Matching & Merging

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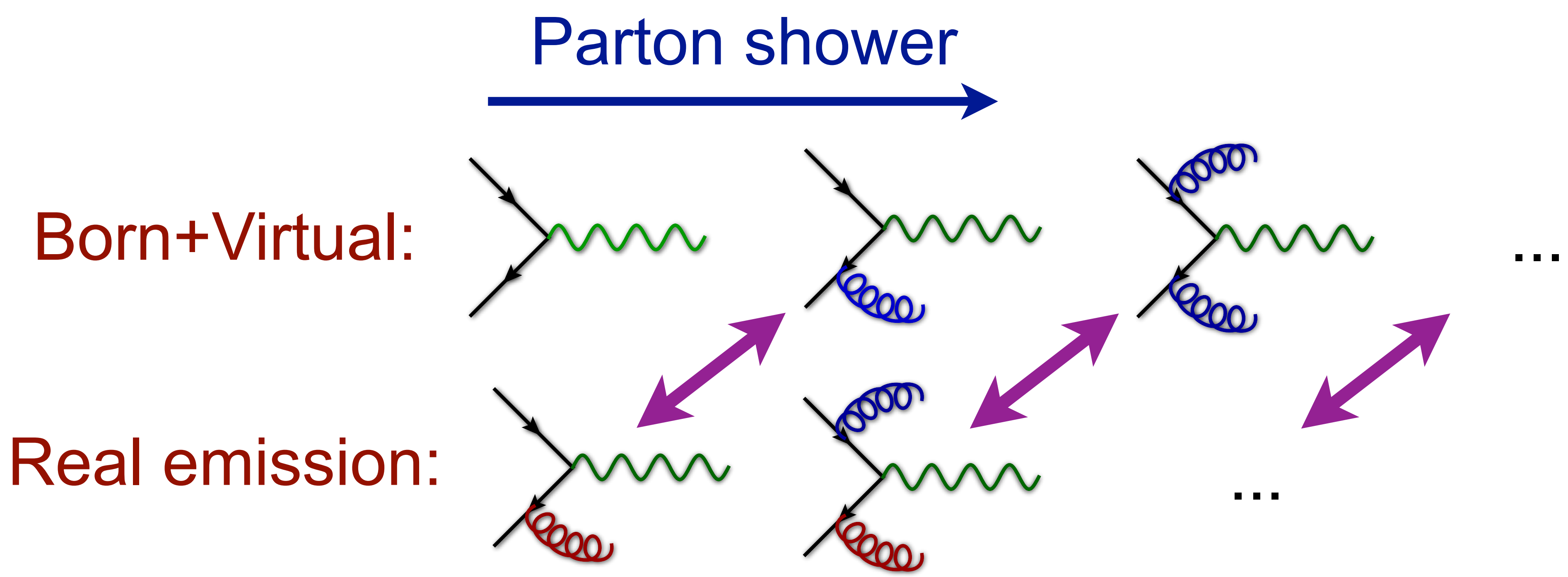
Matrix Elements + Parton Shower Merging

improving MC's

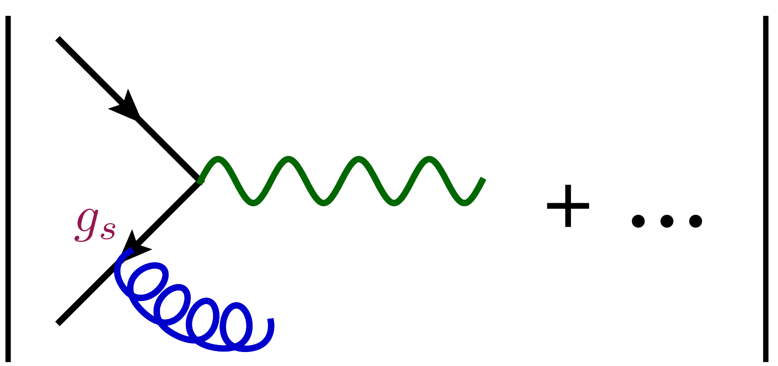
- Parton shower MC programs are only correct in the soft-collinear region. Hard radiation cannot be described correctly
- There are two ways to improve a Parton Shower Monte Carlo event generator with matrix elements:
  - NLO+PS matching: Include full NLO corrections to the matrix elements to reduce theoretical uncertainties in the matrix elements. The real-emission matrix elements will describe the hard radiation
  - ME+PS merging: Include matrix elements with more final state partons to describe hard, well-separated radiation better

Double counting



- There is double counting between the real emission matrix elements and the parton shower: the extra radiation can come from the matrix elements or the parton shower
- There is also an overlap between the virtual corrections and the Sudakov suppression in the zero-emission probability

At NLO

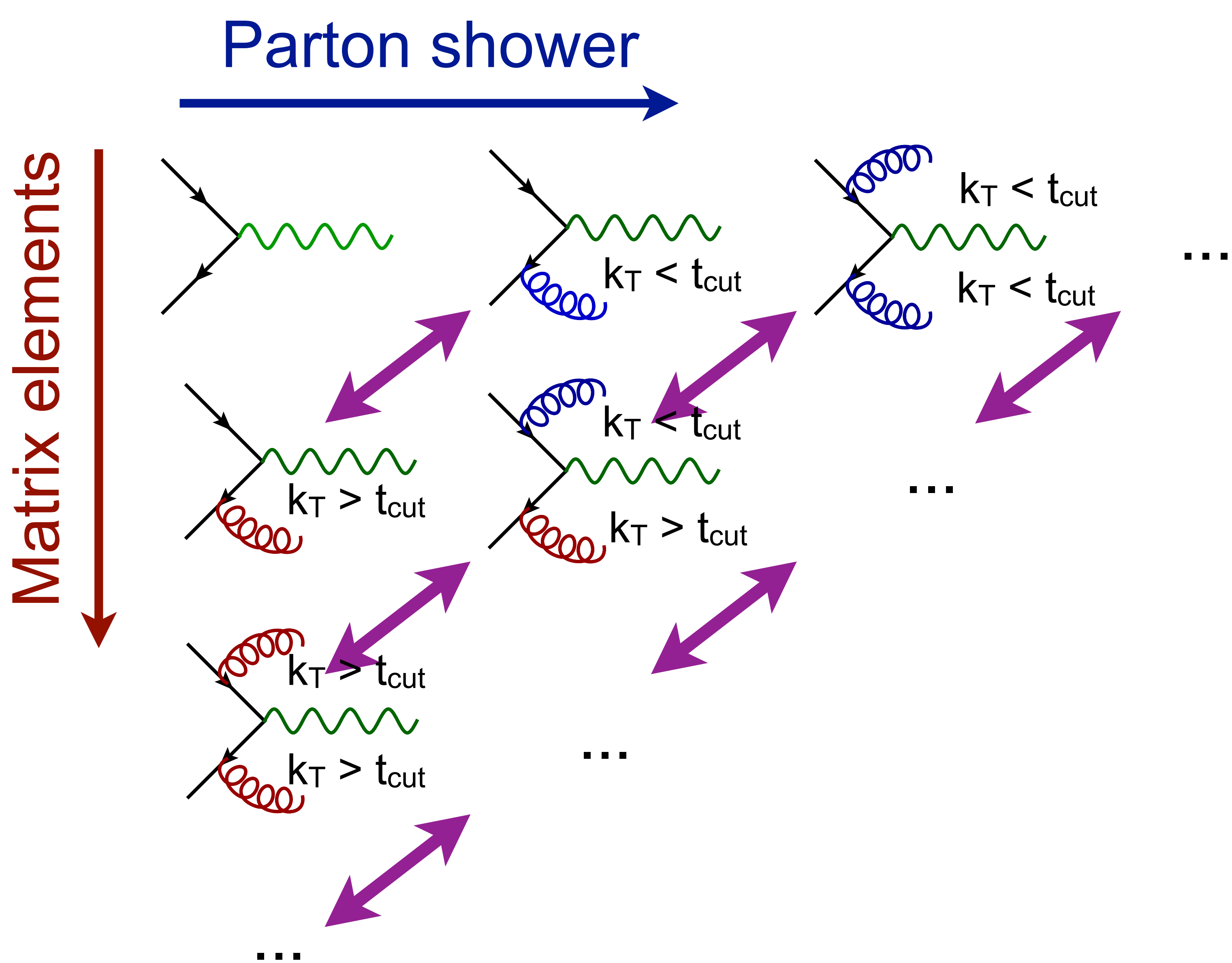


- We have to integrate the real emission over the **complete** phase-space of the one particle that can go soft or collinear to obtain the infra-red poles that will cancel against the virtual corrections

Let us forget about NLO: what about including only Leading Order below the cut, and only real-emission above the cut. No need to integrate real-emission over the whole phase-space, since no need to cancel divergences of the virtual corrections

- Hence, we cannot introduce a "cut" that says that:
  - hard radiation needs to be described by the matrix elements
  - and soft radiation by the parton shower
- We have to invent a new procedure to match NLO matrix elements with parton showers

Possible double counting

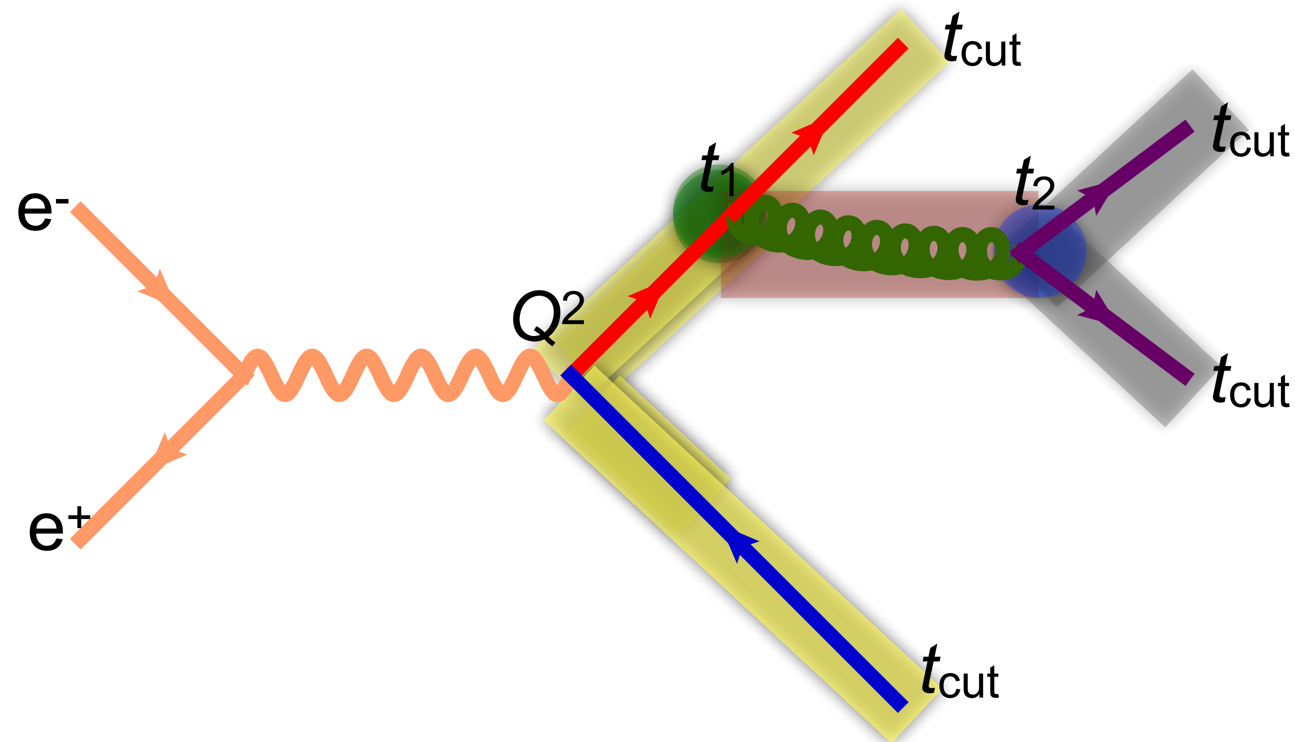


Possible double counting between partons from matrix elements and parton shower easily avoided by applying a cut in phase space

Merging ME with PS

- So double counting no problem, but what about getting smooth distributions that are independent of the precise value of t_{cut}?
- Below cutoff, distribution is given by PS
  - need to make ME look like PS near cutoff
- Let's take look at the PS!

Merging ME with PS



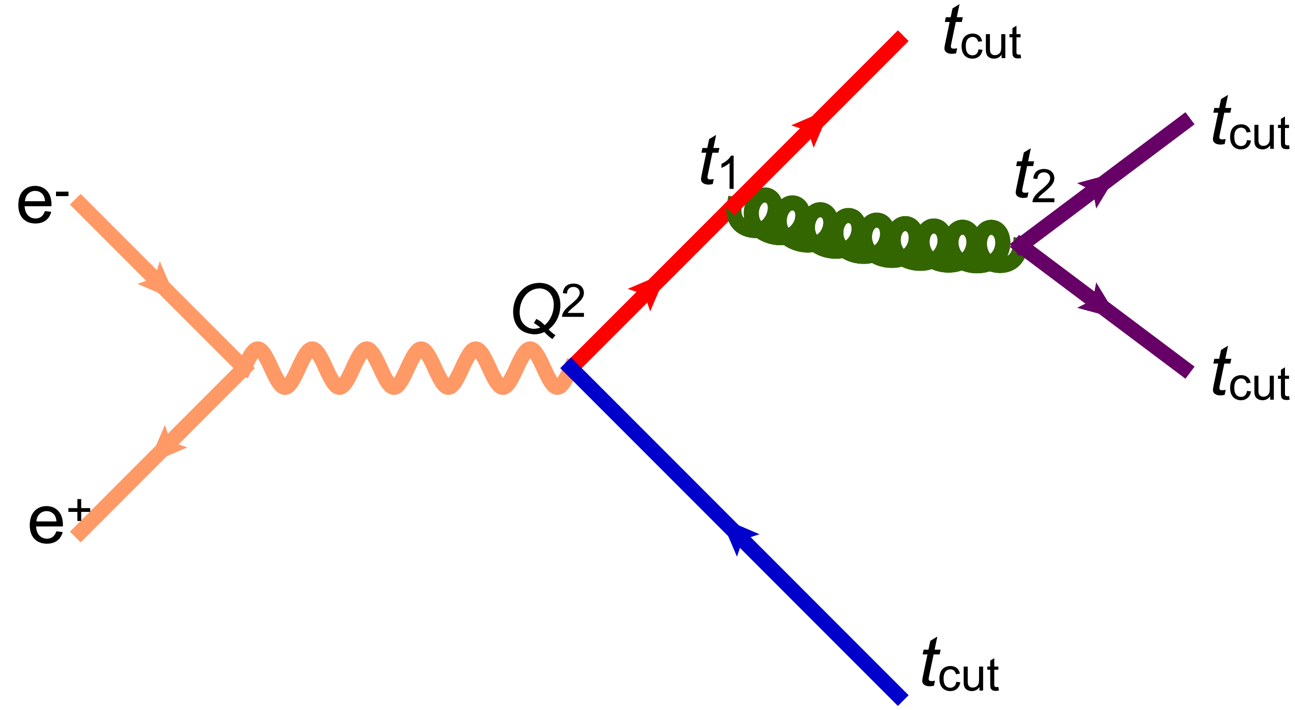
- How does the PS generate the configuration above (i.e. starting from e^+e^- -> qqbar events)?
- Probability for the splitting at t_1 is given by

$$(\Delta_q(Q^2, t_1))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z)$$

and for the whole tree

$$(\Delta_q(Q^2, t_{\mathrm{cut}}))^2 \Delta_g(t_1, t_2) (\Delta_q(t_2, t_{\mathrm{cut}}))^2 \frac{\alpha_s(t_1)}{2\pi} P_{gq}(z) \frac{\alpha_s(t_2)}{2\pi} P_{qg}(z')$$

