Beyond the Standard Model physics
From Lagrangians to events

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Outline

Monte Carlo simulations & new physics

- Towards the characterisation of new physics
  - About the nature of an observation
    - ★ Fitting (and interpreting) deviations
    - ★ Leading order Monte Carlo tools and techniques good enough
  - Final words on the nature of any potential new physics
    - ★ Accurate measurements
    - ★ More precise predictions mandatory

- Monte Carlo tools play a key role at each step
  - Prospective collider studies (*a priori* preparation)
    - ★ Study of various new physics signals
  - Reinterpretation of existing results (*a posteriori* reactions)
    - ★ LHC recasting in new contexts

New physics simulations crucial

From Lagrangians to events

✦ Simulating new physics is standard

- ✤ 20−25 years of developments ↝ LO simulations ≡ bread and butter
- ✤ Simulations at the NLO accuracy in QCD can be easily achieved

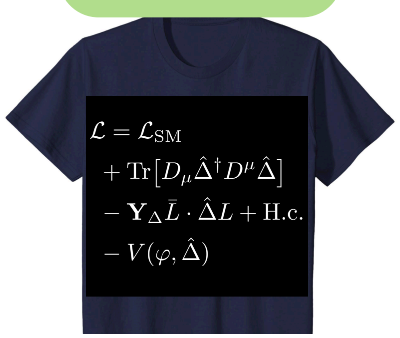
✦ Streamlining the connection of a physics models to events

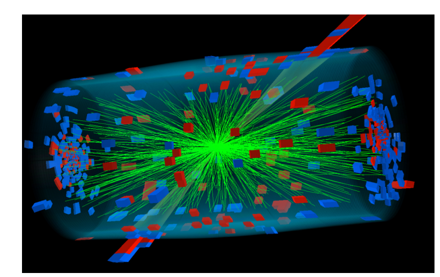
- ✤ Any new physics model can be implemented
- ✤ Easy to validate and maintain

Idea Lagrangian → Simulated collisions → Event analysis

Chain of tools

Other tools

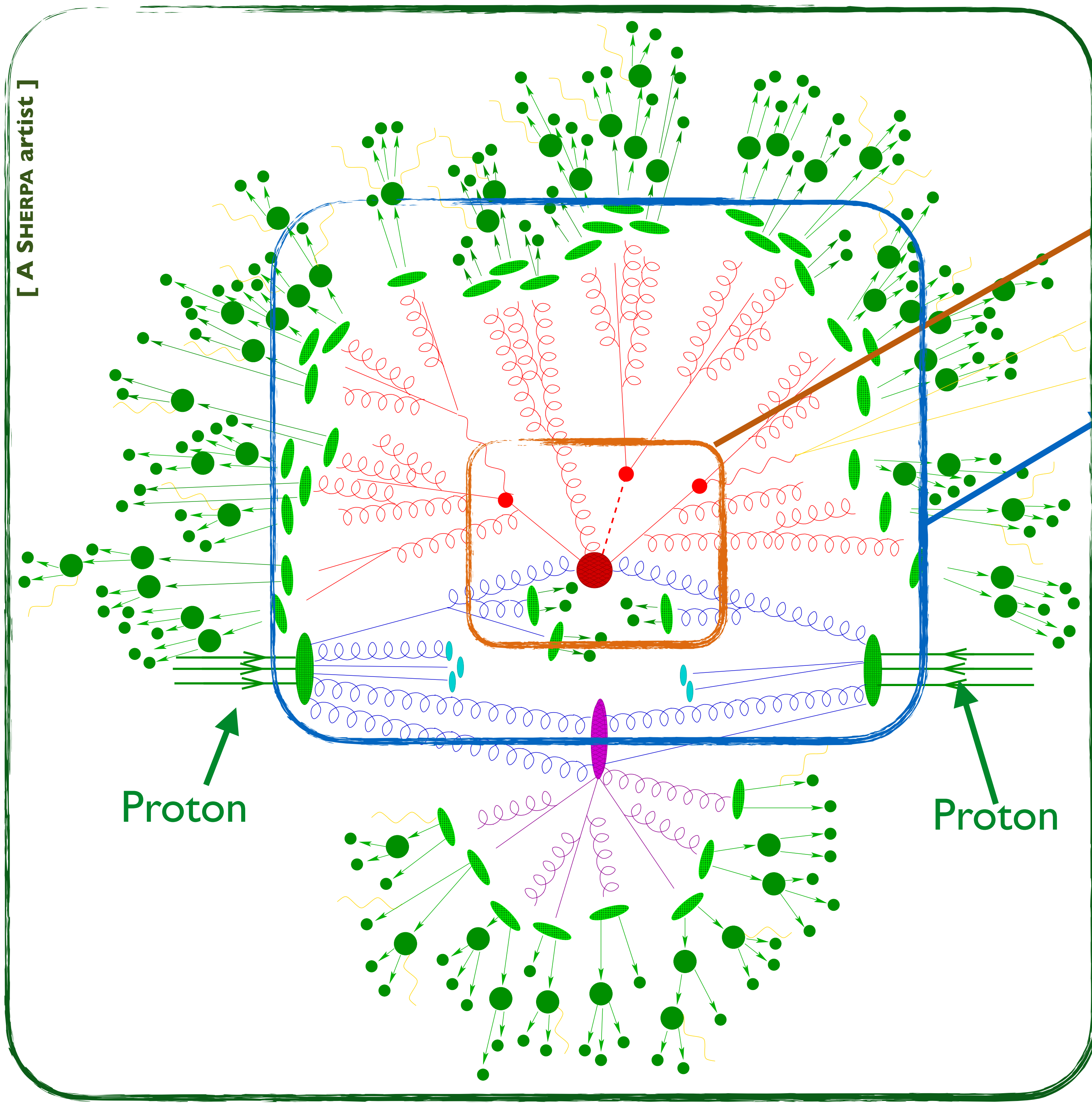




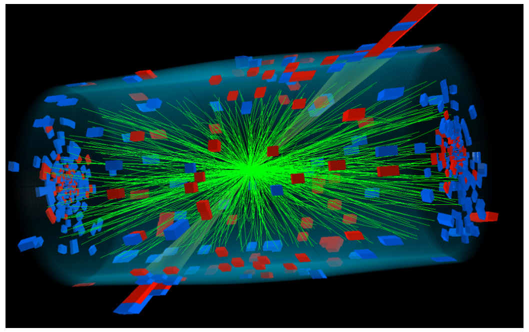
✦ A chain of several tools?

- ✤ Phenomena at different scales ↝ factorisation

Deciphering a proton-proton collision



- ✦ Hard process
  - ✤ Depends on the physics model (SM, BSM,...)
  - ✤ Perturbative QCD
- ✦ Parton showering
  - ✤ Universal (QCD)
- ✦ Hadronisation
  - ✤ Model-based, universal
- ✦ Underlying event
  - ✤ Model-based, non-universal
- ✦ Detector simulation

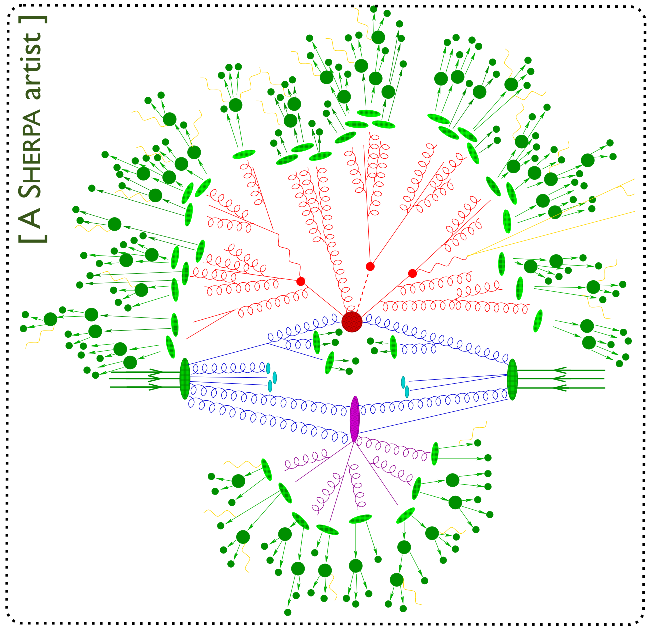


Monte Carlo simulations for proton collisions

✦ Multi-scale problem ↝ factorisation

- ✤ TeV scale: hard scattering (new physics?)
- ✤ Down to $\Lambda_{\rm QCD}$: QCD environment
- ✤ Down to sub-MeV: interactions with a detector

