



Nucleon Resonances and Kaonic Atoms with Hamiltonian Effective Field Theory

Zhan-Wei Liu

School of Physical Science and Technology, Lanzhou University

ZWL Kamleh Leinweber Stokes Thomas Wu, **PRL**116, 082004

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Introduction

Hadron Physics

Hadron physics is mainly focused on hadron scatterings, spectra, structures, interactions, etc.

- Hadron spectra are obtained from experimental Hadron scattering.
- Hadron structures and interactions \Leftrightarrow Hadron spectra and scattering.

Hadron physics lies in the region of low energies with a large α_s , traditional perturbation expansion in series of $(\alpha_s)^n$ cannot work here.

- constituent quark model
- effective field theory —expanded by small momenta
- lattice QCD —discretized QCD
- QCD sum rule —operator product expansion—twist
- large N_c — $1/N_c$
- ...

- LQCD starts from the first principle of QCD
- model independent, reliable
- LQCD gives hadron spectra and quark distribution functions at finite volumes, large quark masses, discrete spaces
- not directly related to physical observables

Connection between Scattering Data and Lattice QCD Data

Lattice QCD

- large pion mass: extrapolation
- finite volume
- discrete space

Lattice QCD Data \rightarrow Physical Data

- Lüscher Formalisms and extensions:
 - Model independent; efficient in single-channel problems
 - Spectrum \rightarrow Phaseshifts; $m_{K_L} - m_{K_S}$ etc.
- Effective Field Theory (EFT), Models, etc
 - with low-energy constants fitted by Lattice QCD data

Physical Data \rightarrow Lattice QCD Data

- EFT: discretization, analytic extension, Lagrangian modification
- various discretization: eg. discretize the momentum in the loop

Effective field theory deals with extrapolation powerfully.

Finite-volume effect can be studied by discretizing the EFT.

discretize the mass equation (in **integral form**)

(most of time, potentials are momentum independent.)

discretize the Hamiltonian equation (in **differential form**)

Discrete spacing effects can also be studied with EFT.

Scattering Data and Lattice QCD data are two important sources for studying resonances.

We should try to analyse them both at the same time.

Hamiltonian Effective Field Theory

Hamiltonian Effective Field Theory (HEFT)

analyses both **experimental data at infinite volume**
and **lattice QCD results at finite volume** at the same time.

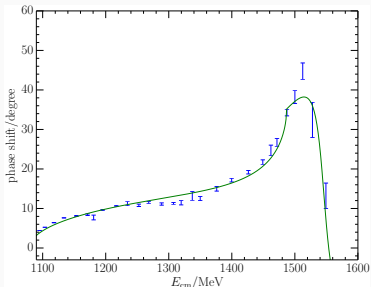
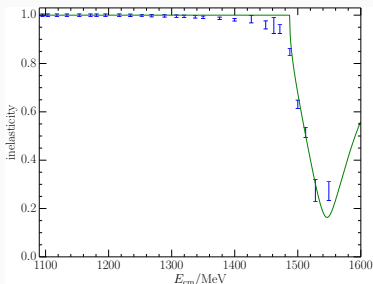
- at infinite volume
 - Lagrangian (via 2-particle irreducible diagrams) \rightarrow
 - potentials (via Bethe-Salpeter Equation) \rightarrow
 - phaseshifts and inelasticities
- at finite volume
 - potentials discretized (via Hamiltonian Equation) \rightarrow spectra
 - wavefunctions: analyse the structure of the eigenstates on the lattice
- finite-volume and infinite-volume results are connected by the coupling constants etc.

Nucleon excitation resonance with Hamiltonian effective field theory

$N^*(1535)$ with πN Scattering

$N^*(1535)$ is the lowest resonance with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- One needs to consider the interactions among the bare baryon N_0^* , πN channel, and ηN channel.
- Phase shifts and inelasticities are obtained by solving Bethe-Salpeter equation with the interactions.



πN Scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$.

