An introduction to large momentum effective (field) theory

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Refs: arXiv: 2007.06613

& arXiv:2004.03543

outline

- Introduction to effective (field) theories
- Partons as an effective theory concept
- laMET: a Euclidean EFT for partons
- Some examples
- conclusion

Introduction to effective (field) theory

Standard methodology in physics

- Ignore unimportant factors, focusing on the most important ones.
- Idealized concepts
  - Point-like particles
  - Frictionless surfaces
  - Ideal gases
  - Ideal fluids
  - Absolute-zero temperature
  - Infinite potential well
  - Harmonic oscillators
  - …

Math methods: Taylor expansion

- A physical quantity f may depend important variable x, less important ones ε, δ, etc

$$f(x, \epsilon, \delta, \dots)$$

- One can simplify the problem by making Taylor expansion

$$f(x, \epsilon, \delta, \dots) = f(x, 0, 0, \dots) + \epsilon f_\epsilon(x, 0, 0, \dots) + \delta f_\delta(x, 0, 0, \dots) + \dots\ .$$

- Most of time, we just care about the first term. But higher order terms can be calculated systematically. This is an effective approach, or theory.

More examples

- Multipole expansion in electrostatics
- Virial expansion for equation of state
- Perturbation theory in celestial mechanics
- Perturbation theory in QM $H = H_0 + H'$

If H_0 contains a cluster of states that have similar eigenvalues and span a subspace P of dimension d_P, then the eigenstates of H with largest overlaps with P can be obtained through an effective Hamiltonian,

$$H_{\rm eff} = PHP + PH'\frac{Q}{E - H_0}H'P + \ldots \qquad (3)$$

A few remarks

- The complementary space P = 1-Q has been summed or “integrated out”
- Expansion parameter is the energy ratio

$$\epsilon = ||H'||/\Delta E$$

where ΔE is the energy difference between P and Q spaces

- To calculate the observables using “effective wave functions”, one needs to have “effective operators”, O_{eff}

Effective field theory

- EFT refers to a theory in which an effective approach has been applied for QFT.
- Some well-know examples
  - Renormalization is an EFT, thus all QFT are EFTs
  - Standard model is an EFT
  - QCD perturbation theory is an EFT
  - Chiral perturbation theory
  - Lattice QCD
  - Heavy quark effective theory (HQET)
  - Soft collinear effective theory (SCET)
  - ….

New feature of EFT: UV divergences

- Field theories have UV divergences, which are generally renormalized to make finite predictions.
- UV divergences make Taylor expansion more complicated.

$f(x, \epsilon, ..., \Lambda)$, which now contains a UV cut-off scale Λ. If one does a Taylor expansion around $\epsilon = 0$, one finds there is an ambiguity. Either you expand after finishing the full calculation, or you take $\epsilon = 0$ beforehand. There is a difference because taking $\epsilon \to 0$ does not commute with $\Lambda \to \infty$ and the function $f(x, \epsilon, ..., \Lambda)$ is nonanalytic at the point $\epsilon = 0$!

$\epsilon = 0$ is not a legitimate point for T-expansion!

Standard EFT approach: Matching

The standard EFT methodology is to take $\epsilon = 0$ before doing any computation. An effective Lagrangian is constructed to evaluate $f(x, \epsilon = 0, ..., \Lambda)$, and this calculation is presumably simpler. However this does not give the right answer $f(x, \epsilon \to 0, ..., \Lambda)$. One needs to figure out what is their difference, and this is very important! This difference is quite often independent of other parameters x. So if one does a calculation for some specific values of x and figures out the difference, the result can be used for all x. Once an effective theory calculation is done, one can get the right Taylor series by adding up the difference. This is called EFT matching! Matching is needed to get the effective Lagrangian as well as effective operators.

Standard EFT approach: Running

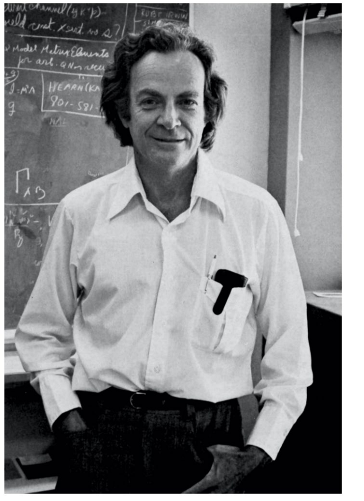
The UV behavior of an EFT at $\epsilon = 0$ is very different from the full theory, and this difference can be exploited for useful purposes. It can help to sum up the so-called large logarithms in the coupling constant expansion of the full theory through the renormalization-group running in the EFT.

Matching and running are standard methods in EFT which have been exploited for various purposes

Large momentum effective theory uses EFT technique and makes parton physics calculable in Euclidean formulation of QCD, such as lattice gauge theory

Partons as an effective theory concept

Feynman's parton model

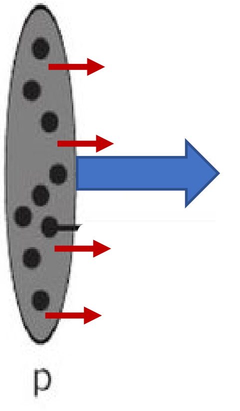


- When a high-energy proton travels at $v \to c$, one can assume the proton travels exactly at v=c, or the proton momentum is

$$p=E=\infty$$

(Infinite momentum frame, IMF)

- The proton may be considered as a collection of interaction-free particles: partons



PHYSICAL REVIEW VOLUME 185, NUMBER 5 25 SEPTEMBER 1969

Inelastic Electron-Proton and γ-Proton Scattering and the Structure of the Nucleon*

J. D. BJORKEN AND E. A. PASCHOS
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 10 April 1969)

A model for highly inelastic electron-nucleon scattering at high energies is studied and compared with existing data. This model envisages the proton to be composed of pointlike constituents ("partons") from which the electron scatters incoherently. We propose that the model be tested by observing γ rays scattered inelastically in a similar way from the nucleon. The magnitude of this inelastic Compton-scattering cross section can be predicted from existing electron-scattering data, indicating that the experiment is feasible, but difficult, at presently available energies.

I. INTRODUCTION

ONE of the most interesting results emerging from the study of inelastic lepton-hadron scattering at high energies and large momentum transfers is the possibility of obtaining detailed information about the structure, and about any fundamental constituents, of hadrons. We discuss here an intuitive but powerful model, in which the nucleon is built of fundamental pointlike constituents. The important feature of this model, as developed by Feynman, is its emphasis on the infinite-momentum frame of reference.

Parton distribution functions (PDF)