Tools and methods for each step



✦ Tools development: a very intense activity

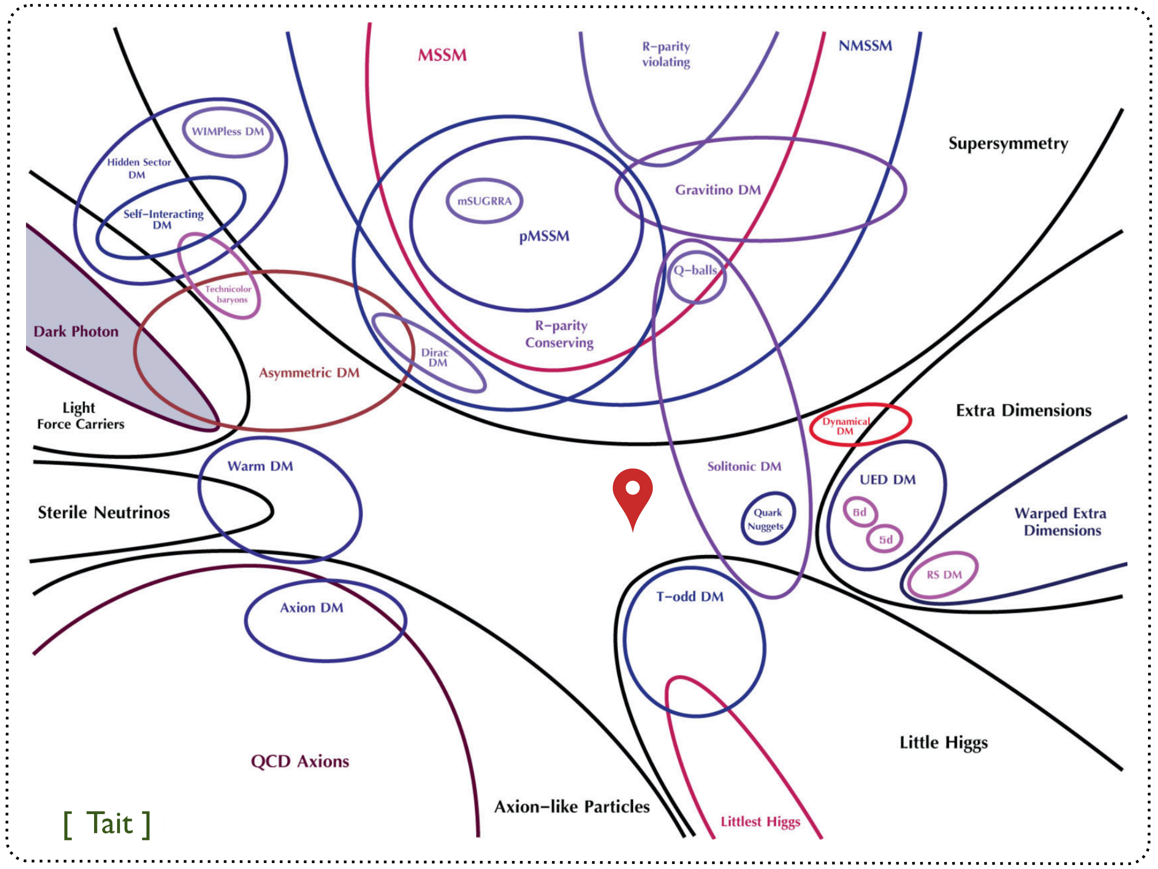
- ✤ Matrix-element generation (CALCHEP, HERWIG, MG5_AMC, SHERPA, WHIZARD, *etc.*)
- ✤ Higher-order computation techniques (MC@NLO, POWHEG, NNLO)
- ✤ Parton showering / hadronisation (PYTHIA, HERWIG, SHERPA)
- ✤ Matrix element - parton shower matching
- ✤ Merging techniques (MLM, CKKW, FxFx, UNLOPS, *etc.*)
- ✤ Detector simulators (DELPHES, RIVET, MADANALYSIS 5)

SM and BSM simulations: the status

- ✦ Standard Model simulations under good control
  - ✤ Relevant LHC processes: known with a very good precision
  - ✤ Further improvements expected in the next few years

- ✦ Different challenges for new physics
  - ✤ No sign of new physics
  - ✤ SM-like measurements
    - ↝ no leading candidate theory
  - ✤ Plethora of models to consider
    - ↝ many implementations in tools

Despite of this, new physics is standard today

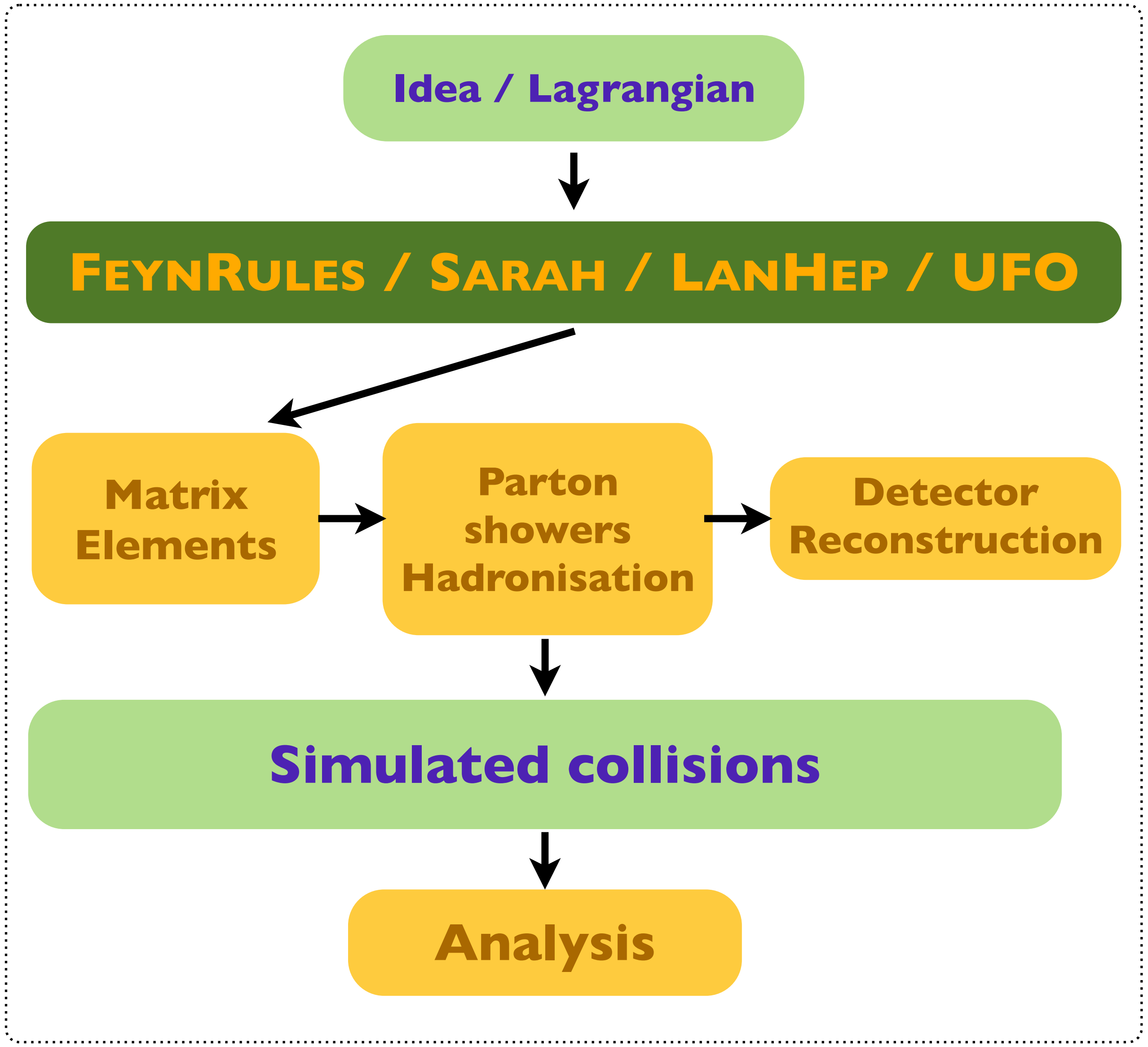


[Tait]

