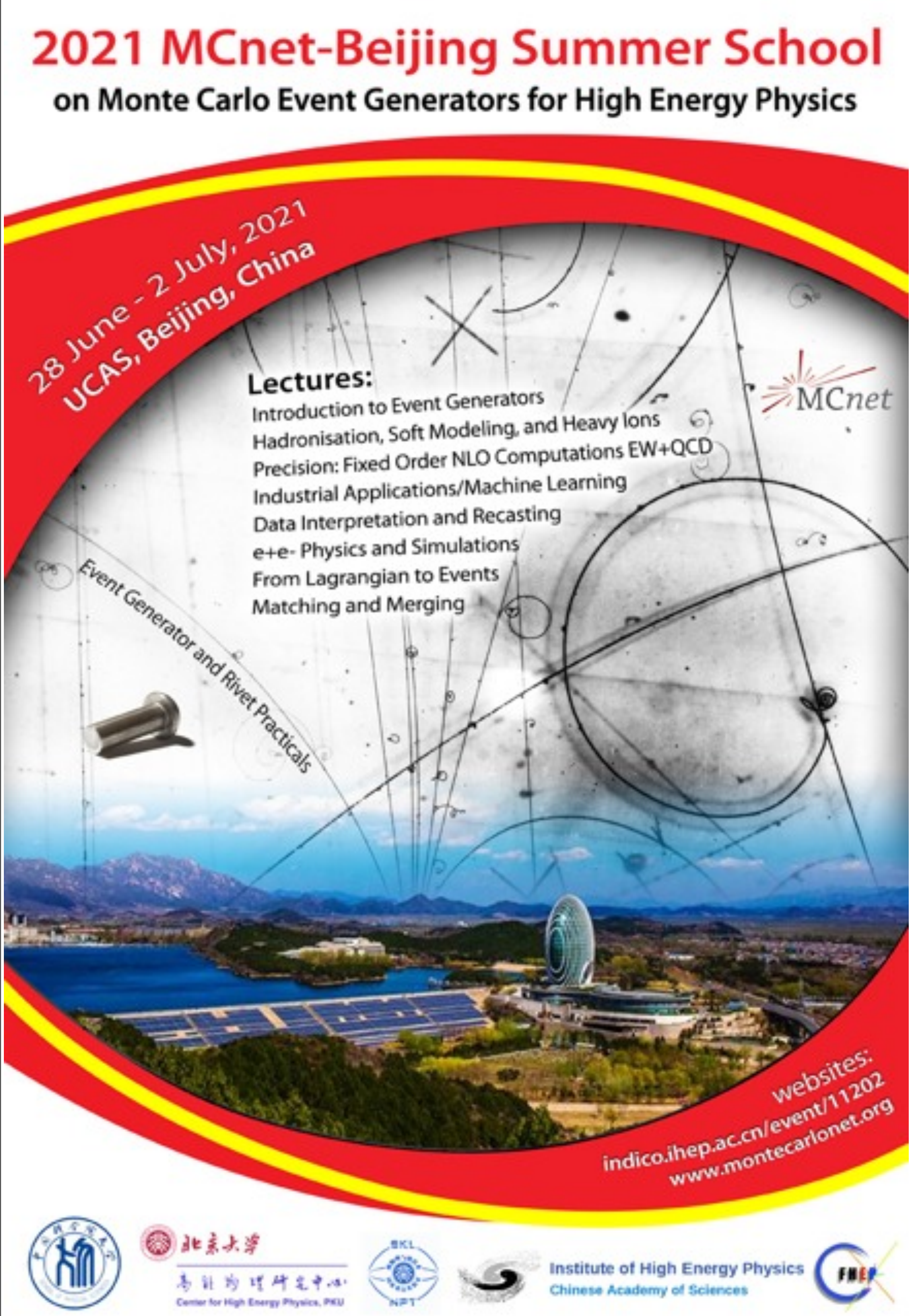
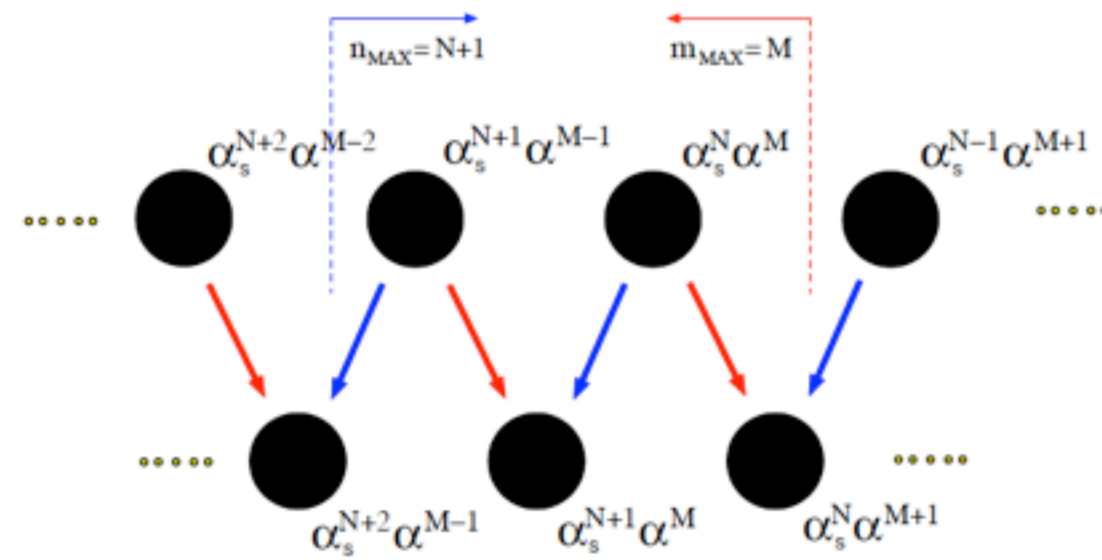


PRECISION :
FIXED ORDER NLO COMPUTATIONS
QCD+EW



HUA-SHENG SHAO

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SORBONNE UNIVERSITÉ
CRÉATEURS DE FUTURS DEPUIS 1257

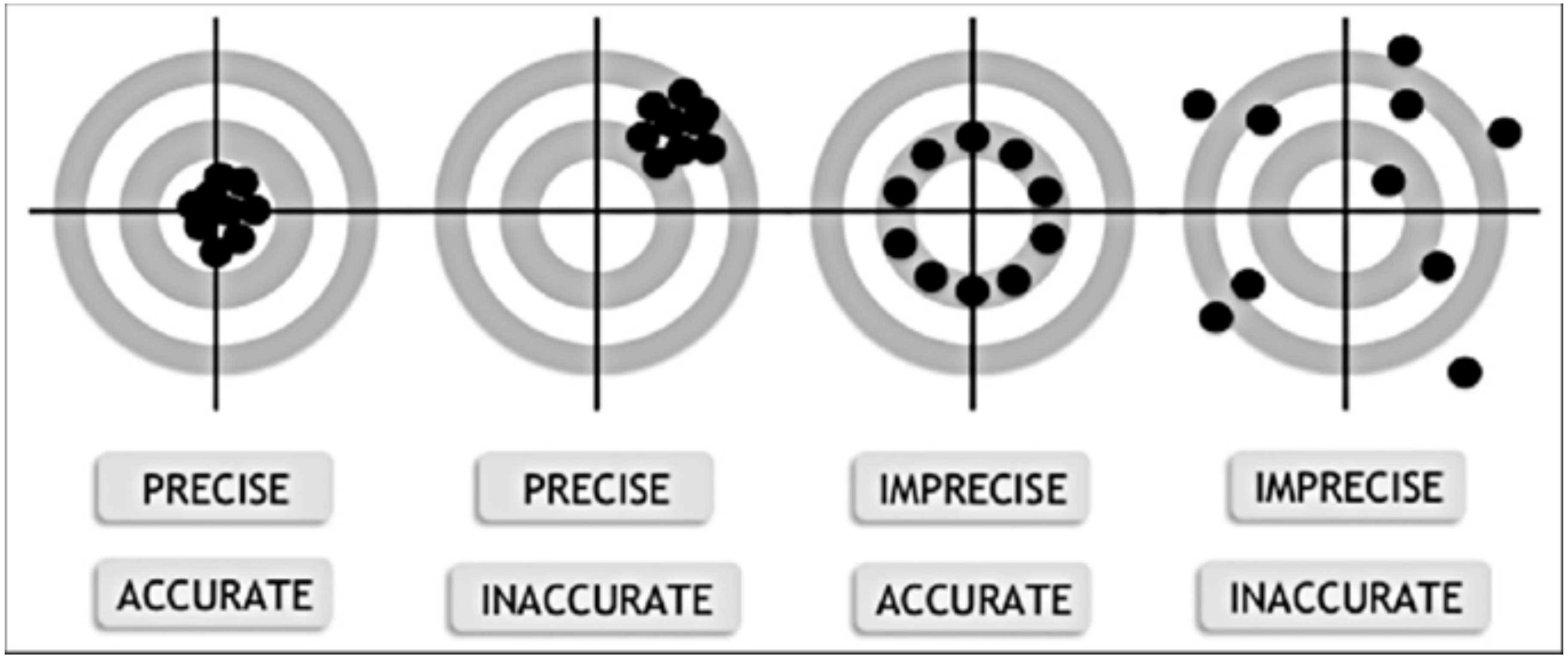
28 JUNE - 02 JULY 2021

Plan

- **Introduction: why we need precision ?**
- **Basics in NLO calculations**
- **Generics in NLO calculations**
- **Advanced NLO topics**

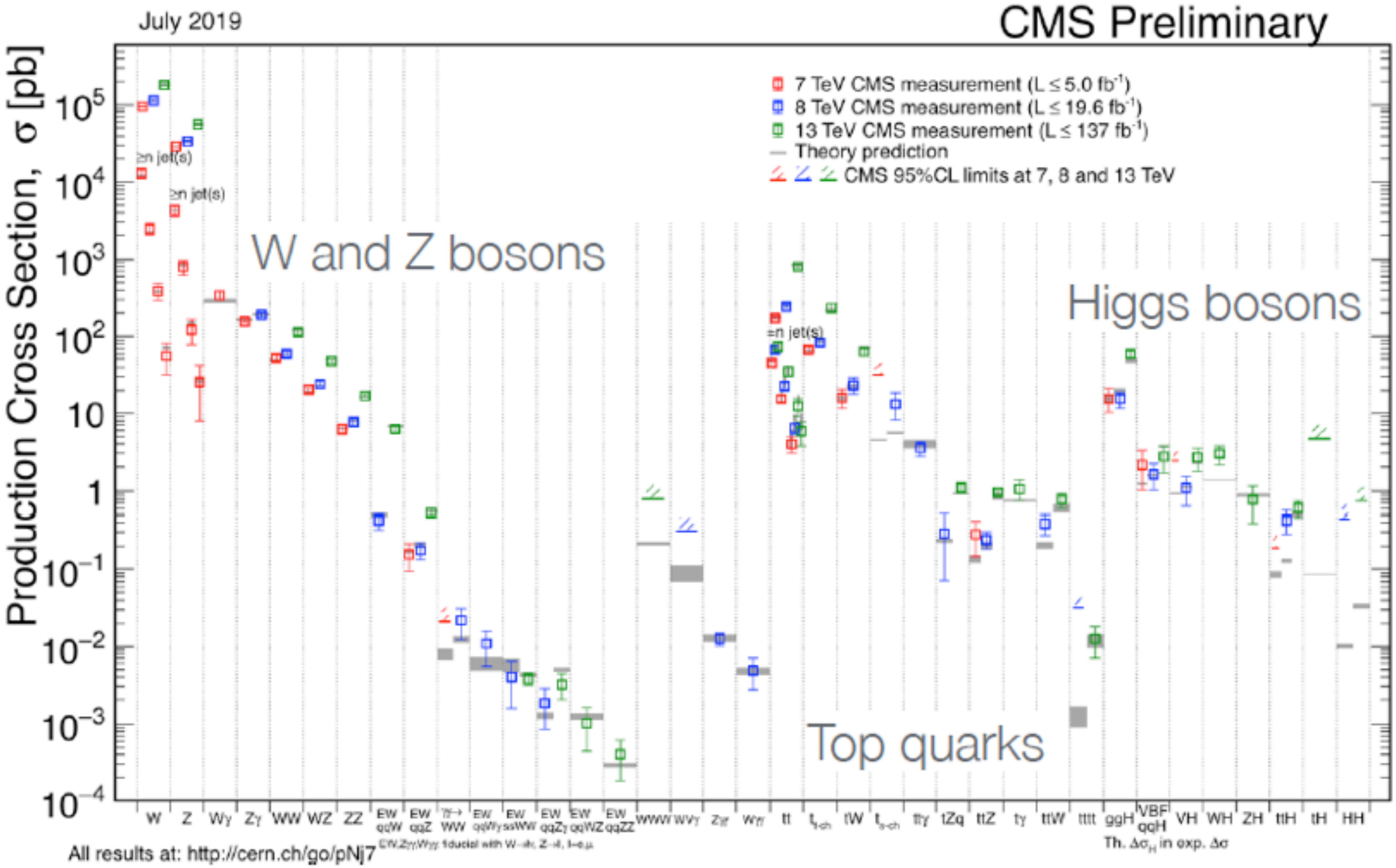
Introduction

Precision & Accuracy



PRECISION MEASUREMENTS AT THE LHC

- **Very impressive SM cross section measurements at the LHC**
  - **many processes are at percent even subpercent level**



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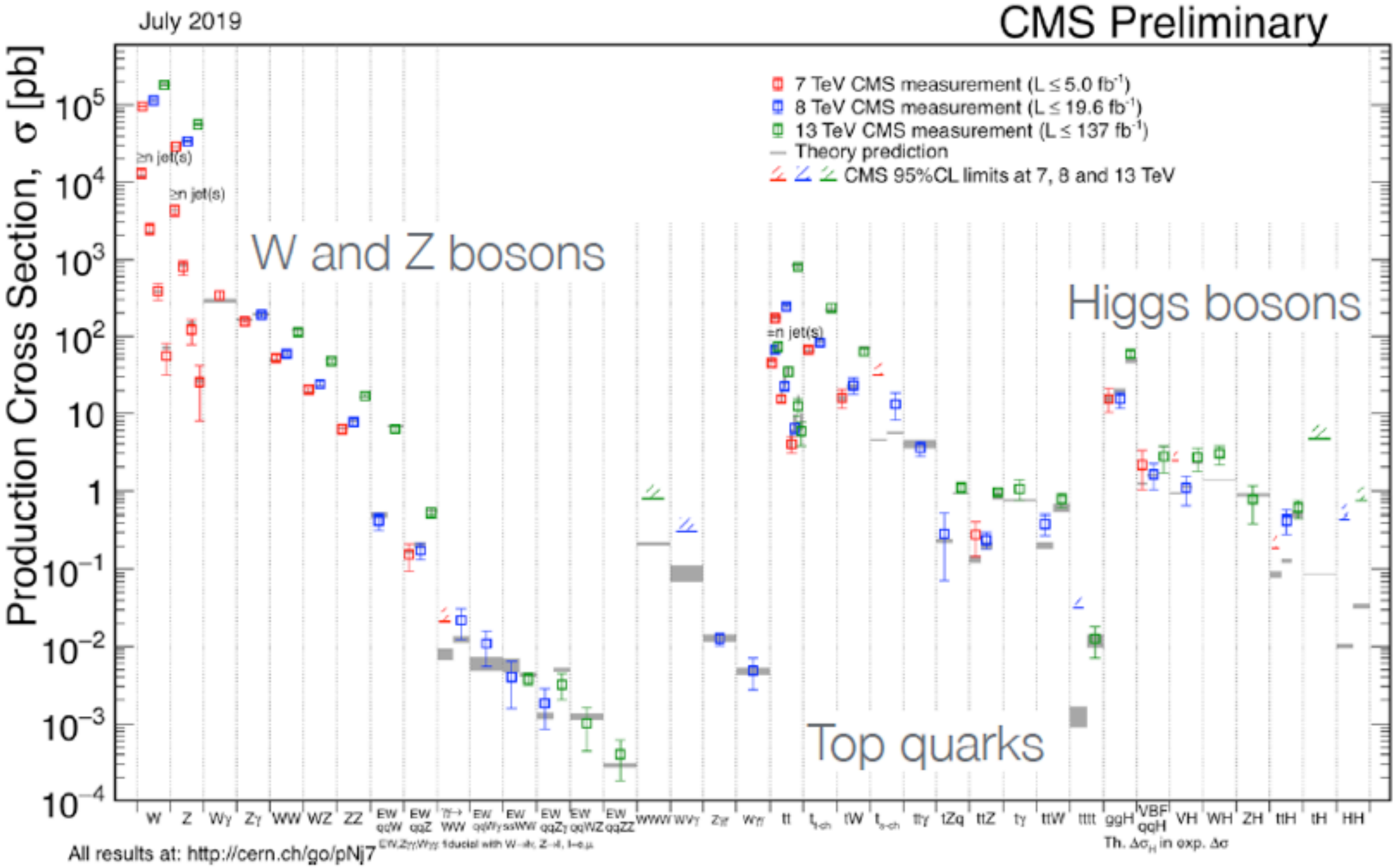
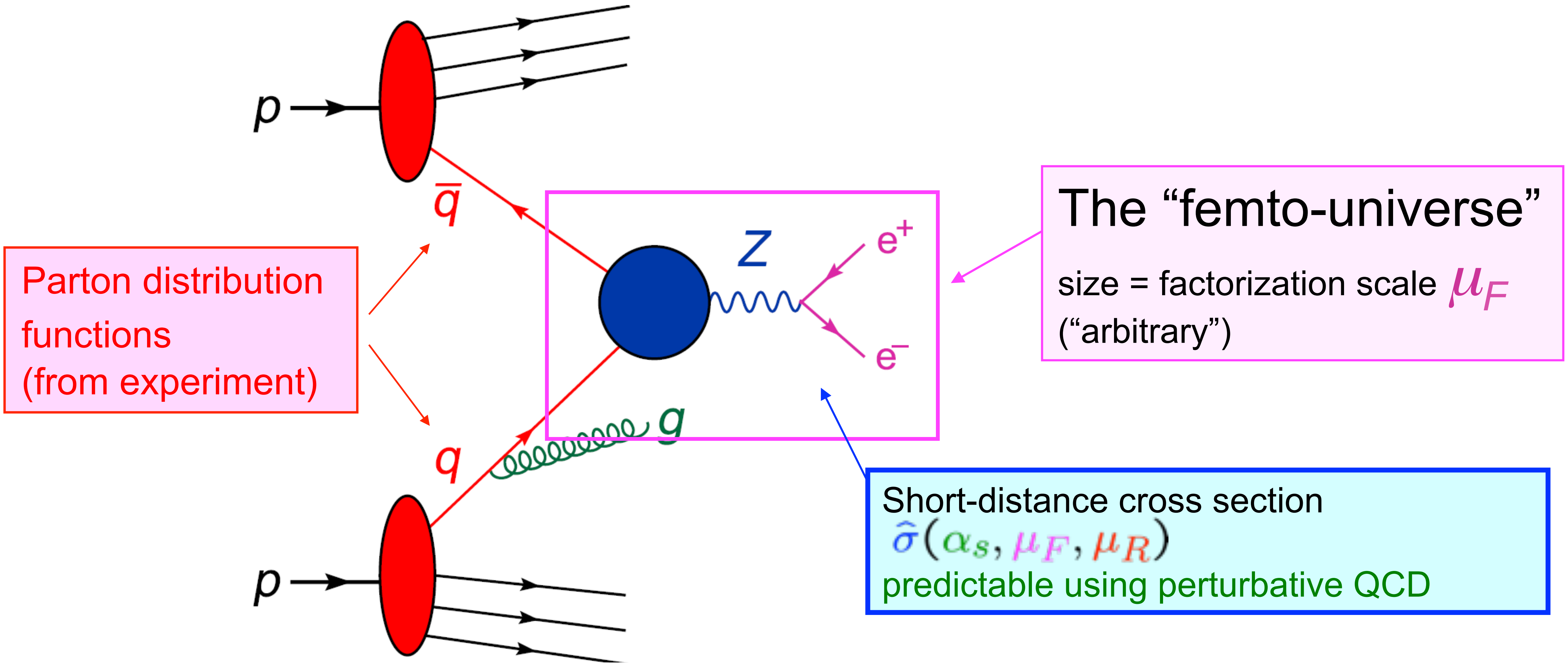


In order to fully exploit these data, theoretical calculations are crucial to keep pace !

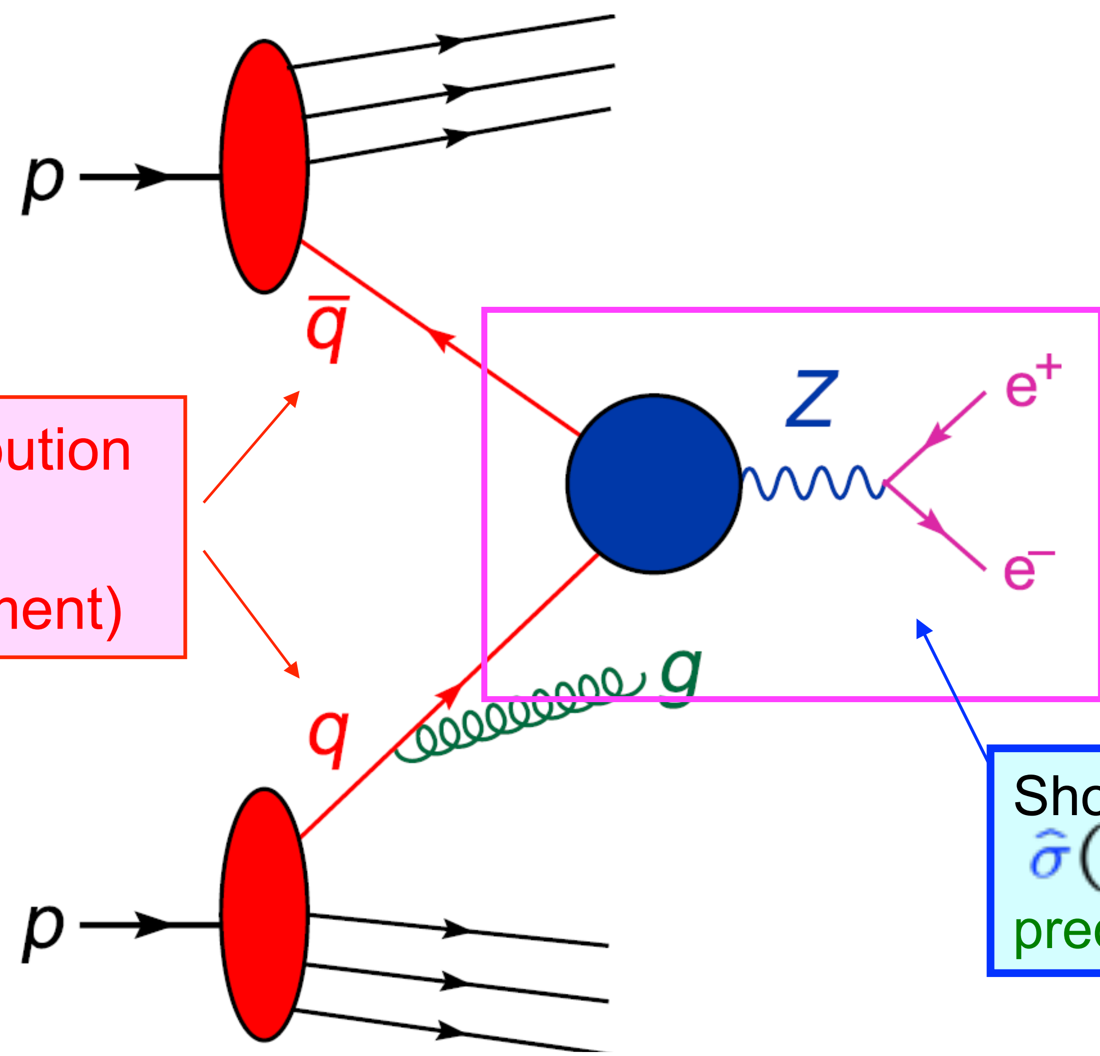
CROSS SECTION @ LHC



$$\sigma(pp \to Z + X) = \int dx_1 dx_2 f(x_1, \mu_F) f(x_2, \mu_F) \hat{\sigma}(\alpha_s, \mu_F, \mu_R)$$

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^n \left[\hat{\sigma}^{(0)} + \frac{\alpha_s}{2\pi}\hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots\right]$$

CROSS SECTION @ LHC



Parton distribution functions (from experiment)

The "femto-universe"

size = factorization scale μ_F ("arbitrary")

Short-distance cross section $\hat{\sigma}(\alpha_s, \mu_F, \mu_R)$ predictable using perturbative QCD

$$\sigma(pp \to Z + X) = \int dx_1 dx_2 f(x_1, \mu_F) f(x_2, \mu_F) \hat{\sigma}(\alpha_s, \mu_F, \mu_R)$$

$$\hat{\sigma}(\alpha_s, \mu_F, \mu_R) = [\alpha_s(\mu_R)]^n \left[\underbrace{\hat{\sigma}^{(0)}}_{\text{LO}} + \frac{\alpha_s}{2\pi} \hat{\sigma}^{(1)}(\mu_F, \mu_R) + \left(\frac{\alpha_s}{2\pi}\right)^2 \hat{\sigma}^{(2)}(\mu_F, \mu_R) + \cdots \right]$$

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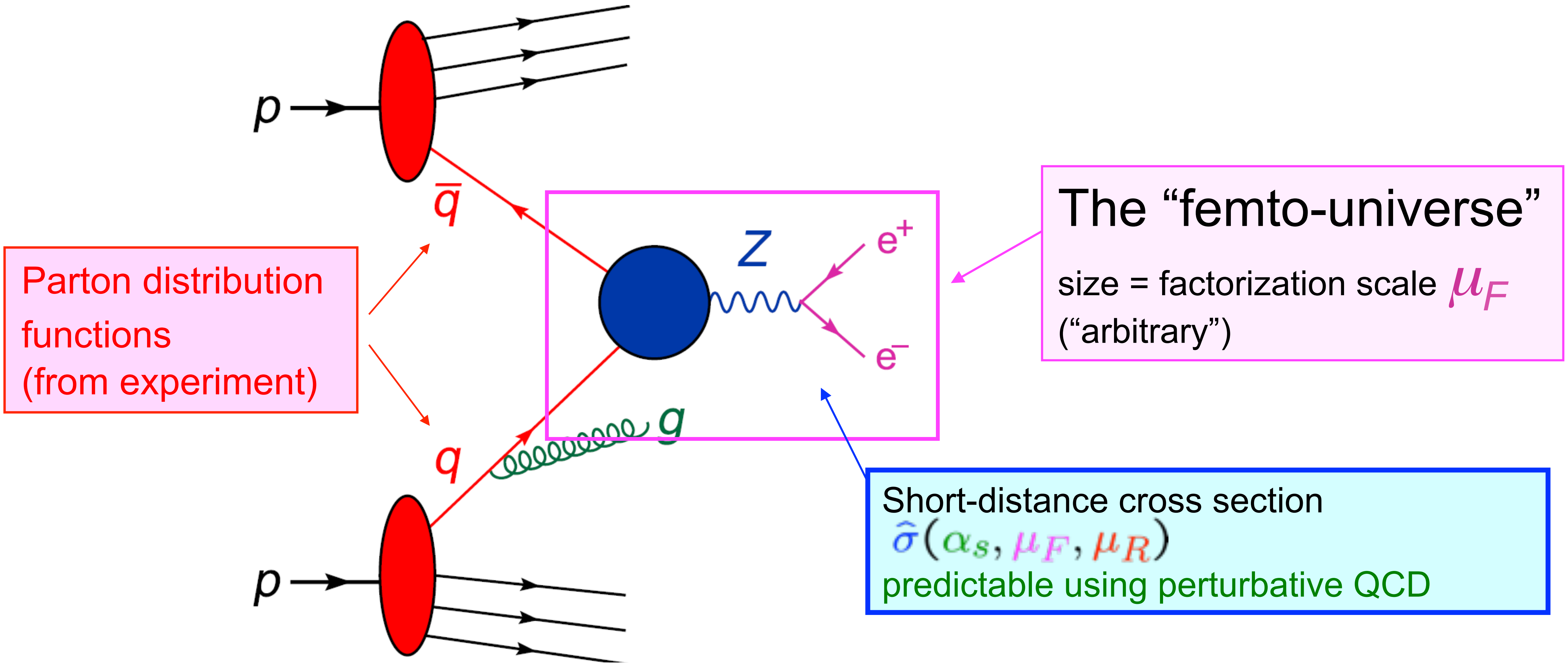

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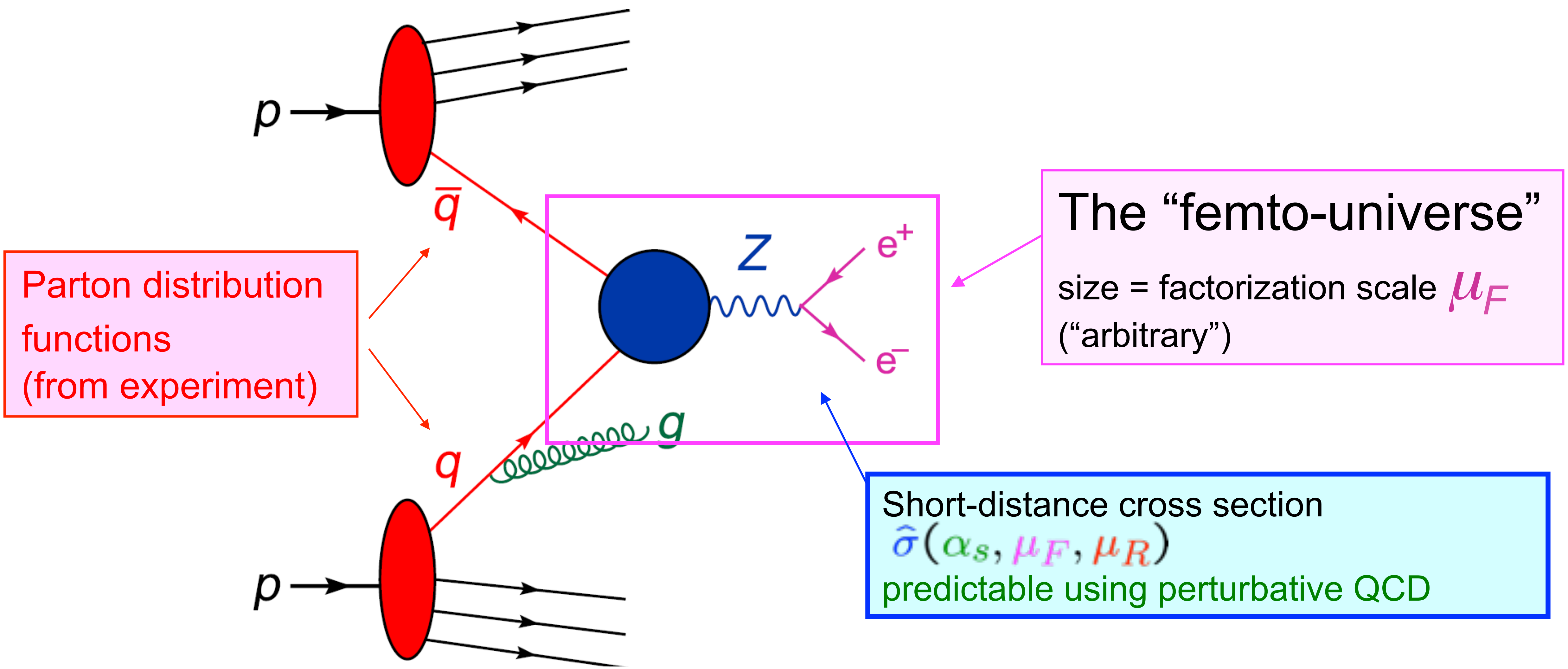
Friday, June 11, 21

CROSS SECTION @ LHC



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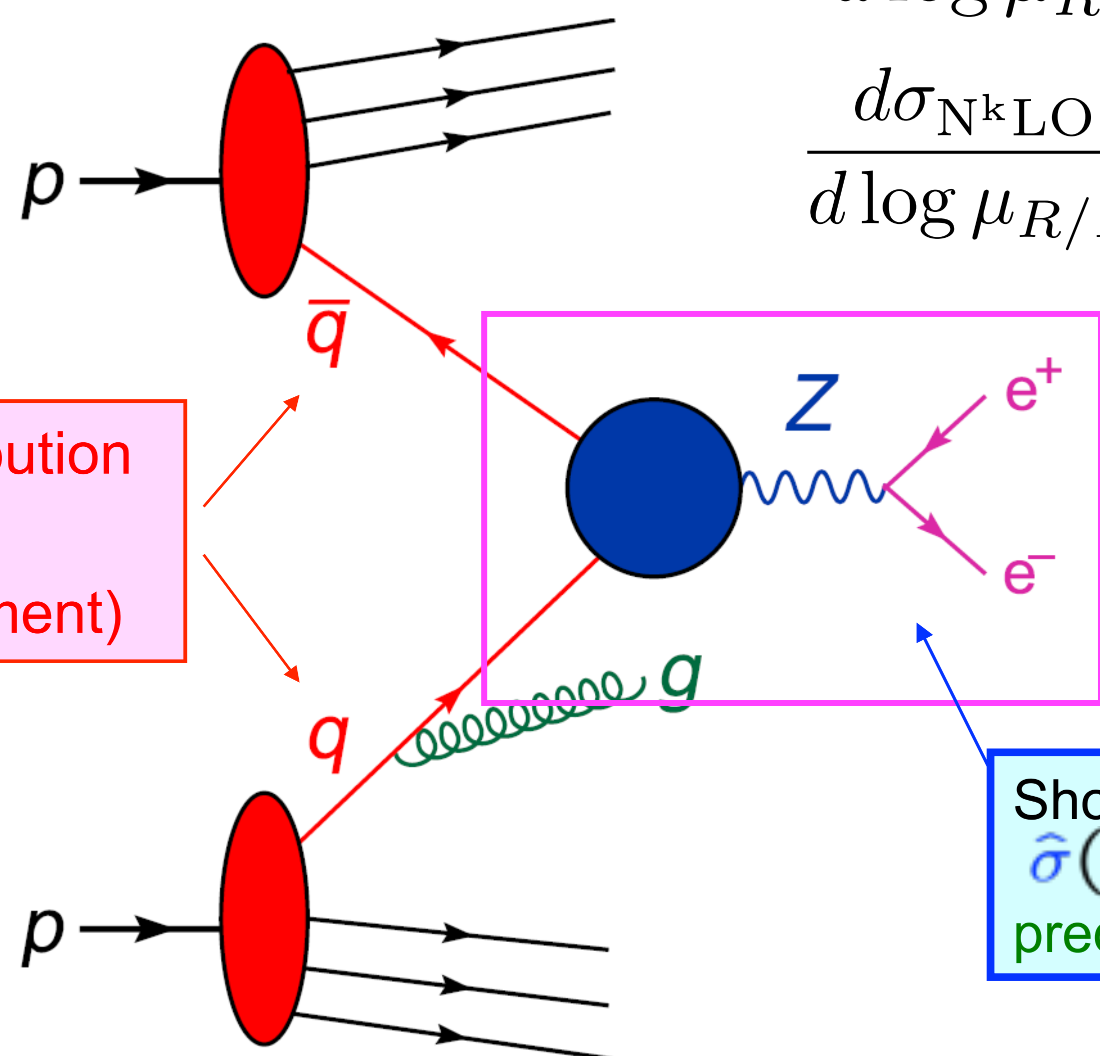
NNLO

CROSS SECTION @ LHC

$$\frac{d\sigma_{\text{all-orders}}}{d\log\mu_{R/F}} = 0$$

$$\frac{d\sigma_{\text{N}^k\text{LO}}}{d\log\mu_{R/F}} \sim \mathcal{O}(\alpha_s^{n+k+1})$$

remaining scale uncertainty



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NNLO

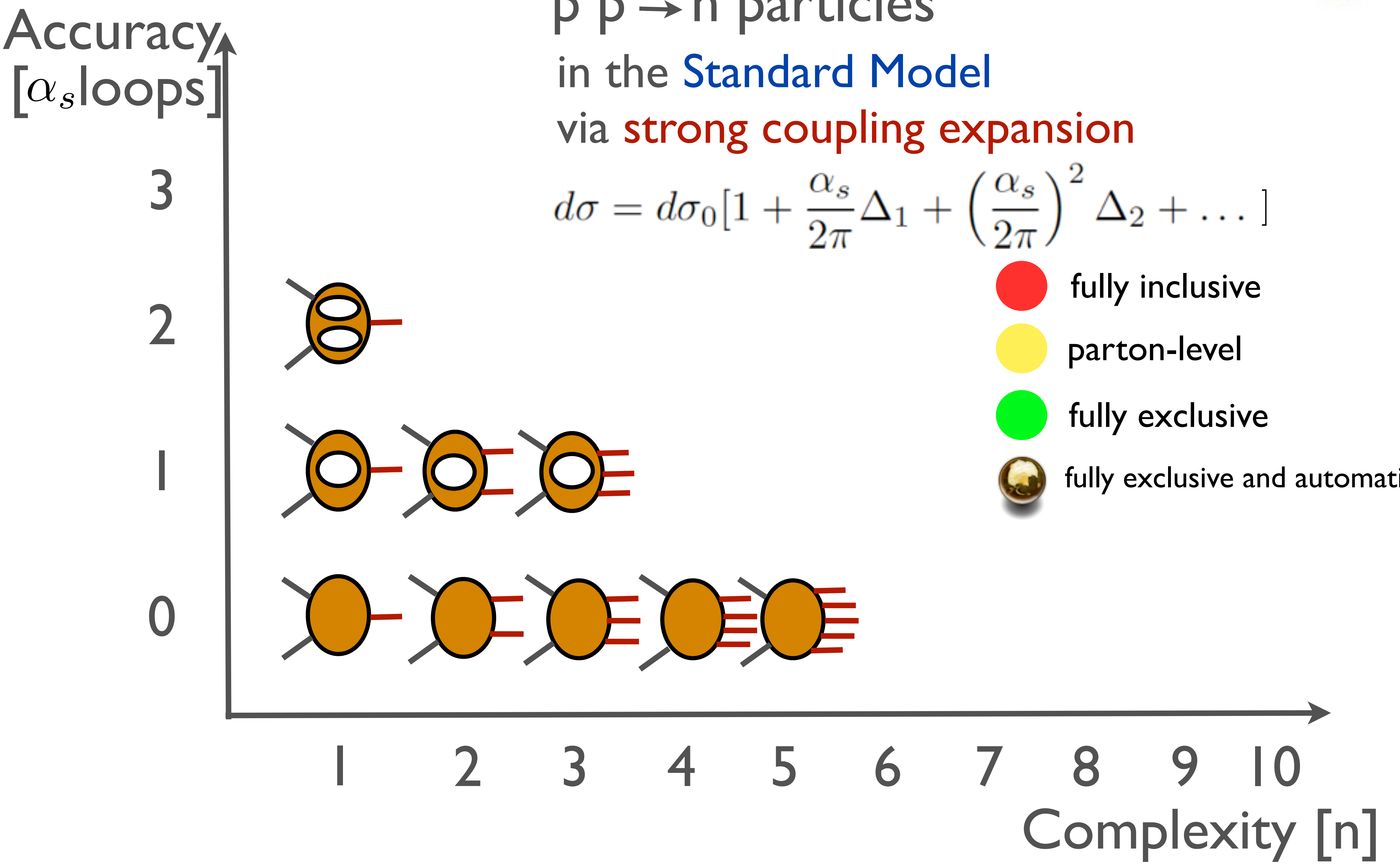
HADRON COLLIDER PHYSICS: PRE-LHC ERA

p p → n particles

in the Standard Model

via strong coupling expansion

$$d\sigma = d\sigma_0[1 + \frac{\alpha_s}{2\pi}\Delta_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta_2 + \ldots]$$



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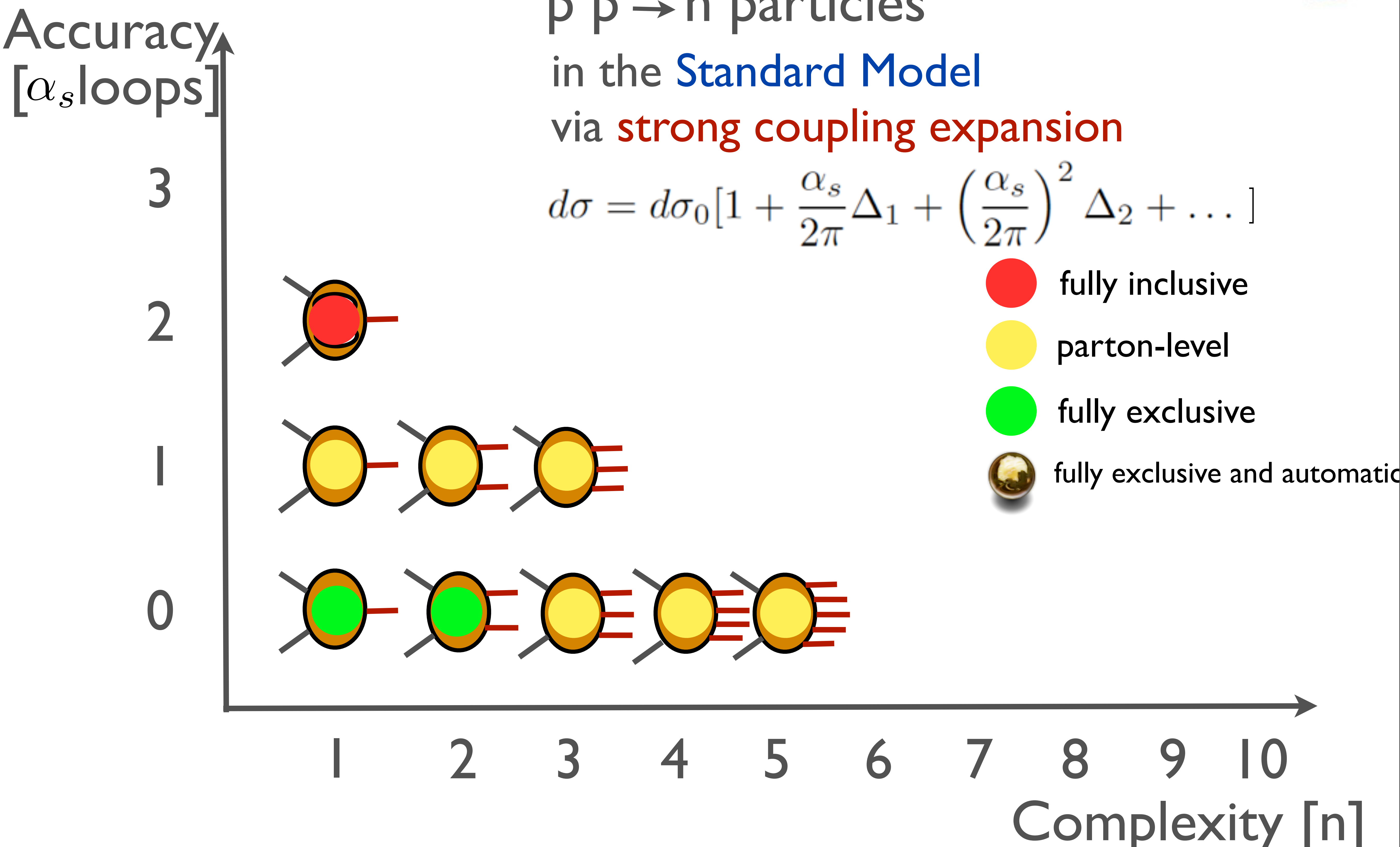

