第五届“强子谱和强子结构研讨会”
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Form Factors and Trace Anomaly of Energy Momentum Tensor

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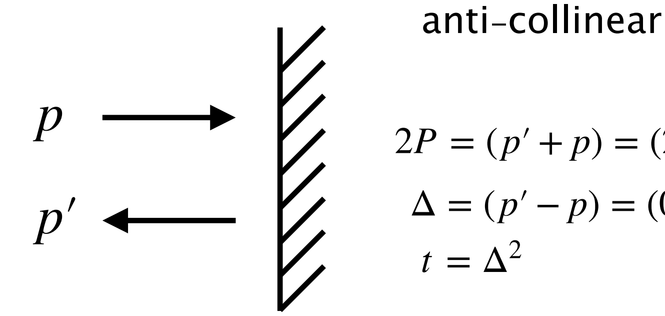
Outline：

- Introduction：nucleon
- GFFs for ρ meson（J=1）and Δ isobar（J=3/2）
  - gravitational form factors (GFFs), sum rules to GPDs
  - mechanical properties, quark model, Skyrme model
- QED EMT Trace anomaly
  - hydrogen mass, Lamb shift
- Summary

Based on:
Polyakov, BDS, PRD**100** (2019) 036003
BDS, Dong, PRD**101** (2020) 096008
Kim, BDS, 2011.00292
BDS, Sun, Zhou, 2012.09443

Gravitational form factors (GFFs)

Breit frame:



$2P = (p'+p) = (2E, \vec{0})$

$\Delta = (p'-p) = (0, \vec{\Delta})$

$t = \Delta^2$

- Energy Momentum Tensor (EMT)

$$\hat{T}^{\mu\nu}_{C} = \sum_n \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_n)} \partial^\nu \phi_n - g^{\mu\nu}\mathcal{L}$$

$$\hat{T}^{\mu\nu}_{\rm grav} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\rm grav}}{\delta g^{\mu\nu}} \longrightarrow$$

- GPDs ↔ GFFs (polynomiality) (Ji, 1996)

$$\int \mathrm{d}x\, x\, H^a(x,\xi,t) = A^a(t) + \xi^2 D^a(t)$$

$$\int \mathrm{d}x\, x\, E^a(x,\xi,t) = B^a(t) - \xi^2 D^a(t)$$

Ji sum $A^q(t) + B^q(t) = 2J^q(t)$

- {EM form factor, PDFs} ∈ GPDs

$$\int \mathrm{d}x\, H^q(x,\xi,t) = F_1^q(t)$$

$$\lim_{\Delta\to 0} H^q(x,\xi,t) = f_1^q(x)$$

DVCS @ EicC (lower Q^2 v.s. US-EIC)

✤ nucleon GFFs: (Kobzarev & Okun 1962; Pagels 1966; Polyakov & Schweitzer, 2018)

$$\langle p', s'|\hat{T}^a_{\mu\nu}(x)|p,s\rangle = \bar{u}'\Big[A^a(t)\frac{\gamma_{\{\mu}P_{\nu\}}}{2} + B^a(t)\frac{i\,P_{\{\mu}\sigma_{\nu\}\rho}\Delta^\rho}{4m} + D^a(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4m} + m\,\bar{c}^a(t)\,g_{\mu\nu}\Big]u\, e^{i(p'-p)x}$$

$(a = q, g)$

$t \to 0$: A^a → mass; B^a → spin (external properties); D^a → D-term ("internal" property)

"Druck" (Polyakov, 1999)

gluon GFFs in pQCD

X.-B. Tong, J.-P. Ma, F. Yuan, 2101.02395

✤ Mellin moments (Diehl, 2003; Belitsky, Radyushkin, 2005)

$$(P^+)^{n+1}\int \mathrm{d}x\, x^n \int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-}\Big[\bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\Big]_{z^+=0,\, \boldsymbol{z}=0}$$

$$= \Big(i\frac{\mathrm{d}}{\mathrm{d}z^-}\Big)^n \Big[\bar{q}(-\tfrac{1}{2}z)\,\gamma^+ q(\tfrac{1}{2}z)\Big]\Big|_{z=0} = \bar{q}(0)\,\gamma^+(i\overleftrightarrow{\partial}^+)^n\, q(0)$$

$n \to 0$: $\hat{J}^\mu_{\rm em}$; $n \to 1$: $\hat{T}^{\mu\nu}_{\rm grav}$

probe $|N\rangle$ by $\hat{J}^\mu_{\rm em}$, $\hat{T}^{\mu\nu}_{\rm grav}$

strong interaction

GFFs of spin-1

- Definition: (Holstein, 2006; Cosyn, Cotogno, Freese, Lorcé, 2019; Cosyn, Freese, Pire, 2019; Polyakov, BDS, 2019)

$$\langle p',\sigma'|\hat{T}^a_{\mu\nu}(x)|p,\sigma\rangle = \Big[2P_\mu P_\nu\Big(-\epsilon'^*\cdot\epsilon\,A^a_0(t) + \frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}A^a_1(t)\Big)$$
$$+2\left[P_\mu(\epsilon'^*_\nu\,\epsilon\cdot P+\epsilon_\nu\,\epsilon'^*\cdot P)+P_\nu(\epsilon'^*_\mu\,\epsilon\cdot P+\epsilon_\mu\,\epsilon'^*\cdot P)\right]\,J^a(t)$$
$$+\frac{1}{2}(\Delta_\mu\Delta_\nu-g_{\mu\nu}\Delta^2)\Big(\epsilon'^*\cdot\epsilon\,D^a_0(t)+\frac{\epsilon'^*\cdot P\,\epsilon\cdot P}{m^2}D^a_1(t)\Big)$$
$$+\Big[\frac{1}{2}(\epsilon_\mu\epsilon'^*_\nu+\epsilon'^*_\mu\epsilon_\nu)\Delta^2-(\epsilon'^*_\mu\Delta_\nu+\epsilon'^*_\nu\Delta_\mu)\,\epsilon\cdot P$$
$$+(\epsilon_\mu\Delta_\nu+\epsilon_\nu\Delta_\mu)\,\epsilon'^*\cdot P-4g_{\mu\nu}\,\epsilon'^*\cdot P\,\epsilon\cdot P\Big]\,E^a(t)$$
$$+\Big(\epsilon_\mu\epsilon'^*_\nu+\epsilon'^*_\mu\epsilon_\nu-\frac{\epsilon'^*\cdot\epsilon}{2}g_{\mu\nu}\Big)\,m^2\,\bar{f}^a(t)$$
$$+g_{\mu\nu}\Big(\epsilon'^*\cdot\epsilon\,m^2\,\bar{c}^a_0(t)\;+\;\epsilon'^*\cdot P\,\epsilon\cdot P\,\bar{c}^a_1(t)\Big)\Big]\,e^{i(p'-p)x}$$

6 conserving

3 non-conserving

- multipole expansion: (Polyakov, BDS, 2019)

$$\langle\hat{T}^{00}_a(0)\rangle = 2m^2\mathcal{E}^a_0(t)\,\delta_{\sigma'\sigma}+\hat{Q}^{kl}\,\Delta^k\Delta^l\,\mathcal{E}^a_2(t)\;,$$
$$\langle\hat{T}^{0j}_a(0)\rangle = i\epsilon^{jkl}\hat{S}^k_{\sigma'\sigma}\Delta^l\,m\,\mathcal{J}^a(t)\;,$$
$$\langle\hat{T}^{ij}_a(0)\rangle = \frac{1}{2}(\Delta^i\Delta^j-\delta^{ij}\vec{\Delta}^2)\mathcal{D}^a_0(t)\,\delta_{\sigma'\sigma}$$
$$+\Big(\Delta^j\Delta^k\hat{Q}^{ik}+\Delta^i\Delta^k\hat{Q}^{jk}-\vec{\Delta}^2\hat{Q}^{ij}-\delta^{ij}\Delta^k\Delta^l\hat{Q}^{kl}\Big)\;\mathcal{D}^a_2(t)$$
$$+\frac{1}{2m^2}(\Delta^i\Delta^j-\delta^{ij}\vec{\Delta}^2)\Delta^k\Delta^l\hat{Q}^{kl}\,\mathcal{D}^a_3(t)$$
$$+\text{non-conserving terms}$$

✤ gravitational multipole form factors

$$\mathcal{E}^a_0(t) = A^a_0(t)-\frac{t}{m^2}\frac{5}{12}A^a_0(t)+\cdots$$
$$\mathcal{E}^a_2(t) = -A^a_0(t)+2J^a(t)-E^a(t)+\cdots$$
$$\mathcal{J}^a(t) = J^a(t)-\frac{t}{4m^2}\Big[J^a(t)-E^a(t)\Big]+\cdots$$
$$\rightarrow\;\mathcal{D}^a_0(t) = -D^a_0(t)+\frac{4}{3}E^a(t)+\cdots$$
$$\mathcal{D}^a_2(t) = -E^a(t)$$
$$\mathcal{D}^a_3(t) = \frac{1}{4}\Big[2D^a_0(t)-2E^a(t)+D^a_1(t)\Big]+\cdots$$

- spin operators, etc. ...