- Pole position for $N^*(1535)$: $1531 \pm 29 - i 88 \pm 2$ MeV.
Particle Data Group (PDG): $1510 \pm 20 - i 85 \pm 40$ MeV.

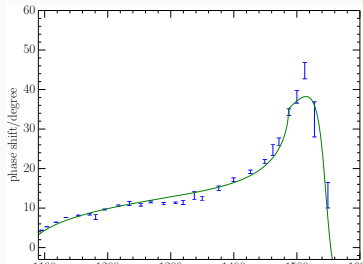
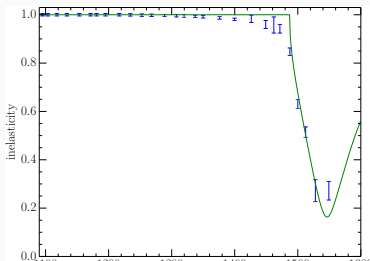
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$$G_{\pi N; N_0^*}^2(k) = \frac{3g_{\pi N; N_0^*}^2}{4\pi^2 f^2} \omega_\pi(k)$$
$$V_{\pi N; \pi N}^S(k, k') = \frac{3g_{\pi N}^S}{4\pi^2 f^2} \frac{m_\pi + \omega_\pi(k)}{\omega_\pi(k)} \frac{m_\pi + \omega_\pi(k')}{\omega_\pi(k')} \quad (1)$$

- Phase shifts and inelasticities are obtained by solving Bethe-Salpeter equation with the interactions.

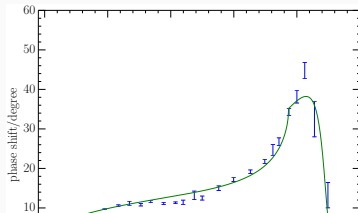
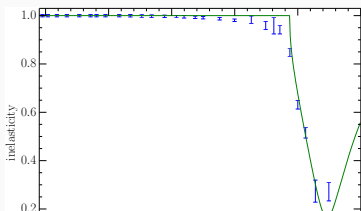


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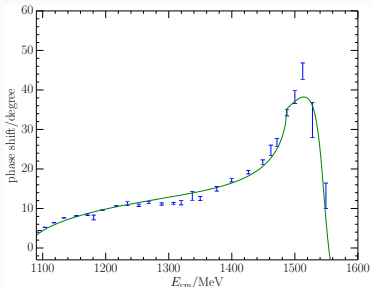
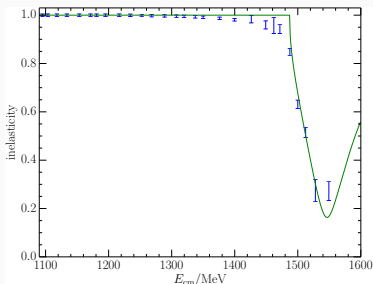
$$T_{\alpha,\beta}(k, k'; E) = V_{\alpha,\beta}(k, k') + \sum_{\gamma} \int q^2 dq$$
$$V_{\alpha,\gamma}(k, q) \frac{1}{E - \sqrt{m_{\gamma 1}^2 + q^2} - \sqrt{m_{\gamma 2}^2 + q^2} + i\epsilon} T_{\gamma,\beta}(q, k'; E)$$



$N^*(1535)$ with πN Scattering

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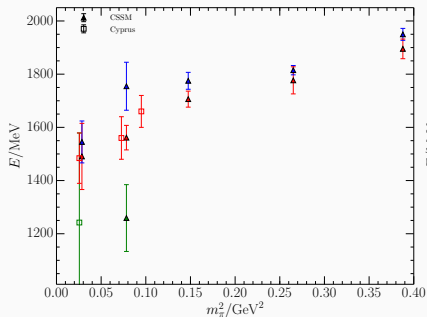
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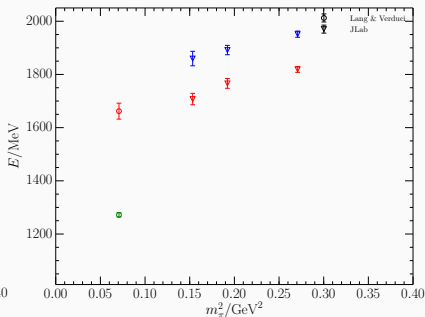
Particle Data Group (PDG): $1510 \pm 20 - i 85 \pm 40$ MeV.

