DDK 3-body system in Lattice QCD

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Based on *PRD 102, 114515* and *on-going work*

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强子谱和强子结构研讨会　Jan. 23, 2021

1. *Motivation*
2. *DDK 3-body effective field theory*
3. *DDK 3-body system in a finite volume*
4. *DDK 3-body lattice spectrum*
5. *Summary and Outlook*

1. Motivation
2. DDK 3-body effective field theory
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4. DDK 3-body lattice spectrum
5. Summary and Outlook

- *$D-K$ interaction*
  - $D_{s0}^{*}(2317)(c\bar{s})$, $I(J^P)=0(0^+)$
    PRL 90,242001(2003)
  - *$D-K$ molecular state*
    EPJ Web Conf. 202,02001(2019)
  - *$D-K$ strongly attractive interaction in S-wave*

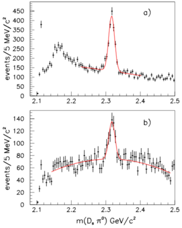


FIG. 2: The $D_s^+\pi^0$ mass distribution for (a) the decay $D_s^+ \to K^+K^-\pi^+$ and (b) the decay $D_s^+ \to K^+K^-\pi^+\pi^0$. The fits to the mass distributions as described in the text are indicated by the curves.

arXiv:hep-ex/0304021

- *DDK 3-body system*
  - *$D-D_{s0}^{*}$ interaction*
    PRD 98(2018),054001
  - *DDK* $(cc\bar{s}\bar{q})$ $\frac{1}{2}(0^-)$, *$DDDK$* $1(0^+)$ *bound states*
    PRD 100(2019)3,034029

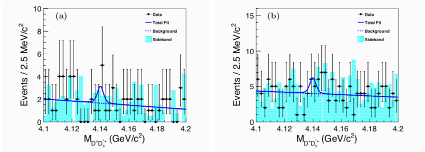
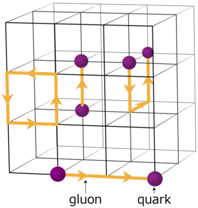


FIG. 4: The invariant-mass spectra of $D^+D_s^{*+}$ in the (a) $\Upsilon(1S)$ and (b) $\Upsilon(2S)$ data samples. The cyan shaded histograms are from the normalized M_{D^+} and $M_{D_s^{*+}}$ sideband events. The blue solid curves show the fitted results with the R^{++} mass fixed at 4.14 GeV/c^2 and width fixed at 2 MeV, and the blue dashed curves are the fitted backgrounds.

arXiv:2008.13341

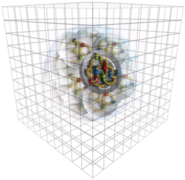
- *Lattice Quantum ChromoDynamics*

$$\mathscr{L}_{\text{QCD}} = \sum_i \bar{\psi}_i (i\not{D}_\mu - m_i)\psi_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$



Unphysical: lattice spacing a, lattice size L

Build operators for DDK system



$$D^0 = \bar{u}(x)\gamma_5 c(x),\ K^+ = \bar{s}(x)\gamma_5 u(x)$$

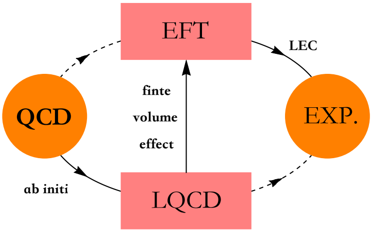
$$\mathscr{O}(\tau) \sim \left[D^0(\tau) D^0(\tau) K^+(\tau)\right]$$

$$C(\tau) = \langle \mathscr{O}(\tau)\mathscr{O}(0)\rangle \sim e^{-E\tau}$$

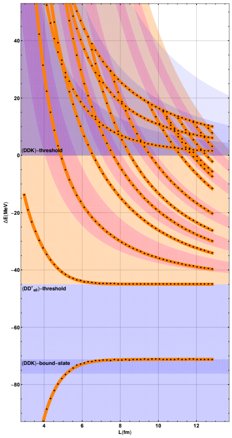
Eigen-energy of 3-body system

$E(L)$ is function of lattice size

- *DDK effective field theory*



Hammer, JYP and Rusetsky. JHEP 10(2017)



- *DDK lattice spectrum and physical quantities*
  - *DDK 3-body bound state*
  - $D - D^*_{s0}(2317)$ *scattering length*
  - *DDK 3-body decay Dalitz plot*

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- $D-K$ *potential* L.-S. Geng et al. PRD 100(2019)3,034029

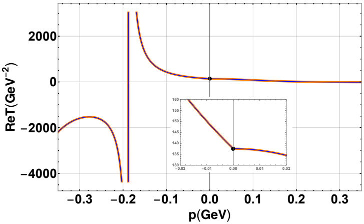
$$\tilde{V}_{DK}(r) = C_L e^{-(r/R_c)^2} + C_S e^{-(r/R_s)^2}, \quad R_c = 1\text{fm},\ R_s = 0.5\text{fm} \tag{1}$$

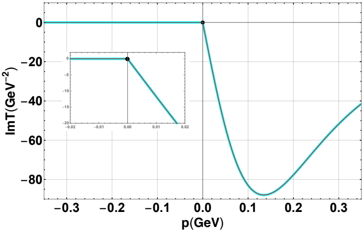
- $D-K$ *scattering equation*

$$T(\mathbf{p},\mathbf{q}) = V(\mathbf{p},\mathbf{q}) + \int \frac{d^3k}{(2\pi)^3} V(\mathbf{p},\mathbf{k})\, G(\mathbf{k})\, T(\mathbf{k},\mathbf{q}) \tag{2}$$

