# Leading and higher twist TMDs in a spectator model

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# OUTLINE

# Foundations

- The quark-quark corrector

$$\Phi(x,P,S) = \int \frac{d^4\xi}{(2\pi)^4} \exp^{ip\cdot\xi} \langle PS|\overline{\Psi}(0)\Psi(\xi)|PS\rangle,$$

$$= \frac{1}{(2\pi)^4}\sum_X \int \frac{d^3 \boldsymbol{P}_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^{(4)}(P-p-P_X)\langle PS|\overline{\Psi}(0)|X\rangle\langle X|\Psi(0)|PS\rangle,$$

- In the spectator model, the sum of all possible intermediate states in the definition of the correlator is effectively replaced by the spectator diquark

$$\Phi(x,P,S) = \frac{1}{(2\pi)^4}\sum_X \int \frac{d^3 \boldsymbol{P}_X}{(2\pi)^3 2P_X^0}(2\pi)^4\delta^{(4)}(P-p-P_X)\langle PS|\overline{\Psi}(0)|X\rangle\langle X|\Psi(0)|PS\rangle,$$

$$= \int \frac{d^4 P_X}{(2\pi)^4}(2\pi)\delta(P_X^2 - M_X^2)\theta(P_X^+)\delta^{(4)}(P_X-(P-p))\langle PS|\overline{\Psi}(0)|P_X\rangle\langle P_X|\Psi(0)|PS\rangle,$$

$$= \frac{1}{(2\pi)^3}\delta((P-p)^2 - M_X^2)\theta(P^+ - p^+)\langle PS|\overline{\Psi}(0)|P-p\rangle\langle P-p|\Psi(0)|PS\rangle.$$

# Foundations

- A more common used quark-quark correlator (at first leading order)

$$\Phi(x,\boldsymbol{p}_T,S) = \int dp^- \Phi(x,P,S)\Big|_{p^+=xP^+},$$
$$= \frac{1}{(2\pi)^3}\frac{1}{2(1-x)P^+}\overline{\mathcal{M}}^{(0)}(S)\mathcal{M}^{(0)}(S)\Big|_{p^2=\tau(x,\boldsymbol{p}_T),p^+=xP^+},$$

- The nucleon-quark-diquark scattering amplitude

$$\mathcal{M}^{(0)} = \langle P-p|\Psi(0)P\rangle = \begin{cases} \frac{i}{\not p - m}\Upsilon_s U(P,S) & \text{scalar diquark,} \\ \frac{i}{\not p - m}\varepsilon_\mu^*(P-p,\lambda_a)\Upsilon_a^\mu U(P,S) & \text{axial-vector diquark,} \end{cases}$$

$$\mathcal{M}^{(1)}(S) = \begin{cases} \int \frac{d^4 l}{(2\pi)^4}\frac{ie_q\Gamma_s^+(\not p-\not l+m)\Upsilon_s U(P,S)}{-(D_1+i\varepsilon)(D_2-i\varepsilon)(D_3+i\varepsilon)(D_4+i\varepsilon)} & \text{scalar diquark,} \\ \int \frac{d^4 l}{(2\pi)^4}\frac{ie_q\varepsilon_\sigma^*(P-p,\lambda_a))\Gamma_{a,+}^{\nu\sigma}(\not p-\not l+m)d_{\mu\nu}(P-p+l)\Upsilon_a^\mu U(P,S)}{-(D_1+i\varepsilon)(D_2-i\varepsilon)(D_3+i\varepsilon)(D_4+i\varepsilon)} & \text{axial-vector diquark,} \end{cases}$$

