



Standard Model Effective Field Theory

Jiang-Hao Yu (于江浩)

Institute of Theoretical Physics, Chinese Academy of Science

jhyu@itp.ac.cn

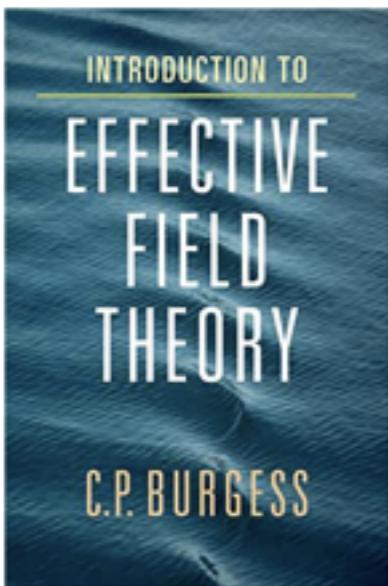
Summer School on Collider Phenomenology

July 8, 2021 at Shandong U, Qingdao

Outline

- Basics: EFT why, what and how
- Proficient: SMEFT construction
- Application: dim-6 SMEFT
- Summary and outlook

Recommended Reading



[Introduction to Effective Field Theory: Thinking Effectively about Hierarchies of Scale](#)
by C. P. Burgess | Jan 21, 2021

[Hardcover](#)

\$80⁸²

Introduction to Effective Field Theories

#3

[Aneesh V. Manohar \(UC, San Diego\)](#) (Apr 16, 2018)

Published in: *Les Houches Lect.Notes* 108 (2020) • Contribution to: [Les Houches summer school](#) • e-Print: [1804.05863](#) [hep-ph]

Effective Field Theory and Precision Electroweak Measurements

[Witold Skiba \(Yale U.\)](#) (Jun, 2010)

Published in: • Contribution to: [TASI 2009](#), 5-70 • e-Print: [1006.2142](#) [hep-ph]

As Scales Become Separated: Lectures on Effective Field Theory

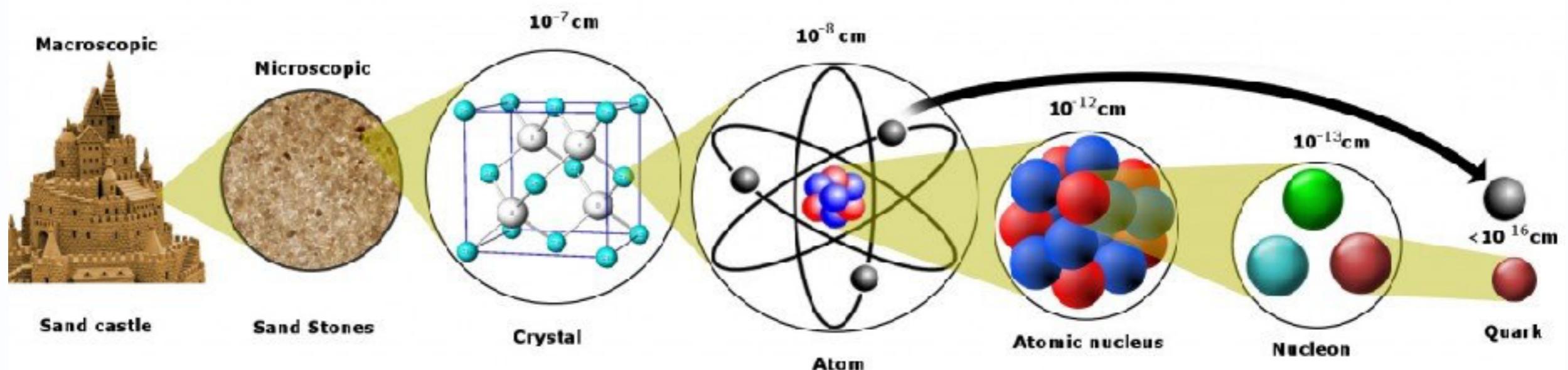
[Timothy Cohen \(Oregon U.\)](#) (Mar 8, 2019)

Published in: *PoS TASI2018* (2019) 011 • Contribution to: [TASI 2018](#), 011 • e-Print: [1903.03622](#)

Basic: EFT Why/What

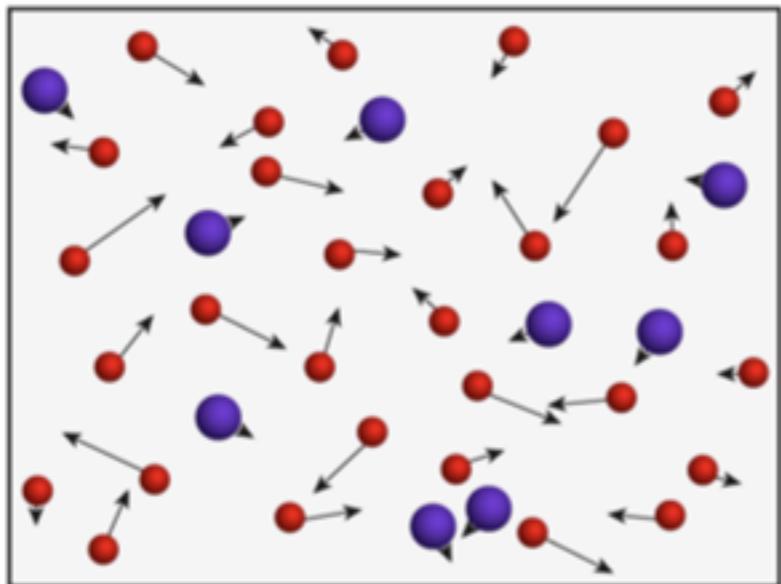
Scales in Physics

Nature at various scales



Decouple among different scales

Statistical and Thermal Physics



$$\text{H} = 10^{-10} \text{ m}$$

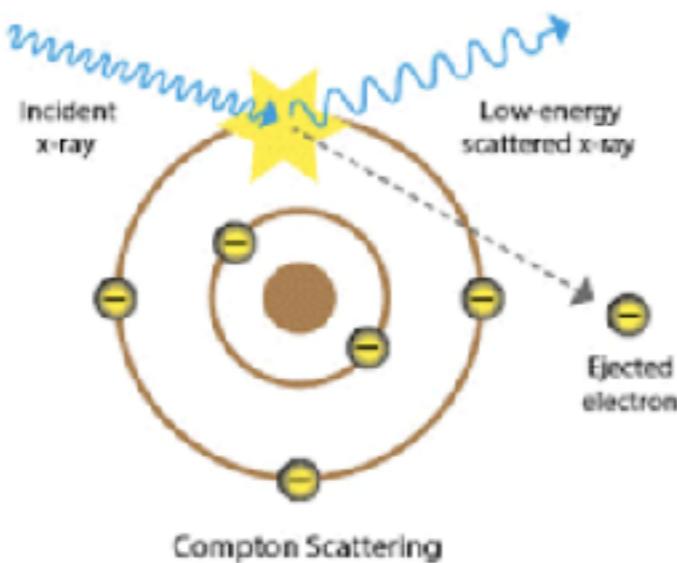
**At small scales,
the degrees of freedom of gas
are positions and velocities
of its component atoms**

$$\rho, p, T, s$$

$$\text{H} = 10^{-2} \text{ m}$$

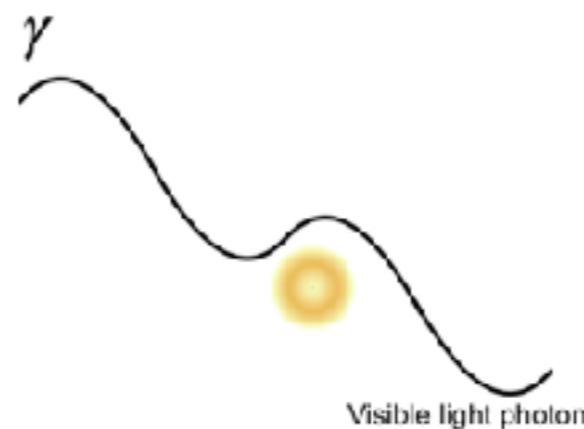
**At large scales,
the useful degrees of freedom
are its macroscopic properties
like density, pressure,
temperature, or entropy**

Scales in microscopic world



$$\text{---} = \frac{1}{m_e \alpha}$$

**X-ray photons see
the atomic structure
and scatter on
the orbiting electrons**

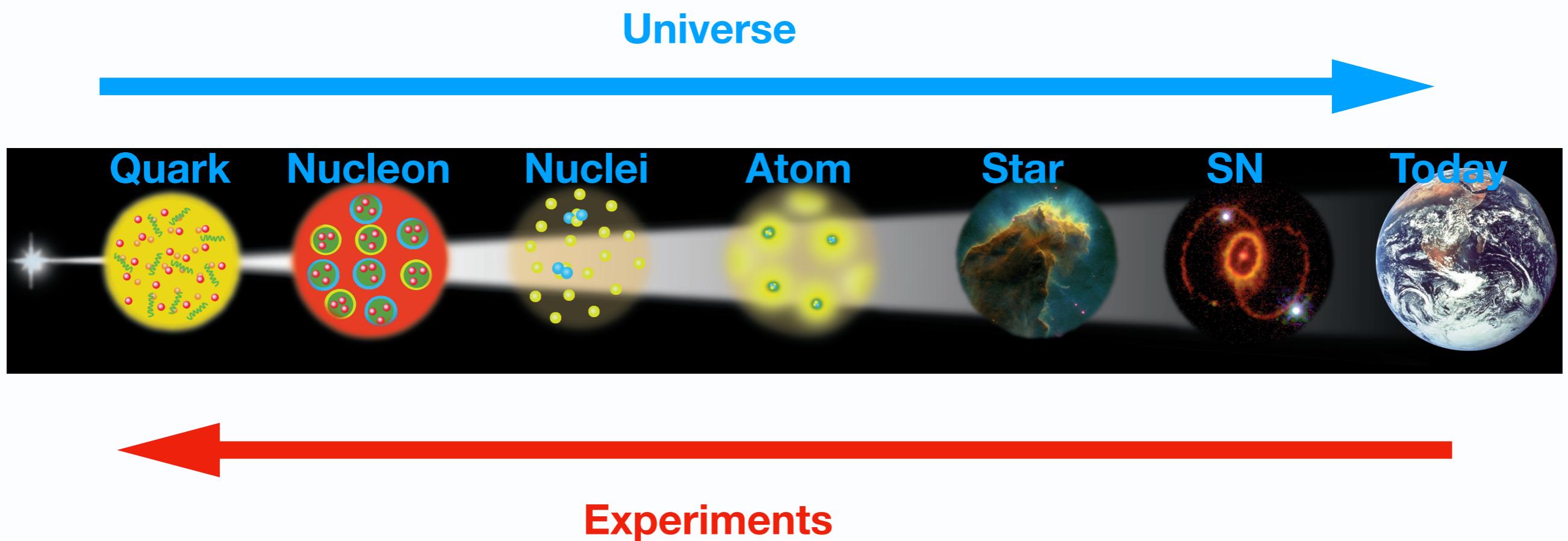


$$\text{---} = \frac{10}{m_e \alpha}$$

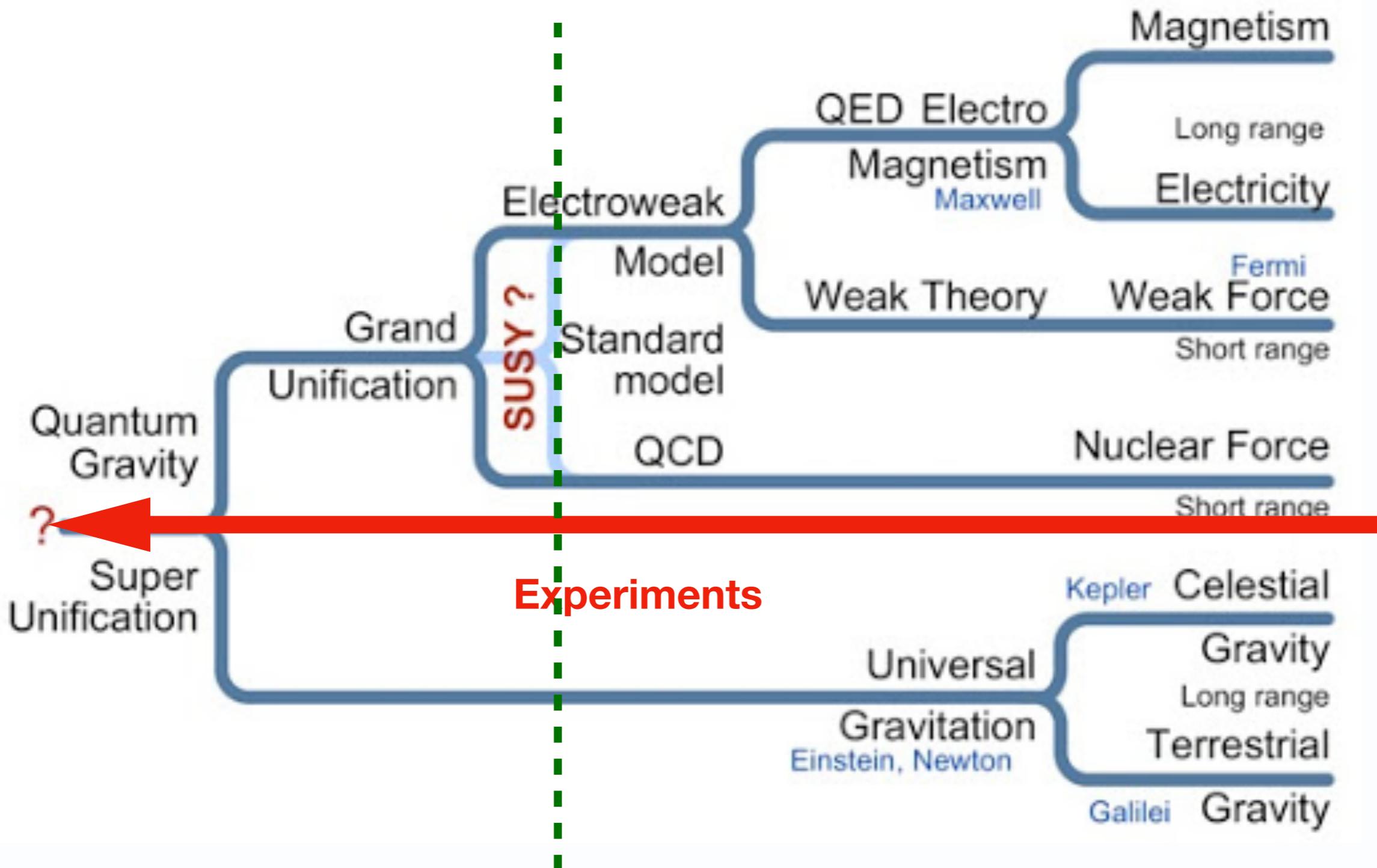
**Lower-energy photons
see atoms as neutral objects
which are basically transparent**