Spectra at Finite Volumes

3 sets of lattice data at different pion masses and finite volumes



$L \approx 3 \text{ fm}$



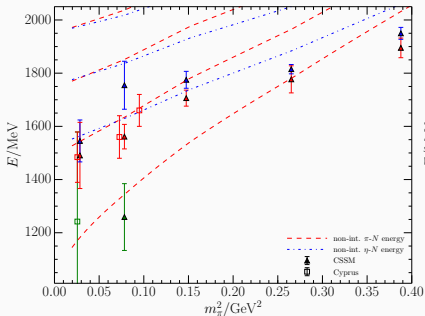
$L \approx 2 \text{ fm}$

Spectra with $I(J^P) = \frac{1}{2}(1_2^-)$ at finite volumes

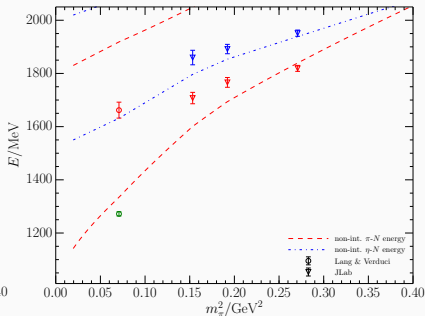
Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes

Non-interacting energies of the two-particle channels



$L \approx 3 \text{ fm}$

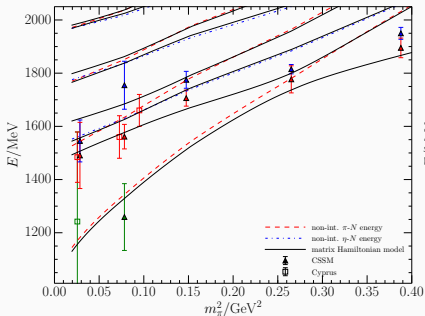


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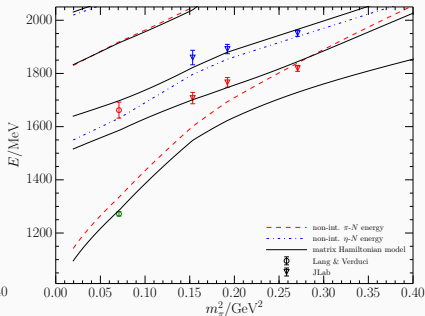
Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

Spectra at Finite Volumes

3 sets of lattice QCD data at different pion masses and finite volumes
Non-interacting energies of the two-particle channels
Eigenenergies of Hamiltonian effective field theory



$L \approx 3$ fm



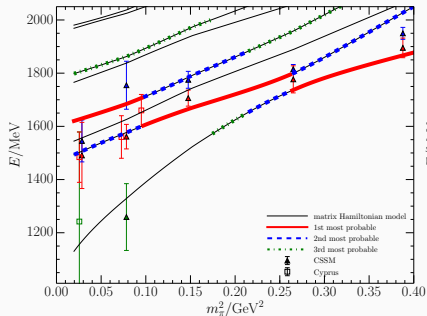
$L \approx 2$ fm

Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ at finite volumes

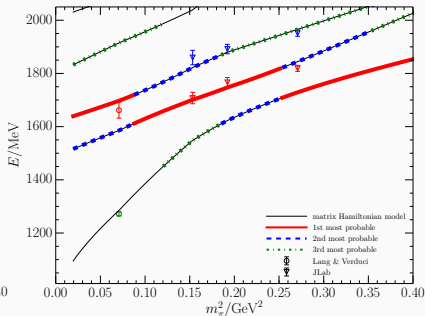
Spectra at Finite Volumes

3 sets of lattice data at different pion masses and finite volumes
Eigenenergies of Hamiltonian effective field theory