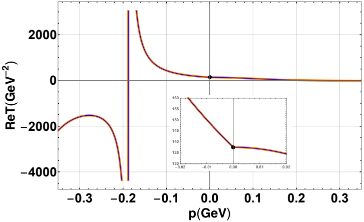
$$\text{where } V(\mathbf{p},\mathbf{k}) = \int d^3x \tilde{V}(\mathbf{x}) e^{-i(\mathbf{p}-\mathbf{k})\mathbf{x}} \tag{3}$$

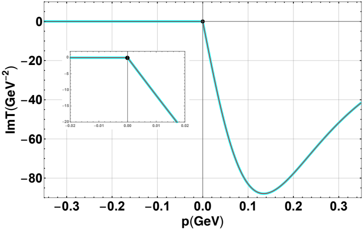
- $D-K$ *scattering amplitude*





- $D-K$ *scattering amplitude*





- $D-K$ *scattering phase shift*

$$T(p) = -\frac{2\pi}{\mu}\frac{1}{p\cot\delta - ip} \tag{4}$$

$$\text{where } p\cot\delta = -a_{DK}^{-1} + \frac{1}{2}r_{DK}p^2, \quad a_{DK} = 1.683\text{fm}, \; r_{DK} = 0.792\text{fm} \tag{5}$$

- *Analytic continuation below threshold* $(\kappa = -ip)$ $D_{s0}^*(2317)$

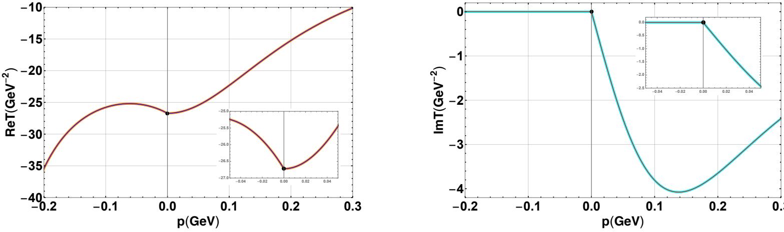
$$T(\kappa) = -\frac{2\pi}{\mu}\left[\frac{C_{DK}}{\kappa - \kappa_{DK}} + R_{DK} + R_{DK}^{(1)}\kappa + R_{DK}^{(2)}\kappa^2\right], \quad \kappa_{DK} = 187.795\text{MeV}. \tag{6}$$

- $D-D$ *potential* L.-S. Geng et al. PRD 100(2019)3,034029

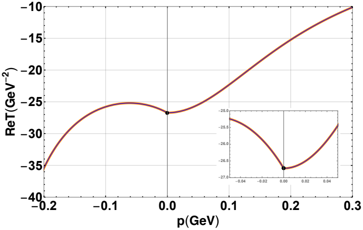
$$\tilde{V}_{DD}(\mathbf{p},\mathbf{k}) = \sum_{V=\sigma,\rho,\omega} C_{\text{iso.}}(V)\frac{g_V^2}{(\mathbf{p}-\mathbf{k})^2+m_V^2}\left(\frac{\Lambda^2-m_V^2}{(\mathbf{p},\mathbf{k})^2+\Lambda^2-q_0^2}\right)^2 \quad (7)$$

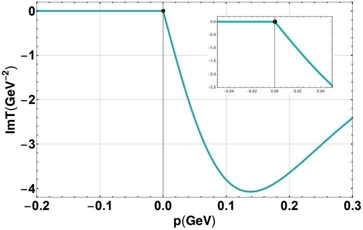
where $C_{\text{iso.}}(\sigma)=-1, C_{\text{iso.}}(\rho)=1, C_{\text{iso.}}(\omega)=+1$ and $\Lambda=1$GeV, $g_\sigma=3.4, g_\rho=g_\omega=2.6$.

- $D-D$ *scattering amplitude*



- $D-D$ *scattering amplitude*





- $D-D$ *scattering phase shift*

$$T(p) = -\frac{8\pi}{m_D} \frac{1}{p\cot\delta - ip} \tag{8}$$

$$\text{where } p\cot\delta = -a_{DD}^{-1} + \frac{1}{2} r_{DD} p^2, \quad a_{DD} = -0.392\text{fm},\ r_{DD} = 3.236\text{fm} \tag{9}$$

- *Analytic continuation below threshold* $(\kappa = -ip)$

$$T(\kappa) = -\frac{8\pi}{m_D}\left[\frac{C_{DD}}{\kappa - \kappa_{DD}} + R_{DD} + R_{DD}^{(1)}\kappa + R_{DD}^{(2)}\kappa^2\right], \quad \kappa_{DD} = -195.166\text{MeV}. \tag{10}$$

	a (fm)	r (fm)	κ (MeV)	C	R (MeV^{-1})	$R^{(1)}$ (MeV^{-1})	$R^{(2)}$ (MeV^{-1})
DK	1.683 ($1.58^{+0.22}_{-0.17}$)	0.791	187.795	3.881	0.0121	3.73×10^{-5}	1.11×10^{-7}
DD	−0.392 ($-0.4^{+0.1}_{-0.2}$)	3.236	−195.166	0.243	7.43×10^{-4}	2.43×10^{-6}	7.52×10^{-9}

PRD 87, 014058 (2013); JHEP 11 (2015) 058; PRD 96, 074501 (2017); EPJC 79, 13 (2019);

$$T_{DK}(E) = -\frac{2\pi}{\mu}\begin{cases} \dfrac{1}{-a_{DK}^{-1}+\frac{1}{2}r_{DK}p^2-ip}, & p=\sqrt{+2\mu E}, \quad E>0; \\ \dfrac{C_{DK}}{\kappa-\kappa_{DK}}+R_{DK}+R_{DK}^{(1)}\kappa+R_{DK}^{(2)}\kappa^2, & \kappa=\sqrt{-2\mu E}, \quad E<0. \end{cases} \tag{11}$$

$$T_{DD}(E) = -\frac{8\pi}{m_D}\begin{cases} \dfrac{1}{-a_{DD}^{-1}+\frac{1}{2}r_{DD}p^2-ip}, & p=\sqrt{+m_D E}, \quad E>0; \\ \dfrac{C_{DD}}{\kappa-\kappa_{DD}}+R_{DD}+R_{DD}^{(1)}\kappa+R_{DD}^{(2)}\kappa^2, & \kappa=\sqrt{-m_D E}, \quad E<0. \end{cases} \tag{12}$$