This is how the universe becomes transparent after recombination

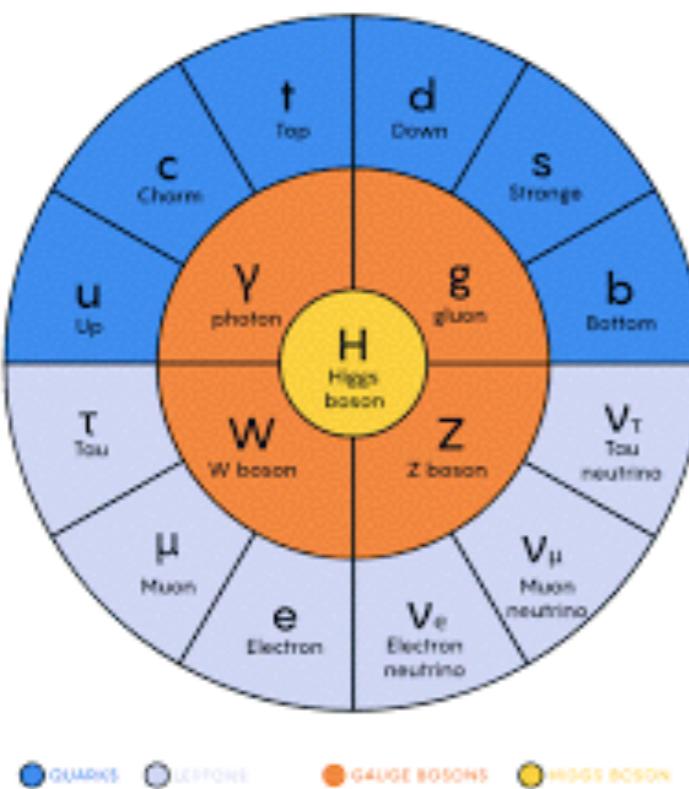
Probing Higher Scales



Probing Higher Scales



The Standard Model



$$\begin{aligned}\mathcal{L}_{SM} = & \underbrace{\frac{1}{4}W_{\mu\nu}W^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}G_{\mu\nu}^aG_a^{\mu\nu}}_{\text{kinetic energies and self-interactions of the gauge bosons}} \\ & + \underbrace{\bar{L}\gamma^\mu\left(i\partial_\mu - \frac{1}{2}g\tau\cdot W_\mu - \frac{1}{2}g'YB_\mu\right)L + \bar{R}\gamma^\mu\left(i\partial_\mu - \frac{1}{2}g'YB_\mu\right)R}_{\text{kinetic energies and electroweak interactions of fermions}} \\ & + \underbrace{\frac{1}{2}\left(i\partial_\mu - \frac{1}{2}g\tau\cdot W_\mu - \frac{1}{2}g'YB_\mu\right)\phi^2}_{W^\pm, Z, \gamma \text{ and Higgs masses and couplings}} - V(\phi) \\ & + \underbrace{g''(\bar{q}\gamma^\mu T_3 q)G_\mu^a}_{\text{interactions between quarks and gluons}} + \underbrace{(G_1\bar{L}\phi R + G_2\bar{L}\phi_c R + h.c.)}_{\text{fermion masses and couplings to Higgs}}\end{aligned}$$

17 elementary particles

19 parameters

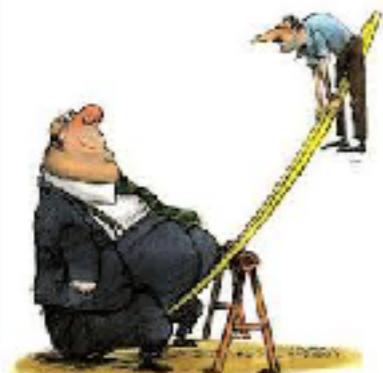
Very successful!

Motivate for New Physics

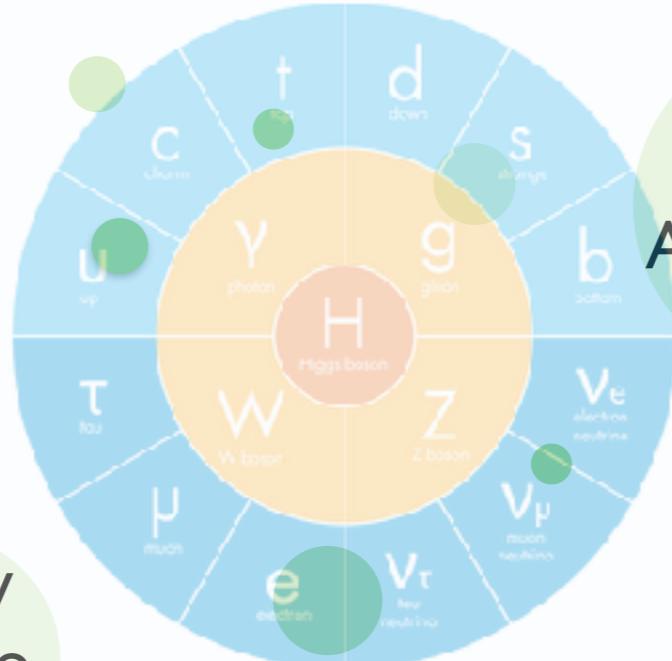
Origin of Mass

Why Higgs
mass hierarchy
so large?

$$m_{\text{Higgs}}^2 = \dots + y_t \cdot t$$

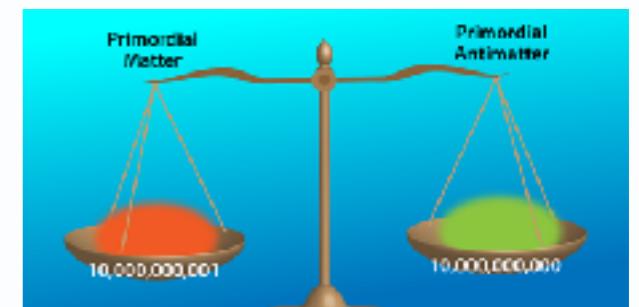


Why ν
mass so
tiny?

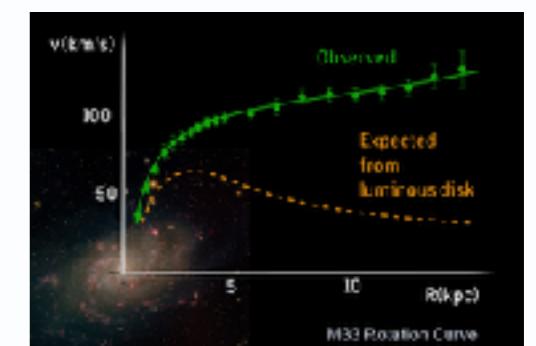


Origin of Matter

Baryon
Asymmetry

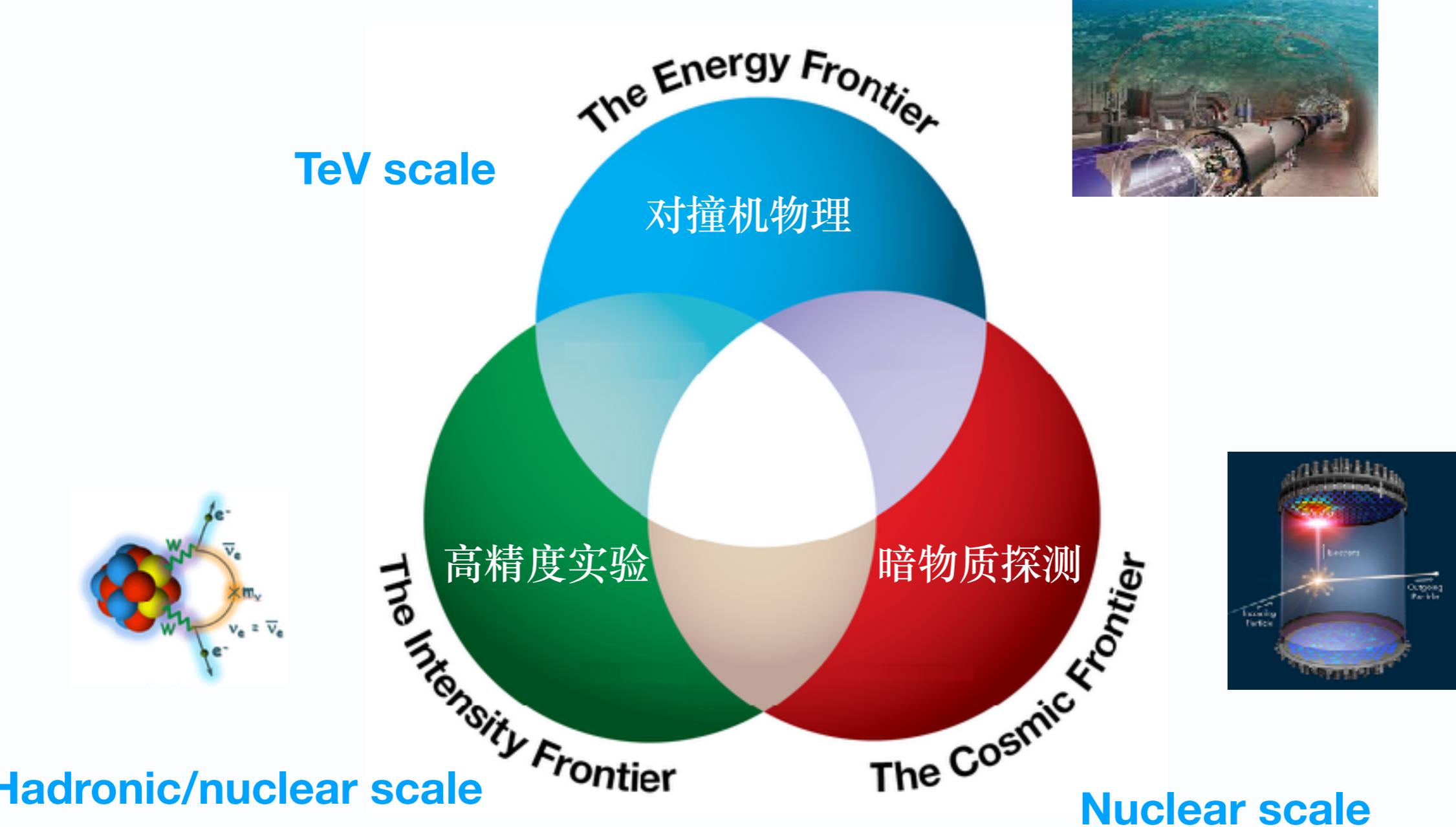


Dark
matter

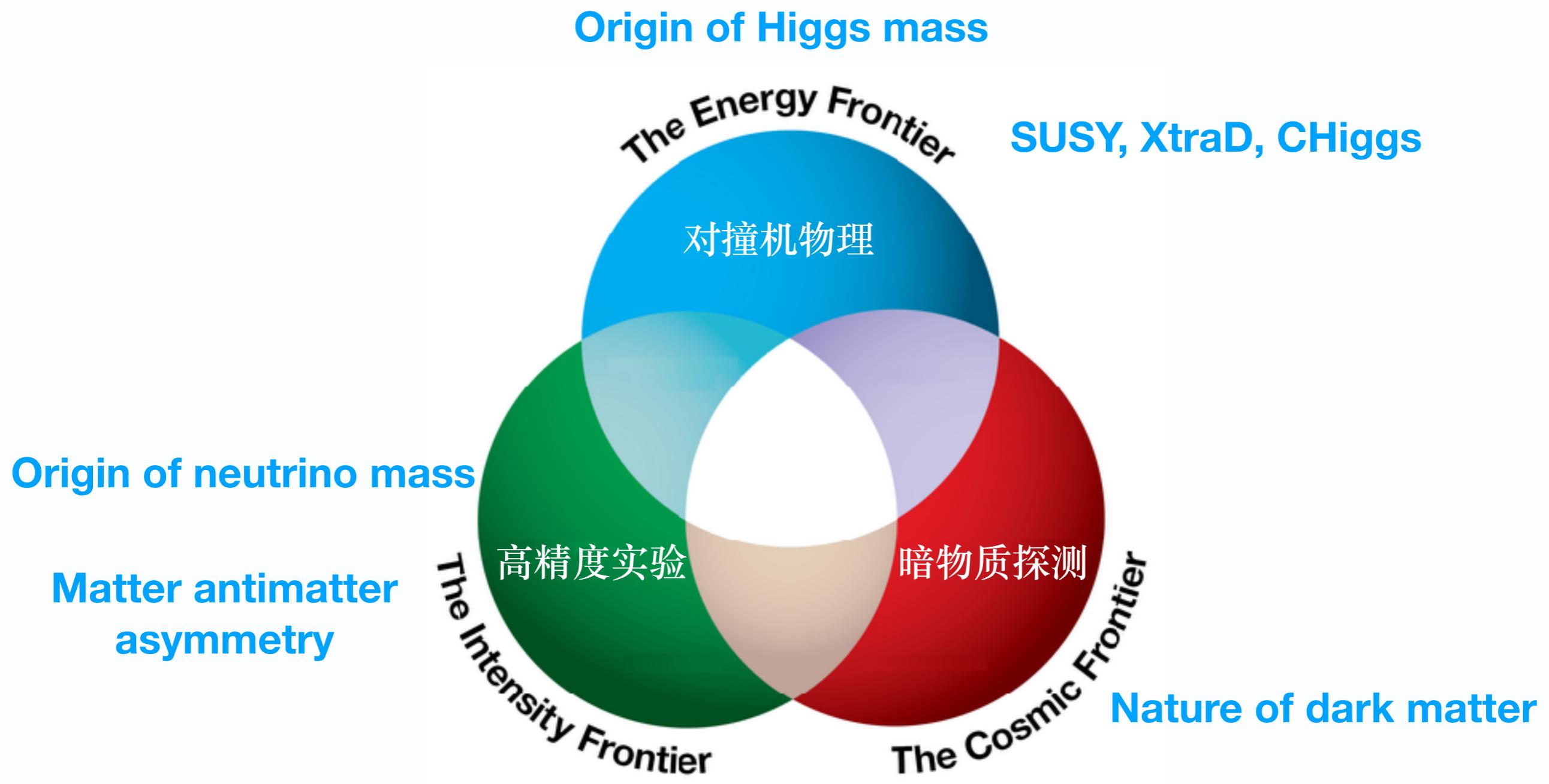


Search For New Physics

at various scales



Many Theories, Many Particles



Results at Energy Frontier

ATLAS SUSY Searches^{*} - 95% CL Lower Limits

March 2021

**Only a selection of the available mega units on non stated or postorder is shown. Many of the units are based on different models, e.g. see the separate notes.*

