Towards the understanding of fully-heavy tetraquark states from various models

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1. Introduction

Experimental side

Very recently, the LHCb Collaboration researched the J/Ψ pairs invariant mass spectrum and observed two structure,

The broad structure ranging from 6.2 to 6.8 GeV,

The narrow structure around 6.9 GeV, denoted as X(6900).

The structures are made up of four charm quarks. However, their properties and spin-parity quantum numbers are not completely clear so far.

Theoretical side

The dynamics is very simple, one gluon exchange (OGE), color confinement potential, weak Relativistic effects.

More than 40 years, various theoretical frameworks: QCD sum, Bethe-Salpeter equation lattice QCD, MIT bag model, ...

2. Three models

A. Color-magnetic interaction model

OGE interaction:

$$V_{ij}^{oge} = \frac{\alpha_s}{4}\lambda_i^c \cdot \lambda_j^c \left(\frac{1}{r_{ij}} - \frac{2\pi\delta(\mathbf{r}_{ij})\sigma_i \cdot \sigma_j}{3m_i m_j}\right) + \cdots$$

color-coulomb:

$$V_{ij}^{clb} = \frac{\alpha_s \lambda_i^c \cdot \lambda_j^c}{4r_{ij}},$$

color-magnetic:

$$V_{ij}^{\mathrm{cm}} = -\frac{\pi\alpha_s\delta(\mathbf{r}_{ij})\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j}{6m_i m_j},$$

Under the assumption of the same size, the meson and baryon mass splitting among different spin is determined by the color-magnetic term.

$$H_{\mathrm{cm}}^n = -\sum_{i<j}^{n} C_{ij}\boldsymbol{\lambda}_i^c \cdot \boldsymbol{\lambda}_j^c \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j, \quad C_{ij} = \frac{\pi\alpha_s\delta(\mathbf{r}_{ij})}{6m_i m_j},$$

Mass formula : $M = \sum_{i=1}^{n} m_i + \langle H_{\mathrm{cm}}^n \rangle.$

Other dynamic effects are assumed to be absorbed by the effective quark masses!

Ground state meson spectrum

Comparison for meson masses measured by experiments and calculated by using the Mass formula. For mesons $\langle \lambda_i^c \cdot \lambda_j^c \rangle = -\frac{16}{3}$, $\langle \sigma_i \cdot \sigma_j \rangle = -3$ and 1 for $S = 0$ and $S = 1$, respectively.

Hadron	CMI	Th.	Ex.	(Th.-Ex.)	Hadron	CMI	Th.	Ex.	(Th.-Ex.)
π	$-16C_{n\bar{n}}$	246.6	139.6	107	ρ	$\frac{16}{3}C_{n\bar{n}}$	882.3	775.3	107
K	$-16C_{n\bar{s}}$	602.8	493.7	109	K^*	$\frac{16}{3}C_{n\bar{s}}$	1001.7	891.8	110
					ω	$\frac{16}{3}C_{n\bar{n}}$	882.3	782.7	100
					ϕ	$\frac{16}{3}C_{s\bar{s}}$	1136.7	1019.5	117
D	$-16C_{c\bar{n}}$	1980.7	1869.7	111	D^*	$\frac{16}{3}C_{c\bar{n}}$	2121.5	2010.3	111
D_s	$-16C_{c\bar{s}}$	2157.7	1968.3	189	D_s^*	$\frac{16}{3}C_{c\bar{s}}$	2300.6	2112.2	188
B	$-16C_{b\bar{n}}$	5380.9	5279.5	102	B^*	$\frac{16}{3}C_{b\bar{n}}$	5425.7	5324.7	101
B_s	$-16C_{b\bar{s}}$	5556.3	5366.9	189	B_s^*	$\frac{16}{3}C_{b\bar{s}}$	5605.4	5415.4	190
η_c	$-16C_{c\bar{c}}$	3364.4	2983.9	381	J/ψ	$\frac{16}{3}C_{c\bar{c}}$	3477.5	3096.9	381
η_b	$-16C_{b\bar{b}}$	10059.2	9399.0	660	Υ	$\frac{16}{3}C_{b\bar{b}}$	10121.1	9460.3	661
B_c	$-16C_{\bar{c}b}$	6724.6	6274.9	450	B_c^* [33]	$\frac{16}{3}C_{\bar{c}b}$	6795.0		