- *Lagrangian*

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_2 + \mathscr{L}_3 \tag{13}$$

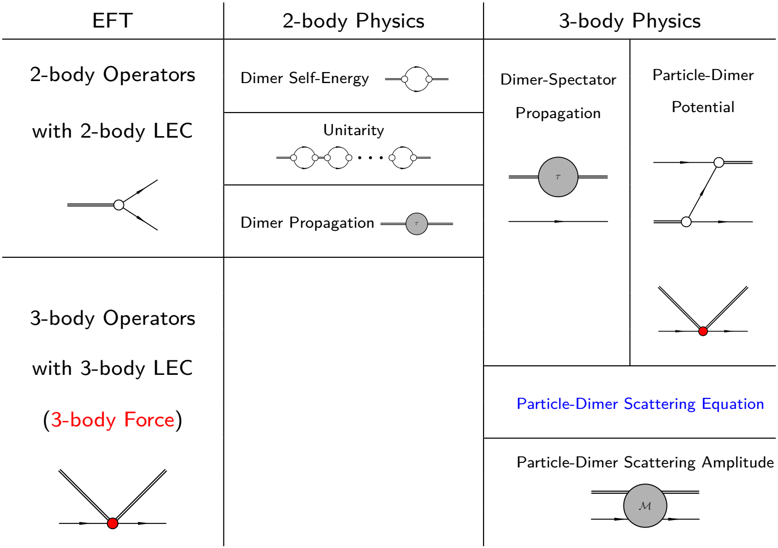
where

$$\mathscr{L}_0 = D^\dagger(i\partial_0 + \frac{\nabla^2}{2m_D})D + K^\dagger(i\partial_0 + \frac{\nabla^2}{2m_K})K + T^\dagger_{DK}\sigma_{DK}T_{DK} + T^\dagger_{DD}\sigma_{DD}T_{DD} \tag{14}$$

$$\mathscr{L}_2 = T^\dagger_{DK}[D\mathscr{F}_{DK}K] + T^\dagger_{DD}[D\mathscr{F}_{DD}D] + \text{h.c.} \tag{15}$$

$$\mathscr{L}_3 = \left[T^\dagger_{DK}D^\dagger\right][T_{DK}\mathscr{H}_D D] + \left[T^\dagger_{DD}K^\dagger\right][T_{DD}\mathscr{H}_K K] + \text{h.c.} \tag{16}$$

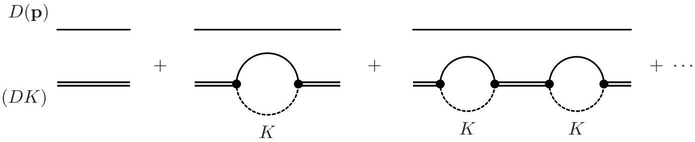
  - *Non-relativistic kinematics of D-meson and K-menson*
  - *2-body sub-system(dimer):* T_{DK}, T_{DD}
  - *operator and LEC:* $\mathscr{F} = f_0 + f_2 \overleftrightarrow{\nabla}^2$, $\mathscr{H} = h_0 + h_2\nabla^2$


EFT
2-body Physics
3-body Physics
2-body Operators
with 2-body LEC
Dimer Self-Energy
Unitarity
Dimer Propagation
Dimer-Spectator
Propagation
Particle-Dimer
Potential
3-body Operators
with 3-body LEC
(3-body Force)
Particle-Dimer Scattering Equation
Particle-Dimer Scattering Amplitude
τ
$\mathcal{M}$

2-body matching

- *(DK)-dimer propagator*

$\tau_{DK}(p;E) =$



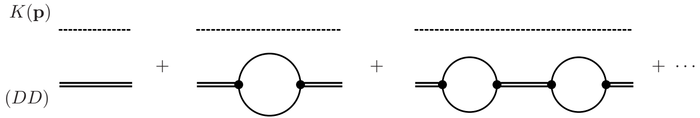
$$= \frac{1}{p_* \cot\delta_{DK}(p_*) - ip_*}, \quad \text{with } p_*^2 = 2\mu E - (1 - m_D^2/M^2)p^2 + i\epsilon$$

- *$D-K$ scattering phase shift*
  - *on-shell relative momentum in (DK) sub-system:* $p_*^2 = 2\mu E - (1 - m_D^2/M^2)p^2 + i\varepsilon$
  - *phase shift above threshold:* $p_* \cot\delta_{DK}(p_*) = -a_{DK}^{-1} + \frac{1}{2} r_{DK} p_*^2, (p_*^2 \geq 0)$
  - *analytic continuation:* $\tau_{DK}(p;E) = \frac{C_{DK}}{\kappa - \kappa_{DK}} + R_{DK} + R_{DK}^{(1)}\kappa + R_{DK}^{(2)}\kappa^2, (p_*^2 = -\kappa^2 < 0)$

2-body matching

- *(DD)-dimer propagator*

$\tau_{DD}(p;E) =$



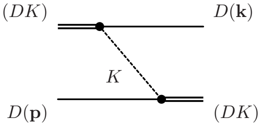
$$= \frac{4(1-m_D/M)}{p_* \cot\delta_{DD}(p_*) - ip_*}, \quad \text{with } p_*^2 = m_D E - \frac{m_K + 2m_D}{4m_K} p^2 + i\epsilon$$

