$B^0_{(s)} \to \pi\pi(KK)$ form factors with the width effect

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Outline

Physics Motivation

- SM works wonderful, but an effective theory valid up to some scale.
- BSM are needed to explained the matter antimatter asymmetry.
- At the current scale, the precision examination in SM is more important.
- Precision study

  beta decay, (1983, UA1,UA2, W boson).

  kaon physics, (1974, BNL/SLAC, charm quark; 1977, Fermilab E288, beauty quark).

  1987, ARGUS, large $\Delta m_{B_d,B_s}$ (1994, CDF/D0, top quark).

  2001, B factories, $A_{CP}(t,f)$ in B decays (irreducible phase in CKM paradigm).

  $\cdots$ CPV in $\eta \to \pi\pi\pi$ presented by Jun Shi (SCNU)

  $\cdots$ Semi-leptonic $B_{(s)}, D_{(s)}$ decays presented by Lu Zhang (BNU)

Motivation-$|V_{ub}|$ and $|V_{cb}|$

† The most important CKM unitarity test is the unitarity triangle (UT),

$|V_{cb}|$: $|\epsilon_K| \propto |V_{cb}|^2$, and FCNC $\propto |V_{tb}V_{ts}^*|^2 \sim |V_{cb}|^2 \left(1+\lambda^2\right)$,
a dominate source of theoretical uncertainty.

$|V_{ub}|$: accompanied with the weak phase angle $\gamma \Rightarrow$ *CPV* in *B* physics,
$|V_{ub}/V_{cb}|$ constrains directly for UT.

† $|V_{cb}|$: $3.4\sigma \sim 8\%$ tension between exclu. and inclu. determination (HFLAV2019),

$|V_{cb}^{B\to X_c l\nu}| = 42.19(0.78)\times 10^{-3}$, $|V_{cb}^{B\to D}| = 39.58(1.31)\times 10^{-3}$,

$|V_{cb}^{B\to D^*}| = 38.76(0.97)\times 10^{-3}$, $\Rightarrow$ R_D, R_{D^*} puzzels ?

$|V_{cb}^{B_s\to D_s^*}| = 41.3(1.3)\times 10^{-3}$ (LHCb2020),

† $|V_{ub}|$: $2\sigma \sim 15\%$ tension between exclu. and inclu. determination (HFLAV2019),

$|V_{ub}^{B\to X_u l\nu}| = 45.2(2.9)\times 10^{-3}$, $|V_{ub}^{B\to\pi}| = 37.3(1.4)\times 10^{-3}$,

$|V_{ub}^{B\to\tau\nu}| f_B = 0.72(9)$(Belle) and $1.01(14)$(BABAR), less precise, disagreement.

† Enlarge the set of exclusive processes to determine $|V_{ub}|$

A natural choice is $B \to \rho l \nu$, but how to identify ρ ?
$\pi\pi$ invariant mass spectral, width effect/nonresonant contribution ?

† The underlying consideration is $B \to \pi\pi l \bar{\nu}_l \, (B_{l4})$
[S. Faller, T. F, A. K, T. M and Danny v Dyk, Phys.Rev.D89(2014)014015]

- QCD and Phenomenology

A competitive determination of the $|V_{ub}|$ with precise form factors (FFs).

Precise FFs needs more knowledge of QCD.

QCD includes hadron inner structures and transition hard amplitudes.

FCNC $B \to \pi\pi l^+ l^-$ and $B \to \pi\pi\pi$.

More phenomenological interesting is the $B \to K\pi ll$ related to R_{K^*}.

QCD sum rules approach: twofold way of treating correlation fucntion.

Mutually versions for different types of hadronic matrix elements: LCSRs(hadronic form factors), 2pSRs(decay constants).

LCSRs

† Non-local correlation function, putting the external meson/state on-shell, expanding on LC to get LCDAs;

† Quark level: OPE evaluation in terms of LCDAs,
Hadron level: intermediate hadrons interpolating between two decoupled quark currents.

† Quark-hadron duality: equate hadron dispersion integral to the OPE calculation (threshold s_0).

† Borel transformation: mitigate the harassment of ultraviolet subtraction scheme from the OPE side & suppress the contributions from higher excited and continuum states from the hadron aspect (M^2).

$$
\begin{aligned}
i\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle &= F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\,i\epsilon_{\nu\alpha\beta\gamma}\,q^\alpha\,k^\beta\,\bar{k}^\gamma \\
&+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right) \\
&+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right). \qquad (1)
\end{aligned}
$$

$\cdots$ Nucleon form factor presented by Bao-dong Sun (SDU)

- For definiteness and conciseness, take $\bar{B}^0 \to \pi^+\pi^0 l\bar{\nu}$ for example.
- Iso-vector dipion final state.
- How much of the dominant intermediate ρ contribution to the FFs? accurate interpretation (10%) of the $B \to \pi\pi(\rho)l\bar{\nu}_l$ measurements.
- † We are now at LO for hard amplitude,
  partial three particle DAs corrections in the $B-$meson LCSRs (soft),
  Leding twist level in the dipion LCSRs.

  † Our predictions are mainly effected/constrained by hadronic inputs:
  dipion DAs (twist, strong phase) / B meson DAs (λ_B).