$m_n = 361.7$ MeV ($n = u, d$), $m_s = 540.3$ MeV, $m_c = 1724.6$ MeV, and $m_b = 5052.8$ MeV

$C_{n\bar{n}} = 29.8$ $C_{n\bar{s}} = 18.7$ $C_{n\bar{c}} = 6.6$ $C_{n\bar{b}} = 2.1$ $C_{s\bar{s}} = 10.5$ $C_{s\bar{c}} = 6.7$ $C_{s\bar{b}} = 2.3$ $C_{c\bar{c}} = 5.3$ $C_{b\bar{b}} = 2.9$ $C_{c\bar{b}} = 3.3$

Taken from Yan-Rui Liu, et al, Prog. Part. Nucl. Phys. 107 (2019) 237-320

The predicted masses are generally overestimated! Therefore, the mass formular is modified in the generalization from conventional hadrons to multiquark states.

$$M = M_{\mathrm{ref}} - \langle H^n_{\mathrm{cm}} \rangle_{\mathrm{ref}} + \langle H^n_{\mathrm{cm}} \rangle.$$

M_{ref} and $\langle H^n_{\mathrm{cm}} \rangle_{\mathrm{ref}}$ are the physical mass of the reference system and its color-magnetic interaction energy, respectively. Di-meson, meson-baryon, di-baryon. One can define the binding energy as

$$\Delta E = M - M_{\mathrm{ref}} = \langle H^n_{\mathrm{cm}} \rangle - \langle H^n_{\mathrm{cm}} \rangle_{\mathrm{ref}}$$

The mass formula has been widely used to study the multiquark states, see Prog. Part. Nucl. Phys. 107 (2019) 237-320.

No dynamical effect! Everything depends only on the color-spin algebra.

B. Constituent Quark Model (Isgur-Karl model)

OGE interaction: $$V_{ij}^{oge} = \frac{\alpha_s}{4}\lambda_i^c \cdot \lambda_j^c \left(\frac{1}{r_{ij}} - \frac{2\pi\delta(\mathbf{r}_{ij})\sigma_i \cdot \sigma_j}{3m_i m_j}\right) + \cdots$$

$$\delta(\mathbf{r}_{ij}) \to \frac{1}{4\pi r_{ij} r_0^2(\mu_{ij})} e^{-r_{ij}/r_0(\mu_{ij})}, \quad \alpha_s(\mu_{ij}^2) = \frac{\alpha_0}{\ln \frac{\mu_{ij}^2}{\Lambda_0^2}},$$

Confinement: $$V^{\mathrm{con}} = -a_c \sum_{i<j}^{n} \lambda_i^c \cdot \lambda_j^c r_{ij}^2,$$

Total Hamiltonian: $$H_n = \sum_{i=1}^{n}\left(m_i + \frac{\mathbf{p}_i^2}{2m_i}\right) - T_c + \sum_{i<j}^{n} V_{ij}^{\mathrm{oge}} + V^{\mathrm{con}}.$$

Dynamical calculation

The Gaussian expansion method (GEM) has been proven to **be a rather high precision computational method**. According to the GEM, the relative motion wave function between the quark and antiquark can be written as

$$\phi_{lm}^{G}(\mathbf{r}) = \sum_{n=1}^{n_{max}} c_n N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}})$$

Gaussian size parameters are taken as geometric progression,

$$\nu_n = \frac{1}{r_n^2}, \quad r_n = r_1 a^{n-1}, \quad a = \left(\frac{r_{n_{max}}}{r_1}\right)^{\frac{1}{n_{max}-1}}$$

With r_1 = 0.2 fm, r_nmax = 2.0 fm and **n_max = 7**, the converged numerical results can be achieved.