HADRON COLLIDER PHYSICS: NOW

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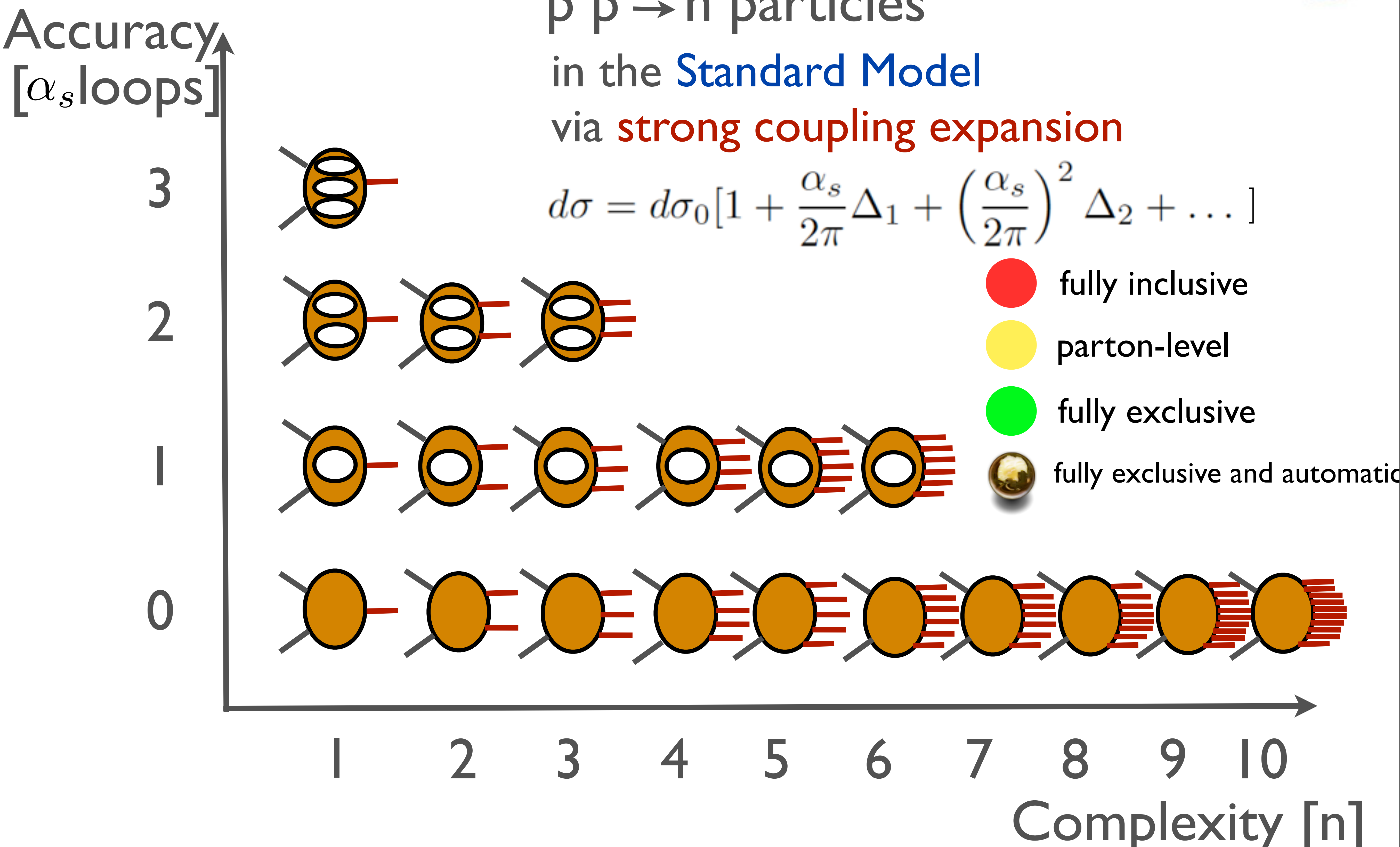

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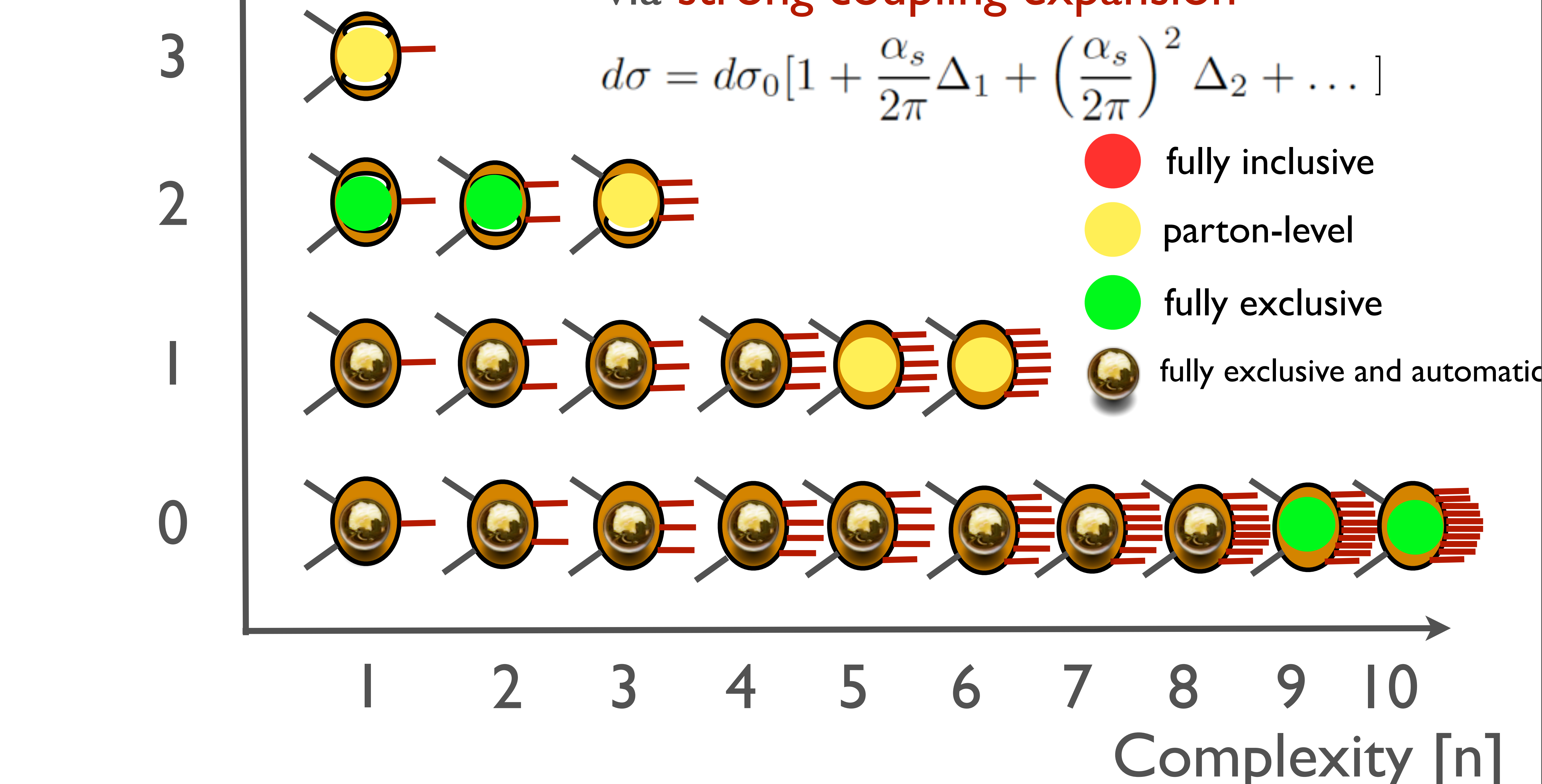

Friday, June 11, 21

HADRON COLLIDER PHYSICS: NOW

p p → n particles

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via electromagnatic coupling expansion

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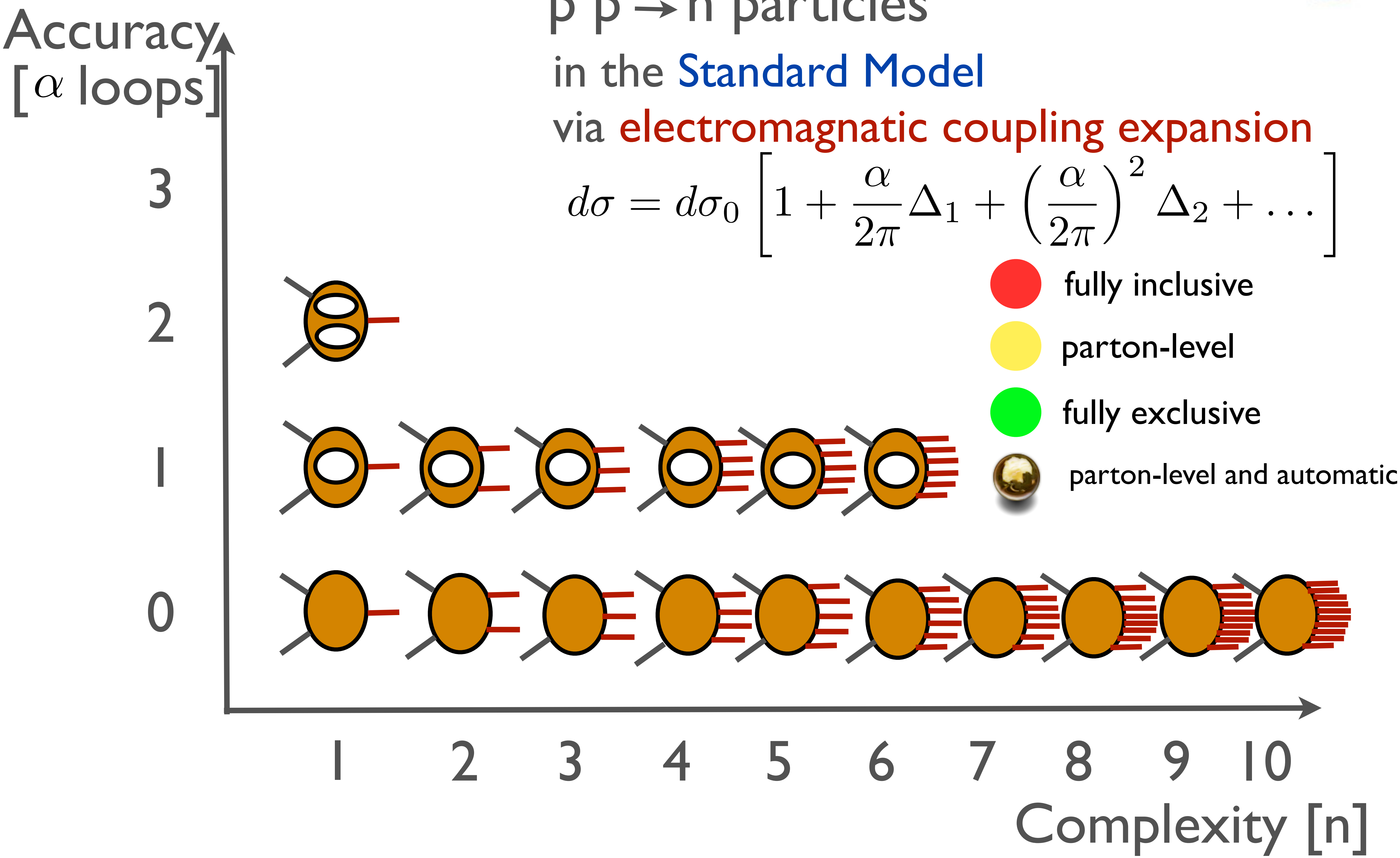

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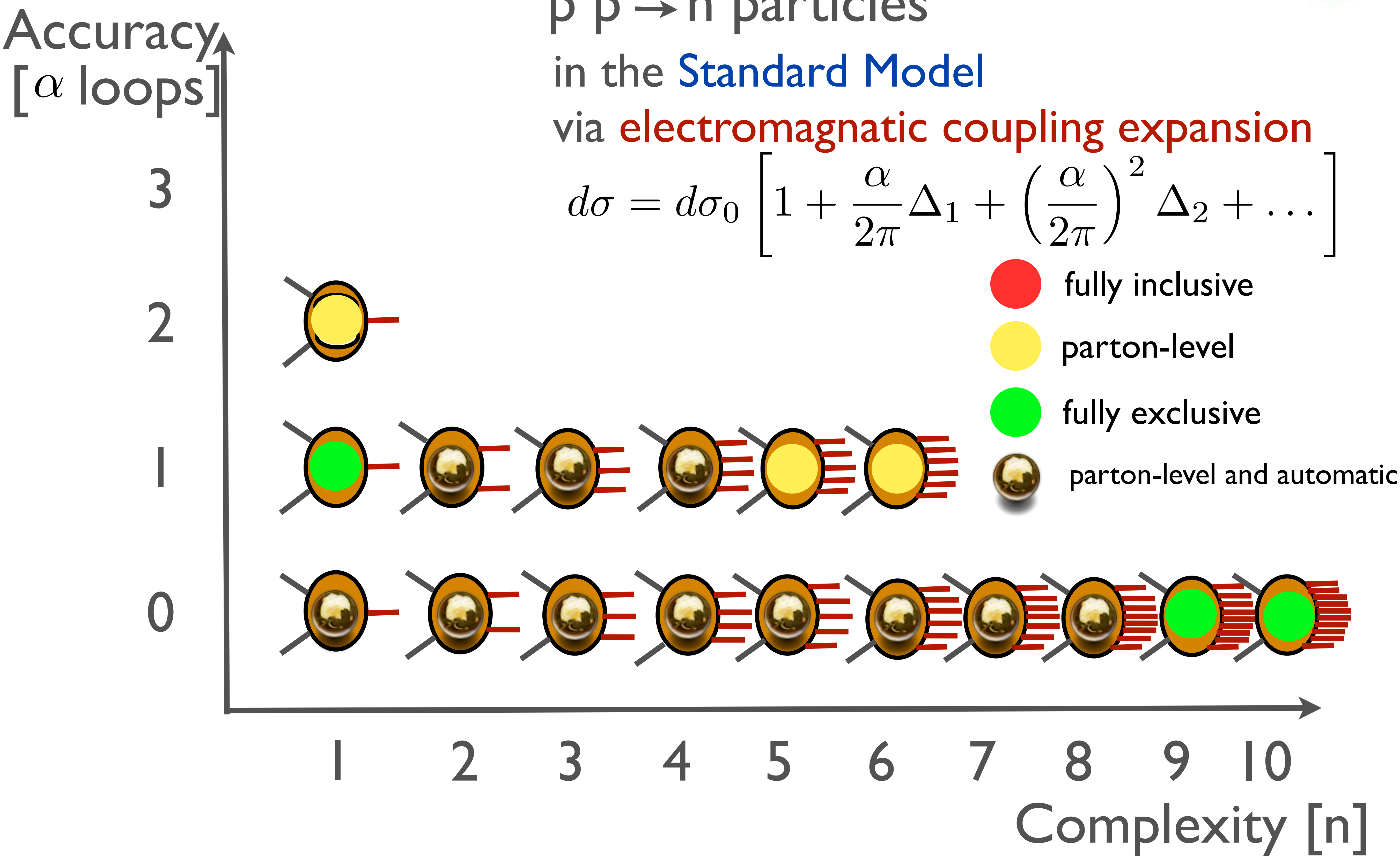


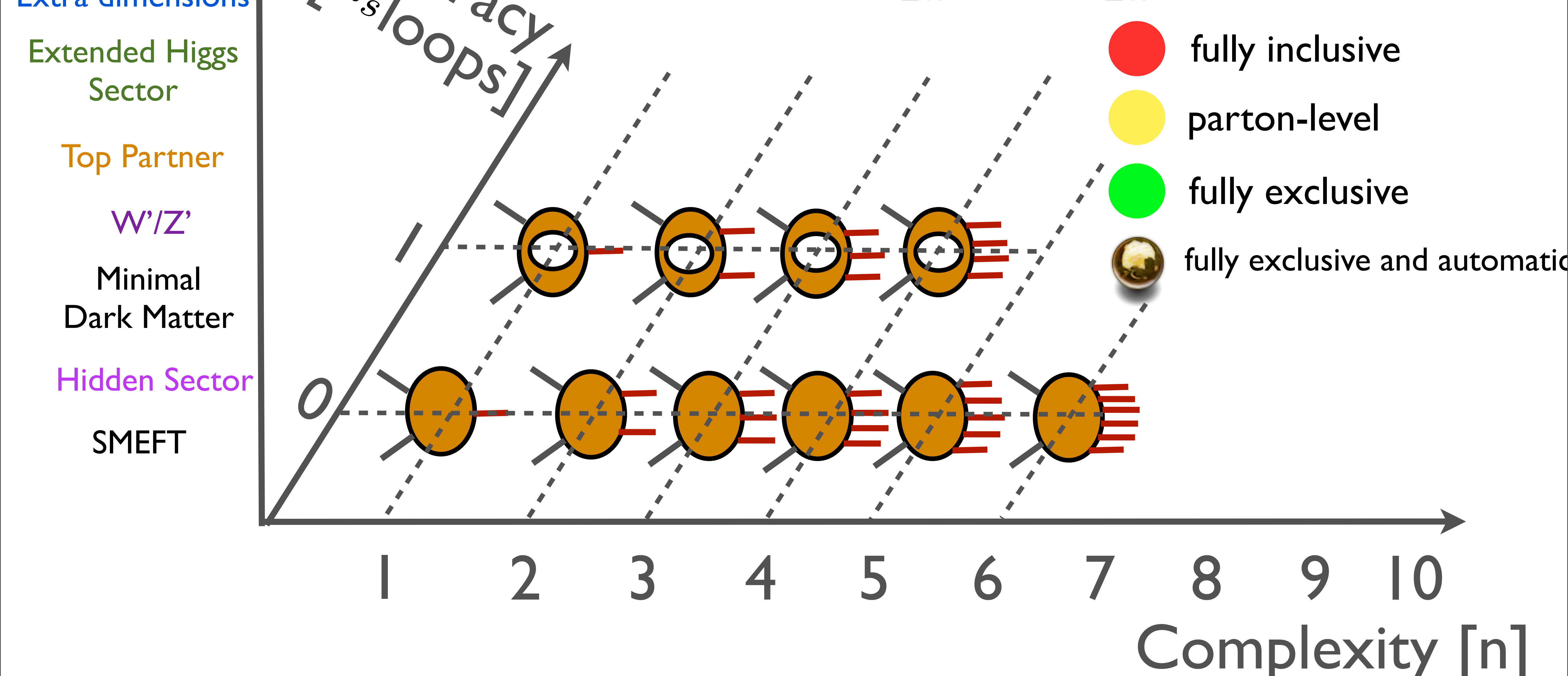
HADRON COLLIDER PHYSICS: NOW

p p → n particles

in Beyond the Standard Model

via strong coupling expansion

$$d\sigma = d\sigma_0[1 + \frac{\alpha_s}{2\pi}\Delta_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 \Delta_2 + \ldots]$$



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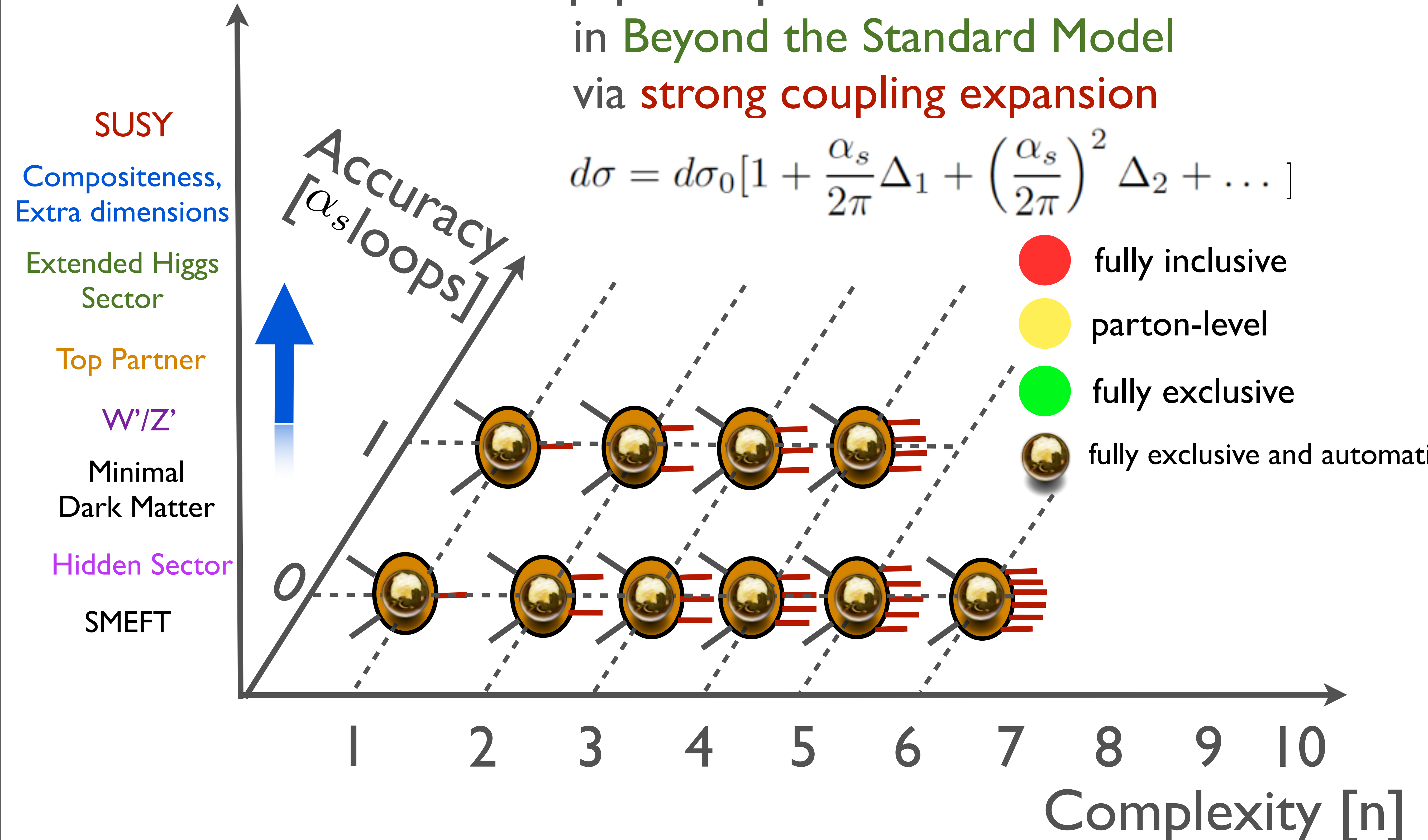
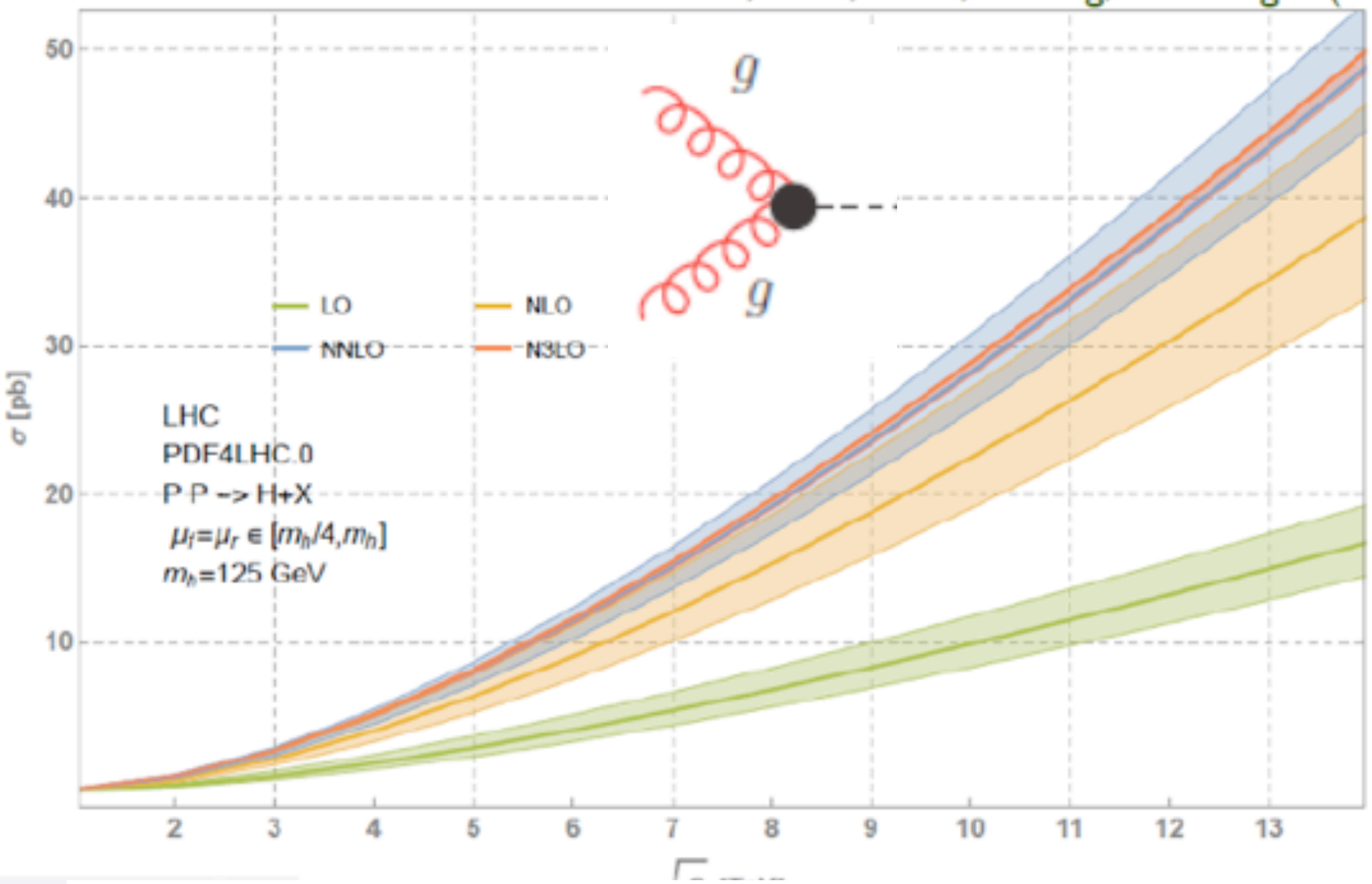


N3LO HIGGS PRODUCTION: HIGHEST ACCURACY

- Percent level inclusive ggF Higgs cross section

Anastasiou, Duhr, Dulat, Herzog, Mistlberger (PRL'15)



Integral Statistics

	NNLO	N3LO
#diagrams	~1.000	~100.000
#integrals	~50.000	517.531.178
#masters	27	1.028
#soft masters	5	78

- Reverse Unitarity
- Differential equations
- Mellin Barnes Representations
- Hopf Algebra of Generalized Polylogs
- Number Theory
- Soft Expansion by Region
- Optimised Algorithm for IBP reduction and powerful computing resources

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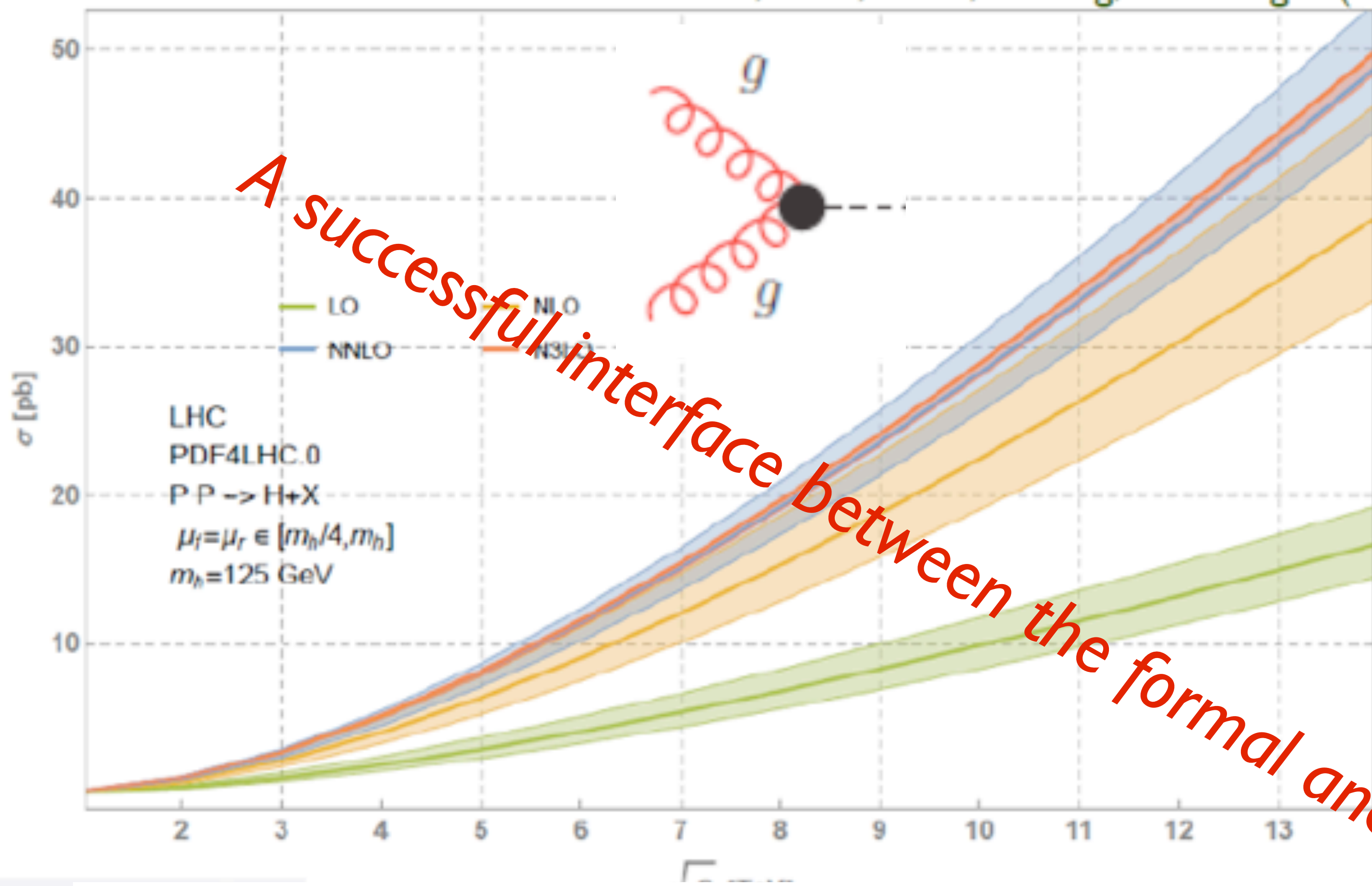

A successful interface between the formal and the phenomenology communities !

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NLO W+JETS: HIGHEST JET MULTIPLICITY AT NLO

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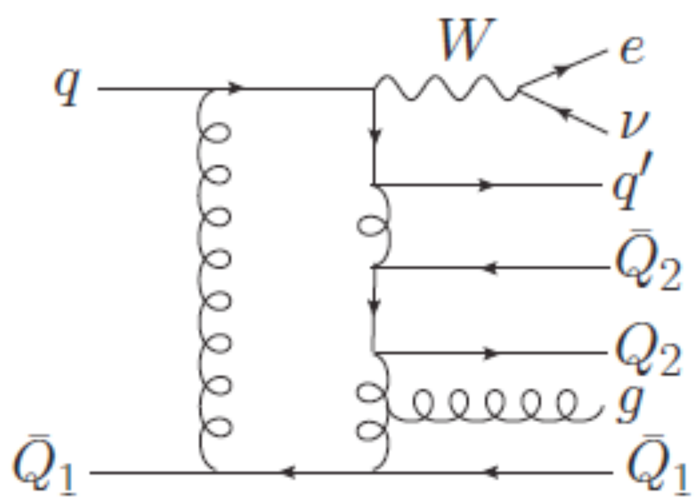

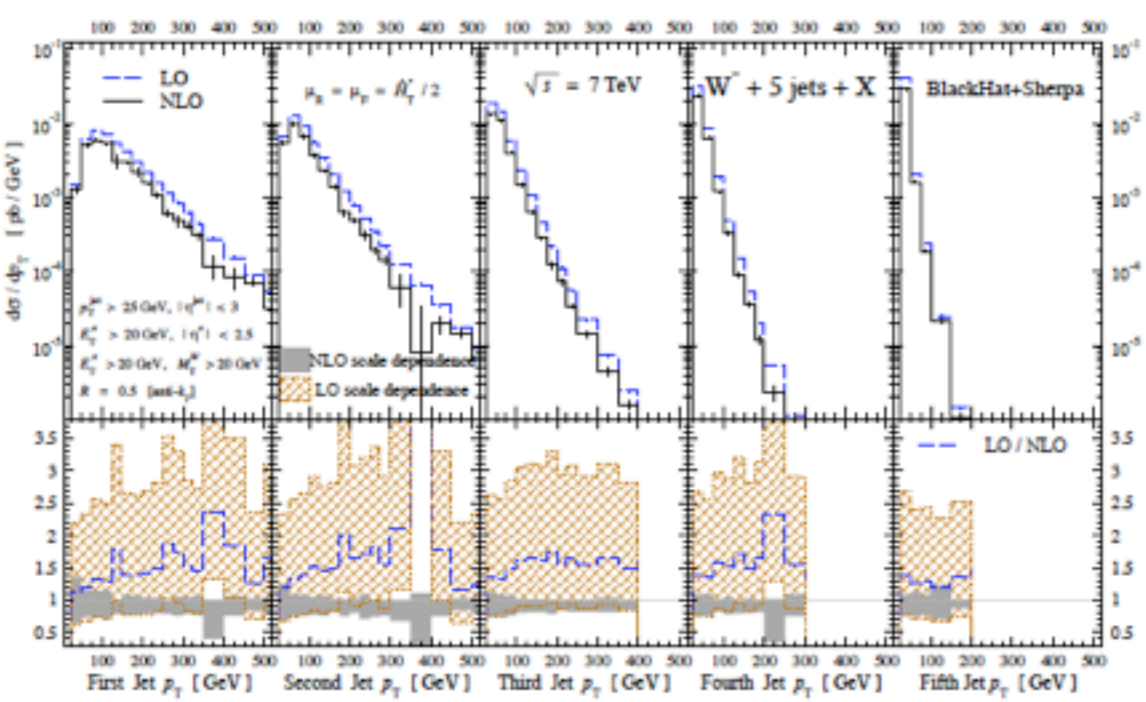

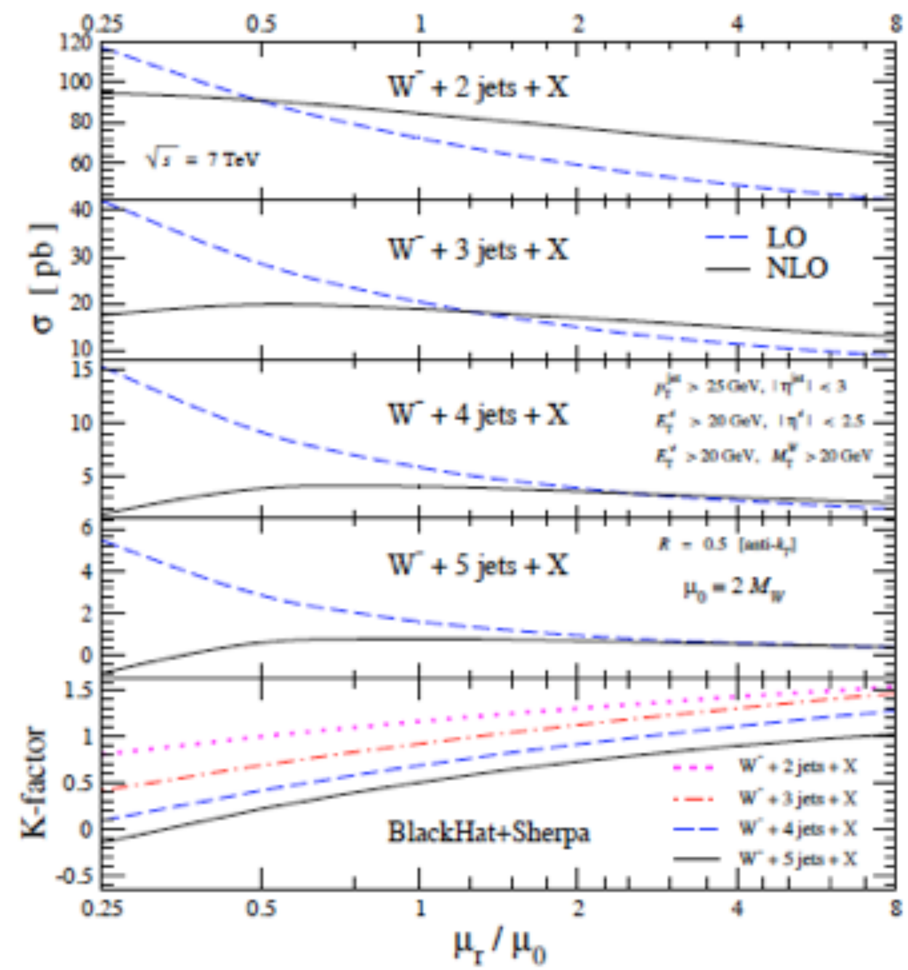

Technique improvements:

- Unitarity cuts
- Integrand reduction
- Recursion relations
- Local IR subtraction

NLO W+JETS: HIGHEST JET MULTIPLICITY AT NLO

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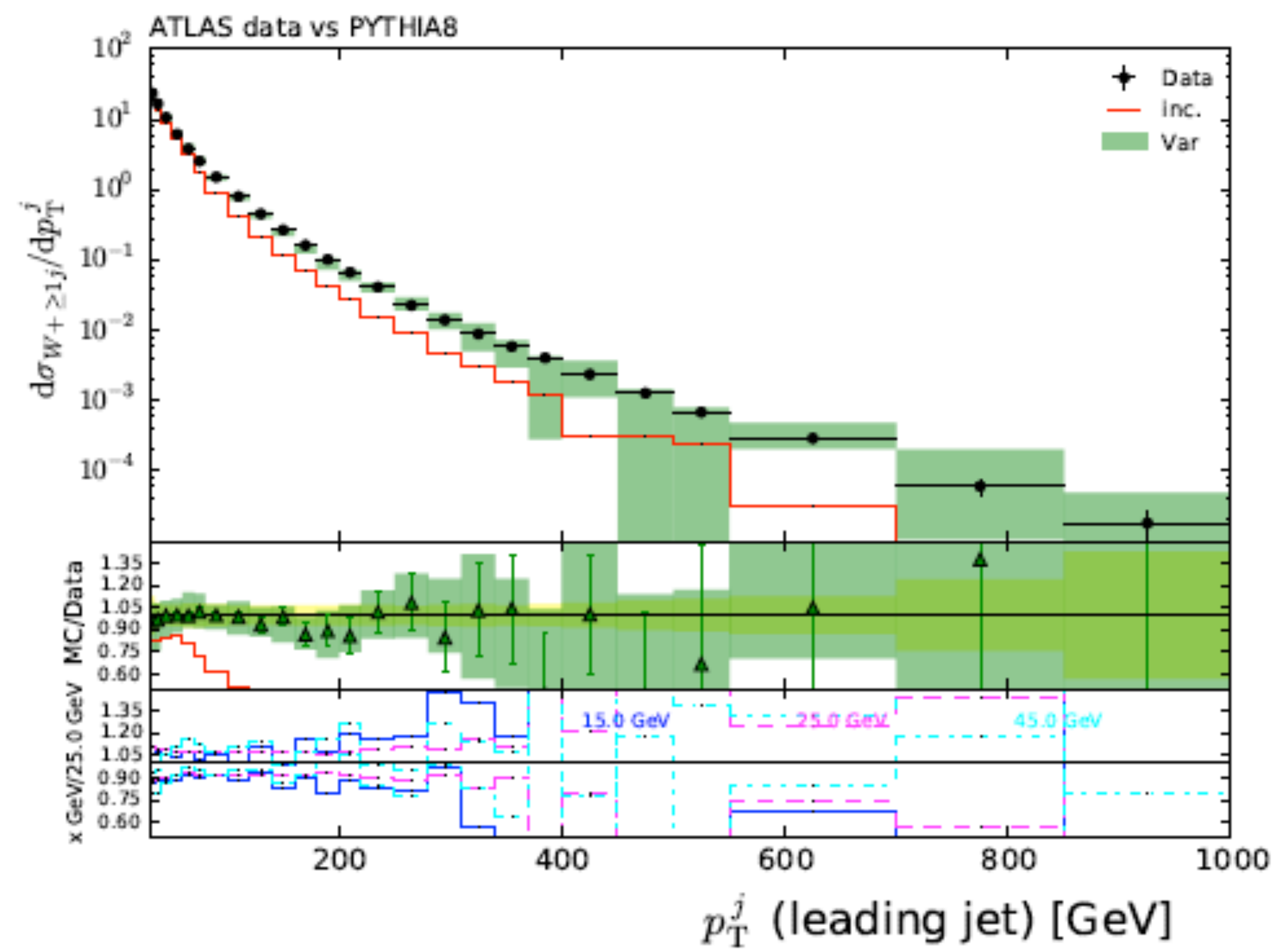
Commands:

```
./bin/mg5_aMC
MG5_aMC > import model loop_sm-no_b_mass
MG5_aMC > define p = p b b~; define j = p
MG5_aMC > define l = e+ mu+ e- mu-
MG5_aMC > define vl = ve vm ve~ vm~
MG5_aMC > generate p p > l vl [QCD] @ 0
MG5_aMC > generate p p > l vl j [QCD] @ 1
MG5_aMC > generate p p > l vl j j [QCD] @ 2
MG5_aMC > output; launch
```



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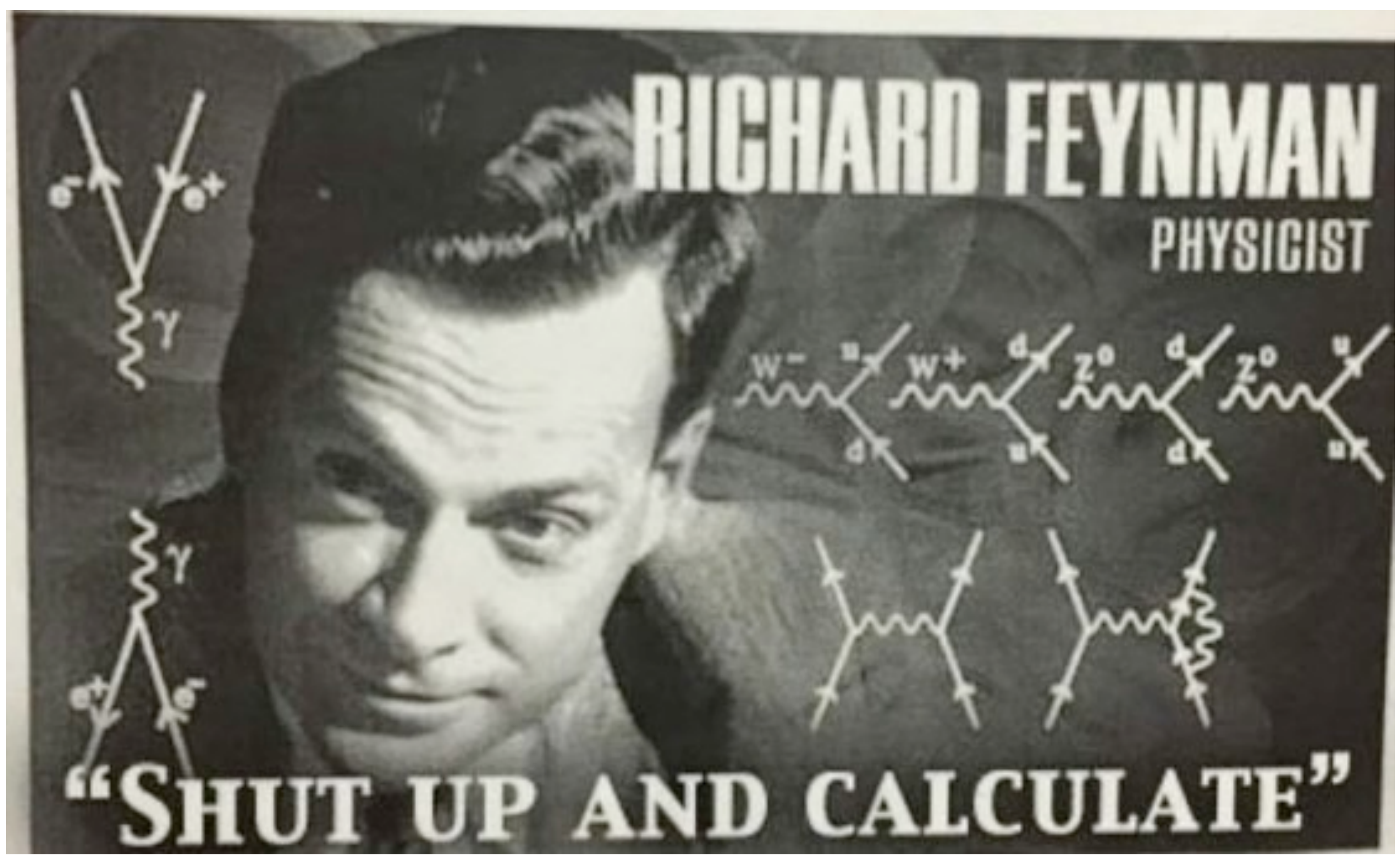
Technique improvements:
- Matured automated framework
- Methods of matching ME to PS
- Merging of multi-jet ME with PS

Alwall et al. (JHEP'14)