With dipion DAs

Correlation Function

[C. Hambrock and A. Khodjamirian, NPB905(2016)373]

$$F_\mu(k_1,k_2,q) = i\int d^4x e^{iq\cdot x}\langle\pi^+(k_1)\pi^0(k_2)|\mathrm{T}\{j_\mu^{V-A}(x),j_5(0)\}|0\rangle$$

$\downarrow$ *Lorentz decomposition*

$$\equiv \varepsilon_{\mu\nu\rho\sigma}q^\nu k_1^\rho k_1^\sigma F^V + q_\mu F^{(A,q)} + k_\mu F^{(A,k)} + \bar{k}_\mu F^{(A,\bar{k})} \quad (2)$$

- $j_\mu^{V-A}(x) \equiv \bar{u}(x)\gamma_\mu(1-\gamma_5)b(x), \qquad j_5(0) \equiv im_b\bar{b}(0)\gamma_5 d(0)$
- Kinematics: $k = k_1 + k_2, \quad \bar{k} = k_1 - k_2, \quad p = k + q$
  Four independent invariant variables: $p^2, q^2, k^2, q\cdot\bar{k}$
- $q\cdot\bar{k} = \frac{1}{2}\sqrt{\lambda}\ \beta_\pi(k^2)\cos\theta_\pi$ with $\beta_\pi(k^2) = \sqrt{1-4m_\pi^2/k^2}$.
- Källén function is $\lambda \equiv \lambda(m_B^2,q^2,k^2) = m_B^4 + q^4 + k^4 - 2(m_B^2q^2 + m_B^2k^2 + q^2k^2)$.
- θ_π is the angle between the 3-momentum of π^0 and B in the dipion rest frame.
- $p^2, q^2 \ll m_b^2$, to guarantee the validity of OPE near the LC ($x^2 \sim 0$).
- $k^2 \lesssim 1\mathrm{GeV}^2 \ll m_b^2$, to avoid generic $\mathcal{O}(k^2x^2)$ terms in LC expansion.

Dipion DAs [M.V. Polyakov, NPB555(1999)231]

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\gamma_\mu[x,0]d(0)|0\rangle = -\sqrt{2}k_\mu \int_0^1 du e^{iu(k\cdot x)} \Phi_{\parallel}^{I=1}(u,\zeta,k^2), \quad (3)$$

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}(x)\sigma_{\mu\nu}[x,0]d(0)|0\rangle = 2\sqrt{2}\frac{k_{1\mu}k_{2\nu} - k_{1\nu}k_{2\mu}}{2\zeta - 1} \int_0^1 du e^{iu(k\cdot x)} \Phi_{\perp}^{I=1}(u,\zeta,k^2), \quad (4)$$

- Chiral-even and -odd LC expansion respectively with gauge factor $[x,0]$.
- u quark carries longitudinal momentum faction,
  $2q\cdot \bar{k}(\propto 2\zeta - 1)$ determines the LC momentum distribution carried by two pions.
- Normalization conditions
  $\int_0^1 \Phi_{\parallel}^{I=1}(u,\zeta,k^2) = (2\zeta-1)F_\pi^{em}(k^2)$, $\int_0^1 \Phi_{\perp}^{I=1}(u,\zeta,k^2) = (2\zeta-1)F_\pi^{t}(k^2)$.
  $F_\pi^{em}(0) = 1$ and $F_\pi^t(0) = 1/f_{2\pi}^{\perp}$.
- Higher twist proportional to $1, \gamma_\mu\gamma_5$ are neglected, γ_5 vanishes because of P-parity conservation.

Dipion DAs - LO and twist-2 appro. [M.V. Polyakov, NPB555(1999)231]

- Double expansion of Legendre and Gegenbauer polynomials $C_l^{1/2}(2\zeta-1)$ & $C_n^{3/2}(2u-1)$.

  Partial wave & eigenfunction of evolution equation:

$$\Phi_{\perp/\parallel}(u,\zeta,k^2) = \frac{6u(1-u)}{f_{2\pi}^{\perp}/1} \sum_{n=0,2,\cdots}^{\infty} \sum_{l=1,3,\cdots}^{n+1} B_{nl}^{\perp/\parallel}(k^2) C_n^{3/2}(2u-1) C_l^{1/2}(2\zeta-1),$$
$$C_l^{1/2}(2\zeta-1) = \beta_\pi P_l^{(0)}(\frac{2\zeta-1}{\beta_\pi}), \tag{4}$$

- $B_{nl}^{\perp/\parallel}(k^2)$: renormalizable coefficients, $B_{01}^{\perp/\parallel}(0)=1$, $B_{01}^{\parallel}(k^2)=F_\pi^{em}(k^2)$.
- $n \geq 2$ at low k^2 determine the non-asymptotic part of DAs, decrease logarithmically at large scale.
- With truncating at a given n_{max}, l is restricted to $n_{max}+1$.
- unitarity relation, $B_{nl}^{\perp}(k^2)$ are complex functions at $k^2 > 4m_\pi^2$.

At twist-2 accuracy

- $p^2 = p^2 - s + s \to s, \quad s = s(u) = (m_b^2 - \bar{u}q^2 + u\bar{u}k^2)/u.$
- $F^{(A,k)}(s, q^2, k^2, \zeta)$:

  $q \cdot \bar{k}$ generate a cut at the real axis to avoid imaginary part.
  $(\sqrt{q^2} - \sqrt{k^2})^2 < p^2 < (\sqrt{q^2} + \sqrt{k^2})^2$.
  non physical intermediate state, a typical kinematic singularity.
- After Borel trans., this cut is enhanced respecting to the b-quark spectral.
- New correlation function Π_5 for this amplitude: PS current ($j_\mu^{V-A} \to j_5$), Multiplying q_μ on Eq.(1), solid at leading power precision.

$$F_\mu(q, k_1, k_2) = \frac{\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}^0(p)\rangle f_B m_B^2}{m_B^2 - p^2} + \cdots, \quad (5)$$

$$\Pi_5(p^2, q^2, k^2, \zeta) = \frac{\sqrt{q^2} F_t(q^2, k^2, \zeta) f_B m_B^2}{m_B^2 - p^2} + \ldots, \quad (6)$$