$$\hat{S}^\lambda_{\sigma'\sigma} = \sqrt{S(S+1)}\;C^{S\sigma'}_{S\sigma 1\lambda}$$
$$\hat{Q}^{ij} = \frac{1}{2}\Big[\hat{S}^i\hat{S}^j+\hat{S}^j\hat{S}^i-\frac{2}{3}S(S+1)\delta^{ij}\Big]$$
$$\epsilon^\mu(p,\sigma) = \Big(\frac{\vec{p}\cdot\hat{\epsilon}_\sigma}{m},\hat{\epsilon}_\sigma+\frac{\vec{p}\cdot\hat{\epsilon}_\sigma}{m(m+E)}\vec{p}\Big)$$ (for $S=1$)

GFFs of spin-3/2

Rarita-Schwinger spinor:
$$u^{\mu}=\sum C^{\frac{3}{2}\sigma}_{1\lambda\frac{1}{2}s}u_s(p)\epsilon^{\mu}_{\lambda}$$

- **Definition:** (Cotogno, Lorcé, Lowdon, Morales, 2020; Kim, BDS, 2020)

$$\begin{aligned}\langle \hat{T}^{\mu\nu}_a(0)\rangle = -\bar{u}^{\alpha'}(p')\Bigg[&\frac{P^\mu P^\nu}{m}\left(g_{\alpha'\alpha}F^a_{1,0}(t)-\frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{1,1}(t)\right)\\
&+\frac{(\Delta^\mu\Delta^\nu-g^{\mu\nu}\Delta^2)}{4m}\left(g_{\alpha'\alpha}F^a_{2,0}(t)-\frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{2,1}(t)\right)\\
&+mg^{\mu\nu}\left(g_{\alpha'\alpha}F^a_{3,0}(t)-\frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{3,1}(t)\right)\\
&+\frac{i}{2}\frac{(P^\mu\sigma^{\nu\rho}+P^\nu\sigma^{\mu\rho})\Delta_\rho}{m}\left(g_{\alpha'\alpha}\underbrace{F^a_{4,0}(t)}_{\text{(spin)}}-\frac{\Delta_{\alpha'}\Delta_\alpha}{2m^2}F^a_{4,1}(t)\right)\\
&-\frac{1}{m}(\Delta^\mu g^\nu_{\alpha'}\Delta_\alpha+\Delta^\nu g^\mu_{\alpha'}\Delta_\alpha+\Delta^\mu g^\nu_{\alpha}\Delta_{\alpha'}+\Delta^\nu g^\mu_{\alpha}\Delta_{\alpha'}\\
&-2g^{\mu\nu}\Delta_{\alpha'}\Delta_\alpha-g^\mu_{\alpha'}g^\nu_\alpha\Delta^2-g^\nu_{\alpha'}g^\mu_\alpha\Delta^2)F^a_{5,0}(t)\\
&+m(g^\mu_{\alpha'}g^\nu_\alpha+g^\nu_{\alpha'}g^\mu_\alpha)F^a_{6,0}(t)\Bigg]u^\alpha(p,\sigma)\end{aligned}$$

7 conserving

3 non-conserving

- **multipole expansion:** (Cotogno, Lorcé, Lowdon, Morales, 2020) (Kim, BDS, 2020)

$$\langle \hat{T}^{00}_a(0)\rangle = 2mE\left[\mathcal{E}^a_0(t)\delta_{\sigma'\sigma}+\left(\frac{\sqrt{-t}}{m}\right)^2\hat{Q}^{kl}_{\sigma'\sigma}Y^{kl}_2\mathcal{E}^a_2(t)\right]$$

$$\langle \hat{T}^{0i}_a(0)\rangle = 2mE\left[\frac{\sqrt{-t}}{m}i\epsilon^{ikl}Y^l_1\hat{S}^k_{\sigma'\sigma}\mathcal{J}^a_1(t)+\left(\frac{\sqrt{-t}}{m}\right)^3 i\epsilon^{ikl}Y^{lmn}_3\hat{O}^{kmn}_{\sigma'\sigma}\mathcal{J}^a_3(t)\right]$$

$$\begin{aligned}\langle \hat{T}^{ij}_a(0)\rangle = 2mE\Bigg[&\frac{1}{4m^2}(\Delta^i\Delta^j+\delta^{ij}\Delta^2)D^a_0(t)\delta_{\sigma'\sigma}\\
&+\frac{1}{4m^4}\hat{Q}^{kl}_{\sigma'\sigma}(\Delta^i\Delta^j+\delta^{ij}\Delta^2)\Delta^k\Delta^l D^a_3(t)\\
&+\frac{1}{2m^2}\left(\hat{Q}^{ik}_{\sigma'\sigma}\Delta^j\Delta^k+\hat{Q}^{jk}_{\sigma'\sigma}\Delta^i\Delta^k+\hat{Q}^{ij}_{\sigma'\sigma}\Delta^2-\delta^{ij}\hat{Q}^{kl}_{\sigma'\sigma}\Delta^k\Delta^l\right)D^a_2(t)\\
&+\text{non-conserving terms}\Bigg]\end{aligned}$$

✤ gravitational multipole form factors

$$\mathcal{E}^a_0(t)=F^a_{1,0}(t)+F^a_{3,0}(t)-\frac{t}{m^2}\frac{5}{12}F^a_{1,0}(t)+\cdots$$
$$\mathcal{E}^a_2(t)=-\frac{1}{6}F^a_{1,0}(t)-\frac{1}{6}F^a_{1,1}(t)+\cdots$$
$$\mathcal{J}^a_1(t)=\frac{1}{3}F^a_{4,0}(t)-\frac{1}{3}F^a_{6,0}(t)+\cdots$$
$$\mathcal{J}^a_3(t)=-\frac{1}{6}\left[F^a_{4,0}(t)+F^a_{4,1}(t)\right]+\frac{t}{24m^2}F^a_{4,1}(t)$$
$$D^a_0(t)=F^a_{2,0}(t)-\frac{16}{3}F^a_{5,0}(t)+\cdots$$
$$D^a_2(t)=\frac{4}{3}F^a_{5,0}(t)$$
$$D^a_3(t)=-\frac{1}{6}F^a_{2,0}(t)-\frac{1}{6}F^a_{2,1}(t)+\cdots$$

- **octupole operator:**

$$\begin{aligned}\hat{O}^{ijk}=\frac{1}{6}\Bigg[&\hat{S}^i\hat{S}^j\hat{S}^k+\hat{S}^j\hat{S}^i\hat{S}^k+\hat{S}^k\hat{S}^j\hat{S}^i\\
&+\hat{S}^j\hat{S}^k\hat{S}^i+\hat{S}^i\hat{S}^k\hat{S}^j+\hat{S}^k\hat{S}^i\hat{S}^j\\
&-\frac{6S(S+1)-2}{5}(\delta^{ij}\hat{S}^k+\delta^{ik}\hat{S}^j+\delta^{kj}\hat{S}^i)\Bigg]\end{aligned}$$

- **n-rank irreducible tensors:**

$$Y^{i_1 i_2\ldots i_n}_n(\Omega_p)=\frac{(-1)^n}{(2n-1)!!}p^{n+1}\partial^{i_1}\partial^{i_2}\ldots\partial^{i_n}\frac{1}{p}$$

Static EMT

- Definition (Polyakov, 2003)

$$T^{\mu\nu}(\boldsymbol{r},\sigma',\sigma) = \sum_a T^{\mu\nu}_a(\boldsymbol{r},\sigma',\sigma)$$

$$= \sum_a \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} \langle p',\sigma'|\hat{T}^{\mu\nu}_a(0)|p,\sigma\rangle$$

- energy(mass) densities

$$T^{00}(\boldsymbol{r},\sigma',\sigma) = \varepsilon_0(r)\delta_{\sigma'\sigma} + \varepsilon_2(r)\hat{Q}^{ij}_{\sigma'\sigma}Y^{ij}_2(\Omega_r)$$

- spin density

$$J^i(\boldsymbol{r},\sigma',\sigma) = \sum_a J^i_a(\boldsymbol{r},\sigma',\sigma) = \epsilon^{ijk}r^j \sum_a T^{0k}_a(\boldsymbol{r},\sigma',\sigma)$$

$$\rho_J(r) = -r\frac{d}{dr}\int \frac{d^3\Delta}{(2\pi)^3} e^{-\boldsymbol{\Delta}\cdot\boldsymbol{r}} \mathcal{J}_1(t) \quad \text{(averaged)}$$

(Kim, BDS, 2020)

- pressure and shear forces: ("mechanical properties")

$$T^{ij}(\boldsymbol{r},\sigma',\sigma) = p_0(r)\delta^{ij}\delta_{\sigma'\sigma} + s_0(r)Y^{ij}_2\delta_{\sigma'\sigma} + \left(p_2(r) + \frac{1}{3}p_3(r) - \frac{1}{9}s_3(r)\right)\hat{Q}^{ij}_{\sigma'\sigma}$$

$$+ \left(s_2(r) - \frac{1}{2}p_3(r) + \frac{1}{6}s_3(r)\right) 2\left[\hat{Q}^{ip}_{\sigma'\sigma}Y^{pj}_2 + \hat{Q}^{jp}_{\sigma'\sigma}Y^{pi}_2 - \delta^{ij}\hat{Q}^{pq}_{\sigma'\sigma}Y^{pq}_2\right]$$

$$+ \hat{Q}^{pq}_{\sigma'\sigma}Y^{pq}_2\left[\left(\frac{2}{3}p_3(r) + \frac{1}{9}s_3(r)\right)\delta^{ij} + \left(\frac{1}{2}p_3(r) + \frac{5}{6}s_3(r)\right)Y^{ij}_2\right]$$