# Foundations

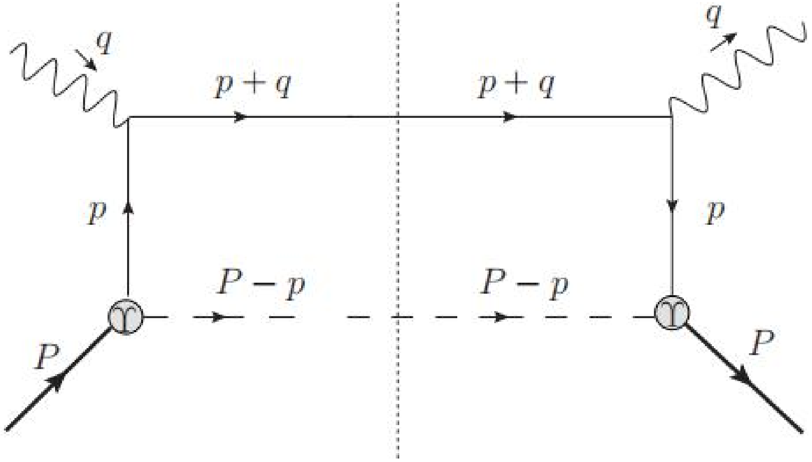


Figure 1: Cut diagram for $\mathcal{M}^{(0)}$

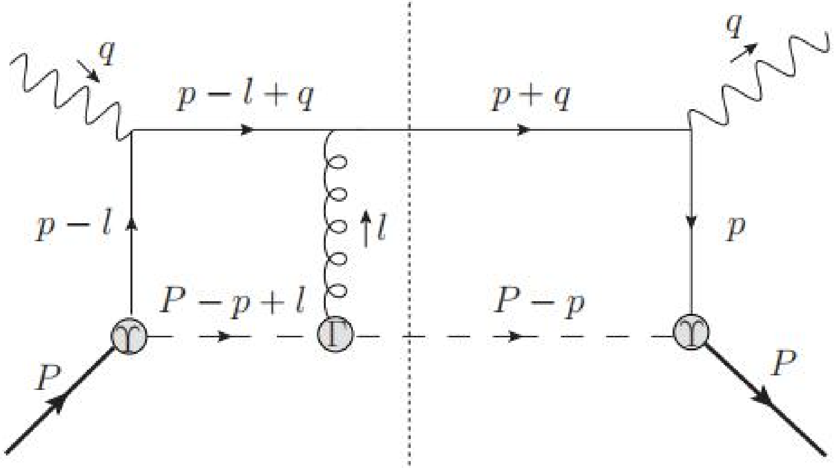


Figure 2: Cut diagram for $M^{(1)}$

# Foundations

- When summing over all polarization states $d^{\mu\nu} = \sum_{\lambda_a} \varepsilon_\mu^* \varepsilon_\nu$:

$$d^{\mu\nu}(P-p) = \begin{cases} -g^{\mu\nu} + \frac{(P-p)^\mu n_-^\nu + (P-p)^\nu n_-^\mu}{(P-p)\cdot n_-} - \frac{M_a^2}{[(P-p)\cdot n_-]^2} n_-^\mu n_-^\nu \\ -g^{\mu\nu} + \frac{(P-p)^\mu (P-p)^\nu}{M_a^2} \\ -g^{\mu\nu} + \frac{P^\mu P^\nu}{M_a^2} \\ -g^{\mu\nu} \end{cases}$$

- The nucleon-quark-diquark vertex

$$\Upsilon_s(p^2) = g_s(p^2)\mathbf{1}(\text{identity matrix}), \quad \Upsilon_a^\mu(p^2) = \frac{g_a(p^2)}{\sqrt{2}}\gamma^\mu\gamma^5,$$

- The coupling factor of the nucleon-quark-diquark vertex

$$g_X(p^2) = g_X \frac{p^2 - m^2}{|p^2 - \Lambda_X^2|^2} = g_X \frac{(p^2 - m^2)(1-x)^2}{(\mathbf{p}_T^2 + L_X^2(\Lambda_X^2))^2} \quad (X = s \text{ or } a),$$