[C. Hambrock and A. Khodjamirian, NPB905(2016)373]
[SC, A. Khodjamirian and J. Virto, PRD96(2017)051901(R)]

$$F_\perp^{(l)}(k^2,q^2) = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^\perp}\frac{\sqrt{\lambda_B}m_b}{m_B^2 f_B}e^{\frac{m_B^2}{M^2}}\sum_{n=0,2,\cdots}\sum_{l'=1,3}^{n+1} I_{ll'}B_{nl'}^\perp(k^2)J_n^\perp(q^2,k^2,M^2,s_0^B), \quad (7)$$

$$F_\parallel^{(l)}(k^2,q^2) = \frac{\sqrt{k^2}}{\sqrt{2}f_{2\pi}^\perp}\frac{m_b^3}{m_B^2 f_B}e^{\frac{m_B^2}{M^2}}\sum_{n=0,2,\cdots}\sum_{l'=1,3}^{n+1} I_{ll'}B_{nl'}^\perp(k^2)J_n^\parallel(q^2,k^2,M^2,s_0^B), \quad (8)$$

$$\sqrt{q^2}F_t^{(l)}(k^2,q^2) = -\frac{6m_b^2}{\sqrt{2}f_B m_B^2}\frac{\beta_\pi(k^2)}{\sqrt{2l+1}}\exp\left(\frac{m_B^2-s}{M^2}\right)$$
$$\times\sum_{n=l-1,l+1,\cdots}^{\infty} B_{nl}^\parallel(k^2)\int_{u_0}^1\frac{du}{u}\bar{u}\,(m_b^2-q^2+u^2k^2)\,C_n^{3/2}(u-\bar{u}), \quad (9)$$

$$I_{ll'} \equiv -\frac{\sqrt{2l+1}(l-1!)}{2(l+1)!}\int_{-1}^1\frac{dz}{z}\sqrt{1-z^2}P_l^{(1)}(z)P_{l'}^{(0)}(z), \quad (10)$$

$$J_n^\perp(q^2,k^2,M^2,s_0^B) = \int_{u_0}^1 du\,e^{\frac{-s}{M^2}}6(1-u)C_n^{3/2}(2u-1), \quad (11)$$

$$J_n^\parallel(q^2,k^2,M^2,s_0^B) = \int_{u_0}^1\frac{du}{u}e^{\frac{-s}{M^2}}6(1-u)C_n^{3/2}(2u-1)\left(1-\frac{q^2-u^2k^2}{m_b^2}\right), \quad (12)$$

[C. Hambrock and A. Khodjamirian, NPB905(2016)373]
[SC, A. Khodjamirian and J. Virto, PRD96(2017)051901(R)]

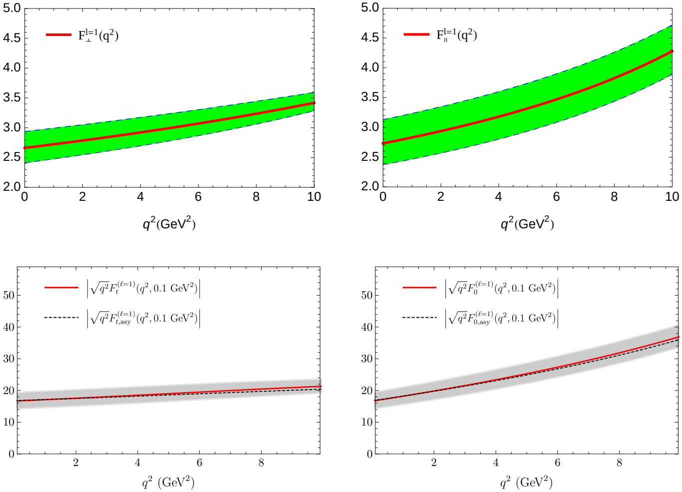
- $I_{ll'} = 0$ when $l > l'$, $I_{11} = 1/\sqrt{3}, I_{13} = -1/\sqrt{3}, I_{15} = 4/(5\sqrt{3})$;
  $l = 1$, asymptotic DAs, partial $P-$wave term remains in the FFs.
- Short-distance part of the correlator: $\mu = 3\,\mathrm{GeV}$ without NLO correction.
- Two-point QCD sum rules' prediction for $f_B = 207^{+9}_{-17}\mathrm{MeV}$.
  [P. Gelhausen, et.al., PRD88(2013)014015[Erratum ibid. D89(2014)099901]]
- $M^2 = 16.0 \pm 4.0\mathrm{GeV}^2$ corresponding to $s_0^B = 37.5 \pm 2.5\mathrm{GeV}^2$.

  - ♦ How large of $P-$wave contribution to $B \to \pi\pi$ FFs (higher partial wave) ?
  - ♦ How much ρ contained in $P-$wave $B \to \pi\pi$ FFs ? Resonance model.
  - ♦ ρ meson: $a_2^\perp = 0.2 \pm 0.1, \quad a_{n>2} = 0, \quad f_\rho^\perp = 160 \pm 10\mathrm{MeV}$.

