Exploring possible triangle singularities in the $\Xi_b^- \to K^- J/\psi \Lambda$ decay

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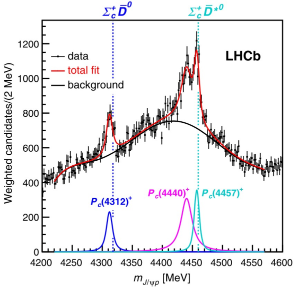
January 24, 2021

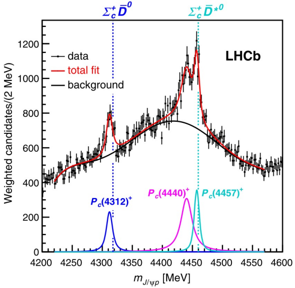
Outline

J. J. Wu, R. Molina, E. Oset and B. S. Zou, "Prediction of narrow N^* and Λ^* resonances with hidden charm above 4 GeV," Phys. Rev. Lett. **105** (2010), 232001; Phys. Rev. C **84** (2011), 015202



- $\bar{D}\Sigma_c(2520)$, $\bar{D}^*\Sigma_c(2455)$ or $\bar{D}^*\Sigma_c(2520)$ hadronic molecules
- antiquark-diquark–diquark model, i.e. $\bar{c}[cu][ud]$
- diquark-triquark model, i.e. $[cu][ud\bar{c}]$
- threshold effect and triangle singularity

J. J. Wu, R. Molina, E. Oset and B. S. Zou, "Prediction of narrow N^* and Λ^* resonances with hidden charm above 4 GeV," Phys. Rev. Lett. **105** (2010), 232001; Phys. Rev. C **84** (2011), 015202



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M. Bayar, F. Aceti, F. K. Guo and E. Oset, "A Discussion on Triangle Singularities in the $\Lambda_b \to J/\psi K^- p$ Reaction," Phys. Rev. D **94** (2016) no.7, 074039

F. K. Guo, X. H. Liu and S. Sakai, "Threshold cusps and triangle singularities in hadronic reactions," Prog. Part. Nucl. Phys. **112** (2020), 103757.

We are interested in the possible triangle singularity effects in the $J/\psi\Lambda$ invariant mass distribution.

- $\Lambda_b \to J/\psi K^0 \Lambda$

  J. X. Lu, E. Wang, J. J. Xie, L. S. Geng and E. Oset, Phys. Rev. D **93** (2016) 094009.
- $\Lambda_b \to J/\psi \eta \Lambda$

  A. Feijoo, V. K. Magas, A. Ramos and E. Oset, Eur. Phys. J. C **76** (2016) no.8, 446.
- $\Xi_b^- \to J/\psi K^- \Lambda$

  H. X. Chen, L. S. Geng, W. H. Liang, E. Oset, E. Wang and J. J. Xie, Phys. Rev. C **93** (2016) no.6, 065203.
- $\Lambda_b \to \pi^+\pi^- J/\psi \Lambda$

It has already been measured by the LHCb Collaboration.

R. Aaij *et al.* [LHCb Collaboration], "Observation of the $\Xi_b^- \to J/\psi \Lambda K^-$ decay," Phys. Lett. B **772** (2017) 265

R. Aaij *et al.* [LHCb Collaboration], "Evidence of a $J/\psi\Lambda$ structure and observation of excited Ξ^- states in the $\Xi_b^- \to J/\psi\Lambda K^-$ decay," arXiv:2012.10380.

$$P_{cs}(4459): \; M = 4458.8 \pm 2.9^{+4.7}_{-1.1} \text{ MeV},$$
$$\Gamma = 17.3 \pm 6.5^{+8.0}_{-5.7} \text{ MeV},$$

The spin-parity quantum number was not determined since the statistic is not enough.