- Every parton has $k=\infty$, however,

  $x=k/p$ = finite, $\in [0,1]$

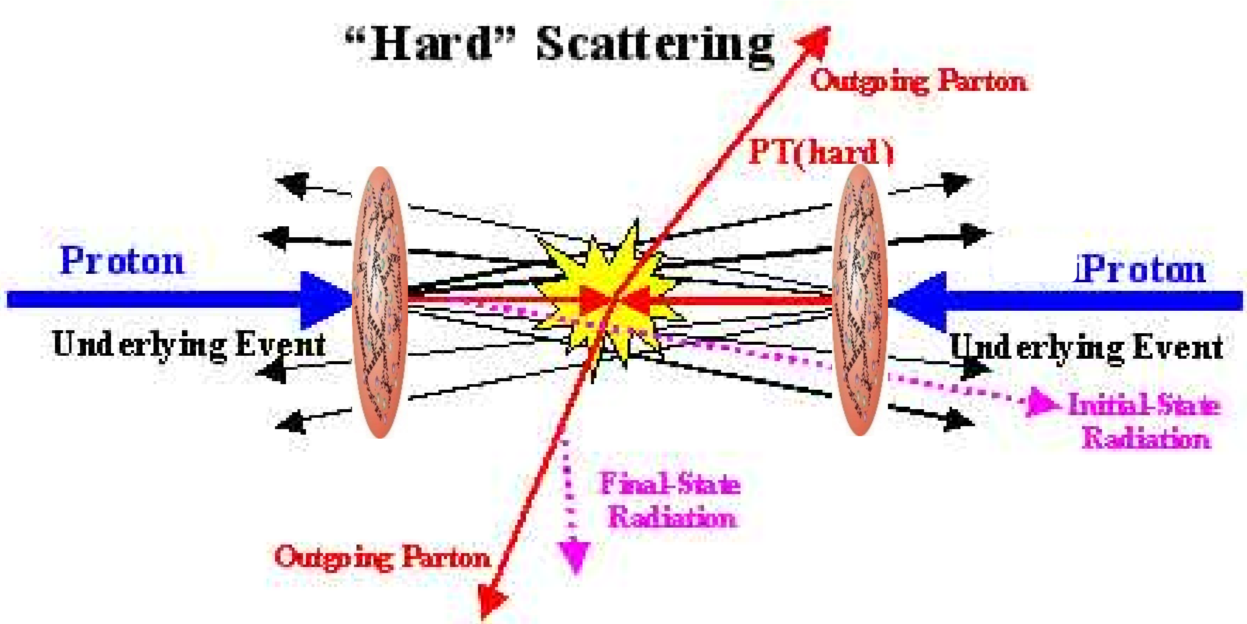
- Parton distribution function

  $$f(x)$$

is the probability of finding parton in a proton, carrying x fraction of the momentum of the parent.

- PDF is a bound state property of the proton, essential to explain the results of high-energy collisions.

EFT for hard scattering: factorization

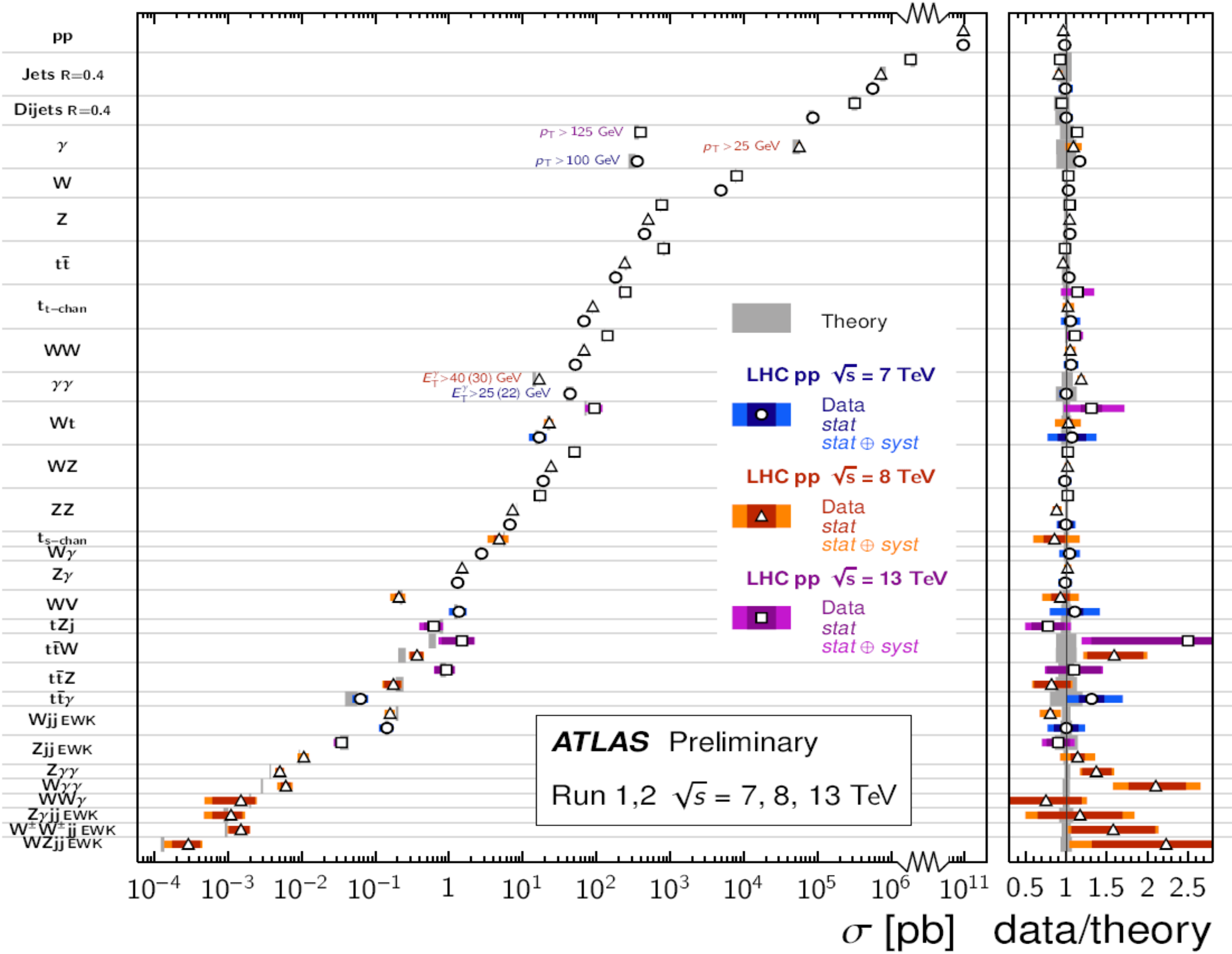


- Factorization theorems or EFT for scattering: The scattering cross sections are factorized in terms of PDFs and parton x-section.

$$\sigma = \int dx_a dx_b f_{a/A}(x_a) f_{b/B}(x_b) \hat{\sigma}$$

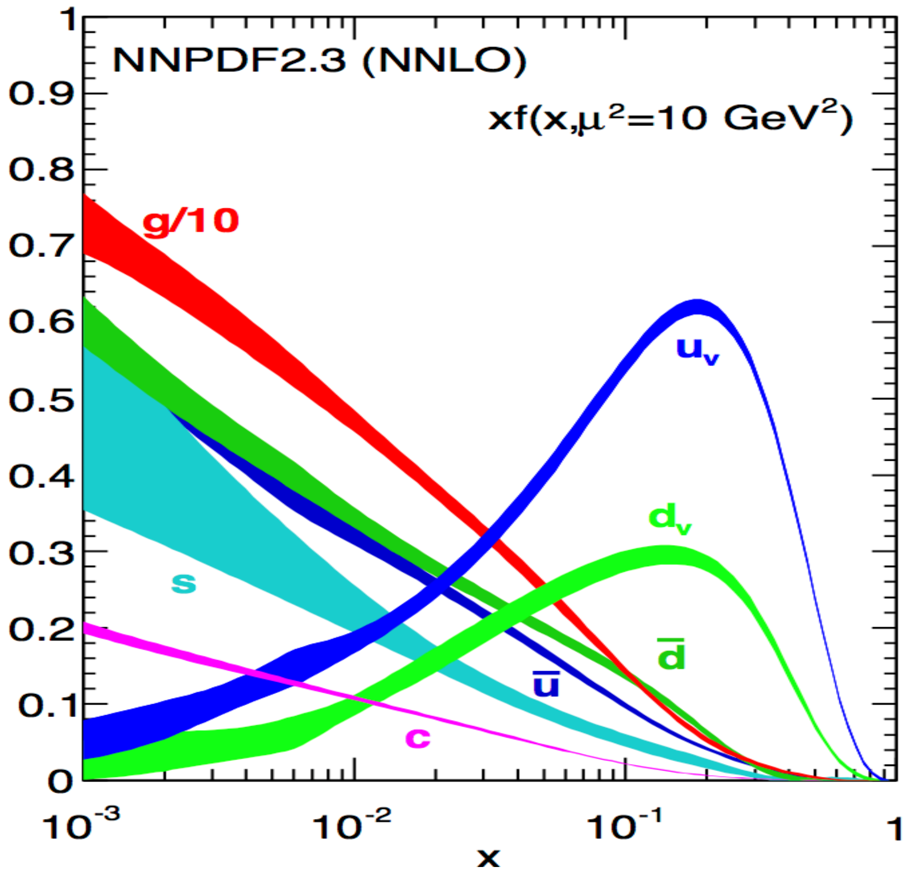
Standard Model Production Cross Section Measurements