- *$D-D$ scattering phase shift*
  - *on-shell relative momentum in (DD) sub-system:* $p_*^2 = m_D E - \frac{m_K+2m_D}{4m_K} p^2 + i\varepsilon$
  - *phase shift above threshold:* $p_* \cot\delta_{DD}(p_*) = -a_{DD}^{-1} + \frac{1}{2} r_{DD} p_*^2, \quad (p_*^2 \geq 0)$
  - *analytic continuation:*

$$\tau_{DD}(p;E) = 4(1-m_D/M)\left[\frac{C_{DD}}{\kappa-\kappa_{DD}} + R_{DD} + R_{DD}^{(1)}\kappa + R_{DD}^{(2)}\kappa^2\right], \quad (p_*^2 = -\kappa^2 < 0)$$

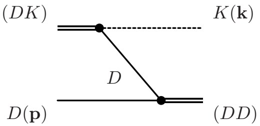
Particle-dimer scattering potential

- *K-hopping* $(DK)D \to (DK)D$



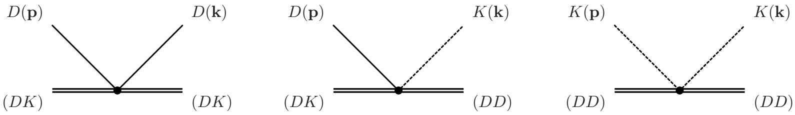
$$= \frac{1}{p^2 + k^2 + 2(m_D/M)\mathbf{pk} - 2\mu E}$$

- *D-hopping* $(DK)D \to (DD)K$



$$= \frac{M/(2m_K)}{p^2 + Mk^2/(2m_K) + \mathbf{pk} - m_D E}$$

- *Contact potential:* $D^\dagger D^\dagger K^\dagger DDK$ *encoding high-momentum physics*



  - *DDK 3-body coupling:* $h(D^\dagger D^\dagger K^\dagger)(DDK)$.
  - *Operators in particle-dimer formalims:*

$$h_0\left(T^\dagger_{DK}D^\dagger\right)(T_{DK}D)+h_0'\left(T^\dagger_{DD}K^\dagger\right)(T_{DD}K)+h_0''\left(\left(T^\dagger_{DD}K^\dagger\right)(T_{DK}D)+\text{h.c.}\right). \tag{17}$$

  - *Integrating out dimer fields*

$$h=\frac{f^2_{0,DK}}{\sigma^2_{DK}}h_0+\frac{f^2_{0,DD}}{\sigma^2_{DD}}h_0'+\frac{2f_{0,DD}f_{0,DK}}{\sigma_{DK}\sigma_{DD}}h_0''. \tag{18}$$

- *Particle-dimer scattering equation*

$$\begin{pmatrix} \mathscr{M}_1 & \mathscr{M}_{12} \\ \mathscr{M}_{21} & \mathscr{M}_2 \end{pmatrix}(\mathbf{p},\mathbf{q}) = \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix}(\mathbf{p},\mathbf{q}) + 4\pi \int^{\Lambda} \frac{d^3k}{(2\pi)^3} \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix}(\mathbf{p},\mathbf{k}) \times \begin{pmatrix} \tau_1(\mathbf{k}) & \\ & \tau_2(\mathbf{k}) \end{pmatrix} \begin{pmatrix} \mathscr{M}_1 & \mathscr{M}_{12} \\ \mathscr{M}_{21} & \mathscr{M}_2 \end{pmatrix}(\mathbf{k},\mathbf{q}). \tag{19}$$

  - *2 coupled channels: $(DK)D$ and $(DD)K$*
  - $\mathscr{M}$*: particle-dimer scattering amplitude*
  - *Z: particle-dimer potential*

$$Z_1(\mathbf{p},\mathbf{q}) = \frac{1}{p^2+q^2+2(m_D/M)\mathbf{pq}-2\mu E} + \frac{H_0}{\Lambda^2}, \tag{20}$$

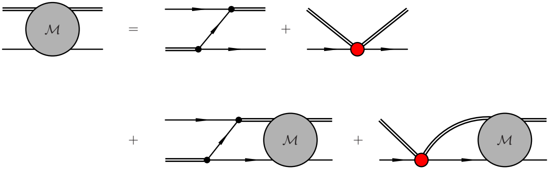
$$Z_{12}(\mathbf{p},\mathbf{q}) = \frac{M/(2m_K)}{p^2+Mq^2/(2m_K)+\mathbf{pq}-m_D E} \tag{21}$$

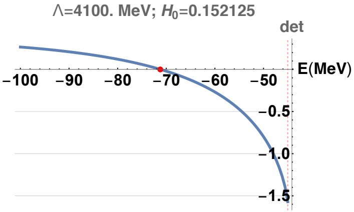
  - *τ: particle-dimer propagator*

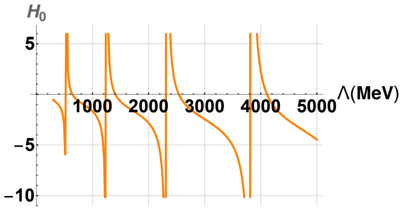
- DDK *3-body bound state* L.-S. Geng et al. PRD 100(2019)3,034029

C_S (MeV)	C_L (MeV)	E_2 (MeV)	E_3 (only DK)	E_3 (both DK and DD)
$R_c = 1$fm, $R_s = 0.5$fm				
0	−320.1	−45.0	−65.8	−71.2
500	−455.4	−45.0	−65.8	−70.4