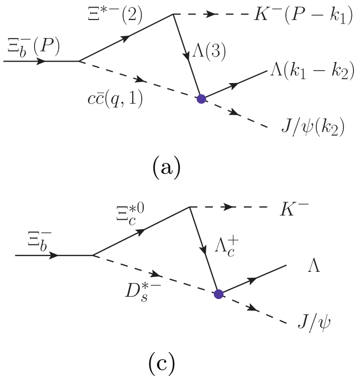
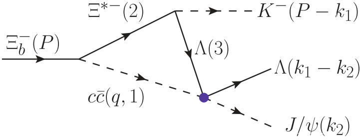


Figure: Four possible kinds of triangle diagrams for the $\Xi_b^- \to K^-\Lambda J/\psi$ decay that may have a triangle singularity of interest.

Theory

For Fig.1(a):



- $c\bar{c}$: $\eta_c, J/\psi, \chi_{c0}, \chi_{c1}, \chi_{c2}, h_c, \eta_c(2S), \Psi(2S)$.
- Ξ^{*-}: all 11 Ξ states in PDG.

For Fig.1(b),(c), all the D_s^{*-} and Ξ_c^{*0} in PDG are considered.

We find there are in total 11 cases for Fig.1(a) and 7 cases for Fig.1(b),(c) that would have singularities in the physical region.

No.	$c\bar{c}$	Ξ^{*-}	Position of TS (MeV)
1	J/ψ	$\Xi(2500)$	4232
2	χ_{c0}	$\Xi(2250)$	4546
3	χ_{c0}	$\Xi(2370)$	4665
4	χ_{c1}	$\Xi(2120)$	4628
5	χ_{c1}	$\Xi(2250)$	4696
6	χ_{c2}	$\Xi(2120)$	4680
7	h_c	$\Xi(2120)$	4644
8	h_c	$\Xi(2250)$	4730
9	$\eta_c(2S)$	$\Xi(2030)$	4754
10	$\eta_c(2S)$	$\Xi(2120)$	4797
11	$\psi(2S)$	$\Xi(2030)$	4810

No.	D_s^{*-}	Ξ_c^{*0}	Intermediate particle 3	**Position of TS (MeV)**
1	$\bar{D}^*_{s1/s3}(2860)$	$\Xi_c(2930)$	$\bar{D}^0$	4838
2	$\bar{D}^*_{s1}(2700)$	$\Xi_c(3055)$	$\bar{D}^0$	4922
3	$\bar{D}^*_{s1}(2700)$	$\Xi_c(3080)$	$\bar{D}^0$	4957
4	$\bar{D}^*_{s1/s3}(2860)$	$\Xi_c(2930)$	$\bar{D}^{*0}$	4959
5	$\bar{D}^*_{s1}(2700)$	$\Xi_c(3080)$	$\bar{D}^{*0}$	5089
6	$\bar{D}^*_{s1}(2700)$	$\Xi_c(3080)$	Λ_c^+	4990
7	$\bar{D}^*_{s1/s3}(2860)$	$\Xi_c(2930)$	Λ_c^+	5149

- The $\bar{D}_{sJ}(3040)\Xi_c(3645)\bar{D}^0$ loop also can give rise to a triangle singularity at 4511 MeV. However, so far the $D_{sJ}(3040)$ has only been observed in D^*K, suggesting that it might have an unnatural parity and thus could not decay into DK.
- the $D^*_{s1}(2700)$ was observed in $B \to D\bar{D}K$.
- the $D^*_{s1/s3}(2860)$ were seen in the decays of $B_s \to \bar{D}\bar{K}\pi$.
- Once a structure is observed in the $J/\psi\Lambda$ spectrum above 4.9 GeV in a future experiment, such triangle singularities need to be considered.

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We first turn to the three vertices:

$\Xi_b \to \Xi^* c\bar{c}$: Its strength is comparable with that of $B \to K c\bar{c}$ for the same charmonium.

Table: The branching ratios for different charmonia in $B \to Kc\bar{c}$.