Coloured lines indicating most probable states observed in LQCD



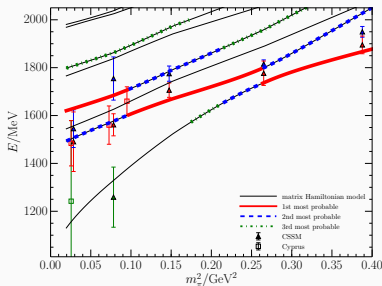
$L \approx 3$ fm



$L \approx 2$ fm

Spectra with $I(J^P) = \frac{1}{2}(1/2^-)$ at finite volumes

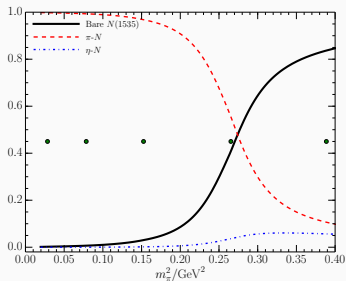
Components of Eigenstates with $L \approx 3$ fm



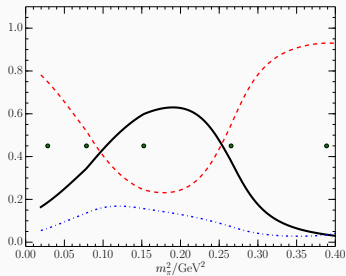
Spectra with $I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ and $L \approx 3$ fm

- The 1st eigenstate at light quark masses is mainly πN scattering states.
- The most probable state at physical quark mass is the 4th eigenstate. It contains about 60% bare $N^*(1535)$, 20% πN and 20% ηN .

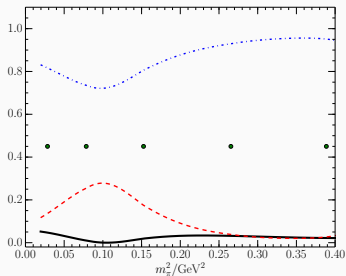
Components of Eigenstates with $L \approx 3$ fm



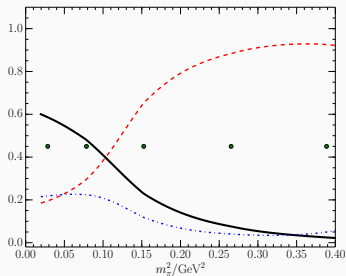
1st eigenstate



2nd eigenstate



3rd eigenstate



4th eigenstate

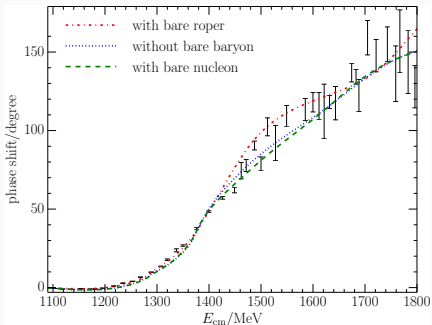
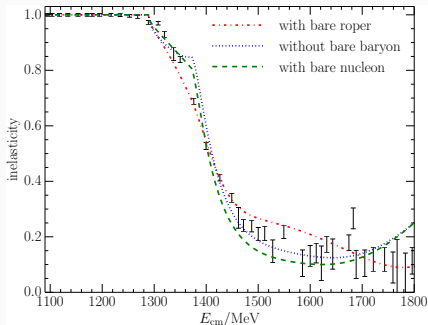
$N^*(1440)$ Resonance

- $N^*(1440)$, usually called Roper, is the excited state $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$
- Naive quark model predicts $m_{N^*(1440)} > m_{N^*(1535)}$ if they are both dominated by 3-quark core. But contrary to experiment.