Ground state heavy-meson spectrum

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States	PDG	E_2	$\langle E_k \rangle$	$\langle V^{con} \rangle$	$\langle V^{cm} \rangle$	$\langle V^{clb} \rangle$	$\langle r^2 \rangle^{\frac{1}{2}}$
$D^{\pm}$	1869	1886	737	200	−92	−937	0.50
D^*	2007	2000	633	226	27	−862	0.53
$D_s^{\pm}$	1969	1982	693	151	−105	−914	0.43
D_s^*	2112	2109	560	179	29	−816	0.47
η_c	2980	2965	679	75	−123	−995	0.31
J/Ψ	3097	3103	488	97	29	−838	0.35
B^0	5280	5261	664	197	−34	−885	0.50
B^*	5325	5305	623	207	11	−855	0.51
B_s^0	5366	5346	612	143	−42	−868	0.42
B_s^*	5416	5399	555	155	13	−824	0.44
B_c	6277	6244	644	54	−79	−1044	0.26
B_c^*	⋯	6336	502	65	20	−921	0.29
η_b	9391	9376	740	24	−96	−1305	0.17
$\Upsilon(1S)$	9460	9486	560	30	24	−1140	0.19

Para.	$m_{u,d}$	m_s	m_c	m_b	a_c	α_0	Λ_0	r_0
Valu.	313	494	1664	5006	−150	4.25	40.85	119.3

Different size, such as D and D^*.

The ratio of the color-magnetic term is **not strict 3：1** but **between 3：1 and 4：1**.

The **Coulomb interaction** provides an extremely strong short-range attraction.

C. Multi-quark color flux-tube model

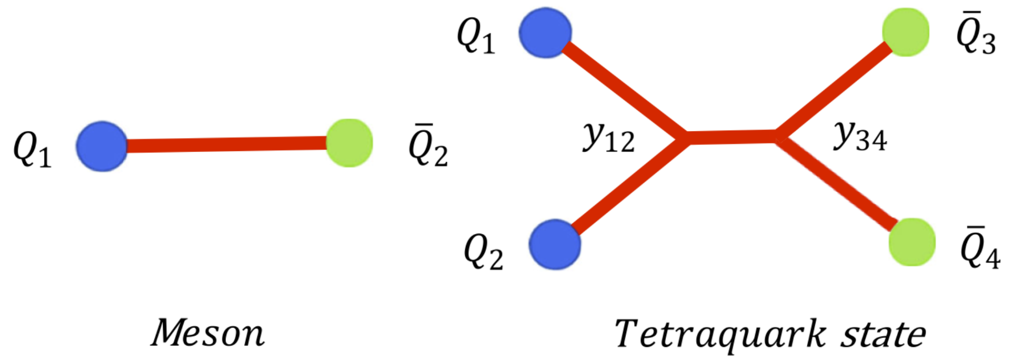

FIG. 1. Color flux-tube structures.

$$V^{\rm con}_{\rm min}(2) = K r_{ij}^2,$$

$$V^{\rm con}(4) = K[(\mathbf{r}_1 - \mathbf{y}_{12})^2 + (\mathbf{r}_2 - \mathbf{y}_{12})^2 + (\mathbf{r}_3 - \mathbf{y}_{34})^2 + (\mathbf{r}_4 - \mathbf{y}_{34})^2 + \kappa_d(\mathbf{y}_{12} - \mathbf{y}_{34})^2],$$