(Polyakov, BDS, 2019, Panteleeva, Polyakov, 2020)

✤ radii: (energy, spin, mechanical)

$$\langle r^2_E\rangle = \frac{1}{m}\int d^3r\; r^2\varepsilon_0(r) \qquad \left.\frac{dF_r}{dS_r}\right|_{\text{unp}} > 0$$

$$\langle r^2_J\rangle = \frac{\int d^3r\, r^2\rho_J(r)}{\int d^3r\, \rho_J(r)} \qquad (n=0)$$

$$\langle r^2_n\rangle_{\text{mech}} = \frac{\int d^3r\, r^2\left[p_n(r) + \frac{2}{3}s_n(r)\right]}{\int d^3r\, \left[p_n(r) + \frac{2}{3}s_n(r)\right]}$$

✤ energy deform by spin: (Kim, BDS, 2020)

$$\mathcal{Q}^{ij}_{\sigma'\sigma} = \frac{2}{15}\hat{Q}^{ij}_{\sigma'\sigma}\int d^3r\; r^2\varepsilon_2(r)$$

✤ generalized D-terms: (Panteleeva, Polyakov, 2020)

$$\mathcal{D}_n = m\int d^3r\, r^2 p_n(r) = -\frac{4}{15}m\int d^3r\, r^2 s_n(r)$$

(Kim, BDS, 2020):

$$\mathcal{D}_0 = D_0(0) \quad (<0 \text{ for stability!})$$

$$\mathcal{D}_2 = D_2(0) + \frac{2}{m^2}\int_{-\infty}^{0} dt\, D_3(t)$$

$$\mathcal{D}_3 = -\frac{5}{m^2}\int_{-\infty}^{0} dt\, D_3(t)$$

$p(r)$ and $s(r)$, normal/tangential force, stability conditions

- force acting on the area element $dS = \mathbf{dS_r}\hat{\mathbf{e}}_\mathbf{r} + \mathbf{dS}_\theta\hat{\mathbf{e}}_\theta + \mathbf{dS}_\phi\hat{\mathbf{e}}_\phi$

$$\frac{dF_r}{dS_r} = \delta_{\sigma'\sigma}\left(p_0(r) + \frac{2}{3}s_0(r)\right) + \hat{Q}^{rr}_{\sigma'\sigma}\left(p_2(r) + \frac{2}{3}s_2(r) + p_3(r) + \frac{2}{3}s_3(r)\right),$$ → normal force:

$$\frac{dF_\theta}{dS_r} = \hat{Q}^{\theta r}_{\sigma'\sigma}\left(p_2(r) + \frac{2}{3}s_2(r)\right), \qquad \frac{dF_\phi}{dS_r} = \hat{Q}^{\phi r}_{\sigma'\sigma}\left(p_2(r) + \frac{2}{3}s_2(r)\right),$$ → tangential force:

- stability condition (von Laue 1911): $\int d^3r\, p_n(r) = 0$

equilibrium relation ($\partial_\mu \hat{T}^{\mu\nu} = 0$):

$$\frac{2}{3}\frac{ds_n(r)}{dr} + 2\frac{s_n(r)}{r} + \frac{dp_n(r)}{dr} = 0$$

↓

$$\int dr\, r^N s_n(r) = -\frac{3(N+1)}{2(N-2)}\int dr\, r^N p_n(r)$$

(for $N > -1$)

(Goeke, et al, 2007)

- local stability condition : $\left.\frac{dF_r}{dS_r}\right|_{\text{unp}} = p_0(r) + \frac{2}{3}s_0(r) \geq 0$

(unpolarized / spherically symmetric hadron)

(Polyakov & Schweitzer, 2018)

↓

- D-term(unp): $\mathcal{D}_0 = m\int d^3r\, r^2 p_0(r) = -\frac{4}{15}m\int d^3r\, r^2 s_0(r) \leq 0$

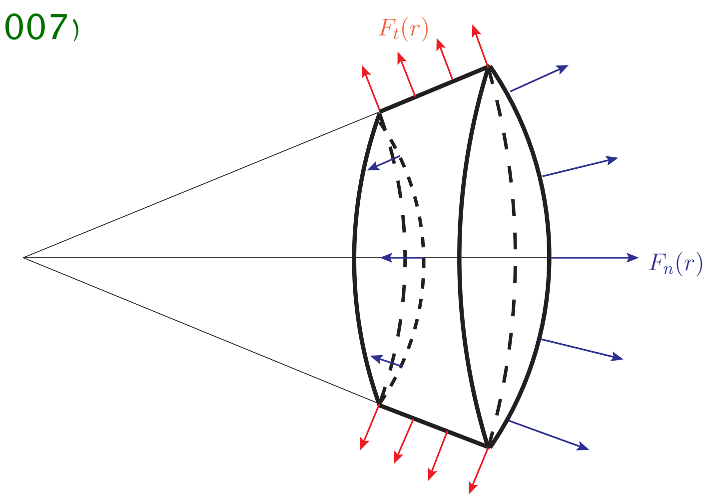
- dispersion relations (Polyakov 2003; Teryaev, 2005; Anikin, Teryaev , 2007; Diehl, Ivanov, 2007)

$$\mathcal{H}(\xi,t) = \int_{-1}^{1} dx\left(\frac{1}{\xi - x - i0} - \frac{1}{\xi + x - i0}\right) H(x,\xi,t)$$

$$\text{Re}\mathcal{H}(\xi,t) = \Delta(t) + \frac{1}{\pi}\text{p.v.}\int_0^1 d\xi'\ \text{Im}\mathcal{H}(\xi',t)\left(\frac{1}{\xi-\xi'} - \frac{1}{\xi+\xi'}\right)$$

$$\Delta(t) = \frac{4}{5}\sum_q e_q^2 D^q(t) + \sum_q e_q^2 d_3^q(t) + \ldots$$

(Gegenbauer polynomials)

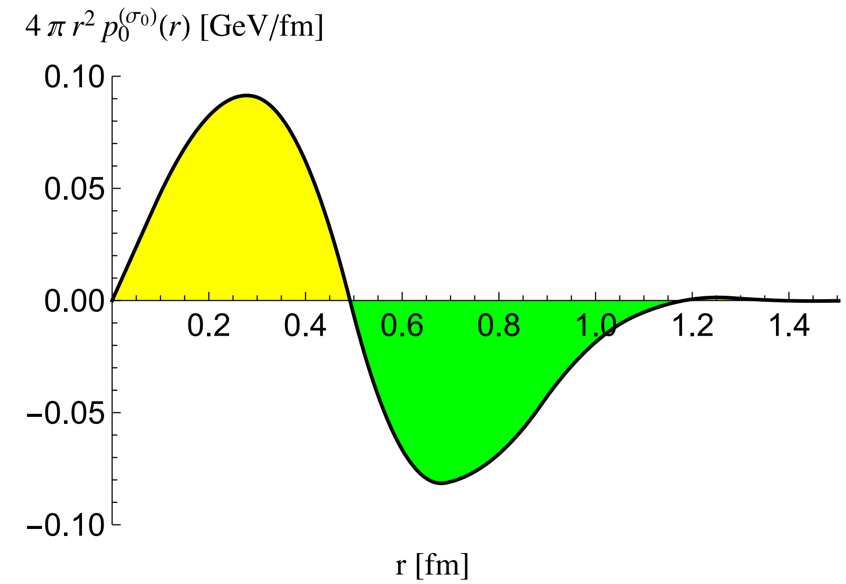


$p(r)$ and $s(r)$: spin 1, 3/2

- equilibrium relation: $\frac{2}{3}\frac{ds_n(r)}{dr} + 2\frac{s_n(r)}{r} + \frac{dp_n(r)}{dr} = 0$

- solution in general (Polyakov & Schweitzer, 2018):

$$p_n(r) = \frac{1}{6m}\partial^2\tilde{\mathcal{D}}_n(r) = \frac{1}{6m}\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\tilde{\mathcal{D}}_n(r),$$
$$s_n(r) = -\frac{1}{4m}r\frac{d}{dr}\frac{1}{r}\frac{d}{dr}\tilde{\mathcal{D}}_n(r),$$



for ρ meson in a quark model (BDS, Dong, 2020)

- solution for spin 1, 3/2, (BDS, Dong, 2020; Kim, BDS, 2020)

$$\tilde{\mathcal{D}}_0(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} D_0(t),$$
$$\tilde{\mathcal{D}}_2(r) = \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} D_2(t) + \frac{1}{m^2}\left(\frac{d}{dr}\frac{d}{dr} - \frac{2}{r}\frac{d}{dr}\right)\int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} D_3(t),$$
$$\tilde{\mathcal{D}}_3(r) = -\frac{2}{m^2}\left(\frac{d}{dr}\frac{d}{dr} - \frac{3}{r}\frac{d}{dr}\right)\int \frac{d^3\Delta}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} D_3(t)$$

→ (valid for $J \geq 2$?)