SUSY Extra dim

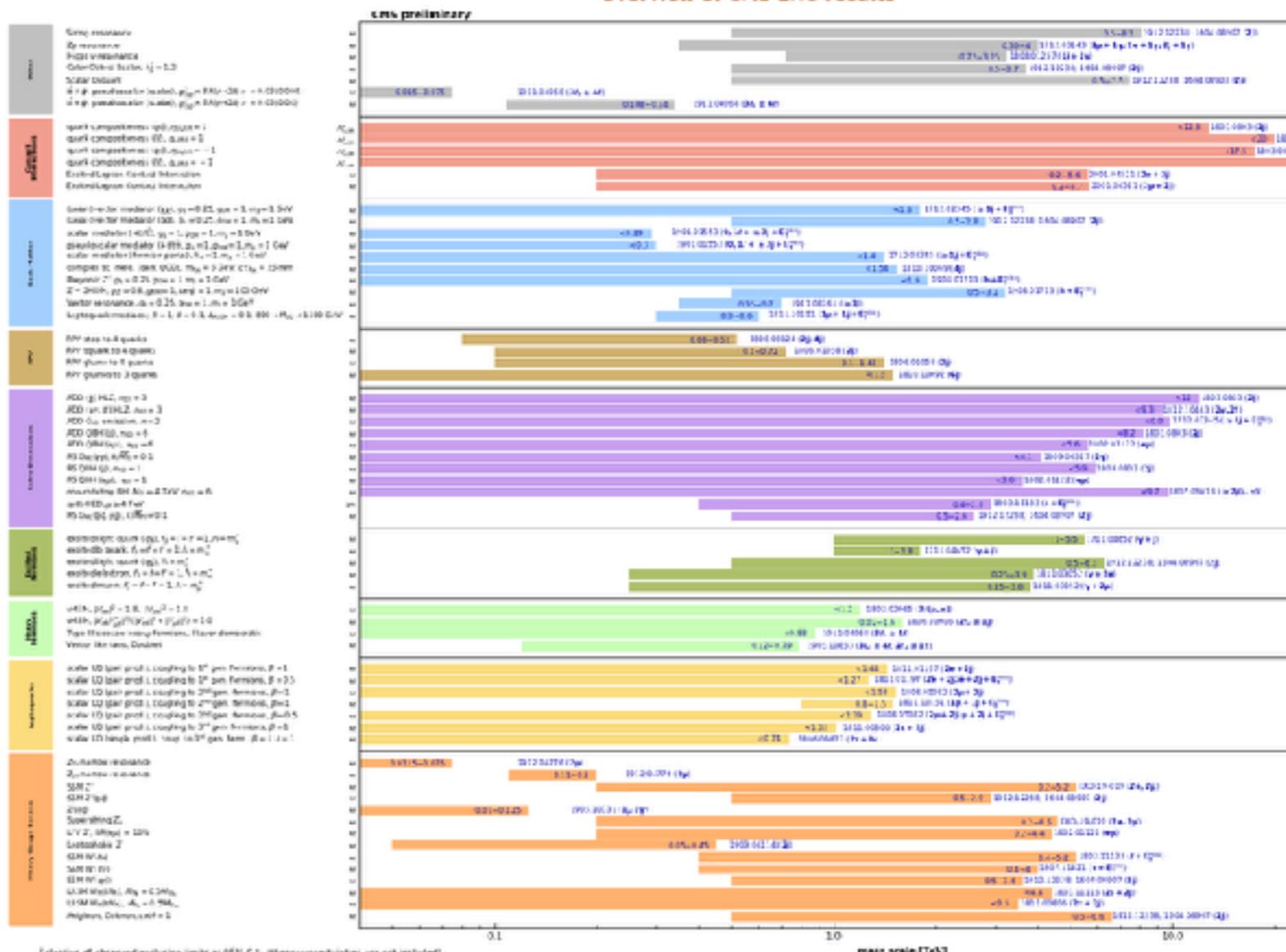
Exotics

ATLAS Preliminary

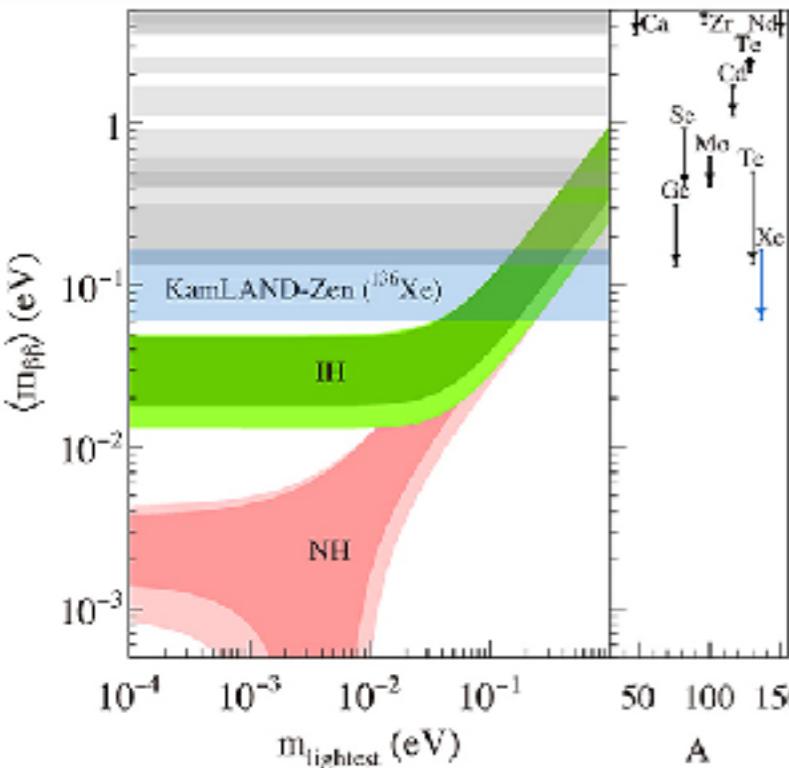
$$\sqrt{G} = 13 \text{ TeV}$$

Reference	
22	0.11602
21	0.12138/4
23	0.14603
20	0.18303
21H	0.1429
1645	0.1181
2048	0.06033
1650	0.0457
ATLAS-CDF-IT	214 214
	1699 0.0457
21C1	1.0527
21C1	1.1527
1699	0.0123
ATLAS-CDF-IT-2020-31	300
3001.1	0.045 2001.03700
22	0.07169
ATLAS-CDF-IT	246 300
1688	0.01440
21F1	2.18/4

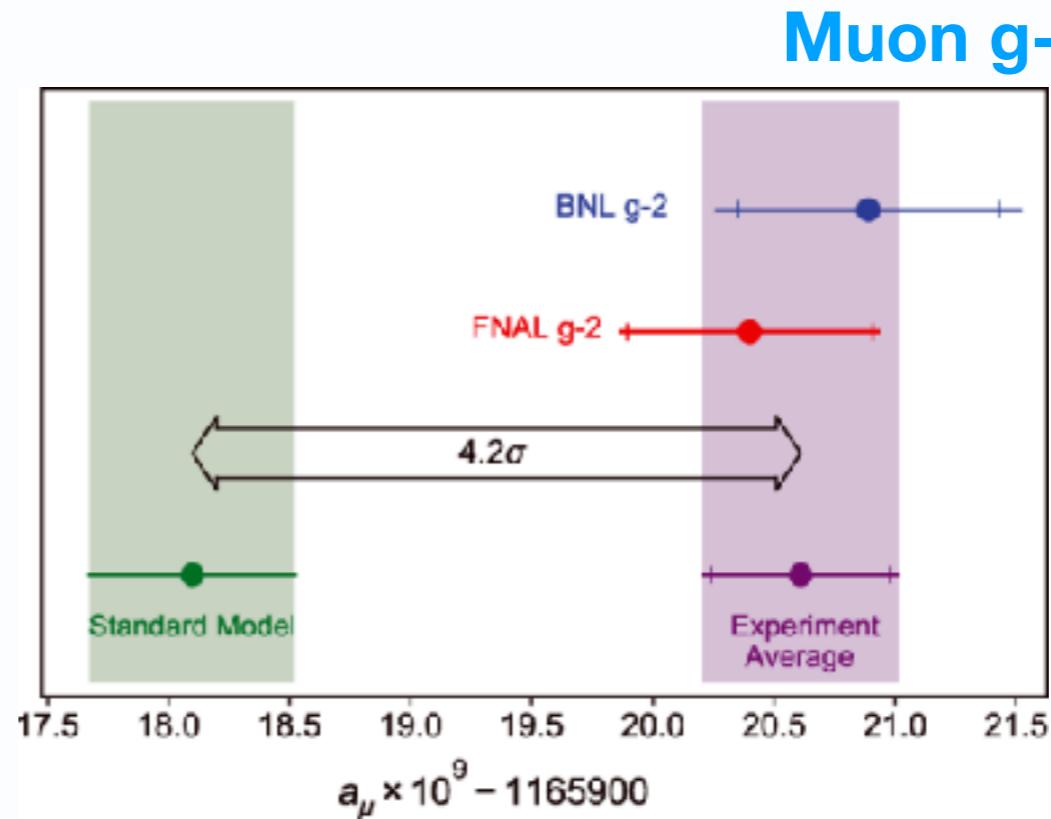
Overview of CMS EXO results



Results at Luminosity Frontier



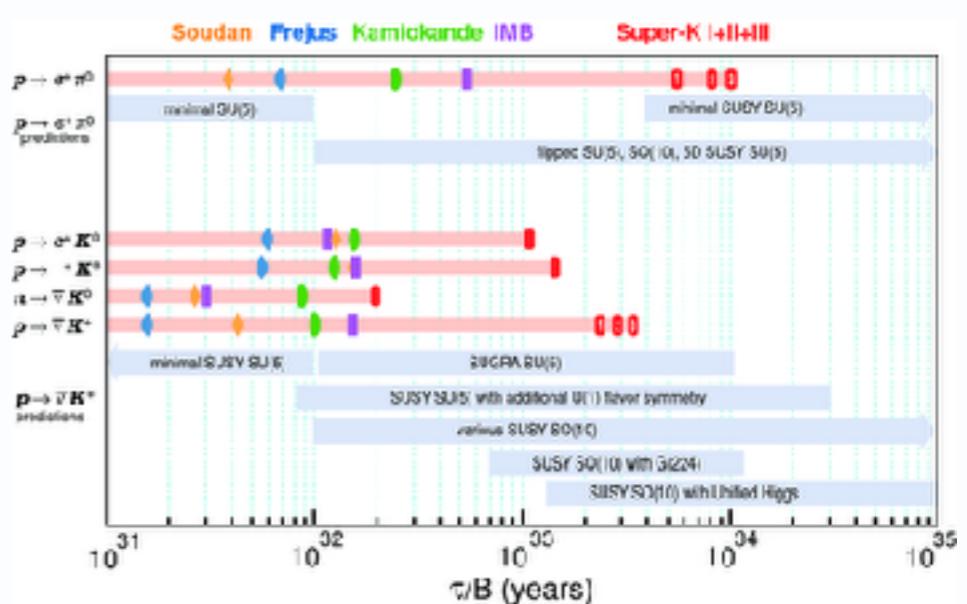
0vbb



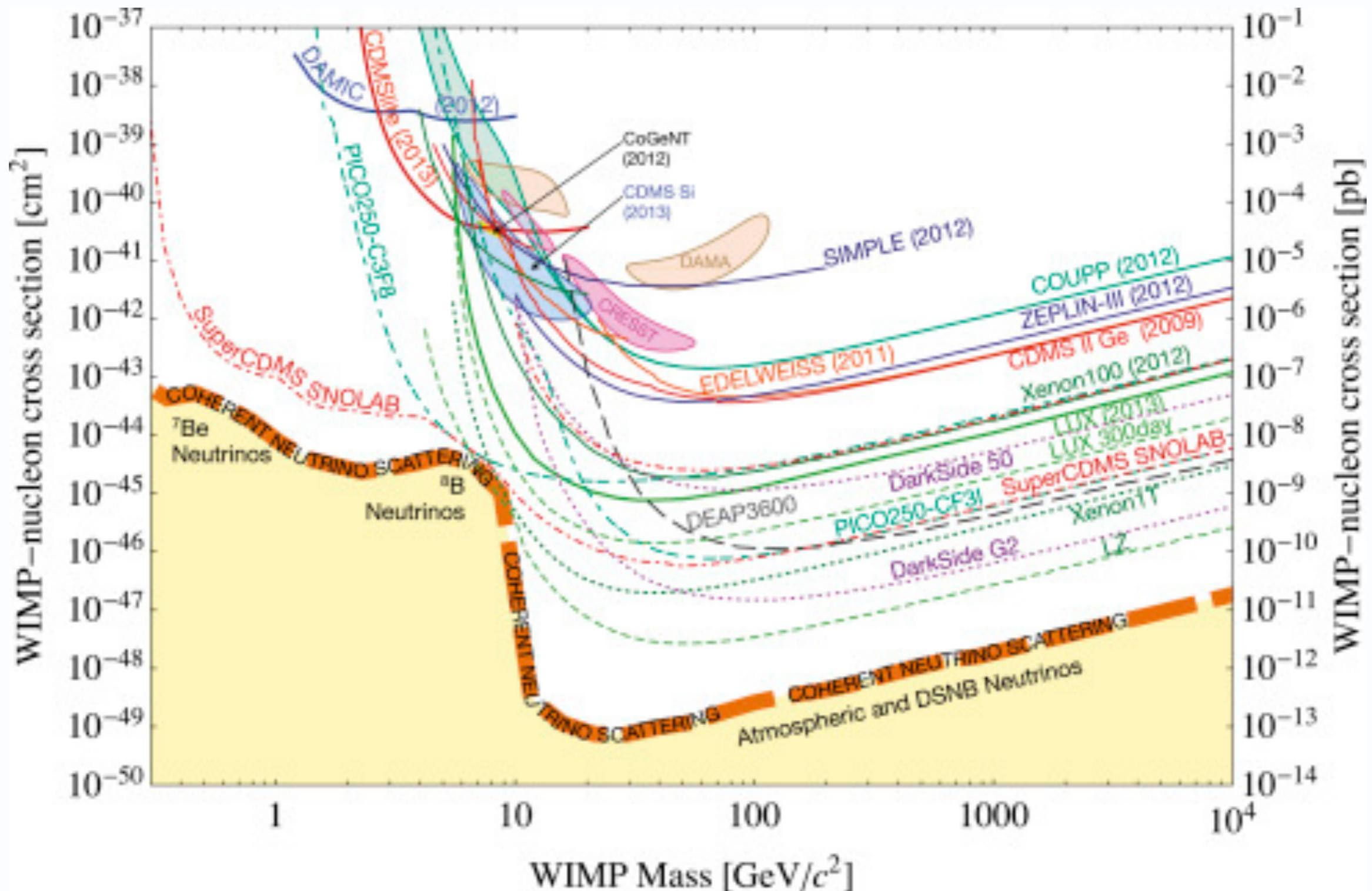
Muon g-2

EDM

	d_{SM}	d_{exp}	d_{future}
e^-	$< 10^{-40}$	$< 1.6 \times 10^{-27}$	$\rightarrow 10^{-31}$
n	$< 10^{-30}$	$< 6.3 \times 10^{-26}$	$\rightarrow 10^{-29}$
^{199}Hg	$< 10^{-31}$	$< 2.1 \times 10^{-28}$	$\rightarrow 10^{-32}$
μ	$< 10^{-28}$	$< 1.1 \times 10^{-18}$	$\rightarrow 10^{-24}$



Results at Cosmic Frontier



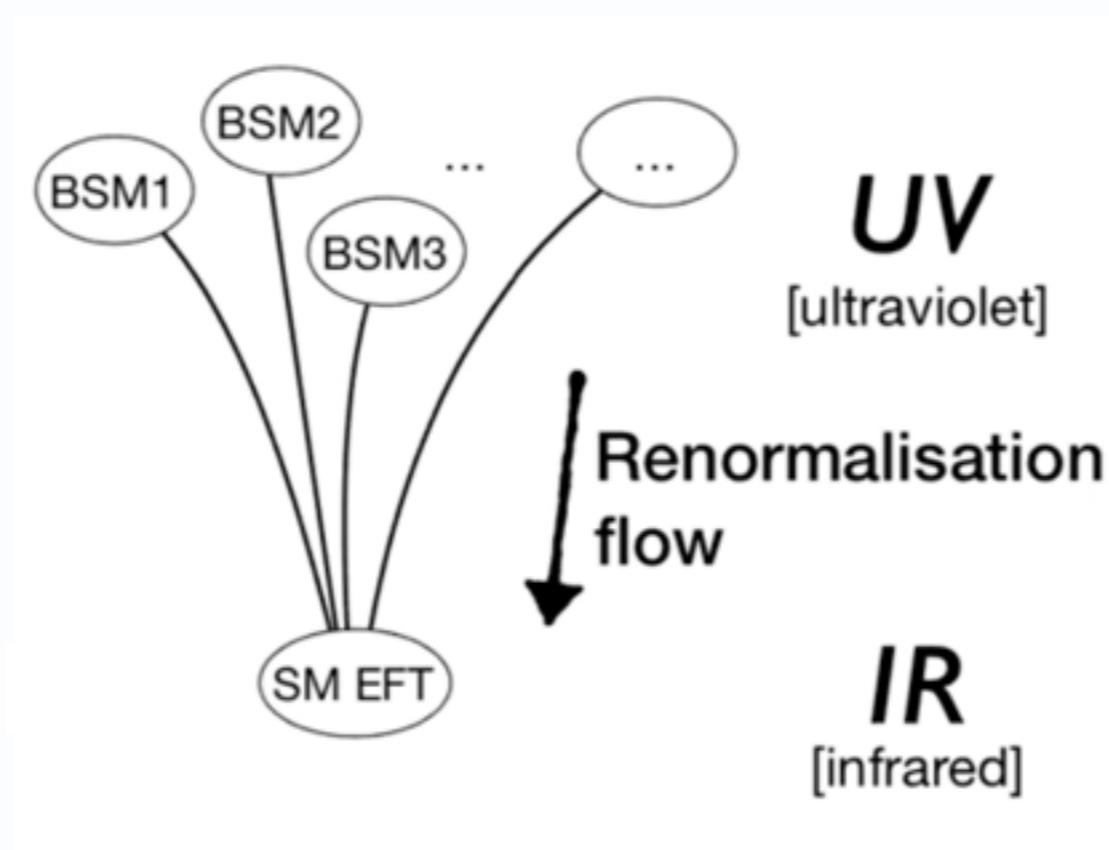
No Sign of New Physics!