- DDK *3-body force*







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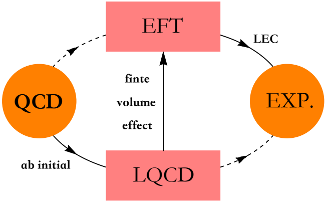
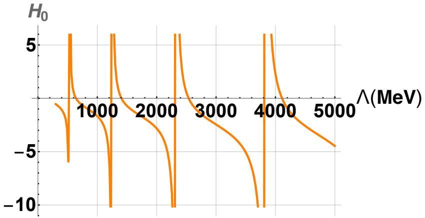
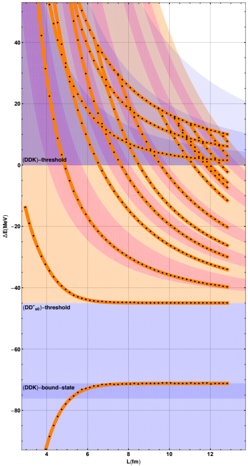
- *Lattice QCD and effective field theory* Hammer, JYP and Rusetsky. JHEP 10(2017)

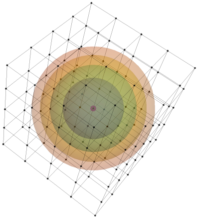


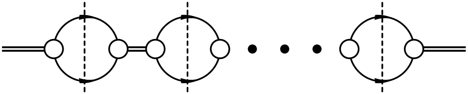
- *Lattice spectrum and 3-body force*





- *Finite Volume Correction* (*Discrete momentum and Invariant 3-body force*)





- *Loop summation*

$$\Sigma(\mathbf{p};E) = \frac{i}{2}\int \frac{d^4q}{(2\pi)^4} \frac{f^*(\mathbf{p},\mathbf{q})f(\mathbf{p},\mathbf{q})}{[q_0-\omega_{D,\mathbf{q}}+i\varepsilon]\,[(E-\omega_{K,\mathbf{p}}-q_0)-\omega_{D,-\mathbf{p}-\mathbf{q}}+i\varepsilon]}$$

$$\to \quad \frac{i}{2}\int \frac{dq_0}{2\pi}\frac{1}{L^3}\sum_{\mathbf{q}} \frac{f^*(\mathbf{p},\mathbf{q})f(\mathbf{p},\mathbf{q})}{[q_0-\omega_{D,\mathbf{q}}+i\varepsilon]\,[(E-\omega_{K,\mathbf{p}}-q_0)-\omega_{D,-\mathbf{p}-\mathbf{q}}+i\varepsilon]} \tag{22}$$

- (DK)*-dimer in a box*

$$\tau_{DK,L}(\mathbf{p};E) = \frac{1}{p_* \cot\delta_{DK}(p_*) - 4\pi S_{DK,L}(\mathbf{p};E)} \tag{23}$$

where $p_*^2 = 2\mu E - (1 - m_D^2/M^2)p^2$ and finite volume correction is

$$S_{DK,L}(\mathbf{p};E) = \left(\frac{1}{L^3}\sum_{\mathbf{q}} - \text{PV}\int \frac{d^3q}{(2\pi)^3}\right)\frac{1}{(\mathbf{q}+(m_D/M)\mathbf{p})^2 - p_*^2} \tag{24}$$

2-body input: $p_* \cot\delta_{DK}(p_*) = -a_{DK}^{-1} + \frac{1}{2} r_{DK} p_*^2, \quad (p_*^2 \geq 0)$

Analytic continuation: $p_* \cot\delta_{DK}(p_*) = \frac{\kappa - \kappa_{DK}}{C_{DK} + [R_{DK} + R'^2_{DK}\kappa](\kappa - \kappa_{DK})} - \kappa, \quad (p_*^2 = -\kappa^2 < 0)$

- *(DD)-dimer in a box*

$$\tau_{DD,L}(\mathbf{p};E) = \frac{4(1-m_D/M)}{p_* \cot\delta_{DD}(p_*) - 4\pi S_{DD,L}(\mathbf{p};E)} \tag{25}$$

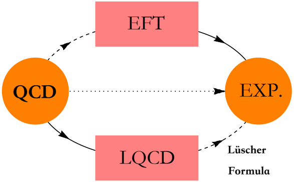
where $p_*^2 = m_D E - \frac{m_K + 2m_D}{4m_K} p^2$ and finite volume correction is

$$S_{DD,L}(\mathbf{p};E) = \left(\frac{1}{L^3} \sum_{\mathbf{q}} - \mathrm{PV} \int \frac{d^3 q}{(2\pi)^3} \right) \frac{1}{(\mathbf{q}+\mathbf{p}/2)^2 - p_*^2} \tag{26}$$

2-body input: $p_* \cot\delta_{DD}(p_*) = -a_{DD}^{-1} + \frac{1}{2} r_{DD} p_*^2, \quad (p_*^2 \geq 0)$

Analytic continuation: $p_* \cot\delta_{DD}(p_*) = \frac{\kappa - \kappa_{DD}}{C_{DD} + [R_{DD} + R_{DD}'^2 \kappa](\kappa - \kappa_{DD})} - \kappa, \quad (p_*^2 = -\kappa^2 < 0)$

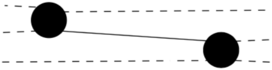
- *Lüscher Formula* M. Lüscher, NPB 354(1991) 531



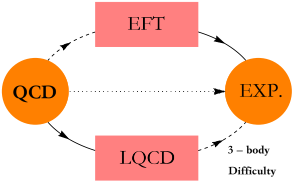
$$\rho \cot \delta(E) = \mathscr{Z}_{00,00}(E, L)$$

- *3-body problem*

$3 \to 3$ scattering process

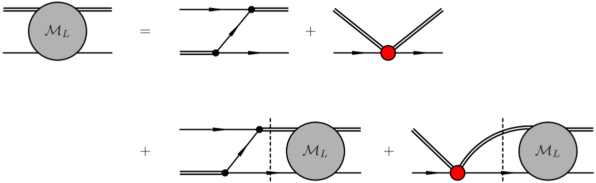


M. Hansen et al. , Phys. Rev. D90(2014) 116003



Scattering equation in a box

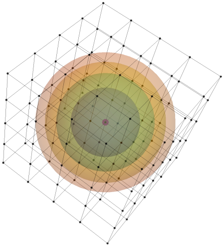
- *Particle-dimer scattering equation in a box*