A comprehensive approach to MC simulations

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

✦ Tools connecting an idea to simulated collisions

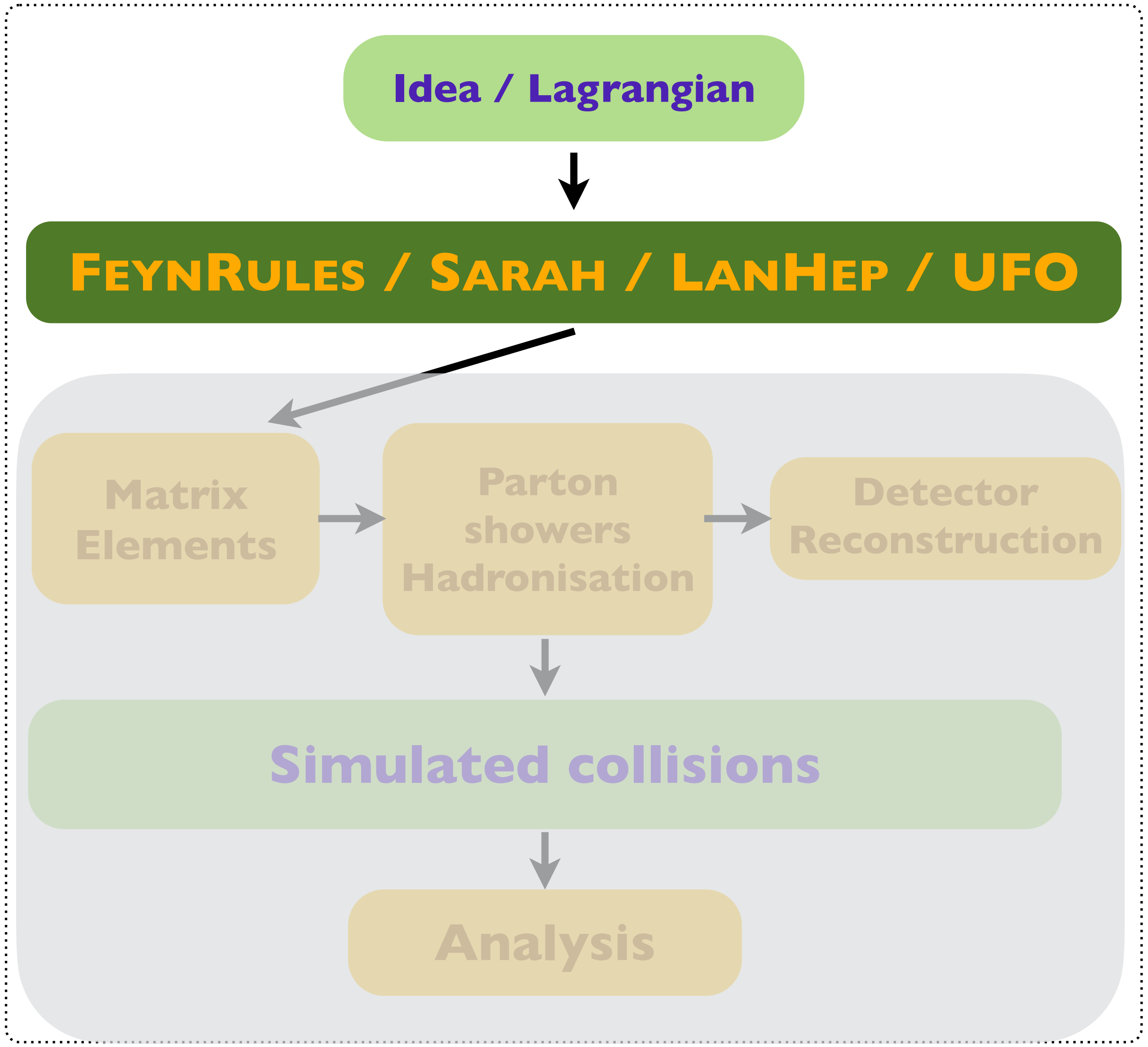


- ✤ Model building
- ✤ Hard scattering
  - ★ Feynman diagram and amplitude generation
  - ★ Monte Carlo integration
  - ★ Event generation
- ✤ QCD environment
  - ★ Parton showering
  - ★ Hadronisation
  - ★ Underlying event
- ✤ Detector simulation
  - ★ Simulation of the detector response
  - ★ Object reconstruction
- ✤ Event analysis
  - ★ Signal/background analysis
  - ★ LHC recasting

A comprehensive approach to MC simulations

[Christensen, de Aquino, Degrande, Duhr, BF, Herquet, Maltoni & Schumann (EPJC`11)]

✦ Tools connecting an idea to simulated collisions



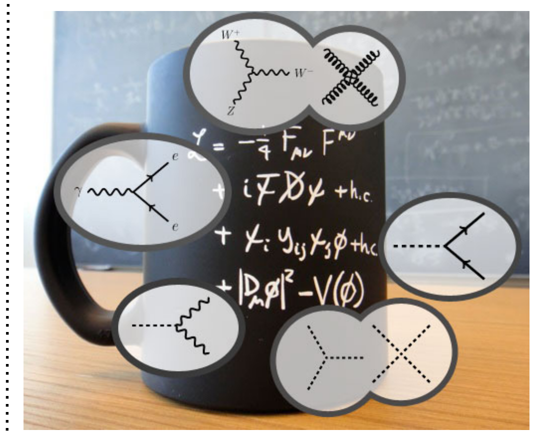
- Model building
  - ★ FeynRules, LanHEP, SARAH
  - ★ UFO
- Matrix element generation
  - ★ CalcHEP, Herwig++, MG5_aMC, Sherpa, Whizard, …
- QCD environment
  - ★ Herwig, Pythia, Sherpa
- Detector simulation
  - ★ Delphes / PGS
  - ★ Rivet / MadAnalysis 5
- Event analysis
  - ★ Rivet / MadAnalysis 5

Outline

New physics simulations: the 'how-to'

✦ How to implement a new physics model in a Monte Carlo program?

- ★ Definition: particles, parameters & vertices (≡ Lagrangian)
  - ↝ translated in some programming language
- ★ Tedious, time-consuming, error prone
- ★ Beware of restrictions/conventions



- ★ Highly redundant (each tool, each model)
- ★ No-brainer tasks (from Feynman rules to codes)

Systematisation
Automation

Existing programs

✦ The FEYNRULES platform (since 2009)

- ✤ Automatic linking of Lagrangians to files in a given programming language
  - ★ With all model particles, interactions, *etc.*
- ✤ Working environment: MATHEMATICA
  - ★ Flexibility, symbolic manipulations, easy implementation of new methods, *etc.*
  - ★ Shipped with many computation platforms (superspace, spectrum, decays, NLO, *etc.*)
- ✤ Interfaced to many Monte Carlo tools
  - ★ Dedicated interfaces (CALCHEP, FEYNARTS, and more in the past)
  - ★ Interfaced to more tools via the UFO (HERWIG, MG5_AMC, SHERPA, WHIZARD,…)
- ✤ Very few limitations on models
  - ★ Higher-dimensional operators supported
  - ★ Spins (up to 2); colour structures (1, 3, 6, 8)

[Christensen & Duhr (CPC '09)]
[Alloul, Christensen, Degrande, Duhr & BF (CPC'14)]

✦ LANHEP (since 1997)

- ✤ Automatic linking of Lagrangians to files in a given programming language
- ✤ Working environment: C
- ✤ Initially restricted to CALCHEP/COMPHEP
- ✤ Now: FEYNARTS and UFO outputs

[Semenov (NIMA'97; CPC'98; CPC'09; CPC'16)]

✦ The SARAH package (since 2013)

- ✤ Automatic linking of Lagrangians to files in a given programming language
- ✤ Working environment: MATHEMATICA
- ✤ Spectrum generator, indirect constraints
- ✤ Interfaced to many tools (CALCHEP, COMPHEP, FEYNARTS, UFO, WHIZARD)

[Staub (CPC'13; CPC'14)]

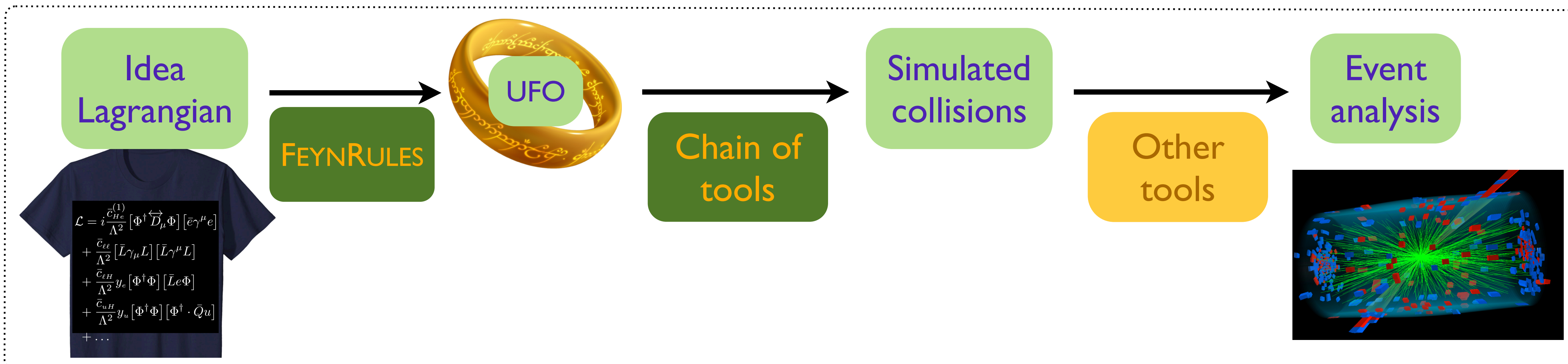
More about interfaces

✦ Each interface dedicated to a given tool is specific

- ✤ Removal of vertices not compliant with the tool
  - ★ Colour structures
  - ★ Lorentz structures
- ✤ Translation to a specific format and programming language