Basics

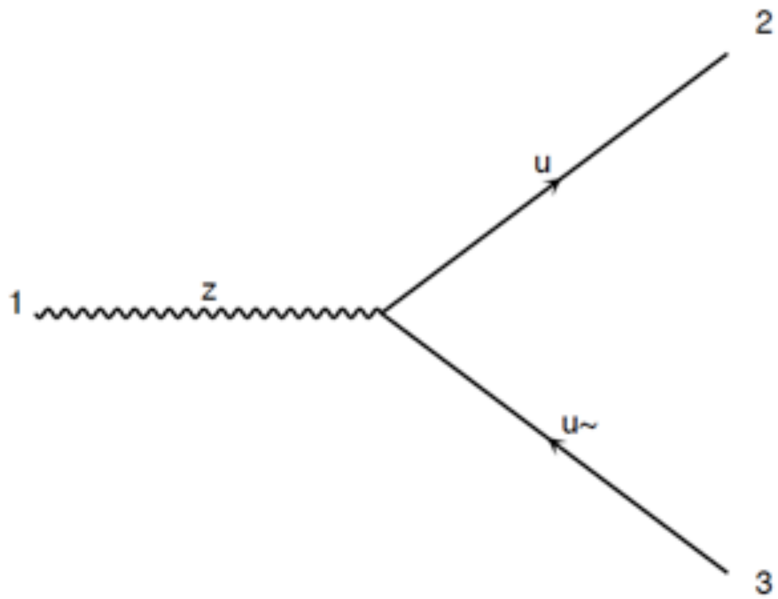


Basics

A NLO example

A NLO EXAMPLE: BORN

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Writing down Born amplitude according to Feynman rules**



For simplicity, we assume quarks are massless

$$\mathcal{A}_{\mathrm{Born}} = -\delta_{c_q c_{\bar{q}}} \varepsilon_\mu(p_Z)\bar{u}(p_q).\Gamma^\mu_{Zq\bar{q}}.v(p_{\bar{q}})$$

$$\Gamma^\mu_{Zq\bar{q}} = ie\left(\frac{I_q}{\cos\theta_w \sin\theta_w} - Q_q\frac{\sin\theta_w}{\cos\theta_w}\right)\gamma^\mu P_L - ieQ_q\frac{\sin\theta_w}{\cos\theta_w}\gamma^\mu P_R$$

  - **Squaring amplitude, summing over colours and spins, and averaging the spin of the initial state**

$$\boxed{\alpha = \frac{e^2}{4\pi}}$$

$$\overline{\sum}|\mathcal{A}_{\mathrm{Born}}|^2 = 8\pi\alpha m_Z^2\left(2Q_q^2\left(\frac{\sin\theta_w}{\cos\theta_w}\right)^2 - 2\frac{I_qQ_q}{\cos^2\theta_w} + \frac{I_q^2}{\cos^2\theta_w\sin^2\theta_w}\right)$$

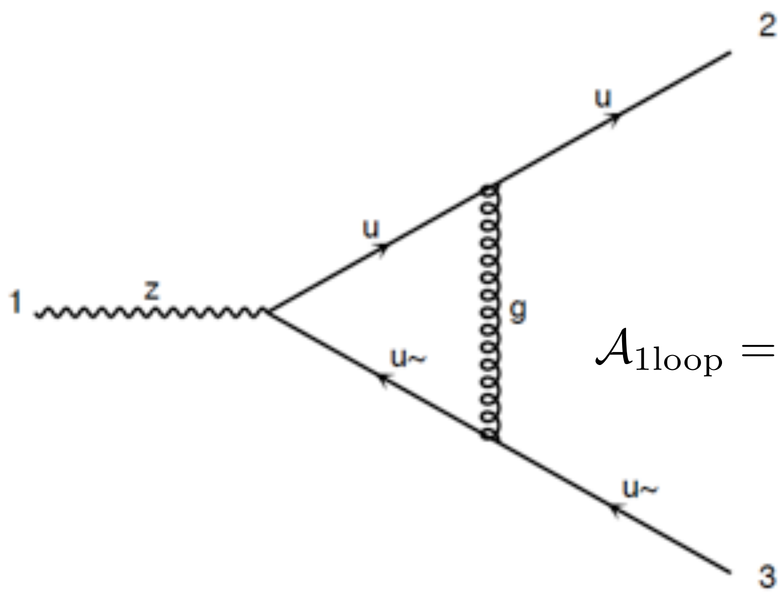
  - **Phase-space integration**

$$\Gamma_{\mathrm{Born}}(Z\to q\bar{q}) = \frac{1}{2m_Z}\int (2\pi)^4\delta^4(p_Z - p_q - p_{\bar{q}})\frac{1}{(2\pi)^{3\times 2}}\frac{d^3p_q}{2E_q}\frac{d^3p_{\bar{q}}}{2E_{\bar{q}}}\overline{\sum}|\mathcal{A}_{\mathrm{Born}}|^2$$
$$= \alpha m_Z\left(Q_q^2\frac{\sin^2\theta_w}{\cos^2\theta_w} - \frac{Q_qI_q}{\cos^2\theta_w} + \frac{I_q^2}{2\cos^2\theta_w\sin^2\theta_w}\right)$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Writing down one-loop amplitude according to Feynman rules**

For simplicity, we assume quarks are massless



$$\mathcal{A}_{1\text{loop}} = i g^{\nu\rho}\varepsilon_\mu(p_Z)\bar{u}(p_q).\left(-i g_s \gamma_\nu T^a_{c_q c}\right).\int \frac{d^d\bar{l}}{(2\pi)^d}\frac{\bar{l\!\!/}.\Gamma^\mu_{Zq\bar{q}}.(\bar{l\!\!/}-p\!\!/_Z)}{\bar{l}^2\left(\bar{l}-p_q\right)^2\left(\bar{l}-p_Z\right)^2}.\left(-i g_s\gamma_\rho T^a_{cc_{\bar{q}}}\right).v(p_{\bar{q}})$$

  - **Need to evaluate two tensor integrals**

$$I_1^\mu = \int \frac{d^d\bar{l}}{(2\pi)^d}\frac{\bar{l}^\mu}{\bar{l}^2\left(\bar{l}-p_q\right)^2\left(\bar{l}-p_Z\right)^2} \qquad I_2^{\mu\nu} = \int \frac{d^d\bar{l}}{(2\pi)^d}\frac{\bar{l}^\mu\bar{l}^\nu}{\bar{l}^2\left(\bar{l}-p_q\right)^2\left(\bar{l}-p_Z\right)^2}$$

according to Lorentz structures

$$I_1^\mu = p_q^\mu B_1 + p_Z^\mu B_2 \qquad I_2^{\mu\nu} = g^{\mu\nu}B_{00} + p_q^\mu p_q^\nu B_{11} + p_Z^\mu p_Z^\nu B_{22} + \left(p_q^\mu p_Z^\nu + p_Z^\mu p_q^\nu\right)B_{12}$$

Solving the coefficients B, e.g.

$$p_q\cdot I_1 = p_q^2 B_1 + p_q\cdot p_Z B_2 = p_q\cdot p_Z B_2 \qquad p_Z\cdot I_1 = p_q\cdot p_Z B_1 + p_Z^2 B_2 = p_q\cdot p_Z B_1 + m_Z^2 B_2$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Solving the coefficients B, e.g.

$$B_2 = \frac{p_q \cdot I_1}{p_q \cdot p_Z} \qquad B_1 = \frac{p_Z \cdot I_1 - m_Z^2 B_2}{p_q \cdot p_Z}$$

$$\begin{aligned} p_q \cdot I_1 &= \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{p_q \cdot \bar{l}}{\bar{l}^2 \left(\bar{l} - p_q\right)^2 \left(\bar{l} - p_Z\right)^2} \\ &= \frac{1}{2} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{\bar{l}^2 - \left(\bar{l} - p_q\right)^2}{\bar{l}^2 \left(\bar{l} - p_q\right)^2 \left(\bar{l} - p_Z\right)^2} \\ &= \frac{1}{2} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{1}{\left(\bar{l} - p_q\right)^2 \left(\bar{l} - p_Z\right)^2} - \frac{1}{2} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l} - p_Z\right)^2} \\ &= \frac{1}{2} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l} - p_{\bar{q}}\right)^2} - \frac{1}{2} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l} - p_Z\right)^2} \end{aligned}$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d}\frac{1}{\bar{l}^2\left(\bar{l}-p_{\bar{q}}\right)^2} = \int_0^1 dx \int \frac{d^d\bar{l}}{(2\pi)^d}\frac{1}{\left[x\bar{l}^2+(1-x)\left(\bar{l}-p_{\bar{q}}\right)^2\right]^2}$$

Feynman parameterization !

$$= \int_0^1 dx \int \frac{d^d\bar{l}}{(2\pi)^d}\frac{1}{\left(\bar{l}-(1-x)p_{\bar{q}}\right)^4}$$

Using on-shell condition !

$$= \int_0^1 dx \int \frac{d^d\bar{l}}{(2\pi)^d}\frac{1}{\left(\bar{l}^2\right)^2}$$

Translational invariance !

$$= \int \frac{d^d\bar{l}}{(2\pi)^d}\frac{1}{\left(\bar{l}^2\right)^2}$$

Integration over x !

$$= \int \frac{d\bar{l}_0 d^{d-1}\vec{\bar{l}}}{(2\pi)^d}\frac{1}{\left(\bar{l}_0^2-|\vec{\bar{l}}|^2\right)^2}$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d}\frac{1}{\bar{l}^2\left(\bar{l}-p_{\bar{q}}\right)^2} \overset{\bar{l}_0\to i\bar{l}_0}{=} \frac{i}{(2\pi)^d}\int d\Omega_d \int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Wick rotation & spherical coordinate !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d}\int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Integration over solid angle !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d}\left(\int_0^{1} d|\bar{l}||\bar{l}|^{d-5}+\int_1^{+\infty} d|\bar{l}||\bar{l}|^{d-5}\right)$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

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Wick rotation & spherical coordinate !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d}\int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Integration over solid angle !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d}\left(\underline{\int_0^1 d|\bar{l}||\bar{l}|^{d-5}} + \underline{\int_1^{+\infty} d|\bar{l}||\bar{l}|^{d-5}}\right)$$

$|\bar{l}|\to 0$ (IR): the integral is divergent when $d\leq 4$

$|\bar{l}|\to +\infty$(UV): the integral is divergent when $d\geq 4$

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A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d}\frac{1}{\bar{l}^2\left(\bar{l}-p_{\bar{q}}\right)^2} \overset{\bar{l}_0\to i\bar{l}_0}{=} \frac{i}{(2\pi)^d}\int d\Omega_d \int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Wick rotation & spherical coordinate !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d}\int_0^{+\infty} d|\bar{l}||\bar{l}|^{d-5}$$

Integration over solid angle !

$$= \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d}\left(\underline{\int_0^1 d|\bar{l}||\bar{l}|^{d-5}} + \underline{\int_1^{+\infty} d|\bar{l}||\bar{l}|^{d-5}}\right)$$

$|\bar{l}|\to 0$ (IR): the integral is divergent when $d\leq 4$

$|\bar{l}|\to +\infty$(UV): the integral is divergent when $d\geq 4$

Regularisations:

$$d = 4-2\epsilon_{\mathrm{IR}}, \epsilon_{\mathrm{IR}}\to 0-$$

$$d = 4-2\epsilon_{\mathrm{UV}}, \epsilon_{\mathrm{UV}}\to 0+$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d}\frac{1}{\bar{l}^2\left(\bar{l}-p_{\bar{q}}\right)^2}=\frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d}\left(-\frac{1}{2\epsilon_{\mathrm{IR}}}+\frac{1}{2\epsilon_{\mathrm{UV}}}\right)$$

  - **Squaring with Born amplitude, summing over colours and spins, and averaging the spin of the initial state**

$$\overline{\sum}2\Re\{\mathcal{A}_{1\mathrm{loop}}\mathcal{A}^*_{\mathrm{Born}}\}=\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\overline{\sum}|\mathcal{A}_{\mathrm{Born}}|^2\right)\frac{\alpha_s}{\pi}\left[\frac{2}{3\epsilon_{\mathrm{UV}}}-\frac{4}{3\epsilon^2_{\mathrm{IR}}}-\frac{4}{3\epsilon_{\mathrm{IR}}}\left(1-\log\frac{m_Z^2}{4\pi^2\mu_R^2}\right)-\frac{2}{3}\left(5-\pi^2-\log\frac{m_Z^2}{4\pi^2\mu_R^2}+\log^2\frac{m_Z^2}{4\pi^2\mu_R^2}\right)\right]$$

$$\boxed{\alpha_s=\frac{g_s^2}{4\pi}}$$

  - **The UV divergence needs renormalisation**

$$\overline{\sum}2\Re\{\mathcal{A}_{\mathrm{UV}}\mathcal{A}^*_{\mathrm{Born}}\}=\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\overline{\sum}|\mathcal{A}_{\mathrm{Born}}|^2\right)\frac{\alpha_s}{\pi}\left[-\frac{2}{3\epsilon_{\mathrm{UV}}}+\frac{2}{3\epsilon_{\mathrm{IR}}}\right]$$

A NLO EXAMPLE: VIRTUAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Need to evaluate two tensor integrals**

Evaluating the scalar integrals, e.g.

$$\int \frac{d^d\bar{l}}{(2\pi)^d} \frac{1}{\bar{l}^2 \left(\bar{l}-p_{\bar{q}}\right)^2} = \frac{i2\pi^{d/2}}{\Gamma(d/2)(2\pi)^d}\left(-\frac{1}{2\epsilon_{\rm IR}}+\frac{1}{2\epsilon_{\rm UV}}\right)$$

  - **Squaring with Born amplitude, summing over colours and spins, and averaging the spin of the initial state**

$$\overline{\sum} 2\Re\{\mathcal{A}_{\rm 1loop}\mathcal{A}^*_{\rm Born}\} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\frac{\alpha_s}{\pi}\left[\cancel{\frac{2}{3\epsilon_{\rm UV}}} - \frac{4}{3\epsilon^2_{\rm IR}} - \frac{4}{3\epsilon_{\rm IR}}\left(1-\log\frac{m_Z^2}{4\pi^2\mu_R^2}\right)\right.$$
$$\left.-\frac{2}{3}\left(5-\pi^2-\log\frac{m_Z^2}{4\pi^2\mu_R^2}+\log^2\frac{m_Z^2}{4\pi^2\mu_R^2}\right)\right]$$

$$\boxed{\alpha_s = \frac{g_s^2}{4\pi}}$$

  - **The UV divergence needs renormalisation**

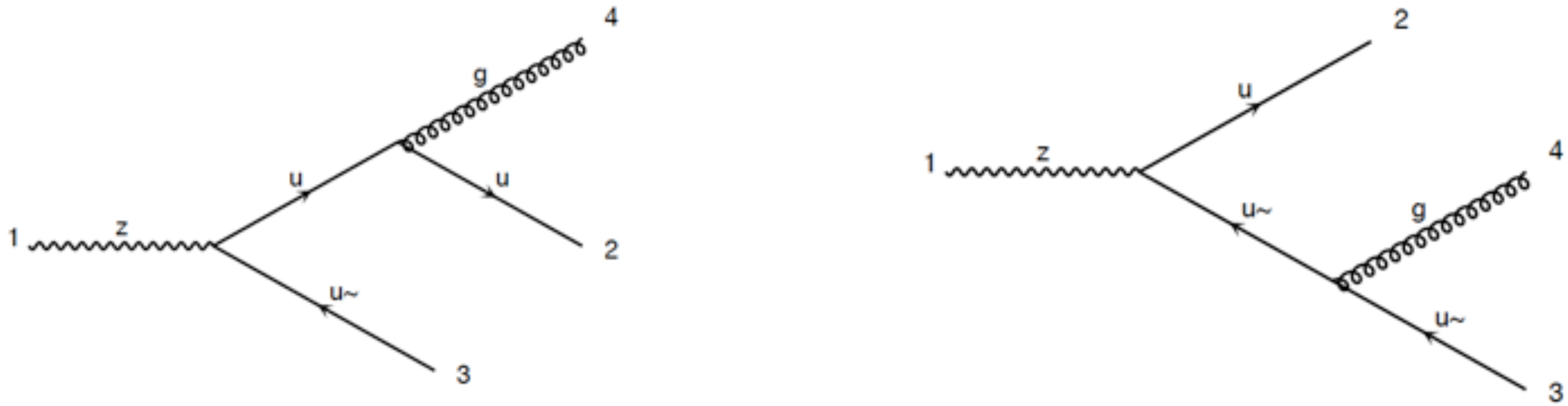
$$\overline{\sum} 2\Re\{\mathcal{A}_{\rm UV}\mathcal{A}^*_{\rm Born}\} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\frac{\alpha_s}{\pi}\left[-\cancel{\frac{2}{3\epsilon_{\rm UV}}}+\frac{2}{3\epsilon_{\rm IR}}\right]$$

  - **The virtual matrix element is:**

$$\mathcal{V} = \overline{\sum} 2\Re\{\mathcal{A}_{\rm 1loop}\mathcal{A}^*_{\rm Born}\} + \overline{\sum} 2\Re\{\mathcal{A}_{\rm UV}\mathcal{A}^*_{\rm Born}\}$$

A NLO EXAMPLE: REAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Writing down real amplitude according to Feynman rules**



- **Squaring amplitude, summing over colours and spins, and averaging the spin of the initial state**

$$\overline{\sum}|\mathcal{A}_{\rm real}|^2 = \left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\alpha_s\frac{8\pi(d-2)}{3m_Z^2 s_{24}s_{34}}$$

$$\times\left[(d-2)s_{24}^2+2(d-4)s_{24}s_{34}+(d-2)s_{34}^2-4m_Z^2(s_{24}+s_{34})+4m_Z^4\right]$$

$$s_{24}=(p_q+p_g)^2, s_{34}=(p_{\bar{q}}+p_g)^2$$

A NLO EXAMPLE: REAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **3-body phase-space integration**

$$\Gamma_{\rm real} = \frac{1}{2m_Z}\int (2\pi)^d\,\delta^d\left(p_Z - p_q - p_{\bar{q}} - p_g\right)\frac{1}{(2\pi)^{3(d-1)}}\frac{d^{d-1}\vec{p}_q}{2E_q}\frac{d^{d-1}\vec{p}_{\bar{q}}}{2E_{\bar{q}}}\frac{d^{d-1}\vec{p}_g}{2E_g}\overline{\sum}|\mathcal{A}_{\rm real}|^2$$

$$y = \frac{s_{34}}{m_Z^2}, 1 - y - z = \frac{s_{24}}{m_Z^2}$$

$$d\Phi^{(2)}(p_Z \to p_q, p_{\bar{q}}) = (2\pi)^d\delta^d(p_Z - p_q - p_{\bar{q}})\frac{1}{(2\pi)^{2(d-1)}}\frac{d^{d-1}\vec{p}_q}{2E_q}\frac{d^{d-1}\vec{p}_{\bar{q}}}{2E_{\bar{q}}}$$
$$= \frac{(4\pi)^{2\epsilon}}{8(2\pi)^2}\frac{1}{m_Z^{2\epsilon}}d\Omega_d$$

$$d\Phi^{(3)}(p_Z \to p_q, p_{\bar{q}}, p_g) = (2\pi)^d\delta^d(p_Z - p_q - p_{\bar{q}} - p_g)\frac{1}{(2\pi)^{3(d-1)}}\frac{d^{d-1}\vec{p}_q}{2E_q}\frac{d^{d-1}\vec{p}_{\bar{q}}}{2E_{\bar{q}}}\frac{d^{d-1}\vec{p}_g}{2E_g}$$
$$= \frac{(4\pi)^{3\epsilon}}{32(2\pi)^4\Gamma(1-\epsilon)}(m_Z^2)^{1-2\epsilon}d\Omega_d$$
$$\times\int_0^1 dz z^{-\epsilon}\int_0^{1-z} dy y^{-\epsilon}(1-z-y)^{-\epsilon}$$
$$= d\Phi^{(2)}(p_Z \to p_q, p_{\bar{q}}) \times \frac{(4\pi)^{\epsilon}}{16\pi^2\Gamma(1-\epsilon)}(m_Z^2)^{1-\epsilon}$$
$$\times\int_0^1 dz z^{-\epsilon}\int_0^{1-z} dy y^{-\epsilon}(1-z-y)^{-\epsilon}$$

A NLO EXAMPLE: REAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **3-body phase-space integration**

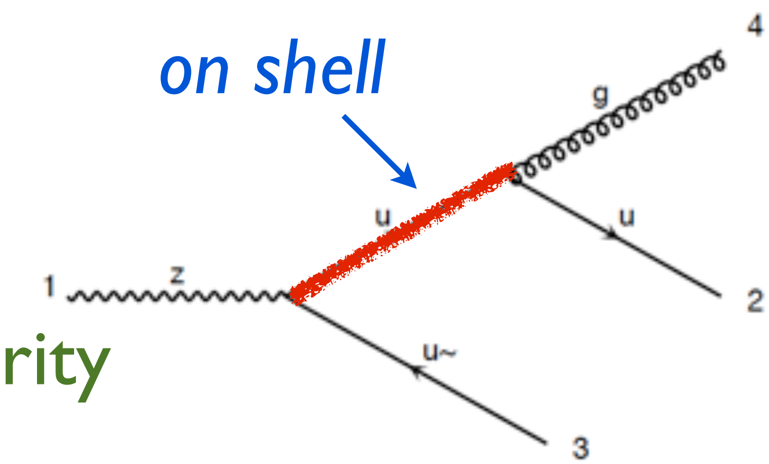
$$\overline{\sum}|\mathcal{A}_{\rm real}|^2 = \left(\overline{\sum}|\mathcal{A}_{\rm Born}|^2\right)\alpha_s \frac{8\pi(d-2)}{3m_Z^2 y(1-z-y)}\left[(d-2)(1-z)^2+4y^2-4y(1-z)+4z\right]$$

The integration over y is divergent when $d\leq 4\ (\epsilon\geq 0)$

$$y\to 1-z\ (s_{24}\to 0)$$

(1) $p_g\to 0\ (z\to 0)$ soft singularity

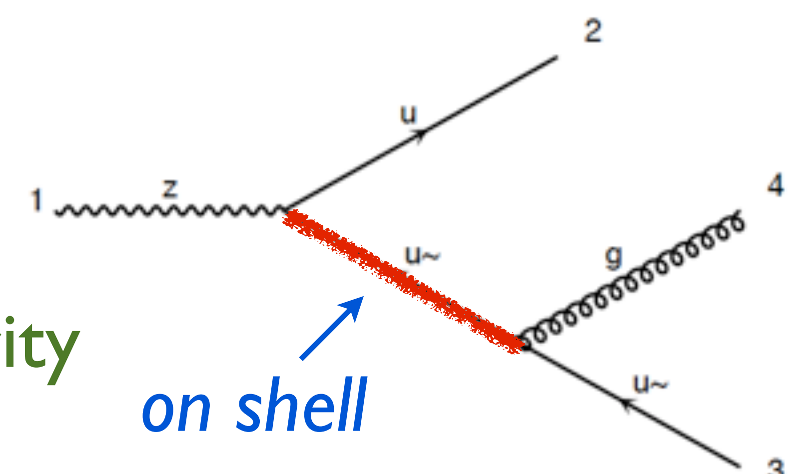
(2) $p_g||p_q$ collinear singularity



$$y\to 0\ (s_{34}\to 0)$$

(1) $p_g\to 0\ (z\to 0)$ soft singularity

(2) $p_g||p_{\bar{q}}$ collinear singularity



A NLO EXAMPLE: REAL

- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **3-body phase-space integration**

$$\begin{aligned}\Gamma_{\mathrm{real}} &= \frac{1}{2m_Z}\int d\Phi^{(3)}(p_Z \to p_q, p_{\bar{q}}, p_g)\overline{\sum}|\mathcal{A}_{\mathrm{real}}|^2 \\ &= \frac{1}{2m_Z}\int d\Phi^{(2)}(p_Z \to p_q, p_{\bar{q}})\left(\overline{\sum}|\mathcal{A}_{\mathrm{Born}}|^2\right) \\ &\times \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\frac{\alpha_s}{\pi}\left[\frac{4}{3\epsilon_{\mathrm{IR}}^2} + \frac{2}{3\epsilon_{\mathrm{IR}}}\left(1 - 2\log\frac{m_Z^2}{4\pi^2\mu_R^2}\right)\right. \\ &\left. + \frac{1}{3}\left(2\log^2\frac{m_Z^2}{4\pi^2\mu_R^2} - 2\log\frac{m_Z^2}{4\pi^2\mu_R^2} - 2\pi^2 + 13\right)\right]\end{aligned}$$

  - **Sum real and virtual**

$$\Gamma_{\mathrm{virtual}} = \frac{1}{2m_Z}\int d\Phi^{(2)}(p_Z \to p_q, p_{\bar{q}})\mathcal{V}$$

$$\Gamma_{\mathrm{virtual}} + \Gamma_{\mathrm{real}} = \frac{1}{2m_Z}\int d\Phi^{(2)}(p_Z \to p_q, p_{\bar{q}})\left(\overline{\sum}|\mathcal{A}_{\mathrm{Born}}|^2\right)\frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\frac{\alpha_s}{\pi}$$

A NLO EXAMPLE: NLO

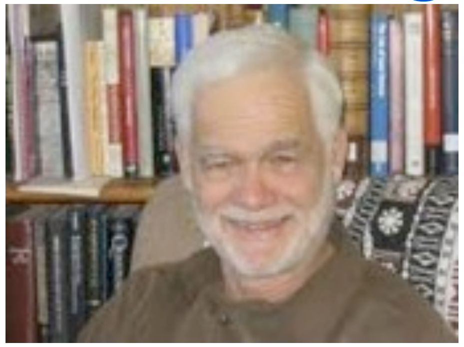
- **Let us calculate NLO QCD of Z -> q qbar decay**
  - **Sum real and virtual**

All remaining IR poles cancel (in general KLN theorem)

Kinoshita

Lee

Nauenberg



$$\Gamma_{\text{virtual}} + \Gamma_{\text{real}} = \frac{1}{2m_Z} \int d\Phi^{(2)}(p_Z \to p_q, p_{\bar{q}}) \left(\overline{\sum} |\mathcal{A}_{\text{Born}}|^2\right) \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)} \frac{\alpha_s}{\pi}$$

$$\stackrel{\epsilon \to 0}{=} \Gamma_{\text{Born}}(Z \to q\bar{q}) \frac{\alpha_s}{\pi}$$

$$\boxed{\Gamma_{\text{NLO}}(Z \to q\bar{q} + X) = \Gamma_{\text{Born}}(Z \to q\bar{q}) \left(1 + \frac{\alpha_s}{\pi}\right)}$$

We finally get a well-known result !

Generics

Modern Techniques

Friday, June 11, 21

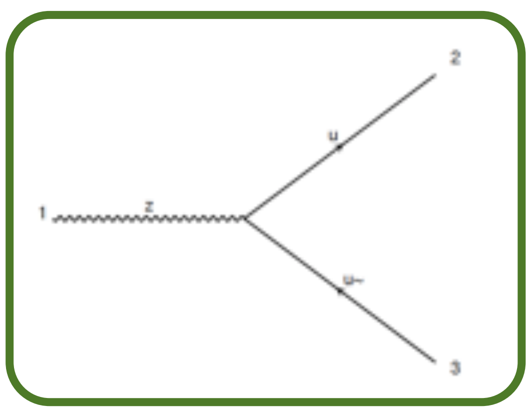
NLO ANATOMY

- **Three parts need to be computed in a NLO calculation**

$$\sigma_{\text{NLO}} = \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\mathcal{V} + \int d\Phi^{(n+1)}\mathcal{R}$$

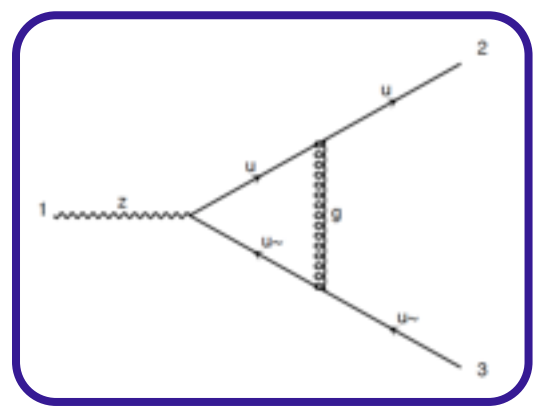
$$\mathcal{O}(\alpha_s^b) \qquad \mathcal{O}(\alpha_s^{b+1}) \qquad \mathcal{O}(\alpha_s^{b+1})$$

Born cross section



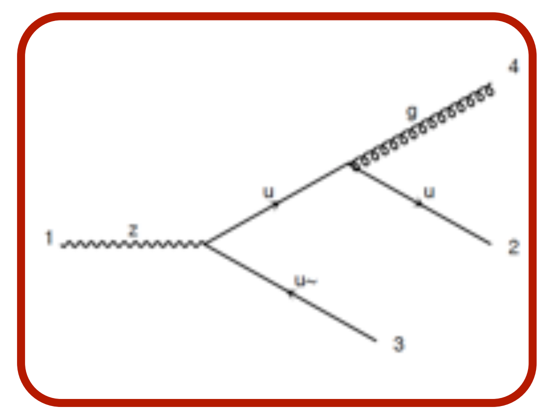
Finite

Virtual correction



Divergent

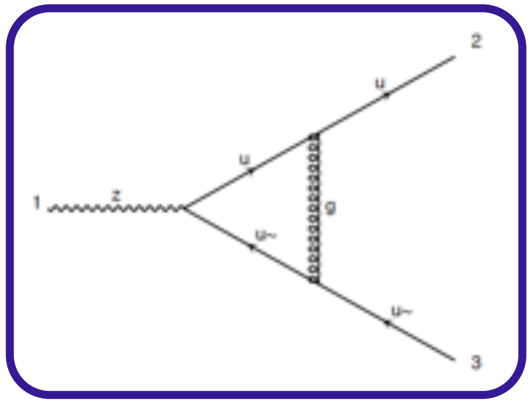
Real correction



Divergent

Generics

Virtual=Loop+UV



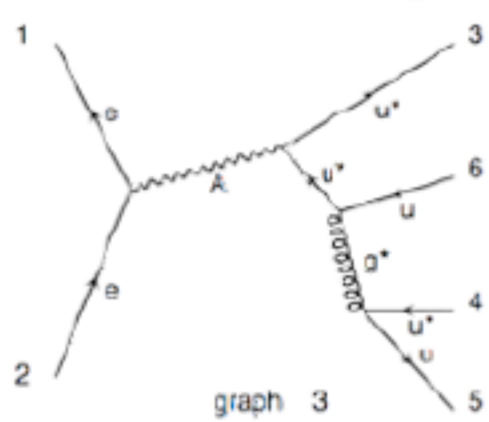
ONE-LOOP DIAGRAM GENERATION

- No external tool for loop diagram generation:
  Reuse MG5_aMC efficient tree level diagram generation!

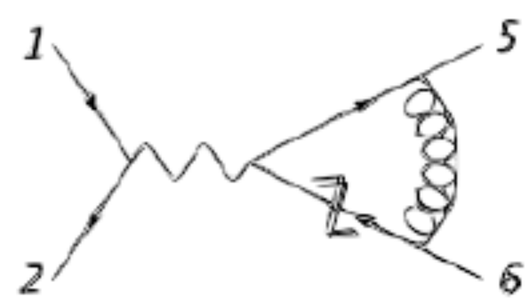
- Cut loops have two extra external particles

Trees ($e^+e^- \rightarrow$ u u~ u u~) ≡ Loops ($e^+e^- \rightarrow$ u u~)

≡

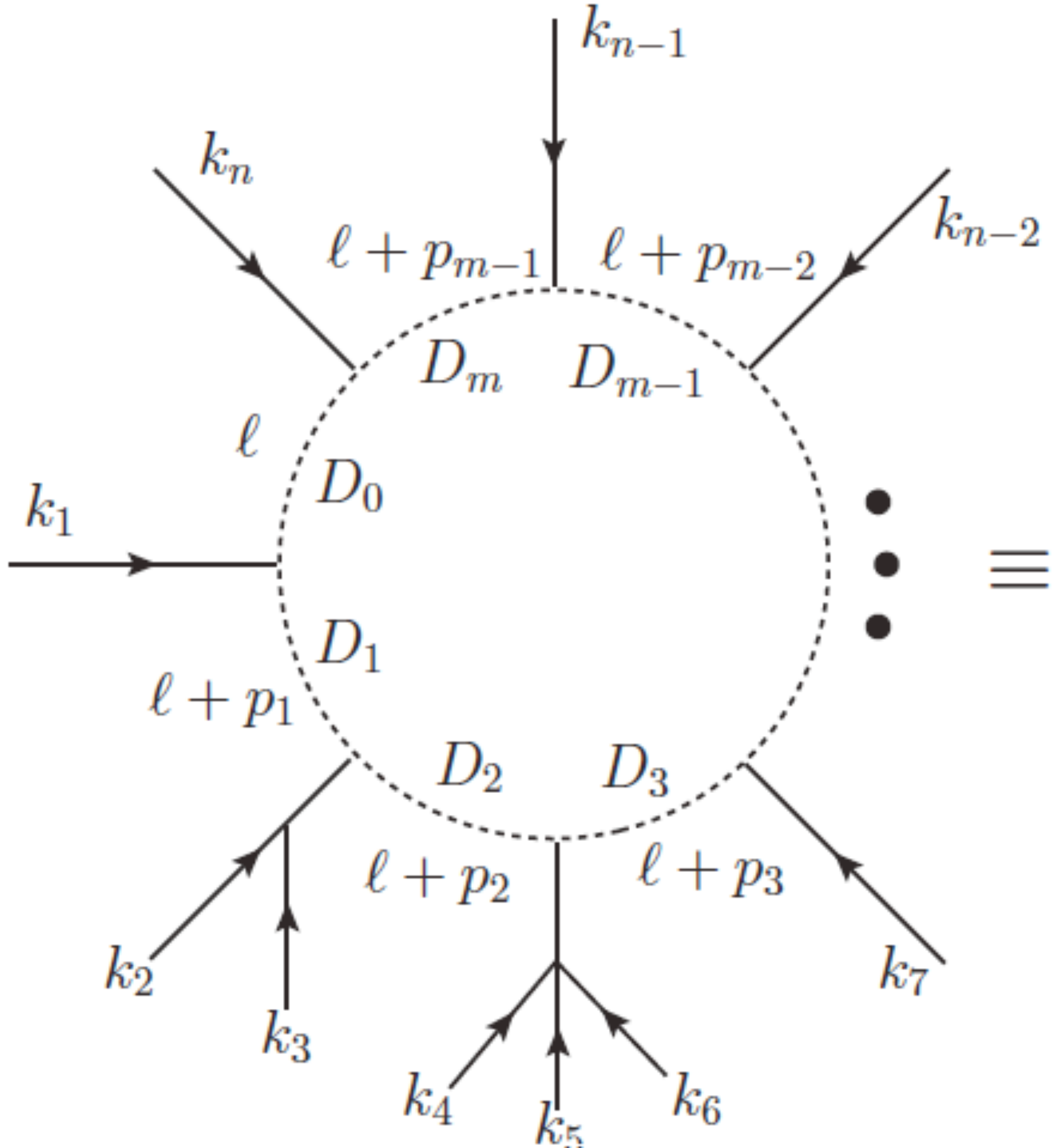


≡



Friday, June 11, 21

ONE-LOOP INTEGRAL EVALUATION



- Consider this m-point loop diagram with n external momenta

$$\equiv \int \frac{d^d\ell}{(2\pi)^d} \frac{\mathcal{N}(\ell)}{D_0 D_1 D_2 D_3 \cdots D_{m-2} D_{m-1}}$$

with $D_i = (\ell + p_i)^2 - m_i^2$

We will denote by $\mathcal{C}$ this integral.

ONE-LOOP INTEGRAL EVALUATION

$$\mathcal{C}^{\text{1-loop}} = \sum_{i_0<i_1<i_2<i_3} d_{i_0i_1i_2i_3}\mathrm{Box}_{i_0i_1i_2i_3} + \sum_{i_0<i_1<i_2} c_{i_0i_1i_2}\mathrm{Triangle}_{i_0i_1i_2} + \sum_{i_0<i_1} b_{i_0i_1}\mathrm{Bubble}_{i_0i_1} + \sum_{i_0} a_{i_0}\mathrm{Tadpole}_{i_0} + R + \mathcal{O}(\epsilon)$$

$$\mathrm{Box}_{i_0i_1i_2i_3} = \int d^dl \frac{1}{D_{i_0}D_{i_1}D_{i_2}D_{i_3}}$$

$$\mathrm{Triangle}_{i_0i_1i_2} = \int d^dl \frac{1}{D_{i_0}D_{i_1}D_{i_2}}$$

$$\mathrm{Bubble}_{i_0i_1} = \int d^dl \frac{1}{D_{i_0}D_{i_1}}$$

$$\mathrm{Tadpole}_{i_0} = \int d^dl \frac{1}{D_{i_0}}$$

The a, b, c, d and R coefficients depend only on external parameters and momenta.

Reduction of the loop to these scalar coefficients can be achieved using either Tensor Integral Reduction or Reduction at the integrand level

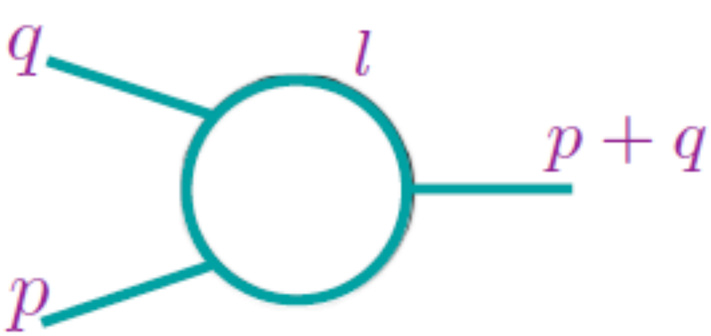
TENSOR INTEGRAL REDUCTION

- Passarino-Veltman reduction:

$$\int d^d l \, \frac{N(l)}{D_0 D_1 D_2 \cdots D_{m-1}} \to \sum_i \mathrm{coeff}_i \int d^d l \, \frac{1}{D_0 D_1 \cdots}$$

  - Reduce a general integral to "scalar integrals" by "completing the square"

- Example:
  Application of PV to this triangle rank-1 integral



$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2 - m_1^2)((l+p)^2 - m_2^2)((l+q)^2 - m_3^2)}$$

- Implemented in codes such as:

COLLIER [A. Denner, S .Dittmaier, L. Hofer, 1604.06792]
GOLEM95 [T. Binoth, J.Guillet, G. Heinrich, E.Pilon, T.Reither, 0810.0992]

TENSOR INTEGRAL REDUCTION

$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2-m_1^2)((l+p)^2-m_2^2)((l+q)^2-m_3^2)}$$

- The only independent four vectors are p^μ and q^μ . Therefore, the integral must be proportional to those. We can set-up a system of linear equations and try to solve for C_1 and C_2

$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2-m_1^2)((l+p)^2-m_2^2)((l+q)^2-m_3^2)} = \begin{pmatrix} p^\mu & q^\mu \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

We can solve for C_1 and C_2 by contracting with p and q

$$\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} [2l\cdot p] \\ [2l\cdot q] \end{pmatrix} = G \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \equiv \begin{pmatrix} 2p\cdot p & 2p\cdot q \\ 2p\cdot q & 2q\cdot q \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

where $[2l\cdot p] = \int \frac{d^n l}{(2\pi)^n} \frac{2l\cdot p}{l^2(l+p)^2(l+q)^2}$ (For simplicity, the masses are neglected here)

- By expressing $2l.p$ and $2l.q$ as a sum of denominators we can express R_1 and R_2 as a sum of simpler integrals, *e.g.*

$$R_1 = \int \frac{d^n l}{(2\pi)^n} \frac{2l\cdot p}{l^2(l+p)^2(l+q)^2} = \int \frac{d^n l}{(2\pi)^n} \frac{(l+p)^2-l^2-p^2}{l^2(l+p)^2(l+q)^2}$$
$$= \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l+q)^2} - \int \frac{d^n l}{(2\pi)^n} \frac{1}{(l+p)^2(l+q)^2} - p^2 \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l+p)^2(l+q)^2}$$

TENSOR INTEGRAL REDUCTION

- And similarly for R_2

$$R_2 = \int \frac{d^n l}{(2\pi)^n} \frac{2l\cdot q}{l^2(l+p)^2(l+q)^2} = \int \frac{d^n l}{(2\pi)^n} \frac{(l+q)^2 - l^2 - q^2}{l^2(l+p)^2(l+q)^2}$$
$$= \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l+p)^2} - \int \frac{d^n l}{(2\pi)^n} \frac{1}{(l+p)^2(l+q)^2} - q^2 \int \frac{d^n l}{(2\pi)^n} \frac{1}{l^2(l+p)^2(l+q)^2}$$

- Now we can solve the equation

$$\begin{pmatrix} R_1 \\ R_2 \end{pmatrix} = \begin{pmatrix} [2l\cdot p] \\ [2l\cdot q] \end{pmatrix} = G \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} \equiv \begin{pmatrix} 2p\cdot p & 2p\cdot q \\ 2p\cdot q & 2q\cdot q \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

by inverting the "Gram" matrix G

$$\begin{pmatrix} C_1 \\ C_2 \end{pmatrix} = G^{-1} \begin{pmatrix} R_1 \\ R_2 \end{pmatrix}$$

- We have re-expressed, reduced, our original integral

$$\int \frac{d^n l}{(2\pi)^n} \frac{l^\mu}{(l^2 - m_1^2)((l+p)^2 - m_2^2)((l+q)^2 - m_3^2)} = \begin{pmatrix} p^\mu & q^\mu \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \end{pmatrix}$$

in terms of known, simpler *scalar* integrals

INTEGRAND REDUCTION

Ossola, Papadopulos, Pittau (NPB'06)

TIR

- The decomposition to the basis scalar integrals works at the level of the integrals

$$\mathcal{C}^{\text{1-loop}} = \sum_{i_0<i_1<i_2<i_3} d_{i_0i_1i_2i_3}\,\text{Box}_{i_0i_1i_2i_3} + \sum_{i_0<i_1<i_2} c_{i_0i_1i_2}\,\text{Triangle}_{i_0i_1i_2} + \sum_{i_0<i_1} b_{i_0i_1}\,\text{Bubble}_{i_0i_1} + \sum_{i_0} a_{i_0}\,\text{Tadpole}_{i_0} + R + \mathcal{O}(\epsilon)$$

OPP

- Knowing a relation directly at the integrand level, we would be able to manipulate the reduction without doing the the integrals

$$\begin{aligned} N(l) = &\sum_{i_0,i_1,i_2,i_3} (d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}) \prod_{i\neq i_0,i_1,i_2,i_3} D_i \\ &+ \sum_{i_0,i_1,i_2} (c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}) \prod_{i\neq i_0,i_1,i_2} D_i \\ &+ \sum_{i_0,i_1} (b_{i_0i_1} + \tilde{b}_{i_0i_1}) \prod_{i\neq i_0,i_1} D_i \\ &+ \sum_{i_0} (a_{i_0} + \tilde{a}_{i_0}) \prod_{i\neq i_0} D_i \\ &+ \tilde{P}(l) \prod_i D_i + \mathcal{O}(\varepsilon) \end{aligned}$$

INTEGRAND REDUCTION

Ossola, Papadopulos, Pittau (NPB'06)

TIR

- The decomposition to the basis scalar integrals works at the level of the integrals

$$\mathcal{C}^{\text{1-loop}} = \sum_{i_0<i_1<i_2<i_3} d_{i_0 i_1 i_2 i_3} \mathrm{Box}_{i_0 i_1 i_2 i_3} + \sum_{i_0<i_1<i_2} c_{i_0 i_1 i_2} \mathrm{Triangle}_{i_0 i_1 i_2} + \sum_{i_0<i_1} b_{i_0 i_1} \mathrm{Bubble}_{i_0 i_1} + \sum_{i_0} a_{i_0} \mathrm{Tadpole}_{i_0} + R + \mathcal{O}(\epsilon)$$

OPP

- Knowing a relation directly at the integrand level, we would be able to manipulate the reduction without doing the the integrals

$$\begin{aligned} N(l) = & \sum_{i_0,i_1,i_2,i_3} (d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}) \prod_{i\neq i_0,i_1,i_2,i_3} D_i \\ & + \sum_{i_0,i_1,i_2} (c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}) \prod_{i\neq i_0,i_1,i_2} D_i \\ & + \sum_{i_0,i_1} (b_{i_0i_1} + \tilde{b}_{i_0i_1}) \prod_{i\neq i_0,i_1} D_i \\ & + \sum_{i_0} (a_{i_0} + \tilde{a}_{i_0}) \prod_{i\neq i_0} D_i \\ & + \tilde{P}(l) \prod_i D_i + \mathcal{O}(\varepsilon) \end{aligned}$$