# Foundations

- $D_i$ in the denominator are propagators

$$D_1 = l^2 - m_g^2,$$

$$D_2 = l^+,$$

$$D_3 = (p-l)^2 - m^2,$$

$$D_4 = (P-p+l)^2 - M_X^2,$$

- The vertex between the gluon and the scalar or axial-vector diquark

$$\Gamma_s^\mu = ie_s(2P-2p+l)^\mu,$$

$$\Gamma_{a,\mu}^{\nu\sigma} = -ie_a[(2P-2p+l)_\mu g^{\nu\sigma} - (P-p+l)^\sigma g_\mu^\nu - (P-p)^\nu g_\mu^\sigma],$$

# Formulas

- For the twist-2 case, there are eight TMDs, six for T-evens and two for T-odds (the red ones)

| $p \backslash N$ | $U$ | $L$ | $T$ |
|---|---|---|---|
| $U$ | $f_1$ | | $f_{1T}^{\perp}$ |
| $L$ | | $g_{1L}$ | $g_{1T}$ |
| $T$ | $h_1^{\perp}$ | $h_{1L}^{\perp}$ | $h_{1T}(h_1),\ h_{1T}^{\perp}$ |

Table 1: TMDs at twist-2

$$\Phi^{[\gamma^+]} = f_1 - \frac{\epsilon_T^{ij} p_{Ti} S_{Tj}}{M} f_{1T}^{\perp},$$

$$\Phi^{[\gamma^+\gamma_5]} = \lambda g_{1L} + \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M} g_{1T},$$

$$\Phi^{[i\sigma^{i+}\gamma_5]} = S_T^i h_{1T} + \frac{p_T^i}{M}\left(\lambda h_{1L}^{\perp} + \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}_T}{M} h_{1T}^{\perp}\right) - \frac{\epsilon_T^{ij} p_{Tj}}{M} h_1^{\perp},$$

$$= S_T^i h_1 + \lambda \frac{p_T^i}{M} h_{1L}^{\perp} + \frac{(p_T^i p_T^j - \frac{1}{2} p_T^2 g_T^{ij}) S_{Tj}}{M^2} h_{1T}^{\perp} - \frac{\epsilon_T^{ij} p_{Tj}}{M} h_1^{\perp},$$

# Formulas

- For the twist-3 case

$$\Phi^{[1]} = \frac{M}{P^+}[e - \frac{\epsilon_T^{ij} p_{Ti} S_{Tj}}{M} e_T^\perp],$$

$$\Phi^{[i\gamma_5]} = \frac{M}{P^+}[\lambda e_L + \frac{p_T \cdot S_T}{M} e_T],$$

$$\Phi^{[\gamma^i]} = \frac{M}{P^+}[-\epsilon_T^{ij} S_{Tj} f_T' - \frac{\epsilon_T^{ij} p_{Tj}}{M}(\lambda f_L^\perp - \frac{p_T \cdot S_T}{M} f_T^\perp) + \frac{p_T^i}{M} f^\perp],$$

$$= \frac{M}{P^+}[-\epsilon_T^{ij} S_{Tj} f_T - \lambda \frac{\epsilon_T^{ij} p_{Tj}}{M} f_L^\perp - \frac{(p_T^i p_T^j + \frac{1}{2} p_T^2 g_T^{ij})\epsilon_{Tjk} S_T^k}{M^2} f_T^\perp + \frac{p_T^i}{M} f^\perp],$$

$$\Phi^{[\gamma^i \gamma_5]} = \frac{M}{P^+}[S_T^i g_T' + \frac{p_T^i}{M}(\lambda g_L^\perp + \frac{p_T \cdot S_T}{M} g_T^\perp) - \frac{\epsilon_T^{ij} p_{Tj}}{M} g^\perp],$$

$$= \frac{M}{P^+}[S_T^i g_T + \lambda \frac{p_T^i}{M} g_L^\perp + \frac{(p_T^i p_T^j - \frac{1}{2} p_T^2 g_T^{ij}) S_{Tj}}{M^2} g_T^\perp - \frac{\epsilon_T^{ij} p_{Tj}}{M} g^\perp],$$