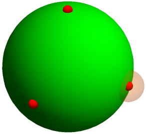


$$\begin{pmatrix} \mathscr{M}_{1,L} & \mathscr{M}_{12,L} \\ \mathscr{M}_{21,L} & \mathscr{M}_{2,L} \end{pmatrix} (\mathbf{p},\mathbf{q}) = \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix} (\mathbf{p},\mathbf{q}) + \frac{4\pi}{L^3} \sum_{\mathbf{k}}^{\Lambda} \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix} (\mathbf{p},\mathbf{k}) \times \begin{pmatrix} \tau_{1,L}(\mathbf{k}) & \\ & \tau_{2,L}(\mathbf{k}) \end{pmatrix} \begin{pmatrix} \mathscr{M}_{1,L} & \mathscr{M}_{12,L} \\ \mathscr{M}_{21,L} & \mathscr{M}_{2,L} \end{pmatrix} (\mathbf{k},\mathbf{q}) \quad (27)$$

- *3-body force $H_0(\Lambda)$ keeps invariant*
- *loop integral $\rightarrow$ loop summation*
- *finite volume dimer τ_L*

- *Cubic periodical boundary condition* $\rightarrow$ *Octahedral group*
  - *Lattice spectrum in irreps.* $\Gamma = A_1^{\pm}, A_2^{\pm}, E^{\pm}, T_1^{\pm}, T_2^{\pm}$
  - *Shell structure of discrete momenta*





- *Projection scheme*

$$\phi(\mathbf{p}) = \phi(g\,\mathbf{p}_0) = \sum_{\Gamma} \mathrm{tr}\left(T^{\Gamma}(g)\phi^{\Gamma}(\mathbf{p}_0)\right) \tag{28}$$

  - g: *symmetry transformation on momentum*
  - $\mathbf{p}_0$: *reference momentum on each shell*
  - $T^{\Gamma}(g)$: *the corresponding transformation matrix in irreps.* Γ
  - $\phi^{\Gamma}(\mathbf{p}_0)$: *matrix component on specific shell in irreps.* Γ

- *Symmetry of equation*

$$\begin{pmatrix} \mathscr{M}_{1,L} & \mathscr{M}_{12,L} \\ \mathscr{M}_{21,L} & \mathscr{M}_{2,L} \end{pmatrix}(\mathbf{p},\mathbf{q}) = \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix}(\mathbf{p},\mathbf{q}) + \frac{4\pi}{L^3}\sum_{\mathbf{k}}^{\Lambda} \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix}(\mathbf{p},\mathbf{k}) \times \begin{pmatrix} \tau_{1,L}(\mathbf{k}) & \\ & \tau_{2,L}(\mathbf{k}) \end{pmatrix} \begin{pmatrix} \mathscr{M}_{1,L} & \mathscr{M}_{12,L} \\ \mathscr{M}_{21,L} & \mathscr{M}_{2,L} \end{pmatrix}(\mathbf{k},\mathbf{q})$$

  - *loop summation is invariant under symmetry transformation*
  - $\tau_L(g\mathbf{k}) = \tau_L(\mathbf{k}) \quad \Rightarrow \quad \tau_L(\mathbf{k}) = \tau_L(g\mathbf{k}_0^{(s)}) = \tau_L(\mathbf{k}_0^{(s)}) = \tau_L(s)$
  - $Z(g\mathbf{p}, g\mathbf{k}) = Z(\mathbf{p},\mathbf{k}) \quad \Rightarrow \quad Z(\mathbf{p},\mathbf{k}) = Z(g'\mathbf{p}_0^{(r)}, g\mathbf{k}_0^{(s)}) = Z(h\mathbf{p}_0^{(r)}, \mathbf{k}_0^{(s)}) = \sum_{\Gamma} s_\Gamma tr\left[T^\Gamma(h) Z^\Gamma(r,s)\right]$

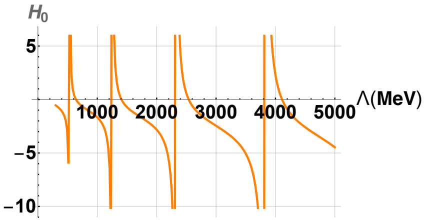
- *``Angular momentum" conservation*

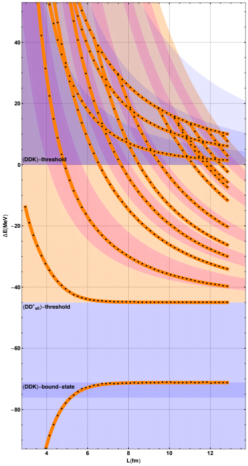
$$\begin{pmatrix} \mathscr{M}_1^\Gamma & \mathscr{M}_{12}^\Gamma \\ \mathscr{M}_{21}^\Gamma & \mathscr{M}_2^\Gamma \end{pmatrix}(r,t) = \begin{pmatrix} Z_1^\Gamma & Z_{12}^\Gamma \\ Z_{21}^\Gamma & 0 \end{pmatrix}(r,t) + \frac{4\pi}{L^3}\sum_{s}^{s_\Lambda} \vartheta_s \begin{pmatrix} Z_1^\Gamma & Z_{12}^\Gamma \\ Z_{21}^\Gamma & 0 \end{pmatrix}(r,s) \times \begin{pmatrix} \tau_{1,L}(s) & \\ & \tau_{2,L}(s) \end{pmatrix} \begin{pmatrix} \mathscr{M}_1^\Gamma & \mathscr{M}_{12}^\Gamma \\ \mathscr{M}_{21}^\Gamma & \mathscr{M}_2^\Gamma \end{pmatrix}(s,t) \quad (29)$$