Merging ME with PS

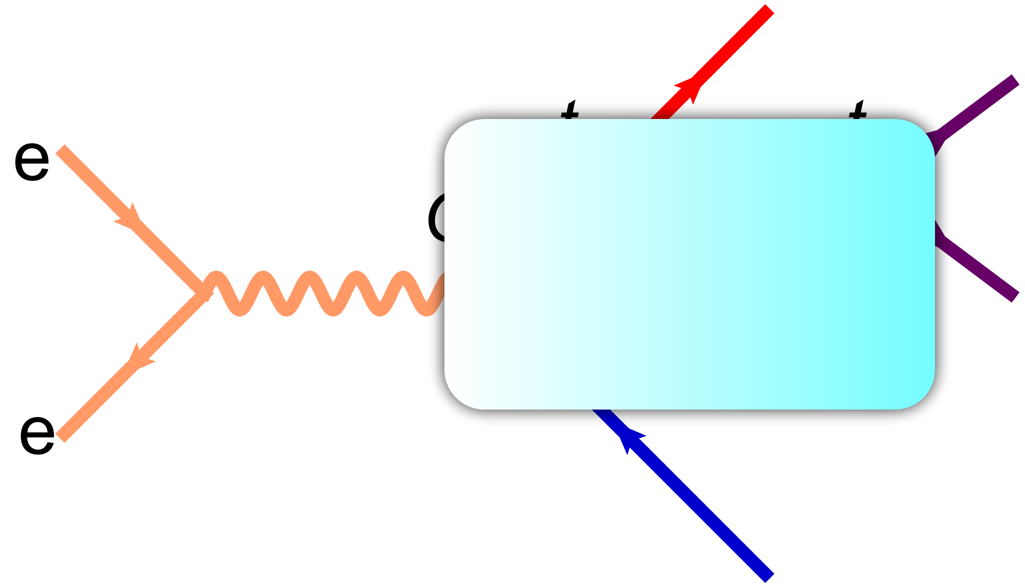


$$\left(\Delta_q(Q^2,t_{\rm cut})\right)^2\Delta_g(t_1,t_2)\left(\Delta_q(t_2,t_{\rm cut})\right)^2 \frac{\alpha_s(t_1)}{2\pi}P_{gq}(z)\frac{\alpha_s(t_2)}{2\pi}P_{qg}(z')$$

Leading Logarithmic approximation of the matrix element BUT with α_s evaluated at the scale of each splitting

Sudakov suppression due to disallowing additional radiation above the scale $t_{\rm cut}$

Merging ME with PS



$$|\mathcal{M}|^2(\hat{s}, p_3, p_4, \ldots)$$

To get an equivalent treatment of the corresponding matrix element, do as follows:

1. Cluster the event using some clustering algorithm
   - this gives us a corresponding "parton shower history"
2. Reweight α_s in each clustering vertex with the clustering scale

$$|\mathcal{M}|^2 \rightarrow |\mathcal{M}|^2 \frac{\alpha_s(t_1)}{\alpha_s(Q^2)} \frac{\alpha_s(t_2)}{\alpha_s(Q^2)}$$

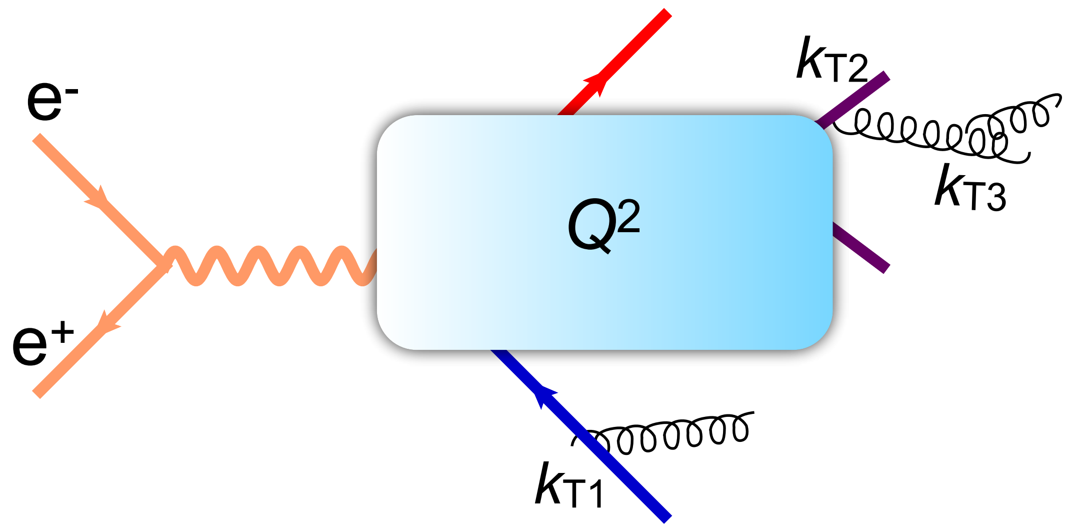
3. Use some algorithm to apply the equivalent Sudakov suppression

$$(\Delta_q(Q^2, t_{\text{cut}}))^2 \Delta_g(t_1, t_2)(\Delta_q(t_2, t_{\text{cut}}))^2$$

MLM matching

[M.L. Mangano, 2002, 2006]
[J. Alwall et al 2007, 2008]

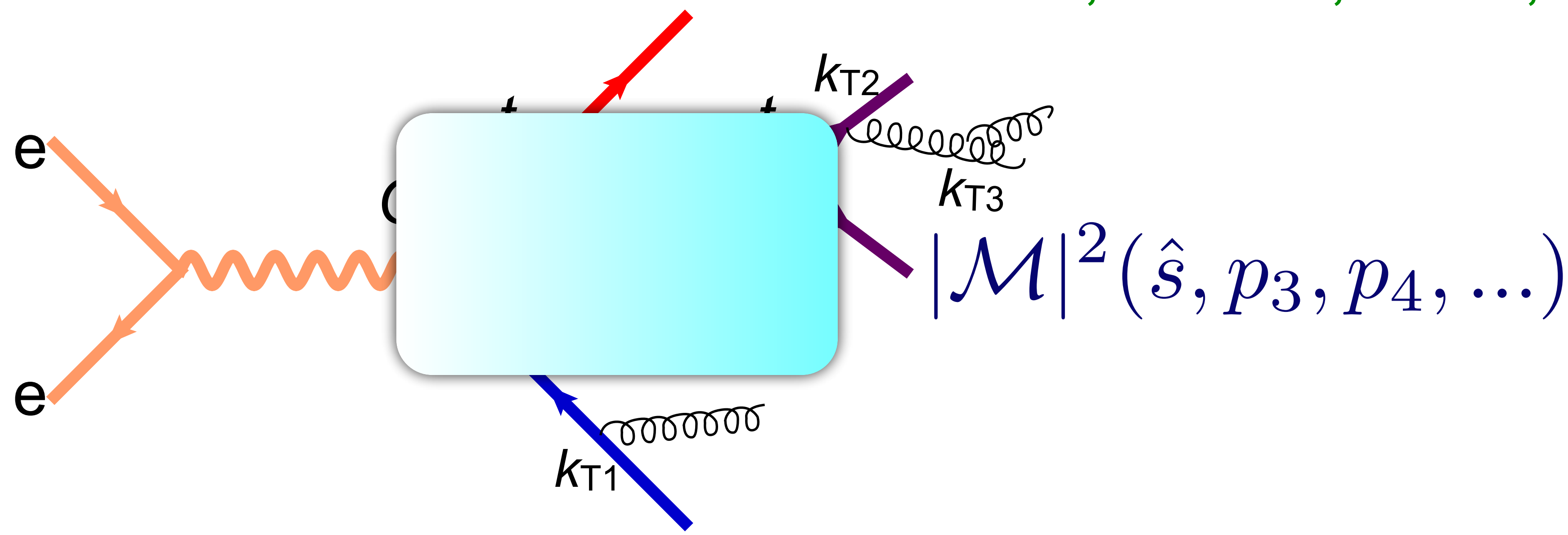
- The simplest way to do the Sudakov suppression is to run the shower on the event, starting from Q^2!



- If hardest shower emission scale $k_{T1} > t_{cut}$, throw the event away, if all $k_{T1,2,3} < t_{cut}$, keep the event
- The suppression for this is $(\Delta_q(Q^2, t_{\mathrm{cut}}))^4$ so the internal structure of the shower history is ignored. In practice, this approximation is still pretty good
- Allows matching with any shower, without modifications!

CKKW matching

Catani, Krauss, Kuhn, Webber [2001]

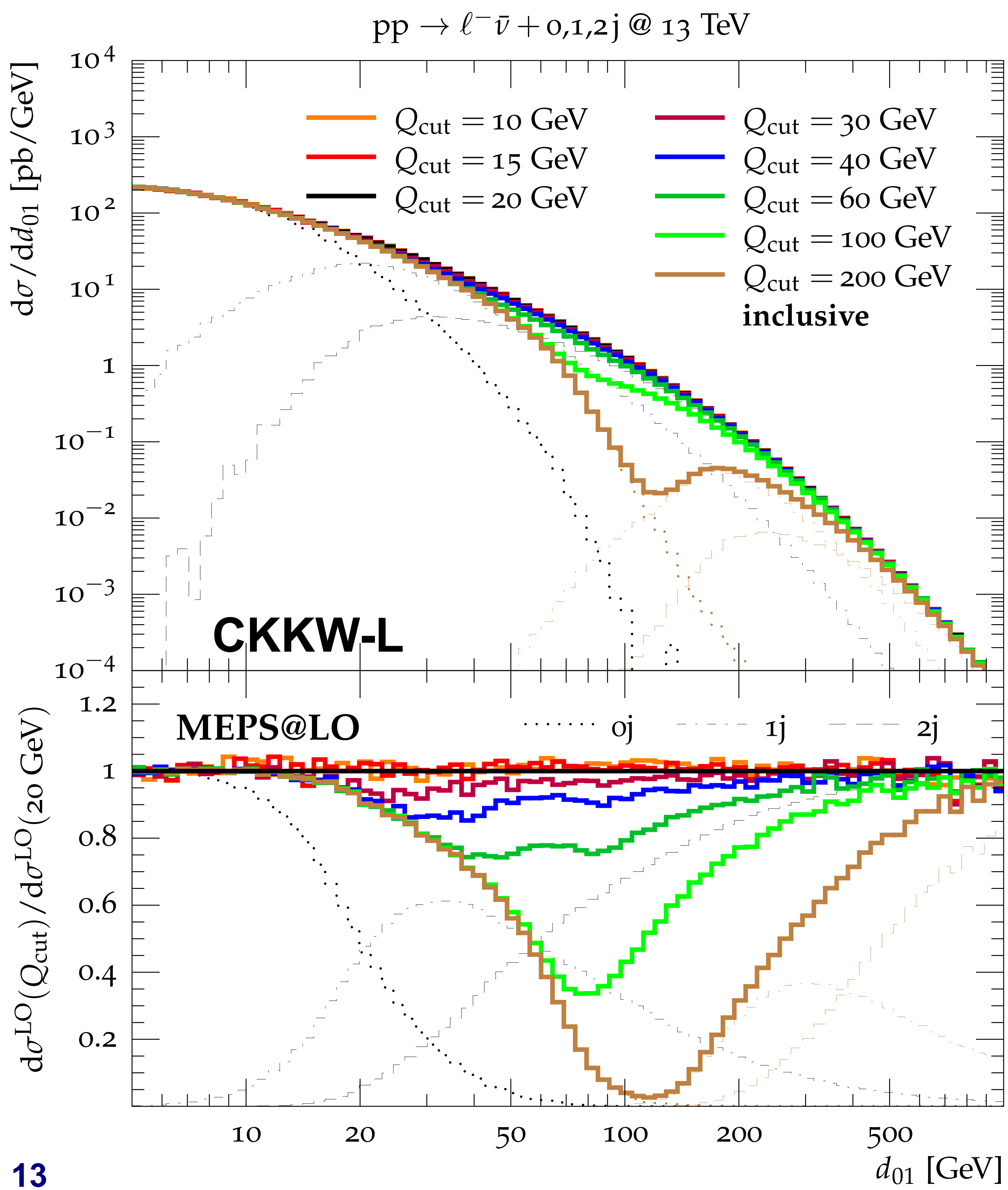


- Once the 'most-likely parton shower history' has been found, one can also reweight the matrix element with the Sudakov factors that give that history

$$(\Delta_q(Q^2, t_{\rm cut}))^2 \Delta_g(t_1, t_2)(\Delta_q(t_2, t_{\rm cut}))^2$$

- To do this correctly, must use same variable to cluster and define this sudakov as the one used as evolution parameter in the parton shower. Parton shower can start at $t_{\rm cut}$

Matching results

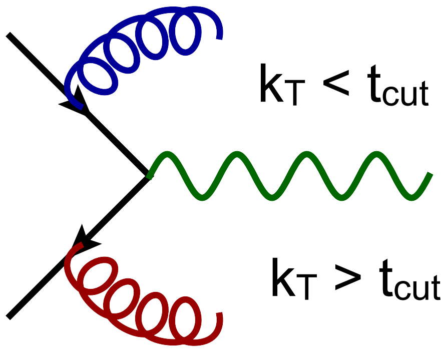


- W+jets production: diff. jet rate for 0➞1 transition (~ p_T of hardest jet)
- Small dependence on the merging scale for small values, ~10%
  - When taken too large, the parton shower cannot fill the region all the way up to the merging scale anymore, leading to large deficits

[Kallweit, Lindert, Maierhöfer, Pozzorini, Schönherr 2016]

Merging LO ME with PS

- Combining LO matrix elements with parton showers.
- Double counting no problem: we simply throw events away when the matrix-element partons are too soft, or when the parton shower generates too hard radiation
- Applying the matrix-element cut is easy: during phase-space integration, we only generate events with partons above the matching scale



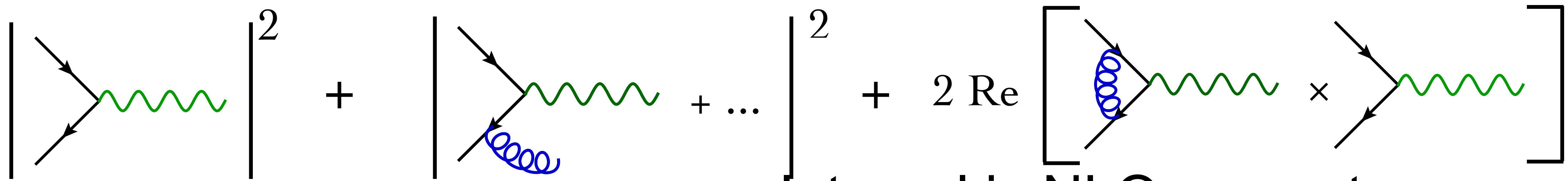
  - For the cut on the shower, there are two methods. Throwing events away after showering is not very efficient, although it is working ("MLM method")
  - Instead we can also multiply the Born matrix elements by suitable product of Sudakov factors (i.e. the no-emission probabilities) $\Delta(Q^{max}, t_{cut})$ and start the shower at the scale t_{cut} ("CKKW(-L) method").