$$\Phi^{[i\sigma^{ij}\gamma_5]} = \frac{M}{P^+}[\frac{S_T^i p_T^j - p_T^i S_T^j}{M} h_T^\perp - \epsilon_T^{ij} h],$$

$$\Phi^{[i\sigma^{+-}\gamma_5]} = \frac{M}{P^+}[\lambda h_L + \frac{p_T \cdot S_T}{M} h_T],$$

| $p\backslash N$ | $U$ | $L$ | $T$ |
|---|---|---|---|
| $U$ | $f^\perp$ | $f_L^\perp$ | $f_T(f_T'),\ f_T^\perp$ |
| $L$ | $g^\perp$ | $g_L^\perp$ | $g_T(g_T'),\ g_T^\perp$ |
| $T$ | $e,\ h$ | $e_L,\ h_L$ | $h_T,\ h_T^\perp,\ e_T,\ e_T^\perp$ |

Table 2: TMDs at twist-3

# Formulas

- For the twist-4 case, it has similar forms as the twist-2 case

$$\Phi^{[\gamma^-]} = \frac{M^2}{(P^+)^2}\Big[f_3 - \frac{\epsilon_T^{ij} p_{Ti} S_{Tj}}{M} f_{3T}^\perp\Big],$$

$$\Phi^{[\gamma^-\gamma_5]} = \frac{M^2}{(P^+)^2}\Big[\lambda g_{3L} + \frac{p_T \cdot S_T}{M} g_{3T}\Big],$$

$$\Phi^{[i\sigma^{i-}\gamma_5]} = \frac{M^2}{(P^+)^2}\Big[S_T^i h_{3T} + \frac{p_T^i}{M}\Big(\lambda h_{3L}^\perp + \frac{p_T \cdot S_T}{M} h_{3T}^\perp\Big) - \frac{\epsilon_T^{ij} p_{Tj}}{M} h_3^\perp\Big],$$

$$= \frac{M^2}{(P^+)^2}\Big[S_T^i h_3 + \lambda \frac{p_T^i}{M} h_{3L}^\perp + \frac{(p_T^i p_T^j - \frac{1}{2} p_T^2 g_T^{ij}) S_{Tj}}{M^2} h_{3T}^\perp - \frac{\epsilon_T^{ij} p_{Tj}}{M} h_3^\perp\Big].$$

# TMDs in light-cone transverse polarized diquark

- The results of TMDs at twist-2 (Bacchetta, Conti, Radici, PRD78, 074010(2008))

**Some symbols:**

$$\mathcal{A}^2_{e,s} = \frac{g_s^2}{(2\pi)^3}\frac{(1-x)^3}{2[\boldsymbol{p}_T^2 + L_s^2]^4},$$

$$\mathcal{A}^2_{o,s} = \frac{g_s^2}{4}\frac{e_q e_s}{(2\pi)^4}\frac{(1-x)^2}{L_s^2[L_s^2 + \boldsymbol{p}_T^2]^3},$$

$$\mathcal{A}^2_{e,a} = \frac{g_a^2}{(2\pi)^3}\frac{(1-x)}{2[\boldsymbol{p}_T^2 + L_a^2]^4},$$

$$\mathcal{A}^2_{o,a} = \frac{g_a^2}{4}\frac{e_q e_a}{(2\pi)^4}\frac{(1-x)^2}{L_a^2[L_a^2 + \boldsymbol{p}_T^2]^3}.$$