↝ not efficient
↝ a unique translation and the tools parse it

✦ One format to rule them all



A step further: the Universal FEYNRULES Output

[Degrande, Duhr, BF, Grellscheid, Mattelaer, Reiter (CPC '12)]
[Degrande, Duhr, BF, Hirschi, Mattelaer, Shao *et al.* (*in prep.*)]

✦ The UFO in a nutshell

- ✤ UFO ≡ Universal FEYNRULES output ↝ Universal Feynman Output
  - ★ Universal as not tied to any specific Monte Carlo program
- ✤ Consists of a set of PYTHON files to be linked to any code
- ✤ This module contains the full model information
  - ★ Generic colour and Lorentz structures
- ✤ Can be employed for next-to-leading order calculations

✦ The UFO is now a standard

ALOHA · GOSAM · HERWIG ++ · MADANALYSIS 5 · SHERPA

MADGRAPH5_aMC@NLO · WHIZARD · LANHEP · SARAH

The UFO in practice

- The UFO is a set of PYTHON files
  - Factorisation of the information: particles, interactions, parameters, NLO, *etc*.

- Example

Parameters

Propagators

```
[fuks@Benjamins-MacBook-Pro-3 ~/Work/tools/FeynRules/trunk/models/SUSYQCD_UFO$] ls
CT_couplings.py     SUSYQCD_UFO.log      couplings.py        object_library.py   propagators.py
CT_parameters.py    __init__.py          function_library.py parameters.py       vertices.py
CT_vertices.py      coupling_orders.py   lorentz.py          particles.py        write_param_card.py
[fuks@Benjamins-MacBook-Pro-3 ~/Work/tools/FeynRules/trunk/models/SUSYQCD_UFO$]
```

NLO

Interactions

Particles

Examples: particles & parameters

- **Particles ≡ instances of the particle class**
  - Attributes: spin, colour representation, mass, width, *etc.*
  - Antiparticles automatically derived

```
go = Particle(pdg_code = 1000021,
              name = 'go',
              antiname = 'go',
              spin = 2,
              color = 8,
              mass = Param.Mgo,
              width = Param.Wgo,
              texname = 'go',
              antitexname = 'go',
              charge = 0)
```

- **Parameters ≡ instances of the parameter class**
  - External parameters: Les Houches-like structure
  - PYTHON-compliant formula for the internal parameters

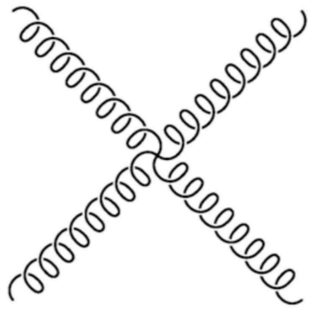
```
aS = Parameter(name = 'aS',
               nature = 'external',
               type = 'real',
               value = 0.1184,
               texname = '\\alpha _s',
               lhablock = 'SMINPUTS',
               lhacode = [ 3 ])

 G = Parameter(name = 'G',
               nature = 'internal',
               type = 'real',
               value = '2*cmath.sqrt(aS)*cmath.sqrt(cmath.pi)',
               texname = 'G')
```

Interactions: the key strategy

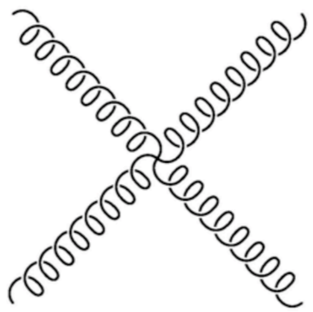
✦ Decomposition in a spin x colour basis (coupling strengths ≡ coordinates)

✤ Example: the quartic gluon vertex



$$\begin{aligned} & i g_s^2\, f^{a_1 a_2 b} f^{b a_3 a_4}\, (\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4}) \\ & \quad + i g_s^2\, f^{a_1 a_3 b} f^{b a_2 a_4}\, (\eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}) \\ & \quad + i g_s^2\, f^{a_1 a_4 b} f^{b a_2 a_3}\, (\eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4}) \end{aligned}$$

✤ UFO version



$$\begin{aligned} & \left(f^{a_1 a_2 b} f^{b a_3 a_4},\ f^{a_1 a_3 b} f^{b a_2 a_4},\ f^{a_1 a_4 b} f^{b a_2 a_3} \right) \\ & \quad \times \begin{pmatrix} i g_s^2 & 0 & 0 \\ 0 & i g_s^2 & 0 \\ 0 & 0 & i g_s^2 \end{pmatrix} \begin{pmatrix} \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} \\ \eta^{\mu_1\mu_4}\eta^{\mu_2\mu_3} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \\ \eta^{\mu_1\mu_3}\eta^{\mu_2\mu_4} - \eta^{\mu_1\mu_2}\eta^{\mu_3\mu_4} \end{pmatrix} \end{aligned}$$

- ★ 3 elements for the colour basis
- ★ 3 elements for the spin (Lorentz structure) basis
- ★ 9 coordinates (6 are zero)

Information encoded in different files

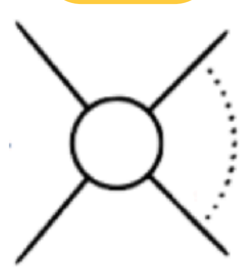
NLO cross sections

✦ Contributions to an NLO result in QCD

✤ Three ingredients: the Born, virtual loop and real emission contributions

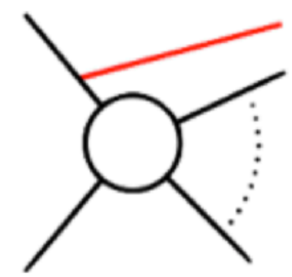
$$\sigma_{NLO} = \int \mathrm{d}^4\Phi_n \mathcal{B} \quad + \boxed{\int \mathrm{d}^4\Phi_n \int_{\text{loop}} \mathrm{d}^d\ell \ \mathcal{V}} \quad + \int \mathrm{d}^4\Phi_{n+1} \ \mathcal{R}$$

Born



Virtuals: one extra power of α_s and divergent

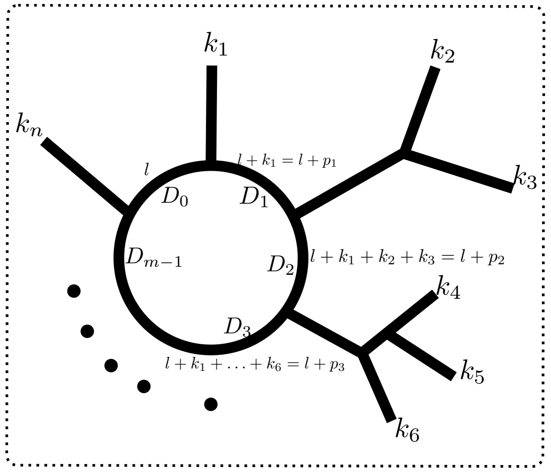
Reals: one extra power of α_s and divergent



Extra information is needed

Loop calculations

- Dimensional regularisation: calculations in $d = 4 - 2\varepsilon$
  - Divergences explicit ($1/\varepsilon^2$, $1/\varepsilon$)
  - Reduction of tensor loop-integrals to scalar integrals
- The reduction must be performed in a *d*-dimensional space
  - Numerical methods work in 4 dimensions ⤳ R_1 and R_2 terms