Search for New Physics

Model building: assume a target (new particle), then shoot (search) for it!

Top-down approach



SMEFT: systematical parametrization of new physics effects order by order

Bottom-up approach

No new particle!

Standard Model EFT

Assumptions: No new light particles, new physics decouples

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \mathcal{L}_{D=5} + \frac{1}{\Lambda^2} \mathcal{L}_{D=6} + \frac{1}{\Lambda^3} \mathcal{L}_{D=7} + \frac{1}{\Lambda^4} \mathcal{L}_{D=8} + \dots$$

Known SM Lagrangian

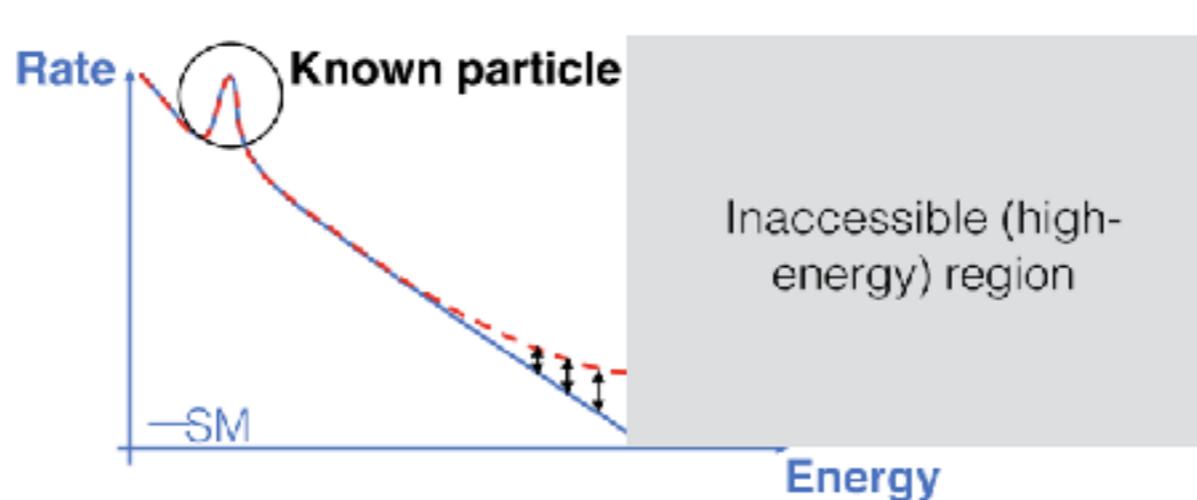
Higher-dimensional $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM

The diagram illustrates the construction of the SMEFT Lagrangian. It begins with the Standard Model Lagrangian (\mathcal{L}_{SM}), indicated by a blue arrow pointing to the first term. This is followed by a series of higher-dimensional terms, each represented by a red arrow pointing to its respective term in the equation: $\mathcal{L}_{D=5}$, $\mathcal{L}_{D=6}$, $\mathcal{L}_{D=7}$, and $\mathcal{L}_{D=8}$. The text "Known SM Lagrangian" is positioned below the first term, and "Higher-dimensional $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM" is positioned below the last term.

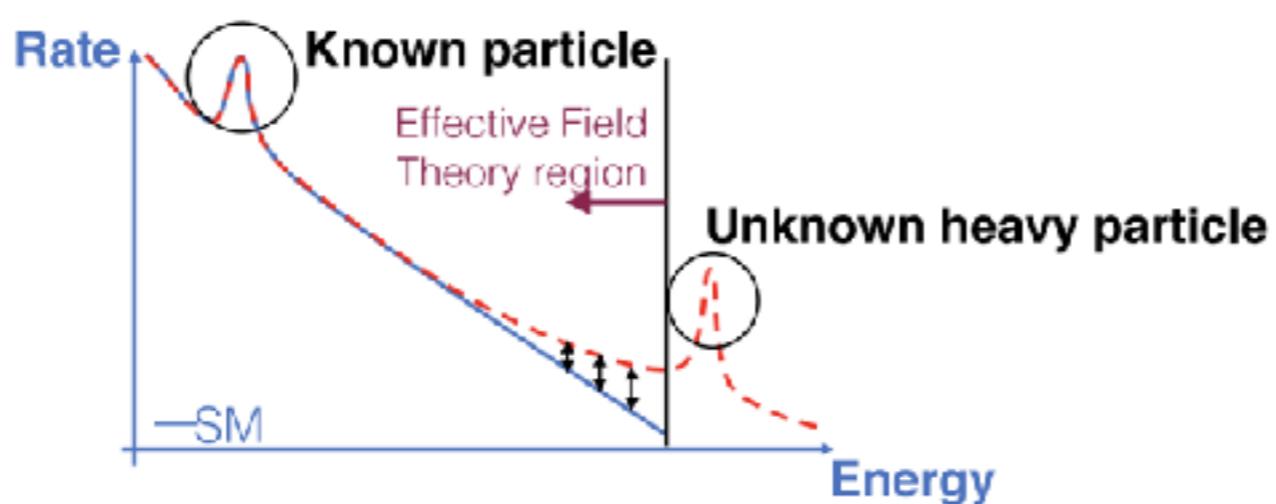
Task: Lorentz and gauge invariant, complete and independent Lagrangian
order by order with power counting canonical dimensions D

Standard Model EFT

Provide universal language to systematize our thinking and design future experiments



Determine Wilson coefficients of higher dim operators from deviations

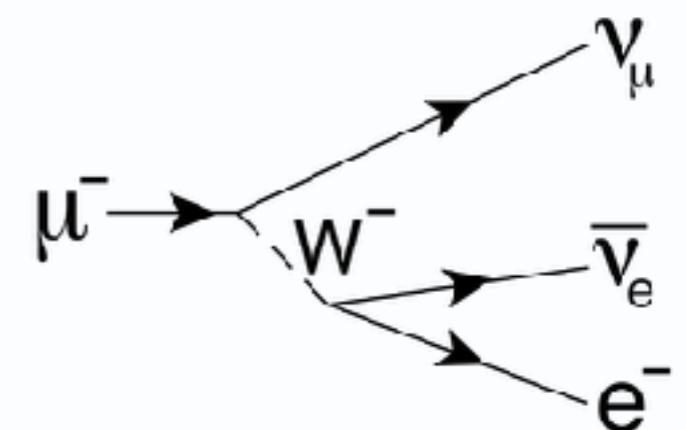
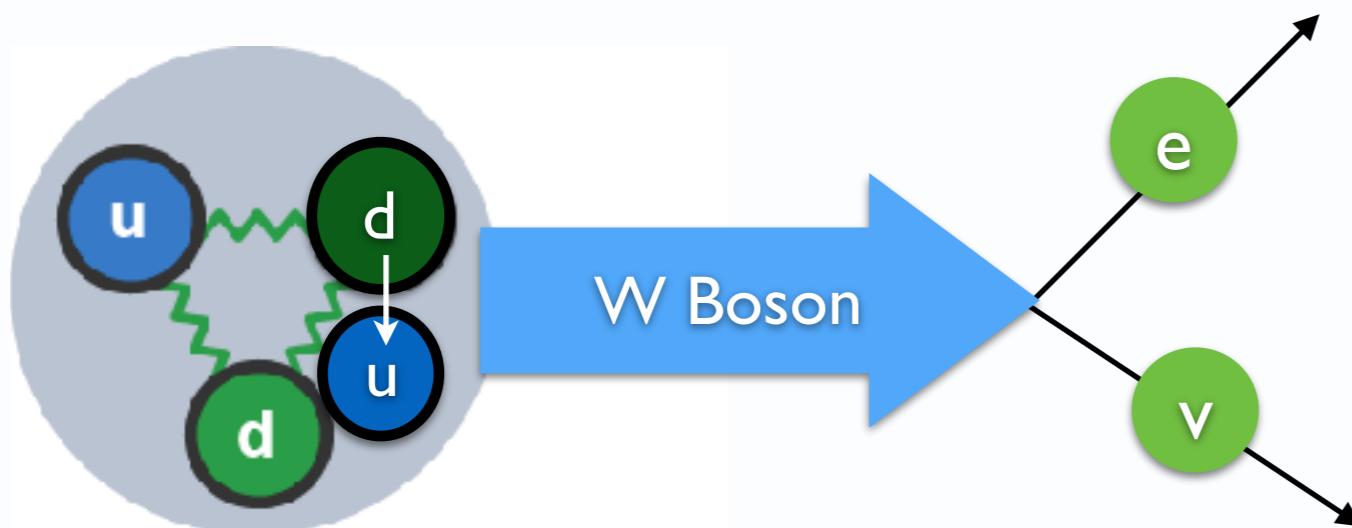


Still very useful even after new physics discovery

Lesson From Weak Decay

$$n \rightarrow p + e^- + \bar{\nu}$$

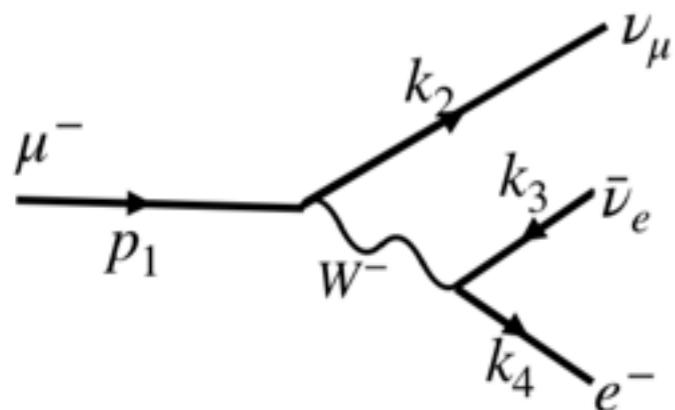
$$\mu^- \rightarrow \nu_\mu + e^- + \bar{\nu}_e$$



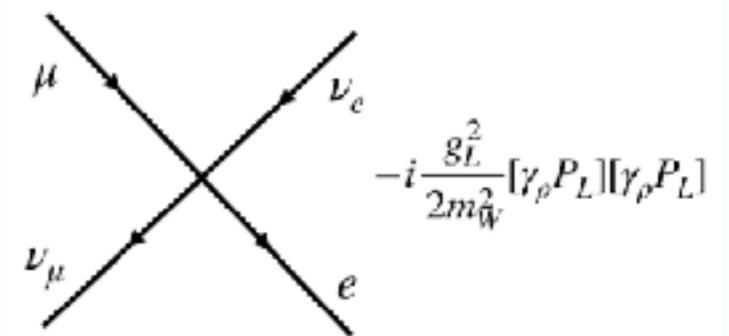
How to describe the weak decays at low energy?

Weak Decay

$$\mathcal{L}_{\text{SM}} \supset \frac{g_L}{\sqrt{2}} [\bar{\nu}_e \gamma_\rho e_L + \bar{\nu}_\mu \gamma_\rho \mu_L] W_\rho^+ + \text{h.c.}$$



$q^2 \lesssim m_\mu^2 \ll m_W^2$



$$\mathcal{M} = \frac{g_L^2}{2} \bar{u}(k_2) \gamma_\rho P_L u(p_1) \frac{1}{q^2 - m_W^2} \bar{u}(k_4) \gamma_\rho P_L v(k_3)$$

$$\mathcal{M} = -\frac{g_L^2}{2m_W^2} \bar{u}(k_2) \gamma_\rho P_L u(p_1) \bar{u}(k_4) \gamma_\rho P_L v(k_3) + \mathcal{O}(q^2/m_W^4)$$

$$\mathcal{L}_{\text{eff}} \supset -\frac{g_L^2}{2m_W^2} (\bar{\nu}_\mu \gamma_\rho \mu_L) (\bar{e}_L \gamma_\rho \nu_e) + \text{h.c.}$$

The Lagrangian coefficient here is called the Wilson coefficient in this context

Top Down Approach

Wilsonian Matching:

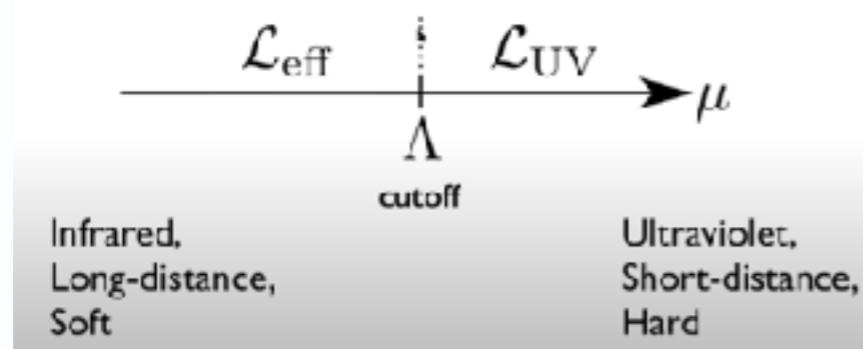
$$Z[J_\Phi, J_\varphi] = \int [D\Phi] [D\varphi] e^{i \int d^d x (\mathcal{L}[\Phi, \varphi] + J_\Phi \Phi + J_\varphi \varphi)}$$



Equation of motion (EOM): $\frac{\delta \mathcal{L}_{\text{UV}}}{\delta \Phi} = 0$.