**2 T-odd:**

$$f_{1T}^{\perp s} = \mathcal{A}^2_{o,s}[M(m + xM)(1-x)],$$

$$h_1^{\perp s} = \mathcal{A}^2_{o,s}[M(m + xM)(1-x)],$$

$$f_{1T}^{\perp a} = \mathcal{A}^2_{o,a}[-xM(m + xM)],$$

$$h_1^{\perp a} = \mathcal{A}^2_{o,a}[M(m + xM)].$$

# TMDs in light-cone transverse polarized diquark

**6 T-even:**

$$f_1^s = \mathcal{A}_{e,s}^2[(m+xM)^2 + \boldsymbol{p}_T^2],$$

$$g_{1L}^s = \mathcal{A}_{e,s}^2[(m+xM)^2 - \boldsymbol{p}_T^2],$$

$$g_{1T}^s = \mathcal{A}_{e,s}^2[2M(m+xM)],$$

$$h_{1L}^{\perp s} = \mathcal{A}_{e,s}^2[-2M(m+xM)],$$

$$h_{1T}^{\perp s} = \mathcal{A}_{e,s}^2[-2M^2],$$

$$h_1^s = \mathcal{A}_{e,s}^2[(m+xM)^2],$$

$$f_1^a = \mathcal{A}_{e,a}^2[\boldsymbol{p}_T^2(1+x^2) + (m+xM)^2(1-x)^2],$$

$$g_{1L}^a = \mathcal{A}_{e,a}^2[\boldsymbol{p}_T^2(1+x^2) - (m+xM)^2(1-x)^2],$$

$$g_{1T}^a = \mathcal{A}_{e,a}^2[2xM(m+xM)(1-x)],$$

$$h_{1L}^{\perp a} = \mathcal{A}_{e,a}^2[2M(m+xM)(1-x)],$$

$$h_{1T}^{\perp a} = 0,$$

$$h_1^a = \mathcal{A}_{e,a}^2[-2x\boldsymbol{p}_T^2].$$

# TMDs in light-cone transverse polarized diquark

- The results of TMDs at twist-3

**Some symbols:**

$$\mathcal{A}^3_{e,s} = \frac{g_s^2}{(2\pi)^3}\frac{(1-x)^2}{2[\boldsymbol{p}_T^2 + L_s^2]^4}, \qquad \mathcal{A}^3_{e,a} = \frac{g_a^2}{(2\pi)^3}\frac{(1-x)^2}{2[\boldsymbol{p}_T^2 + L_a^2]^4},$$

$$\mathcal{A}^3_{o,s} = \frac{g_s^2}{4}\frac{e_q e_s}{2(2\pi)^4}\frac{(1-x)^2}{L_s^2[L_s^2 + \boldsymbol{p}_T^2]^3}, \qquad \mathcal{A}^3_{o,a} = \frac{g_a^2}{4}\frac{e_q e_a}{2(2\pi)^4}\frac{(1-x)}{[L_a^2 + \boldsymbol{p}_T^2]^2}.$$

**8 T-odd:**

$$e_T^{\perp s} = \mathcal{A}^3_{o,s}[(1-x)^2M^2 - L_s^2 - M_s^2],$$

$$e_T^{\perp a} = \mathcal{A}^3_{o,a}[\frac{xM_a^2 - (1-x)(m^2 + 2xmM + xM^2) - L_a^2}{L_a^2(L_a^2 + \boldsymbol{p}_T^2)} + \frac{1}{\boldsymbol{p}_T^2}\log\left(\frac{L_a^2 + \boldsymbol{p}_T^2}{L_a^2}\right)],$$

$$e_L^s = \mathcal{A}^3_{o,s}[(x + \frac{m}{M})(L_s^2 - \boldsymbol{p}_T^2)],$$

$$e_L^a = 0,$$

$$e_T^s = \mathcal{A}^3_{o,s}[L_s^2 + M_s^2 - (1-x)(M^2 + 2mM + xM^2)],$$

$$e_T^a = \mathcal{A}^3_{o,a}[\frac{(1-x)(m^2 - xM^2) - L_a^2 + xM_a^2}{L_a^2(L_a^2 + \boldsymbol{p}_T^2)} + \frac{1}{\boldsymbol{p}_T^2}\log\left(\frac{L_a^2 + \boldsymbol{p}_T^2}{L_a^2}\right)],$$