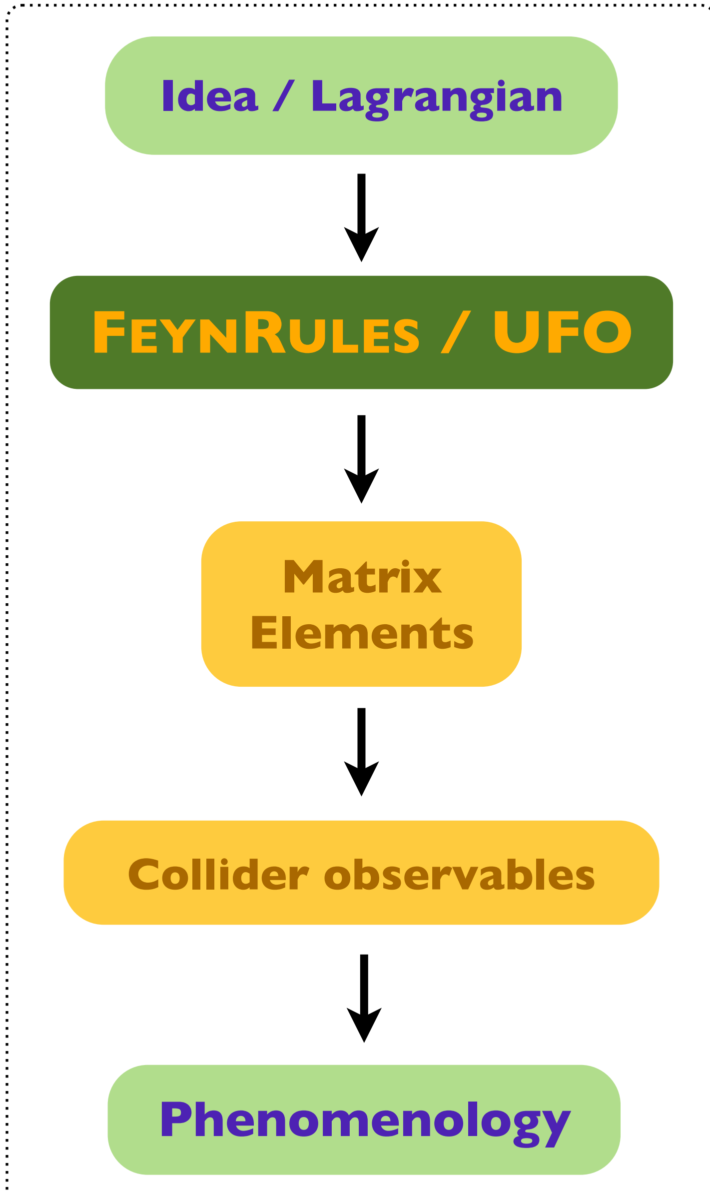
$$\int \mathrm{d}^d \ell \frac{N(\ell, \tilde{\ell})}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}} \quad \text{with } \underbrace{\bar{\ell}}_{\text{D-dim}} = \underbrace{\ell}_{\text{4-dim}} + \underbrace{\tilde{\ell}}_{(-2\varepsilon)\text{-dim}}$$

[Ossala, Papadopoulos, Pittau (NPB'07; JHEP'08)]

- The R_1 terms originate from the denominators
  - Connected to the internal propagators
- The R_2 terms originate from the numerator
  - Process-dependent contributions proportional to $\tilde{\ell}^2$
  - Renormalisable theory: finite number of R_2's
    - ★ Seen as extra diagrams with special Feynman rules (⤳ R_2 Feynman rules)
    - ★ Connected to the UV structure of the integrals (like the UV counterterms)
    - ★ Can be derived from the bare Lagrangian ⤳ NLOCT

UFO @ NLO

Automated NLO simulations with MG5_aMC



✦ Model building: from Lagrangian to tools

- ✤ FeynRules ⊕ MoGRe ⊕ NLOCT ↝ UFO @ NLO
- ✤ General on-shell renormalisation scheme

[Alloul, Christensen, Degrande, Duhr & BF (CPC'14) ; Degrande (CPC'15)]
[Degrande, Duhr, BF, Mattelaer & Reither (CPC'12)]
[Frixione, BF, Hirschi, Mawatari, Shao, Sunder & Zaro (JHEP`19)]

✦ Hard scattering

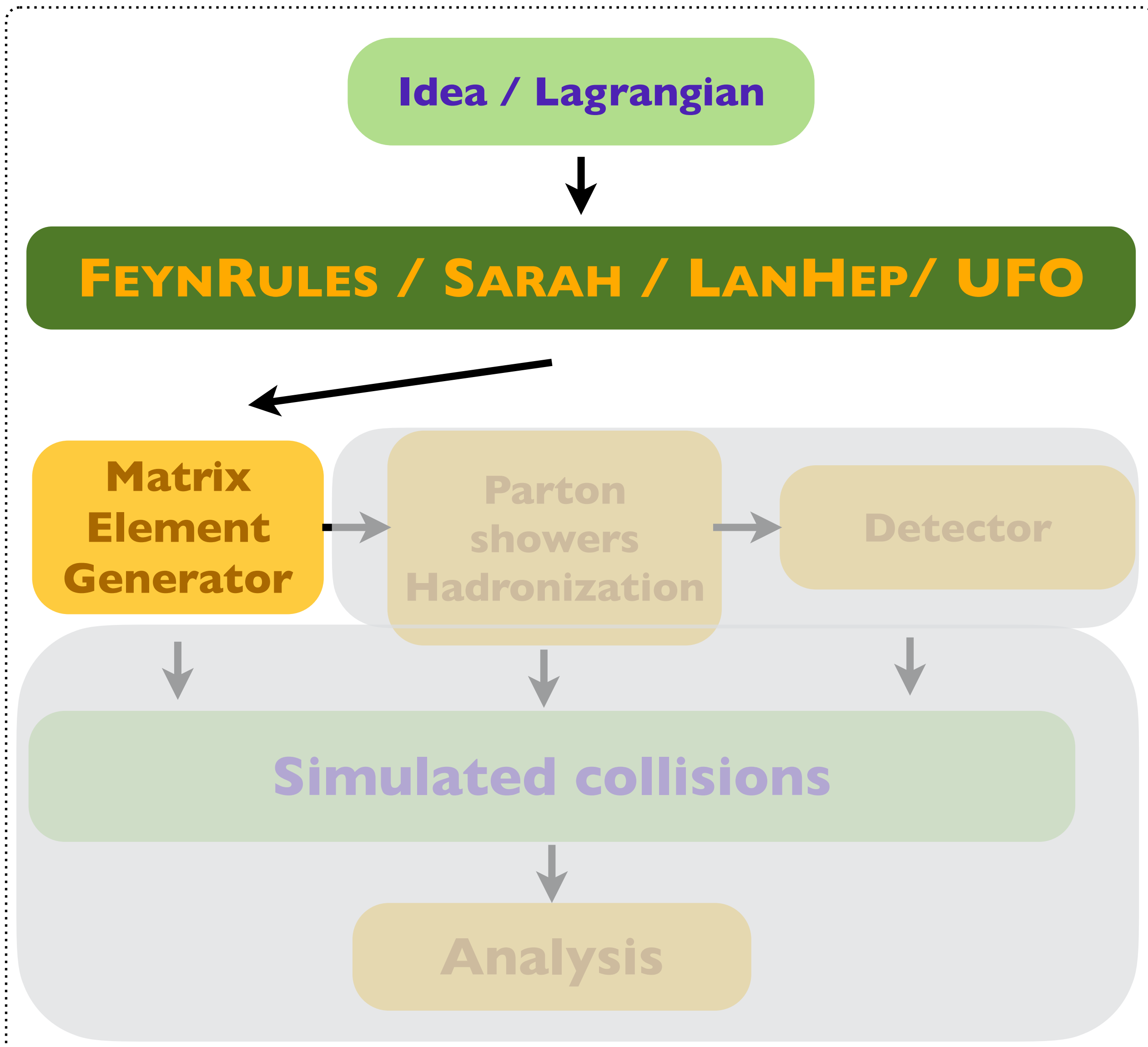
- ✤ Feynman diagram, matrix elements
- ✤ Matching with parton showers
- ✤ MG5aMC ↝ predictions at LO/NLO

[Alwall *et al.* (JHEP'14)]