Spurious term

INTEGRAND REDUCTION

- The functional form of the spurious terms is known (it depends on the rank of the integral and the number of propagators in the loop) [del Aguila, Pittau 2004]
  - for example, a box coefficient from a rank 1 numerator is

$$\tilde{d}_{i_0 i_1 i_2 i_3}(l) = \tilde{d}_{i_0 i_1 i_2 i_3}\, \epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma$$

  (remember that p_i is the sum of the momentum that has entered the loop so far, so we always have $p_0 = 0$)

  - The integral is zero

$$\int d^d l \frac{\tilde{d}_{i_0 i_1 i_2 i_3}(l)}{D_0 D_1 D_2 D_3} = \tilde{d}_{i_0 i_1 i_2 i_3} \int d^d l \frac{\epsilon^{\mu\nu\rho\sigma}\, l^\mu p_1^\nu p_2^\rho p_3^\sigma}{D_0 D_1 D_2 D_3} = 0$$

INTEGRAND REDUCTION

- **Take Box (4-point) coefficients as an example**

$$N(l^{\pm}) = d_{0123} + \tilde{d}_{0123}(l^{\pm}) \prod_{i\neq 0,1,2,3}^{m-1} D_i(l^{\pm})$$

- Two values are enough given the functional form for the spurious term. We can immediately determine the Box coefficient

$$d_{0123} = \frac{1}{2}\left[\frac{N(l^+)}{\prod_{i\neq 0,1,2,3}^{m-1} D_i(l^+)} + \frac{N(l^-)}{\prod_{i\neq 0,1,2,3}^{m-1} D_i(l^-)}\right]$$

- By choosing other values for l, that set other combinations of 4 "denominators" to zero, we can get all the Box coefficients

- **In general:**

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$

$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$

$$+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0i_1} + \tilde{b}_{i_0i_1}(l)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i$$

$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i\neq i_0}^{m-1} D_i$$

$$+\tilde{P}(l) \prod_{i}^{m-1} D_i$$

$$= 0$$

To solve the OPP reduction, choosing special values for the loop momentum helps a lot

For example, choosing *l* such that

$$D_0(l^\pm) = D_1(l^\pm) = \\ = D_2(l^\pm) = D_3(l^\pm) = 0$$

sets all the terms in this equation to zero except the first line

There are two (complex) solutions to this equation due to the quadratic nature of the propagators

INTEGRAND REDUCTION

- **In general:**

$$N(l) = \boxed{\sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i}$$

$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0 i_1 i_2} + \tilde{c}_{i_0 i_1 i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$

$$+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0 i_1} + \tilde{b}_{i_0 i_1}(l)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i$$

$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i\neq i_0}^{m-1} D_i$$

$$+ \tilde{P}(l) \prod_{i}^{m-1} D_i$$

$$= 0$$

Now we choose l such that

$$D_0(l^i) = D_1(l^i) = D_2(l^i) = 0$$

sets all the terms in this equation to zero except the first and second line

Coefficient computed in a previous step

INTEGRAND REDUCTION

- **In general:**

$$
\begin{aligned}
N(l) = & \boxed{\sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i} \\
& \boxed{+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i} \\
& + \sum_{i_0<i_1}^{m-1} \left[b_{i_0i_1} + \tilde{b}_{i_0i_1}(l)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i \\
& \boxed{+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i\neq i_0}^{m-1} D_i \\
+ \tilde{P}(l) \prod_{i}^{m-1} D_i} \\
& = 0
\end{aligned}
$$

Now, choosing *l* such that

$$D_0(l^i) = D_1(l^i) = 0$$

sets all the terms in this equation to zero except the first, second and third line

Coefficient computed in a previous step

Friday, June 11, 21

INTEGRAND REDUCTION

- **In general:**

$$N(l) = \sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}(l)\right] \prod_{i\neq i_0,i_1,i_2,i_3}^{m-1} D_i$$
$$+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}(l)\right] \prod_{i\neq i_0,i_1,i_2}^{m-1} D_i$$
$$+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0i_1} + \tilde{b}_{i_0i_1}(l)\right] \prod_{i\neq i_0,i_1}^{m-1} D_i$$
$$+ \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i\neq i_0}^{m-1} D_i$$
$$+ \tilde{P}(l) \prod_{i}^{m-1} D_i$$

$$= 0$$

Now, choosing l such that

$$D_1(l^i) = 0$$

sets the last line to zero

Coefficient computed in a previous step

INTEGRAND REDUCTION

- **In general:**

$$
\begin{aligned}
N(l) = & \boxed{\sum_{i_0<i_1<i_2<i_3}^{m-1} \left[d_{i_0 i_1 i_2 i_3} + \tilde{d}_{i_0 i_1 i_2 i_3}(l)\right] \prod_{i \neq i_0, i_1, i_2, i_3}^{m-1} D_i} \\
& \boxed{+ \sum_{i_0<i_1<i_2}^{m-1} \left[c_{i_0 i_1 i_2} + \tilde{c}_{i_0 i_1 i_2}(l)\right] \prod_{i \neq i_0, i_1, i_2}^{m-1} D_i} \\
& \boxed{+ \sum_{i_0<i_1}^{m-1} \left[b_{i_0 i_1} + \tilde{b}_{i_0 i_1}(l)\right] \prod_{i \neq i_0, i_1}^{m-1} D_i} \\
& + \sum_{i_0}^{m-1} \left[a_{i_0} + \tilde{a}_{i_0}(l)\right] \prod_{i \neq i_0}^{m-1} D_i \\
& \boxed{+ \tilde{P}(l) \prod_{i}^{m-1} D_i}
\end{aligned}
$$

$$= 0$$

Now, choosing *l* such that

$$D_1(l^i) = 0$$

sets the last line to zero

Coefficient computed in a previous step

D-DIMENSIONAL COMPLEX

- The previous expression should in fact be written in d dimensions

$$\int \frac{d^d\bar{l}}{(2\pi)^d}\frac{N(\bar{l},\epsilon)}{\bar{D}_0\bar{D}_1\bar{D}_2\cdots\bar{D}_{m-1}}$$

$$\bar{D}_i=\left(\bar{l}+p_i\right)^2-m_i^2,\quad p_0=0$$

D-DIMENSIONAL COMPLEX

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$$\bar{D}_i = \left(\bar{l}+p_i\right)^2 - m_i^2, \quad p_0 = 0$$

- In numerical calculations, it is very convenient to perform the following decomposition

$$\underbrace{\bar{l}^\mu}_{d-\text{dim}} = \underbrace{l^\mu}_{4-\text{dim}} + \underbrace{\tilde{l}^\mu}_{(-2\epsilon)-\text{dim}} \qquad \mu = \underbrace{0,1,2,3,}_{\substack{4d\text{ spacetime}\\ \text{physical}}}\underbrace{\cdots,3-2\epsilon}_{\substack{(-2\epsilon)d\text{ space}\\ \text{abstract}}}$$

$$l^\mu = 0, \mu \in (-2\epsilon)d \text{ space} \qquad \tilde{l}^\mu = 0, \mu \in 4d \text{ spacetime}$$

D-DIMENSIONAL COMPLEX

- The previous expression should in fact be written in d dimensions

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$$l^\mu = 0, \mu \in (-2\epsilon)d \text{ space} \qquad \tilde{l}^\mu = 0, \mu \in 4d \text{ spacetime}$$

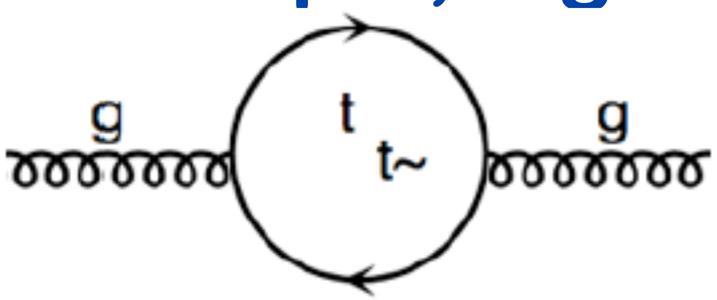
$$N(\bar{l},\epsilon) = \underbrace{N(l)}_{\text{Suitable for numerical calc.}} + \underbrace{\tilde{N}(l,\tilde{l},\epsilon)}_{\text{Complement with special CT } R_2}$$

D-DIMENSIONAL COMPLEX

- Compute the remaining loop part in terms of rational functions of external momentum invariants and masses

$$\boxed{R_2 = \lim_{\epsilon \to 0} \int \frac{d^d \bar{l}}{(2\pi)^d} \frac{\tilde{N}(l, \tilde{l}, \epsilon)}{\bar{D}_0 \bar{D}_1 \cdots \bar{D}_{m-1}}}$$

- For example, a gluon self-energy diagram:



$$N(\bar{l}, \epsilon) = -2\pi\alpha_s\delta_{ab}\mathrm{Tr}\left[\gamma^\mu \left(\bar{\not{l}} + m_t\right) \gamma^\nu \left(\bar{\not{l}} + \not{p}_g + m_t\right)\right] \varepsilon_\mu \varepsilon_\nu$$

  - After performing some Dirac algebra, we have

$$\tilde{N}(l, \tilde{l}, \epsilon) = 8\pi\alpha_s\delta_{ab}g^{\mu\nu}\tilde{l}^2\varepsilon_\mu\varepsilon_\nu$$

  - Using the integration

$$\int \frac{d^d\bar{l}}{(2\pi)^d} \frac{\tilde{l}^2}{\left(\bar{l}^2 - m_t^2\right)\left((\bar{l} + p_g)^2 - m_t^2\right)} = -\frac{i}{32\pi^2}\left(2m_t^2 - \frac{p_g^2}{3}\right) + \mathcal{O}(\epsilon)$$

  - We have R_2 term

$$R_2 = -\frac{i\alpha_s}{4\pi}\delta_{ab}\left(2m_t^2 - \frac{p_g^2}{3}\right) g^{\mu\nu}\varepsilon_\mu\varepsilon_\nu$$

D-DIMENSIONAL COMPLEX

- It has been proven that R_2 is only UV related. Therefore, like renormalisation counterterms, they can be reexpressed into R_2 Feynman rules

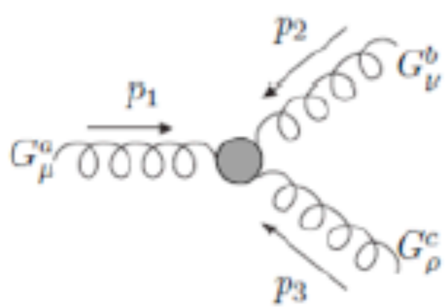
QCD R_2 Feynman Rules


$= \mathrm{Vert}(G^a_\mu, G^b_\nu)$


$= \mathrm{Vert}(Q^i_l, \bar{Q}^j_m)$

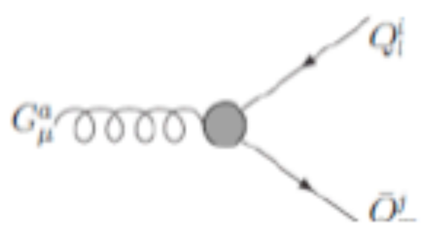
$$\mathrm{Vert}(G^a_\mu, G^b_\nu) = \frac{i g_s^2 N_c}{48\pi^2}\,\delta^{ab}\left[\frac{p^2}{2} g_{\mu\nu} + \lambda_{HV}\left(g_{\mu\nu}p^2 - p_\mu p_\nu\right) + \sum_Q \frac{p^2 - 6m_Q^2}{N_c} g_{\mu\nu}\right]$$

$$\mathrm{Vert}(Q^i_l, \bar{Q}^j_m) = \frac{i g_s^2}{16\pi^2}\,\frac{N_c^2-1}{2N_c}\,\delta^{ij}\,\delta_{lm}\,\left(-\not{p} + 2m_{Q_l}\right)\lambda_{HV}.$$

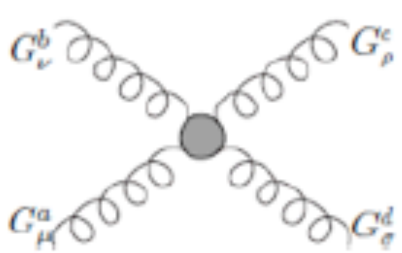

$= \mathrm{Vert}(G^a_\mu, G^b_\nu, G^c_\rho)$

$$\mathrm{Vert}(G^a_\mu, G^b_\nu, G^c_\rho) = -\frac{g_s^3 N_c}{48\pi^2}\left(\frac{7}{4} + \lambda_{HV} + \frac{2N_f}{N_c}\right) f^{abc}\, V_{\mu\nu\rho}(p_1,p_2,p_3)$$

$$V_{\mu\nu\rho}(p_1,p_2,p_3) = g_{\mu\nu}(p_2-p_1)_\rho + g_{\nu\rho}(p_3-p_2)_\mu + g_{\rho\mu}(p_1-p_3)_\nu.$$


$= \mathrm{Vert}(G^a_\mu, Q^i_l, \bar{Q}^j_m)$

$$\mathrm{Vert}(G^a_\mu, Q^i_l, \bar{Q}^j_m) = \delta_{lm}\,\frac{i g_s^3}{16\pi^2}\,T^a_{ji}\,\frac{N_c^2-1}{2N_c}\,\gamma_\mu\left(1+\lambda_{HV}\right)$$


$= \mathrm{Vert}\left(G^a_\mu, G^b_\nu, G^c_\rho, G^d_\sigma\right)$

$$\mathrm{Vert}\left(G^a_\mu, G^b_\nu, G^c_\rho, G^d_\sigma\right) = \frac{i g_s^4}{48\pi^2}\left(C_1\, g_{\mu\nu}g_{\rho\sigma} + C_2\, g_{\mu\rho}g_{\nu\sigma} + C_3\, g_{\mu\sigma}g_{\nu\rho}\right),$$

$$\begin{aligned} C_1 = {} & Tr(\{T^a,T^b\}\{T^c,T^d\})\,(5N_c + 2\lambda_{HV}N_c + 6N_f) \\ & - \left(Tr(T^aT^cT^bT^d) + Tr(T^aT^dT^bT^c)\right)(12N_c + 4\lambda_{HV}N_c + 10N_f) \\ & - \left(\delta^{ab}\delta^{cd} + \delta^{ac}\delta^{bd} + \delta^{ad}\delta^{bc}\right), \quad C_2 = C_1(b \leftrightarrow c) \quad C_3 = C_1(b \leftrightarrow d) \end{aligned}$$

Draggiotis, Garzelli, Papadopoulos, Pittau (JHEP'09); HSS, Zhang, Chao (JHEP'11)

D-DIMENSIONAL COMPLEX

- **In integrand reduction, additional rational terms R_1 are needed !**

$$
\begin{aligned}
N(l) = &\sum_{i_0,i_1,i_2,i_3} (d_{i_0i_1i_2i_3} + \tilde{d}_{i_0i_1i_2i_3}) \prod_{i\neq i_0,i_1,i_2,i_3} D_i \\
&+ \sum_{i_0,i_1,i_2} (c_{i_0i_1i_2} + \tilde{c}_{i_0i_1i_2}) \prod_{i\neq i_0,i_1,i_2} D_i \\
&+ \sum_{i_0,i_1} (b_{i_0i_1} + \tilde{b}_{i_0i_1}) \prod_{i\neq i_0,i_1} D_i \\
&+ \sum_{i_0} (a_{i_0} + \tilde{a}_{i_0}) \prod_{i\neq i_0} D_i \\
&+ \tilde{P}(l) \prod_i D_i + \mathcal{O}(\varepsilon)
\end{aligned}
$$

$$\frac{1}{D_i} \rightarrow \frac{1}{\bar{D}_i} = \frac{1}{D}\left(1 - \frac{\tilde{l}^2}{D_i}\right)$$

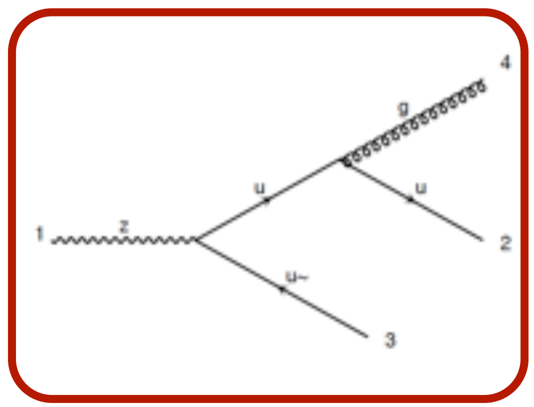
integration of this piece gives rise R_1

$$\boxed{4d \text{ couterparts}}$$

- **Can be included in OPP reduction**
- **Not needed in TIR reduction**

Generics

Real



NLO ANATOMY

- **Three parts need to be computed in a NLO calculation**

$$\sigma_{\rm NLO} = \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\mathcal{V} + \int d\Phi^{(n+1)}\mathcal{R}$$

Born cross section

Virtual correction

Real correction

$$\text{Virtual} = \frac{A}{\epsilon^2} + \frac{B}{\epsilon} + V \qquad \text{Real} = -\frac{A}{\epsilon^2} - \frac{B}{\epsilon} + R$$

NLO ANATOMY

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$$\sigma_{\rm NLO} = \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\mathcal{V} + \int d\Phi^{(n+1)}\mathcal{R}$$

Born cross section

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$$\text{Virtual} = \cancel{\frac{A}{\epsilon^2}} + \cancel{\frac{B}{\epsilon}} + V \qquad \text{Real} = -\cancel{\frac{A}{\epsilon^2}} - \cancel{\frac{B}{\epsilon}} + R$$

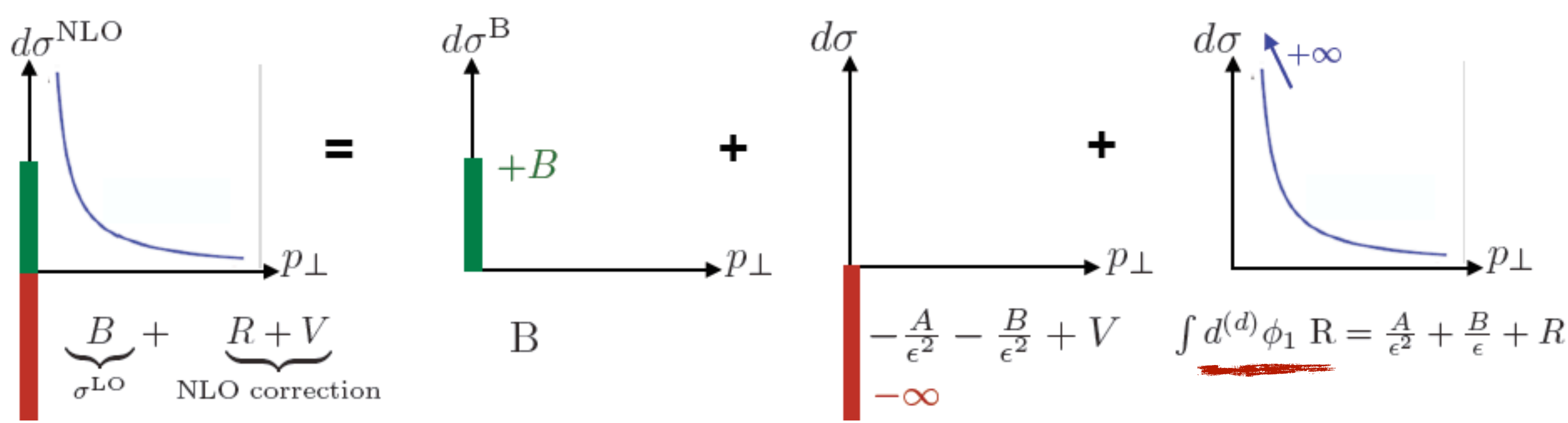
NLO ANATOMY

- **Three parts need to be computed in a NLO calculation**

$$\sigma_{\mathrm{NLO}} = \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\mathcal{V} + \int d\Phi^{(n+1)}\mathcal{R}$$

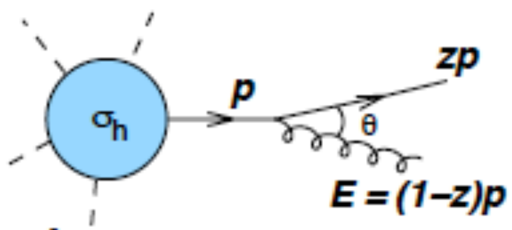
Born cross section | Virtual correction | Real correction

$$\mathrm{Virtual} = \cancel{\frac{A}{\epsilon^2}} + \cancel{\frac{B}{\epsilon}} + V \qquad \mathrm{Real} = -\cancel{\frac{A}{\epsilon^2}} - \cancel{\frac{B}{\epsilon}} + R$$



BRANCHING: TO BE OR NOT TO BE

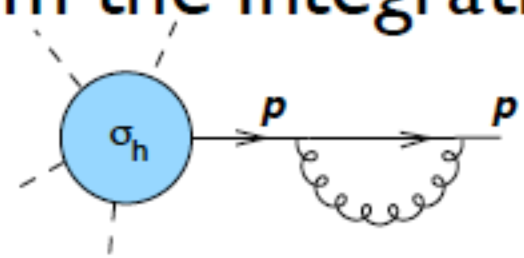
- Let us consider the branching of a gluon from a quark



$$\sigma_{h+g} \simeq \sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

Where k_t is the transverse momentum of the gluon $k_t = E\sin\theta$. It diverges in the soft ($z \to 1$) and collinear ($k_t \to 0$) region

- These singularities cancel with the virtual contribution, which comes from the integration of the loop momentum



$$\sigma_{h+V} \simeq -\sigma_h \frac{\alpha_s C_F}{\pi} \frac{dz}{1-z} \frac{dk_t^2}{k_t^2}$$

- The cancelation happens if we cannot distinguish between the case of no branching, and of a soft or collinear branching

IR SAFETY

- **In order to have meaningful fixed-order predictions in perturbation theory, observables must be IR-safe, i.e. not sensitive to the emission of soft/collinear partons**

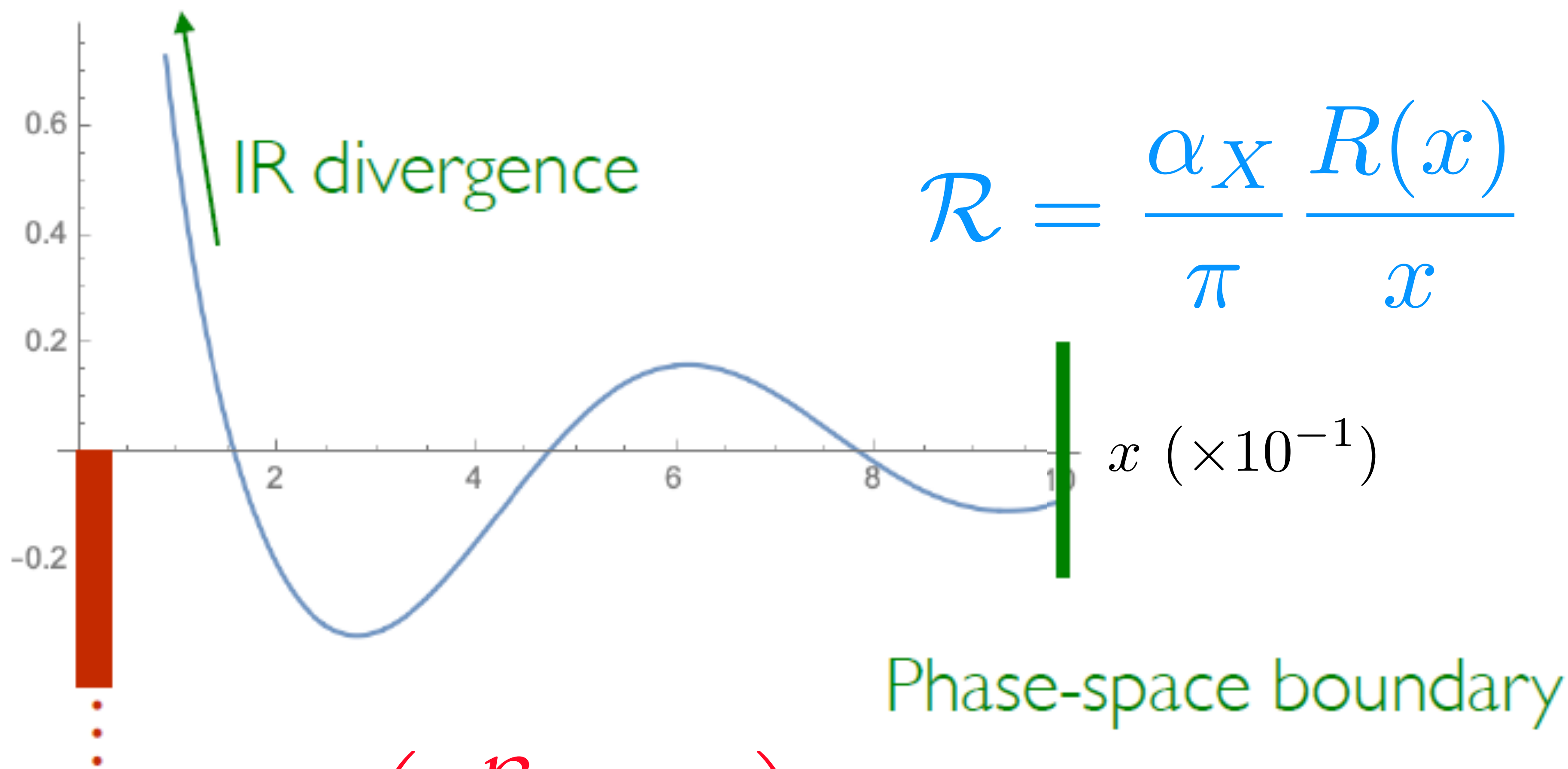
$$\lim_{p_i||p_j} \mathcal{O}\left(1,\cdots,i,\cdots,j-1,j,j+1,\cdots,n\right)=\mathcal{O}\left(1,\cdots,ij,\cdots,j-1,j+1,\cdots,n\right)$$

$$\lim_{p_i\to 0} \mathcal{O}\left(1,\cdots,i-1,i,i+1,\cdots,n\right)=\mathcal{O}\left(1,\cdots,i-1,i+1,\cdots,n\right)$$

- **For example,**
  - **The number of gluons is NOT IR safe.**
  - **The leading p_T/energy particle is NOT IR safe (soft or collinear unsafe ?).**
  - **The colour in a given cone is NOT IR safe (soft or collinear unsafe ?).**
  - **The transverse energy sum is IR safe.**

A TOY EXAMPLE

- **Assuming the phase space integration can be casted into a one-dimensional case** $x \in [0,1]$ **:**



$$\mathcal{R} = \frac{\alpha_X}{\pi}\frac{R(x)}{x}$$

$$\delta(x)\mathcal{V} = \delta(x)\frac{\alpha_X}{\pi}\left(\frac{\mathcal{B}}{2\epsilon_{\rm IR}} + V\right)$$

$\forall\ \mathcal{O}(x)$ IR safety $\lim_{x\to 0}\mathcal{O}(x)R(x) = \mathcal{O}(0)\mathcal{B}$

A TOY EXAMPLE

$$\mathcal{O}(0)\mathcal{V} + \int_0^1 dx x^{-2\epsilon_{\rm IR}}\mathcal{O}(x)\mathcal{R}$$

Dimensionally regularise in x !

$$= \frac{\alpha_X}{\pi}\left[\mathcal{O}(0)\left(\frac{\mathcal{B}}{2\epsilon_{\rm IR}}+V\right)+\int_0^1 dx x^{-1-2\epsilon_{\rm IR}}\mathcal{O}(x)R(x)\right]$$

$$= \frac{\alpha_X}{\pi}\left[\mathcal{O}(0)\left(\frac{\mathcal{B}}{2\epsilon_{\rm IR}}+V\right)+\left(-\mathcal{O}(0)\frac{\mathcal{B}}{2\epsilon_{\rm IR}}+\int_0^1 dx\left(\frac{1}{x}\right)_+\mathcal{O}(x)R(x)\right)\right]$$

$$= \frac{\alpha_X}{\pi}\left[\mathcal{O}(0)V+\int_0^1 dx\left(\frac{1}{x}\right)_+\mathcal{O}(x)R(x)\right]$$

- **We have used:**

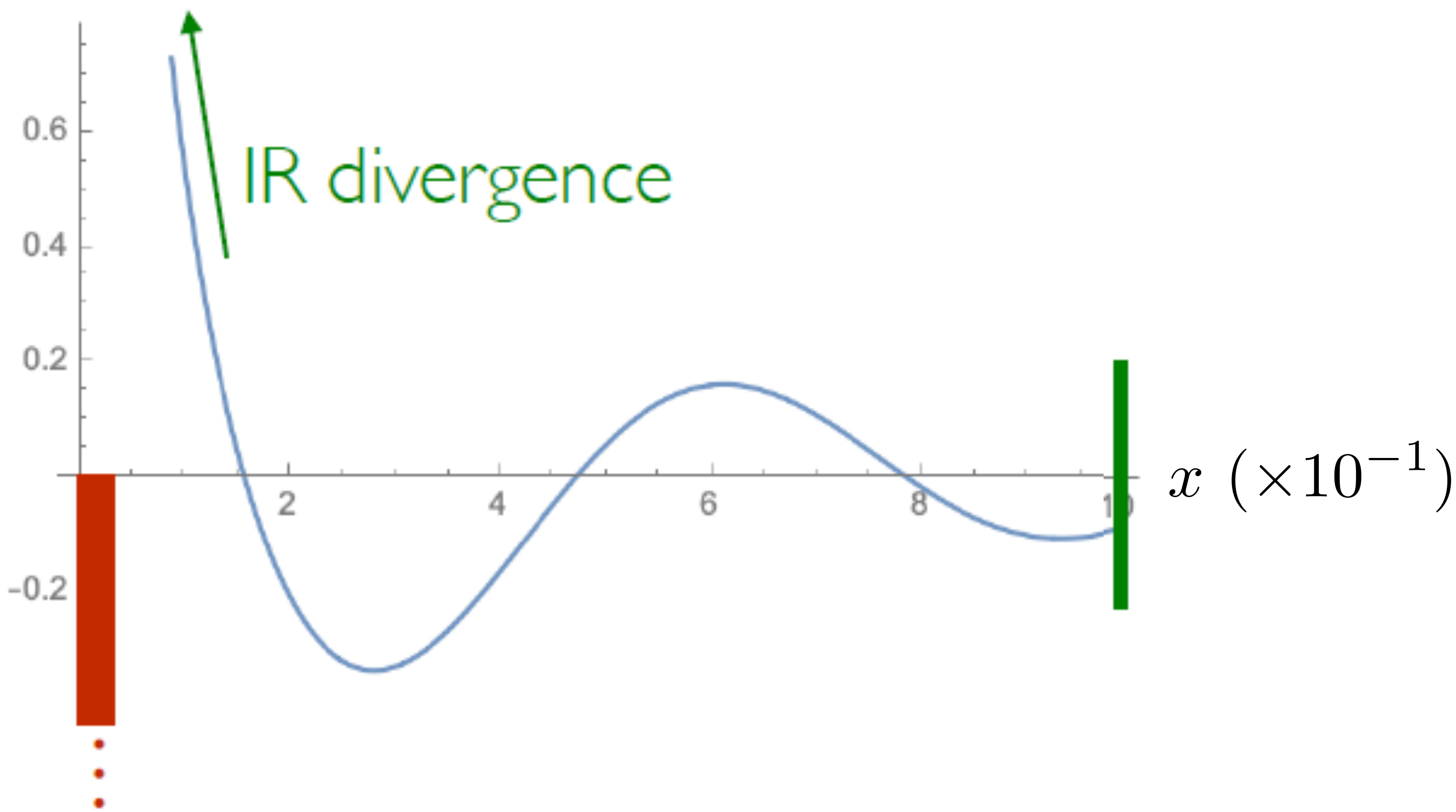
$$x^{-1-2\epsilon_{\rm IR}} = -\frac{1}{2\epsilon_{\rm IR}}\delta(x)+\left(\frac{1}{x}\right)_+ + \epsilon_{\rm IR}\ \text{term}$$

$$\left(\frac{1}{x}\right)_+ f(x) \equiv \frac{f(x)-f(0)}{x} \qquad \forall\ f(x)$$

PHASE-SPACE SLICING

- **In general, the phase-space integration over real matrix element is very hard. Dedicated general approaches are developed !**
  - **Phase-space slicing**

$$\int_0^1 dx x^{-1-2\epsilon_{\rm IR}}\mathcal{O}(x)R(x)$$



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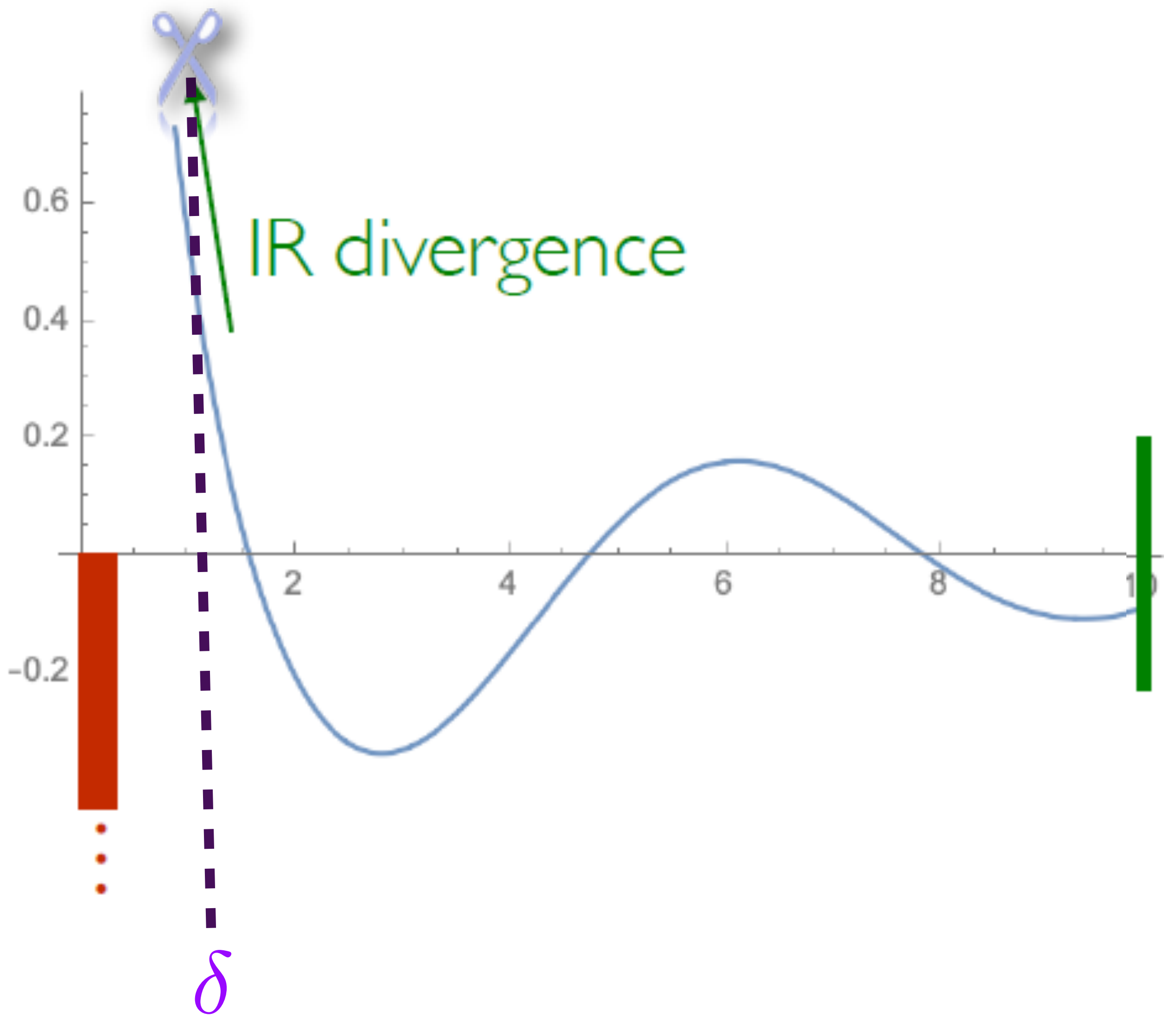

$$\int_0^1 dx x^{-1-2\epsilon_{\rm IR}} \mathcal{O}(x) R(x)$$

$$= \left(\int_0^\delta + \int_\delta^1 \right) dx x^{-1-2\epsilon_{\rm IR}} \mathcal{O}(x) R(x)$$

$$\overset{\delta\to 0}{=} \left(-\mathcal{O}(0) \frac{\mathcal{B}}{2\epsilon_{\rm IR}} \delta^{-2\epsilon_{\rm IR}} \right)$$

$$+ \left(\int_\delta^1 dx x^{-1} \mathcal{O}(x) R(x) + \epsilon_{\rm IR} \text{ term} \right)$$

$$\overset{\delta\to 0}{=} -\mathcal{O}(0) \mathcal{B} \left(\frac{1}{2\epsilon_{\rm IR}} - \log \delta \right)$$

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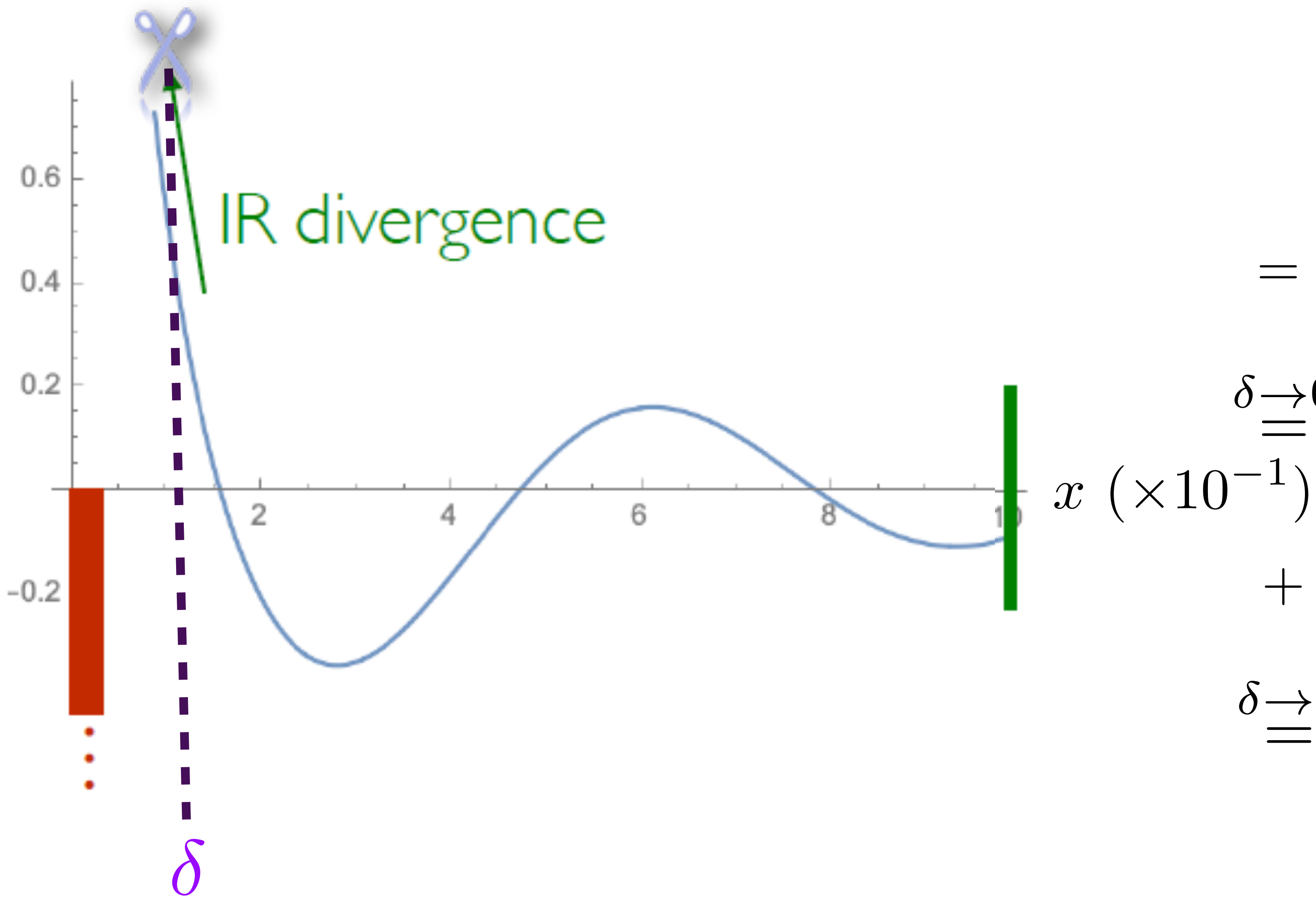

finite integral
(can be computed numerically)

$$
\begin{aligned}
&\int_0^1 dx x^{-1-2\epsilon_{\rm IR}}\mathcal{O}(x)R(x)\\
&=\left(\int_0^\delta+\int_\delta^1\right)dx x^{-1-2\epsilon_{\rm IR}}\mathcal{O}(x)R(x)\\
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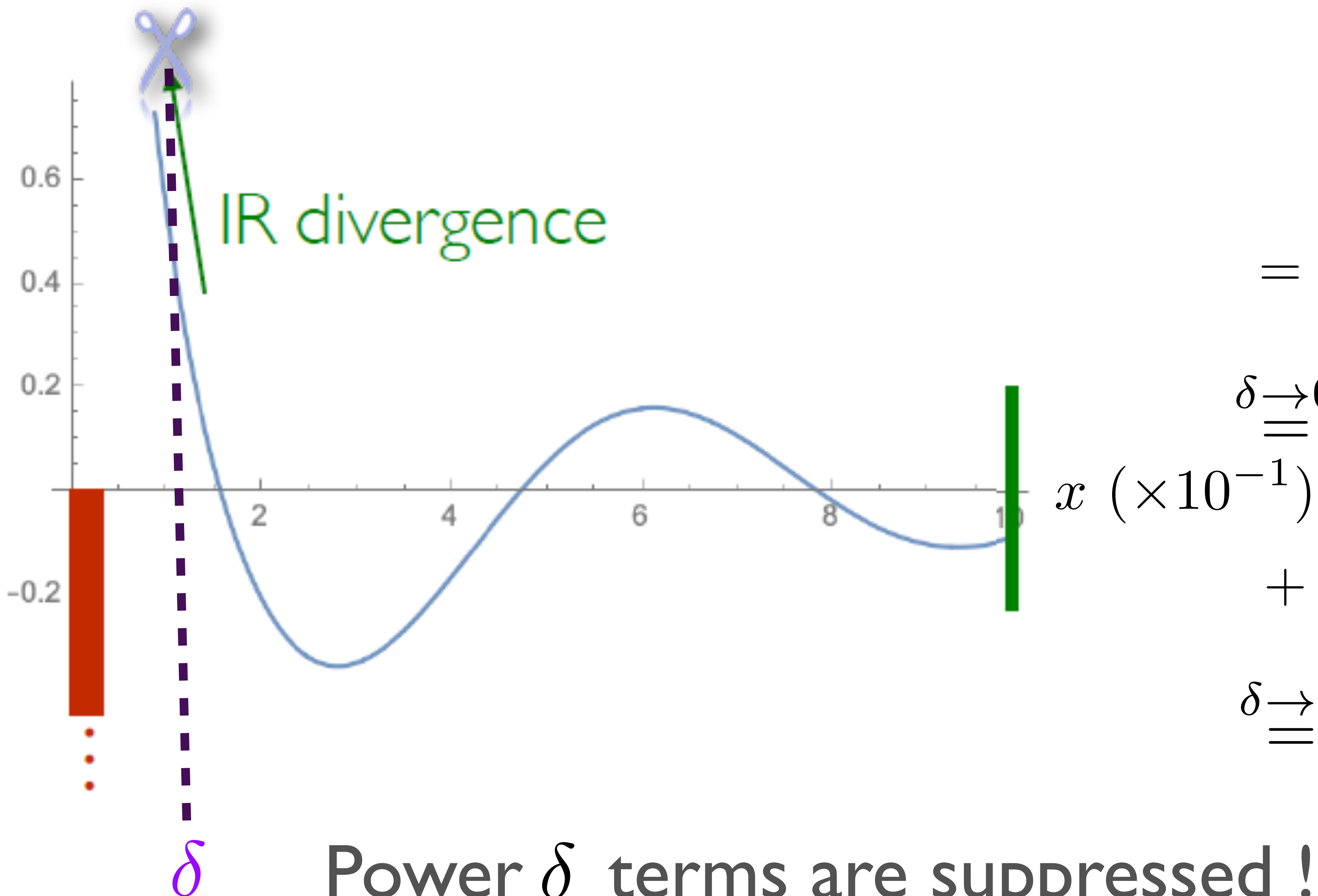

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finite integral
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$$\boxed{+\int_\delta^1 dx x^{-1}\mathcal{O}(x)R(x)}$$

Power δ terms are suppressed !
Large numerical cancellations !