# TMDs in light-cone transverse polarized diquark

$$f_T^s = \mathcal{A}_{o,s}^3[(x+\frac{m}{M})(\boldsymbol{p}_T^2 - L_s^2)],$$

$$f_T^a = 0,$$

$$f_L^{\perp s} = \mathcal{A}_{o,s}^3[L_s^2 + M_s^2 - (1-x)M(2m+M+xM)],$$

$$f_L^{\perp a} = \mathcal{A}_{o,a}^3[\frac{(1-x)^2M(2m+M+xM) - (1-x+\frac{m}{M})\boldsymbol{p}_T^2 + (2x+\frac{m}{M})L_a^2 - (1-x)M_a^2}{L_a^2(L_a^2+\boldsymbol{p}_T^2)} - \frac{x}{\boldsymbol{p}_T^2}\log\left(\frac{L_a^2+\boldsymbol{p}_T^2}{L_a^2}\right)],$$

$$f_T^{\perp s} = 0,$$

$$f_T^{\perp a} = \mathcal{A}_{o,a}^3[\frac{2(L_a^2-\boldsymbol{p}_T^2)}{L_a^2(L_a^2+\boldsymbol{p}_T^2)} - \frac{1}{\boldsymbol{p}_T^2}\log\left(\frac{L_a^2+\boldsymbol{p}_T^2}{L_a^2}\right)]\frac{M(m+xM)(1-x)}{\boldsymbol{p}_T^2},$$

$$g^{\perp s} = \mathcal{A}_{o,s}^3[(1-x)^2M^2 - L_s^2 - M_s^2],$$

$$g^{\perp a} = \mathcal{A}_{o,a}^3[\frac{(1-x)(m+xM)^2 + (1-x)^2M^2 + xL_a^2 - M_a^2}{L_a^2(L_a^2+\boldsymbol{p}_T^2)} - \frac{x}{\boldsymbol{p}_T^2}\log\left(\frac{L_a^2+\boldsymbol{p}_T^2}{L_a^2}\right)],$$

$$h^s = \mathcal{A}_{o,s}^3[(x+\frac{m}{M})(\boldsymbol{p}_T^2 - L_s^2)],$$

$$h^a = 0.$$

# TMDs in light-cone transverse polarized diquark

**8 T-even:**

$$e^{s} = \mathcal{A}^{3}_{e,s}[(1-x)(m+M)(m+xM) - (1+\frac{m}{M})\boldsymbol{p}_T^2 - (x+\frac{m}{M})M_s^2],$$

$$e^{a} = \mathcal{A}^{3}_{e,a}[(1-x)(m+M)(m+xM) - (1+\frac{m}{M})\boldsymbol{p}_T^2 - (x+\frac{m}{M})M_a^2 + \frac{2m\boldsymbol{p}_T^2}{M(1-x)}],$$

$$f^{\perp s} = \mathcal{A}^{3}_{e,s}[2mM(1-x) + M^2(1-x^2) - \boldsymbol{p}_T^2 - M_s^2],$$

$$f^{\perp a} = \mathcal{A}^{3}_{e,a}[2mM(1-x) + M^2(1+x-x^2) - m^2 + \frac{x\boldsymbol{p}_T 2 - M_a^2}{(1-x)}],$$

$$g_L^{\perp s} = \mathcal{A}^{3}_{e,s}[\boldsymbol{p}_T^2 + M_s^2 - (1-x)^2 M^2],$$

$$g_L^{\perp a} = \mathcal{A}^{3}_{e,a}[(m+xM)^2 + (1-x)M^2 + x\boldsymbol{p}_T^2 - M_a^2],$$