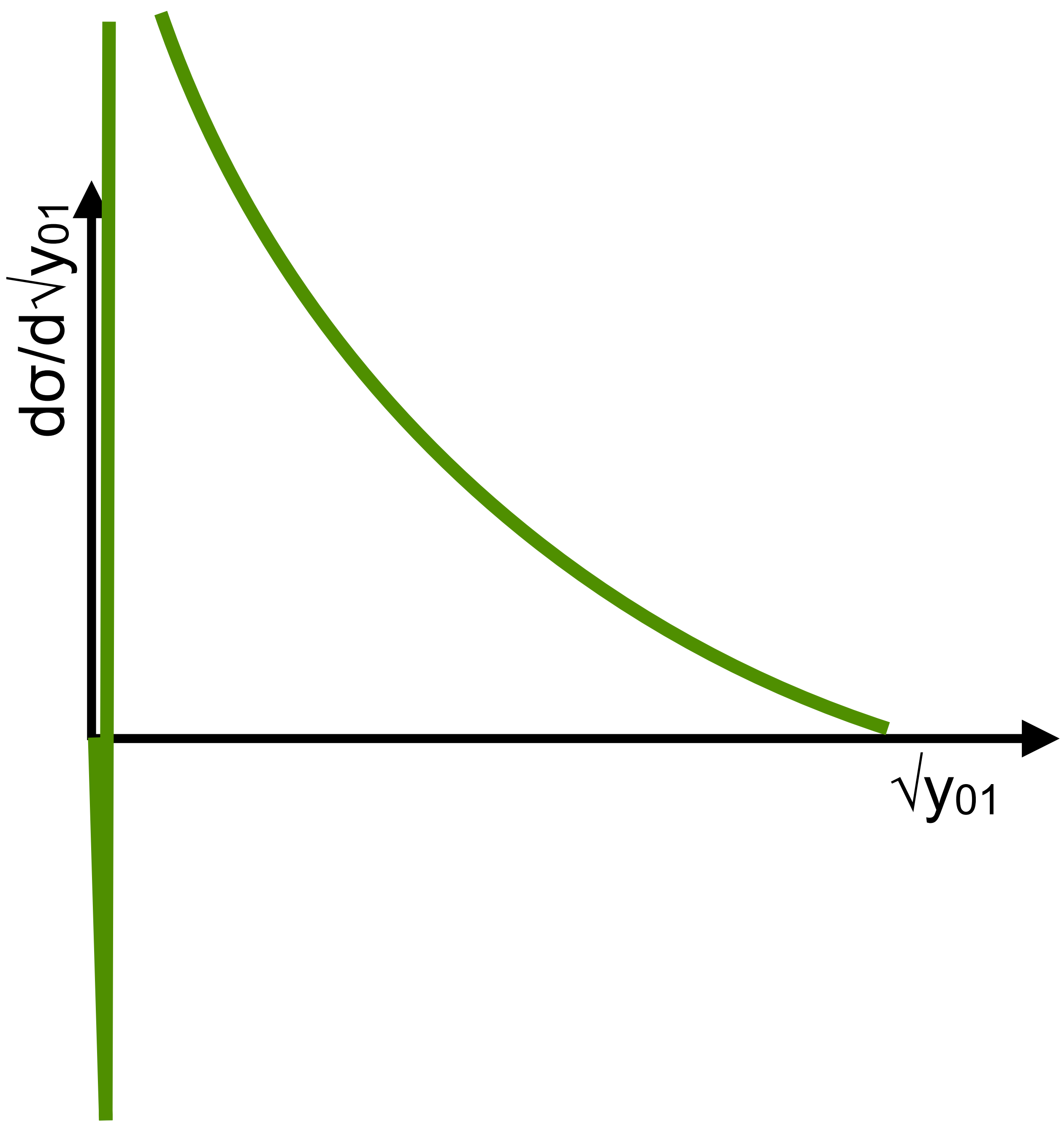
Matrix Elements + Parton Shower Merging at NLO

What to do at NLO

- Let's start very simple and see what to do…
- Let's consider
  - a very simple process: production of a single EW vector boson (or Higgs boson)
  - an observable most-sensitive to QCD radiation: k_T-jet resolution variable (with R=1), $\sqrt{y} \sim p_T(j)$
    [$y_{01} \sim p_T^2(j_1)$; $y_{12} \sim p_T^2(j_2)$; etc]

Next-to-leading order V

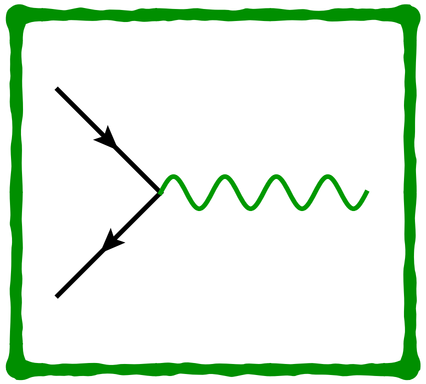


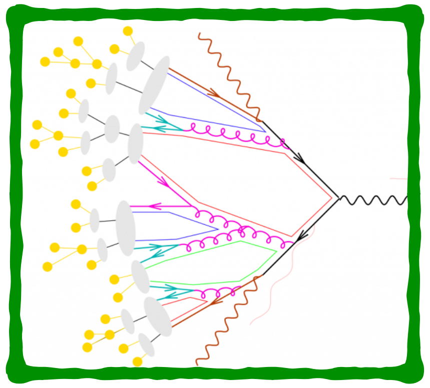


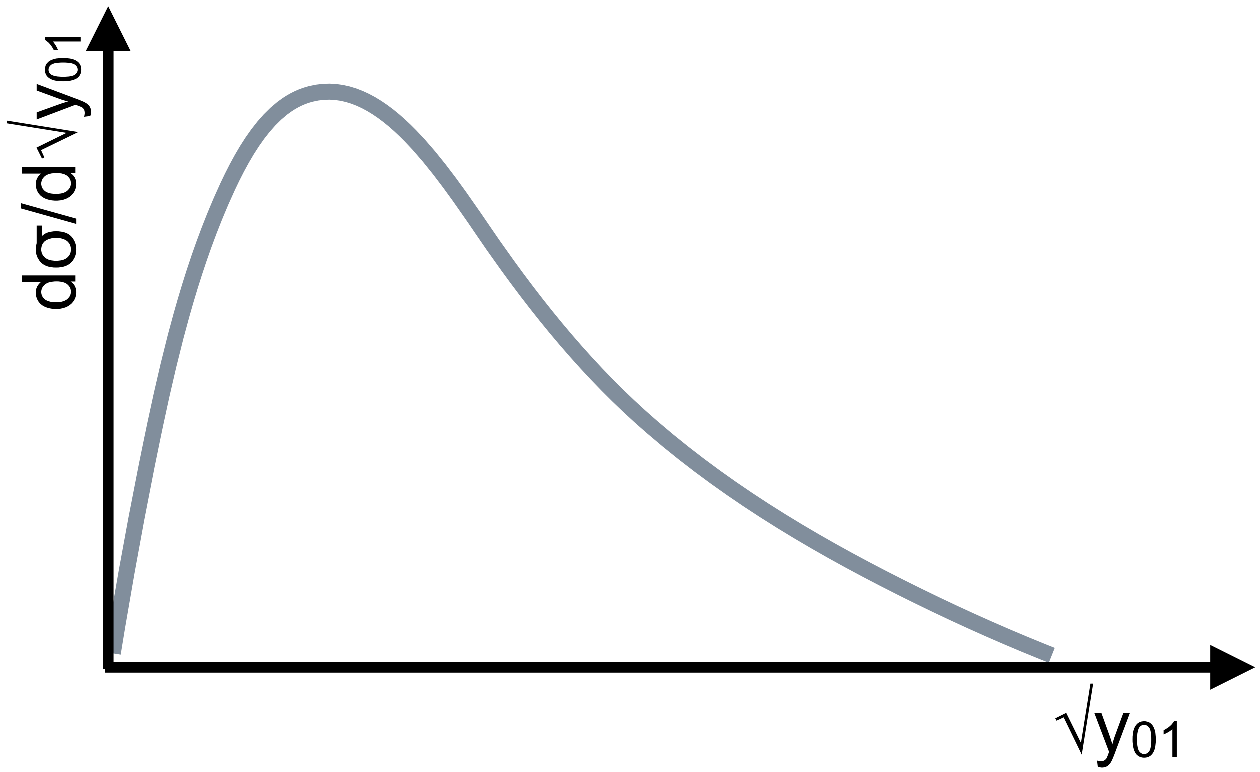
- Integral is NLO accurate
- Curve is non-physical at low p_T: divergent real-emission corrections are compensated for by divergent virtual corrections
- Including higher order corrections (NNLO, etc), does not fix the non-physical behaviour at small pT

Physical curve	Only at high-p_T
Tail	LO
Integral	NLO
Extendible to multi-jet	Yes

NLO+PS V





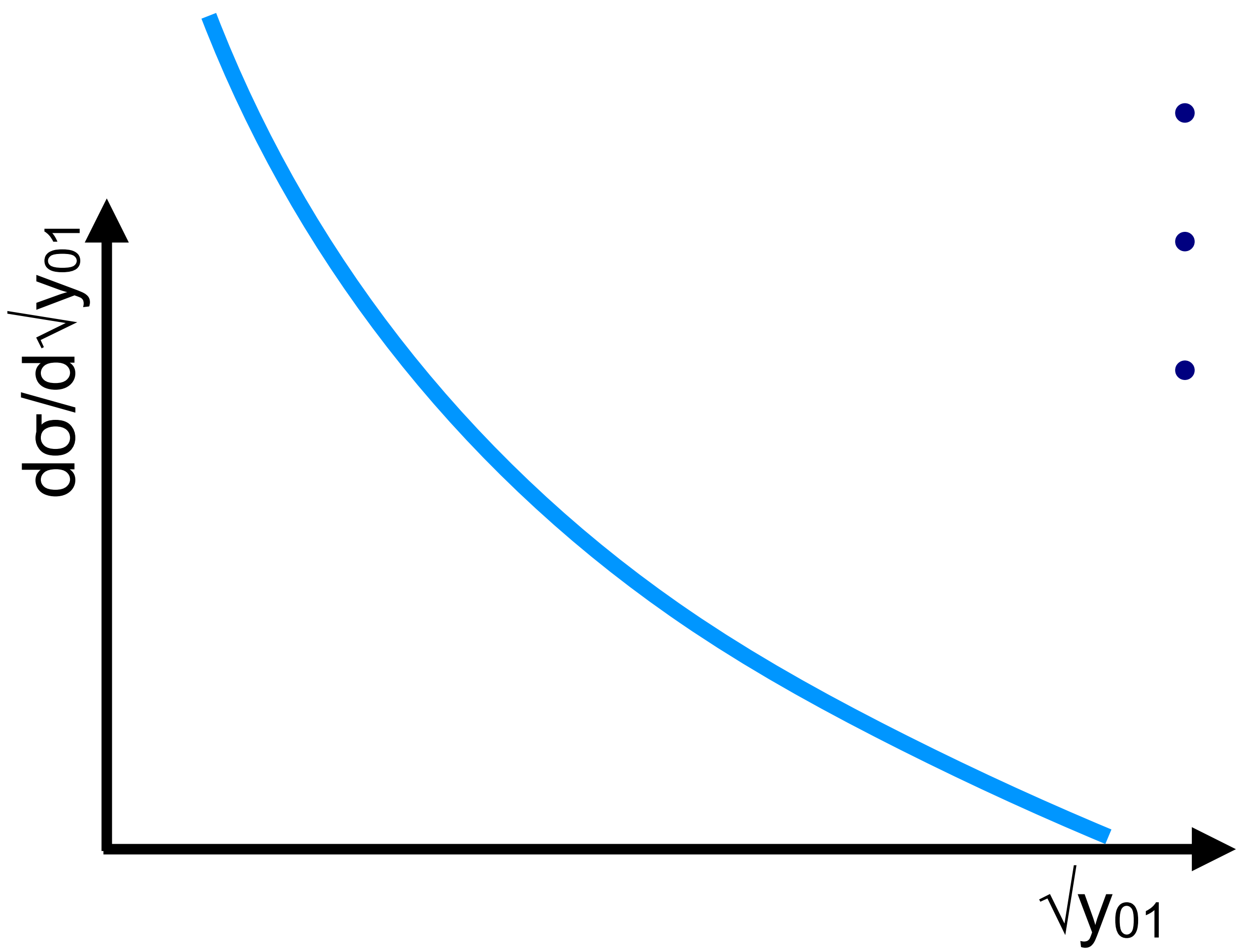


MC@NLO: [Frixione, Webber (2002)]
POWHEG: [Nason (2004)]

- To get a physical shape at low p_T need to resum radiation at all orders
- Can either be done analytically, or with a **parton shower**
- Parton shower also includes hadronisation and other non-factorisable corrections
- Most used methods are MC@NLO and POWHEG

Physical curve	**Yes**
Tail	**LO**
Integral	**NLO**
Extendible to multi-jet	**Yes**

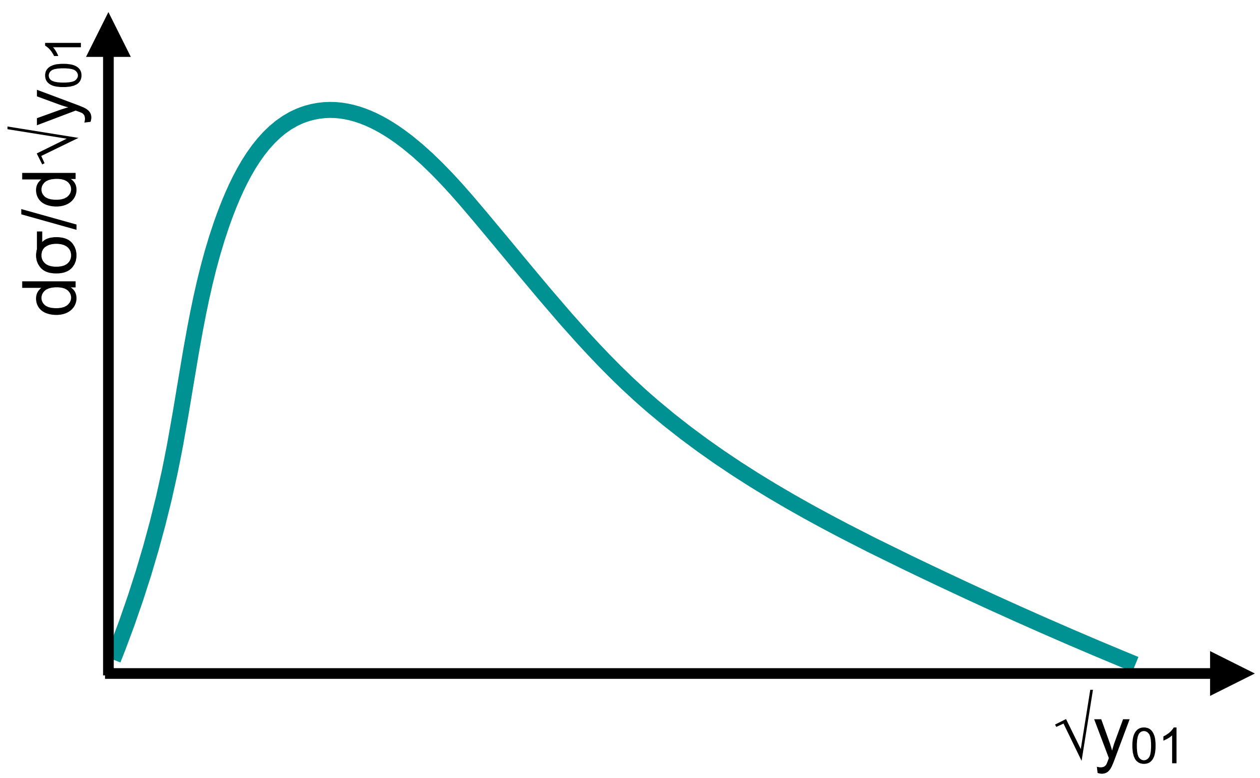
NLO(+PS) V+1 jet



- Distribution diverges at small p_T
- Have to put a generation cut
- Parton shower can easily be added, but this does not solve the low-p_T problem

Physical curve	**Only at high-p_T**
Tail	**NLO**
Integral	**∞**
Extendible to multi-jet	**Yes**