Status: July 2017



Phenomenological PDFs

- Use experimental data (50 yrs) to extract PDFs



J. Gao, et al, Phys. Rept. 742 (2018) 1-121

Partons in QFT are effective DOFs

- Partons are not just the quarks and gluons in the usual QCD lagrangian.
- Partons are a special type of IR collinear modes with momentum

$$k^{\mu} = \left(k^0, k^z, \vec{k}_{\perp}\right)$$

with $k^z \to \infty, k^0 \to \infty, k_{\perp} \sim \Lambda_{QCD}, k_{\mu}^2 \sim \Lambda_{QCD}^2$

Collins, Soper and Sterman, QCD factorization, 70'-80's

Bauer, Stewart et al, Soft-Collinear EFT, 00's

Weinberg's rules

- What does an object look like when travelling at infinite momentum or speed of light?
- S. Weinberg (scalar QFT)

  Dynamics at infinite momentum

  Phys. Rev. 150 (1966) 1313-1318

- All kinematic infinities can be removed from the calculations, resulting a set of rules for Hamiltonian perturbation theory ("old-fashioned p.t.")
- The result is similar to a "non-relativistic" theory.

More Weinberg's rules…
and a discovery

- L. Susskind, K. Bardakci, and M. B.Halpern,…
- S. J. Chang and S. K. Ma (1969)

Feynman rules and quantum electrodynamics at infinite momentum,

Phys.Rev. 180 (1969) 1506-1513

- J. Kogut and D. Soper

Quantum Electrodynamics in the Infinite Momentum Frame,
Phys.Rev.D 1 (1970) 2901-2913

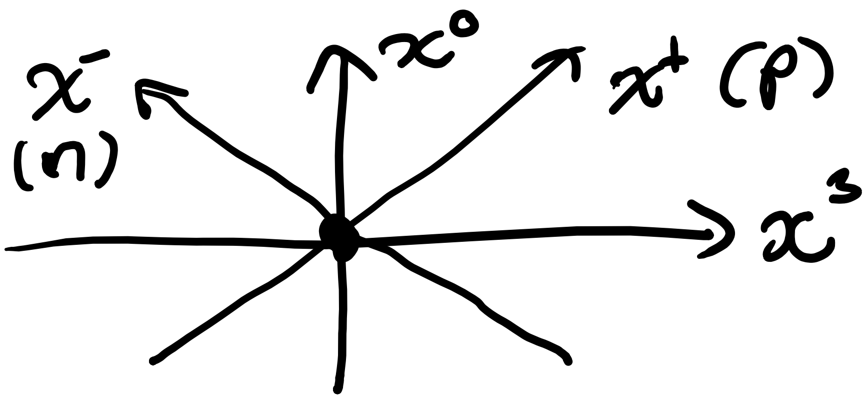
Chang and Ma's discovery

- All Weinberg's rules in the P=∞ limit can be obtained by quantizing the theory with "new coordinates"

$$x^+ = \frac{x^0 + x^3}{\sqrt{2}},\ x^- = \frac{x^0 - x^3}{\sqrt{2}}$$

by treating x^+ as the new "time"

x^- as the new "space" dimension.



Dirac's form of dynamics

- The Weinberg's rules exactly correspond to what Dirac proposed in 1949.
- Paul A.M. Dirac,

Forms of Relativistic Dynamics,

Rev. Mod. Phys. 21 (1949) 392-399.

"Front form"

or Light-front quantization (LFQ)

Why it is hard to solve QCD in LFQ?

- All slow-moving stuff in zero-modes (vacuum).
- EFT has extra rapidity divergences which are not entirely UV nature.
- It is a strongly coupled problem!
- There is no demonstration that the weak coupling expansion actually works for QCD.

K. Wilson et. al. Phys. Rev. D49 (1994)

Light-front collinear modes

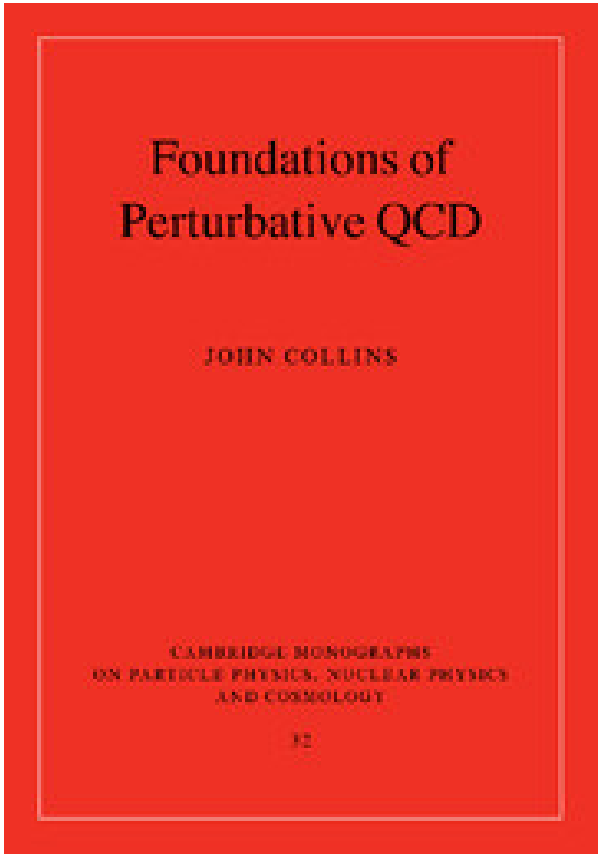
- In lagrangian formulation of parton physics, the partons are represented by collinear modes in QCD

$$\psi(\lambda n)\,,\ n^2 = 0$$

λ is the distance along the LF

- Parton physics is related to correlations of these fields along n with distance λ.

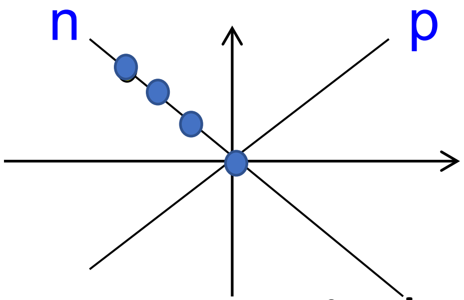
e.g. Soft-collinear Effective Theory (SCET)



Partons as LF correlations

- Probes（operators) are light-cone correlations

$$\hat{O} = \phi_1(\lambda_1 \mathrm{n})\phi_2(\lambda_2 n)\ldots\phi_k(\lambda_k n)$$



- The matrix elements are independent of hadron momentum, and they can be calculated in the states in the rest frame.
- “Heisenberg picture”

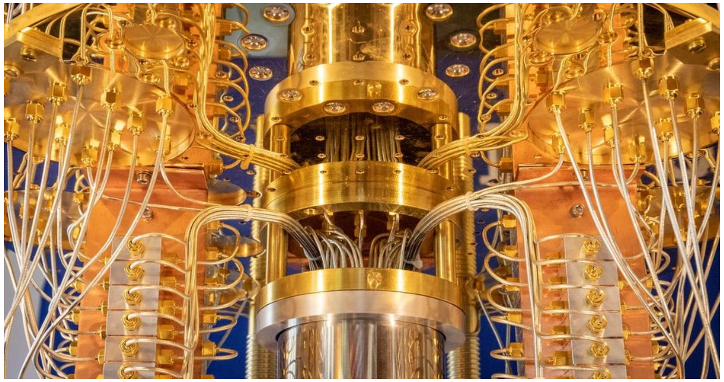
Real-time Monte Carlo in path integrals?