- inverse to get $D_n(t)$

$$D_0(t) = 6m\int d^3r\, \frac{j_0(r\sqrt{-t})}{t} p_0(r),$$
$$D_2(t) = 2m\int d^3r\, \frac{j_2(r\sqrt{-t})}{t}\left(2s_2(r) - \frac{1}{2}p_3(r) + \frac{2}{3}s_3(r)\right),$$
$$D_3(t) = 4m^3\int d^3r\, \frac{j_4(r\sqrt{-t})}{t^2}\left(\frac{1}{2}p_3(r) + \frac{5}{6}s_3(r)\right)$$

→

✤ generalized D-terms (Kim, BDS, 2020)

$$\mathcal{D}_0 = D_0(0) \quad (< 0 \text{ for stability of unp hadron!})$$
$$\mathcal{D}_2 = D_2(0) + \frac{2}{m^2}\int_{-\infty}^{0} dt\, D_3(t)$$
$$\mathcal{D}_3 = -\frac{5}{m^2}\int_{-\infty}^{0} dt\, D_3(t)$$

FREE massive vector particle

(Holstein, 2006; Polyakov, BDS, 2019)

Table II: The free theory values of the total EMT FFs.

EMT FFs	$\mathcal{E}_0(t)$	$\mathcal{E}_2(t)$	$\mathcal{J}(t)$	$\mathcal{D}_0(t)$	$\mathcal{D}_2(t)$	$\mathcal{D}_3(t)$
free theory	1	0	1	$\frac{1}{3}-4h$	-1	0

- Proca Lagrangian + a non-minimal term (?):

$$S_{\rm grav} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu + \frac{1}{2}hRA_\mu A^\mu\right) \longrightarrow$$

- conformal transformation: (Carroll, 2004; Dabrowski, 2009)

$$\tilde{g}_{\mu\nu}(x) = \Omega^2(x) g_{\mu\nu}(x), \quad \widetilde{m} = \Omega^{-1} m,$$
$$\widetilde{A}_\mu = A_\mu, \quad \widetilde{A}^\mu = \tilde{g}^{\mu\nu}\widetilde{A}_\nu = \Omega^{-2}A^\mu,$$
$$\widetilde{U}_{\mu\nu} = U_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

✤ all GFFs are t-independent: free of interaction

✤ $D_\pi = -1 \rightarrow -\frac{1}{3}$: week interaction matters

✤ $D_{\text{fermion}} = 0 \rightarrow \neq 0$: interaction!

✤ $D_\rho \leq 0 \overset{?}{\leftrightarrow} h \geq \frac{1}{12}$: seems NOT allowed ...

Pagels, 1966; Novikov, Shifman, 1980;
Hudson,Schweitzer, 2017;
Polyakov & Schweitzer, 2018, etc.

- choices of S: conformal invariance (CI) (or not)

$$S^0_{\rm grav} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu\right), \quad \text{(CI)}$$

$$S_{\rm grav} = \int d^4x\sqrt{-g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m^2 A_\mu A^\mu + \frac{1}{2}hRA^2_\mu\right), \quad \text{(not CI for } h\neq 0) \longrightarrow$$ Ricci scalar term breaks CI !

$$S^2_{\rm grav} = \int d^4x\sqrt{-g}\left(\frac{1}{2}A_\mu\Box A_\mu - \frac{1}{2}A_\mu\nabla^\mu\nabla^\nu A_\nu + \frac{1}{2}m^2 A^2_\mu\right), \quad \text{(not CI)}$$ (with $\Box = g^{\mu\nu}\nabla_\mu\nabla_\nu$)

$$S^3_{\rm grav} = \int d^4x\sqrt{-g}\left(\frac{1}{2}A_\mu\Box A_\mu - \frac{1}{2}A_\mu\nabla^\mu\nabla^\nu A_\nu + \frac{1}{2}m^2 A^2_\mu - \frac{1}{2}R_{\mu\nu}A^\mu A^\nu\right), \quad \text{(CI and give same } D_0 \text{ as } S^0_{\rm grav}!)$$

- Riemann tensor $R_{\mu\nu\rho\sigma}$, Weyl tensor $C_{\mu\nu\rho\sigma}$, etc., but NO suitable mass-dim-4 terms!

ρ meson GFFs by a quark model

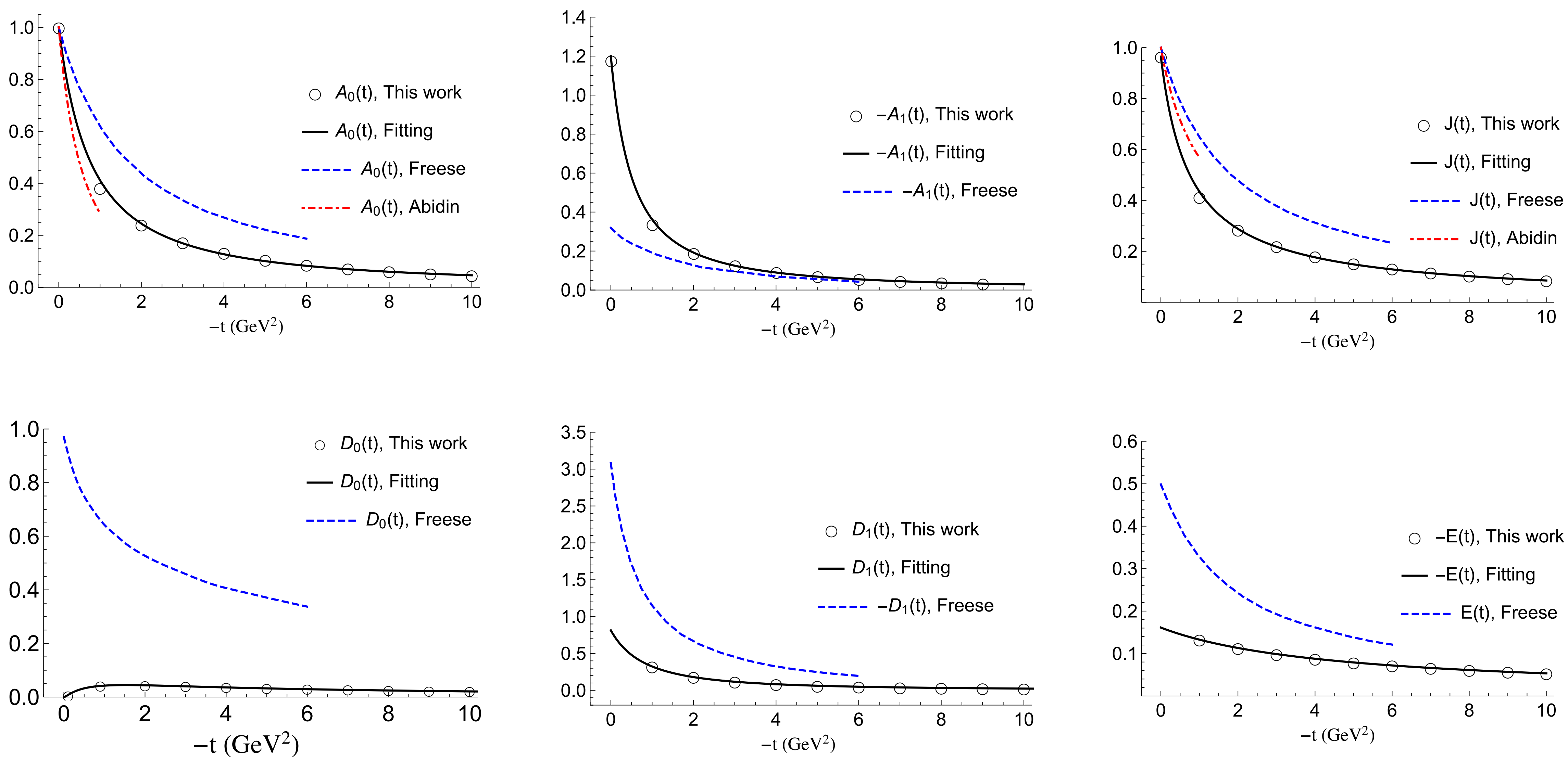
deuteron GPDs: (Berger, Cano, Diehl, Pire, 2001)

sum rules: (Cosyn, Freese, Pire, 2019. etc.)