Outline

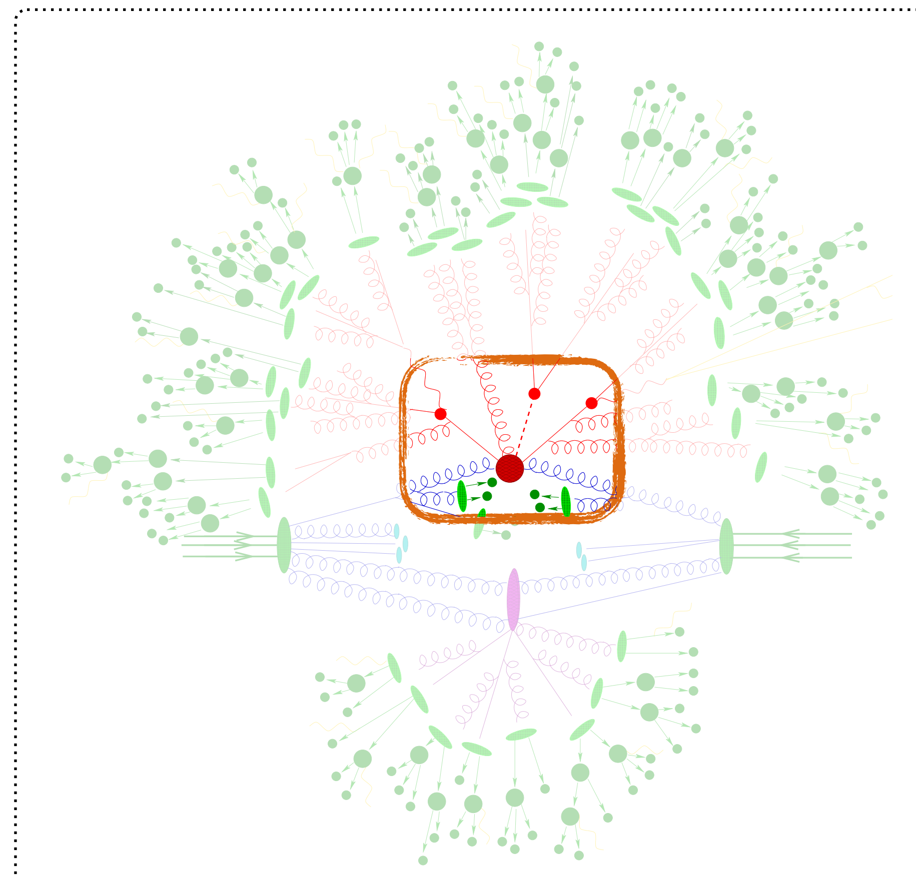
Back to the simulation chain

✦ Tools connecting an idea to simulated collisions



✤ Hard scattering process

- ★ Feynman diagram and amplitude generation
- ★ Monte Carlo integration
- ★ Event generation



QCD 101: predictions at the LHC

✦ Distribution of an observable ω: the QCD factorisation theorem

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = \sum_{ab} \int \mathrm{d}x_a\, \mathrm{d}x_b\, \mathbf{f_{a/p_1}}(x_a; \mu_F)\, \mathbf{f_{b/p_2}}(x_b; \mu_F)\, \frac{\mathrm{d}\sigma_{\mathbf{ab}}}{\mathrm{d}\omega}(\ldots, \mu_F)$$

- Long distance physics: the parton densities
- Short distance physics: the differential parton cross section $d\sigma_{ab}$
- Separation of both regimes through the factorisation scale μ_F
  - ★ Choice of the scale ➢ theoretical uncertainties

✦ Short distance physics: the partonic cross section

- Calculated order by order in perturbative QCD: $d\sigma = d\sigma^{(0)} + \alpha_s\, d\sigma^{(1)} + \ldots$
  - ★ The more orders included, the more precise the predictions
  - ★ Truncation of the series and α_s ➢ theoretical uncertainties

Feynman diagrams (from UFOs)

Feynman diagram calculations

✦ Direct squared matrix element computations

✤ Extraction of the amplitude from the Feynman rules

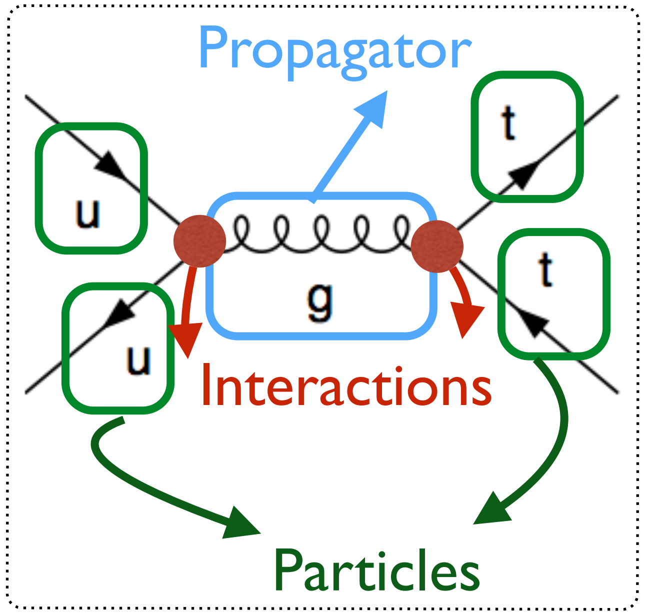
$$i\mathcal{M} = \underline{ig_s^2}\,\left[\boxed{\bar{v}_2}\,\underline{\gamma^\mu}\,\boxed{u_1}\right]\boxed{\frac{\eta_{\mu\nu}}{s}}\left[\boxed{\bar{u}_3}\,\underline{\gamma^\nu}\,\boxed{v_4}\right]\underline{T^a_{c_2c_1}T^a_{c_3c_4}}$$

✤ Squaring with the conjugate amplitude

✤ Algebraic calculation (colour and Lorentz structures)

✤ Sum/average over the external states

$$\overline{|\mathcal{M}|^2} = \frac{1}{36}\frac{2g_s^4}{s^2}\mathrm{Tr}\left[\not{p}_1\gamma^\mu\not{p}_2\gamma^\nu\right]\left[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu\right]$$
$$= \frac{16g_s^4}{9s^2}\left[(p_1\cdot p_3)(p_2\cdot p_4)+(p_1\cdot p_4)(p_2\cdot p_3)\right]$$



✤ The squared matrix element needs to be integrated

✦ The number of diagrams increases with the number of final-state particles

✤ The complexity rises as N^2

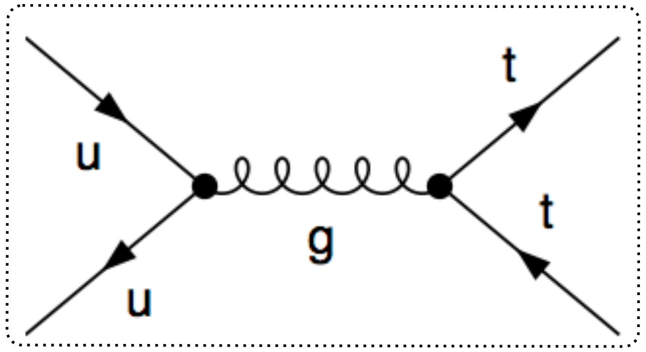
✤ Any calculation beyond 2-to-3 becomes a problem

➢ **Helicity amplitudes**

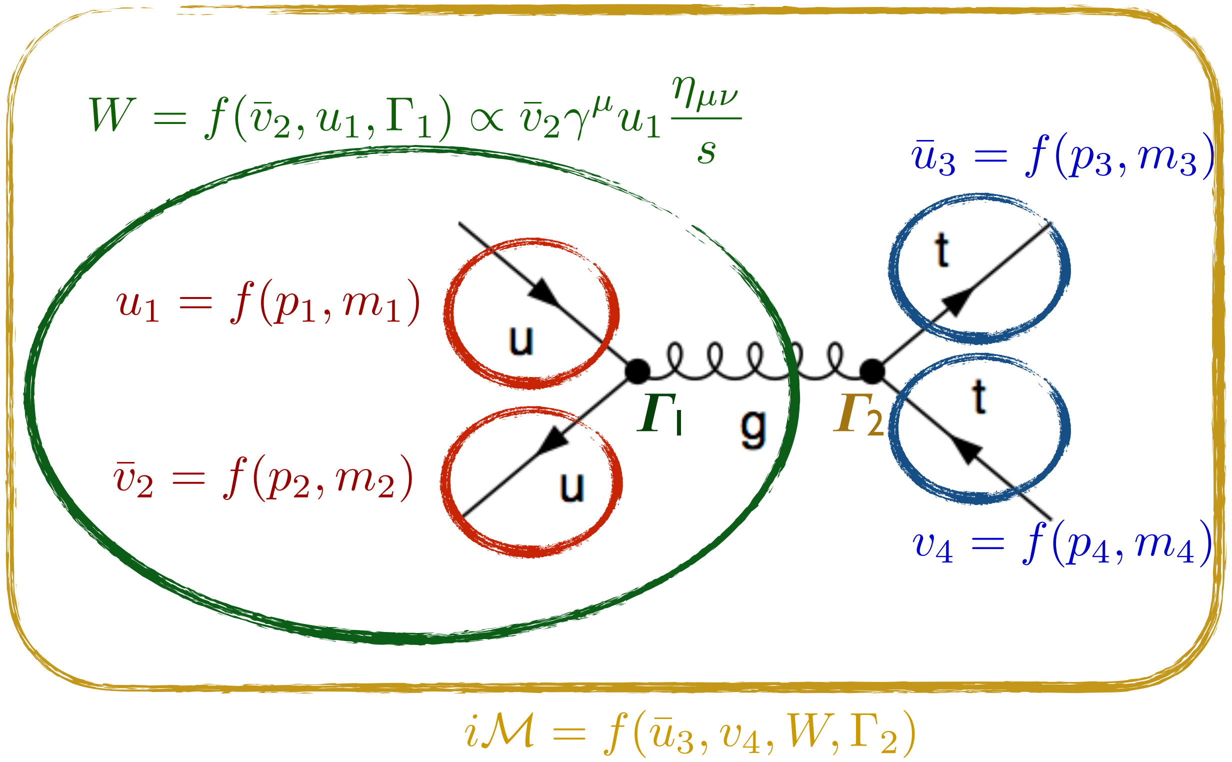
Helicity amplitudes

✦ Principle

- ✤ Evaluation of the amplitude for fixed external helicities
- ✤ Add all amplitudes (we get complex numbers)
- ✤ Squaring
- ✤ Sum/average over the external states