Minlo V+1jet



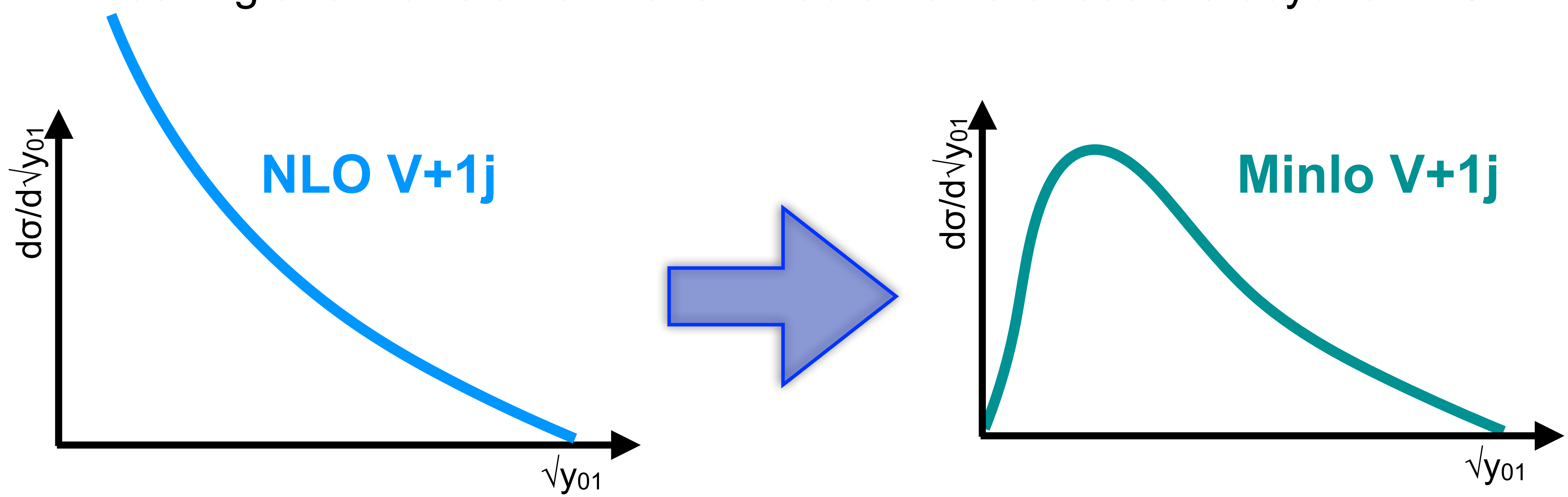
[Hamilton, Nason, Zanderighi (2012)]

- Include suitable Sudakov Form factors in the NLO V+1j predictions
- Distributions is NLO accurate
- Integral is not NLO accurate: the difference starts at $O(\alpha_s^{3/2})$
- Parton shower can easily be attached

Physical curve	**Yes**
Tail	**NLO**
Integral	**LO+**
Extendible to multi-jet	**Yes**

Minlo

- The Minlo approach can be summarised as follows:
  - Renormalisation and factorisation scale setting, a la CKKW
  - Together with matching to the Sudakov form factor, $\exp\left[-R\left(v\right)\right]$
    - Matching requires to subtract the O(alpha_s) expansion of the Sudakov form factor times the Born to prevent double counting with the NLO corrections
  - NLO accuracy of V+1j observables is not hampered by the scale setting and inclusion of the form factor: differences are beyond NLO



Minlo decomposed

$$d\sigma_{\mathcal{M}} = d\sigma_{\mathcal{R}} + d\sigma_{\mathcal{MR}} + d\sigma_{\mathcal{F}}$$

Resummed cross section. (Almost) identical to known LL/NNLL$_\sigma$ results

Logarithmically enhanced terms for y->0 that are not captured by dσ_R

Finite terms in the limit y->0 (coming from real emission corrections)

Resummed cross section

$d\sigma_{\mathcal{M}} = d\sigma_{\mathcal{R}} + d\sigma_{\mathcal{MR}} + d\sigma_{\mathcal{F}}$

[Banfi, Salam, Zanderighi (2005); Dokshitzer, Diakonov, Troian (1980)]

$$\frac{d\sigma_{\mathcal{R}}}{d\Phi dL} = \frac{d\sigma_0}{d\Phi}\left[1 + \bar{\alpha}_s\left(\mu_R^2\right)\mathcal{H}_1\left(\mu_R^2\right)\right]\frac{d}{dL}\left[\exp\left[-R\left(v\right)\right]\mathcal{L}\left(\{x_\ell\},\mu_F,v\right)\right]$$

LO cross section

(Hard) virtual contributions

Sudakov form factor

Luminosity factor

$L = \log(1/v) = \log(Q^2/y)$

- Well-known formula; used e.g. in the Caesar approach
- Sudakov form factor exp[R] not identical to what's (originally) used in Minlo. But Minlo approach can be improved to incorporate these terms (not relevant when colour is trivial)
- Written as **total derivative**: straight-forward to show that this is NLO correct in phase-space Φ up to $d\sigma_F$ after integration over L and expanding in α_S
- However, not NLO correct in the $d\Phi dL$ phase space (i.e., tail is not NLO correct)

Accuracy of Minlo

$d\sigma_{\mathcal{M}} = d\sigma_{\mathcal{R}} + d\sigma_{\mathcal{MR}} + d\sigma_{\mathcal{F}}$

- Explicit derivation, using the general form of the differential NLO V+1j cross sections in the small y limit,

$$\frac{d\sigma_{\mathcal{S}}}{d\Phi dL} = \frac{d\sigma_0}{d\Phi} \sum_{n=1}^{2} \sum_{m=0}^{2n-1} H_{nm} \bar{\alpha}_{\mathrm{S}}^{n} \left(\mu_{R}^{2}\right) L^{m}$$

gives

$$\frac{d\sigma_{\mathcal{MR}}}{d\Phi dL} = \frac{d\sigma_0}{d\Phi} \exp\left[-R\left(v\right)\right] \prod_{\ell=1}^{n_i} \frac{q^{(\ell)}\left(x_\ell, \mu_F^2 v\right)}{q^{(\ell)}\left(x_\ell, \mu_F^2\right)} \left[\bar{\alpha}_{\mathrm{S}}^{2}\left(K_R^2 y\right) \left[\widetilde{R}_{21} L + \widetilde{R}_{20}\right] + \bar{\alpha}_{s}^{3}\left(K_R^2 y\right) L^2 \widetilde{R}_{32}\right]$$

Only non-zero when exp[R] and Minlo Sudakov exponent are different, or when exp[R] is not NNLL$_\sigma$ accurate.

Unknown coefficient!

Known coefficient

Minlo accuracy for (inclusive) 0-jet observables

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$$\frac{d\sigma_{\mathcal{MR}}}{d\Phi dL} = \frac{d\sigma_0}{d\Phi} \exp\left[-R(v)\right] \prod_{\ell=1}^{n_i} \frac{q^{(\ell)}\left(x_\ell, \mu_F^2 v\right)}{q^{(\ell)}\left(x_\ell, \mu_F^2\right)} \left[\bar{\alpha}_{\mathrm{S}}^2\left(K_R^2 y\right)\left[\widetilde{R}_{21}\, L + \widetilde{R}_{20}\right] + \bar{\alpha}_s^3\left(K_R^2 y\right) L^2\, \widetilde{R}_{32}\right]$$

- After integration over the logarithm L (taking R_{21}=0, which is okay for the processes considered here) this results into terms of

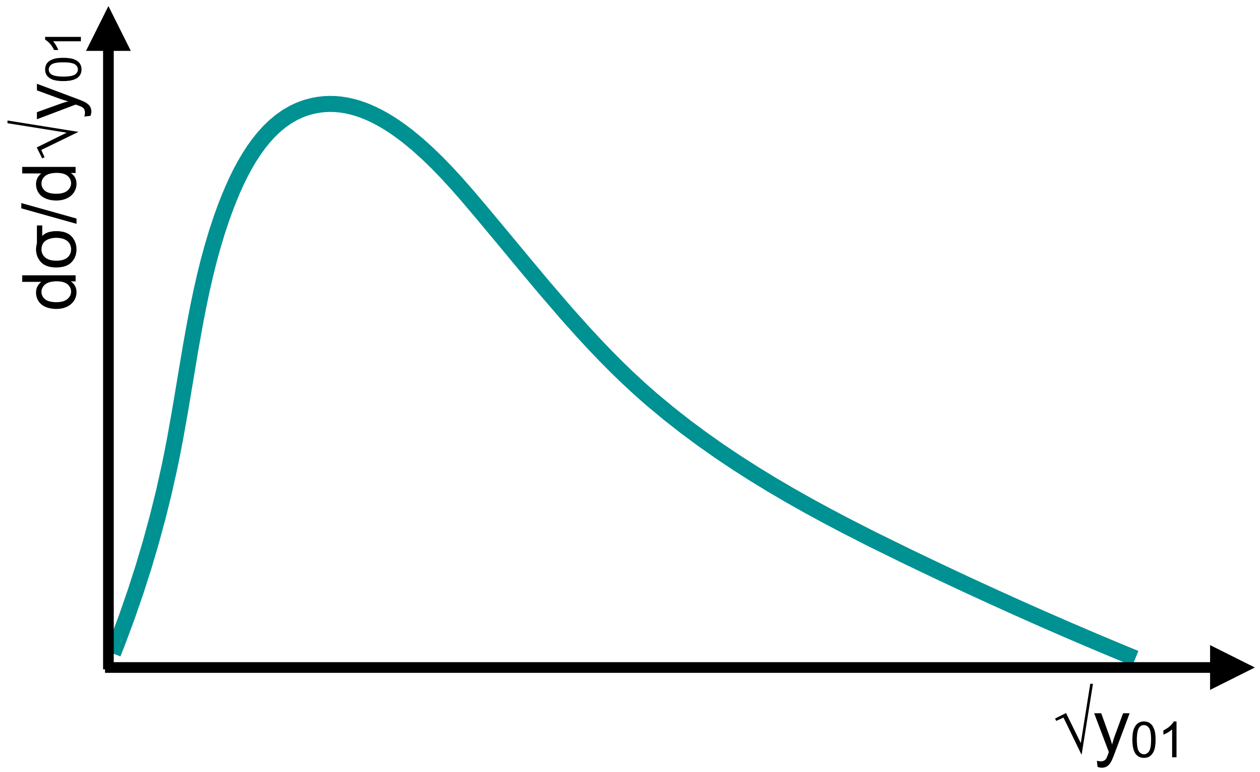
$$\int dL' \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL'} = -\frac{d\sigma_0}{d\Phi}\left[\widetilde{R}_{20} - \bar{\beta}_0 \mathcal{H}_1\left(\mu_R^2\right)\right] \sqrt{\frac{\pi}{2}} \frac{1}{\left|2G_{12}\right|^{1/2}} \bar{\alpha}_{\mathrm{S}}^{3/2}\left(1 + \mathcal{O}\left(\sqrt{\bar{\alpha}_{\mathrm{S}}}\right)\right)$$

- Hence, diff. NLO-0jet cross section not correct with NLO-1jet Minlo

[Hamilton, Nason, Oleari, Zanderighi (2012); RF, Hamilton (2015)]

$$\int_{\Lambda^2}^{Q^2} \frac{\mathrm{d}q_{\mathrm{T}}^2}{q_{\mathrm{T}}^2} \log^m \frac{Q^2}{q_{\mathrm{T}}^2}\, \alpha_{\mathrm{S}}^n\left(q_{\mathrm{T}}^2\right) \exp \mathcal{S}\left(Q, q_{\mathrm{T}}\right) \approx \left[\alpha_{\mathrm{S}}\left(Q^2\right)\right]^{n-\frac{m+1}{2}}$$

Minlo V+1jet

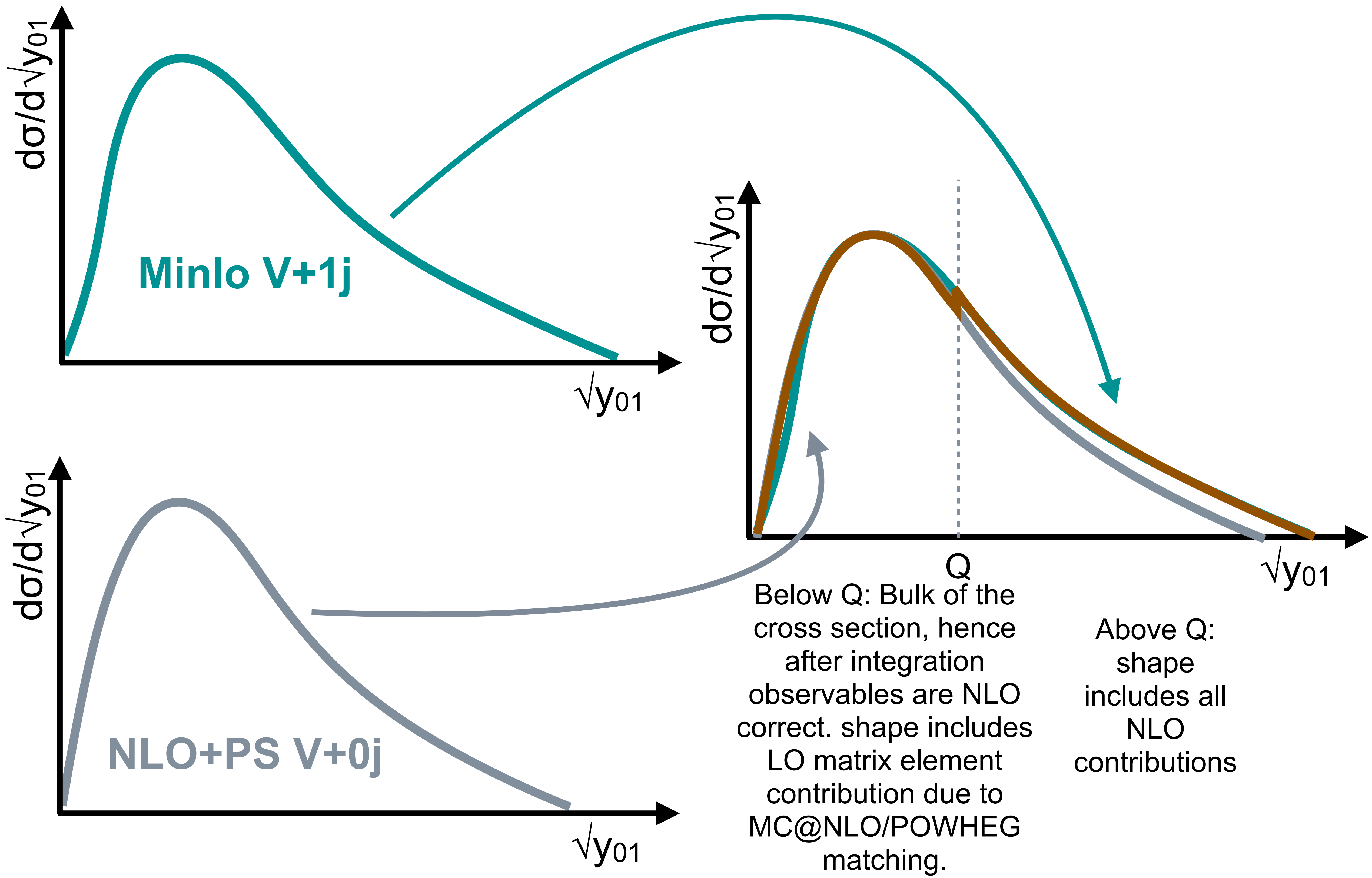


[Hamilton, Nason, Zanderighi (2012)]

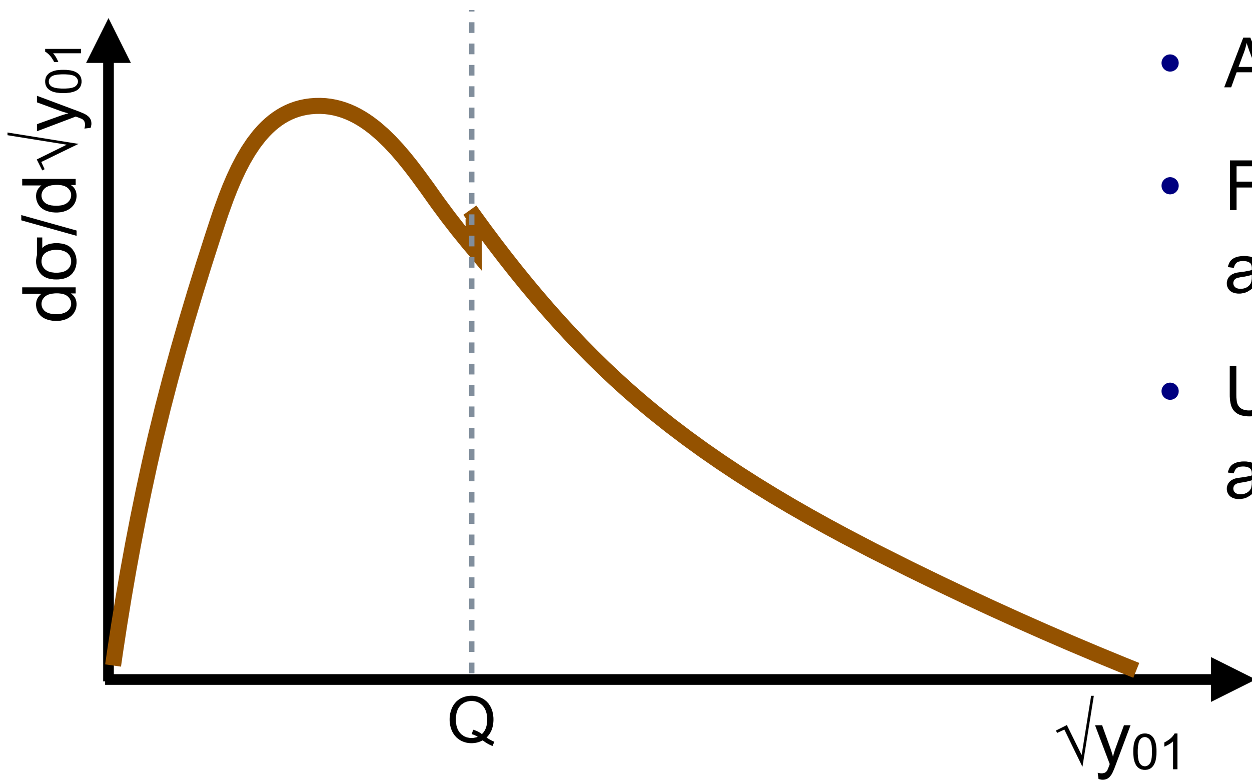
- Include suitable Sudakov Form factors in the NLO V+1j predictions
- Distributions is NLO accurate
- Integral is not NLO accurate: the difference starts at $O(\alpha_s^{3/2})$
- Parton shower can easily be attached