- Monte Carlo simulations have not been very successful with quantum real-time dynamics.

$$\exp(-iHt)$$

  an oscillating phase factor!

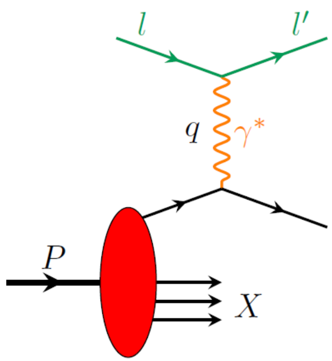
- "Sign problem": Hubbard model for high T_c.
- Signals are exponentially small!
- Quantum computer?



Feynman parton in QFT & connection with other parton EFTs

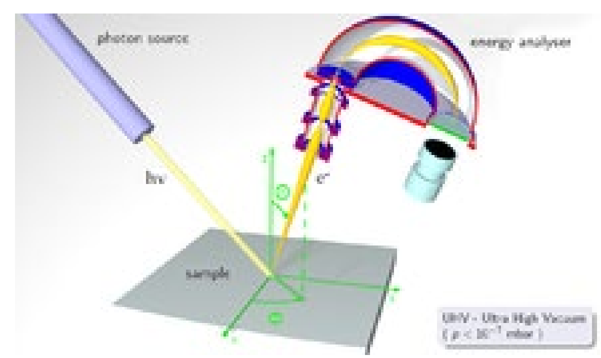
Origin of Parton Model

- Electron-proton deep-inelastic scattering (DIS)



- Knock out scattering in NR systems
  - e-scattering on atoms
  - ARPES in CM systems
  - Neutron scattering on liquid He

  …



Momentum distribution in NR systems

- Knock-out reactions in NR systems probes momentum distribution

$$n\left(\vec{k}\right) = \left|\psi\left(\vec{k}\right)\right|^2$$

$$\sim \int \psi^*(\vec{r})\psi(0)e^{i\vec{k}\vec{r}}d^3r$$

$$\sim \int \langle\Omega\left|\hat{\psi}^+(\vec{r})\hat{\psi}(0)\right|\Omega\rangle e^{i\vec{k}\vec{r}}d^3r$$

- Mom.dis. are related to Euclidean correlations, generally amenable for Monte Carlo simulations.

Difference between relativistic and NR systems

- NR cases, the energy transfer is small.

$$q^0 \sim \frac{1}{M} \sim 0$$

- Relativistic systems:

  In DIS, if we choose a frame in which the virtual photon energy is zero

$$q^\mu = (0, 0, 0, -Q)\,,$$
$$P^\mu = \left(\frac{Q}{2x_B} + \frac{M^2 x_B}{Q}, 0, 0, \frac{Q}{2x_B}\right)\,,$$

In the Bjorken limit, $P^z \sim Q \rightarrow \infty$

Feynman's partons

- Consider the mom.dis. of constituents in a hadron

$$f(k^z, P^z) = \int d^2k_\perp\, f(k^z, k_\perp, P^z)$$

which depends on P^z because of relativity.

(H is not invariant under boost K)

- PDF is a result of the $P^z \to \infty$ limit,

$$f(k^z, P^z) \to_{P^z \to \infty} f(x) \quad with\; x = \frac{k^z}{P^z},$$

Or

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + \ldots$$

Euclidean formulation of partons

- Calculate the Euclidean correlation

$$C(\lambda) = \langle P^z = \infty | \overline{\psi}(z) \Gamma \psi(0) | P^z = \infty \rangle$$

$$\lambda = \lim_{P^z \to \infty, z \to 0} (z P^z).$$

- Parton distribution

$$f(x) = \frac{1}{2P^+} \int \frac{d\lambda}{2\pi} e^{ix\lambda} C(\lambda) \ .$$

Relations among parton formalisms

	States	operators	Time-signature
LFQ	$P=\infty$	LF correlators	Minkowski
SCEF	p=finite	LF correlators	Minkowski
Feynman	$p=\infty$	Equal time	Euclidean

Large-momentum expansion and EFT

Take $P^z \to \infty$ limit

- Highly non-trivial. It must be studied in the context of a QFT (only a field theory can support ∞ momentum modes)
- Assuming the limit exist, the limiting process is controlled by expansion,

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + \ldots$$

where M is a bound-state scale,

P^z is a large-momentum scale.

Naïve dimensional analysis

- $\epsilon = \left(\frac{M}{P^z}\right)^2$ is an expansion parameter

  M =1 GeV, P=2 GeV

  $\epsilon = 1/4$

  the expansion may already work.

Large momentum expansion

- Approximate p=∞ by a large P
- This is what we frequently do in QCD

Lattice QCD: approximate a continuum
theory by a discrete one.
cut-off $\Lambda \to \infty$, on lattice $\Lambda = \pi/a$
0.1 fm ~ 2 GeV

HQET: $\epsilon = \Lambda_{QCD}/m_Q$
using $m_Q = \infty$ to approximate
m_c =1.5 GeV!

QFT subtleties

- There is a UV cut-off Λ_{UV}, $f(k^z, P^z)$ is not analytic at $P^z = \infty$!
- There are two possible $P^z \to \infty$ limits:
  1. $P^z \ll \Lambda_{UV} \to \infty$, IMF limit (lattice QCD)
  2. $P^z \gg \Lambda_{UV} \to \infty$ LFQ limit (HEP PDF)
- Calculating mom.dis. is done with the limit 1) and PDF is defined in limit 2).
- Solution: matching in EFT

  The difference is perturbative!

LaMET expansion

- One needs a proton fast enough

  the control parameter of expansion is

  $(M/P^z)^2 \sim 1/\gamma^2$

- Thus for finite momentum, one can have a factorization formula for large $\gamma \sim (2-5)$:

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x, \mu) dx + \mathcal{O}\Big(\frac{\Lambda^2_{\rm QCD}}{y^2(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(1-y)^2(P^z)^2}\Big),$$

$$\langle\gamma|\hat{O}(z_1, z_2, \ldots, z_k)|\gamma\rangle = Z(\alpha_s, \gamma, z_i, \lambda_i) \otimes \langle 1|\hat{O}(\lambda_1, \ldots, \lambda_k)|1\rangle + + o(1/\gamma^2)$$

Momentum renormalization group equation

- Mom.dis. $f(k^z, P^z)$ has a non-trivial dependence on P^z (H is frame-dependent).
- At large P^z, this dependence shall be calculable in perturbation theory

$$\frac{\partial O(P^z)}{\partial \ln P^z} = \gamma_P(\alpha_s) O(P^z)\,,$$

Momentum RGE

$$P^z \frac{\partial}{\partial P^z} \tilde{q}(y, P^z, \mu) = \int_0^1 \frac{dt}{|t|} P_{qq}(t) \times \tilde{q}\left(\frac{y}{t}, tP^z, \mu\right) - 2\gamma_F \tilde{q}(y, P^z, \mu)\ .$$

- DGLAP evolution is related to the change of mom.dis. with different CoM motion.