To check whether a 3-quark core largely exists in Roper, we consider models

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

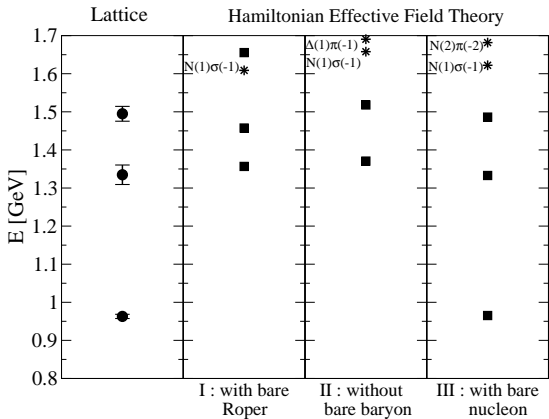
$N^*(1440)$ Resonance



πN scattering with $I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$

- with a bare Roper
- without any bare baryons
- including the effect of the bare nucleon

Our results are verified

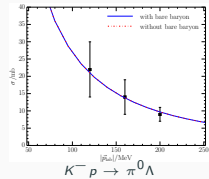
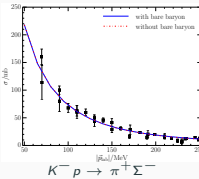
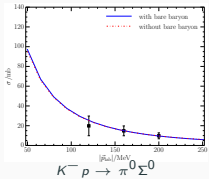
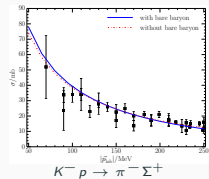
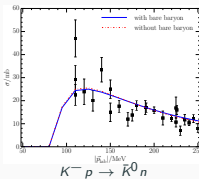
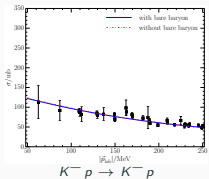


interpolating operators: $N(0)$, $N(0)\sigma(0)$, $N(p)\pi(-p)$, $\Delta(p)\pi(-p)$. from Lang, Leskovec, Padmanath, Prelovsek, [PRD95 \(2017\) no.1, 014510](#).

No these two higher states with $N^{-P}(0)\pi(0)$... from CMMS.

$\Lambda(1405)$ with $K^- p$ scattering

- The well-known Weinberg-Tomozawa potentials are used.
momentum-dependent, non-separable
- We can fit the cross sections of $K^- p$ well
both with and without a bare baryon.



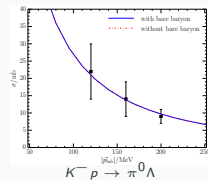
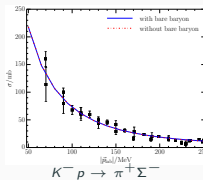
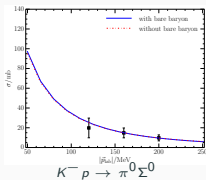
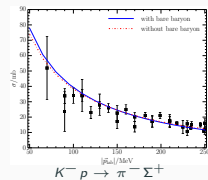
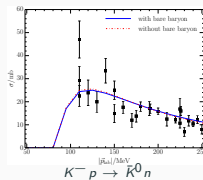
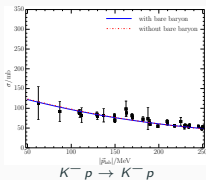
- Two-pole structure of $\Lambda(1405)$
1430 - i 22 MeV, 1338 - i 89 MeV

$\Lambda(1405)$ with K^-p scattering

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momentum-dependent, non-separable