LQCD static potential

$$V_{q\bar{q}} = -\frac{A_{q\bar{q}}}{r} + \sigma_{q\bar{q}} r$$

$$V_{3q} = -A_{3q} \sum_{i>j} \frac{1}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{3q} L_{\rm min}$$

$$V_{4q} = \frac{\alpha_s}{4} \sum_{i>j} \frac{\lambda_i \cdot \lambda_j}{|\mathbf{r}_i - \mathbf{r}_j|} + \sigma_{4q} L_{\rm min}$$

$$L_{\rm min} = \sum_i L_i$$

Coulomb potential + linear confinement

$\kappa_d = \frac{C_d}{C_3}$, **C_d is the eigenvalue of the Casimir operator associated with the SU(3) color representation d, C_3=4/3, C_6=10/3, and C_8=3.**

The minimum of the confinement potential can be obtained by taking the variation with respect to y_12 and y_34,

$$V^{\mathrm{con}}_{\mathrm{min}}(4) = K\left(\mathbf{R}_1^2 + \mathbf{R}_2^2 + \frac{\kappa_d}{1+\kappa_d}\mathbf{R}_3^2\right).$$

The canonical coordinates R_i have the following forms,

$$\begin{aligned}\mathbf{R}_1 &= \frac{1}{\sqrt{2}}(\mathbf{r}_1 - \mathbf{r}_2),\ \mathbf{R}_2 = \frac{1}{\sqrt{2}}(\mathbf{r}_3 - \mathbf{r}_4),\\ \mathbf{R}_3 &= \frac{1}{\sqrt{4}}(\mathbf{r}_1 + \mathbf{r}_2 - \mathbf{r}_3 - \mathbf{r}_4),\ \mathbf{R}_4 = \frac{1}{\sqrt{4}}(\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 + \mathbf{r}_4).\end{aligned}$$

The OGE interaction is also involved in the MCFTM. It is not a completely new model but the updated version of the CQM based on the color flux-tube picture of hadrons in the lattice QCD. In fact, it merely modifies the two-body confinement potential into the multibody one.

3. Wave function

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Total wave function:

$$\Phi^{[Q_1Q_2][\bar{Q}_3\bar{Q}_4]}_{IM_IJM_J} = \sum_\alpha \xi_\alpha \left[\left[\left[\phi^G_{l_am_a}(\mathbf{r})\chi_{s_aM_{s_a}}\right]^{[Q_1Q_2]}_{J_aM_{J_a}} \left[\phi^G_{l_bm_b}(\mathbf{R})\chi_{s_bM_{s_b}}\right]^{[\bar{Q}_3\bar{Q}_4]}_{J_bM_{J_b}}\right]_{J_{ab}M_{J_{ab}}} \phi^G_{l_{ab}m_{ab}}(\mathbf{X})\right]_{JM_J}$$
$$\times \left[\eta^{[Q_1Q_2]}_{i_aM_{i_a}}\eta^{[\bar{Q}_3\bar{Q}_4]}_{i_bM_{i_b}}\right]_{IM_I} \left[\chi^{[Q_1Q_2]}_{[c_a]W_{c_a}}\chi^{[\bar{Q}_3\bar{Q}_4]}_{[c_b]W_{c_b}}\right]_{[C]W_C}$$

Overall color-singlet:

$$\left[[Q_1Q_2]_{\bar{\mathbf{3}}_c} \otimes [\bar{Q}_3\bar{Q}_4]_{\mathbf{3}_c}\right]_{\mathbf{1}} \quad \left[[Q_1Q_2]_{\mathbf{6}_c} \otimes [\bar{Q}_3\bar{Q}_4]_{\bar{\mathbf{6}}_c}\right]_{\mathbf{1}}$$

In the center-of-mass reference frame, the Jacobi coordinates can be defined as

$$\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{R} = \mathbf{r}_3 - \mathbf{r}_4, \quad \mathbf{X} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2} - \frac{m_3\mathbf{r}_3 + m_4\mathbf{r}_4}{m_3 + m_4},$$

The relative motion wave functions should be expressed as the superposition of many different size Gaussian functions,

$$\phi^G_{lm}(\mathbf{r}) = \sum_{n=1}^{n_{max}} c_n N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{\mathbf{r}})$$

The diquark and antidiquark are in the S-wave, angular excitation only occurs between the diquark and antidiquark. Identical particles, Pauli principle.