SUBTRACTION

- **In general, the phase-space integration over real matrix element is very hard. Dedicated general approaches are developed !**
  - **Subtraction method**
    - Find a generic simple function S has exactly same IR singularity as real matrix element

$$\lim_{p_i||p_j} \mathcal{O}(x)S = \lim_{p_i||p_j} \mathcal{O}(x)\mathcal{R} \qquad \lim_{p_i\to 0} \mathcal{O}(x)S = \lim_{p_i\to 0} \mathcal{O}(x)\mathcal{R}$$

    - ... but much easier to integrate analytically.

$$\mathcal{O}(0)\mathcal{V} + \int_0^1 dx x^{-2\epsilon_{\rm IR}} \mathcal{O}(x)\mathcal{R}$$
$$= \left(\mathcal{O}(0)\mathcal{V} + \int_0^1 dx x^{-2\epsilon_{\rm IR}} \mathcal{O}(x)S\right) + \int_0^1 dx x^{-2\epsilon_{\rm IR}} \mathcal{O}(x)\left(\mathcal{R} - S\right)$$

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Analytically known

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Analytically known

Integrating numerically in 4d

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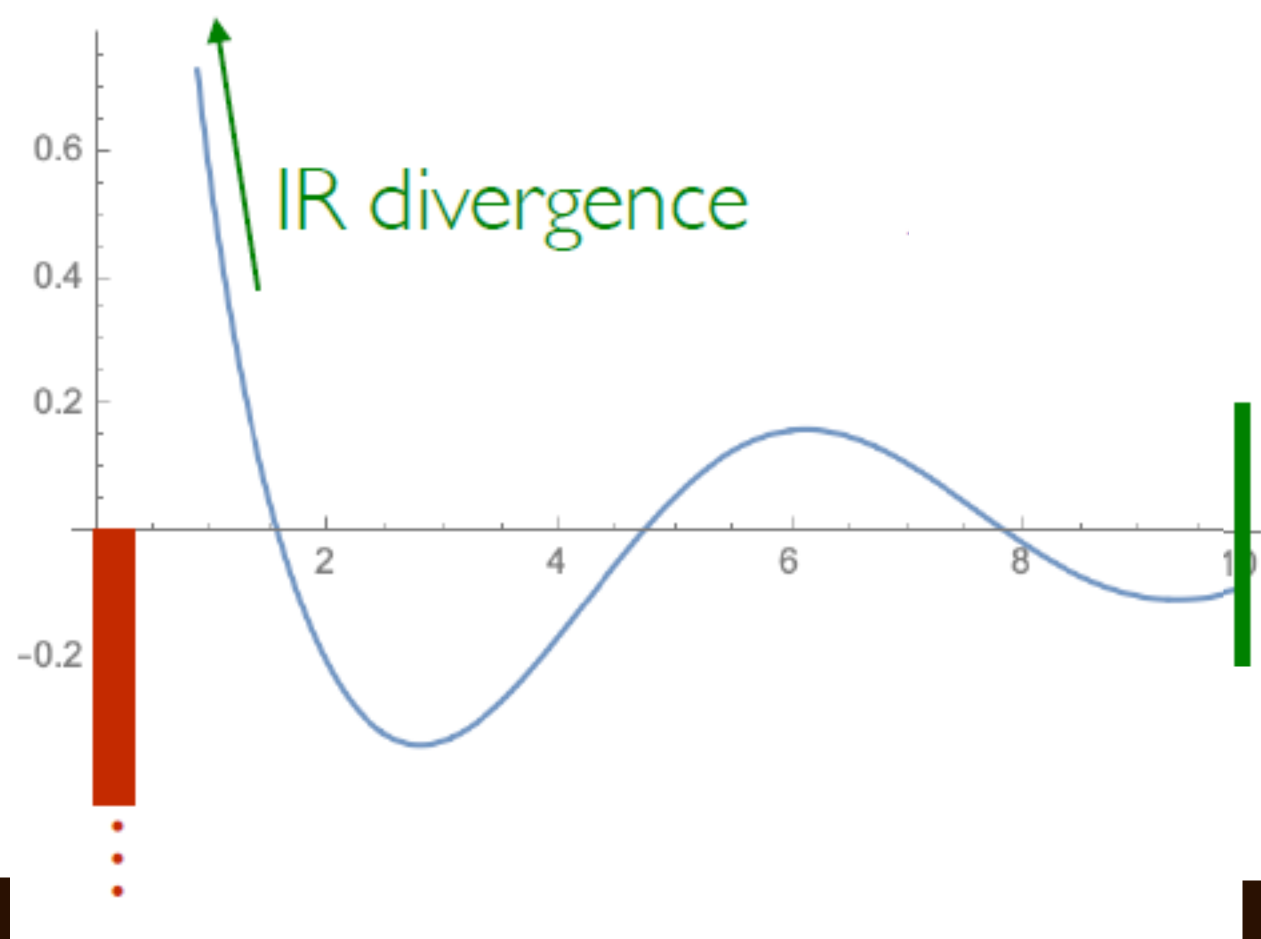

$x\ (\times 10^{-1})$

SUBTRACTION

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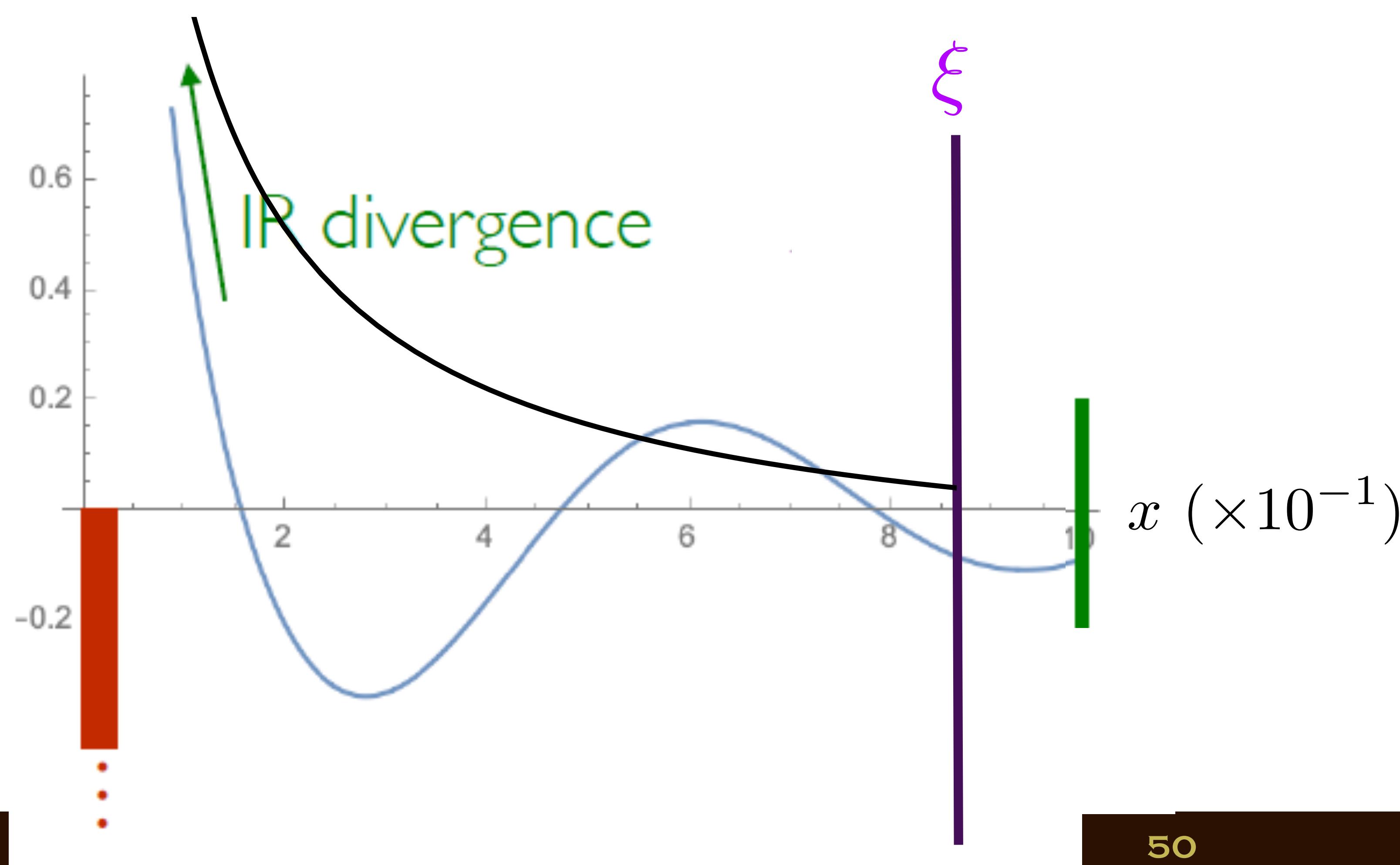
$$S = \frac{\alpha_X}{\pi}\frac{\mathcal{B}\Theta(\xi - x)}{x} \qquad \forall\ \xi \in \ (0,1]$$



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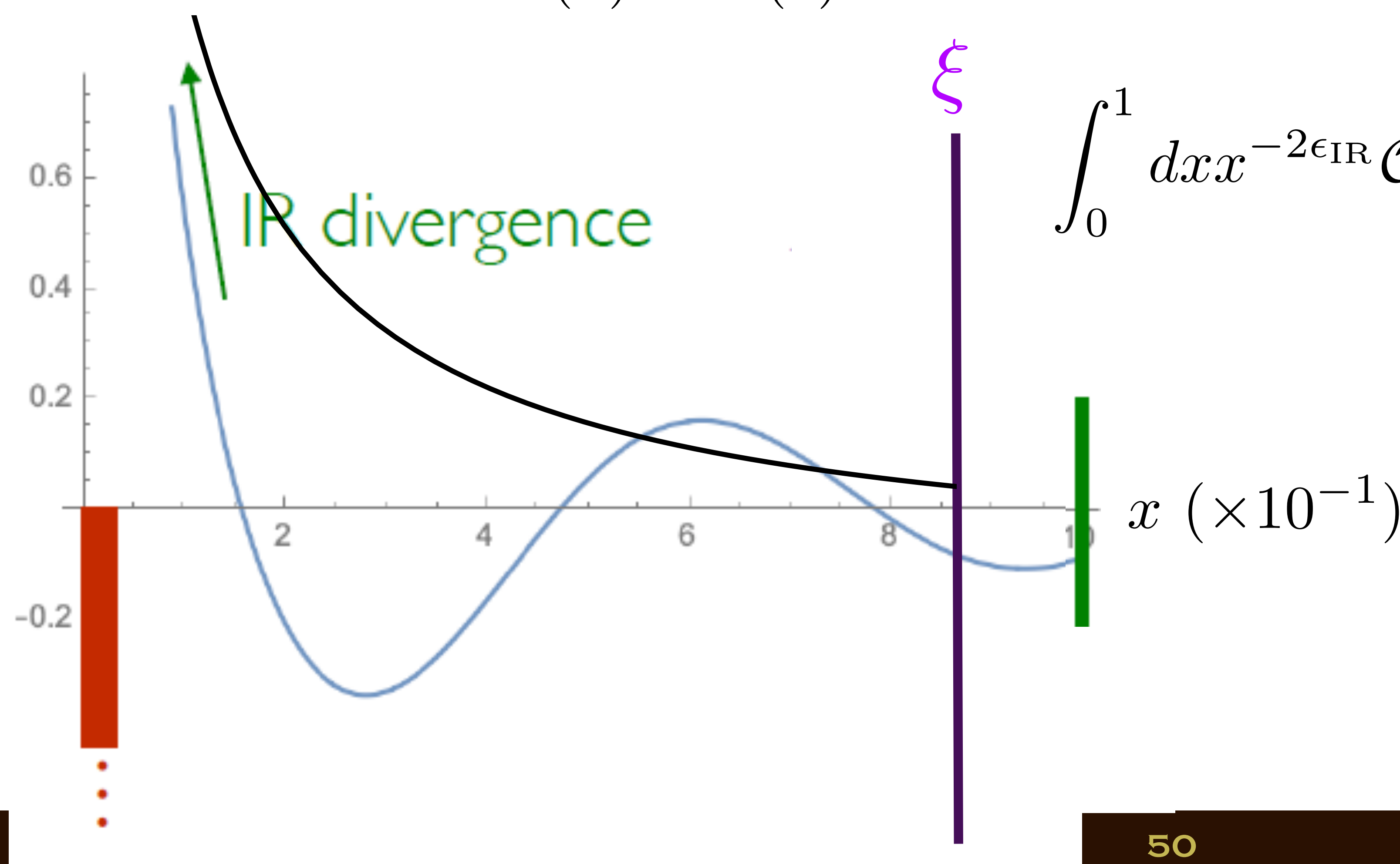

$$\int_0^1 dx x^{-2\epsilon_{\mathrm{IR}}}\mathcal{O}(x)S=-\frac{\alpha_X}{\pi}\mathcal{O}(0)\mathcal{B}\left(\frac{1}{2\epsilon_{\mathrm{IR}}}-\log\xi\right)$$

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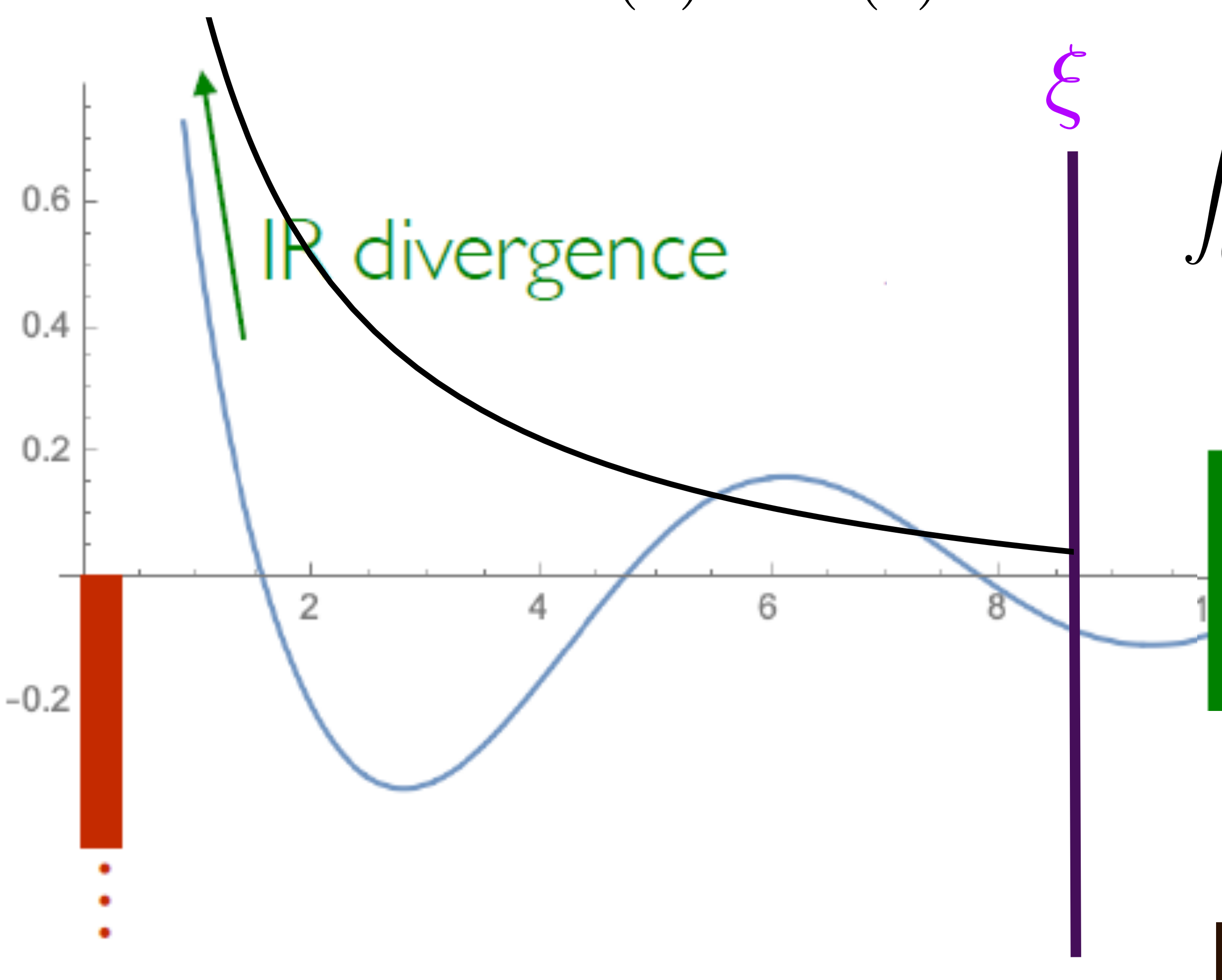


$$\int_0^1 dx x^{-2\epsilon_{\rm IR}}\mathcal{O}(x)S = -\frac{\alpha_X}{\pi}\mathcal{O}(0)\mathcal{B}\left(\frac{1}{2\epsilon_{\rm IR}} - \log\xi\right)$$

No approximation !

Numerical cancellations mitigated !

$x\ (\times 10^{-1})$

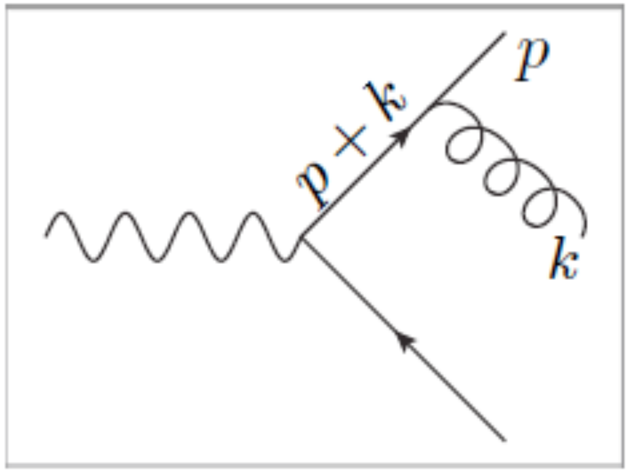
NLO SUBTRACTION

- **Master formula:**

$$\sigma_{\rm NLO} = \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\mathcal{V} + \int d\Phi^{(n+1)}\mathcal{R}$$

$$= \int d\Phi^{(n)}\mathcal{B} + \int d\Phi^{(n)}\left[\mathcal{V} + \int d\Phi^{(1)} S\right] + \int d\Phi^{(n+1)}\left[\mathcal{R} - S\right]$$

- **The subtraction counterterm S should be chosen:**
  - It exactly matches the singular behaviour of real ME
  - It can be integrated numerically in a convenient way
  - It can be integrated exactly in d dimension
  - It is process independent (overall factor times Born ME)
- **In gauge theory, the singular structure is universal**



$$(p+k)^2 = 2E_pE_k(1-\cos\theta_{pk})$$

- Collinear singularity:

$$\lim_{p//k}|M_{n+1}|^2 \simeq |M_n|^2\, P^{AP}(z)$$

- Soft singularity:

$$\lim_{k\to 0}|M_{n+1}|^2 \simeq \sum_{ij}|M_n^{ij}|^2\frac{p_ip_j}{p_ik\; p_jk}$$

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TWO WIDELY-USED SUBTRACTION METHODS

Dipole subtraction

Catani, Seymour, hep-ph/9602277 & hep-ph/9605323

- Most used method
- Recoil taken by one parton
  $\to N^3$ scaling
- Method evolves from cancelation of soft divergences
- Proven to work for simple and complicated processes
- Automated in MadDipole, AutoDipole, Sherpa, Helac-NLO, …

FKS subtraction

Frixione, Kunszt, Signer, hep-ph/9512328

- Less known method
- Recoil distributed among all particles
  $\to N^2$ scaling
- Probably (?) more efficient because less subtraction terms are needed
- Method evolves from cancelation of collinear divergences
- Proven to work for simple and complicated processes
- Automated in MadGraph5_aMC@NLO and in the Powheg box/Powhel

FKS SUBTRACTION

- The real ME singular as

$$\mathcal{R} \xrightarrow{\text{IR limit}} \frac{1}{\xi_i}\frac{1}{1-y_{ij}} \qquad \xi_i = \frac{E_i}{\sqrt{\hat{s}}} \quad y_{ij} = \cos\theta_{ij}$$

- Partition the phase space in order to have at most one soft and/or one collinear singularity

$$\mathcal{R}d\Phi^{(n+1)} = \sum_{ij} S_{ij}\mathcal{R}d\Phi^{(n+1)} \qquad \sum_{ij} S_{ij} = 1$$

$$S_{ij} \to 1 \text{ if } p_i \cdot p_j \to 0$$

$$S_{ij} \to 0 \text{ if } p_m \cdot p_n \to 0,\ \{m,n\} \neq \{i,j\}$$

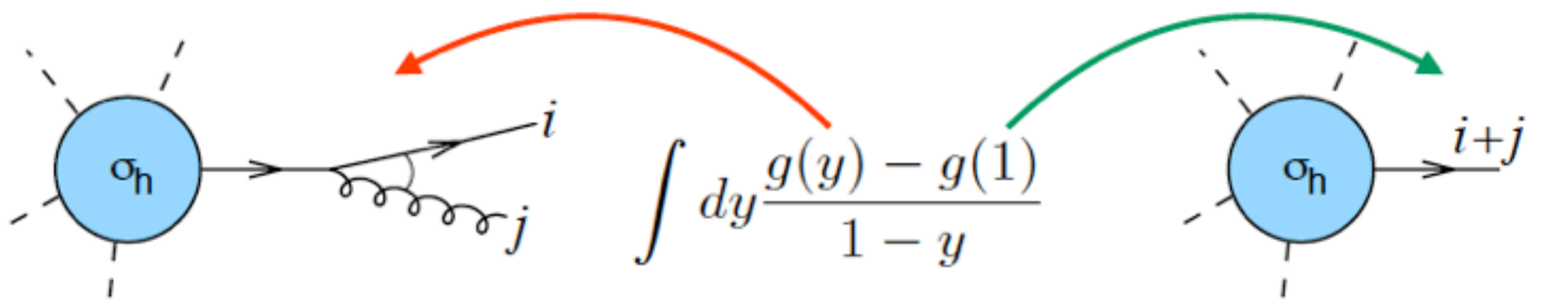
- Use plus prescriptions to subtract the divergences

$$d\sigma_{\tilde{R}} = \sum_{ij}\left(\frac{1}{\xi_i}\right)_+\left(\frac{1}{1-y_{ij}}\right)_+ \xi_i\,(1-y_{ij})\,S_{ij}\mathcal{R}d\Phi^{(n+1)}$$

$$\int d\xi \left(\frac{1}{\xi}\right)_+ f(\xi) = \int d\xi \frac{f(\xi)-f(0)}{\xi} \qquad \int dy \left(\frac{1}{1-y}\right)_+ g(y) = \int dy \frac{g(y)-g(1)}{1-y}$$

FKS SUBTRACTION

- **Counterevents:**
  - Soft counterevent ($\xi_i \to 0$)
  - Collinear counterevents ($y_{ij} \to 1$)
  - Soft-collinear counterevents ($\xi_i \to 0$ and $y_{ij} \to 1$)



$$\int dy \frac{g(y) - g(1)}{1-y}$$

Real emission   Subtraction term

  - If i and j are on-shell in the event, for the counterevent the combined particle $i+j$ must be on shell
  - $i+j$ can be put on shell only be reshuffling the momenta of the other particles
  - It can happen that event and counterevent end up in different histogram bins
    - Use IR-safe observables and don't ask for infinite resolution!

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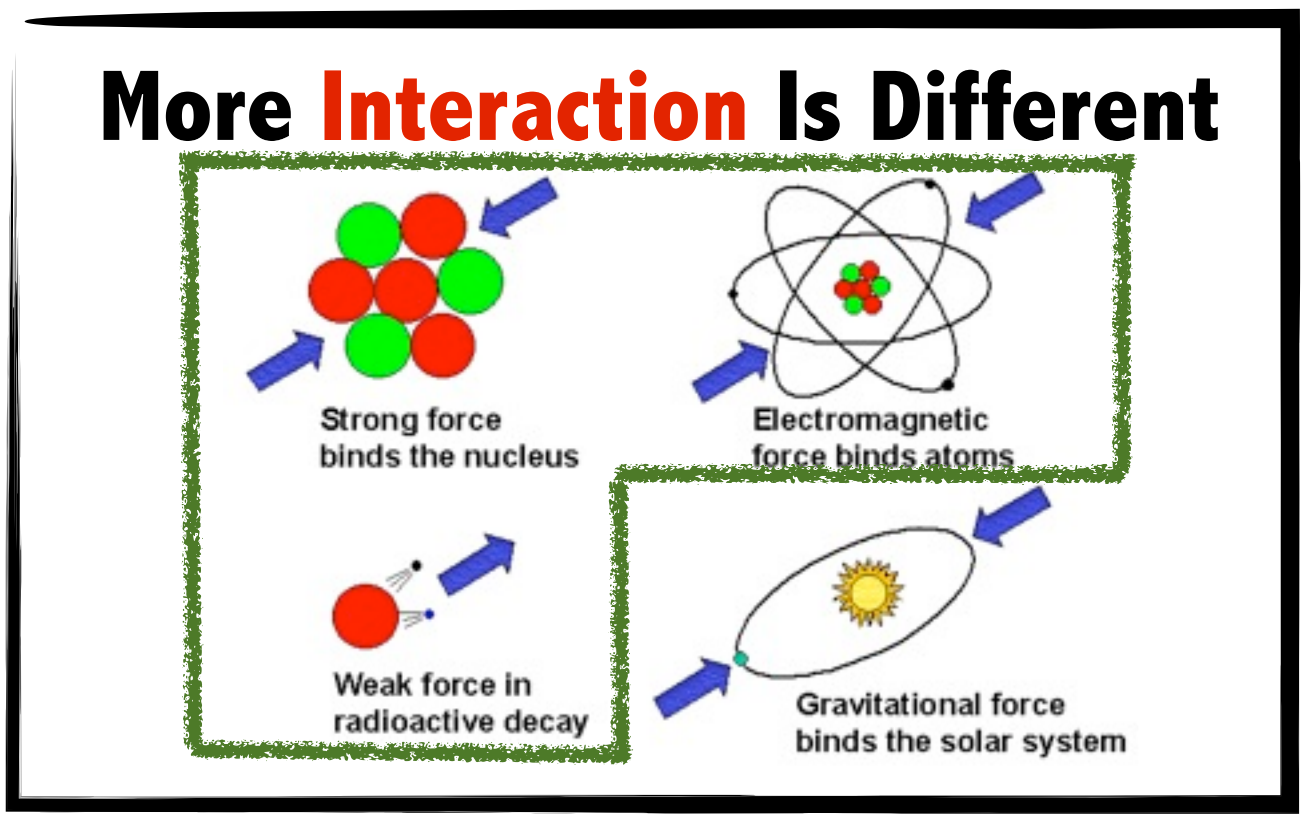
A Few Advanced Topics

More Is Different

Broken symmetry and the nature of the hierarchical structure of science.

P. W. Anderson

A Few Advanced Topics



GENERAL FEATURE OF EW CORRECTIONS

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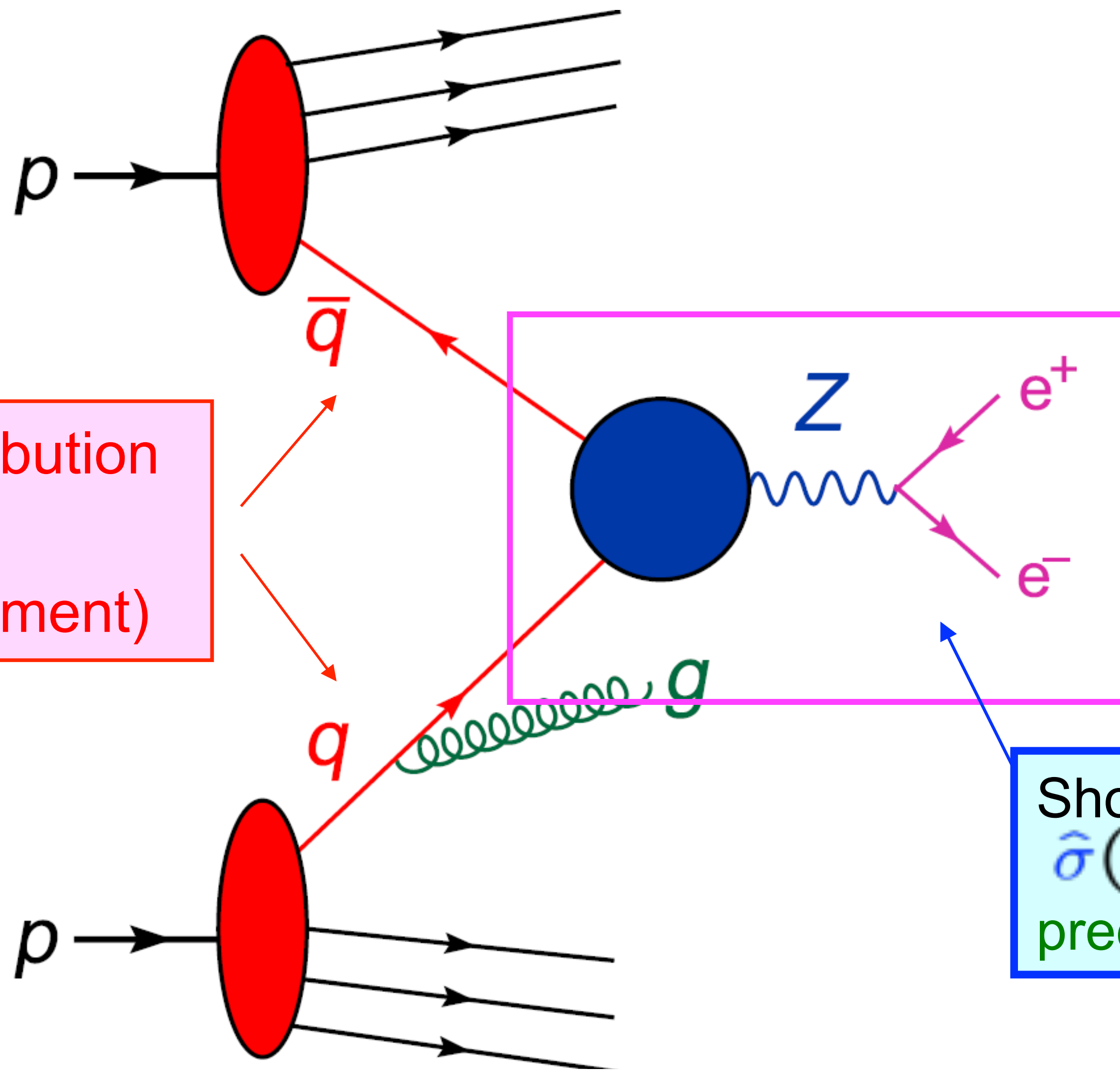

Parton distribution functions (from experiment)

The "femto-universe"

size = factorization scale μ_F ("arbitrary")

Short-distance cross section
$\hat{\sigma}(\alpha_s, \mu_F, \mu_R)$
predictable using perturbative QCD

$$\sigma(pp \to Z + X) = \int dx_1 dx_2 f(x_1, \mu_F) f(x_2, \mu_F) \hat{\sigma}(\alpha_s, \mu_F, \mu_R)$$

GENERAL FEATURE OF EW CORRECTIONS

- Let us start from defining NLO "EW Corrections" (= "EWC")

LO NLO NNLO

$$\hat{\sigma}(\alpha_s,\mu_F,\mu_R)=[\alpha_s(\mu_R)]^n\boxed{\left[\hat{\sigma}^{(0)}+\frac{\alpha_s}{2\pi}\sigma^{(1)}(\mu_F,\mu_R)+\left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}^{(2)}(\mu_F,\mu_R)+\cdots\right]}$$

QCD

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LO NLO NNLO

QCD

EW

$$\hat{\sigma}(\alpha_s,\alpha,\mu_F,\mu_R)=[\alpha_s(\mu_R)]^n\alpha^m\Bigg[\boxed{\hat{\sigma}^{(0,0)}+\frac{\alpha_s}{2\pi}\sigma^{(1,0)}(\mu_F,\mu_R)+\left(\frac{\alpha_s}{2\pi}\right)^2\hat{\sigma}^{(2,0)}(\mu_F,\mu_R)+\cdots}$$

$$\boxed{+\frac{\alpha}{2\pi}\sigma^{(0,1)}(\mu_F,\mu_R)+\left(\frac{\alpha}{2\pi}\right)^2\hat{\sigma}^{(0,2)}(\mu_F,\mu_R)+\cdots}$$

$$+\sum_{i\geq 1}\sum_{j\geq 1}\left(\frac{\alpha_s}{2\pi}\right)^i\left(\frac{\alpha}{2\pi}\right)^j\hat{\sigma}^{(i,j)}(\mu_F,\mu_R)\Bigg]$$

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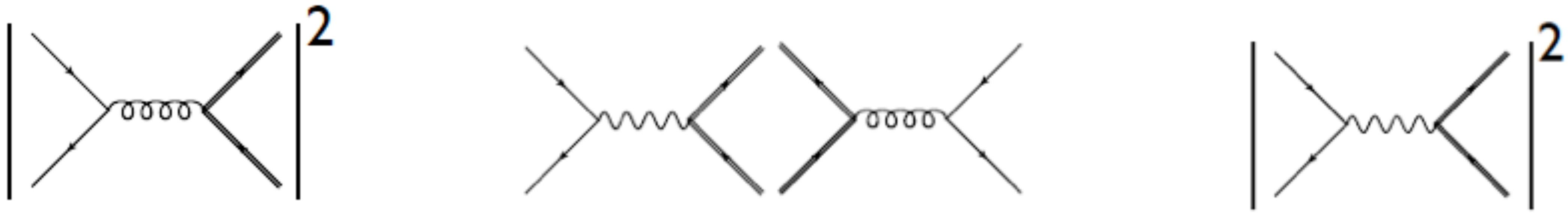

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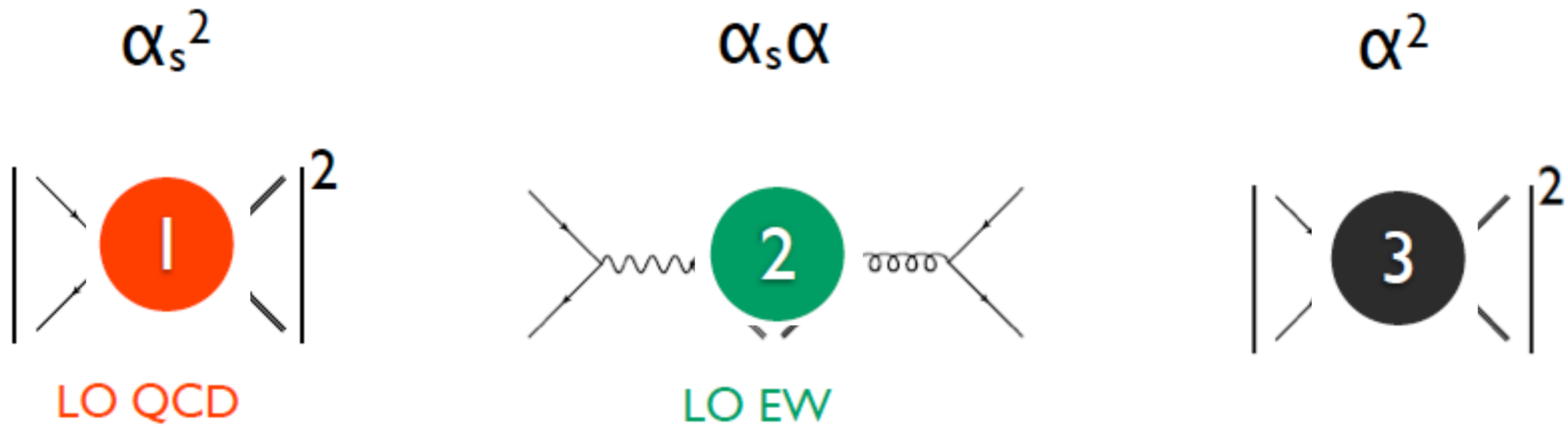

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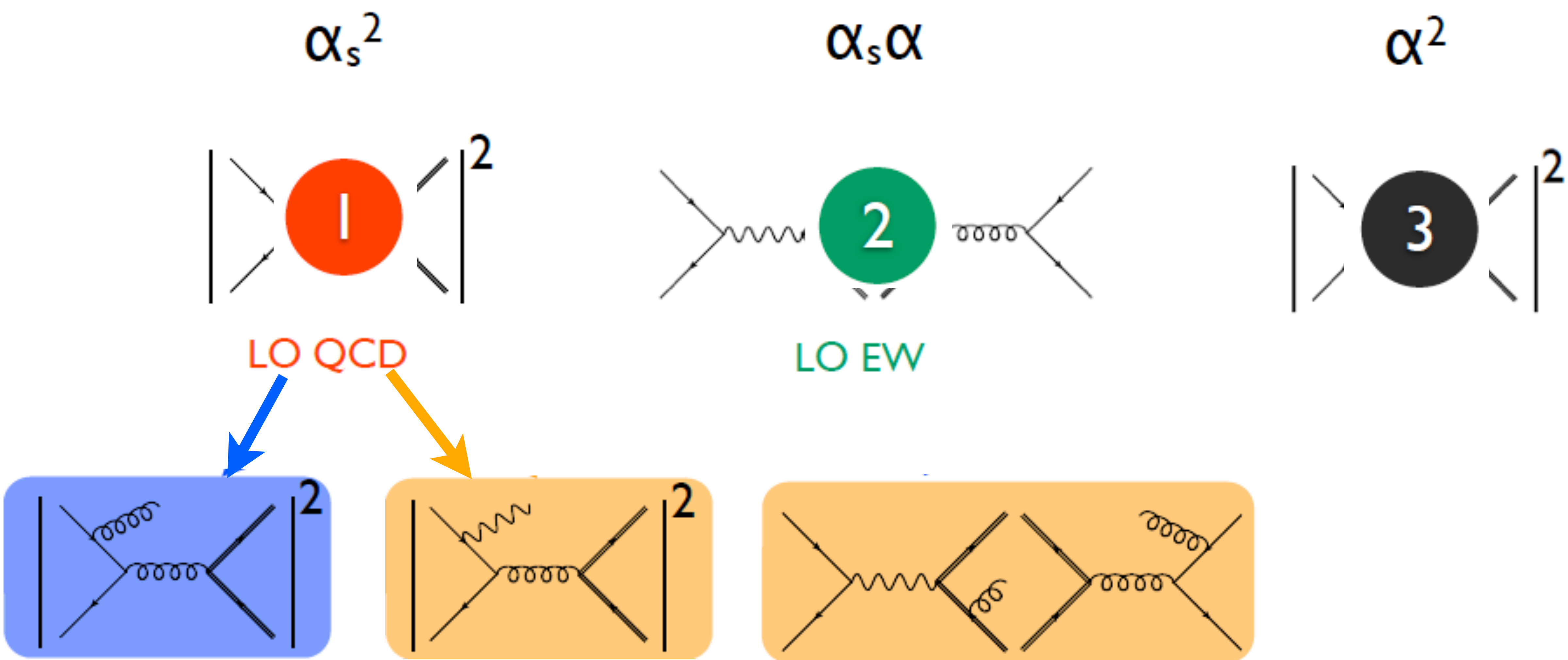

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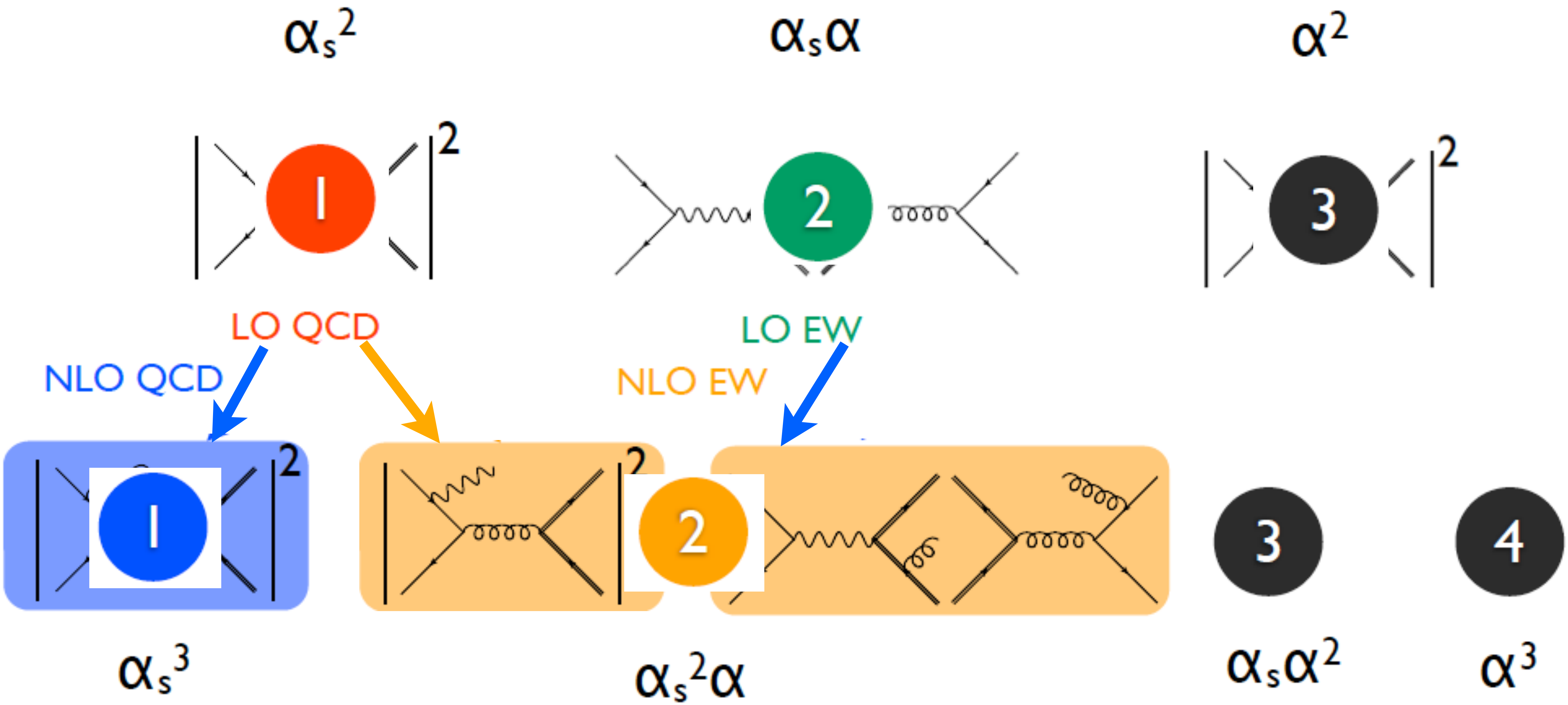

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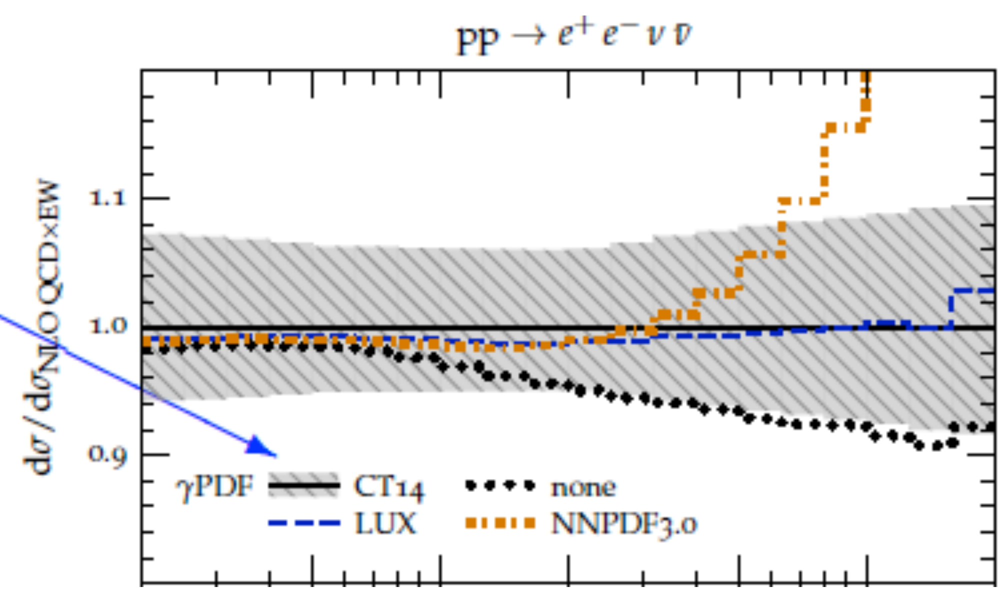
    - **Photon PDF will be quite relevant, which was usually poorly determined until LUXqed**
    - **Photon and jet is not well separated (need fragmentation function or some approximations)**
    - **If phase space is enough, EW boson radiation will be quite often (do we need them ?)**
    - **The general matching between matrix element and parton shower will be difficult**

relative importance of γ-induced channels wrt. NLO QCD×EW

CT14qed (baseline) no γPDF
LUXqed NNPDF3.0qed

Kallweit et al. JHEP’17



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  - $\alpha(0)$ scheme: appropriate for external final photon (see e.g., 2106.02059)
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  - G_μ scheme: works good for weak bosons and well measured

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Shall we use different scheme/renormalization for different vertices in one diagram ?