- The complexity of $B_{nl}^\perp(k^2)$, only predicable at $k^2 = 4m_\pi^2$ within the instanton model of QCD vacuum, WITHOUT phase.
- go to intermediate k^2 region by Watson theorem of $\pi - \pi$ scattering amplitudes in terms of Omnes dispersion relation.
  [SC, Phys.Rev.D99(2019)053005]

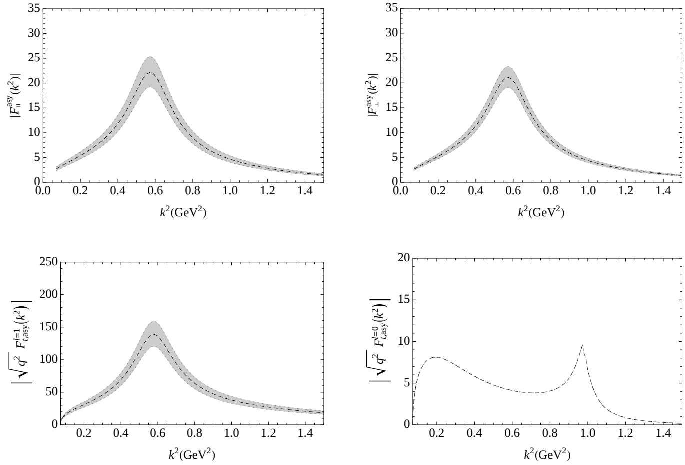
$$B_{n\ell}^l(k^2) = B_{n\ell}^l(0)\,\mathrm{Exp}\left[\sum_{m=1}^{N-1}\frac{k^{2m}}{m!}\frac{d^m}{dk^{2m}}\ln B_{n\ell}^l(0) + \frac{k^{2N}}{\pi}\int_{4m_\pi^2}^\infty ds\,\frac{\delta_\ell^l(s)}{s^N(s-k^2-i0)}\right]. \quad (13)$$

[C. Hambrock and A. Khodjamirian, NPB905(2016)373]
[SC, A. Khodjamirian and J. Virto, PRD96(2017)051901(R)]



With dipion DAs - $F_{\perp,\parallel,t,0}(q^2=0,k^2)$

[SC, Phys.Rev.D99(2019)053005]



† The uncertainty from LCSRs in $F_t^{l=0}(q^2,k^2)$ CANCEL between $e^{m_B^2/M^2}$ and $J_0^t(M^2,s_0^B)$.

At LO and twist-2 approximation of dipion LCDAs

† $F_{\perp,\parallel}(q^2, k^2 = 4m_\pi^2)$:

$F_{\perp,\parallel}(q^2)$ has a smooth evolution.

high partial waves give tiny contribution $\sim 2\%$.

† $\sqrt{q^2}F_{t,0}(q^2, k^2 = 0.1\mathrm{GeV}^2)$:

$\sqrt{q^2}F_{t,0}(q^2)$ is one order larger than $F_{\perp,\parallel}(q^2)$,

high partial waves contribute a few percent.

† $F_{\parallel,\perp}(0, k^2)$, $\sqrt{q^2}F_{t,0}(0, k^2)$

$\sqrt{q^2}F_t(k^2) = \sqrt{q^2}F_0(k^2)$.

high partial wave contribute a few percent.

† ρ', ρ'' and NR background contribute $\sim 20\% - 30\%$ to P-wave.

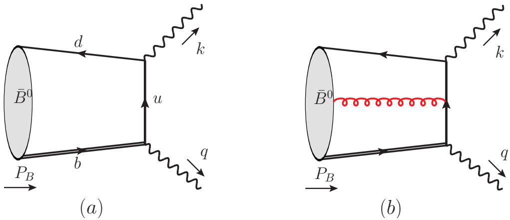
With B meson DAs

Correlation Function [A. Khodjamirian, T. Mannel and Nils Offen, PRD75(2014)054013]

$$F_{\mu\nu}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{\bar{d}(x)\gamma_\mu u(x), \bar{u}(0)\gamma_\nu(1-\gamma_5)b(0)\}|\bar{B}^0(q+k)\rangle$$

$\downarrow$ *Lorentz decomposition*

$$\equiv \varepsilon_{\mu\nu\rho\sigma}q^\rho k^\sigma F_{(\varepsilon)}(k^2,q^2) + ig_{\mu\nu}F_{(g)}(k^2,q^2) + iq_\mu k_\nu F_{(qk)}(k^2,q^2)$$
$$+\ ik_\mu k_\nu F_{(kk)}(k^2,q^2) + iq_\mu q_\nu F_{(qq)}(k^2,q^2) + ik_\mu q_\nu F_{(kq)}(k^2,q^2) \quad (14)$$



- $(q+k)^2 = m_B^2$, $\Lambda^2_{QCD} \ll q^2 \in [0, 10]\,\mathrm{GeV}^2 \ll m_B^2$, $|k^2| \gg \Lambda^2_{QCD}$,
  large virtuality of intrmediate u quark.
  OPE is applicable with DAs defined in HQET ($k \cdot x \sim 0$).
- Take the invariant amplitude $F_\varepsilon(k^2, q^2)$ in Eq. 1 for example to outline.
- At LO, free propagator (u quark) + "Soft" gluon correction.
  heavy-light bilocal $q\bar{q}$ and partial $qG\bar{q}$ matrix elements.

$$F^{\mathrm{OPE}}_{(\varepsilon)}(k^2, q^2) = f_B m_B \int_0^1 d\sigma \, \frac{\phi_+^B(\sigma m_B)}{(1-\sigma)(s-k^2)} + \cdots \tag{15}$$

$$s = s(\sigma, q^2) = \sigma m_B^2 - \sigma q^2/\bar{\sigma}, \qquad \bar{\sigma} \equiv 1 - \sigma \tag{16}$$

- σ : the momentum carried by the light quark in B meson.
  Ellipsis : subleading 3-particle DA contributions (soft gluon correction).
- Dispersion integral formula in k^2 for $F^{\mathrm{OPE}}_{(\varepsilon)}(k^2, q^2)$.

$$F^{\mathrm{OPE}}_{(\varepsilon)}(k^2, q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \, \frac{\mathrm{Im} F^{\mathrm{OPE}}_{(\varepsilon)}(s, q^2)}{s - k^2} \tag{17}$$

$$\frac{1}{\pi} \mathrm{Im} F^{\mathrm{OPE}}_{(\varepsilon)}(s, q^2) = f_B m_B \left[\left(\frac{d\sigma}{ds} \right) \frac{\phi_+^B(\sigma m_B)}{(1-\sigma)} \right]_{\sigma(s)} + \cdots \tag{18}$$