$D_\rho = -0.21 < 0$ ✔

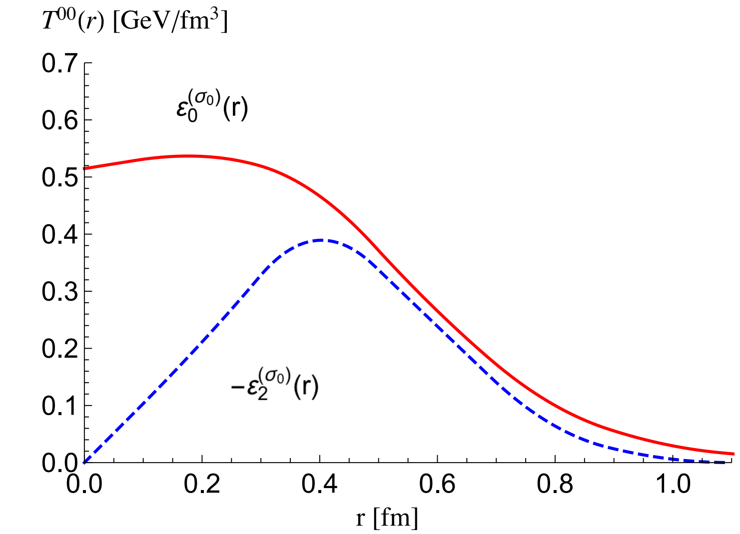
	$\sqrt{\langle r^2\rangle_{\text{mass}}}$	$\sqrt{\langle r^2\rangle_{\text{elec.}}}$	$\mathcal{Q}_{\text{mass}}$	$\mathcal{Q}_{\text{elec.}}$
AdS/QCD [30]	0.46	0.73		
NJL [31], Briet frame	0.45	0.67	−0.0224	−0.0200
NJL [31], Light Cone	0.32	0.45		
this work, Briet frame	0.53	0.72	−0.0322	−0.0212
this work, Light Cone	0.41			

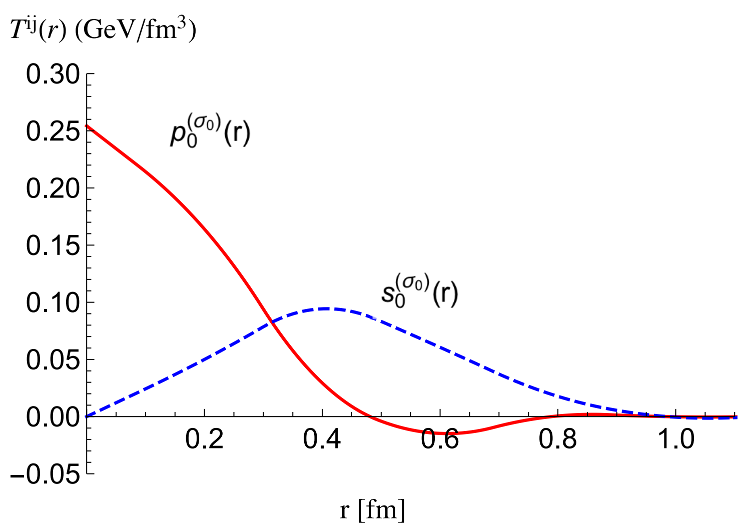
(Abidin et al, 2008; Freese et al, 2019; BDS, Dong, 2019)



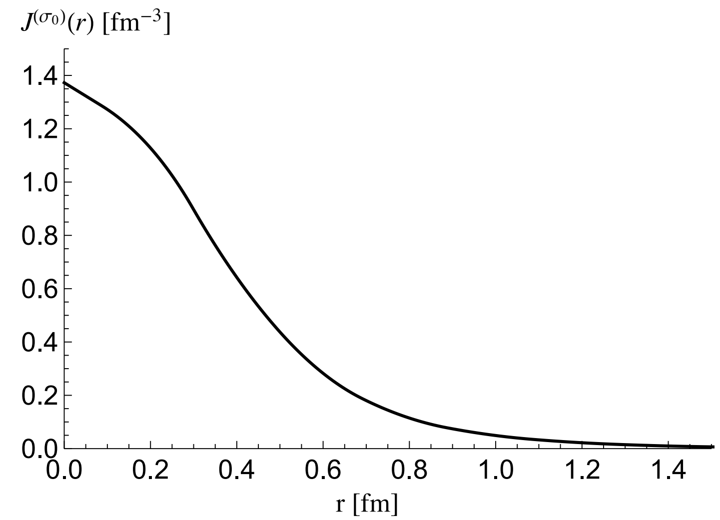
ρ meson densities by a quark model (BDS, Dong, 2017, 2020)

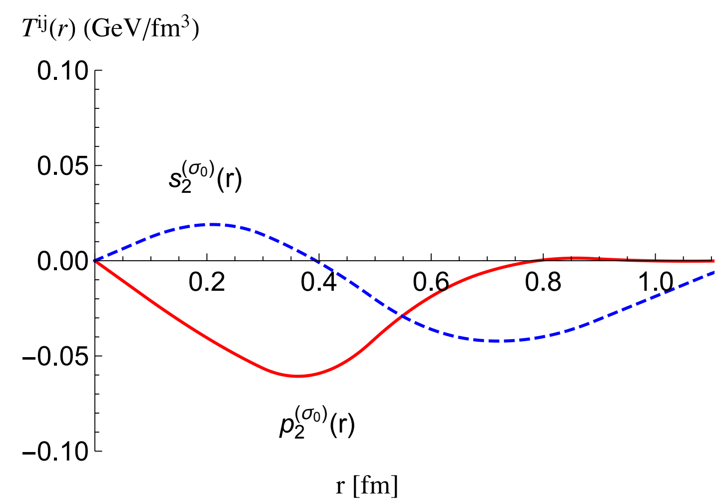
(energy/mass)



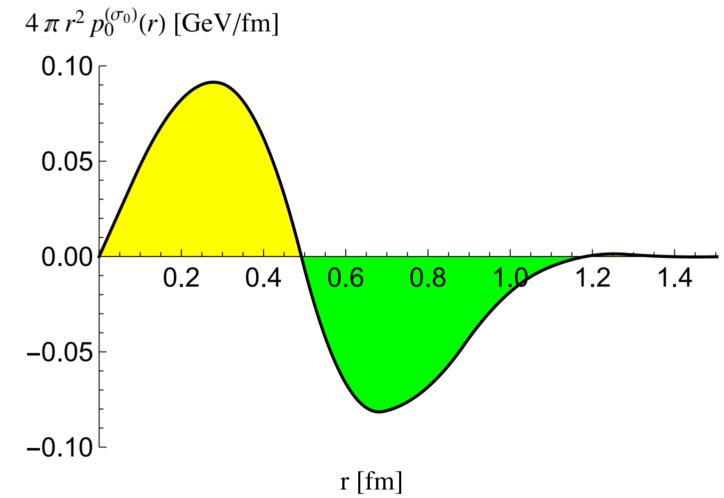


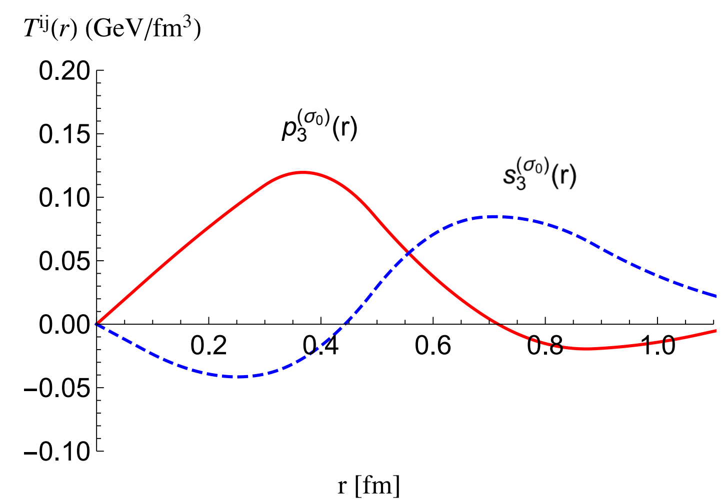
(spin)





(pressure $p_0(r)$)





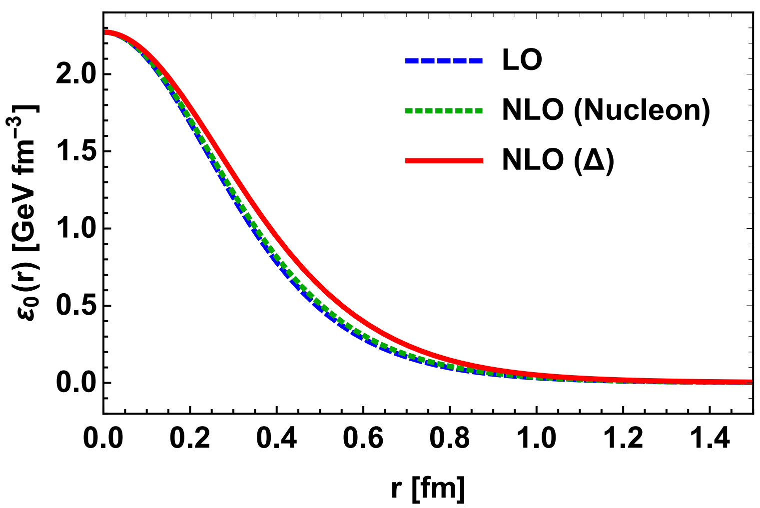
$$\frac{2}{3}\frac{ds_n(r)}{dr} + 2\frac{s_n(r)}{r} + \frac{dp_n(r)}{dr} = 0$$

$$\int d^3r\, p_n(r) = 0$$

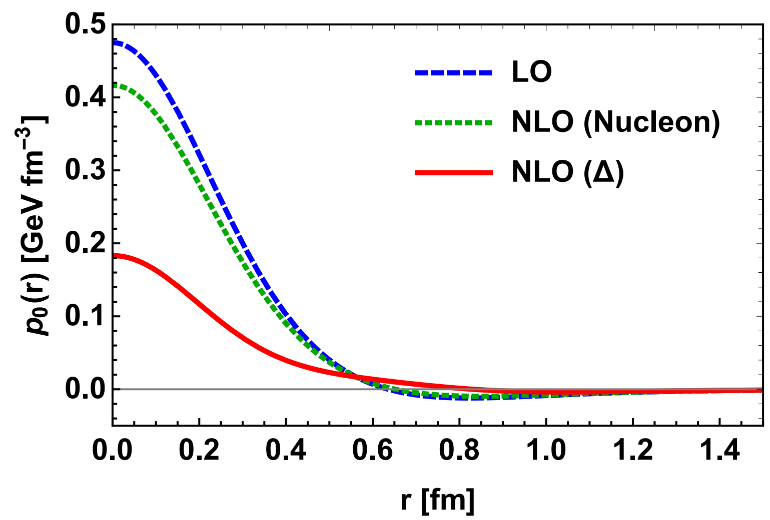
$$\left.\frac{dF_r}{dS_r}\right|_{\rm unp} = p_0(r) + \frac{2}{3}s_0(r) \geq 0$$