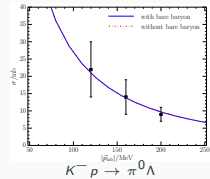
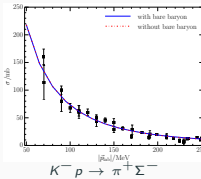
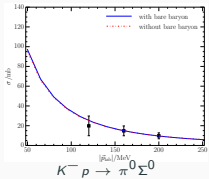
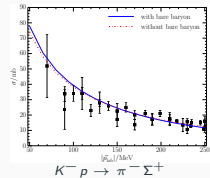
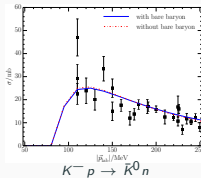
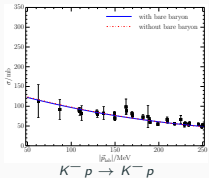
$$V_{\alpha,\beta}(k, k') = g_{\alpha,\beta} \frac{\omega_{\alpha_M}(k) + \omega_{\beta_M}(k')}{8\pi^2 f^2 \sqrt{2\omega_{\alpha_M}(k)} \sqrt{\omega_{\beta_M}(k')}}$$

- We can fit the cross sections of K^-p well
both with and without a bare baryon.



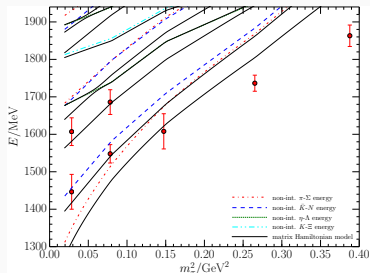
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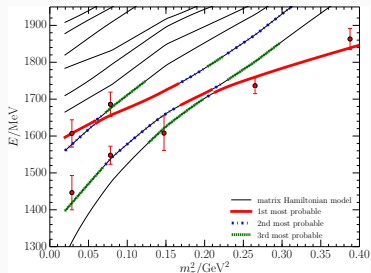


- Two-pole structure of $\Lambda(1405)$
 $1430 - i22$ MeV, $1338 - i89$ MeV

Spectrum on the Lattice



without a bare baryon



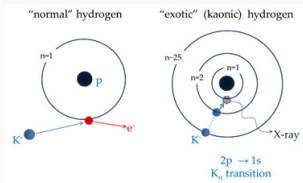
with a bare baryon

Spectra with $S = -1, I(J^P) = 0(\frac{1}{2}^-)$ in the finite volume.

- The bare baryon is important for interpreting the lattice QCD data at large pion masses.
- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state containing very little of bare baryon at physical pion mass.

Kaonic hydrogen and deuteron from Hamiltonian effective field theory

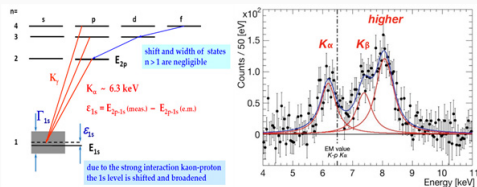
Mesonic Atoms



Experimental progresses

- pionic hydrogen and deuterium, the Paul Scherrer Institute (PSI), Ref[Hauser:1998yd]
- kaonic hydrogen, SIDDHARTA-2, Ref[Curceanu:2013bxa]
- kaonic deuterium, proposed by SIDDHARTA-2 and the J-PARC E57

Kaonic Hydrogen



- energy shift and width of 1s level were measured at SIDDHARTA-2

$$\epsilon_{1S}^p = 283 \pm 36(\text{stat}) \pm 6(\text{sys}) \text{ eV},$$

$$\Gamma_{1S}^p = 541 \pm 89(\text{stat}) \pm 22(\text{sys}) \text{ eV},$$

- they are related to the scattering length of K^-p

$$\epsilon_{1S}^p - \frac{i}{2}\Gamma_{1S}^p = \frac{-2\alpha_e^3 \mu_{K^-p}^2 a_{K^-p}}{1 + 2\alpha_e \mu_{K^-p} (\ln \alpha_e - 1) a_{K^-p}},$$

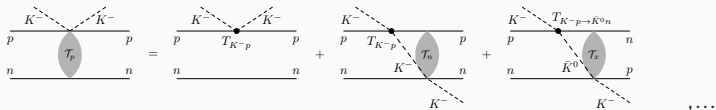
- HEFT provides

$$\epsilon_{1S}^p = 307 \text{ eV}, \quad \Gamma_{1S}^p = 533 \text{ eV},$$

where $\bar{K}N$ interactions are not fine tuned.