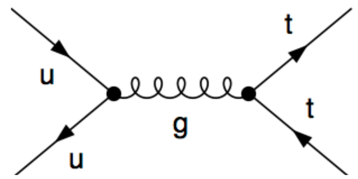
✦ Practical example



1. External incoming particles (numbers)
   - ★ For fixed helicity and momentum
2. Wave function of the gluon propagator
3. External outgoing particles
4. Full amplitude (complex number)

HELAS

✦ The building blocks of the amplitude are the so-called HELAS functions



$$u_1 = f(p_1, m_1)$$
$$\bar{v}_2 = f(p_2, m_2)$$
$$\bar{u}_3 = f(p_3, m_3)$$
$$v_4 = f(p_4, m_4)$$
$$W = f(\bar{v}_2, u_1, \Gamma_1)$$
$$i\mathcal{M} = f(\bar{u}_3, v_4, W, \Gamma_2)$$

✤ HELAS ≡ HELicity Amplitude Subroutine

✤ One specific routine for each Lorentz structure (Γ_i)

✤ Not generic for any model

- ★ SM [Murayama, Watanabe & Hagiwara (KEK-91-11)]
- ★ MSSM [Cho, Hagiwara, Kanzaki, Plehn, Rainwater & Stelzer (PRD`06)]
- ★ HEFT [Frederix (2007)]
- ★ Spin 2 [Hagiwara, Kanzaki, Li & Mawatari (EPJC`08)]
- ★ Spin 3/2 [Mawatari & Takaesu (EPJC`11)]

Sufficient for many models

✦ Generalisation: ALOHA [de Aquino, Link, Maltoni, Mattelaer & Stelzer (CPC`12)]

✤ Translation of any vertex present in a UFO into a HELAS subroutine

✤ Any model supported in MG5_aMC@NLO

✦ Recycling: reusing pieces from one diagram to another

✦ Other MC generators are today connected directly to the UFO

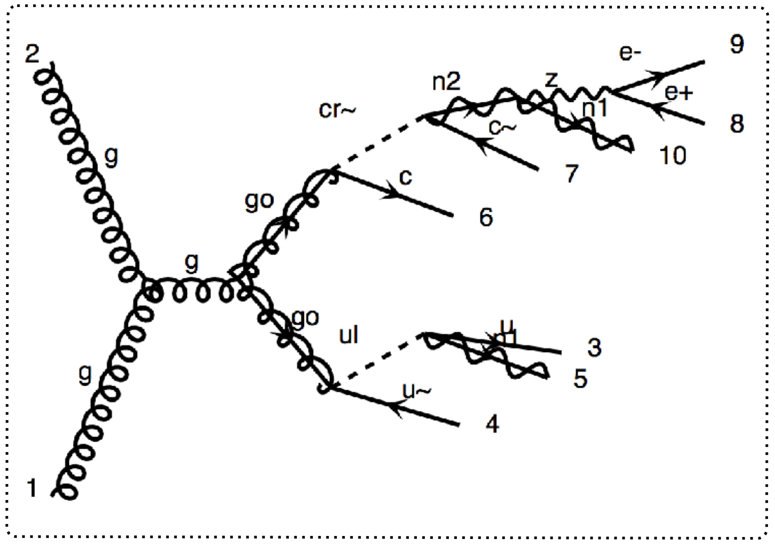
Comparison

	For *M* diags	For *N* particles	$2 \to 6$ example
Analytical	M^2	$(N!)^2$	10^9
Helicity	M	$N!\ 2^N$	10^7
Recycling	M	$(N-1)!\ 2^{N-1}$	$5\mathrm{x}10^5$

Heavy particle decays

- ✦ Concrete BSM models
  - ✤ Many additional new states
    - ★ Usually pair-produced
    - ★ Cascade-decaying into each other
  - ✤ The lightest new state often stable (*cf.* dark matter)

Is the simulation of 2-to-*N* processes (with a large *N*) a problem?



- ✦ 2-to-*N* matrix-element generation is possible
  ↝ computationally challenging
- ✦ The issue is the computing time
  - ✤ Connected to the final-state multiplicity
  - ✤ Diagrams with intermediate resonances dominate
- ✦ Factorisation of the production from the decay

Making decays easy: the key principle

✦ Production and decay processes are factorised

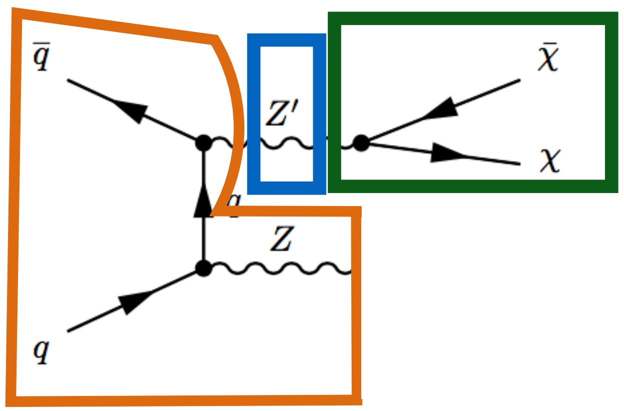
- Propagators can be seen as sums of products of external wave functions
- Example for a vector resonance

$$\mathcal{M} = j_1^\mu \left[\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right] j_2^\nu = \sum_\lambda \boxed{j_1^\mu \varepsilon^*_\mu(\lambda)} \boxed{j_2^\nu \varepsilon_\nu(\lambda)}$$

Propagation

Production of the resonance

Decay of the resonance



- Off-shell effects are lost (as a result of the factorisation)
  - ★ Resonance mass smearing: partial recovery **[Frixione, Laenen, Motylinski, Webber (JHEP '07)]**

Practical implementations of decays

✦ Case 1: loss of spin correlations

✤ Helicity sums performed independently (production ⊕ decays)

$$\mathcal{M} = j_1^\mu \left[\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right] j_2^\nu = \sum_\lambda j_1^\mu \varepsilon_\mu^*(\lambda)\ j_2^\nu \varepsilon_\nu(\lambda) \simeq \underbrace{\boxed{\sum_\lambda j_1^\mu \varepsilon_\mu^*(\lambda)}}_{\text{Production}} \underbrace{\boxed{\sum_{\lambda'} j_2^\nu \varepsilon_\nu(\lambda')}}_{\text{Decay}}$$

✦ Case 2: including spin correlations

✤ Keeping track of helicities

$$\mathcal{M} = \sum_\lambda j_1^\mu \varepsilon_\mu^*(\lambda)\ j_2^\nu \varepsilon_\nu(\lambda)$$

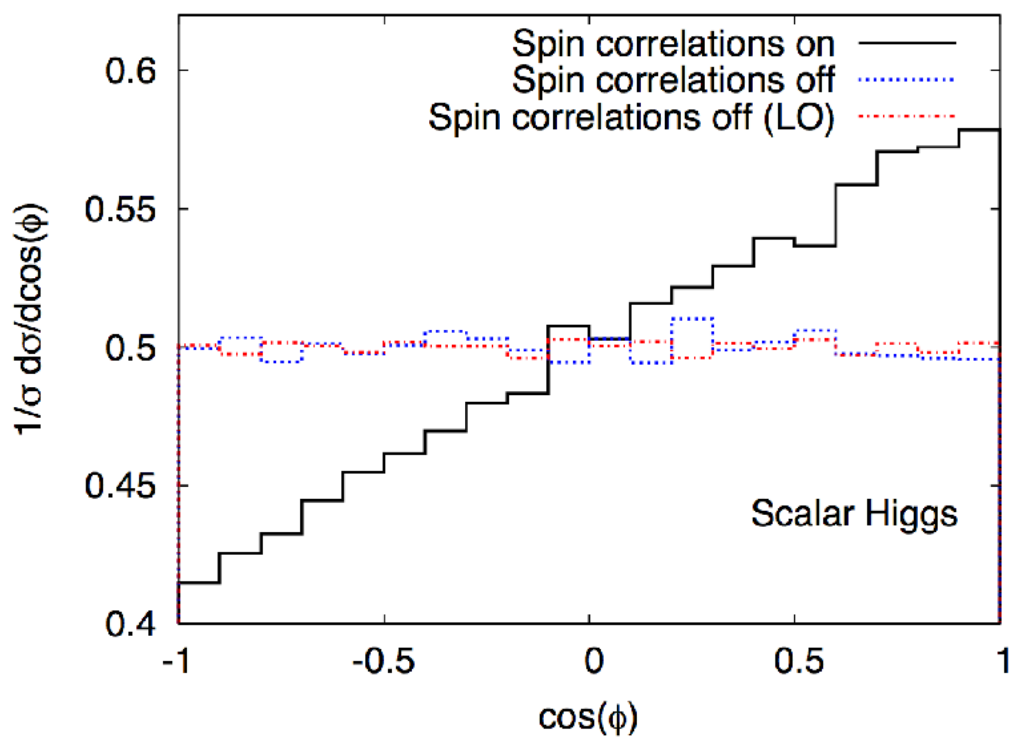
★ Re-weighting from decay matrix element (e.g. MADSPIN)
[Artoisenet *et al.* (JHEP '13)]