Set $\Phi = \Phi_c[\phi]$ (EOM solution)

$$Z[J_\varphi] = \int [D\varphi] e^{i \int d^d x (\mathcal{L}_{\text{eff}}[\varphi, \Phi_c] + J_\varphi \varphi)}$$



The generating functional in the UV theory of light fields ϕ and heavy fields H

$$Z_{\text{UV}}[J_\phi, J_H] = \int [D\phi] [DH] \exp \left[i \int d^d x \left(\mathcal{L}_{\text{UV}}(\phi, H) + J_\phi \phi + J_H H \right) \right]$$

The generating functional in the EFT of light fields ϕ

$$Z_{\text{EFT}}[J_\phi] = \int [D\phi] \exp \left[i \int d^d x \left(\mathcal{L}_{\text{EFT}}(\phi) + J_\phi \phi \right) \right]$$

Matching consists in imposing the condition

$$Z_{\text{EFT}}[J_\phi] = Z_{\text{UV}}[J_\phi, 0]$$

At leading order (tree-level), the field configurations contributing to the path integral are the ones that extremize the action:

$$Z_{\text{UV}}[J_\phi, 0] = \int [D\phi] \exp \left[i \int d^d x \left(\mathcal{L}_{\text{UV}}(\phi, H_{\text{cl}}(\phi)) + J_\phi \phi \right) \right] \quad 0 = \frac{\delta S}{\delta H} |_{H=H_{\text{cl}}(\phi)}$$

that is, $H_{\text{cl}}(\phi)$ solves the classical equations of motion in the UV Lagrangian

Integrate-out and Matching

Apply to the muon decay process:

Starting point: $\mathcal{L}_{\text{UV}} \supset -W_\rho^+(\square - m_W^2)W_\rho^- + \frac{g_L}{\sqrt{2}} [\bar{\nu}_e \gamma_\rho e_L + \bar{\nu}_\mu \gamma_\rho \mu_L] W_\rho^+ + \text{h.c.}$

e.o.m: $-(\square - m_W^2)W_\rho^- + \frac{g_L}{\sqrt{2}} [\bar{\nu}_e \gamma_\rho e_L + \bar{\nu}_\mu \gamma_\rho \mu_L] = 0$

solution: $W_\rho^- = \frac{g_L}{\sqrt{2}} (\square - m_W^2)^{-1} [\bar{\nu}_e \gamma_\rho e_L + \bar{\nu}_\mu \gamma_\rho \mu_L]$

(Non-local) Effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = \frac{g_L^2}{2} [\bar{e}_L \gamma_\rho \nu_e + \bar{\mu}_L \gamma_\rho \nu_\mu] (\square - m_W^2)^{-1} [\bar{\nu}_e \gamma_\rho e_L + \bar{\nu}_\mu \gamma_\rho \mu_L]$$

Leading (local) Effective Lagrangian: $\frac{1}{\square - m_W^2} = -\frac{1}{m_W^2} - \frac{\square}{m_W^4} - \frac{\square^2}{m_W^6} - \dots$

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & -\frac{g_L^2}{2m_W^2} [\bar{e}_L \gamma_\rho \nu_e + \bar{\mu}_L \gamma_\rho \nu_\mu] [\bar{\nu}_e \gamma_\rho e_L + \bar{\nu}_\mu \gamma_\rho \mu_L] + \mathcal{O}\left(\frac{1}{m_W^4}\right) \\ & - \frac{g_L^2}{2m_W^4} [\bar{e}_L \gamma_\rho \nu_e + \bar{\mu}_L \gamma_\rho \nu_\mu] \square [\bar{\nu}_e \gamma_\rho e_L + \bar{\nu}_\mu \gamma_\rho \mu_L] + \dots \end{aligned}$$

Tree-level matching: $\mathcal{M}_{EW} = \mathcal{M}_{\text{Fermi}} \longrightarrow G_F = \frac{g^2}{4\sqrt{2}m_W^2} = \frac{1}{\sqrt{2}v}$

Dimension Analysis

Fermi interaction is a higher dimensional operator

4D QFT functional integral: $Z = \int \mathcal{D}\phi e^{iS[\phi]}, \quad S = \int d^4x \mathcal{L}[\phi(x)]$

Natural units, $\hbar=c=1$: [Length] = Mass⁻¹ From kinetic terms
 $[\mathcal{L}] = 4 : [\phi] = 1, [\psi] = \frac{3}{2}, [D_\mu] = 1, [A_\mu] = 1, [g] = 0$

Renormalisable interactions have couplings $[c] \geq 0$

$$\mathcal{L}_{\text{int.}} = c \mathcal{O}, \quad [\mathcal{O}] \leq 4$$

- Renormalisable: need a **finite number** of counter-terms (CT) to absorb divergences in loop computations to **all orders** in perturbation theory

$$[\mathcal{O}] < 4, [c] > 0$$

‘Relevant’

$$[\mathcal{O}] = 4, [c] = 0$$

‘Marginal’

$$[\mathcal{O}] > 4, [c] < 0$$

‘Irrelevant’

$$I, \phi^2, \phi^3, \bar{\psi}\psi$$

$$\phi^4, \phi\bar{\psi}\psi, V_\mu\bar{\psi}\gamma^\mu\psi$$

$$\bar{\psi}\psi\bar{\psi}\psi, \partial_\mu\phi\bar{\psi}\gamma^\mu\psi, \phi^2\bar{\psi}\psi,$$

Renormalizability

Fermi interaction: $\Psi^4 = \text{dimension-6}$

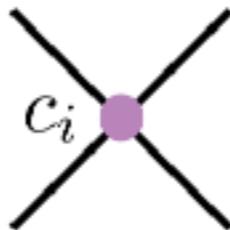
c = 'Wilson coefficient'

$$G_F \bar{\psi} \psi \bar{\psi} \psi \rightarrow \frac{c}{\Lambda_F^2} \bar{\psi} \psi \bar{\psi} \psi, [c] = 0$$

Λ_F = 'cutoff' (nothing to do with loop integration)

Inserting an operator once into a $2 \rightarrow 2$ amplitude:

$$\mathcal{L}_{\text{eff.}} = \sum_i \frac{c_i \mathcal{O}_i^d}{\Lambda^{4-d}}$$



$$[\mathcal{A}] = 0 \rightarrow \mathcal{A} \sim c_i \left(\frac{p}{\Lambda} \right)^{d-4}$$

Expect a power-like dependence on the external momenta

- **At most** equal to the power of Λ in the denominator
- Holds **beyond tree-level**: only physical (IR) scales result from loop integration
- Easily seen using dimensional regularisation, which discards power-like dependence on unphysical scales (unlike, e.g., cut-off regulator)

$$I_{\text{DR}} \sim f \left(p^2, m^2, \log \frac{\mu^2}{p^2}, \log \frac{\mu^2}{m^2}, \frac{1}{\epsilon}, \dots \right)$$

physical
renormalisation
poles

Non-Renormalizable?

Consequence:

- Higher order corrections involving further $d > 4$ operators lead to **higher power** momentum dependence


$$\rightarrow \mathcal{A} \sim \left(\frac{p}{\Lambda}\right)^{n_i+n_j}$$

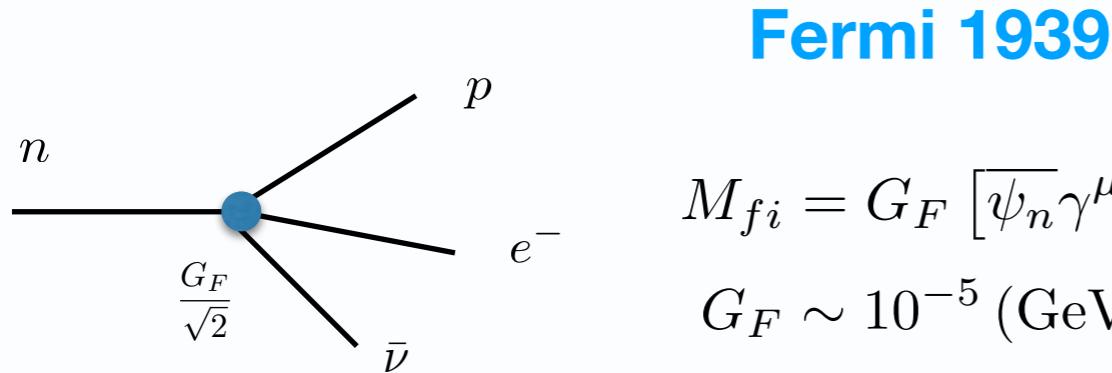
- Renormalisation requires **counter-terms** from higher dimensional operator (d_i+d_j) to cancel divergent piece



- EFTs require an infinite number of CTs to cancel poles to all orders: formally **non-renormalisable**
- Poles can be cancelled (renormalisable) **order-by-order** in Λ

Fermi EFT

Suppose we do not know the underlying theory (SM), how to describe weak decay?



$$M_{fi} = G_F [\bar{\psi}_n \gamma^\mu \psi_p] [\bar{\psi}_e \gamma^\mu \psi_\nu]$$

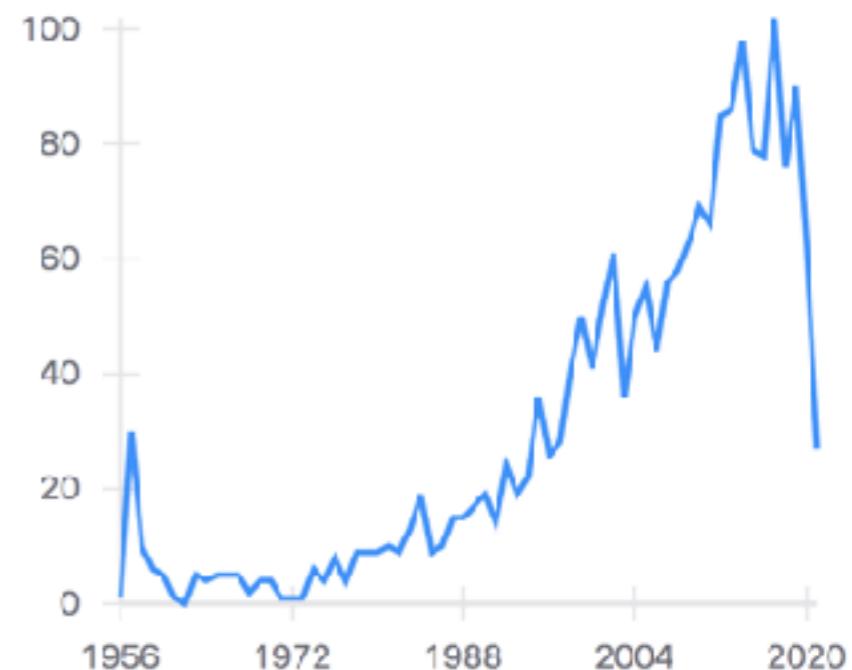
$$G_F \sim 10^{-5} (\text{GeV})^{-2}$$

Lee-Yang 1956

If parity is not conserved in β decay, the most general form of Hamiltonian can be written as

$$\begin{aligned} H_{\text{int}} = & (\psi_p^\dagger \gamma_4 \psi_n) (C_S \psi_e^\dagger \gamma_4 \psi_\nu + C_{S'} \psi_e^\dagger \gamma_4 \gamma_5 \psi_\nu) \\ & + (\psi_p^\dagger \gamma_4 \gamma_\mu \psi_n) (C_V \psi_e^\dagger \gamma_4 \gamma_\mu \psi_\nu + C_{V'} \psi_e^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_\nu) \\ & + \frac{1}{2} (\psi_p^\dagger \gamma_4 \sigma_{\lambda \mu} \psi_n) (C_T \psi_e^\dagger \gamma_4 \sigma_{\lambda \mu} \psi_\nu \\ & + C_{T'} \psi_e^\dagger \gamma_4 \sigma_{\lambda \mu} \gamma_5 \psi_\nu) + (\psi_p^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_n) \\ & \times (-C_A \psi_e^\dagger \gamma_4 \gamma_\mu \gamma_5 \psi_\nu - C_{A'} \psi_e^\dagger \gamma_4 \gamma_\mu \psi_\nu) \\ & + (\psi_p^\dagger \gamma_4 \gamma_5 \psi_n) (C_P \psi_e^\dagger \gamma_4 \gamma_5 \psi_\nu + C_{P'} \psi_e^\dagger \gamma_4 \psi_\nu), \quad (\text{A.1}) \end{aligned}$$

Citations per year



Comprehensive analysis of beta decays
within and beyond the Standard Model

Adam Falkowski,^a Martín González-Alonso,^b and Oscar Naviliat-Cuncic^{c,d}

energies. The general EFT Lagrangian describing these interactions at the leading order was written more than 60 years ago by Lee and Yang [6]:

$$\begin{aligned} \mathcal{L}_{\text{Lee-Yang}} = & -\bar{\nu} T^\mu n (C_V \bar{e} \gamma_\mu \nu - C'_V \bar{e} \gamma_\mu \gamma_5 \nu) + \bar{\nu} T^\mu \bar{n} n (C_A \bar{e} \gamma_\mu \gamma_5 \nu - C'_A \bar{e} \gamma_\mu \nu) \\ & - \bar{\nu} n (C_S \bar{e} \nu - C'_S \bar{e} \gamma_5 \nu) - \frac{1}{2} \bar{\rho} \sigma^{\mu\nu} n (C_T \bar{e} \sigma_{\mu\nu} \nu - C'_T \bar{e} \sigma_{\mu\nu} \gamma_5 \nu) \\ & - \bar{\rho} \gamma_5 n (C_P \bar{e} \gamma_5 \nu - C'_P \bar{e} \nu) + \text{l.c.} \end{aligned} \quad (\text{1.1})$$