$c\bar{c}$	$BR(B^0 \to c\bar{c}K^0)$	$BR(B^+ \to c\bar{c}K^+)$
J/ψ	8.73×10^{-4}	1.01×10^{-3}
χ_{c0}	1.11×10^{-6}	1.49×10^{-4}
χ_{c1}	3.93×10^{-4}	4.84×10^{-4}
χ_{c2}	$< 1.5 \times 10^{-5}$	1.1×10^{-5}
h_c		$< 3.8 \times 10^{-5}$
$\eta_c(2S)$		4.4×10^{-4}
$\psi(2S)$	5.8×10^{-4}	6.21×10^{-4}

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$c\bar{c}\Lambda \to J/\psi\Lambda$: For h_c and $\eta_c(2S)$, the total spin of $c\bar{c}$ is 0, which indicates a spin flip and it is suppressed by the HQSS.

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$\Xi^*\Lambda\bar{K}$: We infer its strength from the $\Xi^* \to \Lambda\bar{K}$ process for each Ξ^* state.

Table: The decay of different Ξ^* states to $\Lambda\bar{K}$.

Ξ^*	$\Xi^* \to \Lambda\bar{K}$
$\Xi(2030)$	$\sim 20\%$
$\Xi(2120)$	The only seen and listed decay mode, but without exact ratio.
$\Xi(2250)$	Not listed. All the decay channels are three-body states.
$\Xi(2370)$	Not listed.
$\Xi(2500)$	Listed.

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We then turn to the character of the intermediate particles in the loop, especially their quantum numbers.

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The spin of $\Xi(2030)$ is equal to or greater than $\frac{5}{2}$. It leads to high partial-wave $(l \geq 2)$ interaction at the $\Xi(2030)\bar{K}\Lambda$ vertex.

$$I = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{(q^2 - m_1^2 + im_1\Gamma_1)[(P-q)^2 - m_2^2 + im_2\Gamma_2][(k_1 - q)^2 - m_3^2 + i\epsilon]},$$

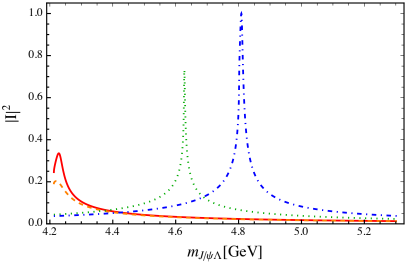


Figure: The value of $|I|^2$ in the $m_{J/\psi\Lambda}$ invariant mass distribution. The red solid (orange dashed), green dotted and blue dotdashed lines correspond to the Nos. 1, 4 and 11 cases, respectively.

Table: The lowest allowed partial waves of the $\Xi_b\Xi(2120)\chi_{c1}$ and $\Xi(2120)\Lambda\bar{K}$ vertices with different spin-parities of $\Xi(2120)$.

J^P	$\Xi_b\Xi(2120)\chi_{c1}$	$\Xi(2120)\Lambda\bar{K}$
$1/2^+$	S-wave	P-wave
$1/2^-$	S-wave	S-wave
$3/2^+$	S-wave	P-wave
$3/2^-$	S-wave	D-wave
$5/2^+$	P-wave	F-wave
$5/2^-$	P-wave	D-wave

In the Lorentz covariant orbital-spin coupling scheme, the interaction of a vertex is directly related to its partial wave L and the relative momentum of the two final particles, i.e., the strength of interaction is proportional to q^L.

For a fixed $J^P(\Xi^*)$, the quantum numbers of the $J/\psi\Lambda$ system:

- $\frac{1}{2}^+$ or $\frac{3}{2}^+$:
  - S-wave in $\chi_{c1}\Lambda$ vertex
  - P-wave in $J/\psi\Lambda$ vertex
- $\frac{1}{2}^-$ or $\frac{3}{2}^-$:
  - P-wave in $\chi_{c1}\Lambda$ vertex
  - S-wave in $J/\psi\Lambda$ vertex

For a fixed $J^P(\Xi^*)$, the quantum numbers of the $J/\psi\Lambda$ system:

- $\frac{1}{2}^+$ or $\frac{3}{2}^+$:
  - S-wave in $\chi_{c1}\Lambda$ vertex
  - P-wave in $J/\psi\Lambda$ vertex $\Longrightarrow$ k_1
- $\frac{1}{2}^-$ or $\frac{3}{2}^-$:
  - P-wave in $\chi_{c1}\Lambda$ vertex $\Longrightarrow$ q
  - S-wave in $J/\psi\Lambda$ vertex

B. S. Zou and F. Hussain, "Covariant L-S scheme for the effective N^*NM couplings," Phys. Rev. C **67** (2003), 015204

Taking the quantum numbers of the $J/\psi\Lambda$ system being $1/2^+$ and $J^P(\Xi(2120)) = 1/2^-$. The vertices are constructed as

$$\begin{aligned}
t_{\Xi_b \Xi^* \chi_{c1}} &= g_a \bar{u}(P-q)\gamma^\mu u(P)\epsilon^*_\mu(q), \\
t_{\Xi^* \Lambda \bar{K}} &= g_b \bar{u}(k_1-q)u(P-q), \\
t_{\Lambda \chi_{c1} \Lambda J/\psi} &= g_c \epsilon_\mu(q)\bar{u}(k_1-k_2)\left(\not{k}_1 + \sqrt{k_1^2}\right)\gamma_5\left(\gamma^\mu - \frac{k_1^\mu \not{k}_1}{k_1^2}\right)u(k_1-q) \\
&\quad \times \epsilon^*_\nu(k_2)k_1^\nu \frac{k_1^2 + m^2_{J/\psi} - m^2_\Lambda}{k_1^2},
\end{aligned}$$

$$\begin{aligned}
\mathcal{M} = g\int \frac{d^4q}{(2\pi)^4}\bar{u}(k_1-k_2)\left(\not{k}_1 + \sqrt{k_1^2}\right)\gamma_5\left(\gamma^\mu - \frac{k_1^\mu \not{k}_1}{k_1^2}\right)\frac{\not{k}_1 - \not{q} + m_3}{(k_1-q)^2 - m_3^2} \\
\times \frac{\not{P} - \not{q} + m_2}{(P-q)^2 - m_2^2 + im_2\Gamma_2}\frac{-\gamma_\mu + \frac{q_\mu \not{q}}{m_1^2}}{q^2 - m_1^2 + im_1\Gamma_1}u(P)\frac{k_1^2 + m^2_{J/\psi} - m^2_\Lambda}{k_1^2}\epsilon^*_\nu(k_2)k_1^\nu .
\end{aligned}$$

Taking the quantum numbers of the $J/\psi\Lambda$ system being $1/2^-$ and $J^P(\Xi(2120)) = 1/2^-$. The amplitude for the $\Lambda\chi_{c1}\Lambda J/\psi$ vertex reads

$$t_{\Lambda\chi_{c1}\Lambda J/\psi} = g_c\bar{u}(k_1-k_2)\gamma_5\left(\gamma^\mu - \frac{k_1^\mu \not{k}_1}{k_1^2}\right)\left(\not{k}_1 + \sqrt{k_1^2}\right)u(k_1-q)\epsilon_\mu^*(k_2) \times\epsilon_\nu(q)k_1^\nu\frac{2k_1\cdot q}{k_1^2},$$

$$\mathcal{M} = g\int\frac{d^4q}{(2\pi)^4}\bar{u}(k_1-k_2)\gamma_5\left(\gamma^\mu - \frac{k_1^\mu \not{k}_1}{k_1^2}\right)\left(\not{k}_1 + \sqrt{k_1^2}\right)\frac{\not{k}_1 - \not{q} + m_3}{(k_1-q)^2 - m_3^2} \times\frac{\not{P} - \not{q} + m_2}{(P-q)^2 - m_2^2 + im_2\Gamma_2}\frac{-\not{k}_1 + \frac{\not{q}}{m_1^2}k_1\cdot q}{q^2 - m_1^2 + im_1\Gamma_1}u(P)\frac{2k_1\cdot q}{k_1^2}\epsilon_\mu^*(k_2).$$