★ Using spin density matrices (e.g. HERWIG, SHERPA).
[Richardson (JHEP '01); Höche *et al.* (EPJC '15)]

Importance of correctly-handled decays

✦Two examples (dependent of the observable)

Angle between the leptons in the respective mother top rest frames



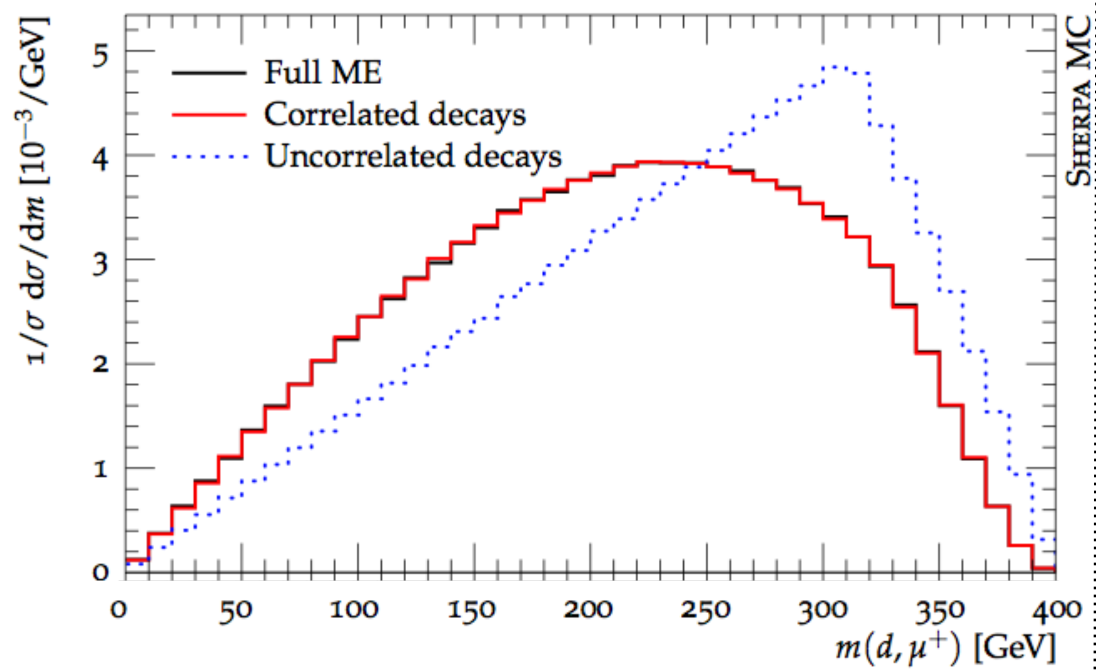
MADSPIN

$t\bar{t}$H production @ (N)LOQCD

[LHC8, dileptonic $t\bar{t}$ decay]

[Artoisenet, Frederix, Mattelaer & Rietkerk (JHEP'13)]

Invariant mass between decay products originating from different cascade steps



SHERPA @ LO[LHC8]

$pp \to \tilde{u}\tilde{u}^\dagger$

$\tilde{u} \to d\tilde{\chi}_1^+ \to d\chi_1^0 W^+ \to d\chi_1^0\mu^+\nu_\mu$

$\tilde{u}^\dagger \to \ldots \to \bar{u}e^+e^-\tilde{\chi}_1^0$

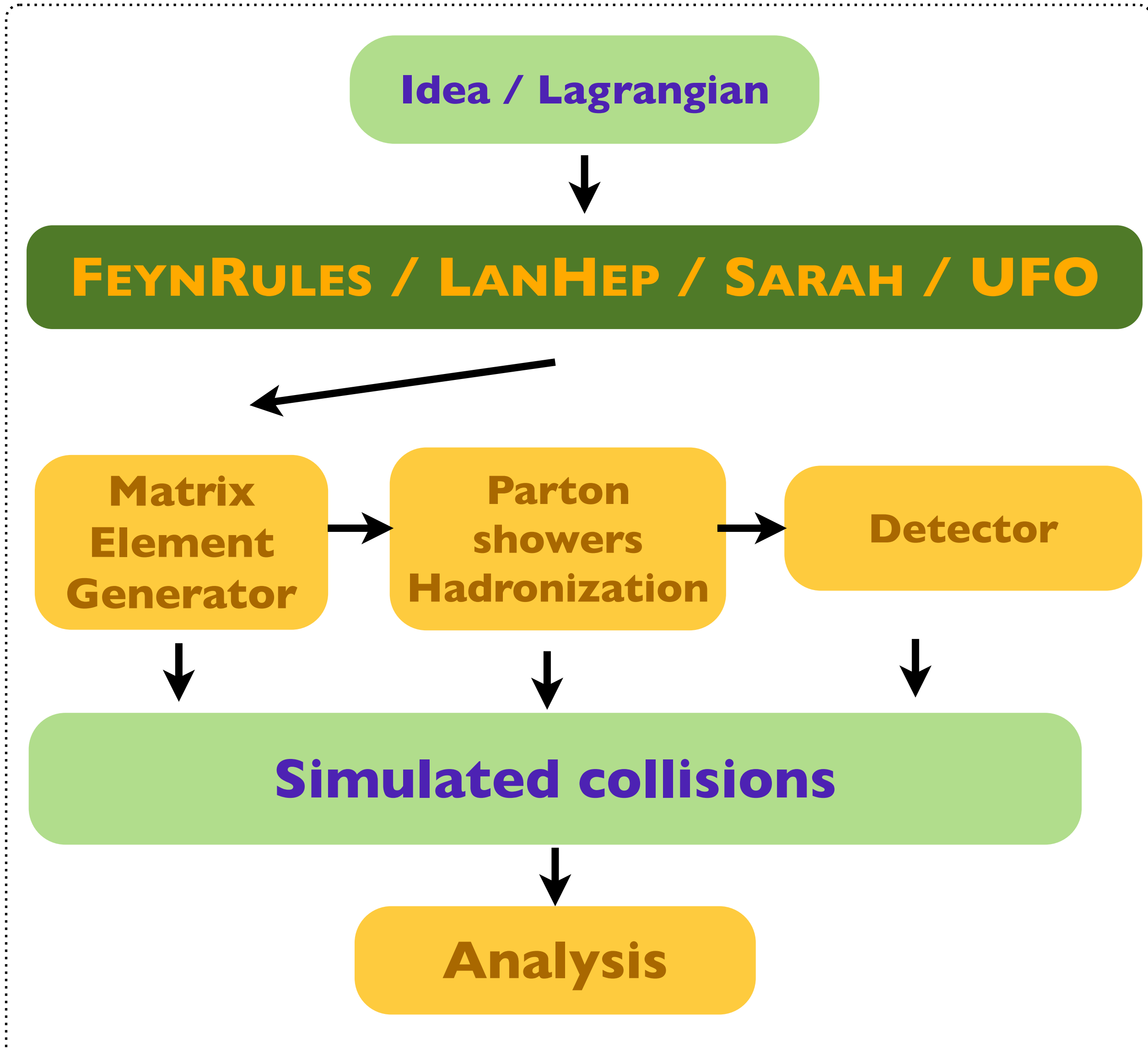
[Höche, Kuttimalai, Schumann & Siegert (EPJC'15)]

[Höche, Kuttimalai, Schumann & Siegert (EPJC'15)]

Outline

1. A comprehensive approach for Monte Carlo simulations
2. Implementing models into Monte Carlo event generators
3. From models to hard-scattering events
4. **Summary**

Summary



- ✦ Event simulation is a complex process
  - ✤ Nature allows us to factorise it into pieces
  - ✤ Event simulation is performed step-by-step
- ✦ This talk: 1st parts of the simulation chain
  - ✤ Connecting models (Lagrangians) to tools
  - ✤ Generating squared matrix elements
  - ✤ Including the decays of heavy particles