Δ densities by SU(2) Skyrme model (Kim, BDS, 2020)

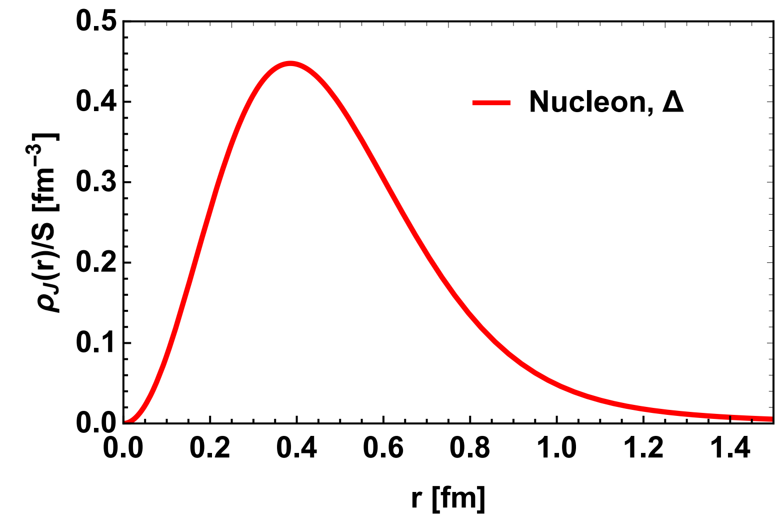
(energy/mass)



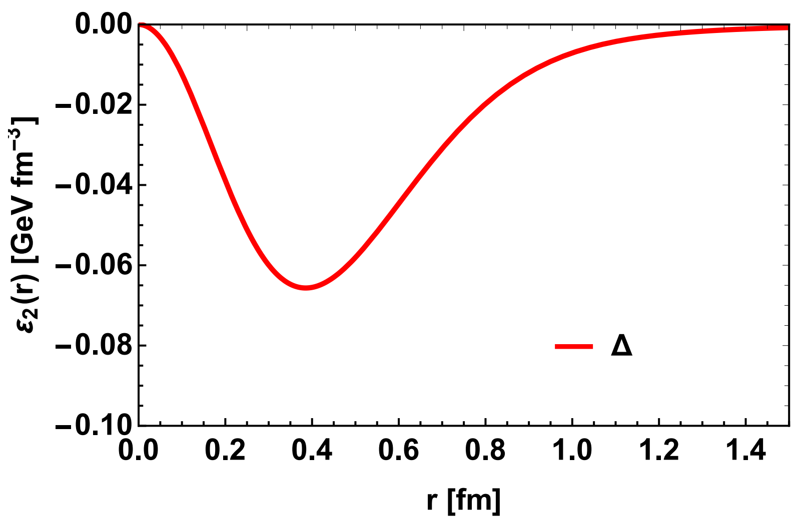
(pressure & shear forces: "mechanical")

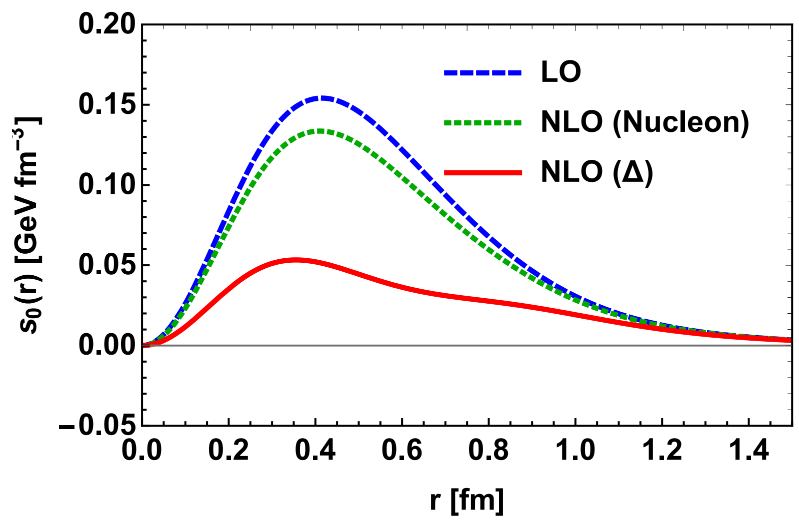


(spin)



$$\langle r_J^2 \rangle_{N,\Delta} = 0.92\,\mathrm{fm}^2$$



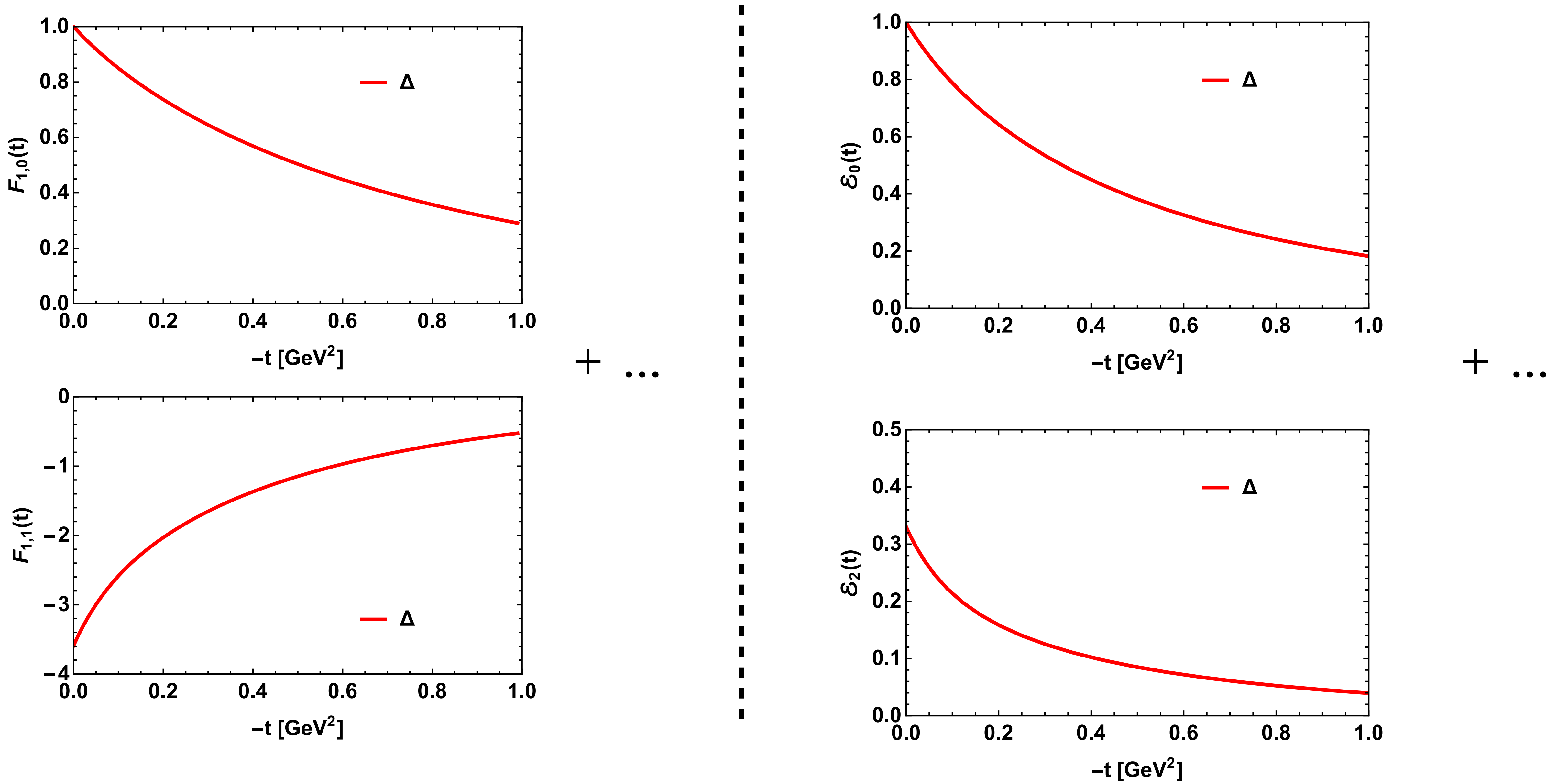


$$\langle r_E^2 \rangle = 0.54\,\mathrm{fm}^2\ (\mathrm{LO})$$
$$\langle r_E^2 \rangle = 0.57\,\mathrm{fm}^2\ (\mathrm{NLO,\ Nucleon})$$
$$\langle r_E^2 \rangle = 0.64\,\mathrm{fm}^2\ (\mathrm{NLO},\ \Delta)$$
$$\mathcal{Q}^{ij}_{\sigma'\sigma} = -0.0181\ Q^{ij}_{\sigma'\sigma}\mathrm{GeV}\cdot\mathrm{fm}^2$$

$$\langle r_0^2 \rangle_{\mathrm{mech}} = 0.61\,\mathrm{fm}^2\ (\mathrm{LO})$$
$$\langle r_0^2 \rangle_{\mathrm{mech}} = 0.63\,\mathrm{fm}^2\ (\mathrm{NLO,\ Nucleon})$$
$$\langle r_0^2 \rangle_{\mathrm{mech}} = 0.85\,\mathrm{fm}^2\ (\mathrm{NLO},\ \Delta)$$
$$\langle r_3^2 \rangle_{\mathrm{mech}} = 0.33\,\mathrm{fm}^2$$

$$\mathcal{D}_0^{\Delta} = -3.53 < 0\ (\mathrm{stable!})$$ ✔
$$\mathcal{D}_0^{N} = -3.63$$
$$\mathcal{D}_2 = 0$$
$$\mathcal{D}_3 = -0.50$$