Bottom-up Approach

Write down the most general Lagrangian with leptons and quarks (a,g) only

Low energy EFT: SU3 x U1 singlet

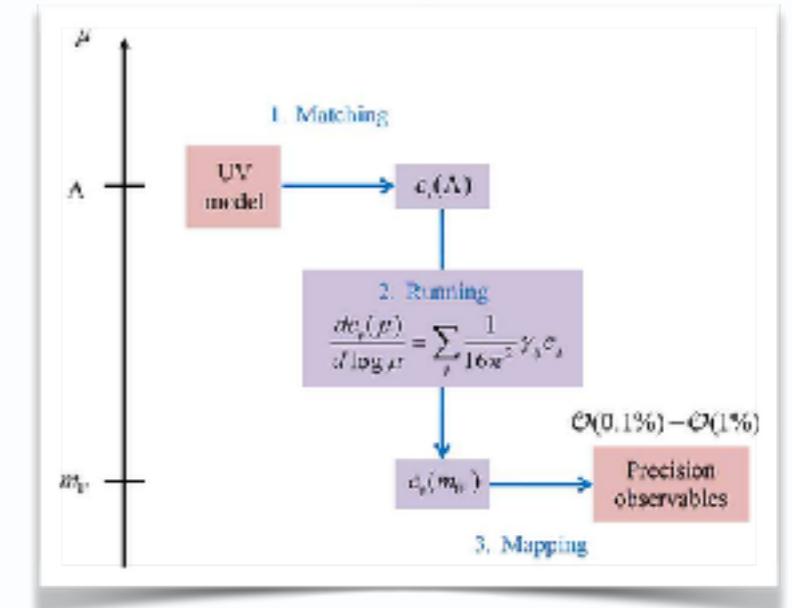
	$(\bar{L}L)(\bar{L}L)$		$(\bar{L}L)(\bar{R}R)$		$(\bar{L}R)(\bar{L}R) + \text{h.c.}$
$\mathcal{O}_{\nu\nu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{\nu}_{Ls}\gamma_\mu\nu_{Lt})$	$\mathcal{O}_{ee}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$	$\mathcal{O}_{ee}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{e}_{Ls}e_{Rt})$
$\mathcal{O}_{ee}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})$	$\mathcal{O}_{ee}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$	$\mathcal{O}_{eu}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{u}_{Ls}u_{Rt})$
$\mathcal{O}_{\nu e}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{e}_{Ls}\gamma_\mu e_{Lt})$	$\mathcal{O}_{eu}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$	$\mathcal{O}_{eu}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{u}_{Ls}\sigma_{\mu\nu}u_{Rt})$
$\mathcal{O}_{\nu u}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$	$\mathcal{O}_{ed}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$	$\mathcal{O}_{ed}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{d}_{Ls}d_{Rt})$
$\mathcal{O}_{\nu d}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu\nu_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$	$\mathcal{O}_{eu}^{V,LR}$	$(e_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$	$\mathcal{O}_{ed}^{T,RR}$	$(\bar{e}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{d}_{Ls}\sigma_{\mu\nu}d_{Rt})$
$\mathcal{O}_{eu}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$	$\mathcal{O}_{ed}^{V,LR}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$	$\mathcal{O}_{edn}^{S,RR}$	$(\bar{e}_{Lp}e_{Rr})(\bar{d}_{Ls}u_{Rt})$
$\mathcal{O}_{ed}^{V,LL}$	$(\bar{e}_{Lp}\gamma^\mu e_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$	$\mathcal{O}_{ue}^{V,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$	$\mathcal{O}_{edn}^{T,RR}$	$(\bar{p}_{Lp}\sigma^{\mu\nu}e_{Rr})(\bar{d}_{Ls}\sigma_{\mu\nu}u_{Rt})$
$\mathcal{O}_{\nu edu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu c_{Lr})(\bar{d}_{Ls}\gamma_\mu u_{Lt}) + \text{h.c.}$	$\mathcal{O}_{dc}^{V,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{c}_{Rs}\gamma_\mu c_{Rt})$	$\mathcal{O}_{uu}^{S,RR}$	$(\bar{u}_{Lp}u_{Rr})(\bar{u}_{Ls}u_{Rt})$
$\mathcal{O}_{\nu euu}^{V,LL}$	$(\bar{\nu}_{Lp}\gamma^\mu u_{Lr})(\bar{u}_{Ls}\gamma_\mu u_{Lt})$	$\mathcal{O}_{\nu edu}^{V,LR}$	$(\bar{\nu}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Rs}\gamma_\mu u_{Rt}) + \text{h.c.}$	$\mathcal{O}_{uu}^{SS,RR}$	$(\bar{u}_{Lp}T^A u_{Rr})(\bar{u}_{Ls}T^A u_{Rt})$
$\mathcal{O}_{dd}^{V,LL}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$	$\mathcal{O}_{\nu dd}^{V1,L,R}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$	$\mathcal{O}_{ud}^{S1,RR}$	$(\bar{u}_{Lp}u_{Rr})(\bar{d}_{Ls}d_{Rt})$
$\mathcal{O}_{ud}^{V1,LL}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Ls}\gamma_\mu d_{Lt})$	$\mathcal{O}_{\nu dd}^{VS,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{u}_{Rs}\gamma_\mu T^A u_{Rt})$	$\mathcal{O}_{ud}^{SS,RR}$	$(\bar{u}_{Lp}T^A u_{Rr})(\bar{d}_{Ls}T^A d_{Rt})$
$\mathcal{O}_{ud}^{V8,LL}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{d}_{Ls}\gamma_\mu T^A d_{Lt})$	$\mathcal{O}_{ud}^{V1,LR}$	$(\bar{u}_{Lp}\gamma^\mu u_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$	$\mathcal{O}_{dd}^{S1,RR}$	$(\bar{d}_{Lp}d_{Rr})(\bar{d}_{Ls}d_{Rt})$
		$\mathcal{O}_{ud}^{V8,LR}$	$(\bar{u}_{Lp}\gamma^\mu T^A u_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$	$\mathcal{O}_{ad}^{SS,RR}$	$(\bar{d}_{Lp}T^A d_{Rr})(\bar{d}_{Ls}T^A d_{Rt})$
	$(\bar{R}R)(\bar{R}R)$				$(\bar{L}R)(\bar{R}L) + \text{h.c.}$
$\mathcal{O}_{ee}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{e}_{Rs}\gamma_\mu e_{Rt})$	$\mathcal{O}_{du}^{V1,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$	$\mathcal{O}_{\nu du}^{S1,RR}$	$(\bar{u}_{Lp}d_{Rr})(\bar{d}_{Ls}u_{Rt})$
$\mathcal{O}_{eu}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$	$\mathcal{O}_{du}^{VS,LR}$	$(\bar{d}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{u}_{Rs}\gamma_\mu T^A u_{Rt})$	$\mathcal{O}_{\nu du}^{SS,RR}$	$(\bar{u}_{Lp}T^A d_{Rr})(\bar{d}_{Ls}T^A u_{Rt})$
$\mathcal{O}_{ed}^{V,RR}$	$(\bar{e}_{Rp}\gamma^\mu e_{Rr})(d_{Rs}\gamma_\mu d_{Rt})$	$\mathcal{O}_{dd}^{V1,LR}$	$(\bar{d}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$		$(\bar{L}R)(\bar{R}L) + \text{h.c.}$
$\mathcal{O}_{eu}^{V,RR}$	$(\bar{u}_{Rp}\gamma^\mu u_{Rr})(\bar{u}_{Rs}\gamma_\mu u_{Rt})$	$\mathcal{O}_{dd}^{VS,LR}$	$(\bar{d}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$	$\mathcal{O}_{eu}^{S,R,L}$	$(\bar{e}_{Lp}e_{Rr})(\bar{u}_{Rs}u_{Lt})$
$\mathcal{O}_{dd}^{V,RR}$	$(\bar{d}_{Rp}\gamma^\mu d_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$	$\mathcal{O}_{u,d,u}^{V1,L,R}$	$(\bar{u}_{Lp}\gamma^\mu d_{Lr})(\bar{d}_{Rs}\gamma_\mu u_{Rt}) + \text{h.c.}$	$\mathcal{O}_{ed}^{S,R,L}$	$(\bar{e}_{Lp}e_{Rr})(\bar{d}_{Rs}d_{Lt})$
$\mathcal{O}_{ud}^{V1,RR}$	$(\bar{u}_{Rp}\gamma^\mu u_{Rr})(\bar{d}_{Rs}\gamma_\mu d_{Rt})$	$\mathcal{O}_{u,d,u}^{VS,L,R}$	$(\bar{u}_{Lp}\gamma^\mu T^A d_{Lr})(\bar{d}_{Rs}\gamma_\mu T^A u_{Rt}) + \text{h.c.}$	$\mathcal{O}_{uedu}^{S,R,L}$	$(\bar{p}_{Lp}e_{Rr})(\bar{d}_{Rs}u_{Lt})$
$\mathcal{O}_{ud}^{V8,RR}$	$(\bar{u}_{Rp}\gamma^\mu T^A u_{Rr})(\bar{d}_{Rs}\gamma_\mu T^A d_{Rt})$				

- (i) muon decay: $g_{e\mu}$
 $\mu^- \rightarrow \nu_\mu e^- \bar{\nu}_e$
- (ii) neutron decay : g_{ud}
 $n \rightarrow p e^- \bar{\nu}_e$ ($d \rightarrow u e^- \bar{\nu}_e$)
- (ii) kaon decay: g_{us}
 $K^- \rightarrow \pi^0 e^- \bar{\nu}_e$ ($s \rightarrow u e^- \bar{\nu}_e$)

Effective Field Theory

Top-down Approach: matching/running among scales

$$e^{\frac{i}{\hbar}S_{IR}[\text{Light}]} = \int D[\text{Heavy}] e^{\frac{i}{\hbar}S_{UV}[\text{Light, Heavy}]}$$



Bottom-up Approach: field d.o.f and symmetry at certain scale

$$W = \int_{k < \Lambda} [Dg][DA][D\psi][D\Phi] \exp \left\{ i \int d^4x \sqrt{-g} \left[\frac{m_p^2}{2} R + \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \mathcal{L}_5 + \frac{1}{\Lambda^2} \mathcal{L}_6 + \dots \right] \right\}$$

[Weinberg 1979, 2009]

one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible *S*-matrix consistent with perturbative unitarity, analyticity, cluster decomposition, and the assumed symmetry properties."

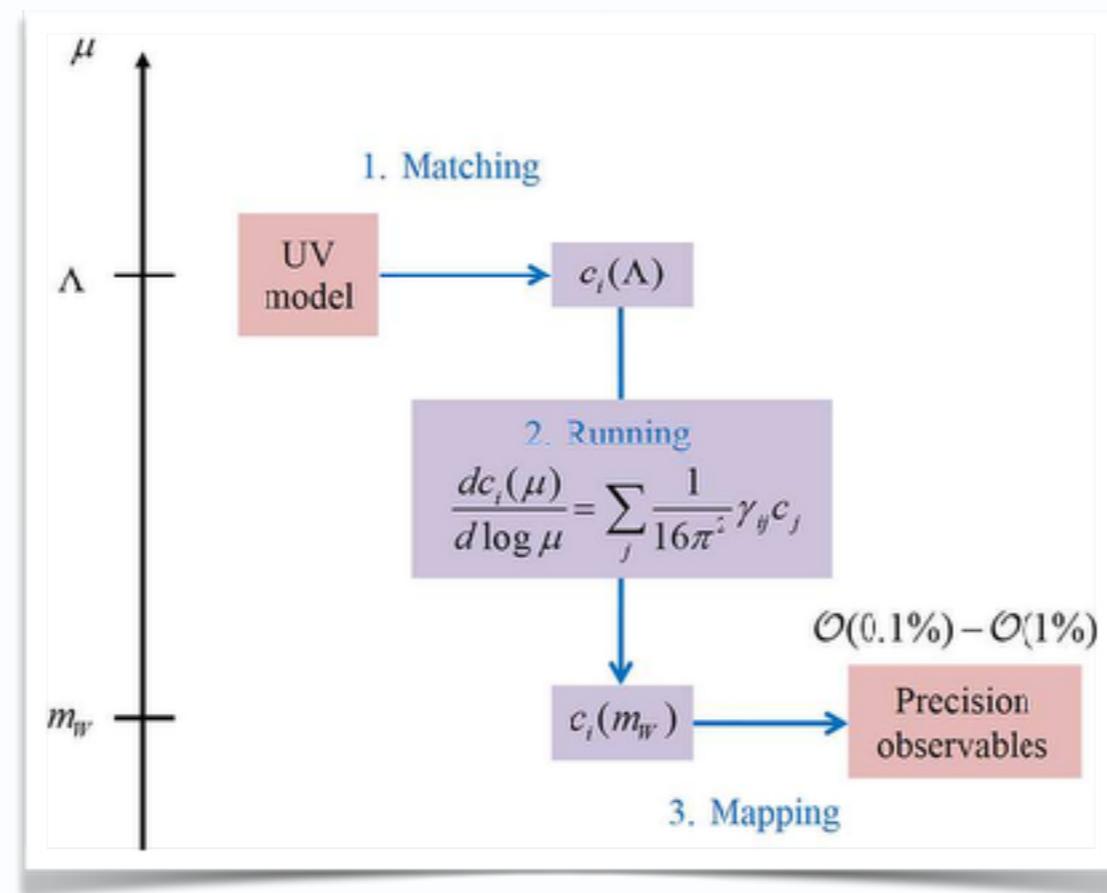
a folk theorem: "if

Weinberg's Folk theorem

Core of EFTs

Scales

Decoupling Theorem



Field d.o.f Power counting

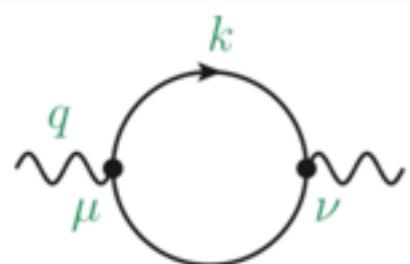
Symmetries

Decoupling Theorem

Decoupling:

Appelquist–Carazzone

The low-energy effects of heavy particles are either suppressed by inverse powers of the heavy masses, or they get absorbed into renormalizations of the couplings and fields of the EFT obtained by removing the heavy particles



$$\Pi(q^2) = -\frac{\alpha Q_f^2}{3\pi} \left\{ \Delta_\infty(\mu) + 6 \int_0^1 dx \, x(1-x) \log \left(\frac{m_f^2 - q^2 x(1-x)}{\mu^2} \right) \right\}$$

$$\alpha_0 \left\{ 1 - \Delta \Pi_\epsilon(\mu^2) - \Pi_R(q^2/\mu^2) \right\} \equiv \alpha_R(\mu^2) \left\{ 1 - \Pi_R(q^2/\mu^2) \right\}$$

$$\frac{\mu}{\alpha} \frac{d\alpha}{d\mu} \equiv \beta(\alpha) = \beta_1 \frac{\alpha}{\pi} + \dots$$

$$\Pi(q^2) \equiv \Delta \Pi_\epsilon(\mu^2) + \Pi_R(q^2/\mu^2)$$

RGE depends on scheme

Decouple or Not?