- In parallel, employ the hadronic DR in k^2 respecting to the $\bar{d}\gamma_\mu u$ interpolation.

$$F_{(\varepsilon)}(k^2, q^2) = \frac{1}{\pi} \int_{4m_\pi^2}^{\infty} ds \, \frac{\mathrm{Im} F_{(\varepsilon)}(s, q^2)}{s - k^2} \tag{19}$$

- Unitarity relation: Inserting the complete set of state with quantum number of the $\bar{d}\gamma_\mu u$ current.

$$2\,\mathrm{Im} F_{\mu\nu}(k, q) = \int d\tau_{2\pi} \langle 0|\bar{d}\gamma_\mu u|\pi^+(k_1)\pi^0(k_2)\rangle \langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(q+k)\rangle + \cdots \tag{20}$$

- Ellipsis : contributions from $4\pi, K\bar{K}, etc.$
- Pion electromagnetic form factor

$$\langle \pi^+(k_1)\pi^0(k_2)|\bar{u}\gamma_\mu d|0\rangle = -\sqrt{2}(k_1 - k_2)_\mu F_\pi(k^2), \tag{21}$$

Normalization $F_\pi^{\rm em}(0) = 1$.
In the isospin symmetry limit $F_\pi(k^2) = F_\pi^{\rm em}(k^2)$.
Available from the Belle Collaboration in $\tau \to \pi^-\pi^0\nu_\tau$ decay up to m_τ^2.
[PRD78(2008)072006; PRD93(2016)032003]

With B meson DAs - Hadron Dispersion Relation

† Legendre expansion of $F_i(k^2, q^2, q \cdot \bar{k})$ defined in Eq. 1

$$F_{0,t}(k^2, q^2, q \cdot \bar{k}) = \sqrt{3}\, F_{0,t}^{(\ell=1)}(k^2, q^2)\, P_1^{(0)}(\cos\theta_\pi) + \cdots ,$$

$$F_{\perp,\parallel}(k^2, q^2, q \cdot \bar{k}) = \sqrt{3}\, F_{\perp,\parallel}^{(\ell=1)}(k^2, q^2)\, \frac{P_1^{(1)}(\cos\theta_\pi)}{\sin\theta_\pi} + \cdots , \quad (22)$$

- (Associated) Legendre polynomials: $P_1^{(0)}(x) = x$, $P_1^{(1)}(x) = -\sqrt{1-x^2}$.
- Only the P-wave components survive $\Leftarrow$ P-wave projector from F_π.
- The imaginary part in hadron dispersion relation Eq. 19

$$\frac{1}{\pi}\,\mathrm{Im}F_{(\varepsilon)}(s, q^2) = \frac{\sqrt{s}\,[\beta_\pi(s)]^3}{4\sqrt{6}\pi^2\sqrt{\lambda}}\, F_\pi^\star(s)\, F_\perp^{(\ell=1)}(s, q^2) + \cdots \quad (23)$$

- Concerning on $F_\pi^\star(s)$ at low $s \lesssim 1.0 - 1.5$ GeV^2

  $4\pi, K\bar{K}$, *etc* contributions are expected suppression.

[S.Eidelman and S.Lukaszuk, PLB582(2004)27]
[X.W. Kang, B. Kubis, C. Hanhart and U-G. Meissner, PRD89(2014)053015]

[SC, A.Khodjamirian and J.Virto, JHEP05(2017)157]

- Semi-local quark-hadron duality approximation to DRs Eqs. (17,19)

$$\int_{s_0^{2\pi}}^{\infty} ds\, e^{-s/M^2}\, \mathrm{Im} F_{(\varepsilon)}(s, q^2) = \int_{s_0^{2\pi}}^{\infty} ds\, e^{-s/M^2}\, \mathrm{Im} F^{\mathrm{OPE}}_{(\varepsilon)}(s, q^2) \tag{24}$$

- B-meson LCSR for $F_\perp^{(\ell=1)}(s, q^2)$

$$\begin{aligned} & f_B m_B \left[\int_0^{\sigma_0^{2\pi}} d\sigma\, e^{-s(\sigma, q^2)/M^2}\, \frac{\phi_+^B(\sigma m_B)}{\bar{\sigma}} + m_B\, \Delta V^{BV}(q^2, \sigma_0^{2\pi}, M^2) \right] \\ = \quad & \int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2}\, \frac{\sqrt{s}\,[\beta_\pi(s)]^3}{4\sqrt{6}\pi^2\sqrt{\lambda}}\, F_\pi^\star(s)\, F_\perp^{(\ell=1)}(s, q^2) \;, \end{aligned} \tag{25}$$

- $\sigma_0^{2\pi} m_B^2 - \sigma_0^{2\pi} q^2/\bar{\sigma}_0^{2\pi} \equiv s_0^{2\pi}$; $\quad \Delta V^{BV}$: three-particle DA contribution
- $F_{(\parallel,0)}(s, q^2)$ are deduced by the Axial-Vector weak curret.
- $F_{(t)}(s, q^2)$, correlation function with current $i m_b \bar{u}(0)\gamma_5 b(0) \cdots$

With B meson DAs - LCSRs for $F_\perp(s, q^2)$

[SC, A.Khodjamirian and J.Virto, JHEP05(2017)157]

$$
\begin{aligned}
& f_B m_B \left[\int_0^{\sigma_0^{2\pi}} d\sigma\; e^{-s(\sigma,q^2)/M^2}\; \frac{\phi_+^B(\sigma m_B)}{\bar\sigma} + m_B \Delta V^{BV}(q^2, \sigma_0^{2\pi}, M^2) \right] \\
= & \int_{4m_\pi^2}^{s_0^{2\pi}} ds\; e^{-s/M^2} \frac{\sqrt{s}\,[\beta_\pi(s)]^3}{4\sqrt{6}\pi^2\sqrt{\lambda}} F_\pi^\star(s)\, F_\perp^{(\ell=1)}(s, q^2) \;, \qquad (26)
\end{aligned}
$$