Δ GFFs/GMFFs by SU(2) Skyrme model (Kim, BDS, 2020)



larg-N_c behaviors: $\mathcal{E}_0(t) \sim \mathcal{O}(N_c^0)$, $\mathcal{E}_2(t) \sim \mathcal{O}(N_c^0)$, $\mathcal{J}_0(t) \sim \mathcal{O}(N_c^0)$, $\mathcal{J}_3(t) \sim \mathcal{O}(N_c^0)$,
(GMFFs) $D_0(t) \sim \mathcal{O}(N_c^2)$, $D_2(t) \sim \mathcal{O}(N_c^0)$, $D_3(t) \sim \mathcal{O}(N_c^2)$ ✔

Trace Anomaly contribution to Hydrogen atom mass

- trace of QED EMT:

$$T^{\mu}_{\mu} = (1+\gamma_m)\, m_0 \bar{\Psi}\Psi + \frac{\beta(e)}{2e}\left[F^{\mu\nu}F_{\mu\nu}\right]_R$$

- hydrogen mass at LO (omitting proton mass / Coulomb potential):

$$M_{H,0} = \frac{\langle H \left| \int d^3x\ m_0 \bar{\Psi}(x)\Psi(x) \right| H\rangle}{\langle H|H\rangle} = m \int d^3x\ \varphi_0^{\dagger}(x)\gamma^0\varphi_0(x)$$

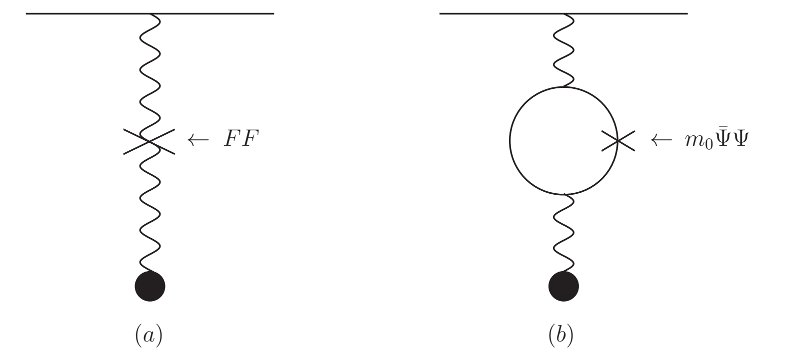
$$\rightarrow M_{H,0} - m = m\sqrt{1-\alpha_{em}^2} - m \approx -13.6\text{eV} \quad \text{(ground state energy)}$$

- subtraction scheme: (Adler et al, 1977; Rodini et al 2020; Metz et al, 2020)

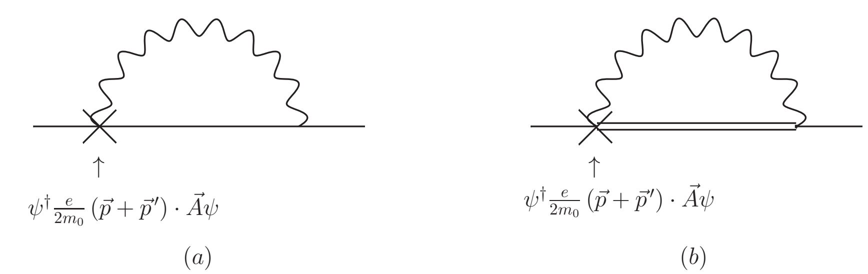
$$\langle e \left| [F^{\mu\nu}(x)F_{\mu\nu}(x)]_R \right| e\rangle = 0$$

$$\langle \gamma \left| [F^{\mu\nu}(x)F_{\mu\nu}(x)]_R \right| \gamma\rangle = \left\langle \gamma \left| Z_3^{-1} F^{\mu\nu}(x)F_{\mu\nu}(x) \right| \gamma\right\rangle_{\text{Tree}}$$

- NLO corrections: (BDS, Sun, Zhou, 2020)



vacuum polarization



self-energy (free, nonrelativistic)

self-energy (bound state)

$\langle F^2\rangle$ **contributes:**

1, most mass splittings in the Charmonium(-like) States:

W. Sun, Y. Chen, P. Sun, Y.-B. Yang, 2012.06228

2, most of the nucleon mass (First Lattice for QCD EMT trace anomaly):

F. He, P. Sun, Y.-B. Yang, 2101.04942

First two terms: (BDS, Sun, Zhou, 2020)

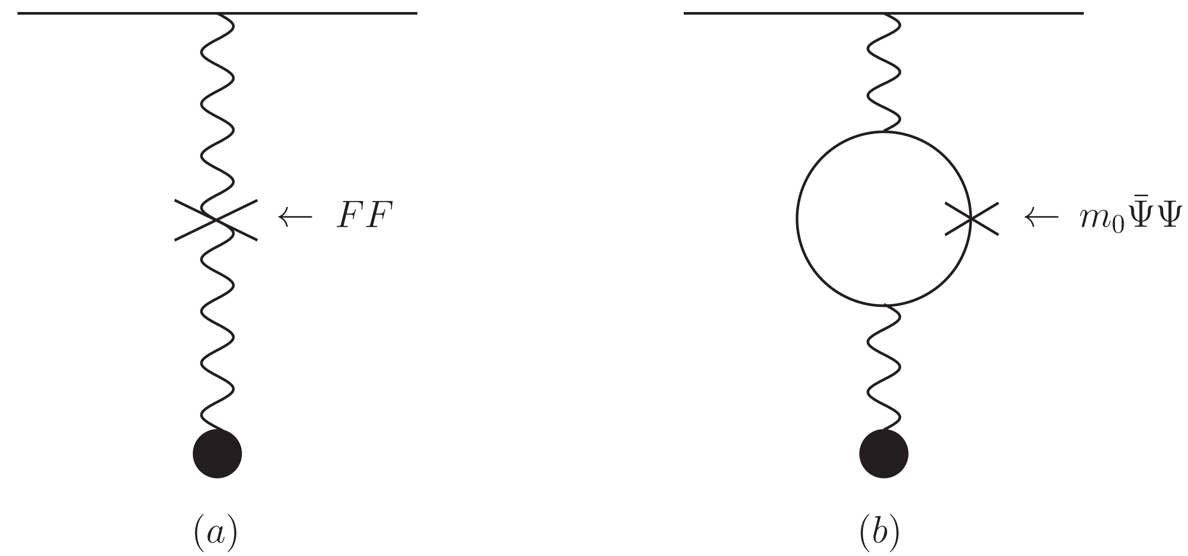


FIG. 1: Trace anomaly contribution(diagram a) and the vacuum polarization diagram with the mass operator insertion(diagram b). Black dots represent the interaction with Coulomb potential.

- in Coulomb gauge:

$$\text{FIG. 1}(a) = \frac{\left\langle H \left| \int d^3x \frac{\beta}{2e} \left[F^{\mu\nu}(x)F_{\mu\nu}(x)\right]_R \mathcal{T} e^{-i\int d^4y H_I(y)} \right| H \right\rangle}{\langle H|H\rangle}$$

$$2\times \text{FIG. 1}(b) = 8\alpha_{em}^2 \int d^3y \int \frac{d^3q}{(2\pi)^3} \frac{e^{i\vec{q}\cdot\vec{y}}}{\vec{q}^2+i\epsilon} \int_0^1 da \frac{a(1-a)m^2}{m^2+a(1-a)\vec{q}^2} \left[\bar{\varphi}_0(y)\gamma^0\varphi_0(y)\right]$$

$$\text{FIG. 1}(a)+2\times\text{FIG. 1}(b) \approx \frac{-4\alpha_{em}^2}{15m^2}\varphi_0^\dagger(0)\varphi_0(0)$$ (Weinberg, 1994; Peskin, 1995; et al)

under non-relativistic limit: $|\vec{q}| \ll m$

Lamb shift by vacuum polarization effect!

self-energy corrections (free):