MSbar scheme

$$\Delta\Pi_\epsilon(\mu^2) \equiv -Q_f^2 \frac{\alpha_0}{3\pi} \Delta_\infty(\mu) \quad \Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{\mu^2} \right]$$

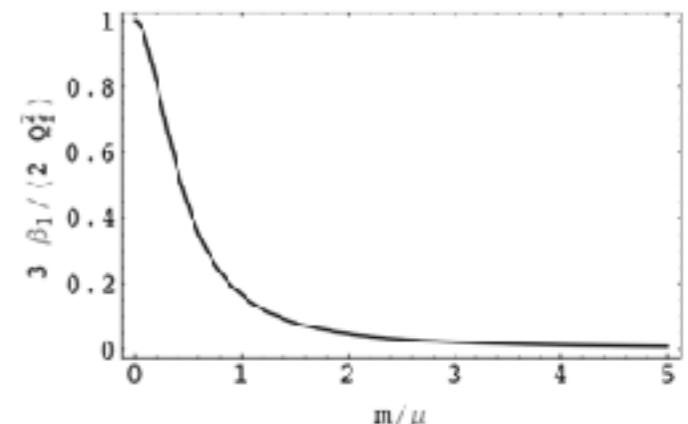
$$\beta_1 = \frac{2}{3} Q_f^2$$

Heavy fermions do not decouple

Mass-dependent scheme

$$\Delta\Pi_\epsilon(\mu^2) \equiv \Pi(-\mu^2) \quad \Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} 6 \int_0^1 dx x(1-x) \log \left[\frac{m_f^2 - q^2 x(1-x)}{m_f^2 + \mu^2 x(1-x)} \right]$$

$$\beta_1 = 4 Q_f^2 \int_0^1 dx \frac{\mu^2 x^2 (1-x)^2}{m_f^2 + \mu^2 x(1-x)}$$



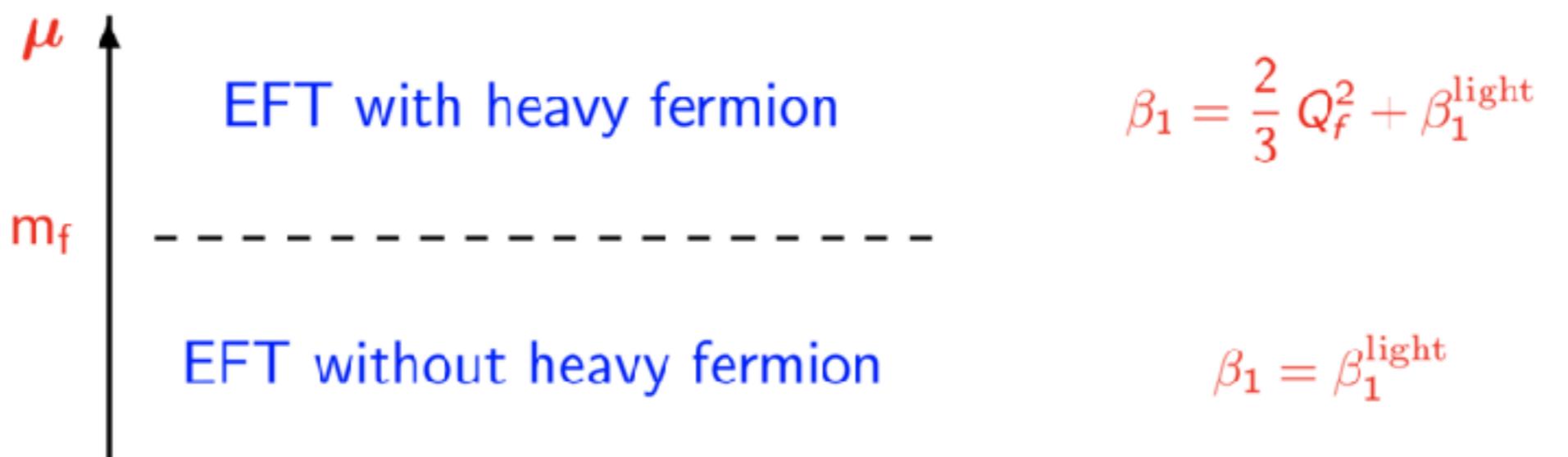
- $m_f^2 \ll \mu^2, q^2$: $\beta_1 = \frac{2}{3} Q_f^2$, $\Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} \log(-q^2/\mu^2)$

- $m_f^2 \gg \mu^2, q^2$: $\beta_1 \sim \frac{2}{15} Q_f^2 \frac{\mu^2}{m_f^2}$, $\Pi_R(q^2/\mu^2) \sim Q_f^2 \frac{\alpha}{15\pi} \frac{q^2 + \mu^2}{m_f^2}$

Heavy fermions decouple

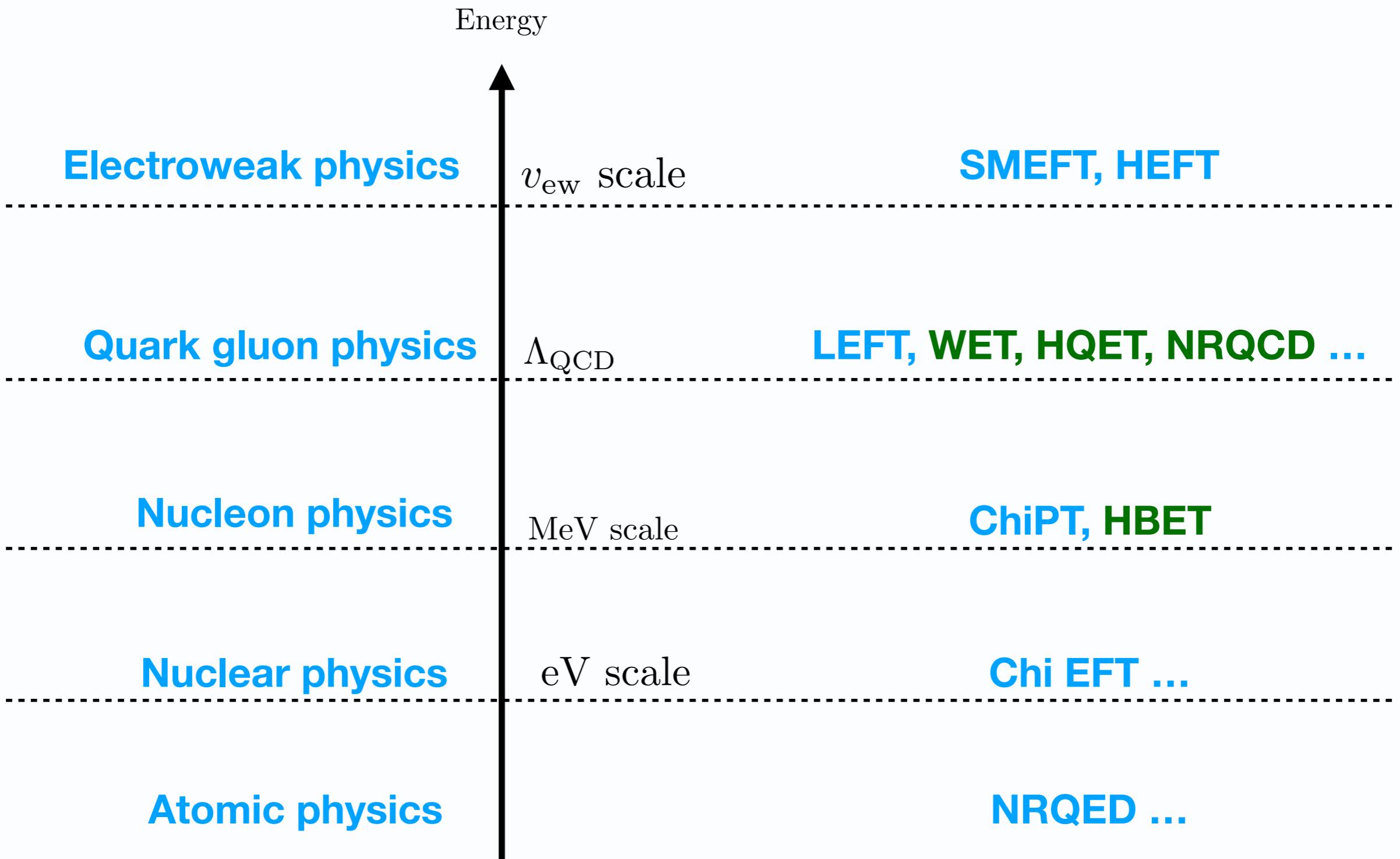
Integrate-out and Matching

Solution in MSbar scheme (do EFT in MSbar scheme)



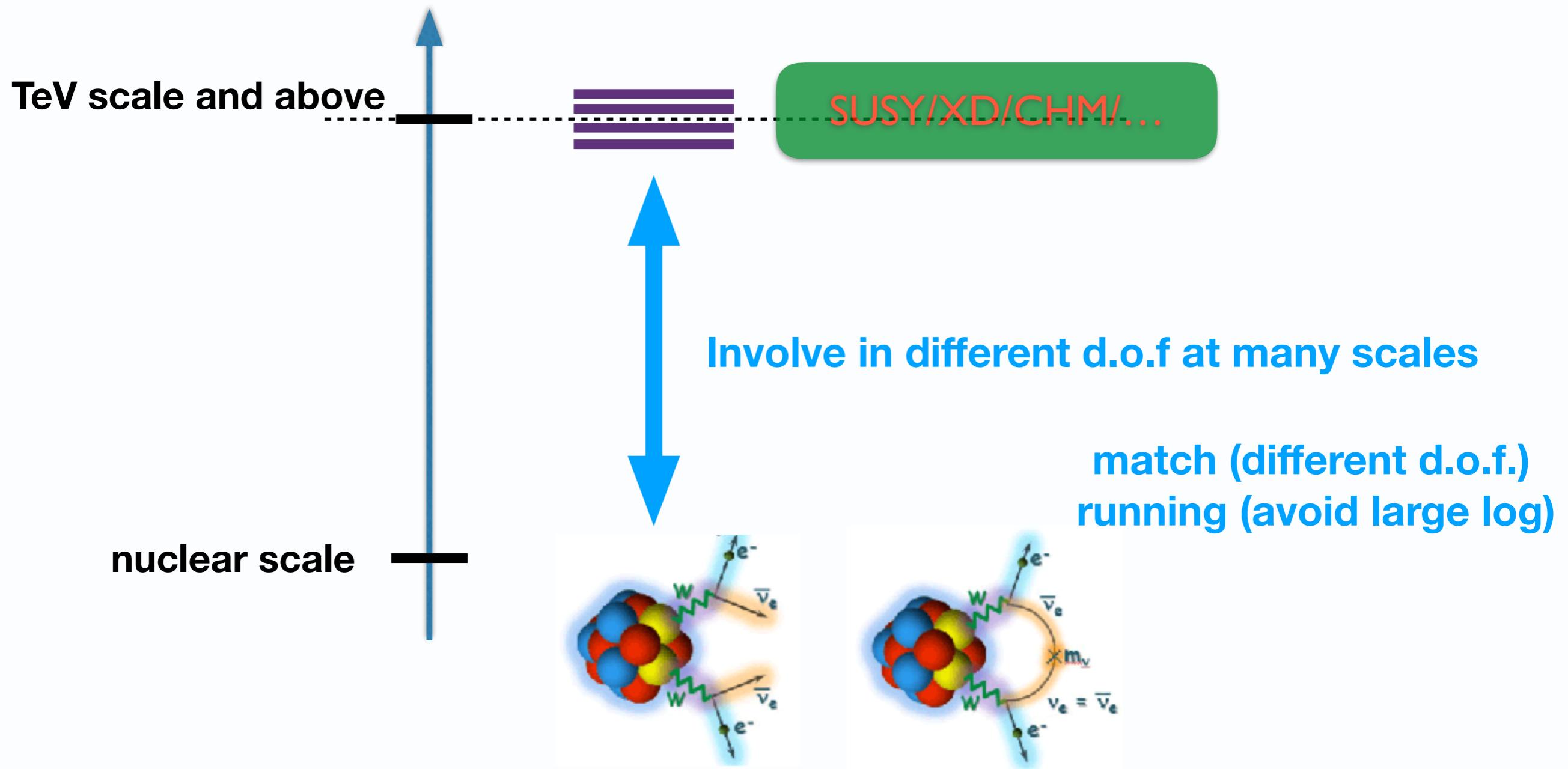
- Two different EFTs (with and without the heavy fermion f)
- Same S-matrix elements for light-particle scattering at $\mu \sim m_f$
- Different β function \rightarrow Different coupling
Couplings are auxiliary parameters (not observables)
- Same IR behaviour. Different UV behaviour

Effective Field Theories



Why Many EFTs?

Low energy probes of high energy physics



Fermi EFT, Again

Beta decay at nucleon level

$$\mathcal{L}_{\text{eff}} \supset -\frac{2V_{ud}}{\sqrt{2}}(\bar{e}_L \gamma_\rho \nu_e)(\bar{u}_L \gamma_\rho d_L) + \text{h.c.} \quad d \rightarrow ue^- \bar{\nu}_e \Rightarrow n \rightarrow pe^- \bar{\nu}_e$$

$$\begin{aligned}
 M(n \rightarrow pe^- \bar{\nu}_e) &= -\frac{2V_{ud}}{\sqrt{2}} \langle pe^- \bar{\nu}_e | (\bar{e}_L \gamma_\rho \nu_e)(\bar{u}_L \gamma_\rho d_L) | n \rangle \\
 &= -\frac{2V_{ud}}{\sqrt{2}} \langle e^- \bar{\nu}_e | (\bar{e}_L \gamma_\rho \nu_e) | 0 \rangle \langle p | (\bar{u}_L \gamma_\rho d_L) | n \rangle \\
 &= -\frac{2V_{ud}}{\sqrt{2}} (\bar{u}(p_e) \gamma_\rho P_L v(p_\nu)) \langle p | (\bar{u}_L \gamma_\rho d_L) | n \rangle \quad P_L \equiv \frac{1 - \gamma_5}{2} \\
 &= -\frac{V_{ud}}{\sqrt{2}} (\bar{u}(p_e) \gamma_\rho P_L v(p_\nu)) \left\{ \langle p | (\bar{u} \gamma_\rho d) | n \rangle - \langle p | (\bar{u} \gamma_\rho \gamma_5 d) | n \rangle \right\} \\
 M(n \rightarrow pe^- \bar{\nu}_e) &= -\frac{V_{ud}}{\sqrt{2}} (\bar{u}(p_e) \gamma_\rho P_L v(p_\nu)) \left\{ \bar{u}(p_p) \gamma_\rho u(p_n) - g_A \bar{u}(p_p) \gamma_\rho \gamma_5 u(p_n) + \mathcal{O}(q) \right\}
 \end{aligned}$$

$$\mathcal{L}_{\text{UV}} \supset -\frac{2V_{ud}}{\sqrt{2}}(\bar{e}_L \gamma_\rho \nu_e)(\bar{u}_L \gamma_\rho d_L) + \text{h.c.}$$



Matching

$$\mathcal{L}_{\text{eff}} \supset -\frac{V_{ud}}{\sqrt{2}}(\bar{e}_L \gamma_\rho \nu_e) \left\{ (\bar{p} \gamma_\rho n) - g_A (\bar{p} \gamma_\rho \gamma_5 n) \right\} + \text{h.c.} + \mathcal{O}\left(\frac{q}{m_n}\right)$$