Physical curve	**Yes**
Tail	**NLO**
Integral	**LO+**
Extendible to multi-jet	**Yes**

Getting 0-jet observables NLO correct



FxFx / Meps@nlo: V & V+1j merging



FxFx: [RF, Frixione (2012)]
MEPS@NLO: [Hoeche, Krauss, Schonherr, Siegert; +Gehrmann (2012)]

- Merge NLO+PS for V with Minlo for V+1j, at "merging scale" Q
- Above Q the tail is NLO accurate
- For not-too-small Q, integral is NLO accurate
- Used by ATLAS & CMS for LHC run II analyses

Physical curve	**Yes**
Tail	**NLO**
Integral	**NLO**
Extendible to multi-jet	**Yes**

Differences between FxFx & MEPS@NLO

- Both FxFx and MEPS@NLO merging are based on making MC@NLO calculation for jet-multiplicities *exclusive* in more jets
  - Veto additional radiation; resum dependence on the veto scale (=merging scale)
- Major difference is in the way this exclusivity is applied
  - CKKW-L approach (i.e. Sudakov rejection based on shower kernels)
    - Used in Sherpa's "MEPS@NLO"
    - Using shower kernels prevents for a direct link with Minlo approach (and comparison to analytic resummation and accuracy), but prevents issues with mismatch in k_T and shower ordering values
  - Minlo (CKKW) from hard scale down to the scale of the softest jet not affected by veto; MLM-type rejection from there down to merging scale
    - Used in MadGraph5_aMC@NLO w/ Pythia/Herwig: "FxFx merging"
    - Direct link with Minlo, but MLM-type rejection prevents mismatches in ordering values

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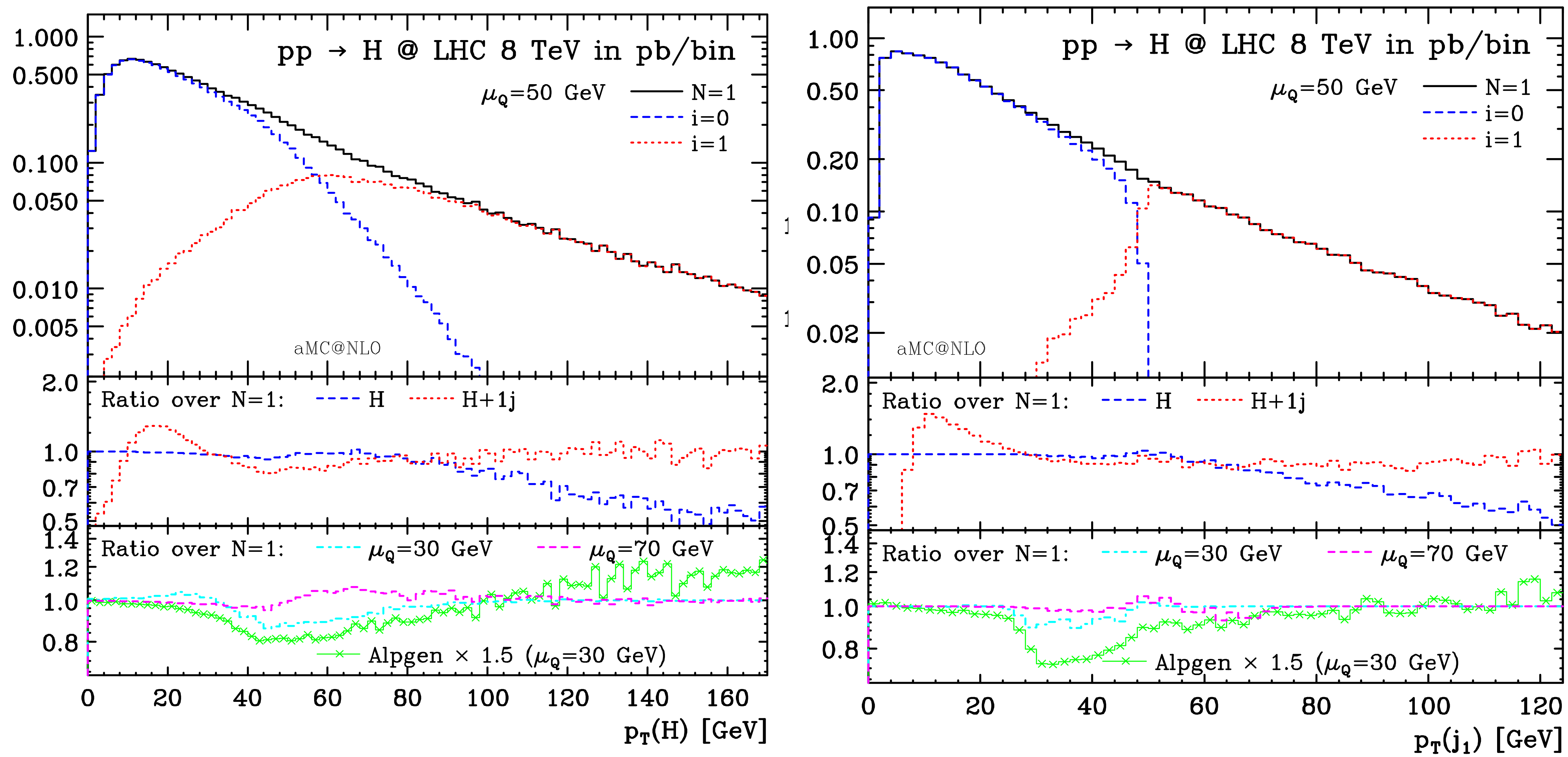
FxFx merging: Higgs boson production

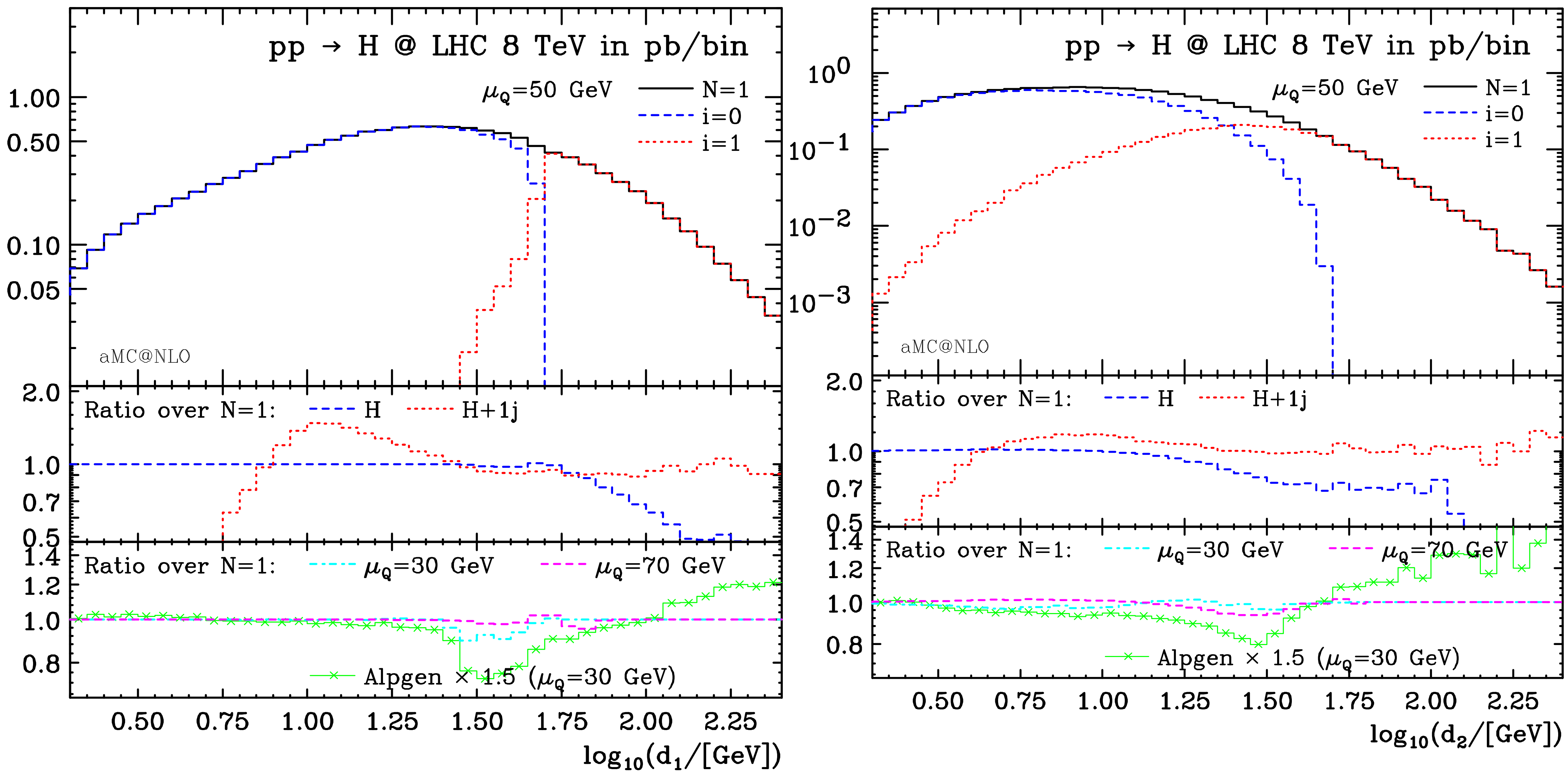
RF & Frixione, 2012



- Transverse momentum of the Higgs and of the 1st jet.
- Agreement with H+0j at MC@NLO and H+1j at MC@NLO in their respective regions of phase-space; Smooth matching in between; Small dependence on matching scale
- Alpgen (LO matching) shows larger kinks