4. Numerical results and discussions

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The mass spectra of the ground states [cc][\barc\barc], unit in MeV.

Model		CMIM			MCFTM			CQM		
Flavor	J^P	$\bar{\mathbf{3}}_c\otimes\mathbf{3}_c$	$\mathbf{6}_c\otimes\bar{\mathbf{6}}_c$	C.C.	$\bar{\mathbf{3}}_c\otimes\mathbf{3}_c$	$\mathbf{6}_c\otimes\bar{\mathbf{6}}_c$	C.C.	$\bar{\mathbf{3}}_c\otimes\mathbf{3}_c$	$\mathbf{6}_c\otimes\bar{\mathbf{6}}_c$	C.C.
	0^+	−28.27, 66%	42.40, 34%	−102.64, 6035	6454, 56%	6467, 44%	6407	6573, 36%	6537, 64%	6491
$[cc][\bar{c}\bar{c}]$	1^+	0.00, 100%	...	0.00, 6139	6463, 100%	...	6463	6580, 100%	...	6580
	2^+	56.53, 100%	...	56.53, 6194	6486, 100%	...	6486	6607, 100%	...	6607

More results can be found in Ref. (Chengrong Deng et al, PRD 103, 014001 (2021)).

In addition, the CMIM motivated by the QCD-string junction picture, 6192 MeV, Karliner et al., PRD 95, 034011 (2017). The CMIM, in which the tetraquark is regarded as point-like diquark (antidiquark) in color three, 5970 MeV, Berezhnoy et al., PRD 84, 094023 (2011) .

The masses given by various CMIMs are around the threshold and generally lower than those in the dynamical models.

The values of various parts of the Hamiltonian in the MCFTM, unit in MeV.

Flavor	J^P	E_4	$\langle E_k\rangle$	$\langle V^{con}_{min}(4)\rangle$	$\langle V^{cm}\rangle$	$\langle V^{clb}\rangle$	$T_{M_1M_2}$	ΔE	$\Delta\langle E_k\rangle$	$\Delta\langle V^{con}_{min}(4)\rangle$	$\Delta\langle V^{cm}\rangle$	$\Delta\langle V^{clb}\rangle$
	0^+	6407	887	192	−51	−1279	$\eta_c\eta_c$	477	−471	42	195	711
$[cc][\bar{c}\bar{c}]$	1^+	6463	800	203	4	−1202	$\eta_c\Psi$	395	−367	32	98	632
	2^+	6486	769	211	27	−1178	$\Psi\Psi$	280	−206	18	−31	499

1. The color-magnetic interaction is **overestimated** in the CMIM.

2. The **dynamical effects** in the meson-meson thresholds and the tetraquark states are **obviously different**, especially the **color-coulomb interaction**, which induces that the masses are **much higher** the threshold in the MCFTM.

3. The CMIMs are **difficult to completely describe the dynamical effects** in the extension from the heavy mesons to fully-heavy tetraquark states.

The values of various parts of the Hamiltonian and average distances in the MCFTM.