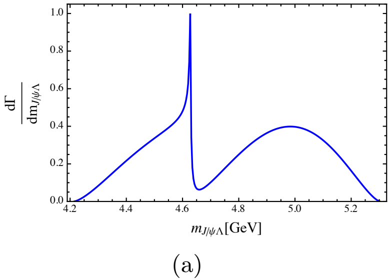
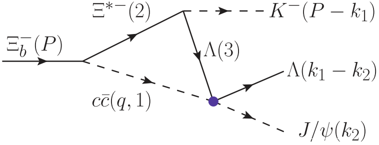


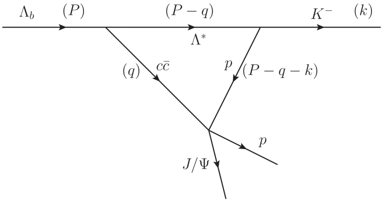
Figure: The $m_{J/\psi\Lambda}$ invariant mass distribution for the $\Xi_b^- \to K^- J/\psi\Lambda$ process via the $\chi_{c1}\Xi(2120)\Lambda$ loop with the quantum numbers of the $J/\psi\Lambda$ system being (a) $1/2^+$, and (b) $1/2^-$.

- In the $1/2^-$ case, when constructing the four-particle vertex, the P-wave interaction in the $\chi_{c1}\Lambda$ system introduces a factor of the momentum difference of the exchanged Λ and χ_{c1}. Thus, the amplitude is directly proportional to the velocity difference between these two particles.
- More specifically, the factor $\left(-\not{k}_1 + \frac{\not{q}}{m_1^2} k_1 \cdot q\right)$ from the amplitude in the triangle singularity region can be derived as follows,

$$-\not{k}_1 + \frac{\not{q}}{m_1^2} k_1 \cdot q \sim \frac{|\vec{q}|}{q^0} - \frac{|\vec{k}_1|}{k_1^0} \sim v_\Lambda - v_{\chi_{c1}} \sim 0.$$

- The threshold of $\chi_{c1}\Lambda$ is at 4.626 GeV, which gives rise to a cusp (for the S-wave) in the distribution of the differential decay width, and the position of the triangle singularity locates about 4.628 GeV. As they are quite close, these two structures would lead to only one peak around this energy.
- The $3/2^+$ and $3/2^-$ cases should be similar to the $1/2^+$ and $1/2^-$ cases, respectively.
- If a structure is observed in this region experimentally in the future, the TS effects need to be taken into account for the $J^P = \frac{1}{2}^+$ or $\frac{3}{2}^+$ quantum numbers.





- The $\Lambda(1890)\chi_{c1}p$ loop diagram is favored over other choices.
- We can roughly regard the strengths of these two diagrams to be of the same order of magnitude.
- Studying possible structures in the $\Xi_b \to K^- J/\psi\Lambda$ decay around 4.63 GeV should also provide useful insights into the possible role of triangle singularity in the $\Lambda_b \to K^- J/\psi p$ decay around 4.45 GeV.

Summary

- There are 11 and 7 possible TS with various combinations of charmonium-Ξ^* states and D_S^*-Ξ_c^* states, respectively.
- For the latter, those with $\bar{D}_{s1}^*(2700)$ should be more important than the others.
- For the former, the $\chi_{c1}\Xi(2120)\Lambda$ loop diagram is expected to be the most promising process to observe the structure, and a narrow peak would be produced around 4.63 GeV in the $J/\psi\Lambda$ invariant mass distribution when $J^P(J/\psi\Lambda) = 1/2^+$ or $3/2^+$.
- Such effects needs to be taken into account in the search of hidden-charm strange pentaquarks, and a study of the $\Xi_b \to K^- J/\psi\Lambda$ will also shed light on the role of triangle singularities in the $\Lambda_b \to K^- J/\psi p$ process.

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Thank You for Your Attention!