Recent developments

Recent progress

- High precision calculations
- GPDs
- TMDs
- Light-Front WFs

How to get high-precision PDF

- Small lattice spacing limit, $a \to 0$
- Renormalization of linear divergences
- Matching up to two loops
- Physical point, $m_\pi \to m_\pi^{phys}$
- Larger momentum P
- …

Continuum limit

- For LaMET application, fine lattice spacing is extremely important
- Effective expansion parameter is

$$r = aP^z \sim a\gamma$$

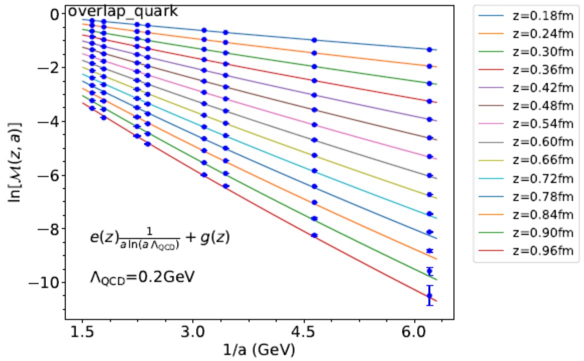
which can be large.

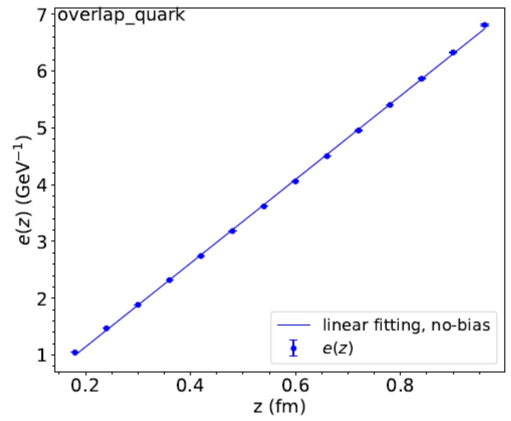
- Controlled extrapolation to continuum…

MSU, EMTC, BNL/ANL, LPC

Renormalization of linear divergences

- Renormalization of linear divergences causes large numerical uncertainties.
- The standard approaches (RI/MOM,ratio) could have large non-pert. effects
- New approaches
  - Auxiliary field approach (Green et al, Ji et al, 2018, C. Alexandrou et al, 2021)
  - Hybrid renormalization (Ji et al, 2021)

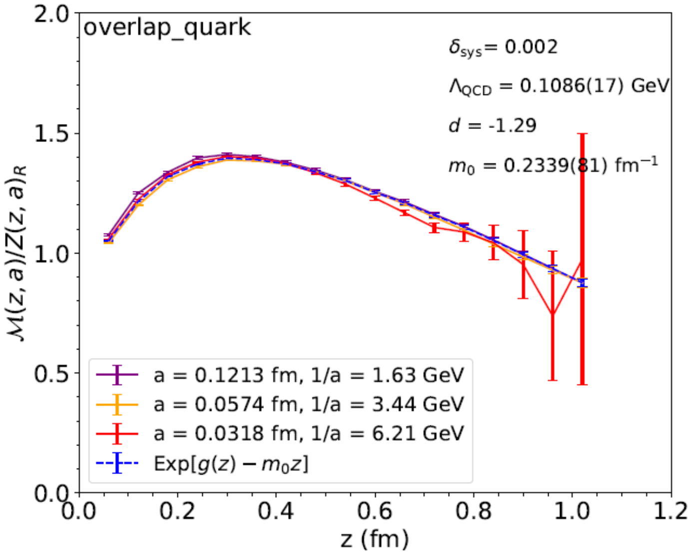




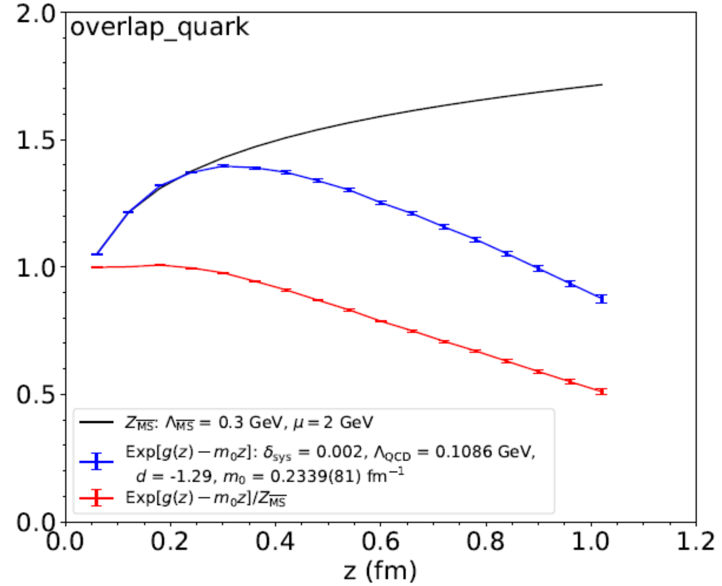
Hybrid Renormalization

Lattice Parton Collaboration (LPC)