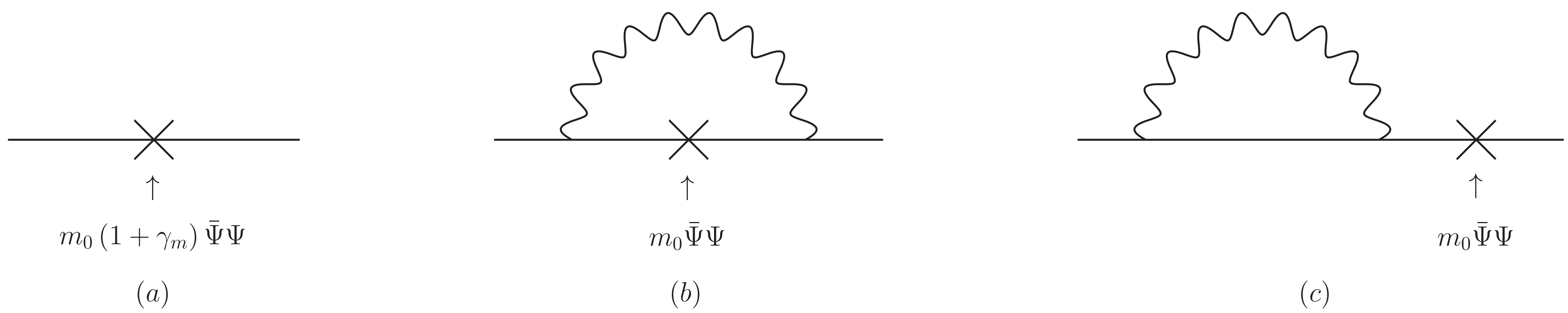


FIG. 2: NLO corrections to the electron mass term.

$$\begin{aligned}
Fig.2(a) &= \gamma_m m_0 + m - \delta m = \gamma_m m_0 + m - \frac{\alpha_{em}}{2\pi} m_0 \int_0^1 da(2-a)\ln\frac{a\Lambda^2}{(1-a)^2 m_0^2} \\
Fig.2(b) &= \frac{\alpha_{em}}{2\pi} m_0 \int_0^1 da \left\{ 2\ln\frac{a\Lambda^2}{(1-a)^2 m_0^2} - \frac{2(2-a)}{(1-a)} \right\} \\
2\times Fig.2(c) &= \frac{\alpha_{em}}{2\pi} m_0 \int_0^1 da \left\{ -a\ln\frac{a\Lambda^2}{(1-a)^2 m_0^2} + \frac{2a(2-a)}{(1-a)} \right\}
\end{aligned}$$

$$\longrightarrow \quad \frac{\langle e \left| (1+\gamma_m)\, m_0 \int d^3x \bar{\Psi}(x)\Psi(x) \right| e \rangle}{\langle e|e \rangle} = m$$

physical mass of a free electron

An all order proof: Callan-Symanzik equation

(Adler, Collins, Duncan, 1977):

self-energy corrections (in bound state): (BDS, Sun, Zhou, 2020; Weinberg, 1994, Chapter 14)

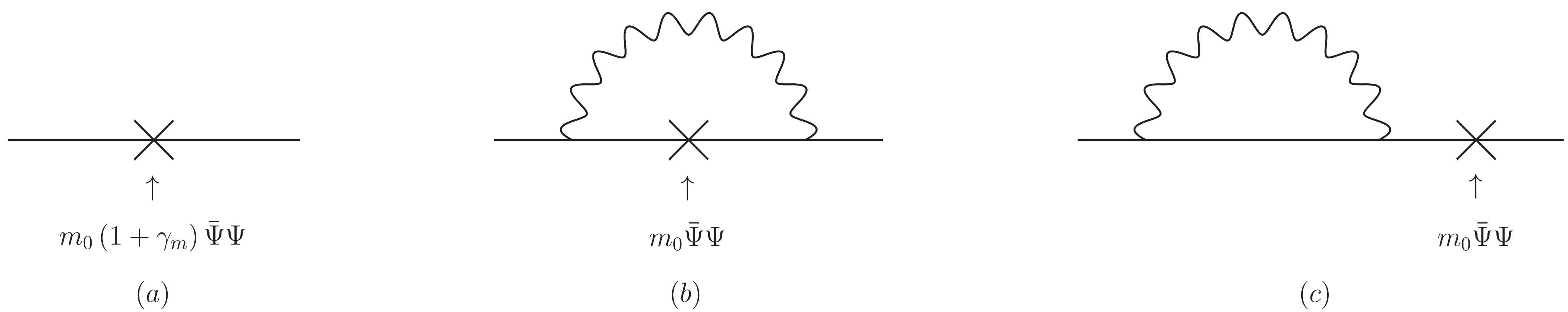


FIG. 2: NLO corrections to the electron mass term.

spinor in nonrelativistic limit:

$$\Psi \approx e^{-im_0 t}\frac{1}{\sqrt{2}}\begin{pmatrix}(1-\frac{\vec{\sigma}\cdot(\vec{p}-e\vec{A})}{2m_0})\psi \\ (1+\frac{\vec{\sigma}\cdot(\vec{p}-e\vec{A})}{2m_0})\psi\end{pmatrix}$$

- NRQED:

$$\mathcal{L} = \psi^\dagger\left(i\partial^0 - eA^0 - \frac{\vec{p}^2}{2m_0} + \frac{e}{2m_0}(\vec{p}'+\vec{p})\cdot\vec{A} - \frac{e^2}{2m_0}\vec{A}^2 - (1+O(\alpha_{em}))\frac{ie}{2m_0}\sigma\cdot[(\vec{p}-\vec{p}')\times\vec{A}]\right)\psi + \ldots$$

$$\text{FIG. } 2(b) + 2\times\text{FIG. } 2(c) = \frac{\left\langle e\left|m_0\int d^3x\left[\bar{\Psi}_R(x)\Psi_R(x) - \Psi_R^\dagger(x)\Psi_R(x)\right]\right|e\right\rangle}{\langle e|e\rangle}$$

$$\approx \frac{\left\langle e\left|\int d^3x\left\{\psi^\dagger\left[\frac{e}{2m_0}(\vec{p}'+\vec{p})\cdot\vec{A} - \frac{\vec{p}^2}{2m_0} - \frac{e^2}{2m_0}\vec{A}^2 - \frac{ie}{2m_0}\sigma\cdot[(\vec{p}-\vec{p}')\times\vec{A}]\right]\psi\right\}\right|e\right\rangle}{\langle e|e\rangle}$$

A trick: $Z_2 = Z_3$
(vector current conservation)

dipole vertex ✔

same as LO ✘
(Weinberg, 1994; et al)

spin-orbital term $\to 0$
$(\vec{p}' \approx \vec{p})$ ✘

self-energy corrections (in bound state):

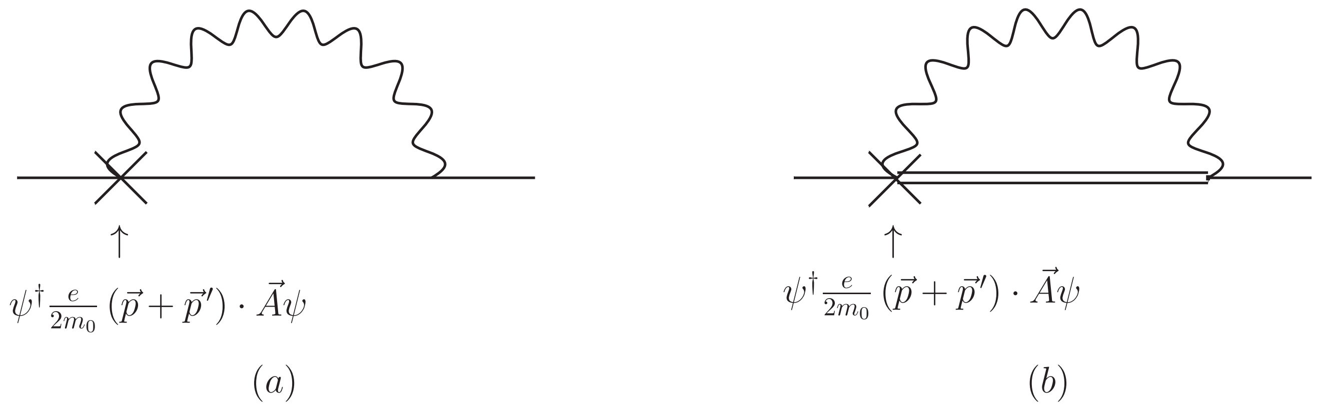


FIG. 3: Part of the NLO correction to the electron mass term in NRQED in vacuum(diagram a), and that in the bound state(diagram b).

- electron propagator: $\sum_M \frac{\varphi_M(x)\varphi_M^\dagger(y)}{\Delta E_M + i\epsilon}$
- photon propagator: $\frac{1}{k^2+i\epsilon} \to \frac{1}{k^2+i\epsilon} - \frac{1}{k^2-\mu^2+i\epsilon}$

(scale cut off $\mu < m$)

- energy shift for ground state:

$$\mathrm{F}ig.3(b) - \mathrm{F}ig.3(a) \approx \frac{4\alpha_{em}^2}{3m^2}\varphi_0^\dagger(0)\varphi_0(0)\left[\ln\frac{\mu}{2\Delta E} + \frac{5}{6}\right]$$

= Bethe 1947's result! (with $\mu = m$)

Conclusion:

Trace anomaly contribution = Lamb shift (part of)!

(Lamb, et al, 1947)

Summary

- Energy Momentum Tensor(EMT), and the gravitational form factors(GFFs) defined by the matrix elements of EMT. The relation (sum-rules) between GFFs and GPDs, and the fundamental properties (mass, spin, D-term & pressure and shear forces).

- Model estimations for spin-1 ρ meson and spin-3/2 Δ isobar.

- QED EMT Trace Anomaly contribution to Hydrogen atom mass, which turns out to be part of Lamb shift.

Thanks! & Happy New Year!