- A_1^+ *Quantization Condition*

$$\det\left[\delta_{rs}\begin{pmatrix}\tau_{1,L}^{-1}(s) & \\ & \tau_{2,L}^{-1}(s)\end{pmatrix} - \frac{4\pi}{L^3}\vartheta_s\begin{pmatrix}Z_1^{\Gamma}(r,s) & Z_{12}^{\Gamma}(r,s)\\ Z_{21}^{\Gamma}(r,s) & 0\end{pmatrix}\right]=0$$

- *From 3-body force to lattice spectrum*

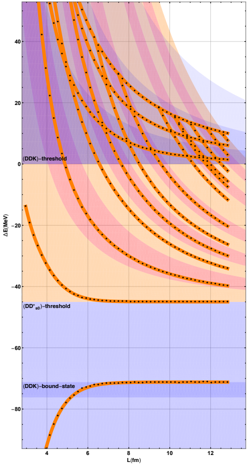




Outline

1. Motivation
2. DDK 3-body effective field theory
3. DDK 3-body system in a finite volume
4. DDK 3-body lattice spectrum
5. Summary and Outlook

- *Spectrum structure*
  - DDK-threshold: $2m_D + m_K \to 0$MeV
  - $DD^*_{s0}(2317)$-threshold: $m_D + m_{D^*} \to -44.5$MeV
  - DDK-bound state: -71.2MeV



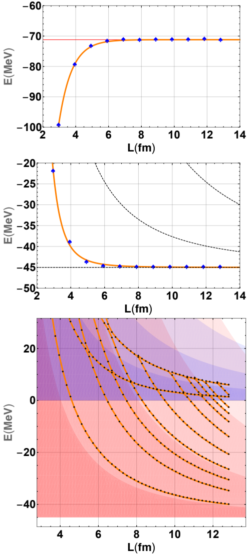
- *Free lines*
  - 3-body (DDK) free lines

$$E_3 = \frac{1}{2m_D}\left(\frac{2\pi\mathbf{n}_1}{L}\right)^2 + \frac{1}{2m_D}\left(\frac{2\pi\mathbf{n}_2}{L}\right)^2 + \frac{1}{2m_K}\left(\frac{2\pi(\mathbf{n}_1+\mathbf{n}_2)}{L}\right)^2$$
$$= \left(\frac{2\pi}{L}\right)^2 \frac{1}{2\mu}\left[n_1^2 + n_2^2 + 2(m_D/M)\mathbf{n}_1\cdot\mathbf{n}_2\right]$$

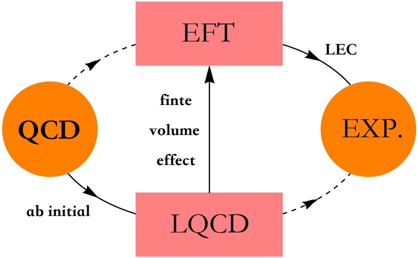
  - 2-body ($DD^*_{s0}(2317)$) free lines

$$\tau^{-1}_{DK}\left(\frac{2\pi n}{L}; E\right) = 0 \quad \Rightarrow \quad E_{1+2} = -\frac{\kappa^2_{DK}}{2\mu} + \frac{1}{2\mu}\left(1 - \frac{m_D^2}{M^2}\right)\left(\frac{2\pi n}{L}\right)^2$$

- *DDK 3-body bound state* (2-body scattering length a ; 3-body binding momentum κ)
  - *DDK 3-body picture:* $\kappa a \gg 1 \Rightarrow \Delta E \sim (\kappa L)^{-3/2} \exp(-\#\kappa L)$
    U.-G. Meißner, PRL 114(9) (2015),091602
  - *$DD^*_{s0}(2317)$ 2-body picture:* $\kappa^2 - a^{-2} \ll \kappa^2$
    $\Rightarrow \Delta E \sim (\kappa L)^{-1} \exp(-\#\sqrt{\kappa^2 - a^{-2}} L)$ M. Lüscher, NPB 354(1991) 531
  - *linear combination of 2 pictures*
- *$DD^*_{s0}(2317)$ 2-body scattering state*
  - *check the scatting length between $D - D^*_{s0}(2317)$*
  - *multiple spectra between two free lines*
- *DDK 3-body scattering state*
  - *avoided level crossing*
  - *perturbative calculation* JYP, J.-J Wu et. al. PRD 99(2019),074513
    $$\Delta E \sim \frac{g_0}{L}\left(1 + \frac{g_1}{L} + \frac{g_2}{L^2} + \frac{g_3}{L^3}\ln\frac{mL}{2\pi} + \frac{g_4}{L^3} + \cdots\right)$$



1. *Motivation*
2. *DDK 3-body effective field theory*
3. *DDK 3-body system in a finite volume*
4. *DDK 3-body lattice spectrum*
5. *Summary and Outlook*

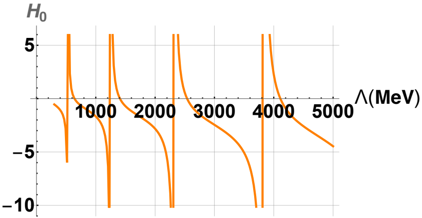


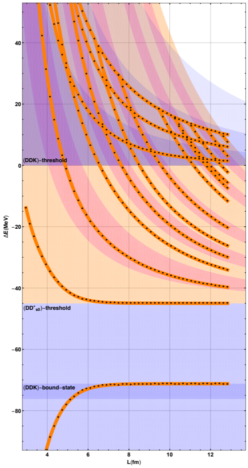
- *Build 3-body EFT for DDK 3-body system* (based on L.-S. Geng et al. PRD 100(2019)3,034029)
- *Construct DDK 3-body quantization condition in A_1^+*