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- Let us start from defining NLO "EW Corrections" (= "EWC")
  - So far, it seems obvious that EWC is just one more α expansion wrt Born
  - Situation may be more complicated
    - There may present several order contributions in Born (e.g. dijet)
    - The usually ignored off-shell effect may be important $\Gamma/M \simeq \alpha$
    - Photon PDF will be quite relevant, which was usually poorly determined until LUXqed
    - Photon and jet is not well separated (need fragmentation function or some approximations)
    - If phase space is enough, EW boson radiation will be quite often (do we need them ?)
    - The general matching between matrix element and parton shower will be difficult
- Three α schemes are frequently used
  - $\alpha(0)$ scheme: appropriate for external final photon (see e.g., 2106.02059)
  - $\alpha(M_Z)$ scheme: works good for internal photon
  - G_μ scheme: works good for weak bosons and well measured

Shall we use different scheme/renormalization for different vertices in one diagram ?

- Use $K_{\text{NLO QCD}} \times K_{\text{NLO EW}}$ to capture the missing higher order ?

ENHANCE EW CORRECTIONS

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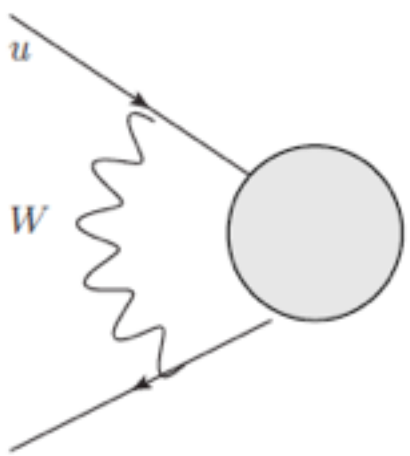

Leading Log Next-to-Leading Log

$$\sim -c_{\rm LL}\frac{\alpha}{\pi s_w^2}\log^2\frac{Q^2}{M_W^2}+c_{\rm NLL}\frac{3\alpha}{\pi s_w^2}\log\frac{Q^2}{M_W^2}+\cdots$$

e.g.

$Q = 1\ \mathrm{TeV}$ $\quad -c_{\rm LL}\times 26\% + c_{\rm NLL}\times 16\%$

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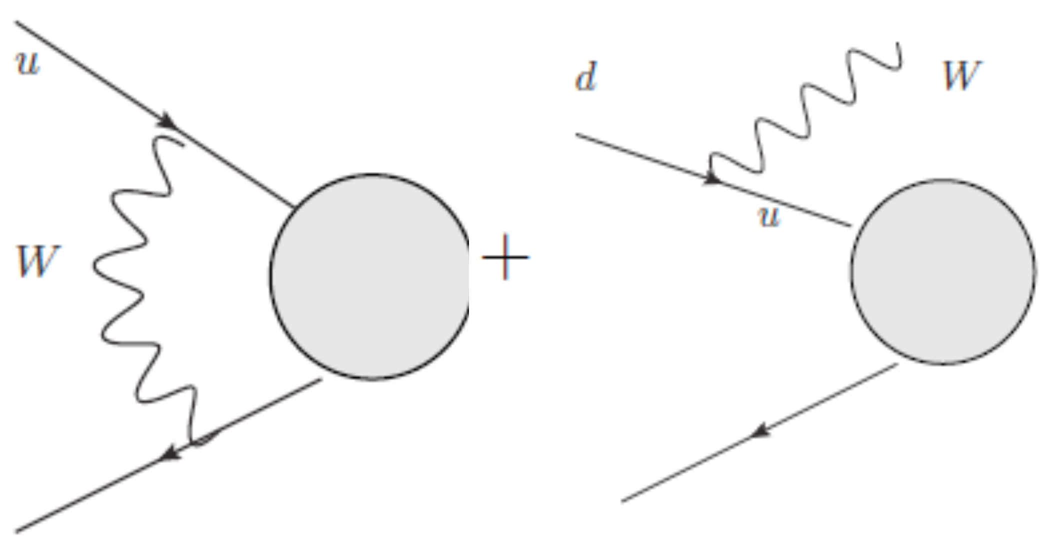
  - Even treat W/Z as inclusive as gluon/photon: initial state is not SU(2) singlet



$$\sim (f_{u/p} - f_{d/p}) \log\frac{Q^2}{M_W^2}$$

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ENHANCE EW CORRECTIONS

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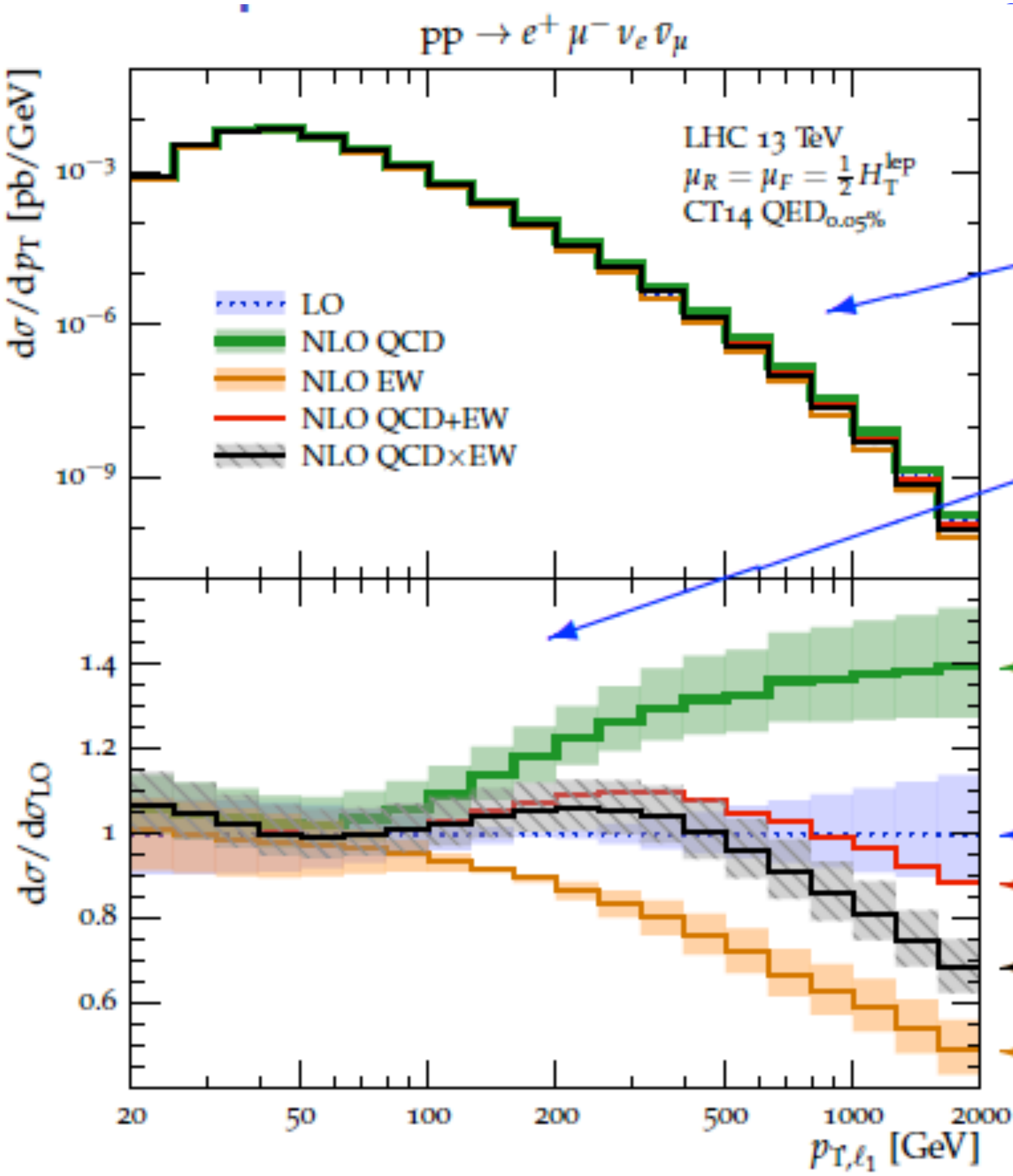

Kallweit et al. JHEP'17

absolute prediction

relative correction wrt. LO

NLO QCD (w/ moderate jet veto)

LO

NLO QCD+EW

NLO QCD×EW

NLO EW

Large +QCD corr. cancel with large -EW corr.

QCDxEW differs significantly wrt QCD+EW

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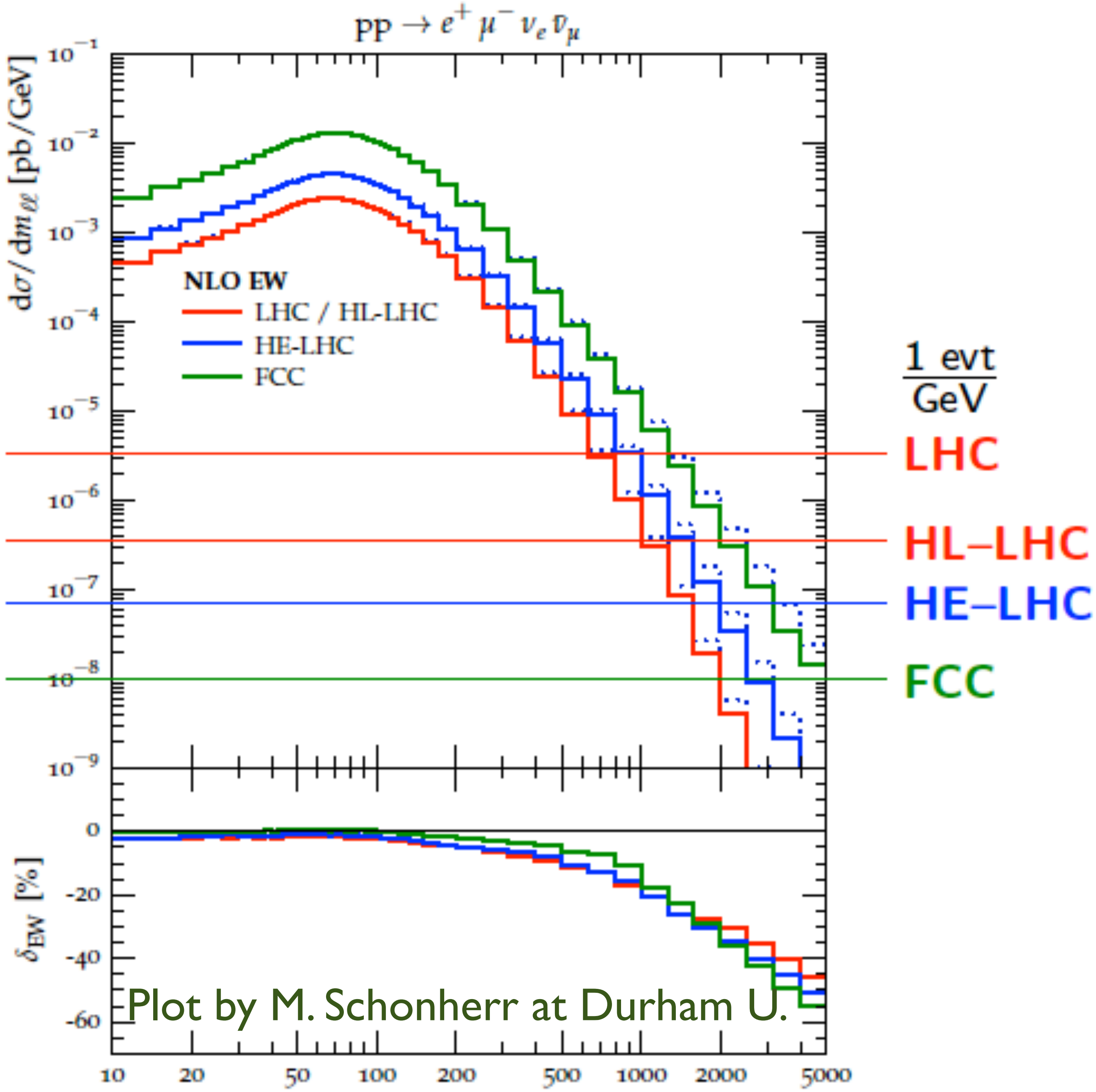
  - One does not treat W/Z inclusively as they can be (at least partially) reconst.
  - Even treat W/Z as inclusive as gluon/photon: initial state is not SU(2) singlet
  - However, EW Sudakov logarithms is not always relevant in Sudakov regime
    - e.g. Drell-Yan at large invariant mass receives large contributions from small t Dittmaier et al.'10

EW IN HIGH-ENERGY SCATTERINGS



- BSM effects are expected to be enhanced in the high-energy scatterings
- -> motivated BSM search go to the tail
- EW corr. increase up to tens of percent due to EW Sudakov logs
- The EW log resummation is still not mandatory@ (HL-)LHC as

$$\alpha L \ll 1$$

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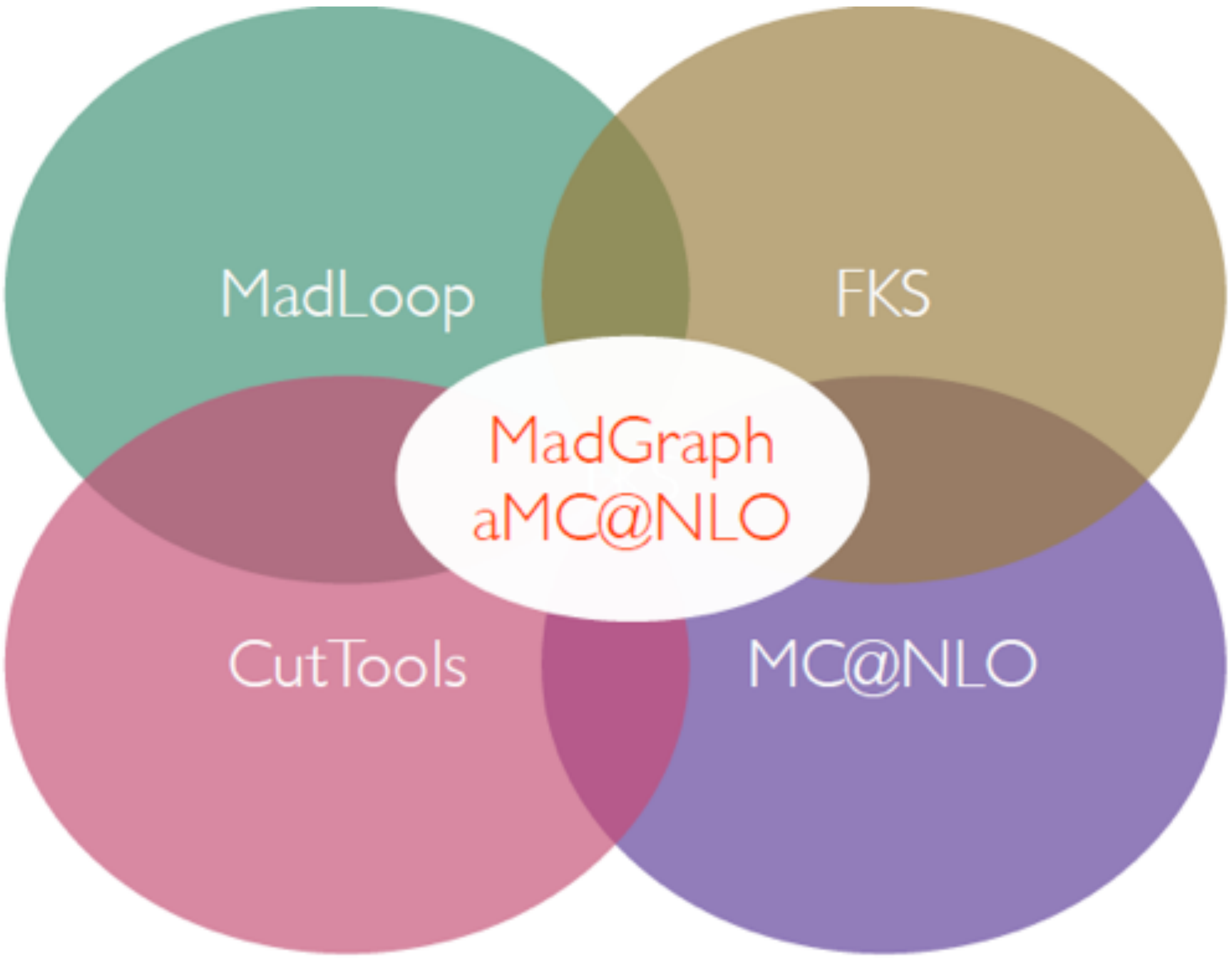
MADGRAPH5_AMC@NLO IN A NUTSHELL

Alwall et al. (JHEP'14)



4 commands for a NLO calculation

```
> ./bin/mg5_aMC
> generate process [QCD]
> output
> launch
```

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> ./bin/mg5_aMC
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Frederix et al. (JHEP'18)

```
> ./bin/mg5_aMC
> generate process [QCD QED]
> output
> launch
```

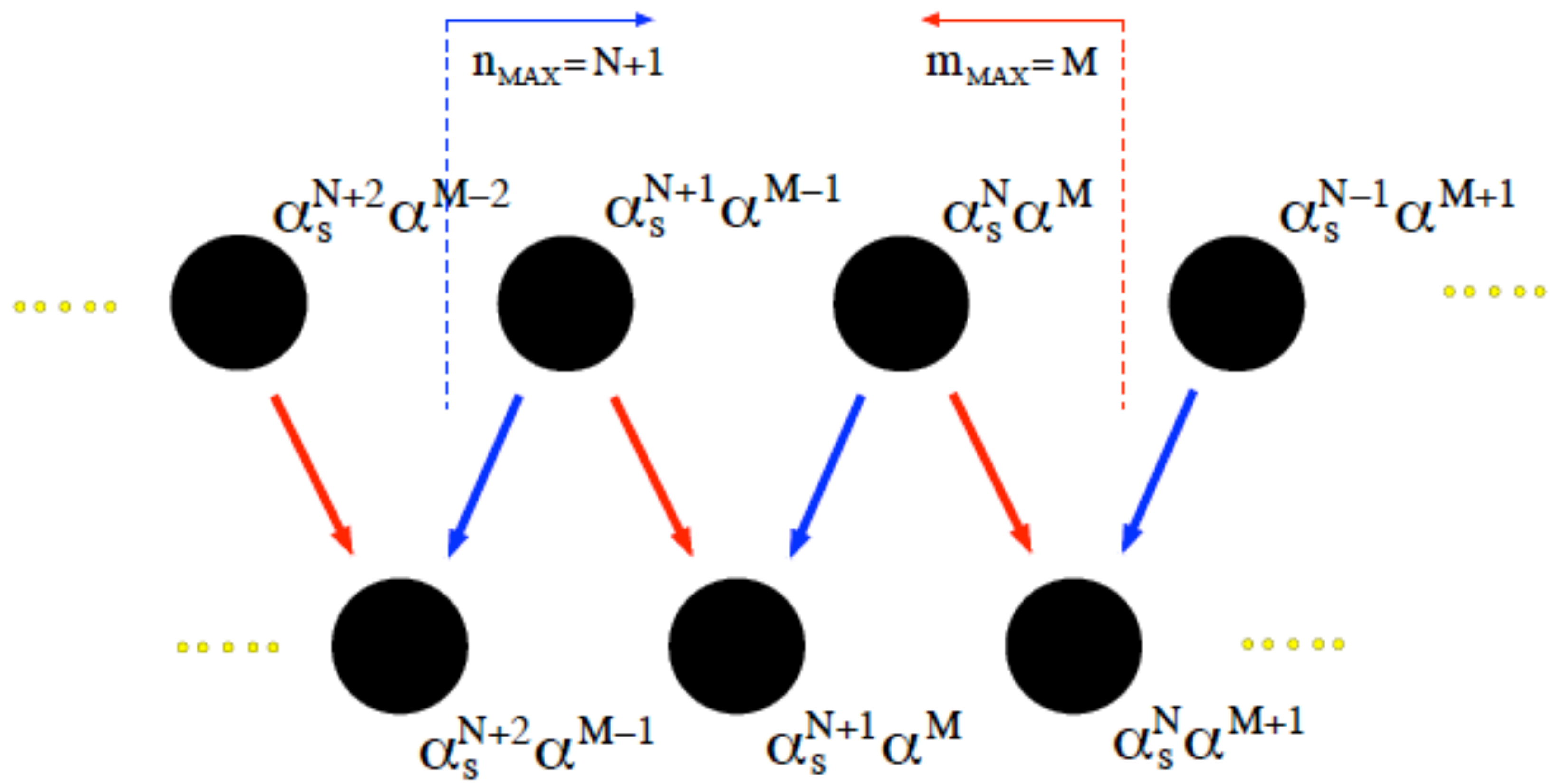
complete automation for QCD+EW

MADGRAPH5_AMC@NLO: COMPLETE NLO

- Generation syntax for any LO and NLO (in v3.X):

Frederix et al. (JHEP'18)

MG5_aMC> generate p1 p2 > p3 p4 p5 p6 QCD=$\mathtt{n_{max}}$ QED=$\mathtt{m_{max}}$ [QCD QED]



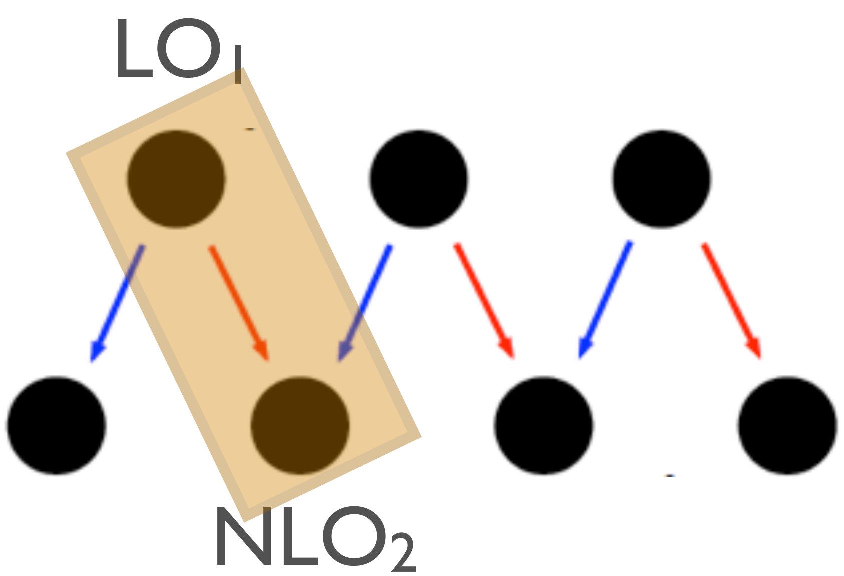
$$\text{LO}: \quad \alpha_s^n\alpha^m, \quad n \le \mathtt{n_{max}}, \quad m \le \mathtt{m_{max}}, \quad n+m = k_0,$$

$$\text{NLO}: \quad \alpha_s^n\alpha^m, \quad n \le \mathtt{n_{max}}+1, \quad m \le \mathtt{m_{max}}+1, \quad n+m = k_0+1$$

Caveat: new generation syntax at http://amcatnlo.web.cern.ch/amcatnlo/co.htm

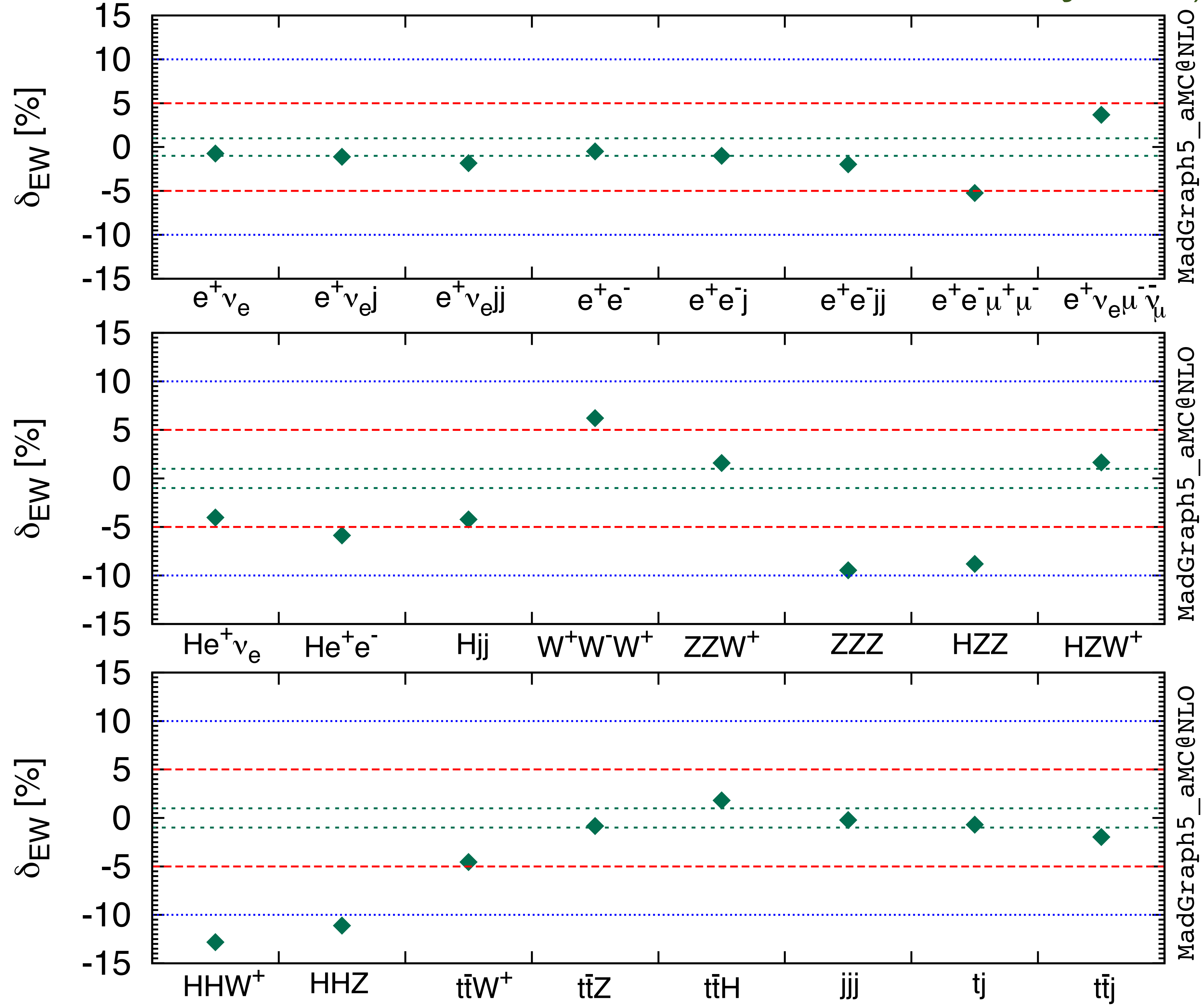
MADGRAPH5_AMC@NLO: NLO EW

- Examples:



$$\delta_{\mathrm{EW}} = \frac{\mathrm{NLO}_2}{\mathrm{LO}_1}$$

Frederix et al. (JHEP'18)



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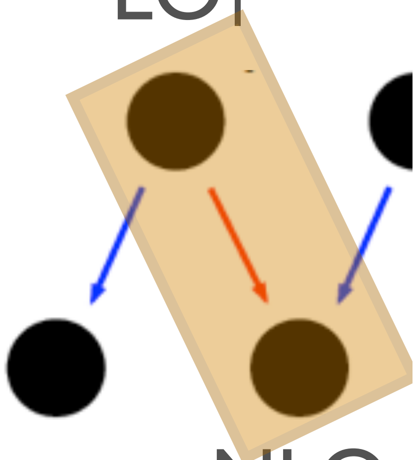
MADGRAPH5_AMC@NLO: NLO EW

- Examples:

Frederix et al. (JHEP'18)



$\delta_{\rm EW} =$

Process	Syntax	Cross section (in pb) LO	NLO	Correction (in %)
$pp \to e^+\nu_e$	`p p > e+ ve QCD=0 QED=2 [QED]`	$5.2498 \pm 0.0005 \cdot 10^3$	$5.2113 \pm 0.0006 \cdot 10^3$	-0.73 ± 0.01
$pp \to e^+\nu_e j$	`p p > e+ ve j QCD=1 QED=2 [QED]`	$9.1468 \pm 0.0012 \cdot 10^2$	$9.0449 \pm 0.0014 \cdot 10^2$	-1.11 ± 0.02
$pp \to e^+\nu_e jj$	`p p > e+ ve j j QCD=2 QED=2 [QED]`	$3.1562 \pm 0.0003 \cdot 10^2$	$3.0985 \pm 0.0005 \cdot 10^2$	-1.83 ± 0.02
$pp \to e^+e^-$	`p p > e+ e- QCD=0 QED=2 [QED]`	$7.5367 \pm 0.0008 \cdot 10^2$	$7.4997 \pm 0.0010 \cdot 10^2$	-0.49 ± 0.02
$pp \to e^+e^- j$	`p p > e+ e- j QCD=1 QED=2 [QED]`	$1.5059 \pm 0.0001 \cdot 10^2$	$1.4909 \pm 0.0002 \cdot 10^2$	-1.00 ± 0.02
$pp \to e^+e^- jj$	`p p > e+ e- j j QCD=2 QED=2 [QED]`	$5.1424 \pm 0.0004 \cdot 10^1$	$5.0410 \pm 0.0007 \cdot 10^1$	-1.97 ± 0.02
$pp \to e^+e^-\mu^+\mu^-$	`p p > e+ e- mu+ mu- QCD=0 QED=4 [QED]`	$1.2750 \pm 0.0000 \cdot 10^{-2}$	$1.2083 \pm 0.0001 \cdot 10^{-2}$	-5.23 ± 0.01
$pp \to e^+\nu_e\mu^-\bar{\nu}_\mu$	`p p > e+ ve mu- vm~ QCD=0 QED=4 [QED]`	$5.1144 \pm 0.0007 \cdot 10^{-1}$	$5.3019 \pm 0.0009 \cdot 10^{-1}$	$+3.67 \pm 0.02$
$pp \to He^+\nu_e$	`p p > h e+ ve QCD=0 QED=3 [QED]`	$6.7643 \pm 0.0001 \cdot 10^{-2}$	$6.4914 \pm 0.0012 \cdot 10^{-2}$	-4.03 ± 0.02
$pp \to He^+e^-$	`p p > h e+ e- QCD=0 QED=3 [QED]`	$1.4554 \pm 0.0001 \cdot 10^{-2}$	$1.3700 \pm 0.0002 \cdot 10^{-2}$	-5.87 ± 0.02
$pp \to Hjj$	`p p > h j j QCD=0 QED=3 [QED]`	$2.8268 \pm 0.0002 \cdot 10^0$	$2.7075 \pm 0.0003 \cdot 10^0$	-4.22 ± 0.01
$pp \to W^+W^-W^+$	`p p > w+ w- w+ QCD=0 QED=3 [QED]`	$8.2874 \pm 0.0004 \cdot 10^{-2}$	$8.8017 \pm 0.0012 \cdot 10^{-2}$	$+6.21 \pm 0.02$
$pp \to ZZW^+$	`p p > z z w+ QCD=0 QED=3 [QED]`	$1.9874 \pm 0.0001 \cdot 10^{-2}$	$2.0189 \pm 0.0003 \cdot 10^{-2}$	$+1.58 \pm 0.02$
$pp \to ZZZ$	`p p > z z z QCD=0 QED=3 [QED]`	$1.0761 \pm 0.0001 \cdot 10^{-2}$	$0.9741 \pm 0.0001 \cdot 10^{-2}$	-9.47 ± 0.02
$pp \to HZZ$	`p p > h z z QCD=0 QED=3 [QED]`	$2.1005 \pm 0.0003 \cdot 10^{-3}$	$1.9155 \pm 0.0003 \cdot 10^{-3}$	-8.81 ± 0.02
$pp \to HZW^+$	`p p > h z w+ QCD=0 QED=3 [QED]`	$2.4408 \pm 0.0000 \cdot 10^{-3}$	$2.4809 \pm 0.0005 \cdot 10^{-3}$	$+1.64 \pm 0.02$
$pp \to HHW^+$	`p p > h h w+ QCD=0 QED=3 [QED]`	$2.7827 \pm 0.0001 \cdot 10^{-4}$	$2.4259 \pm 0.0027 \cdot 10^{-4}$	-12.82 ± 0.10
$pp \to HHZ$	`p p > h h z QCD=0 QED=3 [QED]`	$2.6914 \pm 0.0003 \cdot 10^{-4}$	$2.3926 \pm 0.0003 \cdot 10^{-4}$	-11.10 ± 0.02
$pp \to t\bar{t}W^+$	`p p > t t~ w+ QCD=2 QED=1 [QED]`	$2.4119 \pm 0.0003 \cdot 10^{-1}$	$2.3025 \pm 0.0003 \cdot 10^{-1}$	-4.54 ± 0.02
$pp \to t\bar{t}Z$	`p p > t t~ z QCD=2 QED=1 [QED]`	$5.0456 \pm 0.0006 \cdot 10^{-1}$	$5.0033 \pm 0.0007 \cdot 10^{-1}$	-0.84 ± 0.02
$pp \to t\bar{t}H$	`p p > t t~ h QCD=2 QED=1 [QED]`	$3.4480 \pm 0.0004 \cdot 10^{-1}$	$3.5102 \pm 0.0005 \cdot 10^{-1}$	$+1.81 \pm 0.02$
$pp \to t\bar{t}j$	`p p > t t j QCD=3 QED=0 [QED]`	$3.0277 \pm 0.0003 \cdot 10^2$	$2.9683 \pm 0.0004 \cdot 10^2$	-1.96 ± 0.02
$pp \to jjj$	`p p > j j j QCD=3 QED=0 [QED]`	$7.9639 \pm 0.0010 \cdot 10^6$	$7.9472 \pm 0.0011 \cdot 10^6$	-0.21 ± 0.02
$pp \to tj$	`p p > t j QCD=0 QED=2 [QED]`	$1.0613 \pm 0.0001 \cdot 10^2$	$1.0539 \pm 0.0001 \cdot 10^2$	-0.70 ± 0.02

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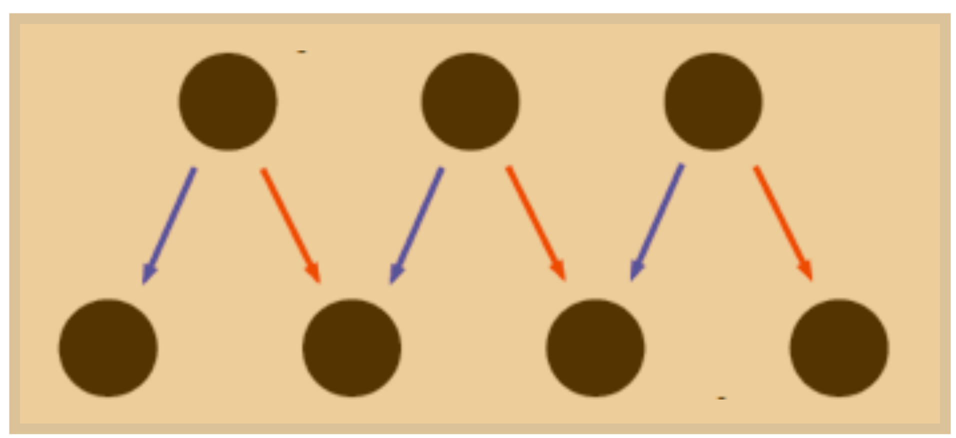
MADGRAPH5_AMC@NLO: COMPLETE NLO

- Examples:

Frederix et al. (JHEP'18)

	$pp \to t\bar{t}$	$pp \to t\bar{t}Z$	$pp \to t\bar{t}W^+$	$pp \to t\bar{t}H$	$pp \to t\bar{t}j$
LO_1	$4.3803 \pm 0.0005 \cdot 10^2$ pb	$5.0463 \pm 0.0003 \cdot 10^{-1}$ pb	$2.4116 \pm 0.0001 \cdot 10^{-1}$ pb	$3.4483 \pm 0.0003 \cdot 10^{-1}$ pb	$3.0278 \pm 0.0003 \cdot 10^2$ pb
LO_2	$+0.405 \pm 0.001$ %	-0.691 ± 0.001 %	$+0.000 \pm 0.000$ %	$+0.406 \pm 0.001$ %	$+0.525 \pm 0.001$ %
LO_3	$+0.630 \pm 0.001$ %	$+2.259 \pm 0.001$ %	$+0.962 \pm 0.000$ %	$+0.702 \pm 0.001$ %	$+1.208 \pm 0.001$ %
LO_4					$+0.006 \pm 0.000$ %
NLO_1	$+46.164 \pm 0.022$ %	$+44.809 \pm 0.028$ %	$+49.504 \pm 0.015$ %	$+28.847 \pm 0.020$ %	$+26.571 \pm 0.063$ %
NLO_2	-1.075 ± 0.003 %	-0.846 ± 0.004 %	-4.541 ± 0.003 %	$+1.794 \pm 0.005$ %	-1.971 ± 0.022 %
NLO_3	$+0.552 \pm 0.002$ %	$+0.845 \pm 0.003$ %	$+12.242 \pm 0.014$ %	$+0.483 \pm 0.008$ %	$+0.292 \pm 0.007$ %
NLO_4	$+0.005 \pm 0.000$ %	-0.082 ± 0.000 %	$+0.017 \pm 0.003$ %	$+0.044 \pm 0.000$ %	$+0.009 \pm 0.000$ %
NLO_5					$+0.005 \pm 0.000$ %



AUTOMATION TOOLS FOR EW CORRECTIONS

- Automation tools for QCD+EW on the market (so far)

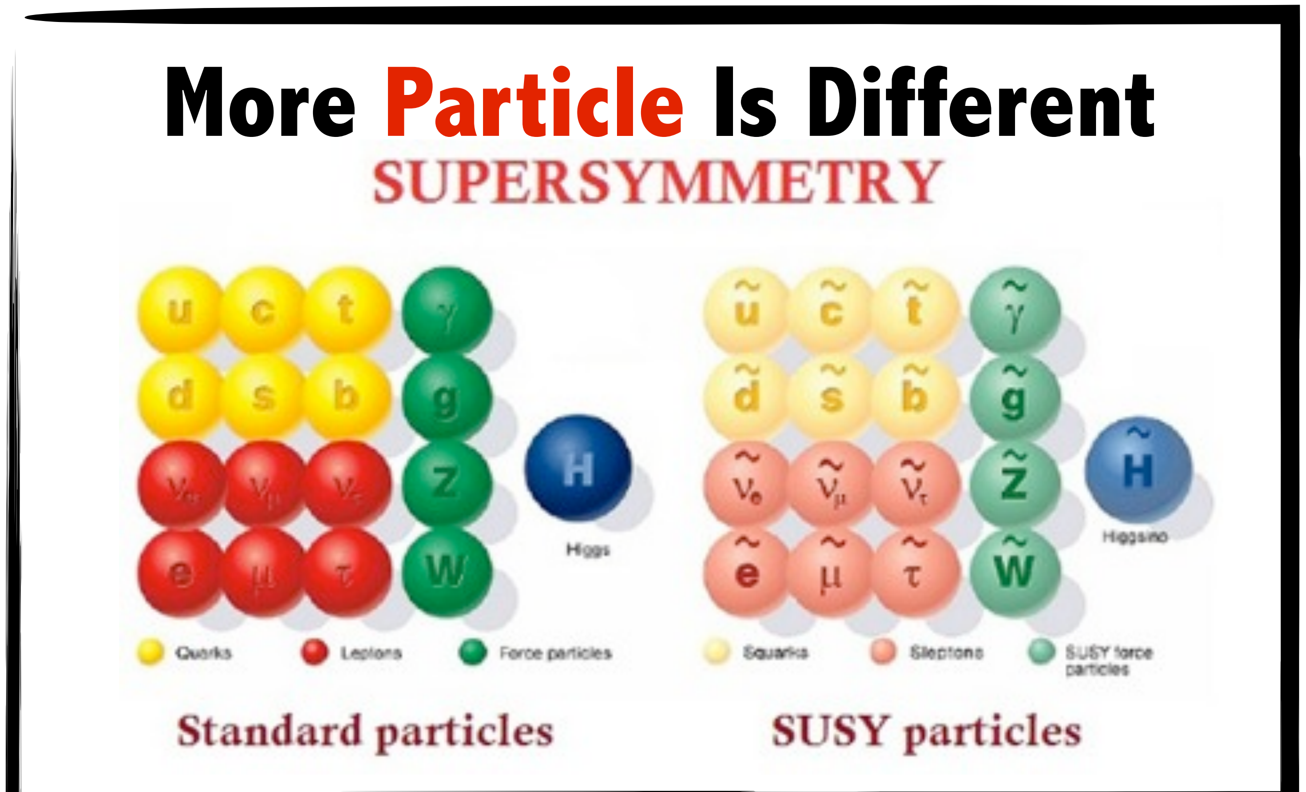
Les Houches SM report 2017 (1803.07977)

  - MadGraph5_aMC@NLO
  - Openloops+Sherpa/Munich
  - Recola+Sherpa/BBMC/MoCaNLO
  - GoSam+Sherpa
  - NLOX (only an one-loop provider)

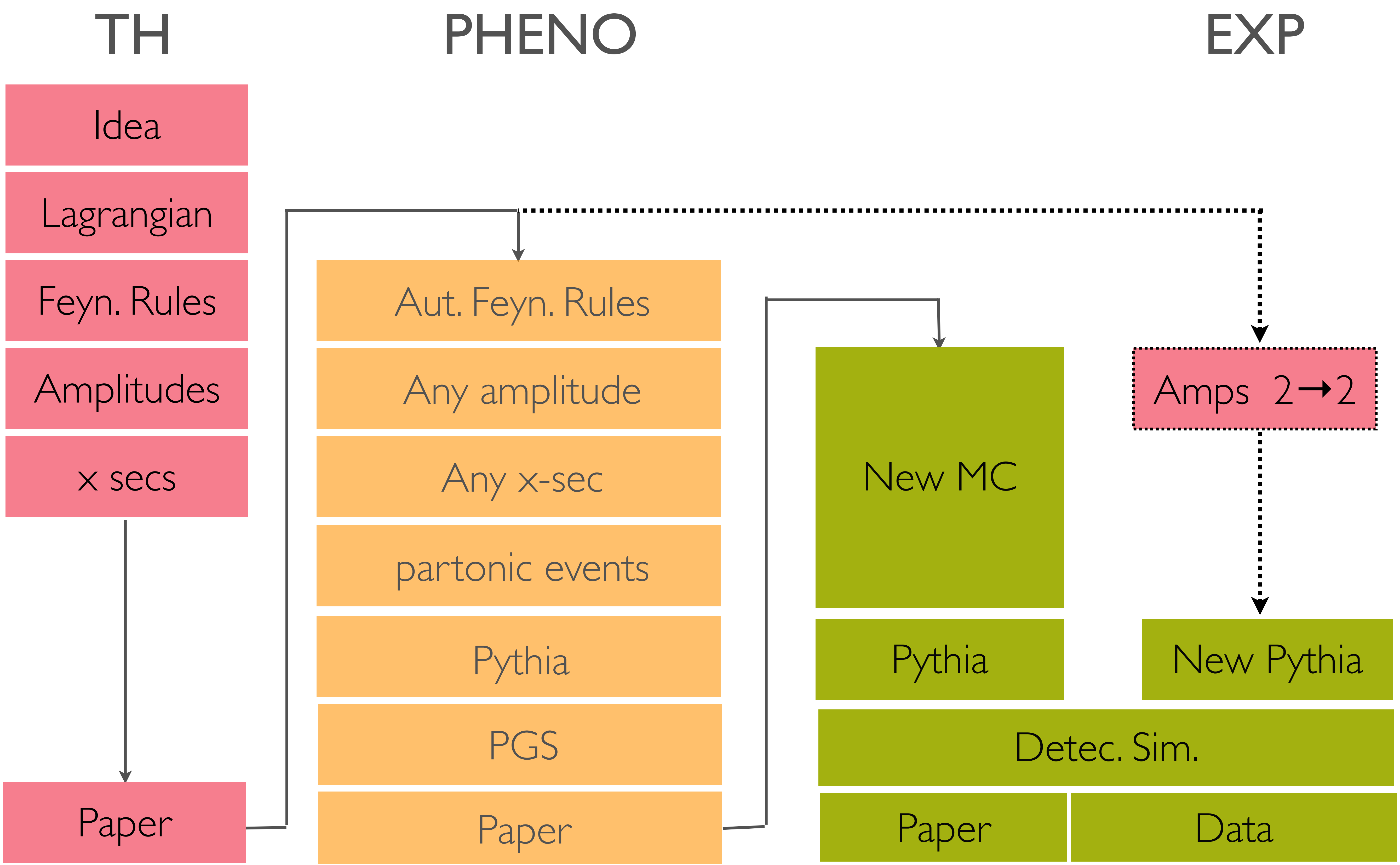
- Extensive validation among various tools is extremely important