FxFx merging: Higgs boson production

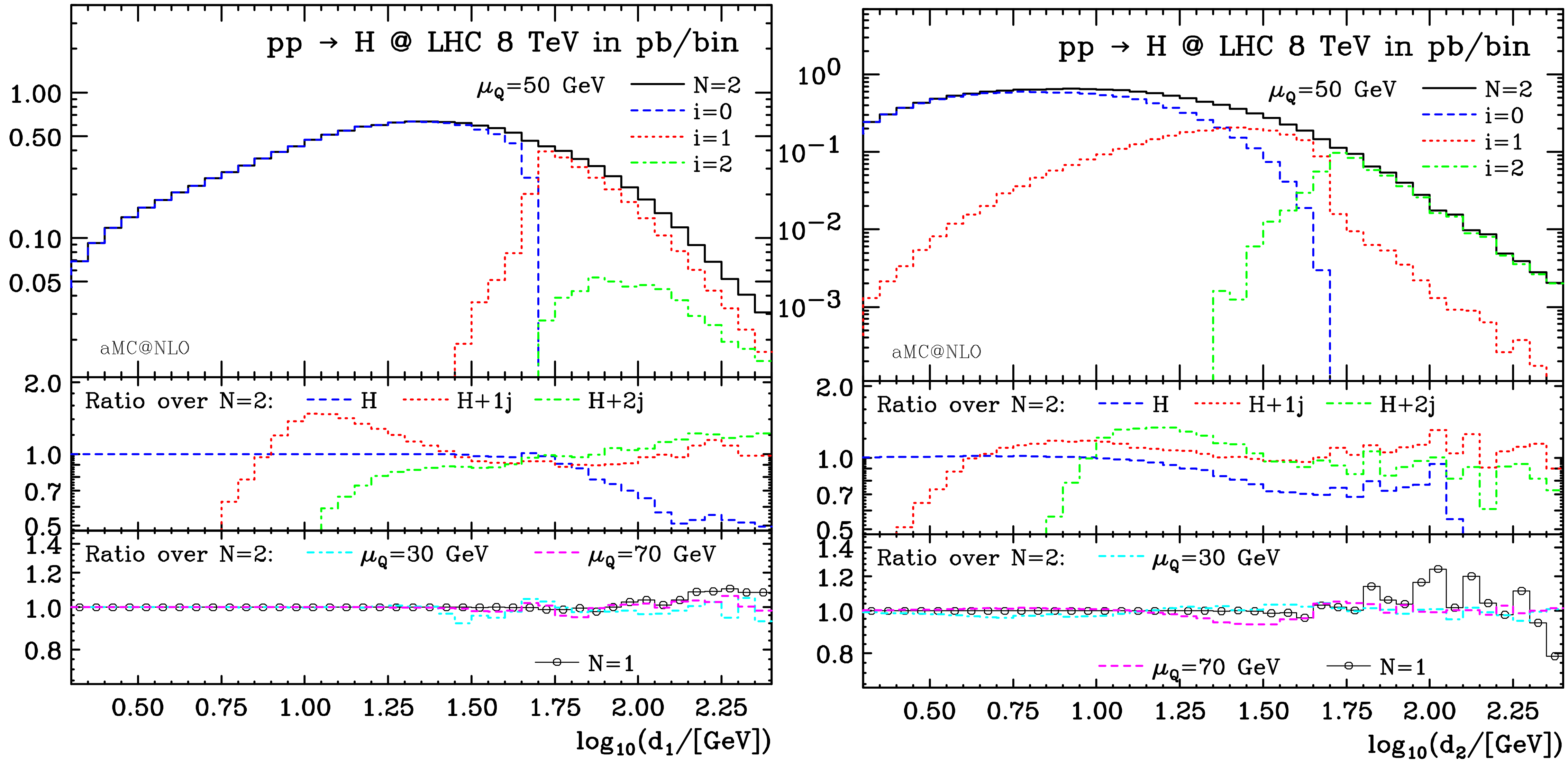
RF & Frixione, 2012



- Differential jet rates for 1->0 and 2->1

FxFx merging: Higgs boson production

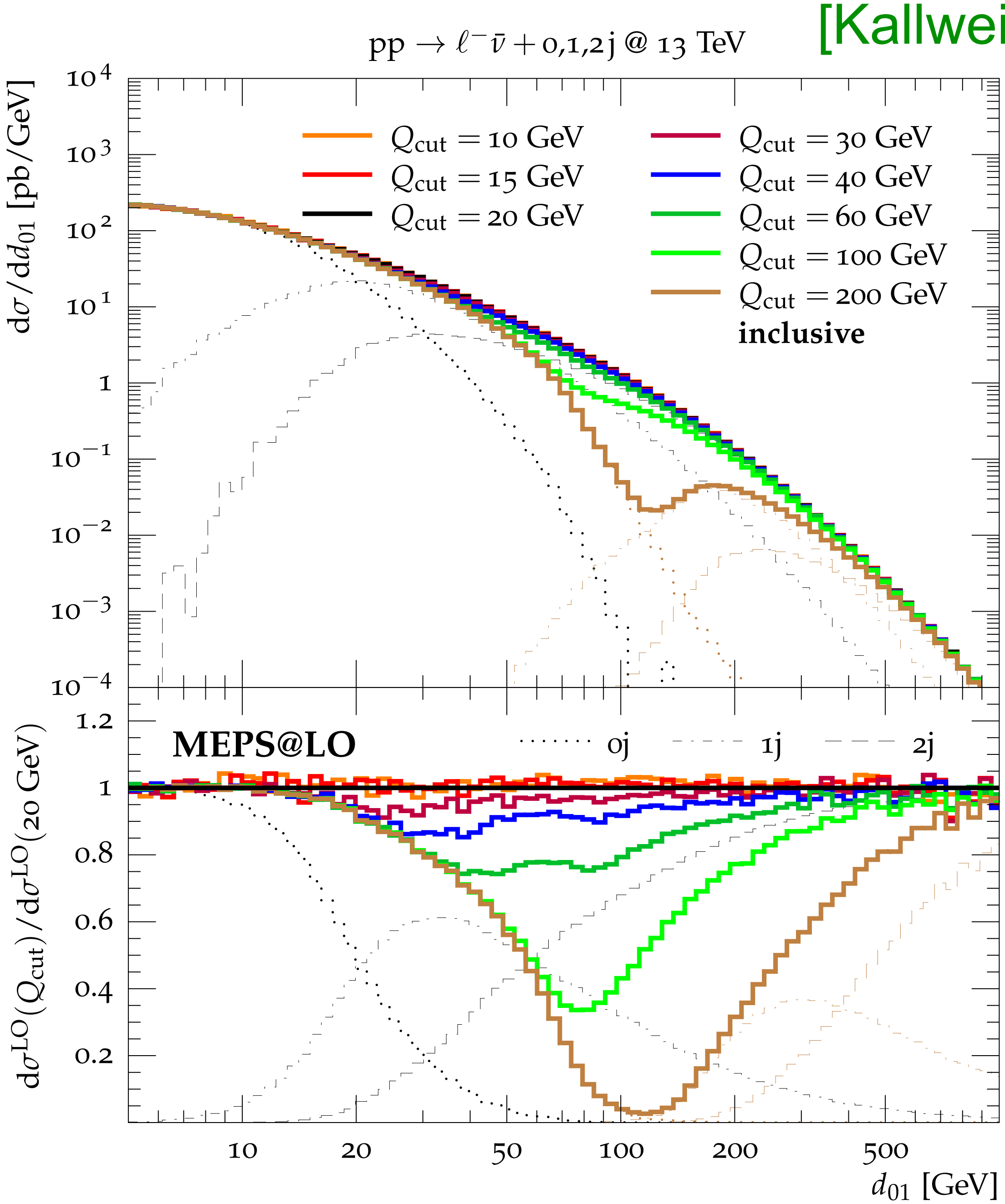
RF & Frixione, 2012

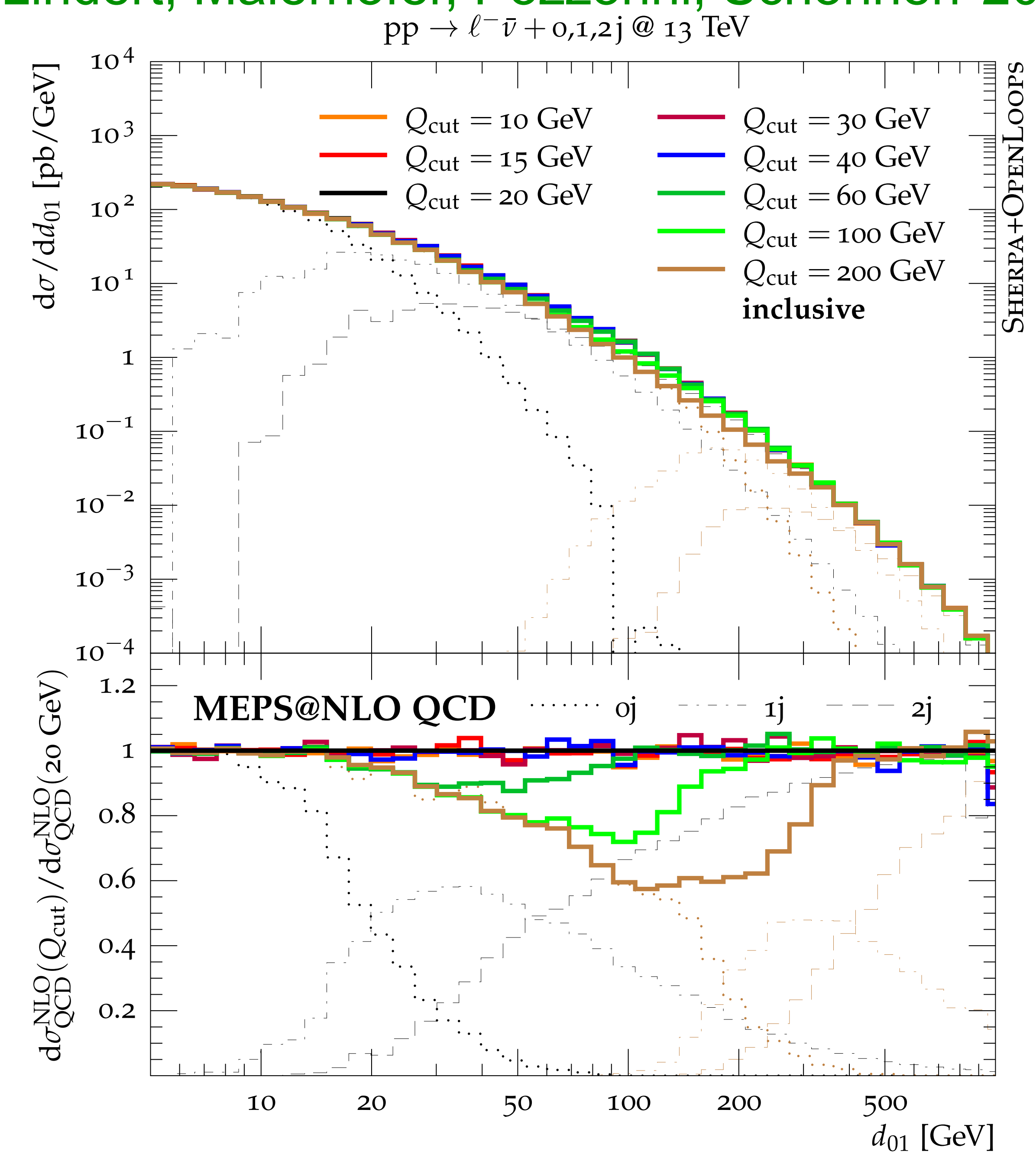


- Differential jet rates
- Matching up to 2 jets at NLO
- Results very much consistent with matching up to 1 jet at NLO

Merging scale dependence

[Kallweit, Lindert, Maierhöfer, Pozzorini, Schönherr 2016]

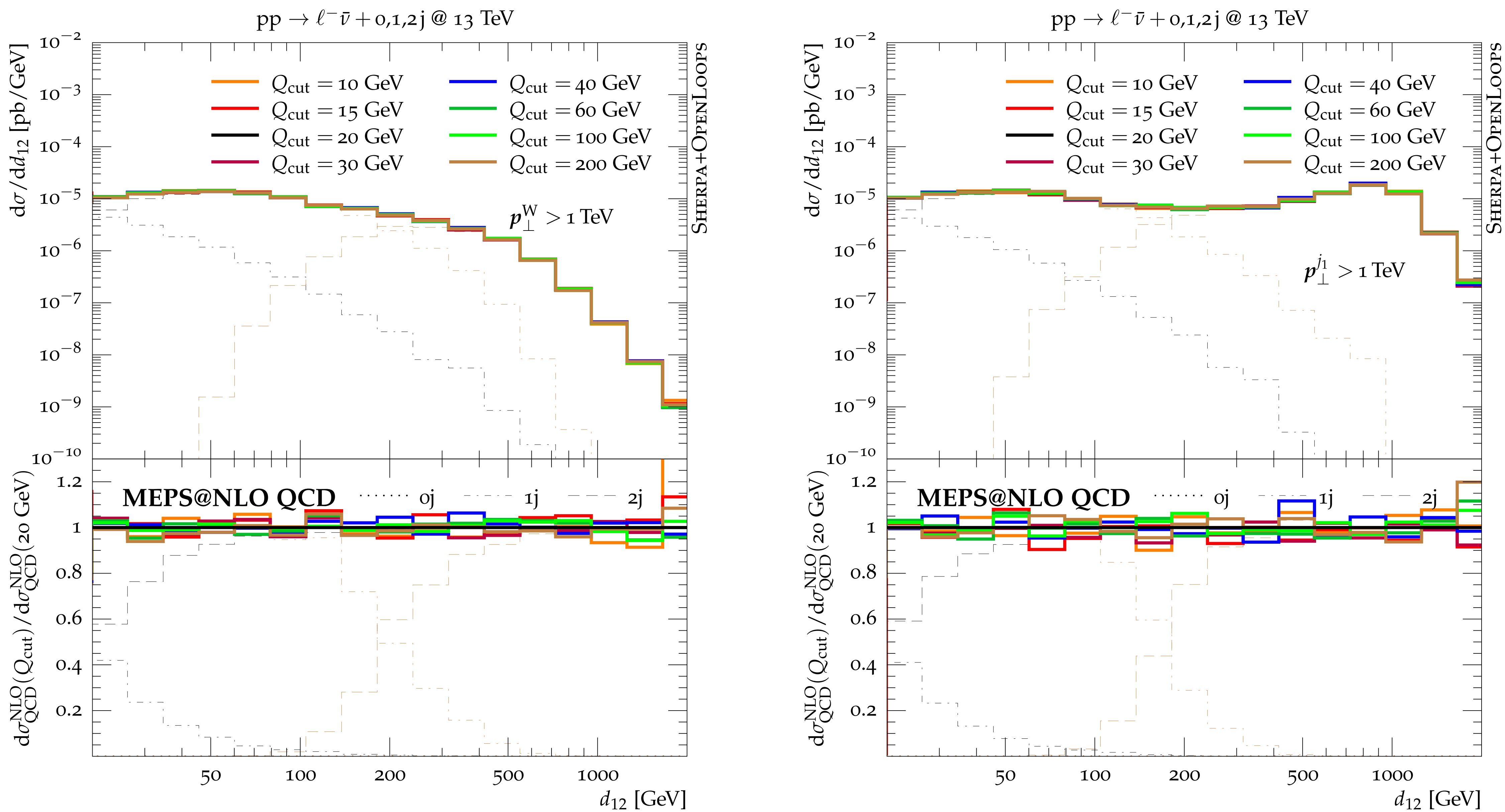




- Smaller merging scale dependence at NLO

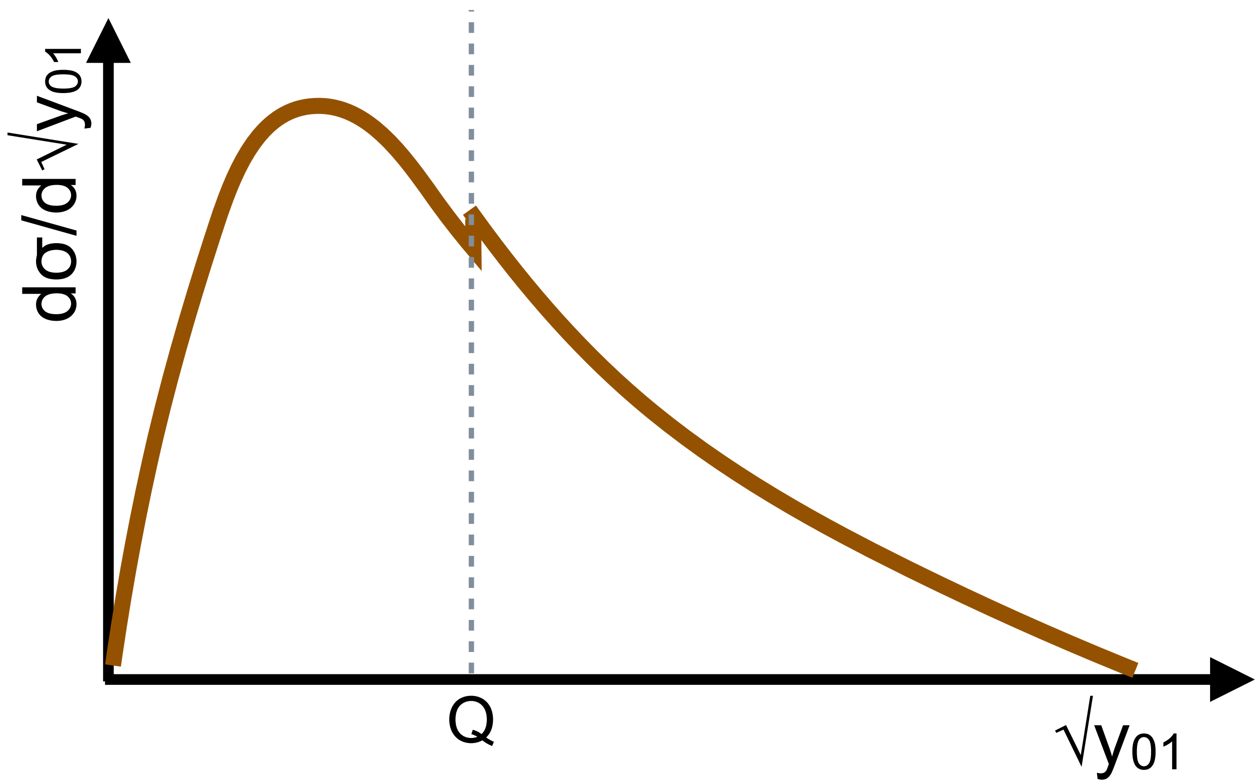
Merging scale dependence

[Kallweit, Lindert, Maierhöfer, Pozzorini, Schönherr 2016]



$\Rightarrow$ dead zones in incl. obs. if Q_{cut} too high

FxFx / Meps@nlo: V & V+1j merging



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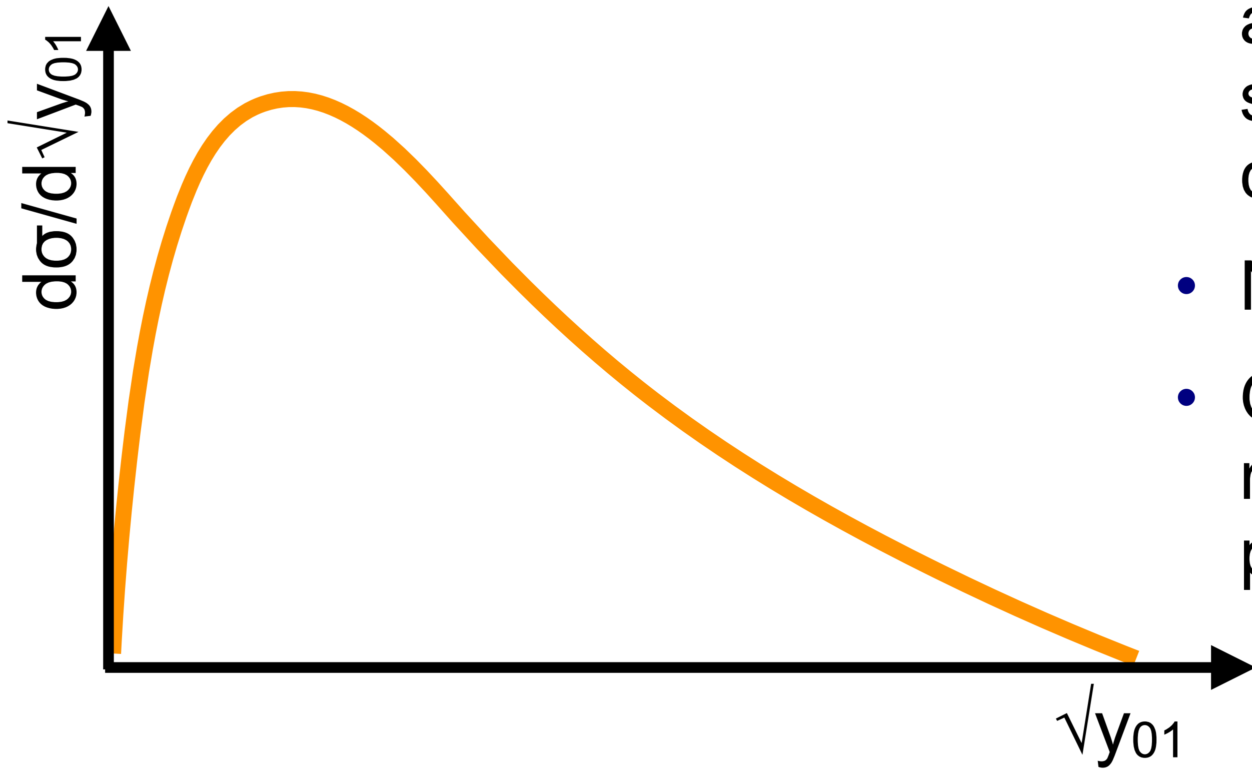
- Merge NLO+PS for V with Minlo for V+1j, at "merging scale" Q
- Above Q the tail is NLO accurate
- For not-too-small Q, integral is NLO accurate
- Used by ATLAS & CMS for LHC run II analyses

