon}$ $\quad \left(\frac{1}{q^2}\right) \xrightarrow{看} a$ $\quad \Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma + O(\varepsilon)$

peskin 7.83

$$= \left(\frac{1}{\varepsilon}-\gamma\right) a^{\varepsilon}$$

$$= \left(\frac{1}{\varepsilon}-\gamma\right) e^{\varepsilon \ln a}$$

$$= \left(\frac{1}{\varepsilon}-\gamma\right)(1+\varepsilon \ln a) = \left(\frac{1}{\varepsilon}-\gamma+\ln a\right).$$