- The s dependence in $F_\perp^{(\ell=1)}(s, q^2)$ is convoluted with $F_\pi^\star(s)$.
- Reality condition: $\mathrm{Im}[$ r.h.s of Eq.(26)$] = 0$.
- An inverse problem to solve $F_\perp^{(\ell=1)}$ directly from Eq.(26), seems impossible.
- Resonance model for $F_\perp^{(\ell=1)}$ should recover the $B \to \rho$ LCSR.
- Constraints $B \to \pi\pi$ FFs in a certain ansatz and/or (resonance) model.

† B meson LCDAs and timelike pion vector FF are the main inputs;

† Validity and consistency with an alternative method/ansatz/model;

With B meson DAs - Resonance ansatz for $F_\perp(s, q^2)$

- $\langle \pi^+(k_1)\pi^0(k_2)|R\rangle = g_{R\pi\pi}(k_1 - k_2)^\alpha \epsilon_\alpha, \quad \langle R^+(k)|\bar{u}\gamma_\nu b|\bar{B}^0(p)\rangle = \epsilon_{\nu\alpha\beta\gamma}\epsilon^{*\alpha} q^\beta k^\gamma \frac{2V^{B\to R}(q^2)}{m_B+m_R}$.

$$F_\perp^{(\ell=1)}(s, q^2) = \frac{\sqrt{s}\sqrt{\lambda}}{\sqrt{3}} \sum_{R=\rho,\,\rho',\,\rho''} \frac{g_{R\pi\pi} V^{B\to R}(q^2) e^{i\phi_R(s)}}{(m_B + m_R)\left[m_R^2 - s - i\sqrt{s}\Gamma_R(s)\right]} \tag{27}$$

- Strong phase $\phi_R(s)$: interactions?
- Tacitly assumed to be same for all $F^{(\ell=1)}_{\perp,\parallel,t,0}(s, q^2)$.
- First ansatz: the simplest way without interaction

$$\phi_R(s, q^2) \to \phi_R(s) \to -\mathrm{Arg}\left[\frac{F_\pi^\star(s)}{m_R^2 - s - i\sqrt{s}\Gamma_R(s)}\right]. \tag{28}$$

† Adopting $\rho \to \pi\pi$ width

$$\Gamma_\rho(s) = \frac{g^2_{\rho\pi\pi}[\beta_\pi(s)]^3\sqrt{s}}{48\pi}\theta(s - 4m_\pi^2) = \Gamma_\rho^{\rm tot}\left[\frac{\beta_\pi(s)}{\beta_\pi(m_\rho^2)}\right]^3 \frac{\sqrt{s}}{m_\rho}\theta(s - 4m_\pi^2), \tag{29}$$

$$F_\pi^\star(s) = \frac{f_\rho g_{\rho\pi\pi} m_\rho}{\sqrt{2}(m_\rho^2 - s + i\sqrt{s}\Gamma_\rho(s))}, \tag{30}$$

† Recover the LCSR of $B \to \rho$ form factors, RHS of Eq. 26

$$\frac{2f_\rho m_\rho V^{B\to\rho}(q^2)}{(m_B + m_\rho)} \int_{4m_\pi^2}^{s_0^{2\pi}} ds\, e^{-s/M^2} \left(\frac{1}{\pi}\frac{\Gamma_\rho(s)\sqrt{s}}{(m_\rho^2 - s)^2 + s\Gamma_\rho^2(s)}\right) \xrightarrow{\Gamma_\rho^{\rm tot}\to 0} \frac{2f_\rho m_\rho V^{B\to\rho}(q^2)}{(m_B + m_\rho)} e^{-m_\rho^2/M^2}. \tag{31}$$

With B meson DAs - Input Parameters

$M^2 = 1.25 \pm 0.25 \mathrm{GeV}^2$:

[P. Colangelo and A. Khodjamirian, hep-ph/0010175]

- Convergence(30%) of OPE is manifested by the relatively small three-parton DA contribution (power counting);
- Higher spectral density contribution does not exceed 40% of the total integral, (quark-hadron duality).

$s_0^{2\pi} = 1.5 \pm 0.1 \mathrm{GeV}^2$

- A separate investigation with using vector $F_\pi(s)$ data from Belle
  [The Belle Collaboration, PRD78(2008)072006, PRD93(2016)032003,]
- Employing 2pSRs (SVZ) for the isospin-1 light-quark vector currents,
  [M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, NPB147(1979)385]
  $\rho \to \pi\pi$, substitute $F_\pi(s)$ in the hadronic part.
- Eq. 29 $\Rightarrow g_{\rho\pi\pi} = 5.94$.