Physical curve	**"Yes"**
Tail	**NLO**
Integral	**"NLO" (depending on Q)**
Extendible to multi-jet	**Yes**

Geneva



[Alioli, Bauer, Berggren, Tackmann, Walsch (2015)]

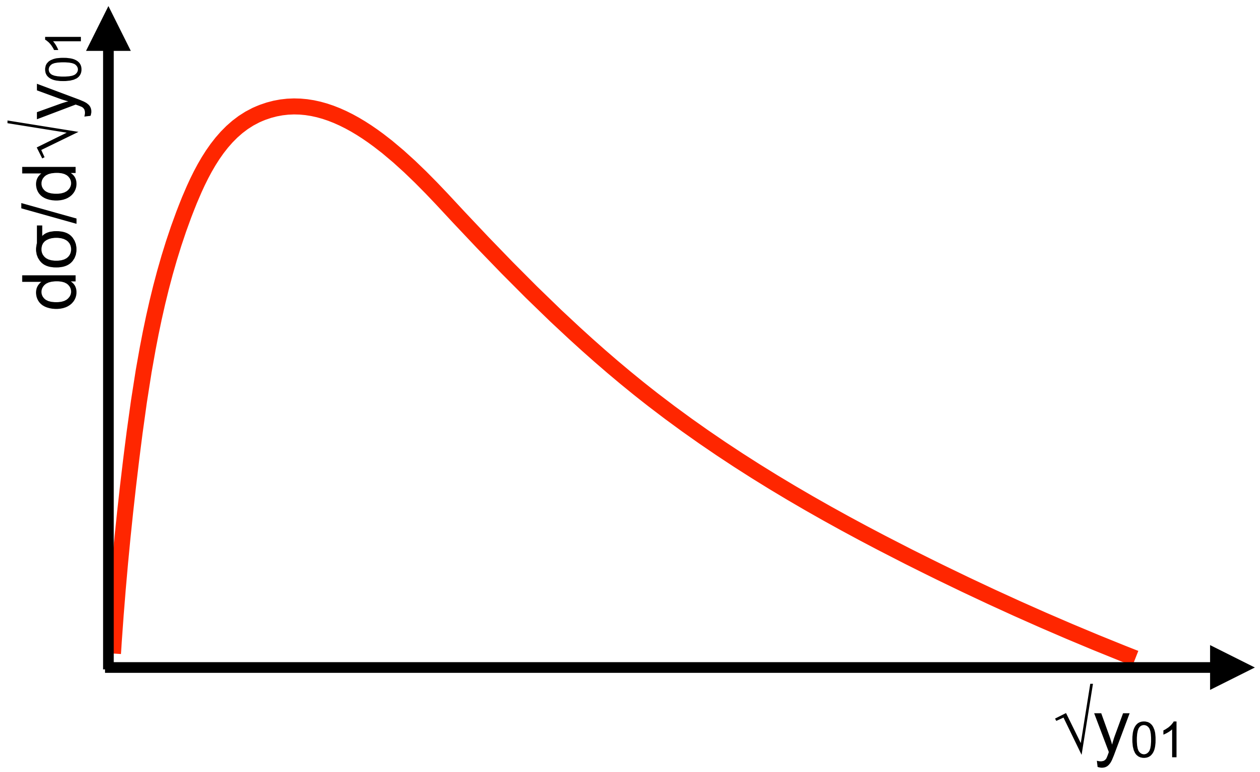
- Start from NNLO for V, add NNLL' analytic resummation
- High-enough orders in resummation accuracy circumvents the need of merging scale: already includes NLO for the complete $p_T(j)$ spectrum
- Non-trivial to attach parton shower
- Only available for W-boson production: rather difficult to extend, even though in principle possible

Physical curve	**Yes**
Tail	**NLO**
Integral	**NNLO**
Extendible to multi-jet	**Tricky**

Geneva

- Not based on MC@NLO or POWHEG for event generation. Rather, just like UNLOPS, use projections to underlying kinematics to allow for event generation
  - No real issues with inefficiencies here: can put this cut to very small value ~1 GeV; similar to a shower cut-off or phase-space slicing parameter in NNLO computations
  - Projections done very carefully. No issues with mismatches
    - First steps to N-jettiness subtraction instead of slicing for NNLO?
- Split phase-space according to variable that is easy to resum: N-jettiness
  - It is known how to resum N-jettiness up to NNLL' accuracy
  - NNLO corrections naturally included in NNLL' resummation
  - N-jettiness and shower evolution are very different: need some gymnastics to attach a parton shower: recent study on underlying event studies shows that this seems to be under control [Alioli, Bauer, Guns, Tackmann (2015)]
- Very powerful approach

Minlo-Revisited V+1j



[Hamilton, Nason, Oleari, Zanderighi (2012);
Hamilton, Nason, Re, Zanderighi (2013);
RF, Hamilton (2015)]

- Much simpler as Geneva
- Like Minlo V+1j, include Sudakov form factors to make distribution physical at low p_T
- **Modify the Sudakov form factors with subleading, process dependent terms such that total integral becomes NLO accurate**
- Can include NNLO corrections for V

Physical curve	**Yes**
Tail	**NLO**
Integral	**(N)NLO**
Extendible to multi-jet	**Yes**

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Minlo accuracy for (inclusive) 0-jet observables

Explicitly compute and remove that term in the Minlo calculation such that the integral $\int_{dL} \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL}$ is zero up to NLO

It's process dependent and not a constant in phase-space

- An explicit comparison between the diff.-jet-rate-resummation formula (which integrates to the correct NLO 0-jet diff. cross section) and Minlo shows that they differ by terms of order

$$\frac{d\sigma_{\mathcal{MR}}}{d\Phi dL} = \frac{d\sigma_0}{d\Phi} \exp\left[-R(v)\right] \prod_{\ell=1}^{n_i} \frac{q^{(\ell)}\left(x_\ell, \mu_F^2 v\right)}{q^{(\ell)}\left(x_\ell, \mu_F^2\right)} \left[\bar{\alpha}_S^2 \left(K_R^2\, y\right) \left[\widetilde{R}_{21} L + \widetilde{R}_{20}\right] + \bar{\alpha}_s^3 \left(K_R^2\, y\right) L^2\, \widetilde{R}_{32}\right]$$

- After integration over the logarithm L (taking R_{21}=0, which is okay for the processes considered here) this results into terms of

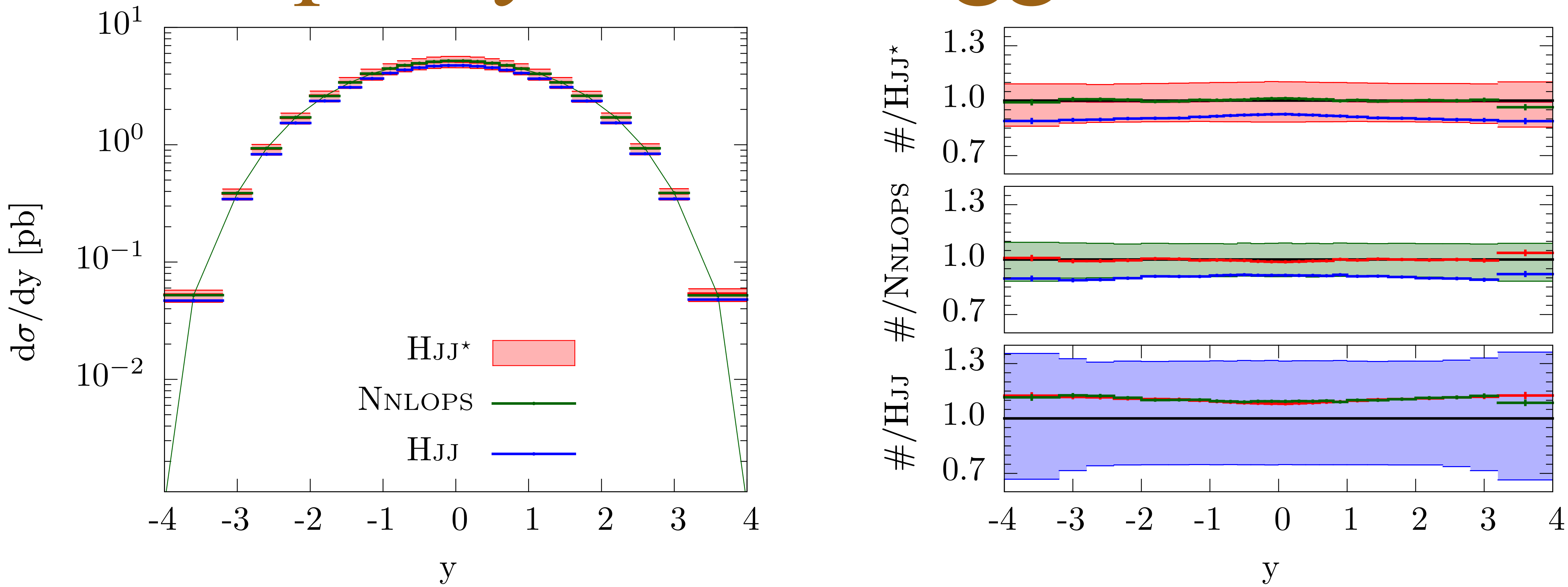
$$\int dL' \frac{d\sigma_{\mathcal{MR}}}{d\Phi dL'} = -\frac{d\sigma_0}{d\Phi} \left[\widetilde{R}_{20} - \bar{\beta}_0 \mathcal{H}_1 \left(\mu_R^2\right)\right] \sqrt{\frac{\pi}{2}} \frac{1}{\left|2G_{12}\right|^{1/2}} \bar{\alpha}_s^{3/2} \left(1 + \mathcal{O}\left(\sqrt{\bar{\alpha}_s}\right)\right)$$

Can either be done analytically or numerically by enforcing unitarity

- Hence, diff. NLO-0jet cross section not correct with NLO-1jet Minlo

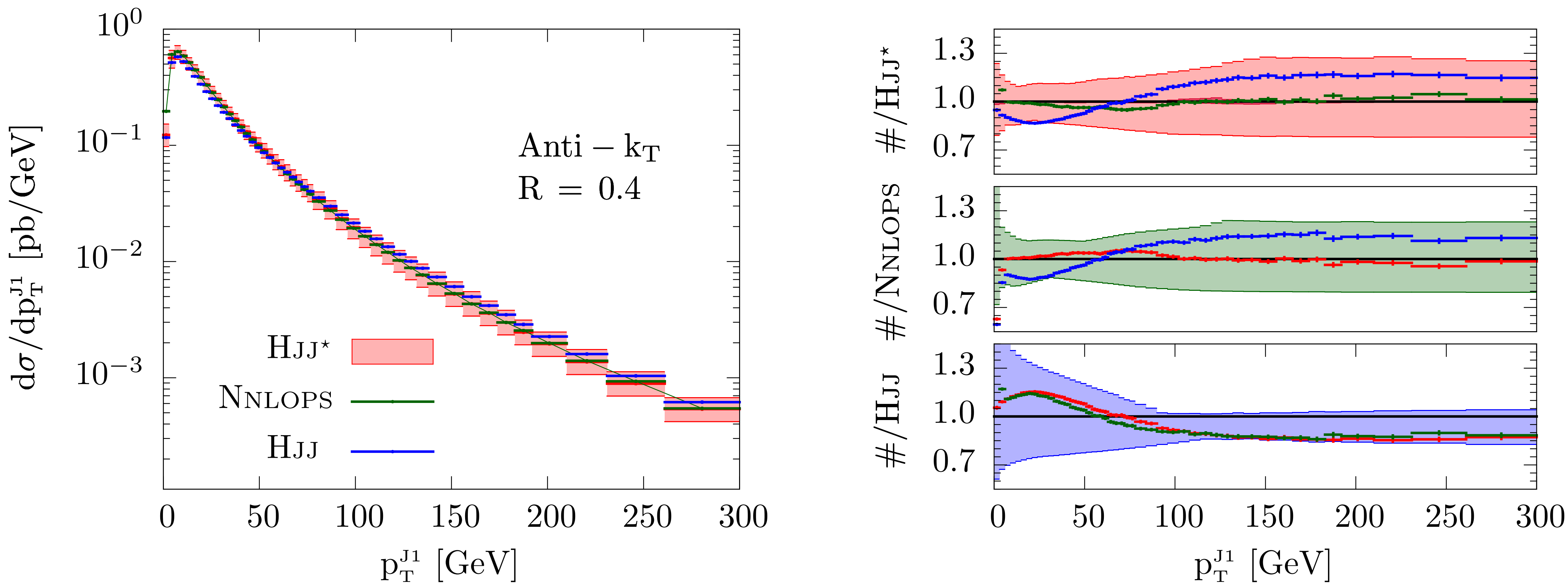
[Hamilton, Nason, Oleari, Zanderighi (2012); RF, Hamilton (2015)]

Rapidity of the Higgs boson



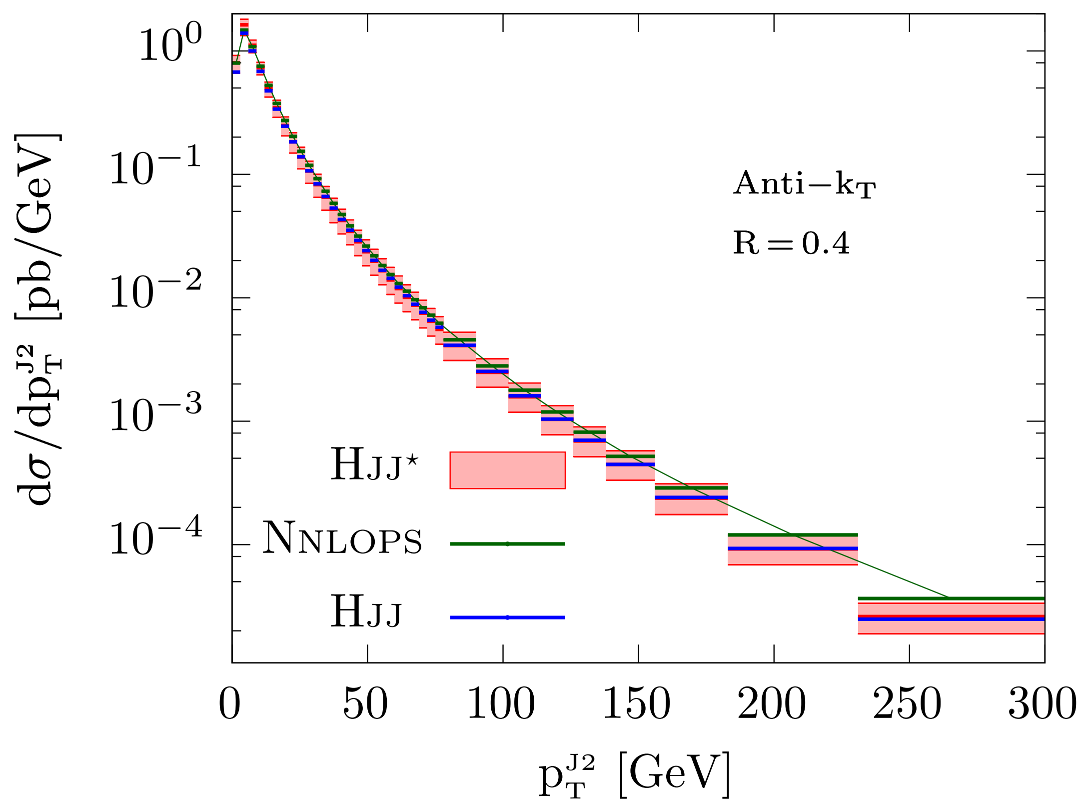
- Only observable truly NNLO correct
- Extended Minlo' method (HJJ★) agrees with NNLOPS by construction
- Normal HJJ Minlo shows larger uncertainty bands and different central value: it's only LO accurate for this observable