	LS	J^P	States	Mass, prop.	$\langle E_k\rangle$	$\langle V^{com}_{min}(4)\rangle$	$\langle V^{cm}\rangle$	$\langle V^{clb}\rangle$	$\langle \mathbf{r}^2_{12}\rangle^{\frac{1}{2}}$	$\langle \mathbf{r}^2_{34}\rangle^{\frac{1}{2}}$	$\langle \mathbf{r}^2_{13}\rangle^{\frac{1}{2}}$	$\langle \mathbf{r}^2_{24}\rangle^{\frac{1}{2}}$	$\langle \mathbf{r}^2_{14}\rangle^{\frac{1}{2}}$	$\langle \mathbf{r}^2_{23}\rangle^{\frac{1}{2}}$	$\langle \mathbf{X}^2\rangle^{\frac{1}{2}}$
			$\bar{\mathbf{3}}_c\otimes\mathbf{3}_c$	6454, 56%	878	188	−11	−1258	0.42	0.42	0.45	0.45	0.45	0.45	0.33
	00	0^+	$\mathbf{6}_c\otimes\bar{\mathbf{6}}_c$	6467, 44%	899	199	17	−1306	0.46	0.46	0.43	0.43	0.43	0.43	0.28
			C.C.	6407	887	192	−51	−1279	0.44	0.44	0.44	0.44	0.44	0.44	0.31
			$\bar{\mathbf{3}}_c\otimes\mathbf{3}_c$	6730, 98%	783	283	4	−997	0.47	0.47	0.61	0.61	0.61	0.61	0.52
$[cc][\bar{c}\bar{c}]$	10	1^-	$\mathbf{6}_c\otimes\bar{\mathbf{6}}_c$	6888, 2%	910	274	12	−966	0.51	0.51	0.54	0.54	0.54	0.54	0.40
			C.C.	6727	785	283	−2	−997	0.47	0.47	0.61	0.61	0.61	0.61	0.51
			$\bar{\mathbf{3}}_c\otimes\mathbf{3}_c$	6945, >99%	802	364	9	−888	0.48	0.48	0.75	0.75	0.75	0.75	0.66
	20	2^+	$\mathbf{6}_c\otimes\bar{\mathbf{6}}_c$	7213, <1%	978	339	10	−772	0.55	0.55	0.63	0.63	0.63	0.63	0.50
			C.C.	6944	802	364	8	−887	0.48	0.48	0.75	0.75	0.75	0.75	0.66

1. The color-coulomb interaction is still **strong** while the color-magnetic one is **weak** .

2. The tetraquark states and mesons (0.30~0.35 fm) do not **share the same size**.

3. **Three-dimensional spatial configuration**. The sizes of the diquark (antidiquark) do **not dramatically change with L** while the distance **X** between the diquark and antidiquark **remarkably change**.

The values of various parts of the Hamiltonian and average distances in the MCFTM.

	LS	J^P	States	Mass, prop.	$\langle E_k \rangle$	$\langle V^{com}_{min}(4) \rangle$	$\langle V^{cm} \rangle$	$\langle V^{clb} \rangle$	$\langle \mathbf{r}^2_{12} \rangle^{\frac{1}{2}}$	$\langle \mathbf{r}^2_{34} \rangle^{\frac{1}{2}}$	$\langle \mathbf{r}^2_{13} \rangle^{\frac{1}{2}}$	$\langle \mathbf{r}^2_{24} \rangle^{\frac{1}{2}}$	$\langle \mathbf{r}^2_{14} \rangle^{\frac{1}{2}}$	$\langle \mathbf{r}^2_{23} \rangle^{\frac{1}{2}}$	$\langle \mathbf{X}^2 \rangle^{\frac{1}{2}}$
			$\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$	6454, 56%	878	188	−11	−1258	0.42	0.42	0.45	0.45	0.45	0.45	0.33
	00	0^+	$\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$	6467, 44%	899	199	17	−1306	0.46	0.46	0.43	0.43	0.43	0.43	0.28
			C.C.	6407	887	192	−51	−1279	0.44	0.44	0.44	0.44	0.44	0.44	0.31
			$\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$	6730, 98%	783	283	4	−997	0.47	0.47	0.61	0.61	0.61	0.61	0.52
$[cc][\bar{c}\bar{c}]$	10	1^-	$\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$	6888, 2%	910	274	12	−966	0.51	0.51	0.54	0.54	0.54	0.54	0.40
			C.C.	6727	785	283	−2	−997	0.47	0.47	0.61	0.61	0.61	0.61	0.51
			$\bar{\mathbf{3}}_c \otimes \mathbf{3}_c$	6945, >99%	802	364	9	−888	0.48	0.48	0.75	0.75	0.75	0.75	0.66
	20	2^+	$\mathbf{6}_c \otimes \bar{\mathbf{6}}_c$	7213, <1%	978	339	10	−772	0.55	0.55	0.63	0.63	0.63	0.63	0.50
			C.C.	6944	802	364	8	−887	0.48	0.48	0.75	0.75	0.75	0.75	0.66

Color matrix elements, $\hat{O}_{ij} = \lambda^c_i \cdot \lambda^c_j$.