  (based on Hammer, JYP and Rusetsky. JHEP 10(2017))
- *Calculate A_1^+ lattice spectrum for DDK 3-body system*

Outlook

- *Consider 3-body force in* $O(p^2)$
- *Resolve the energy shift*
- *Calculate lattice spectrum in irreps.* $A_1^-, E^{\pm}, \cdots$

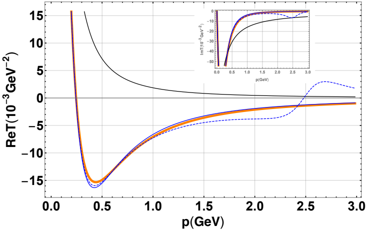




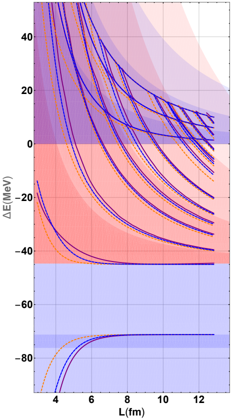
Thank you for your attention!

Appendix: Low momentum expansion near the threshold

- *Power counting in 2-body amplitude*

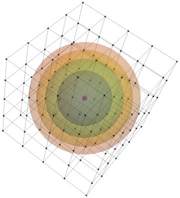


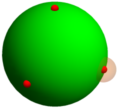
- *Power counting in 3-body lattice spectrum*

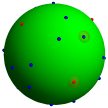


Appendix: Shell Structure and Cubic Irreps. Expansion

- ***Shell Structure*** <u>Shell</u> is a set of momenta with the same $|\mathbf{p}|$, which can be obtained from <u>reference momentum $\mathbf{p}_0$</u>, $\mathbf{p} = g\mathbf{p}_0, \quad g \in O_h$. The momenta unrelated by the O_h, but having $|\mathbf{p}| = |\mathbf{p}'|$, belong to the different shells.



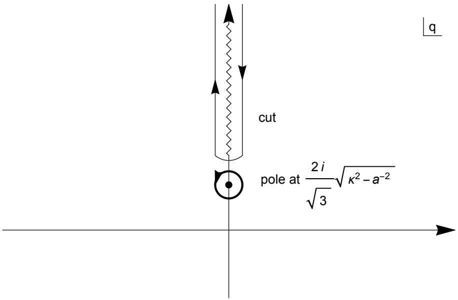




- ***Cubic Irreps. Expansion***

$$f(\mathbf{p}) \quad = f(g\mathbf{p}_0(s)) = \sum_{\Gamma,ij} f_{ij}^{(\Gamma)}(s)\, T_{ij}^{(\Gamma)}(g). \tag{30}$$

$$\sum_{\mathbf{p}} f(\mathbf{p}) = \underbrace{\sum_{s}}_{\text{different shells}} \underbrace{\frac{\vartheta_s}{G}\sum_{g}}_{\text{orientations inside shell } s} f(g\mathbf{p}_0(s)). \tag{31}$$



- *Energy Shifts of Bound States*

$$\Delta E = 8\pi \int \frac{d^3 q}{(2\pi)^3} \phi^\dagger(\mathbf{q}) \sum_{\mathbf{n}\neq 0} e^{i\mathbf{n}\mathbf{q}L} \tau(\mathbf{q}) \phi(\mathbf{q}) + \cdots \tag{32}$$

- *Regular Wave Function* $\phi \sim$ *const.*
- *Cut and Pole of* $\tau(\mathbf{q}; E) = \frac{1}{-a^{-1} + \sqrt{\frac{3}{4}\mathbf{q}^2 - mE - i\varepsilon}}$

The energy of the scattering states vanishes in the infinite volume limit. We quote their finite volume energy E in terms of the quantity $\kappa^2 = L^2 mE/(2\pi)^2$.

The energy shift of the ground state (which resides in the A_1^+ irrep) is:

$$\kappa^2 = \frac{g_0}{L}\left(1 + \frac{g_1}{L} + \frac{g_2}{L^2} + \frac{g_3}{L^3}\ln\frac{mL}{2\pi} + \frac{g_4}{L^3} + \cdots\right), \tag{33}$$

with

$$\begin{aligned} g_0 &= \frac{3}{\pi}\,a\,, \\ g_1 &= 2.837297480\,a\,, \\ g_2 &= 9.725330808\,a^2\,, \\ g_3 &= 8\pi\left(\frac{2\sqrt{3}}{\pi} - \frac{8}{3}\right)a^3\,, \\ g_4 &= \left(-5.159159617 + 6\pi\left(\frac{r}{a}\right) - 8\pi\left(\frac{\hat{\mathscr{M}}}{a^2}\right)\right)a^3\,. \end{aligned} \tag{34}$$

The energy shift of the 1st excited state in the A_1^+ irrep is

$$\kappa^2 - 1 = \frac{h_0}{L}\left(1 + \frac{h_1}{L} + \frac{h_2}{L^2} + \frac{h_3}{L^3}\ln\frac{mL}{2\pi} + \frac{h_4}{L^3} + \cdots\right), \tag{35}$$

with

$$\begin{aligned}
h_0 &= \frac{10}{\pi}\, a, \\
h_1 &= 0.279070\, a, \\
h_2 &= \left(8.494802 + \frac{7\pi^2}{5}\left(\frac{r}{a}\right)\right) a^2, \\
h_3 &= \frac{27}{5} \times 8\pi \left(\frac{2\sqrt{3}}{\pi} - \frac{8}{3}\right) a^3, \\
h_4 &= \left(-172.001650 + 83.745841\left(\frac{r}{a}\right) - \frac{27}{5} \times 8\pi \left(\frac{\hat{\mathscr{M}}}{a^2}\right)\right) a^3.
\end{aligned} \tag{36}$$