Kaonic Deuteron without Recoil Effect

$\bar{K}NN$ scattering amplitude can be solved by the Faddeev equation



With the static approximation,

$$a_{K^-d} = \frac{m_d}{m_K + m_d} \int d^3\vec{r} |\psi_d(\vec{r})|^2 \hat{A}_{K^-d}(r),$$

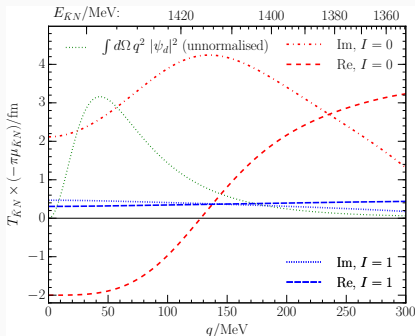
where

$$\hat{A}_{K^-d}(r) = \frac{\tilde{a}_{K^-p} + \tilde{a}_{K^-n} + (2\tilde{a}_{K^-p}\tilde{a}_{K^-n} - b_x^2)/r - 2b_x^2\tilde{a}_{K^-n}/r^2}{1 - \tilde{a}_{K^-p}\tilde{a}_{K^-n}/r^2 + b_x^2\tilde{a}_{K^-n}/r^3}.$$

Our results without recoil effect are similar to others

$$\epsilon_{1S}^d|_{\text{StaticApprox}} = 855 \text{ eV}, \quad \Gamma_{1S}^d|_{\text{StaticApprox}} = 1127 \text{ eV}.$$

Recoil Effect

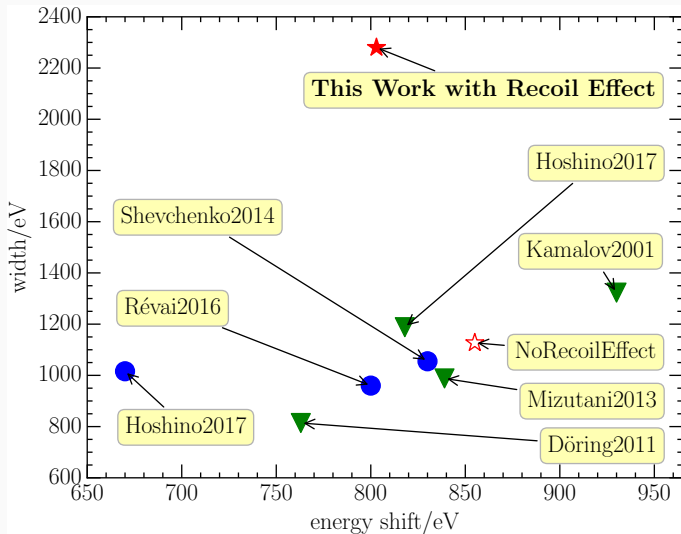


- The recoil effect is mainly from the single scattering process

$$\langle T_{\bar{K}N}^d \rangle \equiv \int d^3 \vec{q} |\psi_d(\vec{q})|^2 T_{\bar{K}N}(\vec{q}).$$

- If no $\Lambda(1405)$ exists,
this kind of recoil effect can be totally neglected.

Comparison



Summary

Summary

In this talk we have introduced the Hamiltonian Effective Field theory and applied it in the low-lying baryons and kaonic atoms.

By analyzing the scattering data and LQCD data,

- $N^*(1535)$ contains a 3-quark core;
- $N^*(1440)$ should contain little of 3-quark consistent;
- $\Lambda(1405)$ is mainly a $\bar{K}N$ molecular state at physical quark mass, while a 3-quark core dominates at large quark masses.

Recoil effect makes kaonic deuteron much short lived because of the close $\Lambda(1405)$ through HEFT study.

欢迎参加 “第二届强子与重味物理理论与实验联合研讨会”
兰州大学 2021年3月25日—28日



<https://indico.ihep.ac.cn/event/13399/overview>