$$g_T^{\perp s} = \mathcal{A}^{3}_{e,s}[2M^2(1-x)],$$

$$g_T^{\perp a} = \mathcal{A}^{3}_{e,a}[2M(m+xM)],$$

# TMDs in light-cone transverse polarized diquark

$$g_T^s = \mathcal{A}_{e,s}^3[(1-x)(m+M)(m+xM)-(x+\frac{m}{M})(\boldsymbol{p}_T^2+M_s^2)],$$

$$g_T^a = \mathcal{A}_{e,a}^3[x-\frac{m(1+x)}{M(1-x)}]\boldsymbol{p}_T^2,$$

$$h_T^{\perp s} = \mathcal{A}_{e,s}^3[(1-x)(M^2+2mM+xM^2)-\boldsymbol{p}_T^2-M_s^2],$$

$$h_T^{\perp a} = \mathcal{A}_{e,a}^3[(m^2-xM^2)+\frac{xM_a^2-\boldsymbol{p}_T^2}{(1-x)}],$$

$$h_L^s = \mathcal{A}_{e,s}^3[(1-x)(m+M)(m+xM)+(1-2x-\frac{m}{M})\boldsymbol{p}_T^2-(x+\frac{m}{M})M_s^2],$$

$$h_L^a = \mathcal{A}_{e,a}^3[-(1-x)(m+M)(m+xM)+(x+\frac{m}{M})M_a^2-(1-\frac{m(1+x)}{M(1-x)})\boldsymbol{p}_T^2],$$

$$h_T^s = \mathcal{A}_{e,s}^3[(1-x)^2M^2-\boldsymbol{p}_T^2-M_s^2],$$

$$h_T^a = \mathcal{A}_{e,a}^3[(m^2+2xmM+xM^2)+\frac{\boldsymbol{p}_T^2-xM_a^2}{(1-x)}].$$

# TMDs in light-cone transverse polarized diquark

- The results of TMDs at twist-4

**Some symbols:**

$$\mathcal{A}^4_{e,s} = \frac{g_s^2}{(2\pi)^3}\frac{(1-x)}{4[\boldsymbol{p}_T^2+L_s^2]^4}, \qquad \mathcal{A}^4_{e,a} = \frac{g_a^2}{(2\pi)^3}\frac{(1-x)}{4[\boldsymbol{p}_T^2+L_a^2]^4},$$

$$\mathcal{A}^4_{o,s} = \frac{g_s^2}{4}\frac{e_q e_s}{2(2\pi)^4}\frac{(1-x)^2}{L_s^2[L_s^2+\boldsymbol{p}_T^2]^3}, \qquad \mathcal{A}^4_{o,a} = \frac{g_a^2}{4}\frac{e_q e_a}{2(2\pi)^4}\frac{(1-x)}{[L_a^2+\boldsymbol{p}_T^2]^2}.$$

**2 T-odd:**

$$f_{3T}^{\perp s} = \mathcal{A}^4_{o,s}[L_s^2+M_s^2-M(m+M)(1-x)],$$

$$f_{3T}^{\perp a} = \mathcal{A}^4_{o,a}[\frac{\frac{m}{M}M_a^2+(1-x+\frac{m}{M})L_a^2-m(m+M)(1-x)}{L_a^2(L_a^2+\boldsymbol{p}_T^2)}+\frac{m}{M\boldsymbol{p}_T^2}\log\left(\frac{L_a^2+\boldsymbol{p}_T^2}{L_a^2}\right)],$$

$$h_3^{\perp s} = \mathcal{A}^4_{o,s}[L_s^2+M_s^2-M(m+M)(1-x)],$$

$$h_3^{\perp a} = \mathcal{A}^4_{o,a}[\frac{(1-x-\frac{m}{M})M_a^2+\frac{m}{M}L_a^2-(m+M)(M-xM-m)(1-x)}{L_a^2(L_a^2+\boldsymbol{p}_T^2)}-\frac{m}{M\boldsymbol{p}_T^2}\log\left(\frac{L_a^2+\boldsymbol{p}_T^2}{L_a^2}\right)].$$