B-meson

- $\lambda_B(1\,\mathrm{GeV}) = 460 \pm 110\,\mathrm{MeV}$ from QCDSRs, [V.M. Braun, et.al., PRD69(2004)034014]
- $f_B = 207^{+17}_{-9}\,\mathrm{MeV}$ from 2pSRs, [P. Gelhausen, et.al., PRD88(2013)014015[Erratum ibid. D89(2014)099901]]

- Sum rules Eq. 26 in a compact version

$$I_V^{\rm OPE}(q^2, M^2, s_0^{2\pi}) = \sum_R X_V^R V^R(q^2) \int_{4m_\pi^2}^{s_0^{2\pi}} ds e^{-s/M^2} \left[F_\pi^\star(s) D^R(s) e^{i\phi_R(s)}\right] \equiv \sum_R X_V^R V^R(q^2) \mathcal{I}^R(M^2, s_0^{2\pi}) \quad (32)$$

- $X_V^R = 1/(m_B + m_R)$, $V^R(q^2)$ in $z-$series formula. [A. Khodjamirian, et.al., JHEP09(2010)089]
- Large hierarchy of $|\mathcal{I}^R(1.0, 1.5)|$ generate large uncertainties in parameters fitting.

$$\text{Abs}\{\mathcal{I}^\rho,\ \mathcal{I}^{\rho'},\ \mathcal{I}^{\rho''}\} \cdot 10^2 = \{2.6,\ 0.43,\ 0.25\}, \quad (33)$$

- Second ansatz to fix the relative weight of different R to $F(s, q^2)$.
- Model-0: single-ρ with width

	$V^{B\rho}(0)$	$A_1^{B\rho}(0)$	$A_2^{B\rho}(0)$	$A_0^{B\rho}(0)$
narrow-ρ	0.34	0.26	0.21	0.30
$F_\pi^{(\rho)}$	0.36 ± 0.17	0.27 ± 0.13	0.22 ± 0.15	0.30 ± 0.06
F_π	0.41 ± 0.11	0.31 ± 0.08	0.25 ± 0.10	0.34 ± 0.04
ρ-DAs	0.33 ± 0.03	0.26 ± 0.03	0.23 ± 0.04	0.36 ± 0.04

† The finite width of ρ does not impact the $B \to \rho$ form factors significant.
† but the higher resonances in F_π does $(15\% - 20\%)$.

Model-I: $\rho + \rho'$

- Assuming the $B \to \rho$ FFs are well determined from the LCSRs with $\rho-$meson DAs.

R	$g_{R\pi\pi} V^{BR}(0)$	$g_{R\pi\pi} A_1^{BR}(0)$	$g_{R\pi\pi} A_2^{BR}(0)$	$g_{R\pi\pi} A_0^{BR}(0)$,
ρ	2.0 ± 0.2	1.6 ± 0.2	1.4 ± 0.2	2.1 ± 0.2
ρ'	3.0 ± 2.5	1.5 ± 1.4	1.0 ± 2.2	-0.3 ± 0.4

- Large uncertainties in the ρ' region, a quite appreciable ρ' contribution.

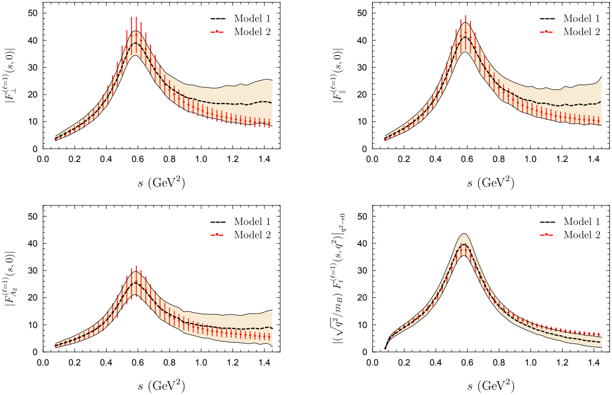
Model-II: $\rho + \rho' + \rho''$

- Assuming the relative size of contributions from R is the same as in F_π.

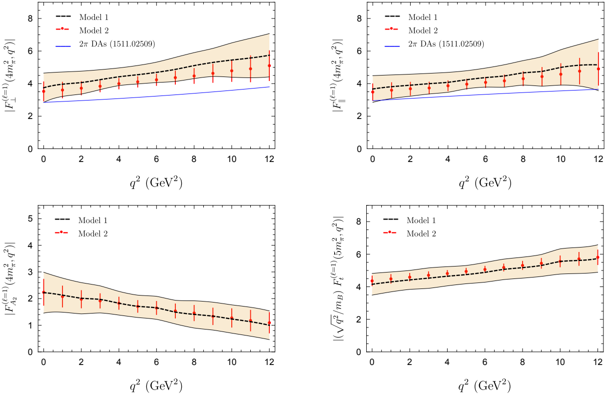
R	$g_{R\pi\pi} V^{BR}(0)$	$g_{R\pi\pi} A_1^{BR}(0)$	$g_{R\pi\pi} A_2^{BR}(0)$	$g_{R\pi\pi} A_0^{BR}(0)$
ρ	2.4 ± 0.4	1.8 ± 0.3	1.5 ± 0.3	1.9 ± 0.1
ρ'	0.35 ± 0.06	0.27 ± 0.04	0.22 ± 0.05	0.29 ± 0.02
ρ''	0.09 ± 0.01	0.07 ± 0.01	0.05 ± 0.01	0.07 ± 0.01

- Uncertainties decrease, ρ, ρ', ρ'' contributions are in the strong hierarchy.

With B meson DAs - $F_\perp(s, q^2)$ numerics [SC, A.Khodjamirian and J.Virto, JHEP05(2017)157]



- Dipion invariant mass k^2 shapes peaks around ρ mass at full recoiled $q^2 = 0$.
- Radially excited resonant states contribute $\sim 20\%$ in our model-II.
- A way to extract $B \to \rho', \rho''$ FFs if we know exactly the couplings $g_{\rho'\pi\pi}, g_{\rho''\pi\pi}$.



- Evolutions of FFs on q^2 are gentle with the acceptable uncertainties at low $k^2 = 4m_\pi^2$.
- The discrepancy between dipion and B-meson LCSRs shows up the contribution from high twist dipion LCDAs.
- Interactions between different resonances $\phi_R(s, q^2)$ should be included.