Continuum limit



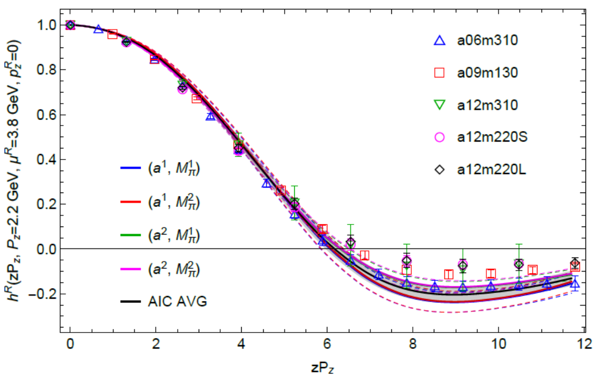
Pert.vs Non-pert Z



Continuum extrapolations

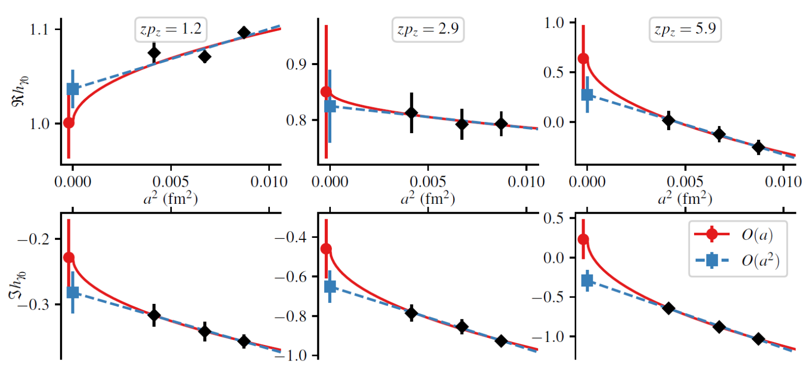
MSU

2011.14971
a=0.06,0.09,0.12



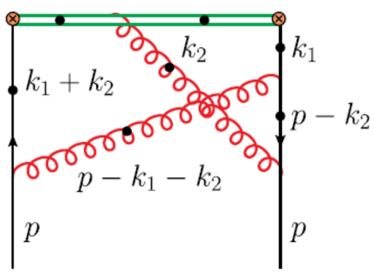
ETMC

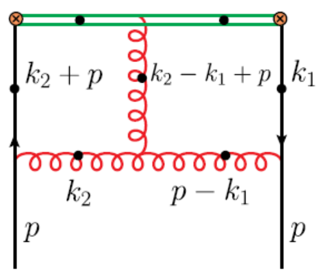
2011.00964
a= 0.064,0.082,0.093



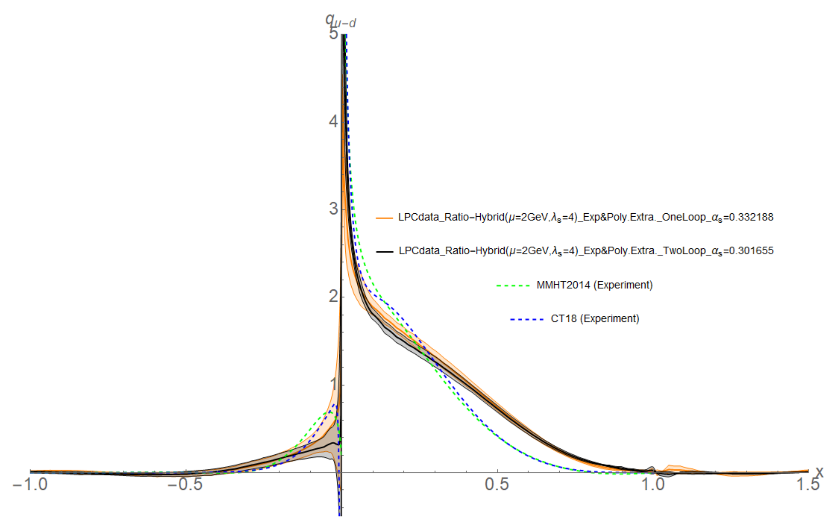
Two-loop matching

- Two-loop matching in the non-singlet sector has been calculated recently, L. Chen et al, PRL 126 (2021); Z. Li et al, PRL 126 (2021);

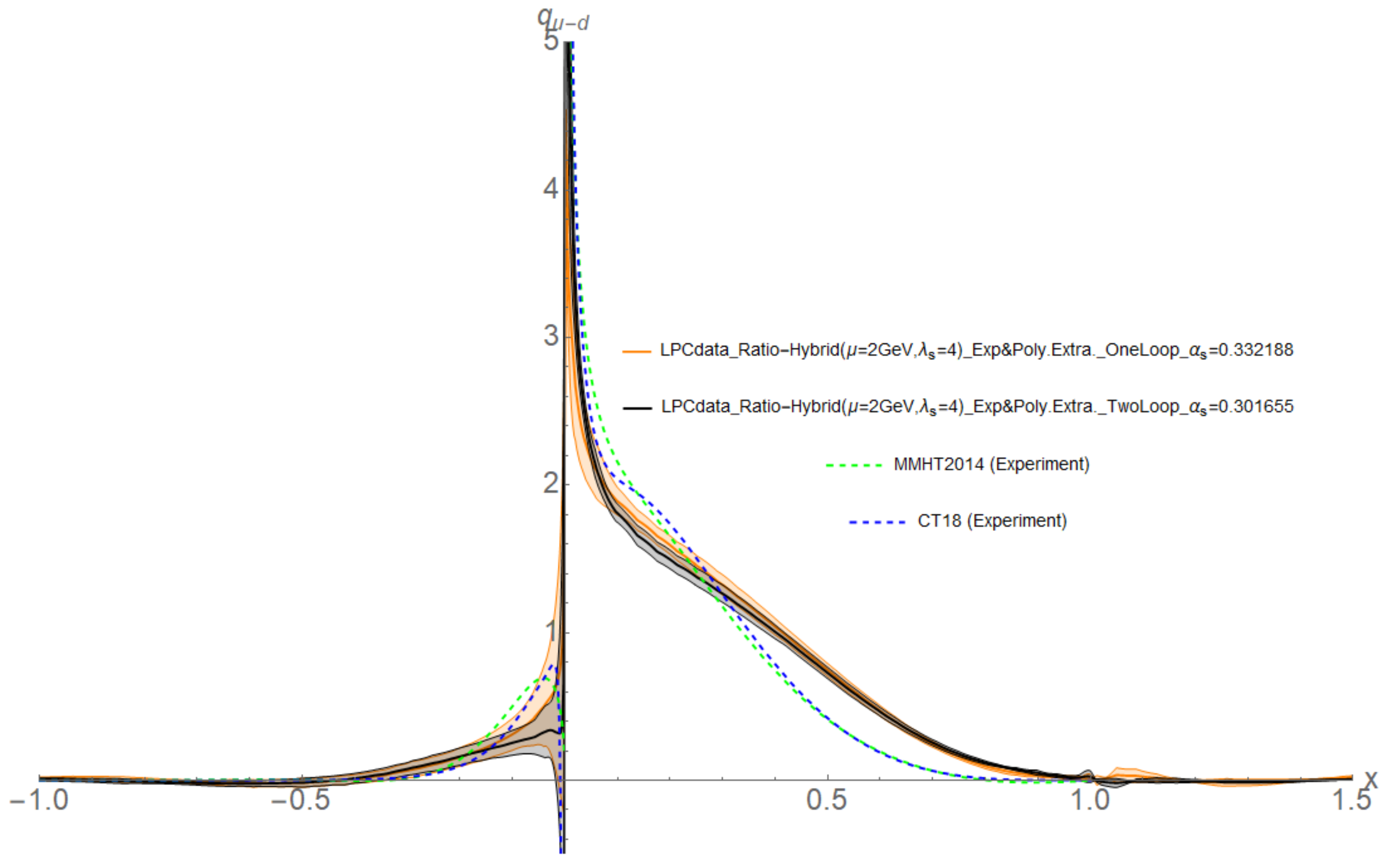




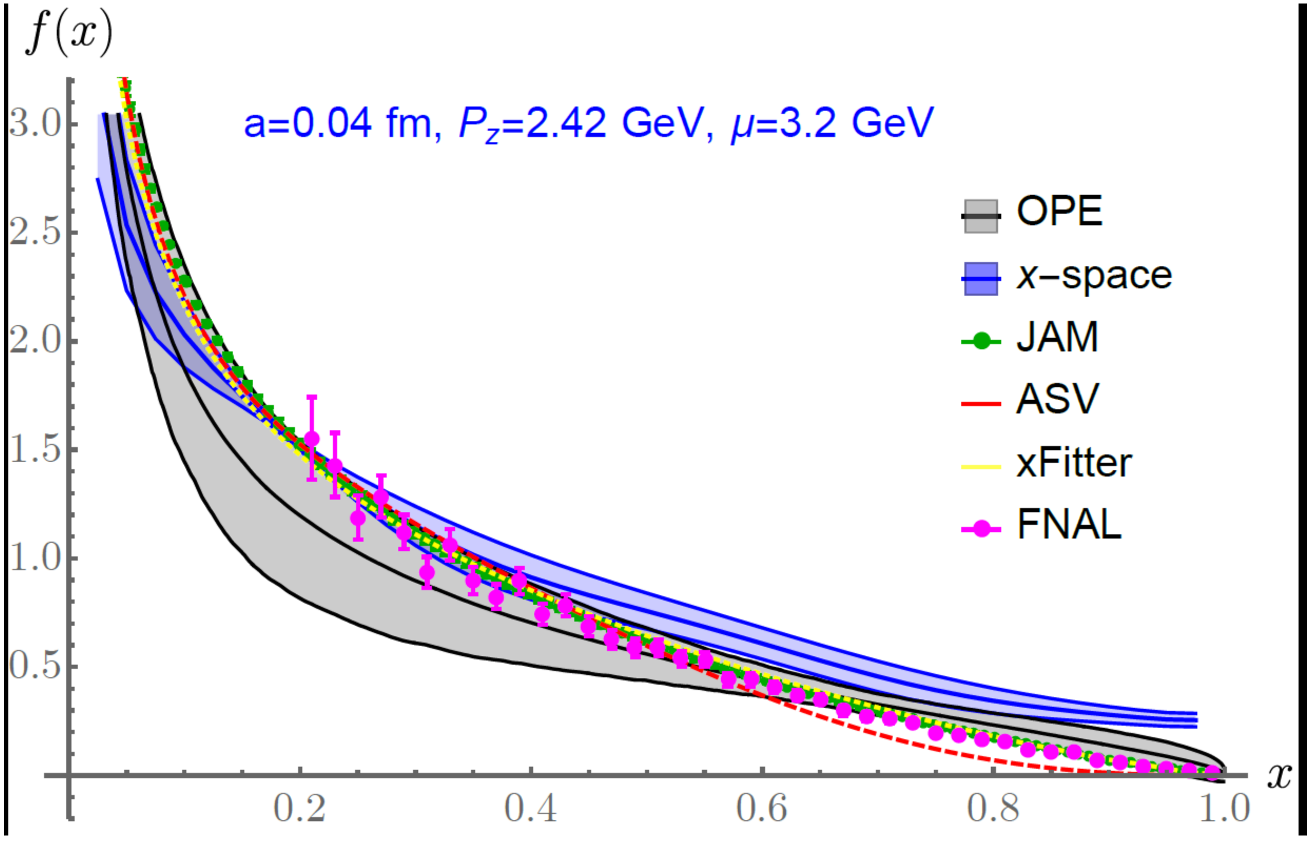
- Singlet part?