# TMDs in light-cone transverse polarized diquark

**6 T-even:**

$$f_3^s = \mathcal{A}_{e,s}^4\{(m+M)^2(1-x)^2 - [(1+x+2\frac{m}{M})\boldsymbol{p}_T^2 + 2(1+\frac{m}{M})M_s^2](1-x) + (\frac{\boldsymbol{p}_T^2+M_s^2}{M})^2\},$$

$$f_3^a = \mathcal{A}_{e,a}^4\{(m+M)^2(1-x)^2 + [2\frac{m}{M}(\frac{m}{M}-1+x) + (1-x)^2]\boldsymbol{p}_T^2 - 2(1-x)(1+\frac{m}{M})M_a^2 + \frac{\boldsymbol{p}_T^4+M_a^4}{M^2}\},$$

$$g_{3L}^s = \mathcal{A}_{e,s}^4\{-(m+M)^2(1-x)^2 + [(3-x+2\frac{m}{M})\boldsymbol{p}_T^2 + 2(1+\frac{m}{M})M_s^2](1-x) - (\frac{\boldsymbol{p}_T^2+M_s^2}{M})^2\},$$

$$g_{3L}^a = \mathcal{A}_{e,a}^4\{(m+M)^2(1-x)^2 - [2\frac{m}{M}(\frac{m}{M}-1+x) + (1-x)^2]\boldsymbol{p}_T^2 - 2(1-x)(1+\frac{m}{M})M_a^2 + \frac{\boldsymbol{p}_T^4+M_a^4}{M^2}\},$$

$$g_{3T}^s = \mathcal{A}_{e,s}^4 2(1-x)[M(m+M)(1-x) - \boldsymbol{p}_T^2 - M_s^2],$$

$$g_{3T}^a = \mathcal{A}_{e,a}^4 2[(1-x-\frac{m}{M})\boldsymbol{p}_T^2 + \frac{m}{M}M_a^2 - m(m+M)(1-x)],$$

# TMDs in light-cone transverse polarized diquark

$$h_{3T}^{s} = \mathcal{A}_{e,s}^{4}\{(m+M)^2(1-x)^2 - [(1+x+2\frac{m}{M})p_T^2 + 2(1+\frac{m}{M})M_s^2](1-x) + (\frac{p_T^2+M_s^2}{M})^2\},$$

$$h_{3T}^{a} = \mathcal{A}_{e,a}^{4}[M_a^2 - m^2 - M^2(1-x)]\frac{p_T^2}{M^2},$$

$$h_{3L}^{\perp s} = \mathcal{A}_{e,s}^{4}2(1-x)[M(m+M)(1-x) - p_T^2 - M_s^2],$$

$$h_{3L}^{\perp a} = \mathcal{A}_{e,a}^{4}\{[m^2 + xM(m+M) - M^2](1-x) + \frac{m}{M}p_T^2 + (1-x-\frac{m}{M})M_a^2\},$$

$$h_{3T}^{\perp s} = \mathcal{A}_{e,s}^{4}[-2M^2(1-x)^2],$$

$$h_{3T}^{\perp a} = \mathcal{A}_{e,a}^{4}2[M(m+M)(1-x) - M_a^2].$$

- The results of TMDs at twist-4 are given for the first time and we find

**T-odd:** $f_{3T}^{\perp s} = h_3^{\perp s}$ **T-even:** $g_{3T}^{s} = h_{3L}^{\perp s}$

# SUMMARY

- We build an analytical framework for the calculations of TMDs under the quark-diquark spectator model.

- We can also calculate the transverse momentum dependent fragmentation functions (FFs) in a similar way.

- Those formulas with model-input of various coupling factors can be used to calculate physical observables in explicit physical process. (see our privious work )

- We can confront theoretical predictions with experimental data for better understanding of the three-dimensional picture of hadron.

# THANK YOU!