$pp \to e^+e^-\mu^+\mu^-$	σ^{LO} [fb]	$\sigma^{\mathrm{NLO}}_{\mathrm{EW}}$ [fb]	$\Delta\sigma^{\mathrm{LO}}$ [σ]	$\Delta\sigma^{\mathrm{LO}}$ [‰]	$\Delta\sigma^{\mathrm{NLO}}_{\mathrm{EW}}$ [σ]	$\Delta\sigma^{\mathrm{NLO}}_{\mathrm{EW}}$ [‰]
average	11.49675[8]	10.88697[15]				
MCBB+Recola	11.49648[12]	10.88669[22]	−2.9	−0.02	−1.7	−0.03
Munich+OpenLoops	11.49702[11]	10.88720[25]	+3.2	+0.02	+1.2	+0.02
MoCaNLO+Recola	11.49666[26]	10.88734[56]	−0.3	−0.01	+0.7	+0.03
Sherpa+GoSam/OpenLoops/Recola	11.49670[34]	10.88737[77]	−0.1	−0.00	+0.5	+0.04
MadGraph5_aMC@NLO+MadLoop	11.4956[22]	10.8860[63]	−0.5	−0.10	−0.1	−0.09

A Few Advanced Topics



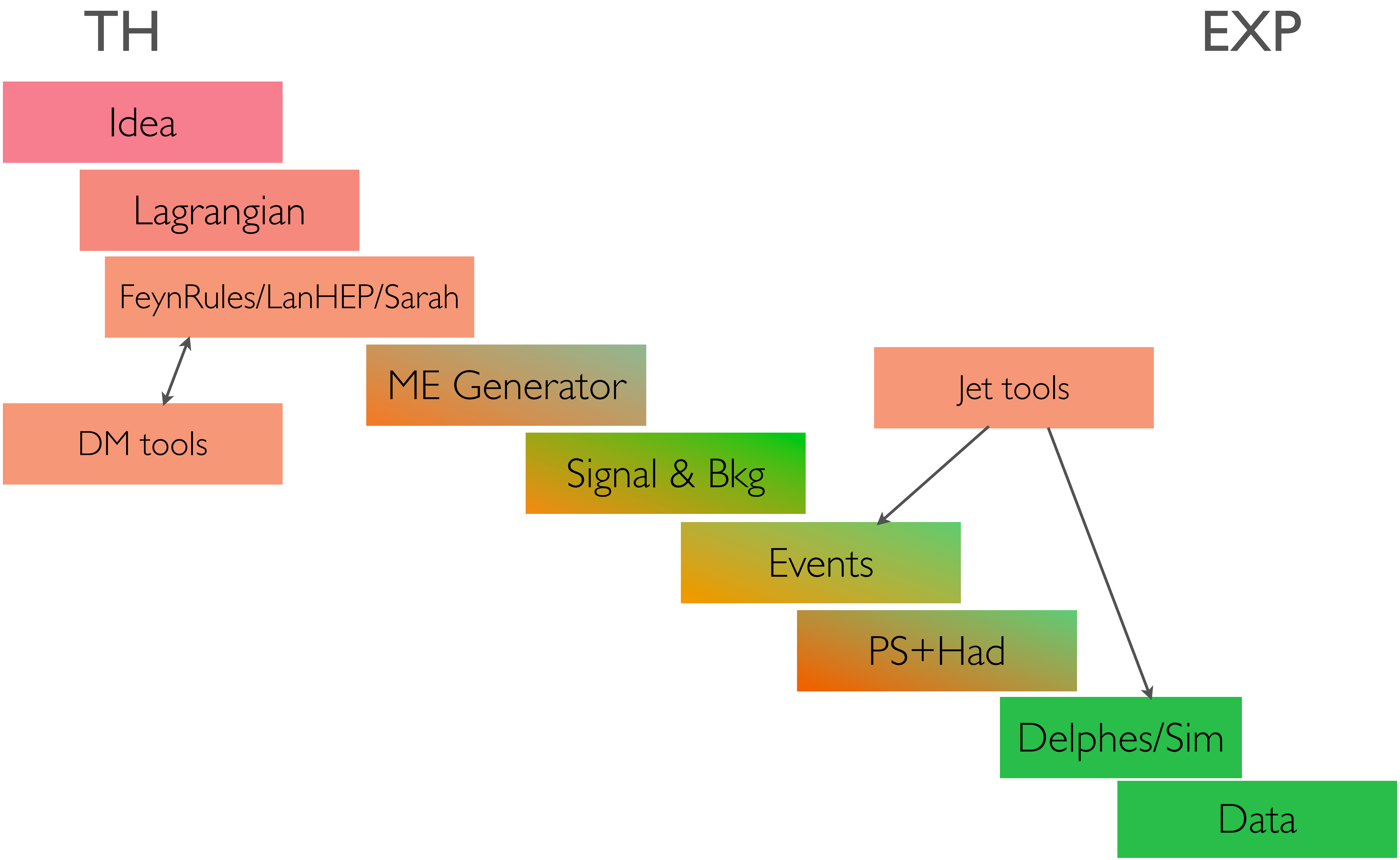
BSM TH/EXP INTERACTIONS: THE OLD WAY



BSM TH/EXP INTERACTIONS: THE OLD WAY

TH	PHENO	EXP
Idea		
Lagrangian		
	Aut. Feyn. Rules	
	Any amplitude	
	Any x-sec	
	partonic events	
		Pythia
		Detec. Sim.
		Data

BSM TH/EXP INTERACTIONS AUGMENTED



BSM TH/EXP INTERACTIONS AUGMENTED

TH

EXP

- One path for all
- Physics and software validations streamlined
- Robust and efficient Th/Exp communication
- It works top-down and bottom-up

Idea

Lagrangian

FeynRules/LanHEP/Sarah

DM tools

ME Generator

Signal & Bkg

Jet tools

Events

PS+Had

Delphes/Sim

Data

BSM TH/EXP INTERACTIONS AUGMENTED

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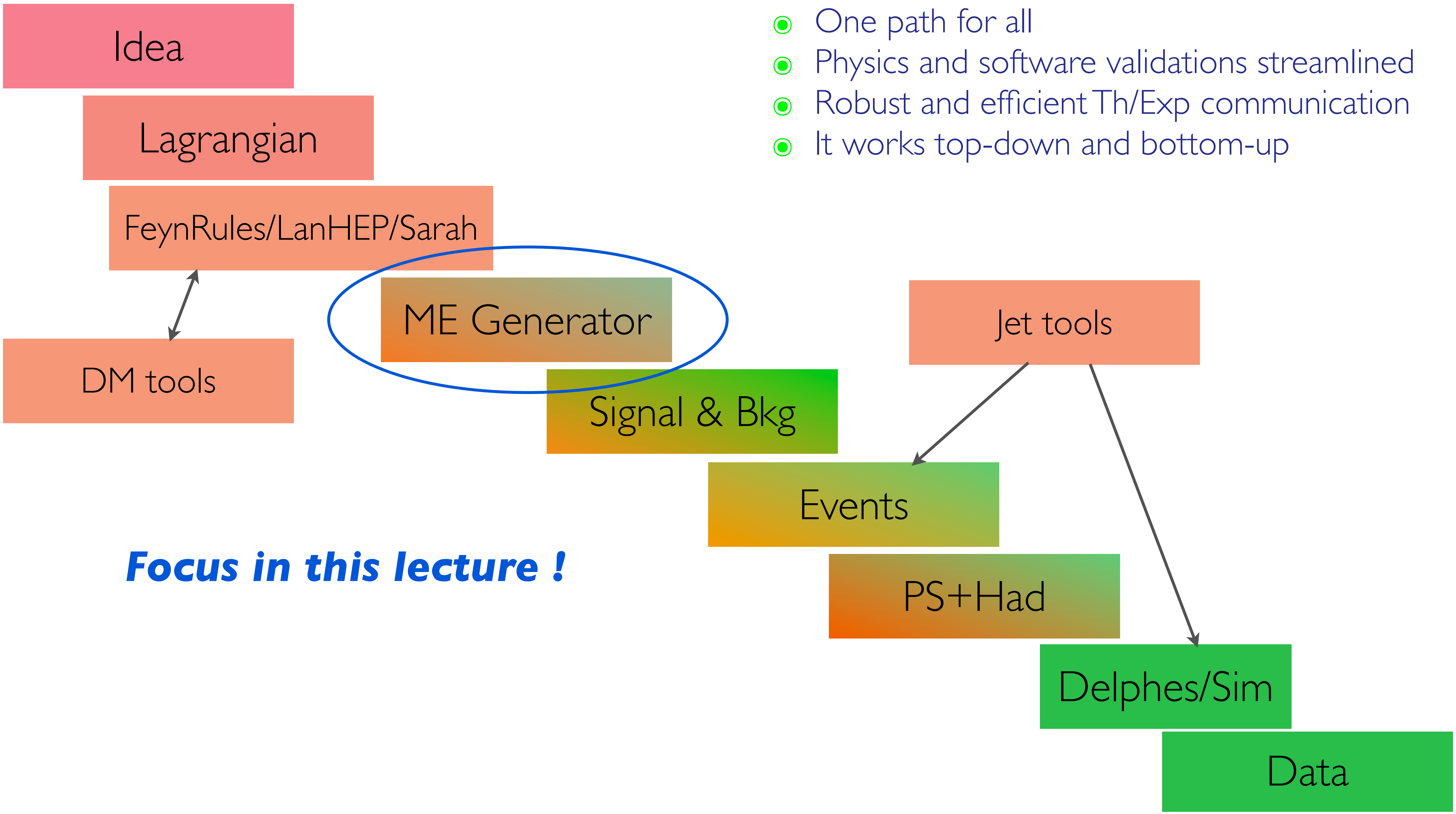


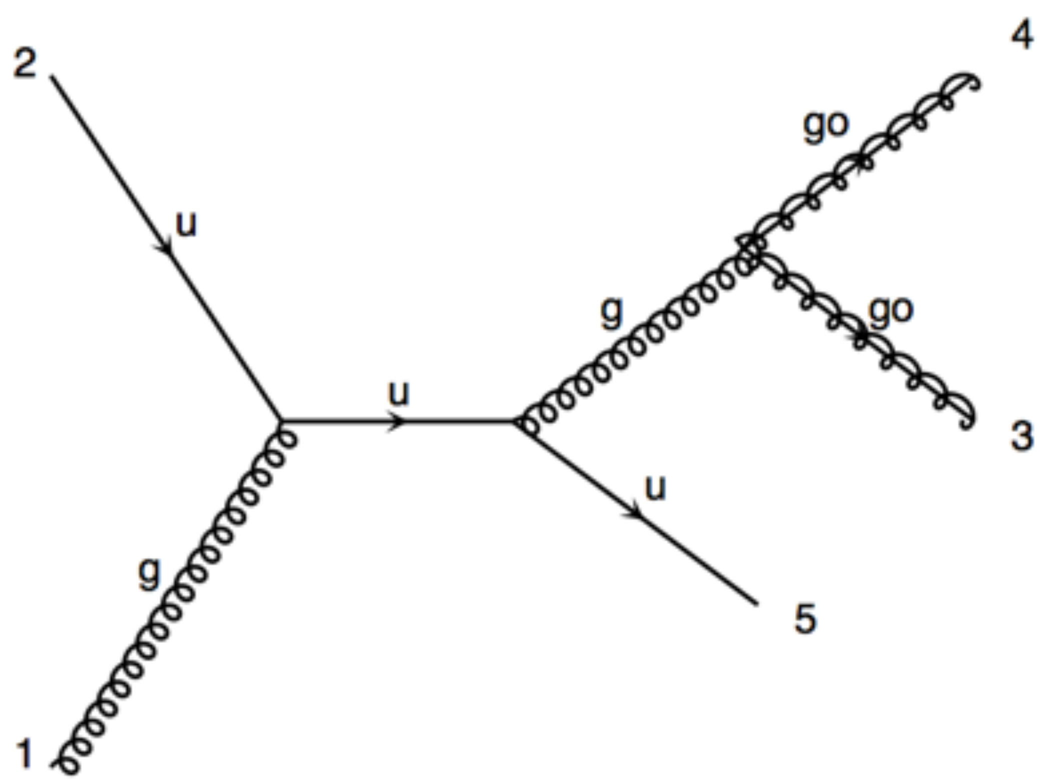
Focus in this lecture !

NEW ISSUES WITH RICH PARTICLE SPECTRUM

- **How to define final states at NLO without spoiling perturbative convergence ?**

NEW ISSUES WITH RICH PARTICLE SPECTRUM

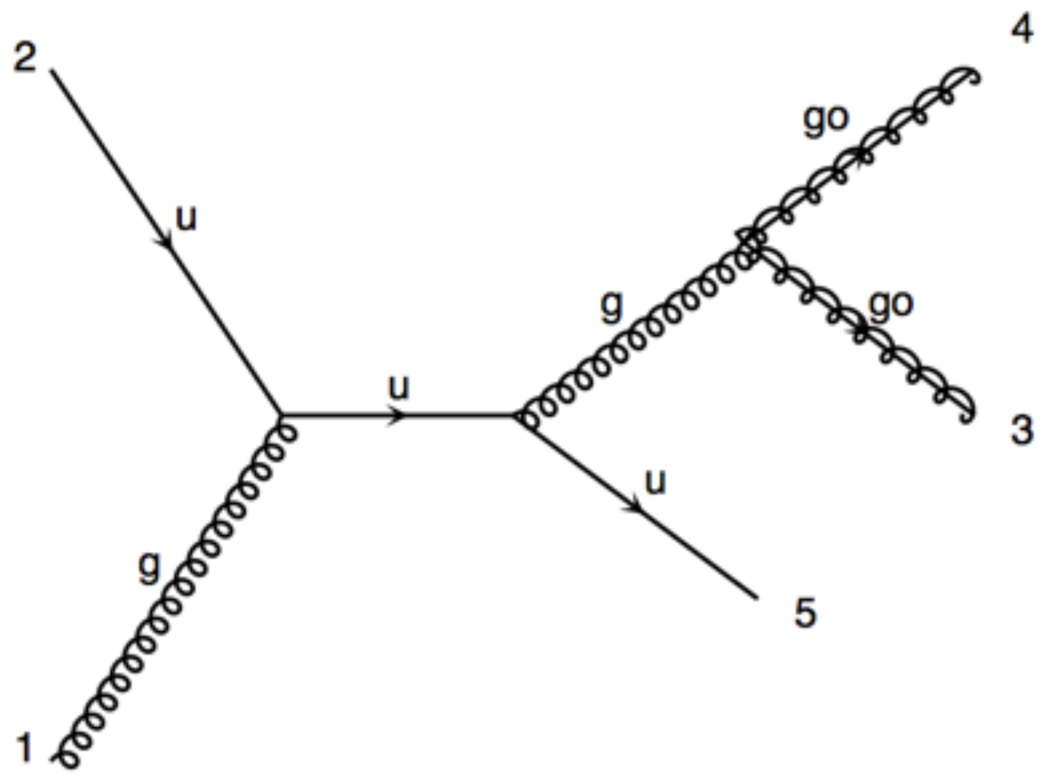
- **How to define final states at NLO without spoiling perturbative convergence ?**
  - Let us consider gluino pair production in SUSY

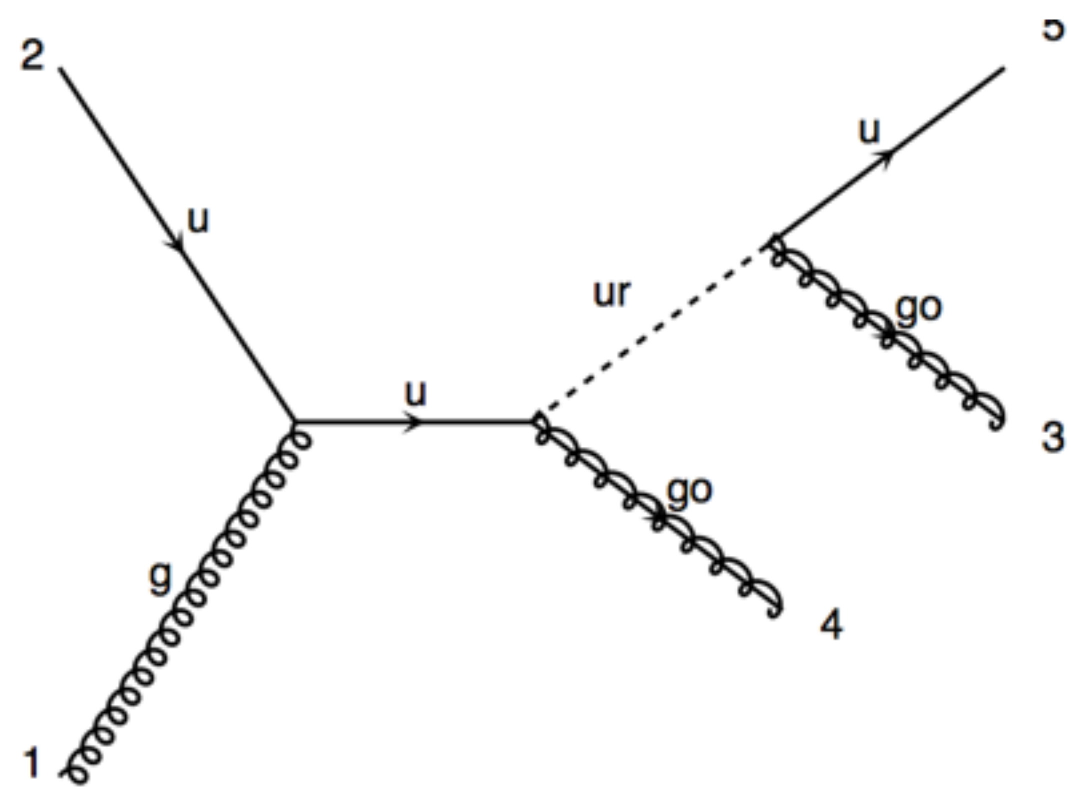


NLO diagram for gluino-pair

NEW ISSUES WITH RICH PARTICLE SPECTRUM

- **How to define final states at NLO without spoiling perturbative convergence ?**
  - **Let us consider gluino pair production in SUSY**





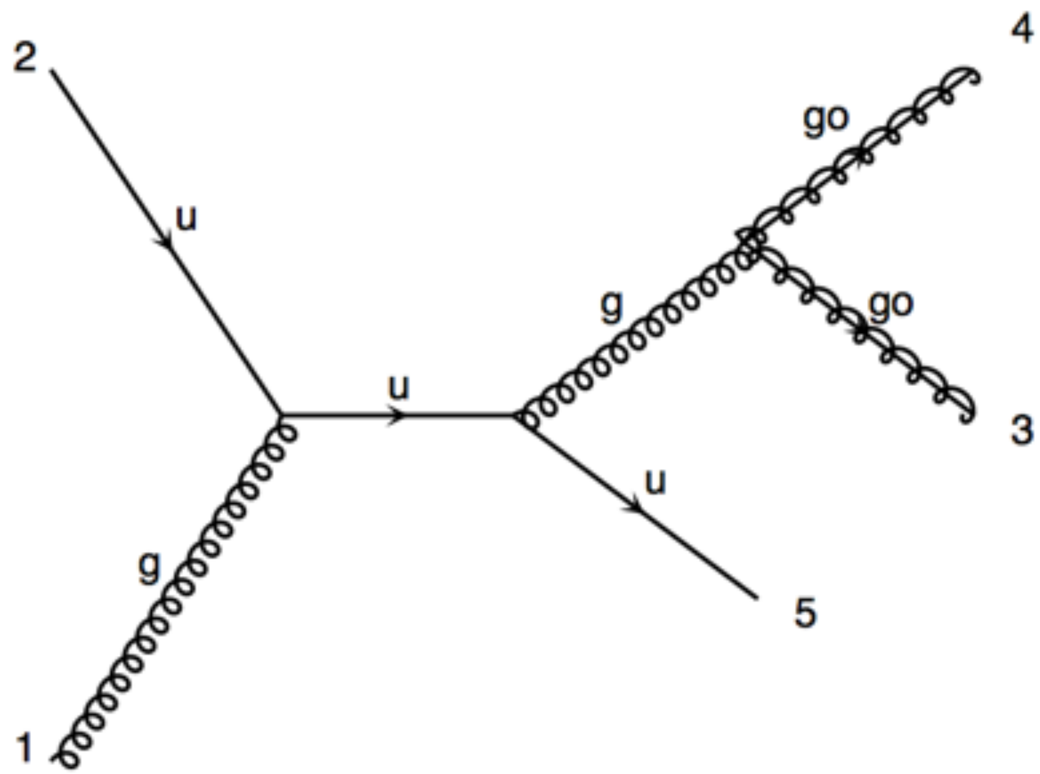
NLO diagram for gluino-pair

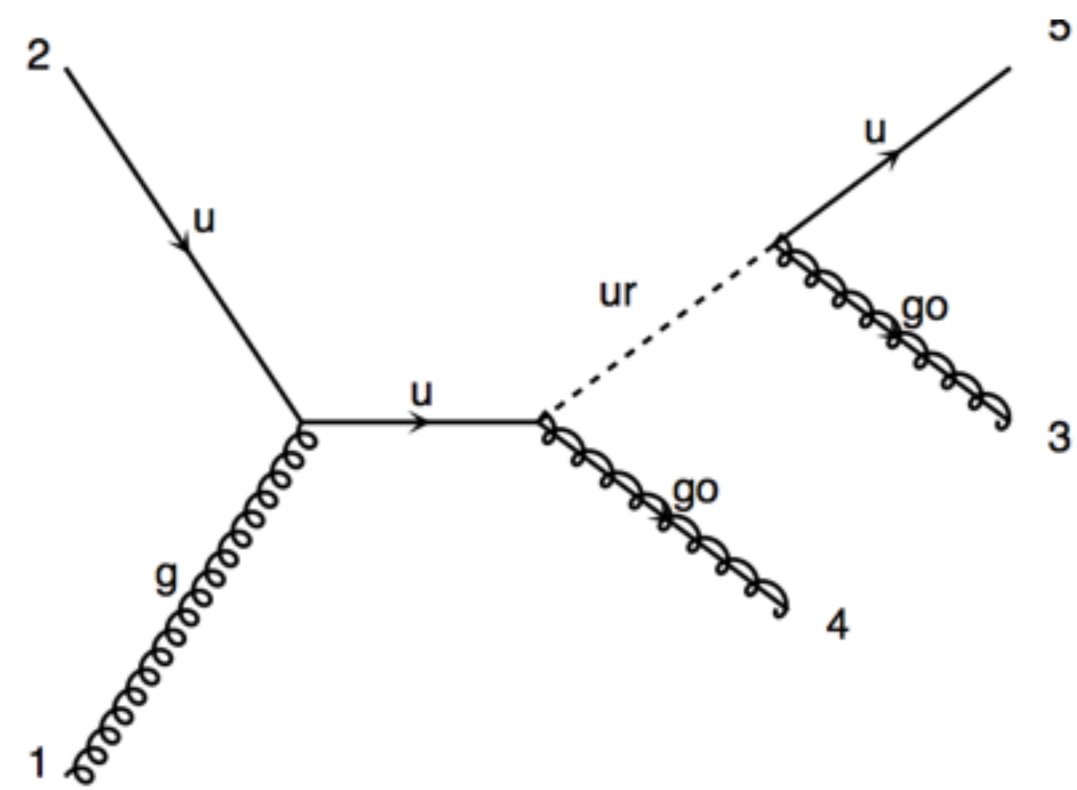
LO diagram for gluino-squark with squark decay

NEW ISSUES WITH RICH PARTICLE SPECTRUM

- **How to define final states at NLO without spoiling perturbative convergence ?**
  - **Let us consider gluino pair production in SUSY**

Frixione et al. (JHEP'19)





NLO diagram for gluino-pair **LO diagram for gluino-squark with squark decay**

Simplified Treatments of Resonances

SIMPLIFIED TREATMENTS OF RESONANCES

Frixione et al. (JHEP'19)

- The formulation of the problem is:

LO: $a + b \longrightarrow \delta + X$

NLO(Real): $a + b \longrightarrow \delta + \gamma + X$ with/without $\beta \longrightarrow \delta + \gamma$

$$\mathcal{A}_{ab\to\delta\gamma X} = \underline{\mathcal{A}^{(\not\beta)}_{ab\to\delta\gamma X}} + \underline{\mathcal{A}^{(\beta)}_{ab\to\delta\gamma X}}$$

non-resonance
resonance

$$\left|\mathcal{A}_{ab\to\delta\gamma X}\right|^2 = \left|\mathcal{A}^{(\not\beta)}_{ab\to\delta\gamma X}\right|^2 + 2\Re\left(\mathcal{A}^{(\not\beta)}_{ab\to\delta\gamma X}\mathcal{A}^{(\beta)\dagger}_{ab\to\delta\gamma X}\right) + \left|\mathcal{A}^{(\beta)}_{ab\to\delta\gamma X}\right|^2$$

- No fully satisfactory solutions but a few proposals:

Diagram Removal

istr=1 • DR: **remove the resonance diagrams/amplitude**

istr=2 • DRI: **remove the resonance amplitude squared**

Diagram Subtraction

$$d\sigma^{(\mathrm{DS})}_{ab\to\delta\gamma X} \propto \left\{\left|\mathcal{A}^{(\not\beta)}_{ab\to\delta\gamma X}\right|^2 + 2\Re\left(\mathcal{A}^{(\not\beta)}_{ab\to\delta\gamma X}\mathcal{A}^{(\beta)\dagger}_{ab\to\delta\gamma X}\right) + \left|\mathcal{A}^{(\beta)}_{ab\to\delta\gamma X}\right|^2\right\} d\phi$$
$$- \; f(m^2_{\delta\gamma})\,\mathbb{P}\left(\left|\mathcal{A}^{(\beta)}_{ab\to\delta\gamma X}\right|^2 d\phi\right), \quad (18)$$

DS subtraction term

istr=6 • DS-finalresh-runBW: **P (FS momenta reshuffling), f (ratio of two BWs with running width**

istr=4 • DS-initresh-runBW: **P (IS momenta reshuffling), f (ratio of two BWs with running width)**

istr=5 • DS-finalresh-stdBW: **P (FS momenta reshuffling), f (ratio of two standard BWs)**

istr=3 • DS-initresh-stdBW: **P (IS momenta reshuffling), f (ratio of two standard BWs)**

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- No fully satisfactory solutions but a few proposals:

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istr=1 • DR: remove the resonance diagrams/amplitude

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} Not gauge invariant

Diagram Subtraction

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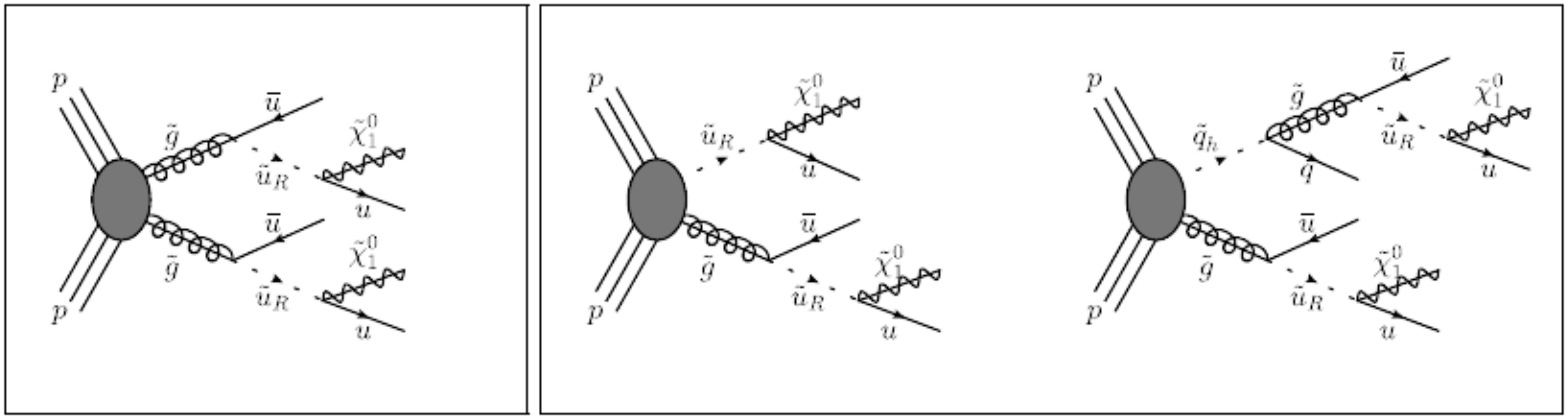
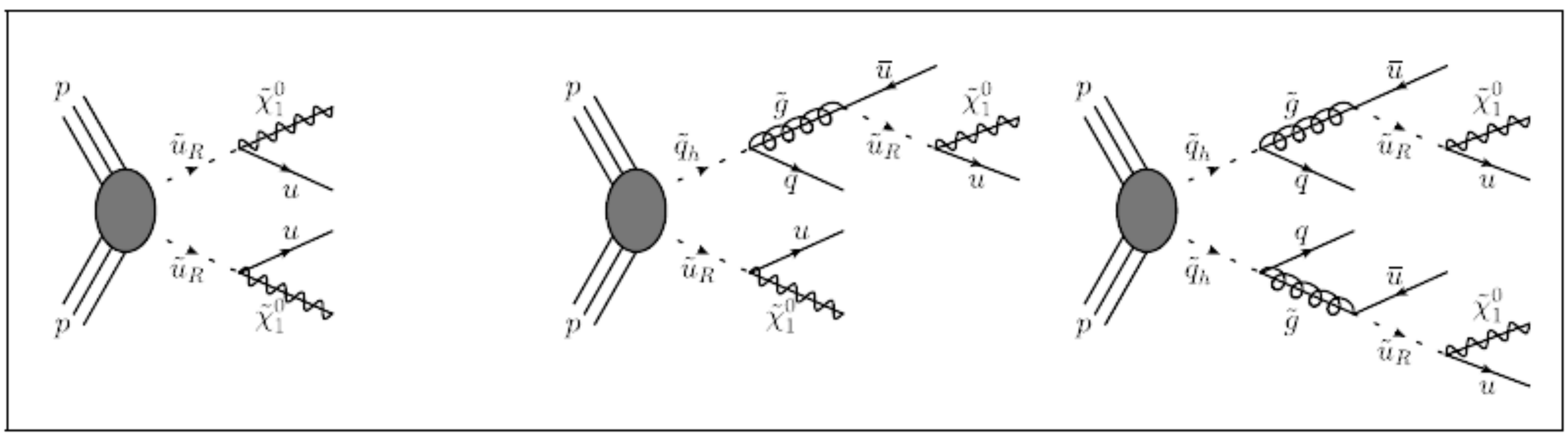
SIMPLIFIED TREATMENTS OF RESONANCES

Frixione et al. (JHEP'19)

- Jets plus missing Et $pp \longrightarrow nj + \not{E}_T$



(a) $\tilde{g}\tilde{g}$ (b) $\tilde{g}\tilde{q}$



(c) $\tilde{q}\tilde{q}$

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SIMPLIFIED TREATMENTS OF RESONANCES

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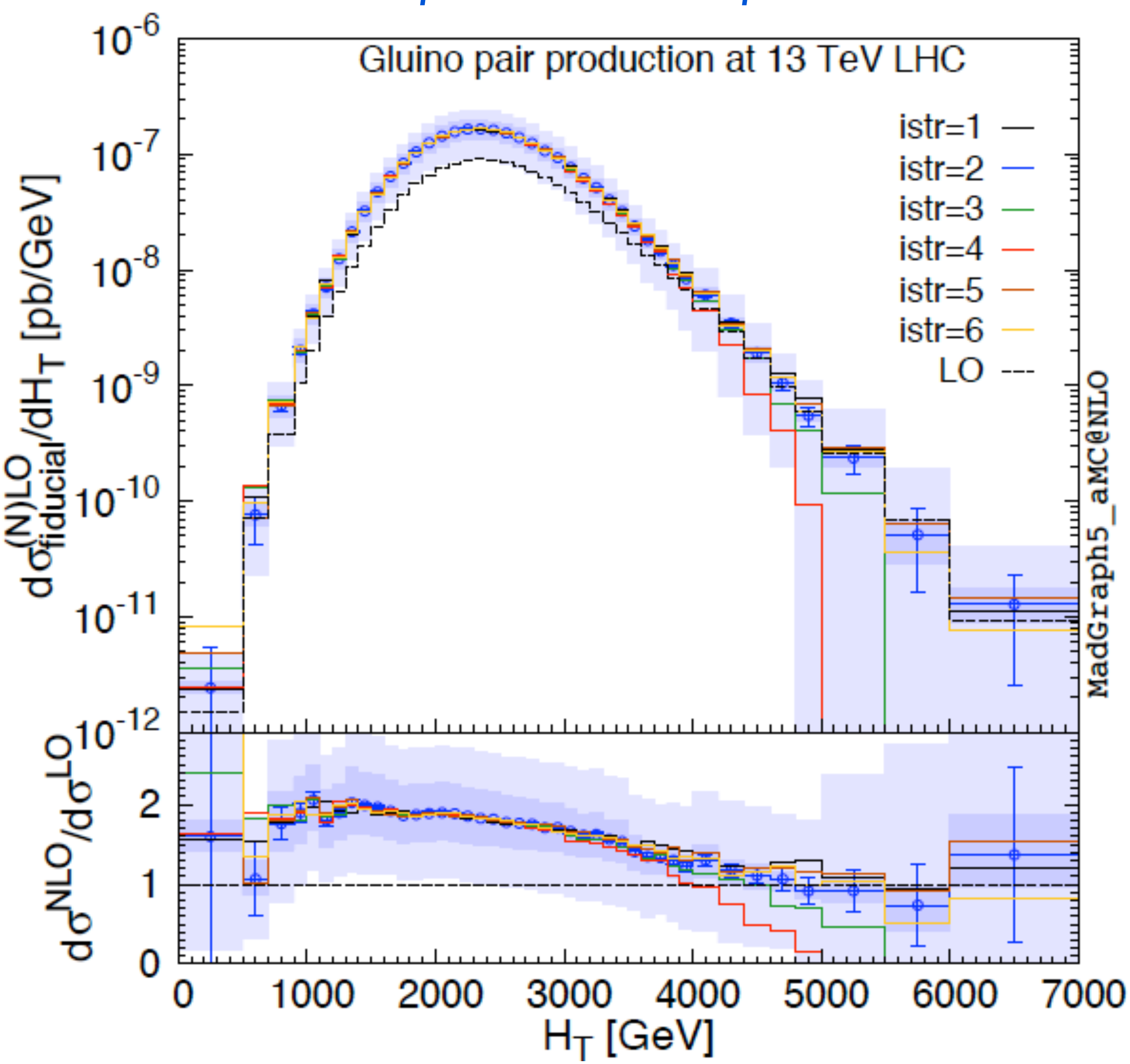
- Jets plus missing Et $pp \longrightarrow nj + \not{E}_T$

https://code.launchpad.net/~maddevelopers/mg5amcnlo/MadSTRPlugin



> ./bin/mg5_aMC --mode=MadSTR
> import model MSSMatNLO_UFO
> generate p p > go go [QCD]
> output; launch

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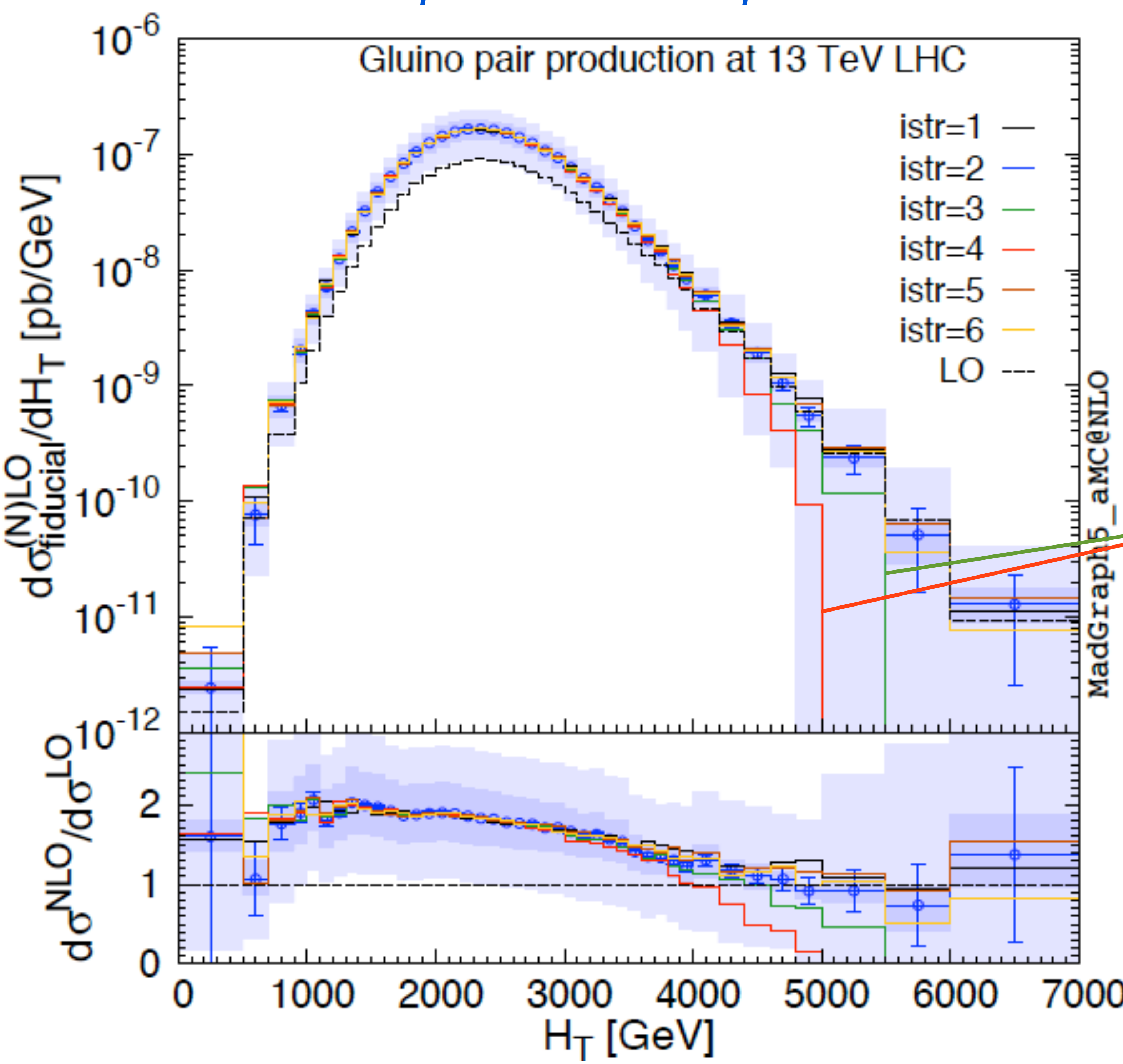

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Sensitive to large x PDF due to initial momenta reshuffling !

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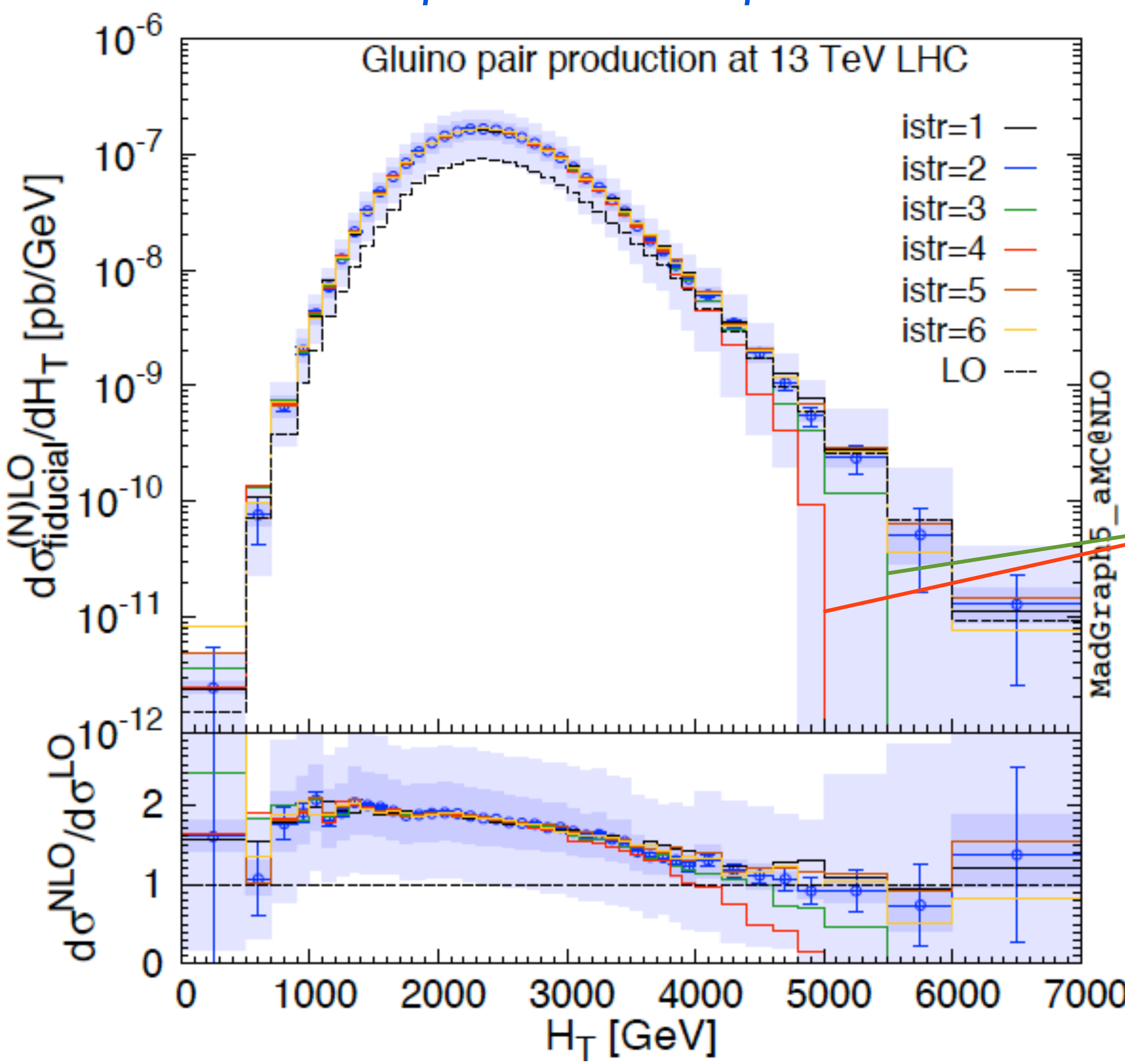

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Important to check the systematic dependencies !

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A Few Advanced Topics

More Operator Is Different

$$\mathcal{L}_{\rm SMEFT} = \mathcal{L}_{\rm SM} + \sum_i \frac{C_i^{(6)} O_i^{(6)}}{\Lambda^2} + \sum_i \frac{C_i^{(8)} O_i^{(8)}}{\Lambda^4} + \ldots$$

Evanescent 4f Operators

Slide by C. Zhang at QCD@LHC-X 2020

- 4F operators that vanish in D=4, need to be introduced so that Dirac algebra is complete and closed in D dimension.
- E.g $E = \left(\bar{u}\gamma^\mu\gamma^\nu\gamma^\rho P_L T^A u\right)\left(\bar{t}\gamma_\mu\gamma_\nu\gamma_\rho P_L T^A t\right)$ $+ (-16 + 4a\varepsilon)\left(\bar{u}\gamma^\mu P_L T^A u\right)\left(\bar{t}\gamma_\mu P_L T^A t\right)$
  - Which vanishes in 4D.
- Loop results can depend on how they are chosen (i.e. evanescent operator basis)
- It means that just "MSbar" is not sufficient to specify a renormalization scheme. Have to use, e.g. "MSbar + some evanescent basis".
- Other equivalent formulation exists. "Greek projections".