SUMMARY-II

Comparison of LCSRs with B and dipion DAs

- They give similar plots for $B \to \pi^+\pi^0$ FFs, at the same order .
- They both predict sizable contribution $(\sim 10\%)$ from $\rho', \rho'', \cdots$ and/or NR background for $P-$wave $B \to \pi^+\pi^0$ FFs.
- B-meson LCSRs does not suggest any higher partial wave contribution, WHILE it indeed exist but tiny in dipion LCSRs.
- B-meson LCSRs is more powerful (k^2, q^2, ϕ) but rely on the resonance model. Dipion LCSRs is limited by the current poor knowledge of dipion system.

Further improvements on this project

- $B_{(s)} \to$ Scalar ($f_0 \to KK$) FFs in B meson LCSRs. [SC and Jian-ming Shen, EPJC80(2020)554]
- $B \to K\pi(K^*)$ FFs. [S.Descotes-Genon and et.al.„ JHEP12(2019)083]
- $\lambda_{B_{(s)}}$. [M. Beneke and et.al., JHEP07(2018)154, A. Khodjimirian and et.al., JHEP10(2020)043]
- Complete twist-3 B-meson DAs and its contributions. [M. Beneke and et.al., JHEP05(2017)022] [N. Gubernari and et.al., JHEP01(2019)150, C.D Lü and et.al., JHEP01(2019)024]
- More data on time-like FFs: $\mathcal{A}\,\&\,\phi$.
- NLO correction on the OPE side (hard part). [Jin Gao and et.al., PRD101(2020)074035]
- Dipion DAs: twist-3, s evolution, strong phase. [SC, PRD99(2019)053005] Need more

TKANKS

Back Slides

The data of pion timelike form factor in Gounaris-Sakuria model

$$F_\pi(s) = \frac{BW_\rho^{GS}(s) + |\beta| e^{i\phi_\beta} BW_{\rho'}^{GS}(s) + |\gamma| e^{i\phi_\gamma} BW_{\rho''}^{GS}(s)}{1 + |\beta| e^{i\phi_\beta} + |\gamma| e^{i\phi_\gamma}} \,, \tag{34}$$

$$BW_R^{GS}(s) = \frac{m_R^2 + m_R \Gamma_R d}{m_R^2 - s + f(s) - i\sqrt{s}\,\Gamma_R(s)} \,, \tag{35}$$

Parameters measured in $\tau^- \to \pi^- \pi^0 \nu_\tau$

Resonance	m_R(MeV)	Γ_R(MeV)	Weight factor
ρ	$774.6 \pm 0.2 \pm 0.5$	$148.1 \pm 0.4 \pm 1.7$	1.0
ρ'	$1446 \pm 7 \pm 28$	$434 \pm 16 \pm 60$	$\lvert\beta\rvert = 0.15 \pm 0.05^{+0.15}_{-0.04}$ $\phi_\beta = 202 \pm 4^{+41}_{-8}$
ρ''	$1728 \pm 17 \pm 89$	$164 \pm 21^{+80}_{-26}$	$\lvert\gamma\rvert = 0.037 \pm 0.006^{+0.065}_{-0.009}$ $\phi_\gamma = 24 \pm 9^{+118}_{-28}$

Back Slides

$z-$parameterization for $B \to R$ decay FFs

$$\begin{aligned}
\mathcal{F}^{B\to R}(q^2) &= \frac{\mathcal{F}^{B\to R}(0)}{1-q^2/m_{\mathcal{F}}^2}\left\{1 + b_{\mathcal{F}}^R\left[z(q^2) - z(0) + \frac{1}{2}(z(q^2)^2 - z(0)^2)\right] + \cdots\right\} \\
&\equiv \frac{\kappa_{\mathcal{F}}^R + \eta_{\mathcal{F}}^R \zeta_R(q^2)}{1-q^2/m_{\mathcal{F}}^2}, \quad \mathcal{F} = V, A_0, A_1, A_2, \qquad (36)
\end{aligned}$$

$$\begin{aligned}
z(q^2) &= \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}, \qquad (37) \\
& t_\pm \equiv (m_B \pm m_R)^2, \quad t_0 = t_+(1 - \sqrt{1 - t_-/t_+}), \\
\kappa_{\mathcal{F}}^R &\equiv g_{R\pi\pi}\mathcal{F}^{B\to R}(0), \quad \eta_{\mathcal{F}}^R \equiv g_{R\pi\pi}\mathcal{F}^{B\to R}(0) b_V^R, \\
X_V^R &= X_{A_2}^R = \frac{1}{m_B + m_R}, \quad X_{A_1}^R = m_B + m_R, \quad X_{A_0}^R = -m_R. \qquad (38)
\end{aligned}$$

Back Slides

Orthogonality of Legendra Polynomials

$$\int_{-1}^{1} dx\, P_l^n(x) P_k^n(x) = \frac{2}{2l+1} \frac{(l+n)!}{(l-n)!} \delta_{kl}, \tag{39}$$

$$\int_{-1}^{1} dx\, \frac{P_l^m(x) P_l^n(x)}{1-x^2} = \frac{(l+m)!}{m(l-m)!} \delta_{mn}, \quad m = n \neq 0, \tag{40}$$

$$P_0^0(x) = 1, \quad P_0^1(x) = x, \quad P_1^1(x) = \sqrt{1-x^2}, \quad \cdots \tag{41}$$

Poisson Kernel

$$\eta_y(x) = \frac{1}{\pi} \frac{y}{x^2+y^2} = \int_{-\infty}^{\infty} d\zeta\, e^{2\pi i \zeta x - |y\zeta|} \tag{42}$$

$$\int_{-\infty}^{\infty} dx\, e^{-2\pi i(\zeta_1 - \zeta_2)x} = \delta(\zeta_1 - \zeta_2), \qquad \int_{-\infty}^{\infty} dx\, e^{-2\pi i \zeta x} = 1, \tag{43}$$