$\langle \hat{O}_{ij} \rangle$	$\langle \hat{O}_{12} \rangle$	$\langle \hat{O}_{34} \rangle$	$\langle \hat{O}_{13} \rangle$	$\langle \hat{O}_{24} \rangle$	$\langle \hat{O}_{14} \rangle$	$\langle \hat{O}_{23} \rangle$
$\langle \bar{\mathbf{3}}_c \otimes \mathbf{3}_c \vert \hat{O}_{ij} \vert \bar{\mathbf{3}}_c \otimes \mathbf{3}_c \rangle$	$-\frac{8}{3}$	$-\frac{8}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$
$\langle \mathbf{6}_c \otimes \bar{\mathbf{6}}_c \vert \hat{O}_{ij} \vert \mathbf{6}_c \otimes \bar{\mathbf{6}}_c \rangle$	$\frac{4}{3}$	$\frac{4}{3}$	$-\frac{10}{3}$	$-\frac{10}{3}$	$-\frac{10}{3}$	$-\frac{10}{3}$
$\langle \bar{\mathbf{3}}_c \otimes \mathbf{3}_c \vert \hat{O}_{ij} \vert \mathbf{6}_c \otimes \bar{\mathbf{6}}_c \rangle$	0	0	$-2\sqrt{2}$	$-2\sqrt{2}$	$2\sqrt{2}$	$2\sqrt{2}$

4. **The color configuration 6 should not be ignored in the ground states due to the strong Coulomb attraction.**

5. **The color configuration 3 is absolutely dominant in the excited states because of the strong Coulomb attraction.**

Masses of the ground state [cc][\barc\barc] in various dynamical models, unit in MeV.

J^P	MCFTM	CQM	[3]	I, II [4]	[5]	[20]	[47]
0^+	6407	6491	6477	6377, 6371	6470	6350	6440 ± 0.15
1^+	6463	6580	6528	6425, 6450	6512	6440	6370 ± 0.18
2^+	6486	6607	6573	6432, 6479	6534	6470	6370 ± 0.19

Masses of the excited state [cc][\barc\barc] in the CQM and MCFTM, unit in MeV..

L		1			2	
S	0	1	2	0	1	2
CQM	6901	6912	6924	7182	7185	7191
MCFTM	6727	6735	6744	6944	6947	6951

1. Various dynamical models present similar mass spectra, which are much higher than the corresponding thresholds. The broad structure locating at around 6490 MeV can be described as a ground state [cc][\barc\barc] in the dynamical models.

2. In the excited states, the difference between two models is obvious, (L=1, 180MeV) , (L=2, 240MeV).

3. The narrow structure X(6900) can be interpreted as a excited state [cc][\barc\barc] with L=1 in the CQM and L=2 in the MCFTM.

5. Summary

1. The CMIMs cannot completely absorb QCD dynamic effects and may overestimate the c-m interaction in the extension from heavy mesons to the fully-heavy states.

2. The Coulomb interaction is very strong while the color-magnetic interaction is weak in the heavy mesons and the fully-heavy states.

3. The color configuration-6 can not be ignored in the ground states owing to the strong Coulomb interaction. However, the color configuration-3 is absolutely dominant in the excited states.

4. The J/Ψ-pair resonances observed recently by the LHCb Collaboration are difficult to be accommodated in the CMIMs.

5. The broad structure locating at around 6490 MeV can be described as a ground state [cc][\barc\barc] in the dynamical models. The narrow structure X(6900) can be interpreted as a excited state with L=1 in the CQM and L=2 in the MCFTM.

Thanks!