E.g. *Nucl. Phys.*, B586:397–426, 2000.

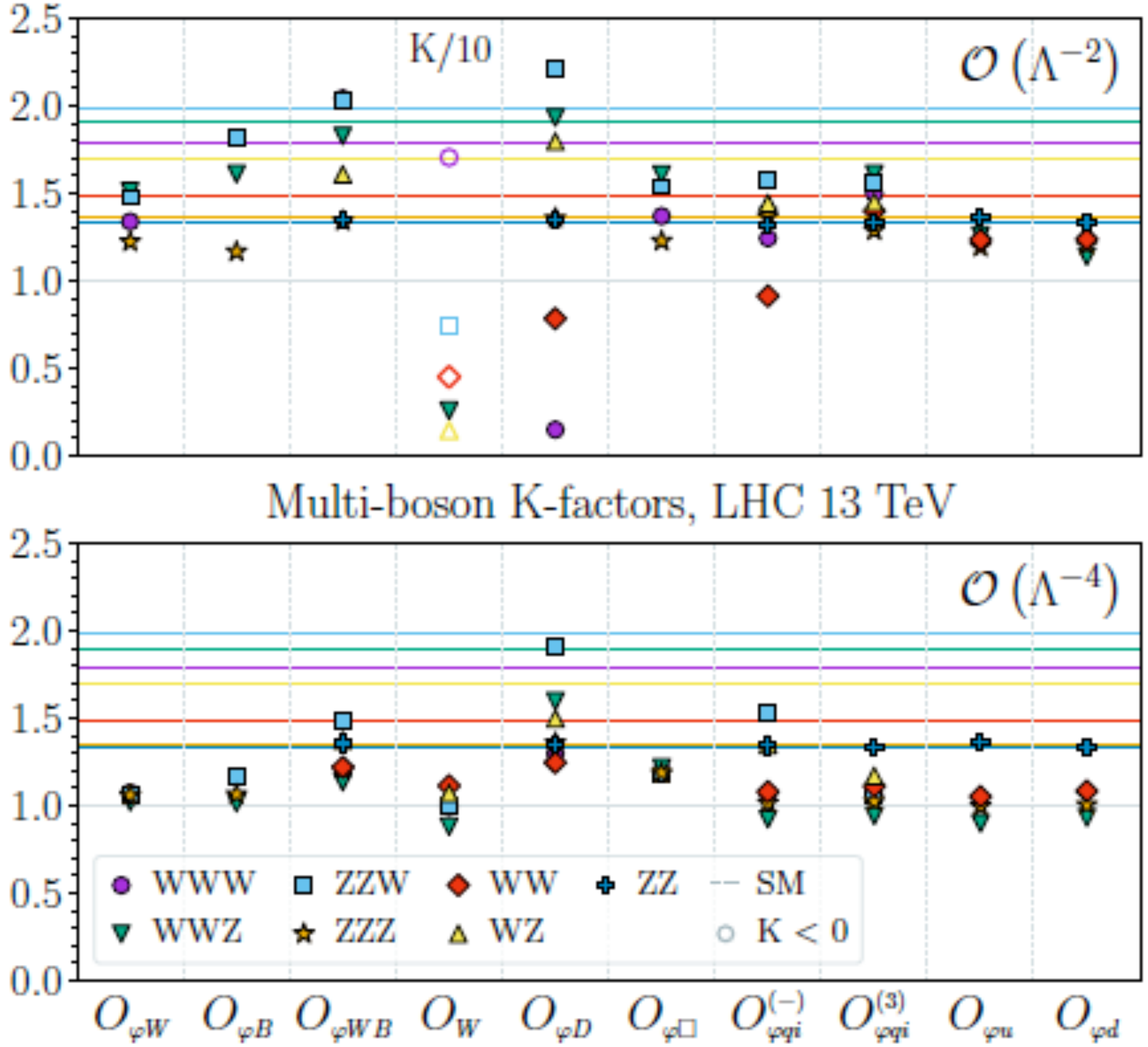
$$
\begin{aligned}
E_1^{\mathrm{VLL}} &= (\bar{s}^\alpha\gamma_\mu P_L d^\beta)(\bar{s}^\beta\gamma^\mu P_L d^\alpha) - Q_1^{\mathrm{VLL}},\\
E_2^{\mathrm{VLL}} &= (\bar{s}^\alpha\gamma_\mu\gamma_\nu\gamma_\rho P_L d^\alpha)(\bar{s}^\beta\gamma^\mu\gamma^\nu\gamma^\rho P_L d^\beta) + (-16+4\epsilon)Q_1^{\mathrm{VLL}},\\
E_3^{\mathrm{VLL}} &= (\bar{s}^\alpha\gamma_\mu\gamma_\nu\gamma_\rho P_L d^\beta)(\bar{s}^\beta\gamma^\mu\gamma^\nu\gamma^\rho P_L d^\alpha) + (-16+4\epsilon)Q_1^{\mathrm{VLL}},\\
E_1^{\mathrm{LR}} &= (\bar{s}^\alpha P_L d^\beta)(\bar{s}^\beta P_R d^\alpha) + \frac{1}{2}Q_1^{\mathrm{LR}},\\
E_2^{\mathrm{LR}} &= (\bar{s}^\alpha\gamma_\mu P_L d^\beta)(\bar{s}^\beta\gamma^\mu P_R d^\alpha) + 2Q_2^{\mathrm{LR}},\\
E_3^{\mathrm{LR}} &= (\bar{s}^\alpha\gamma_\mu\gamma_\nu\gamma_\rho P_L d^\alpha)(\bar{s}^\beta\gamma^\mu\gamma^\nu\gamma^\rho P_R d^\beta) + (-4-4\epsilon)Q_1^{\mathrm{LR}},\\
E_4^{\mathrm{LR}} &= (\bar{s}^\alpha\gamma_\mu\gamma_\nu\gamma_\rho P_L d^\beta)(\bar{s}^\beta\gamma^\mu\gamma^\nu\gamma^\rho P_R d^\alpha) + (8+8\epsilon)Q_2^{\mathrm{LR}},\\
E_5^{\mathrm{LR}} &= (\bar{s}^\alpha\sigma_{\mu\nu} P_L d^\alpha)(\bar{s}^\beta\sigma^{\mu\nu} P_R d^\beta) - 6\epsilon Q_2^{\mathrm{LR}},\\
E_6^{\mathrm{LR}} &= (\bar{s}^\alpha\sigma_{\mu\nu} P_L d^\beta)(\bar{s}^\beta\sigma^{\mu\nu} P_R d^\alpha) + 3\epsilon Q_1^{\mathrm{LR}},\\
E_1^{\mathrm{SLL}} &= (\bar{s}^\alpha P_L d^\beta)(\bar{s}^\beta P_L d^\alpha) + \frac{1}{2}Q_1^{\mathrm{SLL}} - \frac{1}{8}Q_2^{\mathrm{SLL}},\\
E_2^{\mathrm{SLL}} &= (\bar{s}^\alpha\sigma_{\mu\nu} P_L d^\beta)(\bar{s}^\beta\sigma^{\mu\nu} P_L d^\alpha) - 6Q_1^{\mathrm{SLL}} - \frac{1}{2}Q_2^{\mathrm{SLL}},\\
E_3^{\mathrm{SLL}} &= (\bar{s}^\alpha\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma P_L d^\alpha)(\bar{s}^\beta\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma P_L d^\beta) + (-64+96\epsilon)Q_1^{\mathrm{SLL}} + (-16+8\epsilon)Q_2^{\mathrm{SLL}},\\
E_4^{\mathrm{SLL}} &= (\bar{s}^\alpha\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma P_L d^\beta)(\bar{s}^\beta\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma P_L d^\alpha) - 64Q_1^{\mathrm{SLL}} + (-16+16\epsilon)Q_2^{\mathrm{SLL}}.
\end{aligned}
$$

$$
\begin{aligned}
\gamma^\mu\gamma^\nu\gamma^\rho P_L \otimes \gamma_\mu\gamma_\nu\gamma_\rho P_L &= E + (16 - 4a\varepsilon)\gamma^\mu P_L \otimes \gamma_\mu P_L\\
\gamma^\mu\gamma^\nu\gamma^\rho P_L \otimes \gamma_\rho\gamma_\nu\gamma_\mu P_L &= -E + [4 - (12 - 4a)\varepsilon]\gamma^\mu P_L \otimes \gamma_\mu P_L
\end{aligned}
$$

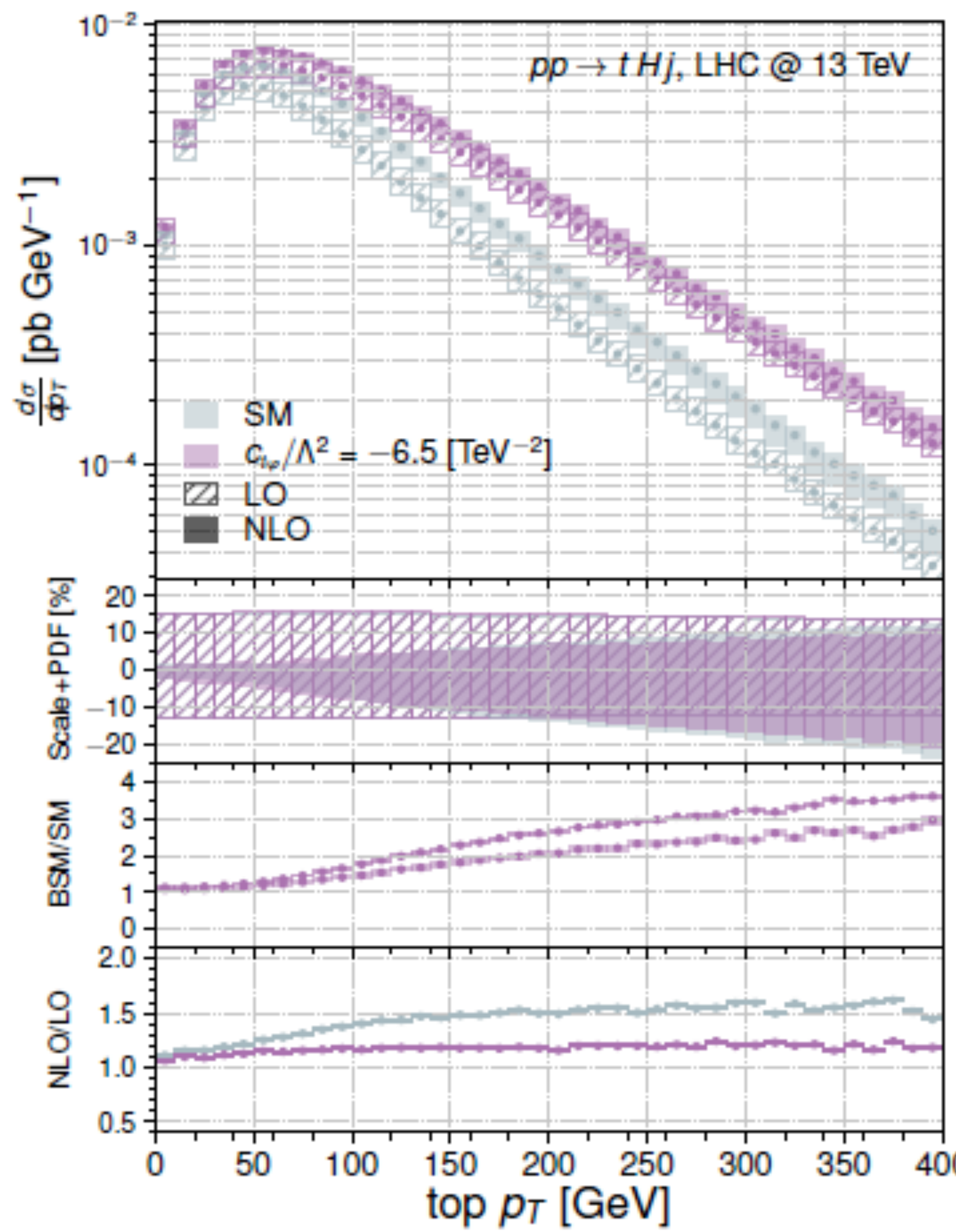
NLO HELPS SMEFT@LHC

- **Precision and accuracy**

Degrande et al. (PRD'21)



Degrande et al. (JHEP'18)



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Large, negative K factors: Non-interference/cancellation at LO breaks at NLO

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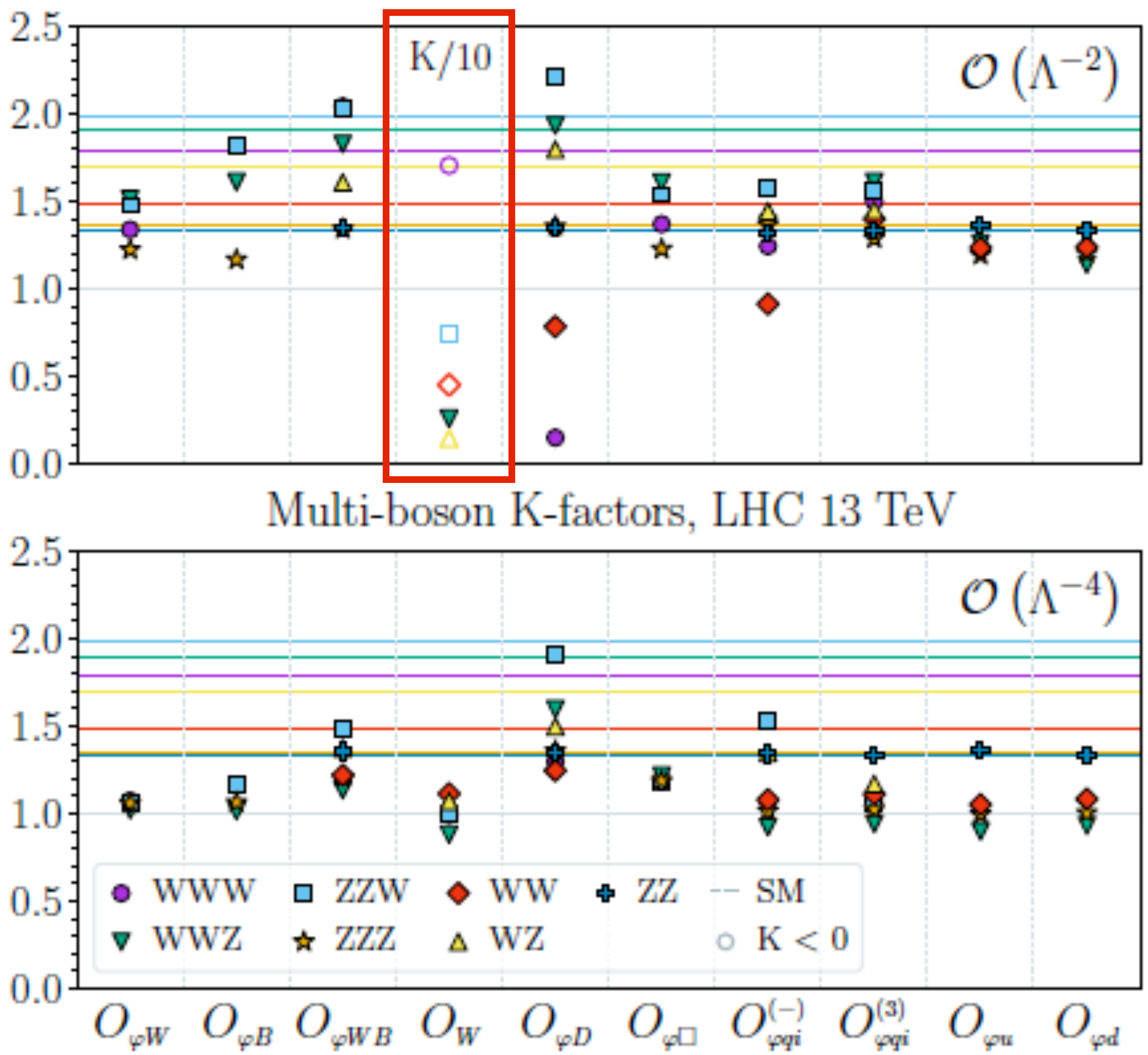

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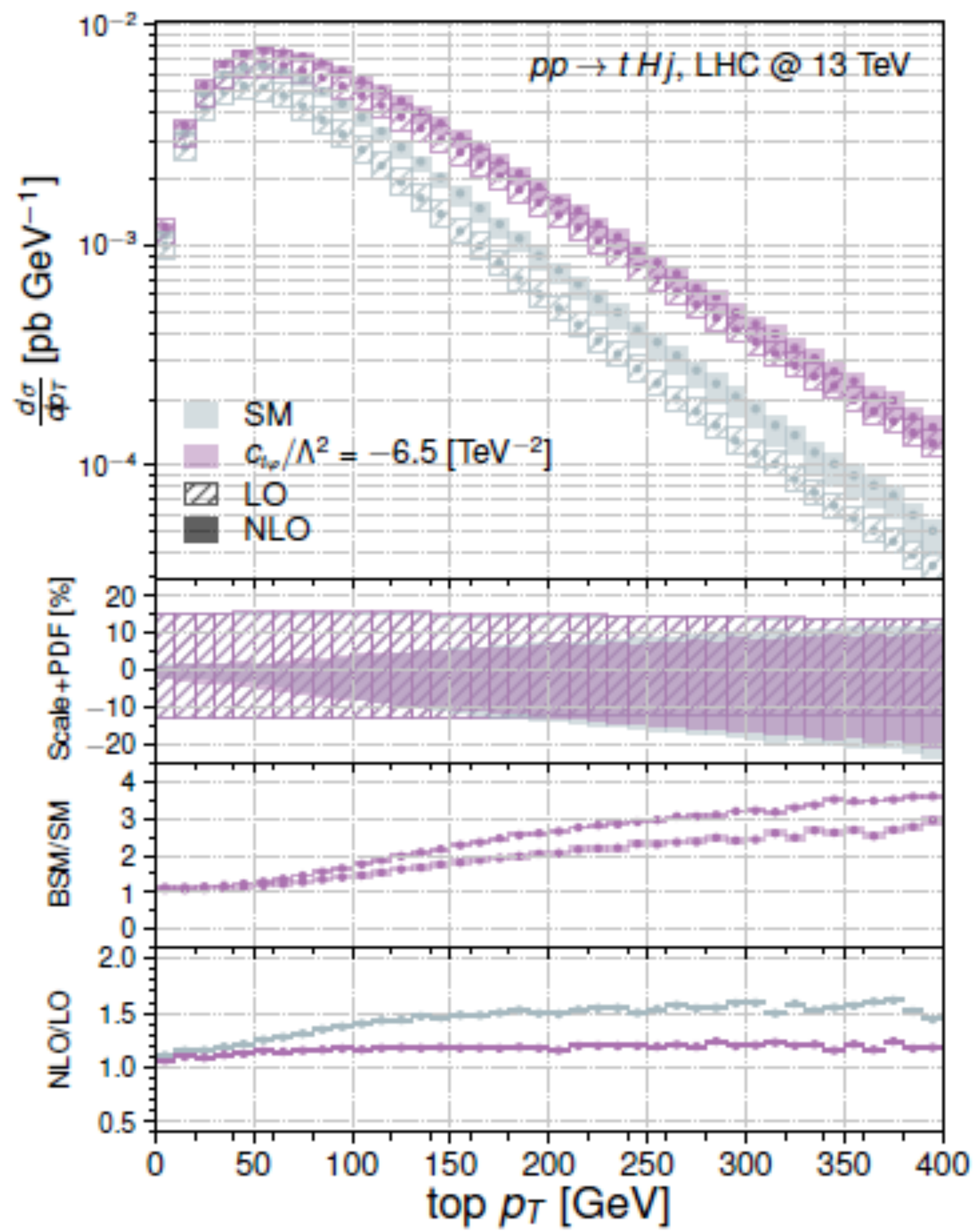

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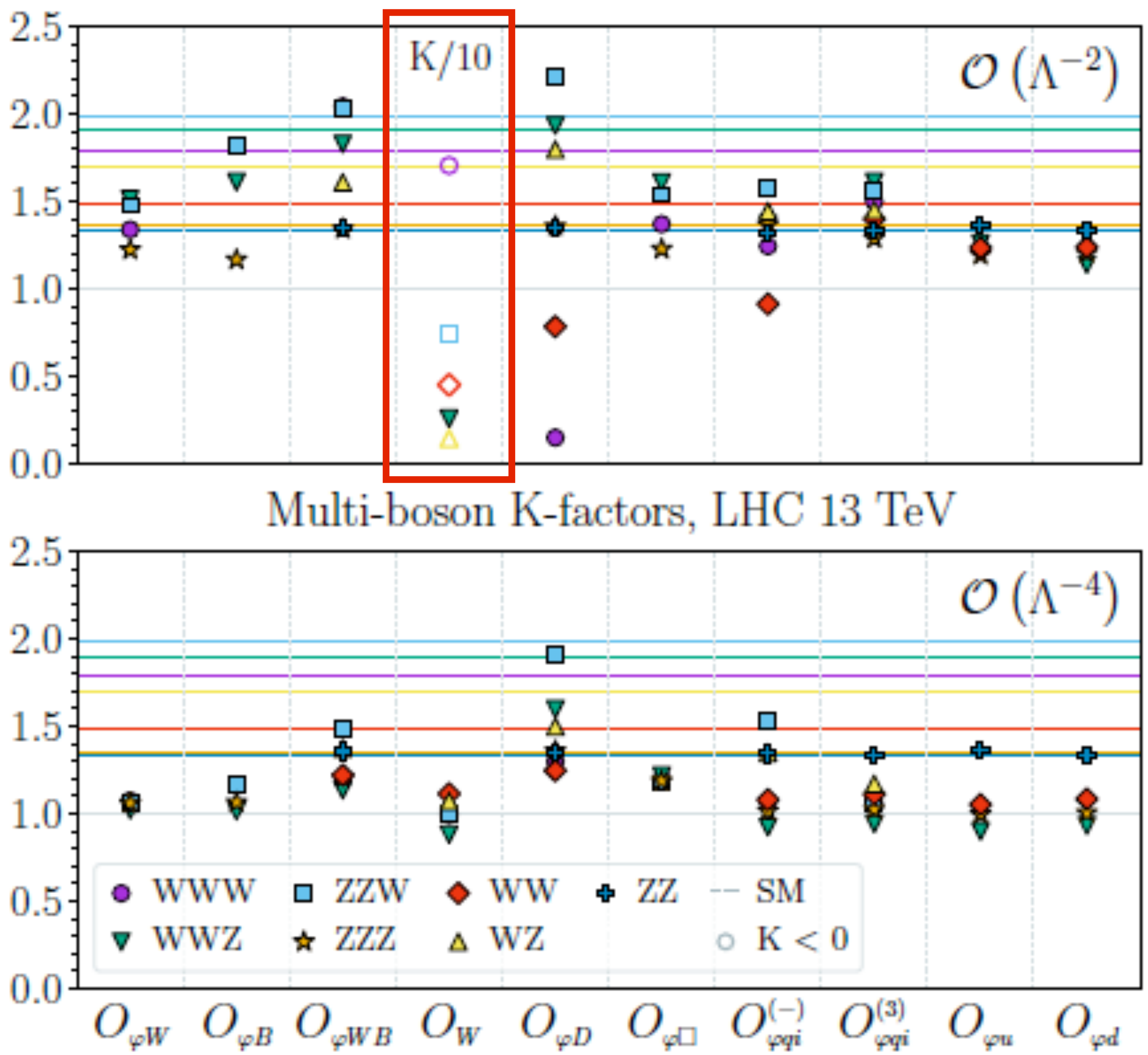

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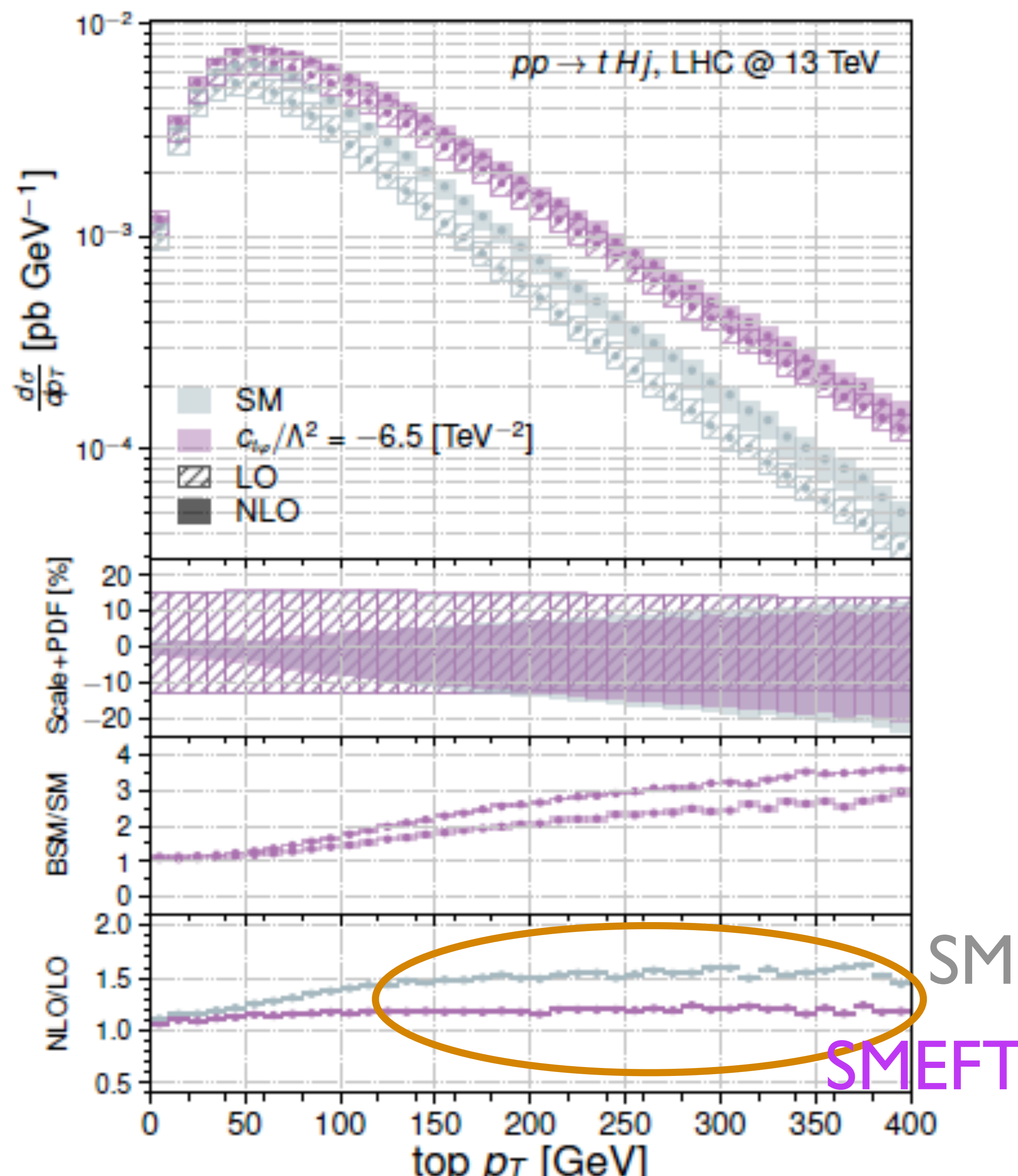

Different and non-uniform K factors

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NLO HELPS SMEFT@LHC

- **Improved sensitivity**
  - New operators opening up at NLO

4-heavy operators in top pair production

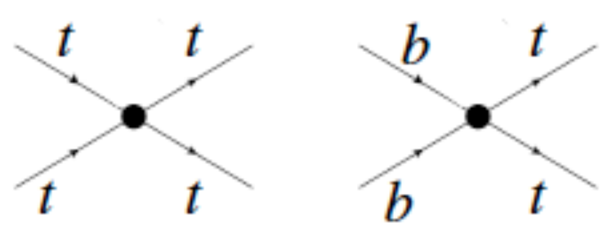
$$\mathcal{O}^{8}_{QQ} = (\bar{Q}\gamma^{\mu}T^{A}Q)(\bar{Q}\gamma_{\mu}T^{A}Q)$$

$$\mathcal{O}^{1}_{QQ} = (\bar{Q}\gamma^{\mu}Q)(\bar{Q}\gamma_{\mu}Q)$$

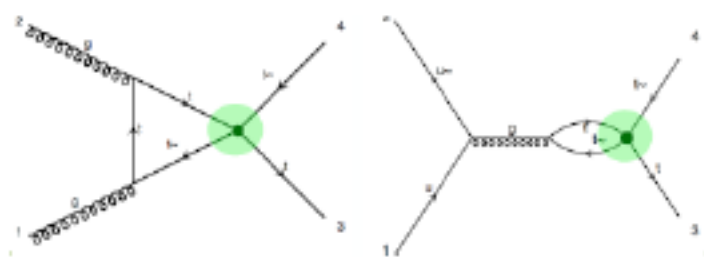
$$\mathcal{O}^{8}_{Qt} = (\bar{Q}\gamma^{\mu}T^{A}Q)(\bar{t}\gamma_{\mu}T^{A}t)$$

$$\mathcal{O}^{1}_{Qt} = (\bar{Q}\gamma^{\mu}Q)(\bar{t}\gamma_{\mu}t)$$

$$\mathcal{O}^{1}_{tt} = (\bar{t}\gamma^{\mu}t)(\bar{t}\gamma_{\mu}t)$$



At NLO:



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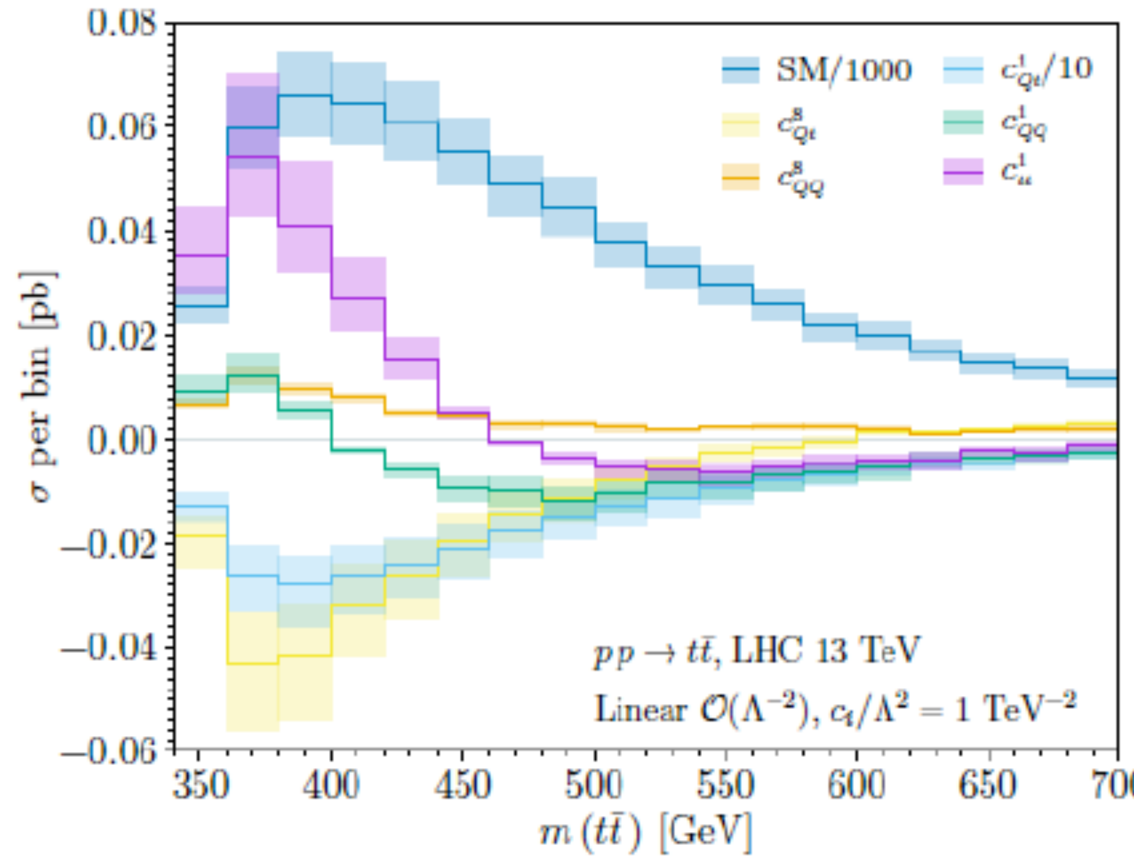

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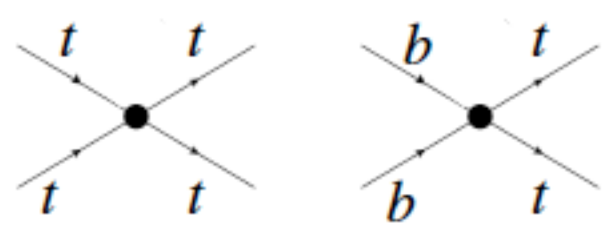

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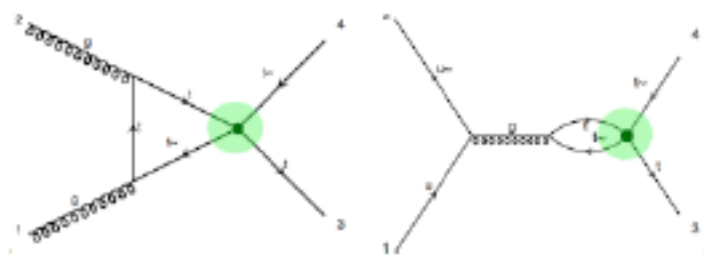

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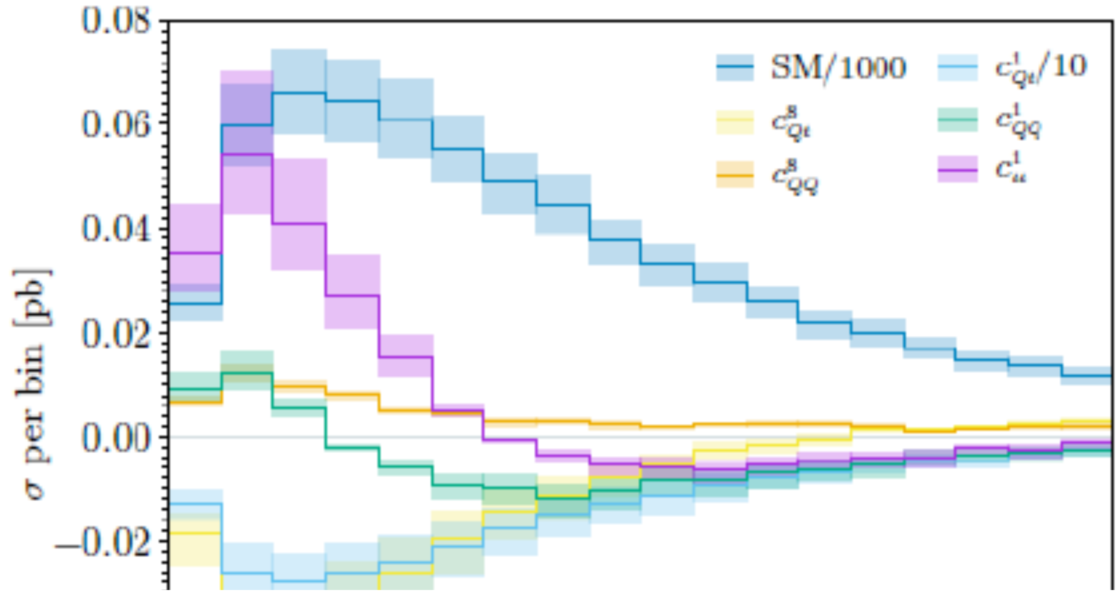
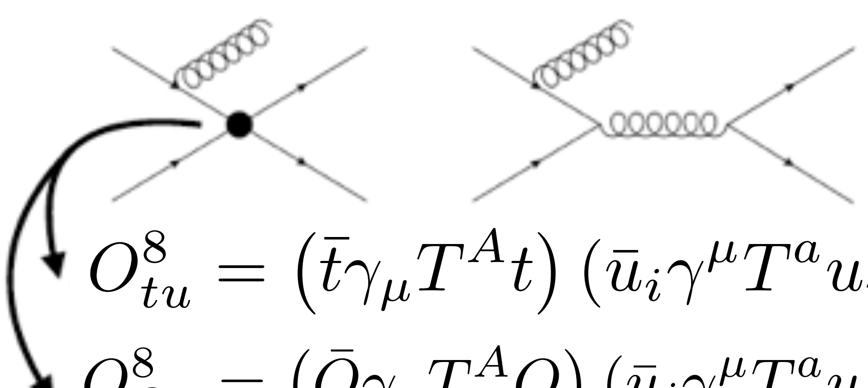
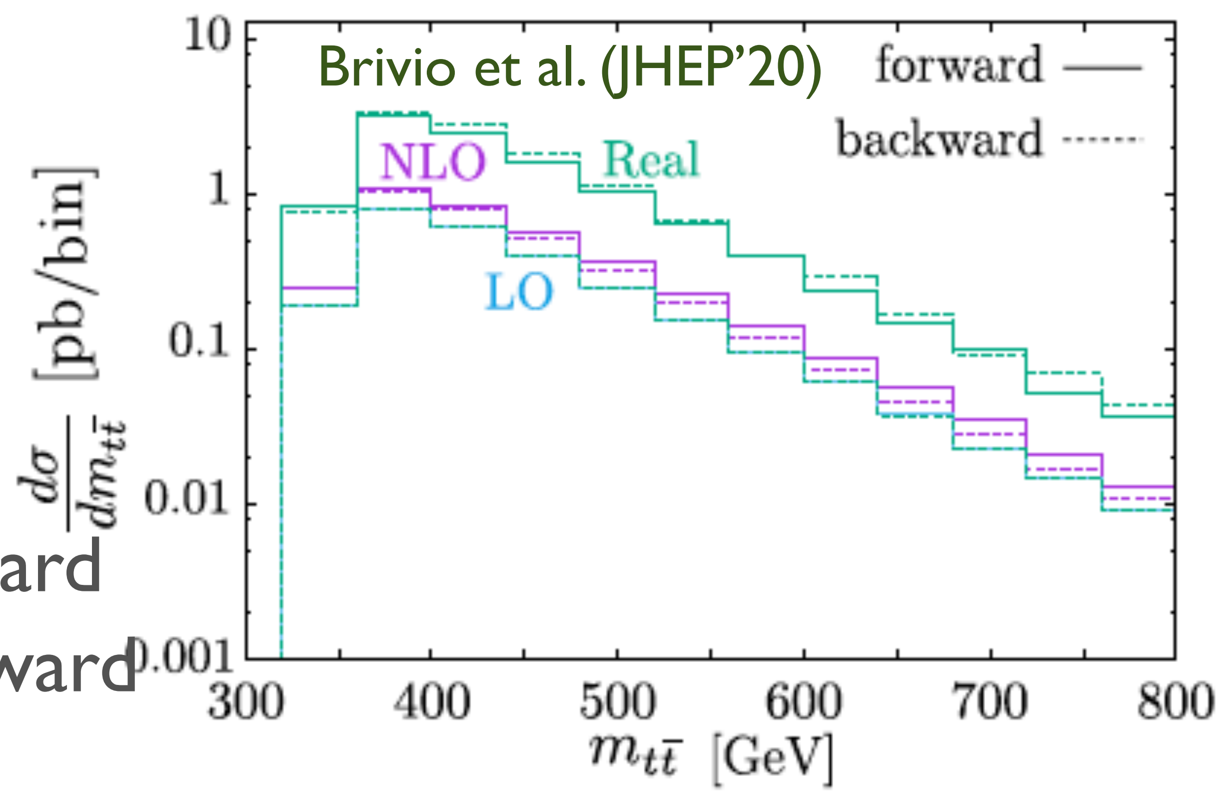


- Breaking degeneracies at NLO



$$O^{8}_{tu} = \left(\bar{t}\gamma_{\mu}T^{A}t\right)\left(\bar{u}_{i}\gamma^{\mu}T^{a}u_{i}\right) \longrightarrow \text{Forward}$$
$$O^{8}_{Qu} = \left(\bar{Q}\gamma_{\mu}T^{A}Q\right)\left(\bar{u}_{i}\gamma^{\mu}T^{a}u_{i}\right) \longrightarrow \text{Backward}$$

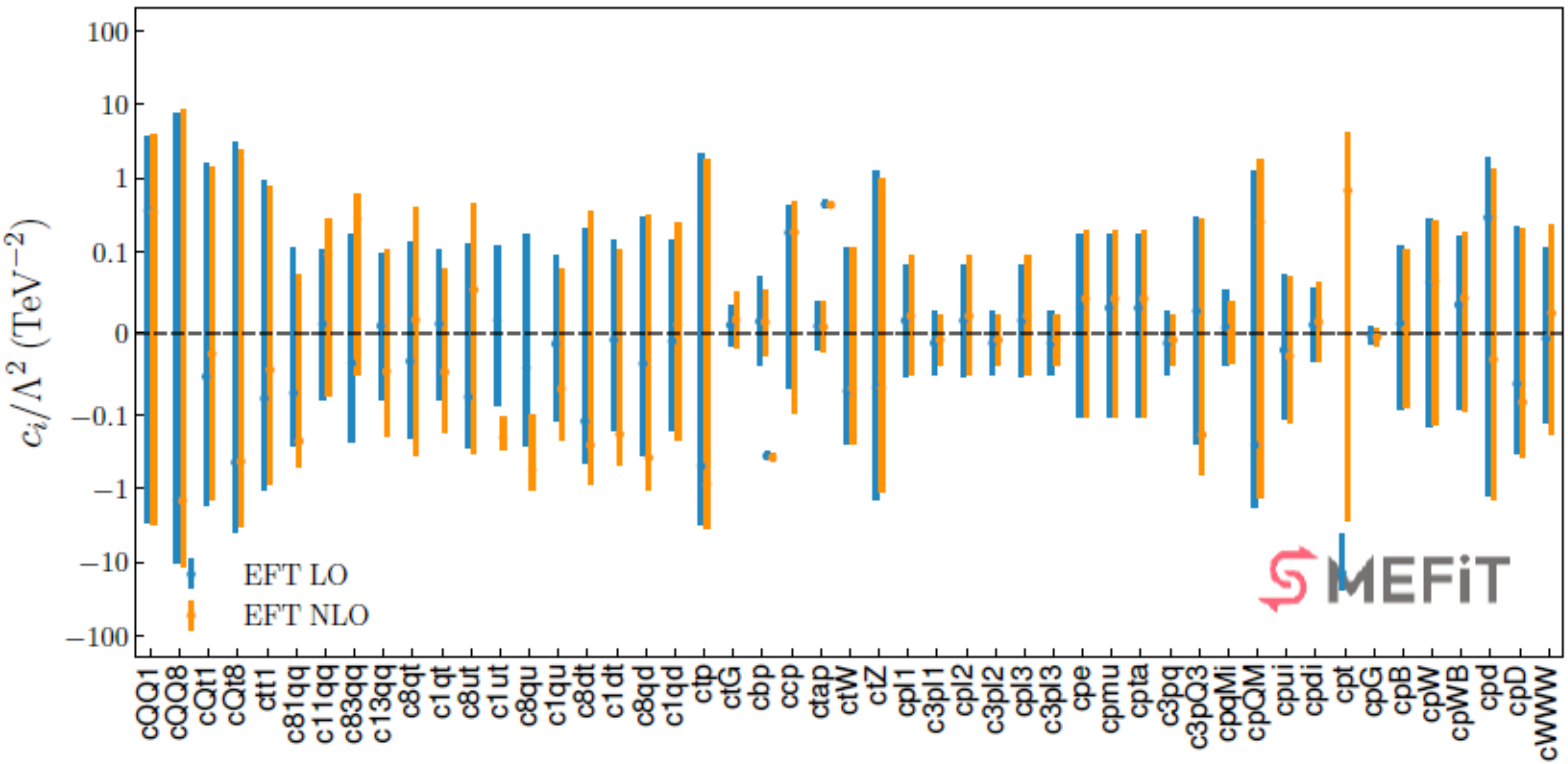
Different top chiralities



NLO HELPS SMEFT@LHC

- **Impact on the global fit**

Ethier et al. (2105.00006)



Friday, June 11, 21

In Memory of Cen Zhang

- A great organiser of the school !
- A talent physicist with several groundbreaking research works
- A very good collaborator and friend
- Our last meeting in person in Beijing (Oct 2019, before Covid confinement)