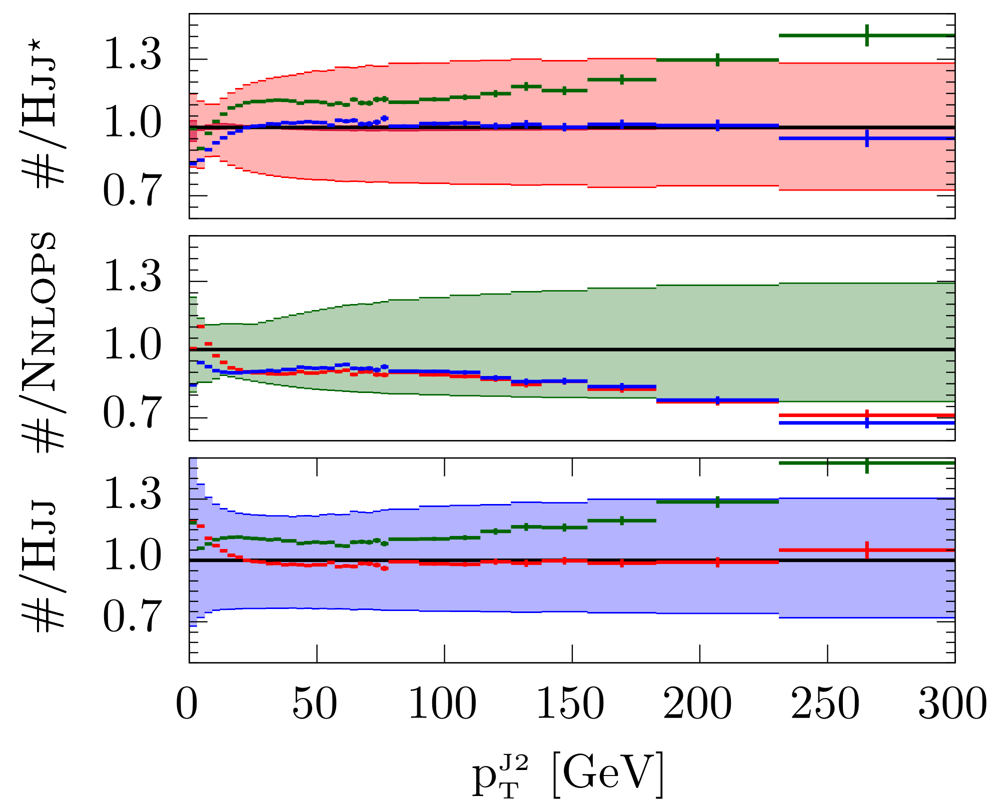
Transverse momentum of the leading jet



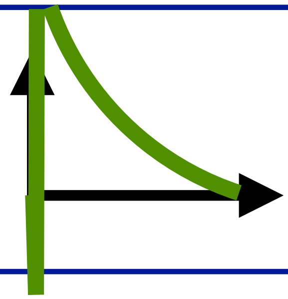
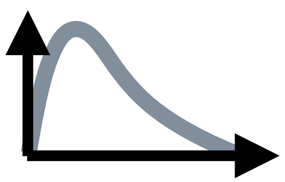
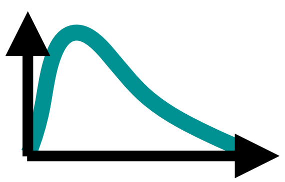
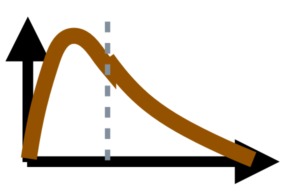
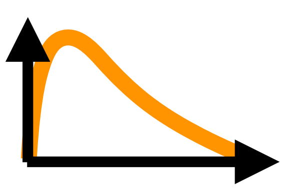
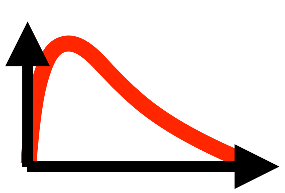
- Extended Minlo’ method (HJJ⋆) agrees with NNLOPS by construction.
- Normal HJJ Minlo shows unphysical uncertainty band. Formally only LO for this observable

Transverse momentum of the second jet

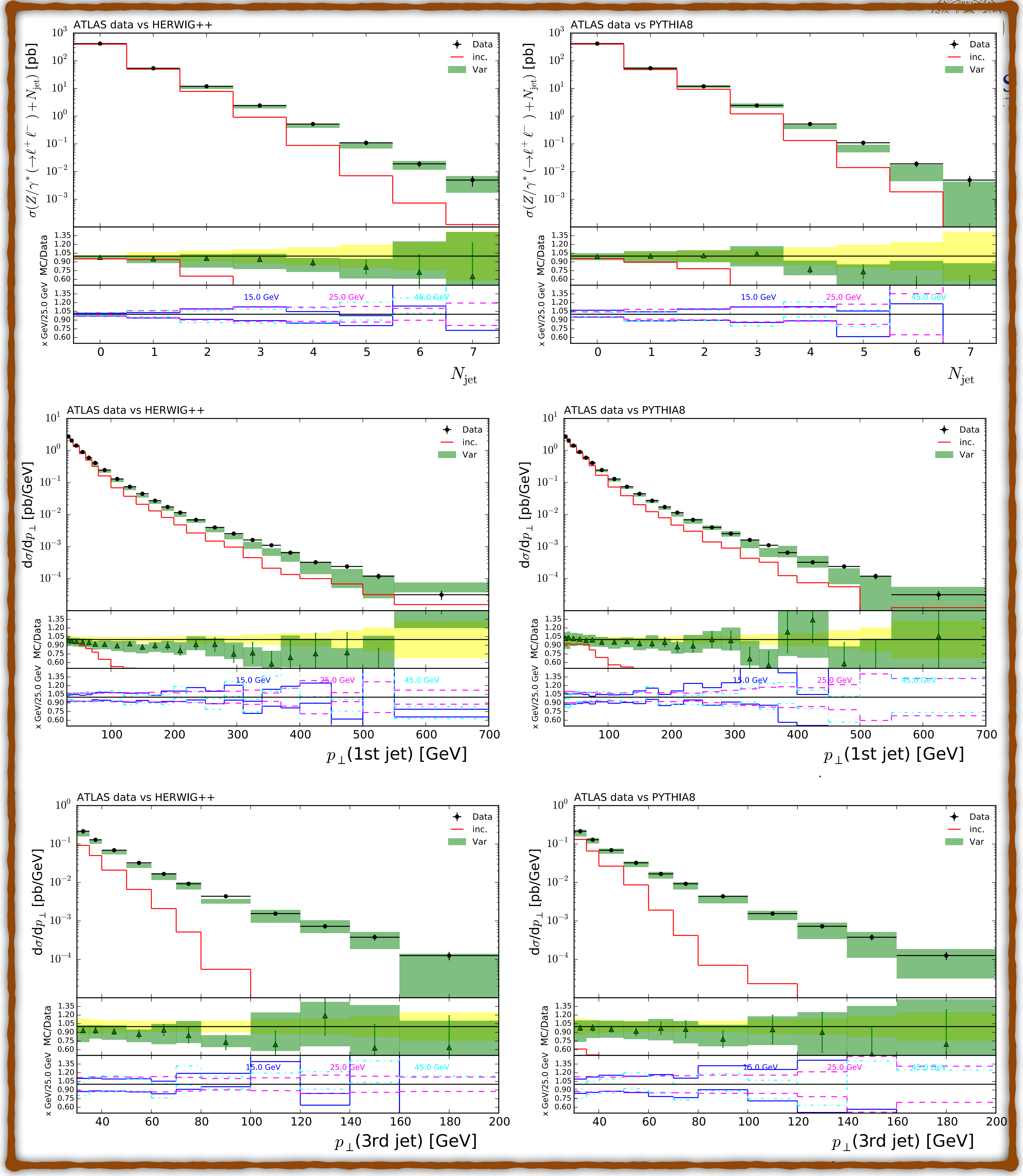




- Extended Minlo' method HJJ⋆ agrees with Minlo HJJ, as expected
  - apart close to the Sudakov peak: the difference between HJJ⋆ and HJJ is beyond LL/NNLL$_\sigma$ accuracy, which is important close to the Sudakov peak
- NNLOPS only LO accurate for this observable: uncertainty band is too small (this is due to the POWHEG method)

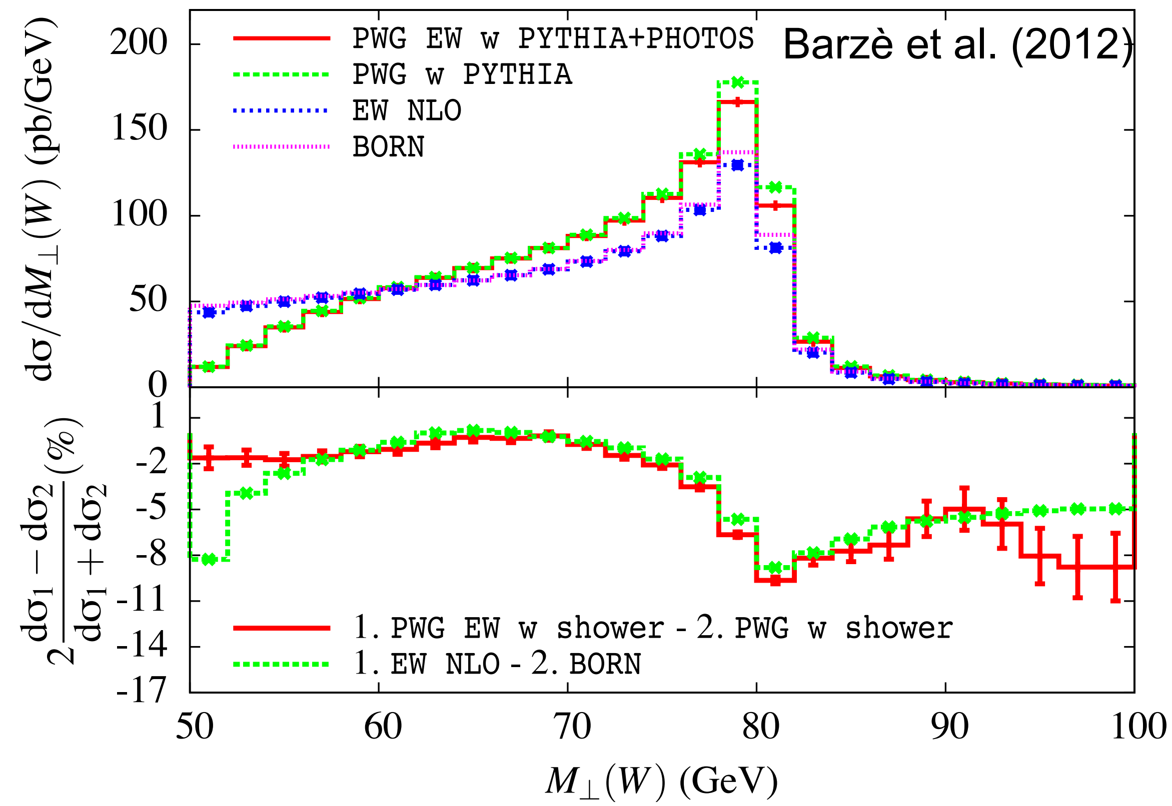
		NLO rate?	NLO tail?	physical?	comment
	NLO V+0j	✓	✗	✗	
	NLO+PS V+0j	✓	✗	✓	fully automated
	Minlo V+1j	✗	✓	✓	
	FxFx/ MEPS@NLO V+0,1j	✓*	✓*	✓	Combines NLO+PS and Minlo
	Geneva	✓	✓	✓	allows for NNLO
	Minlo' V+1j	✓	✓	✓	allows for NNLO

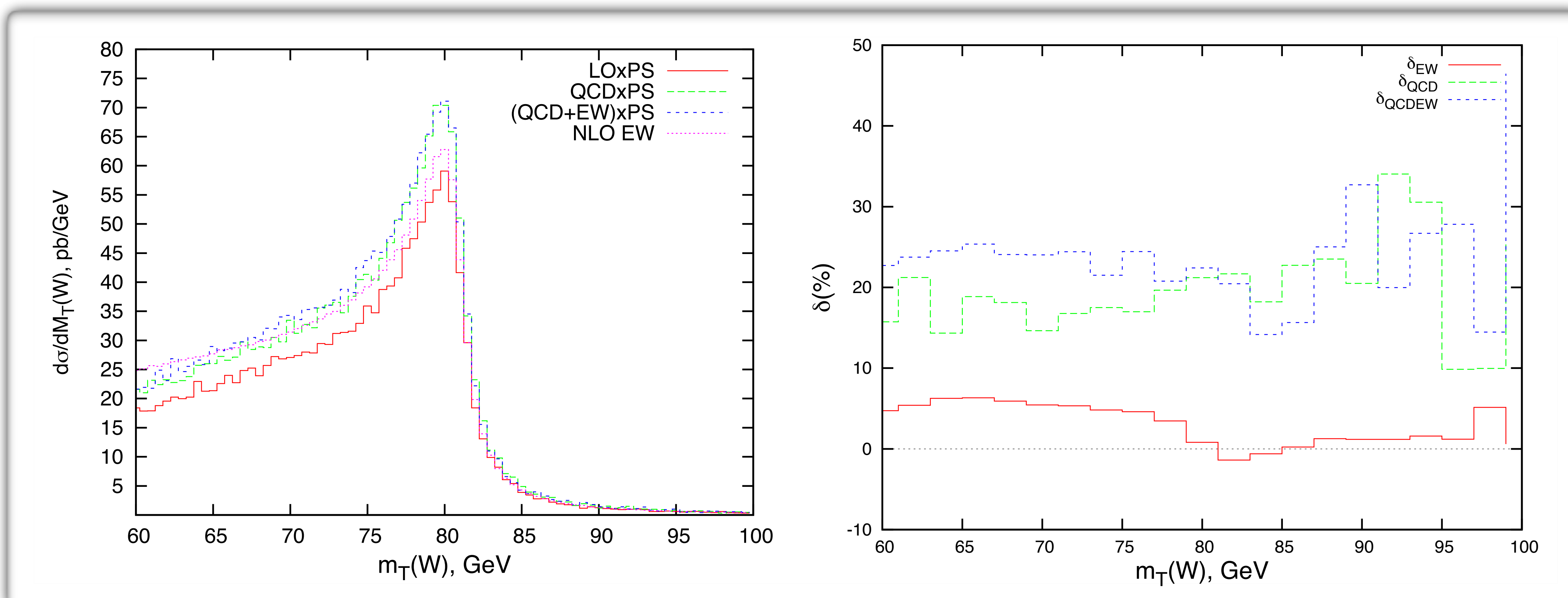
- Comparison to data
- Z+jets
- Exclusive jet multiplicity and hardest and 3rd hardest jet pT spectra
- Uncertainty band contains ren. & fac. scale, PDF & merging scale dependence
- Rather good agreement between data and theory



NLO+PS matching including EW corrections

- In POWHEG, two independent implementations of QCD+EW corrections to W-boson production exist [Bernaciak & Wackeroth (2012); Barzè et al. (2012)]
- MG5_aMC and Sherpa working towards automation. Some first results with Sherpa+OpenLoops have been presented, although they include only EW corrections of virtual origin [Kallweit et al. (2015)]





Conclusions

- The accuracy of event generation has greatly improved since the start of the Large Hadron Collider: NLO merging has become the norm.
  - First results for matching/merging with EW corrections, and also matching NNLO ME to PS
- A lot of freedom in tuning has been replaced by accurate theory descriptions:
  - More predictive power
  - Better control on uncertainties
  - Greater trust in the measurements