An example of precision PDF (LPC)

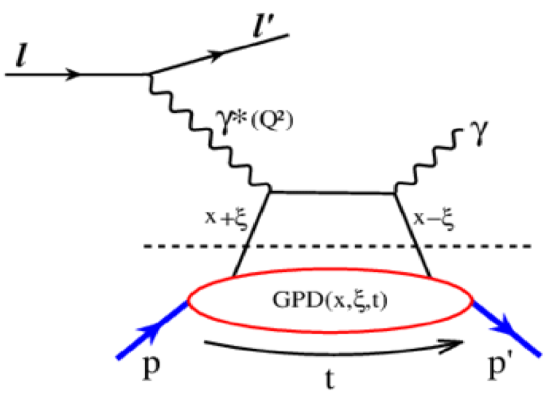


Pion PDF (BNL&Argonne)



Generalized parton distributions (GPD)

- GPD are hybrids of form factors and parton distributions: spin & mass structure, 3D imaging (Mueller et al.'94, Ji'96, Burkardt'00)
- Experimental processes that can be used to measure GPD: DVCS (Ji, 1996) & DVMP (Radyushkin, 1996, Collins et al, 1998)

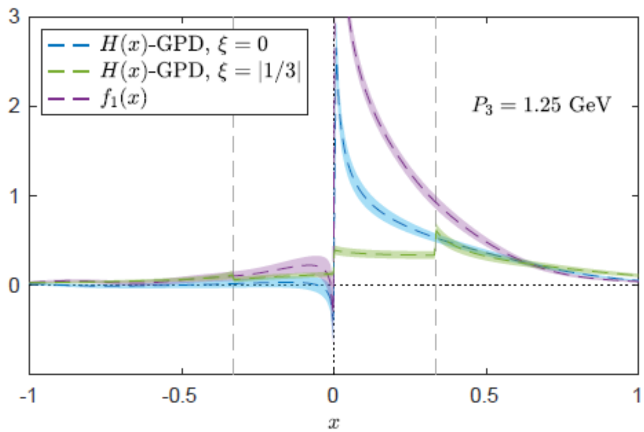


Kinematic variables: x, ξ, t

GPDs via LaMET

ETMC

PRL125 (2020)



MSU

2008.12474 (2020)

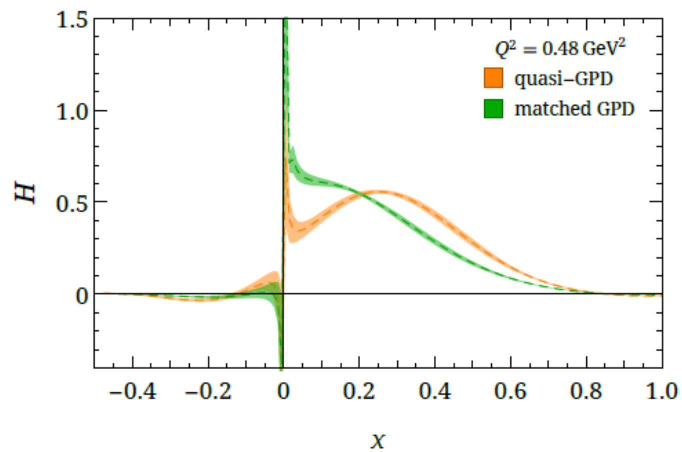


Figure 2: Examples of the first GPD data from lattice QCD. The left plot shows the matched GPD H at $\xi = 0$ and $|\xi| = 1/3$ with momentum transfer $Q^2 = 0.69$ GeV^2 [18]. The right one shows the quasi-GPD and matched GPD H at momentum transfer $Q^2 = 0.48$ GeV^2 and $\xi = 0$ [19].

TMDPDF & Lattice calculations

- Started from A. Schafer et al., invariant functions, matching with continuum quantities?

  Hagler et al, Much et al, Yoon et al. PRD96,094508 (2017)…

- A number of LaMET formulations:

  Ji et al., PRD91,074009 (2015); PRD99,114006(2019)

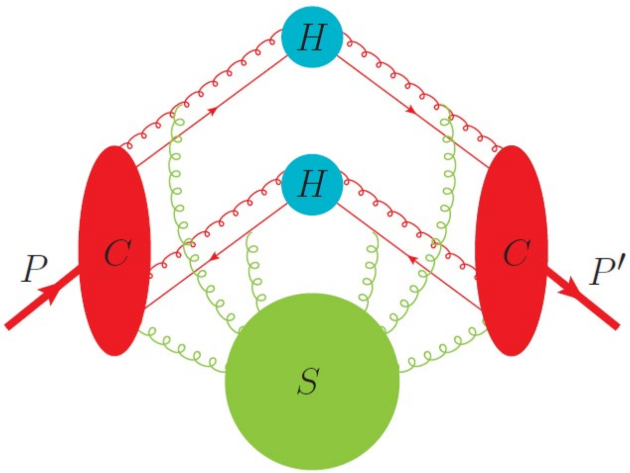
  Ebert, Stewart, Zhao, PRD99,034505 (2019), JHEP09,037(2019)

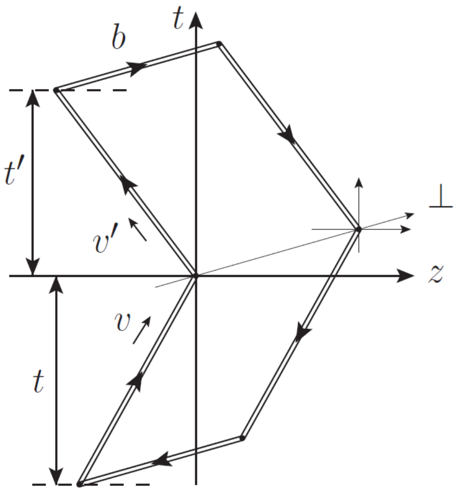
- Collins-Soper kernel can be calculated on lattice

  Ji et al., PRD91,074009 (2015); Ebert, Stewart and YZ, PRD 99(2019).M. Ebert, JHEP 03 (2020) 099

Soft Function

Ji, Liu, Liu, Nucl. Phys. B955 (2020)



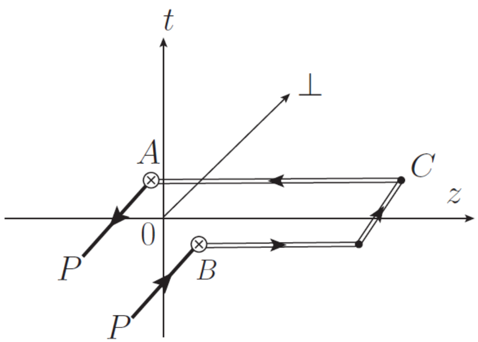


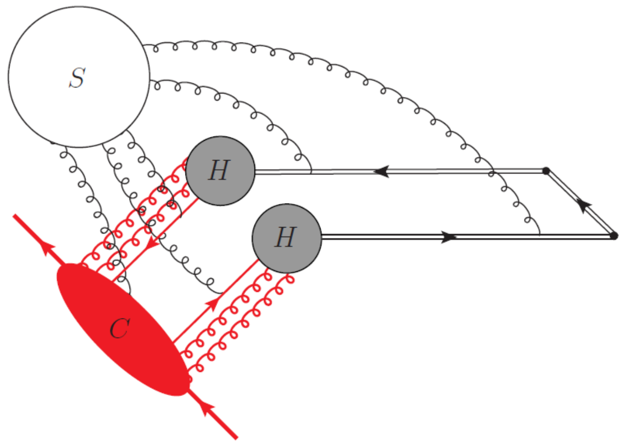
Factorization of form-factor of Light meson

Form-factors of heavy-quark pair

Quasi-TMDPDF Factorization

Ji, Liu, Liu, Phys.Lett.B 811 (2020)

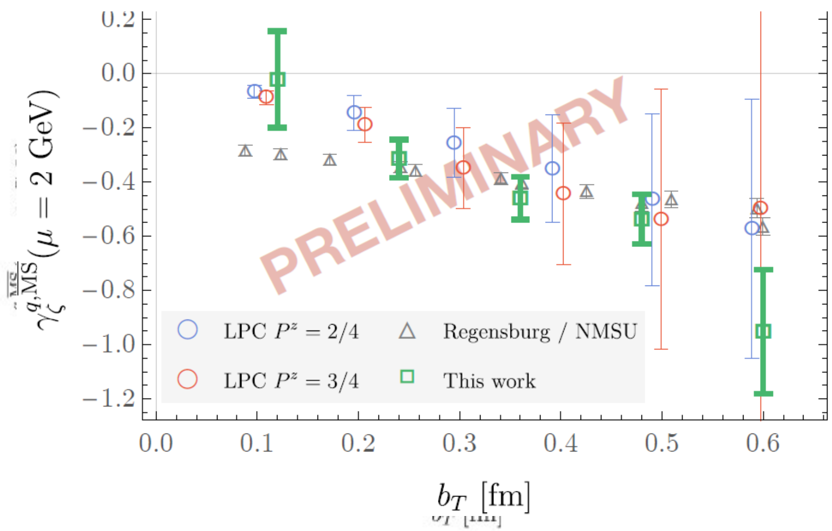


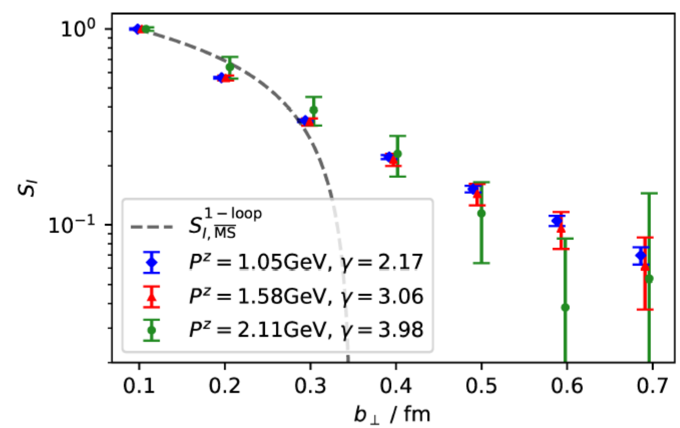


$$\boxed{\begin{aligned}&\tilde{f}(x,b_\perp,\mu,\zeta_z)\sqrt{S_r(b_\perp,\mu)}\\ &=H\left(\frac{\zeta_z}{\mu^2}\right)e^{K(b_\perp,\mu)\ln(\frac{\zeta_z}{\zeta})}f^{\mathrm{TMD}}(x,b_\perp,\mu,\zeta)+\ldots\end{aligned}}$$

$$\mu\frac{d}{d\mu}\ln H\left(\frac{\zeta_z}{\mu^2}\right)=\Gamma_{\mathrm{cusp}}\ln\frac{\zeta_z}{\mu^2}+\gamma_C$$

Collins-Soper Kernel and Soft Factor





P. Shanahan et al, PRD 102 (2020) & unpublished

Q.A.Zhang et al, PRL125 (2020)

M.Schlemmer, 2103.16991 (2021)

Q. A. Zhang et al, PRL125 (2020) (LPC)

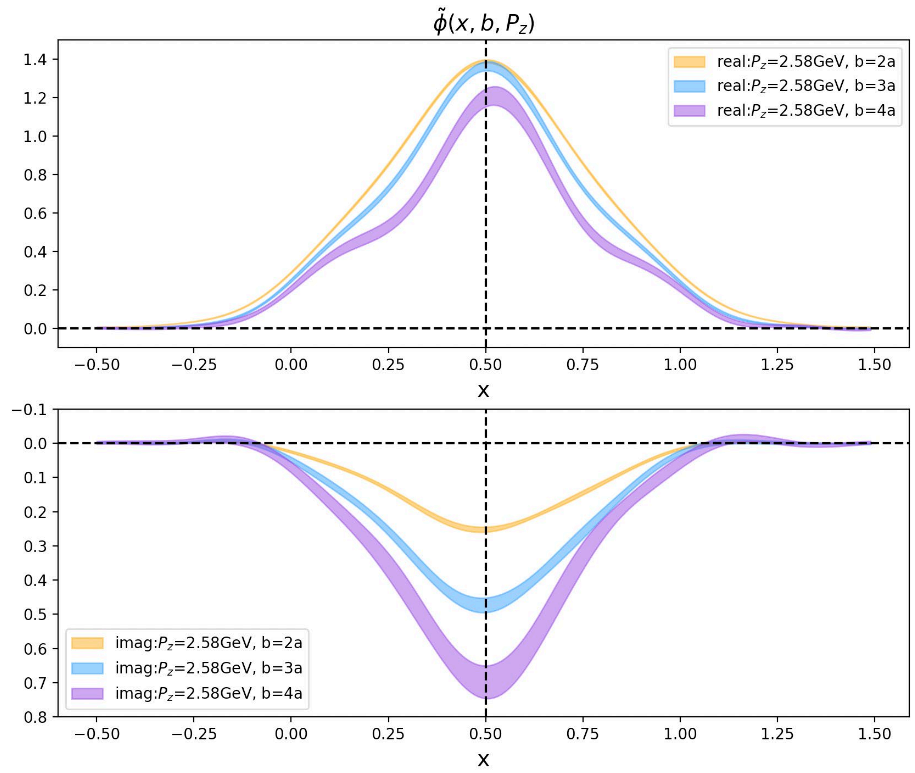
Light-Front Wave-Functions

- LF quantization focuses on the WFs, from which everything can be calculated: a very ambitious goal! Brodsky et al. Phys. Rept. 301 (1998)
- However, there are a number of reasons this approach has not been very successful.
- LaMET provides the practical way to calculate non-perturbative WF, at least for lowest few components. Ji & Liu, to be published.
- All WF can be computed as gauge-invariant matrix elements

$$\left\langle 0 \left| \hat{O}\left(z_1, \vec{b}_1, z_2, \vec{b}_2 \ldots, z_k, \vec{b}_k\right) \right| P \right\rangle$$

LPC result soon.

Meson TMD Light-Front Wave Function (LPC, preliminary)



Important for studying B-meson decays

Outlook

- LaMET is a systematic framework to calculate parton physics (indirectly)
- LaMET3.0 (~ 5% error?)
  - Improved non-pert renormalization at large z.
  - two-loop matching
  - $P \to 3$ GeV
  - Singlet quark and gluon
- GPDs & TMDs & Wigner Functions & LFWF
  - More to be done!