Lattice

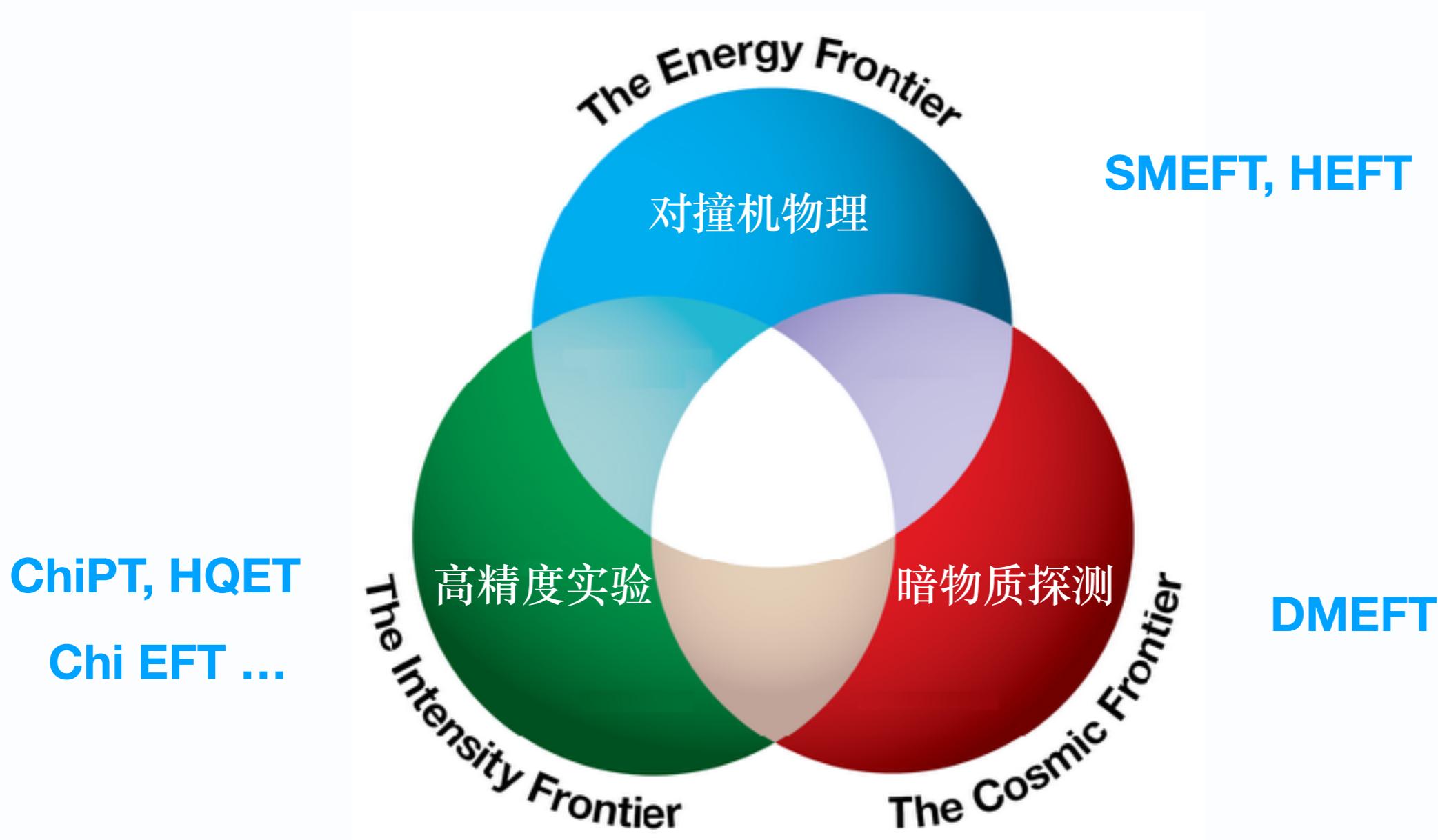
Experiment

$g_A = 1.271 \pm 0.013$

$g_A = 1.27536 \pm 0.00041$

EFT Description

EFT provides systematical description on new physics effects



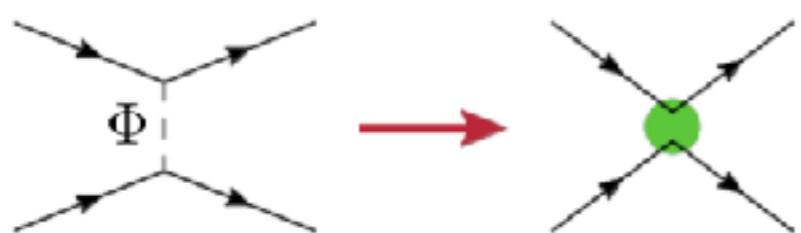
Useful even after new physics is found

Exercise: Matching at tree/loop

Still Fermi theory, but the UV from massive scalar

$$\mathcal{L}_{\text{Full}} = i\bar{\psi}\not{\partial}\psi + \frac{1}{2}(D_\mu\Phi)^2 - \frac{1}{2}M^2\Phi^2 - \lambda\Phi\bar{\psi}\psi$$

Heavy mass scale Yukawa interaction



$$\mathcal{L}_{\text{EFT}} = i\bar{\psi}\partial\psi + \frac{c_S}{M^2} \frac{1}{2}(\bar{\psi}\psi)(\bar{\psi}\psi)$$

Matching condition
 $c_S = \lambda^2$

Lorentz structure does not match cs: new operator @ 1-loop

$$\mathcal{L}_{\text{EFT}} = i\bar{\psi}\not{\partial}\psi - \sigma\bar{\psi}\psi + \frac{c_S}{2M^2}(\bar{\psi}\psi)(\bar{\psi}\psi) + \frac{c_T}{2M^2}(\bar{\psi}\sigma^{\mu\nu}\psi)(\bar{\psi}\sigma_{\mu\nu}\psi)$$

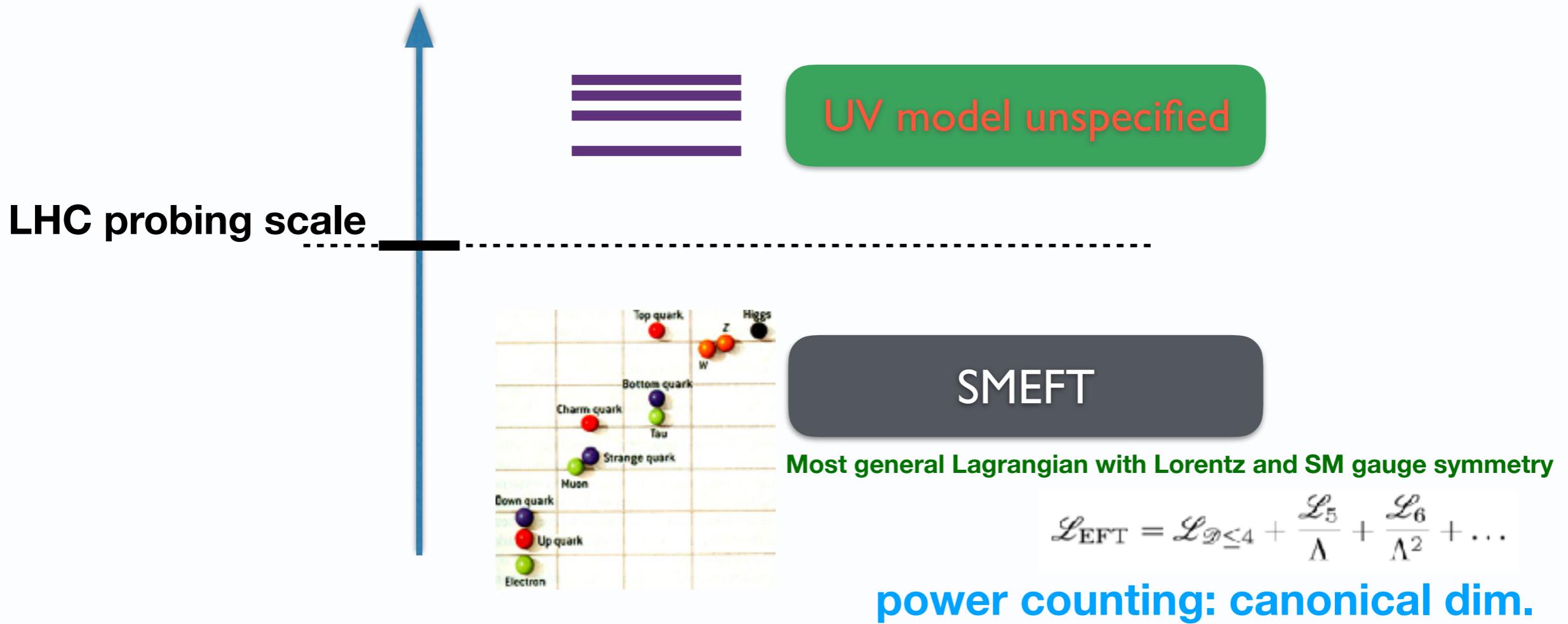
$$c_S = \lambda^2 + \mathcal{O}(\alpha^2) \quad \text{No one loop correction}$$

$$c_T = -\frac{3\alpha Q^2}{8\pi} \lambda^2 + \mathcal{O}(\alpha^2) \quad \text{Leading order one-loop}$$

$$\gamma = \frac{2Q^2\alpha}{\pi} \begin{pmatrix} -3/2 & -12 \\ -1/4 & 1/2 \end{pmatrix} \begin{matrix} \text{cs} \\ \text{ct} \end{matrix} \quad \frac{d}{d \ln \mu} c_i = \gamma_{ij} c_j$$

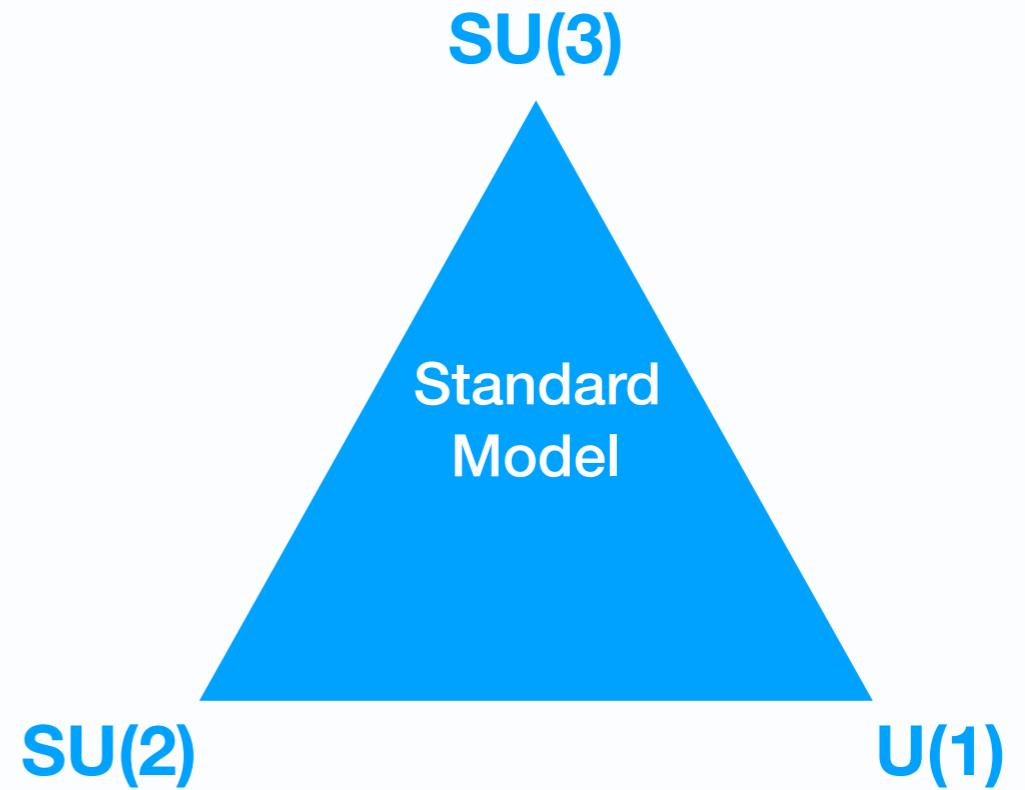
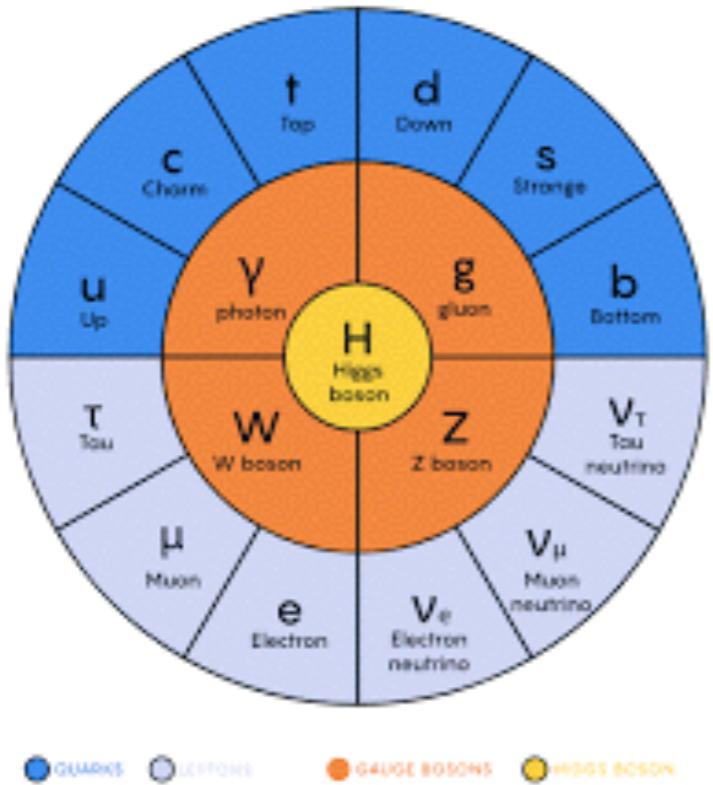
Proficient: SMEFT

SMEFT from Bottom-up



SMEFT provides systematical parametrization of
... all possible Lorentz inv. new physics!

Fields, Symmetry, Expansion



$$\begin{array}{ccc}
 S = \int d^4x \mathcal{L}(x) & \xrightarrow{\text{(Action)}} & [\mathcal{L}] = E^4 \\
 & \xrightarrow{\text{(Lagrangian)}} & \\
 \mathcal{L}_{KG} = \partial_\mu \phi^\dagger \partial^\mu \phi - m^2 \phi^\dagger \phi & \xrightarrow{\text{(scalar)}} & [\phi] = [V^\mu] = [A^\mu] = E \\
 & & \xrightarrow{\text{(vector)}} \\
 \mathcal{L}_{\text{Dirac}} = \bar{\psi} (i\gamma^\mu \partial_\mu - m) \psi & \xrightarrow{\text{(fermion)}} & [\psi] = E^{3/2}
 \end{array}$$

power counting: canonical dim.

SO(N) Group

$$[X_{\mu\nu}, X_{\alpha\beta}] = X_{\mu\nu}X_{\alpha\beta} - X_{\alpha\beta}X_{\mu\nu} \quad X_{\mu\nu} = -X_{\nu\mu}, \quad \mu, \nu = 1, 2, \dots, N$$

$$= i(\delta_{\mu\alpha}X_{\nu\beta} + \delta_{\nu\beta}X_{\mu\alpha} - \delta_{\mu\beta}X_{\nu\alpha} - \delta_{\nu\alpha}X_{\mu\beta}).$$

$$J_{j,k} = \frac{\hbar}{i} \left(x_j \frac{\partial}{\partial x_k} - x_k \frac{\partial}{\partial x_j} \right)$$

$$[J_{ij}, J_{k\ell}] = i\hbar (\delta_{ik}J_{j\ell} + \delta_{j\ell}J_{ik} - \delta_{i\ell}J_{jk} - \delta_{jk}J_{i\ell})$$

$$[J_x, J_y] = iJ_z, [J_y, J_z] = iJ_x, [J_z, J_x] = iJ_y.$$

tensor rep

$$R(\alpha, \beta, \gamma) =$$

$$\begin{pmatrix} \cos\alpha \cos\beta \cos\gamma - \sin\alpha \sin\gamma \\ -\cos\beta \cos\gamma \sin\alpha - \cos\alpha \sin\gamma \\ -\cos\gamma \sin\beta \end{pmatrix}$$

$SU(2)/Z_2$, $Z_2 = \{-1, 1\}$
covering group

$$U(\alpha, \beta, \gamma) = \begin{pmatrix} e^{i(\alpha+\gamma)/2} \cos\beta/2 & e^{i(\alpha-\gamma)/2} \sin\beta/2 \\ -e^{-i(\alpha-\gamma)/2} \sin\beta/2 & e^{-i(\alpha+\gamma)/2} \cos\beta/2 \end{pmatrix}$$

$$= \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}$$

Jiang-Hao Yu

SO(n)

SO(n-1,1)

$$L_{j,k} = \frac{\hbar}{i} \left(x_j \frac{\partial}{\partial x_k} - x_k \frac{\partial}{\partial x_j} \right), \quad L_{0,j} = \frac{\hbar}{i} \left(x_0 \frac{\partial}{\partial x_j} + x_j \frac{\partial}{\partial x_0} \right)$$

$$[L_{0,j}, L_{0,k}] = -i\hbar L_{j,k}, \quad [L_{0,j}, L_{k,n}] = i\hbar (\delta_{jn}L_{0,k} - \delta_{jk}L_{0,n}),$$

$$[L_{k,j}, L_{m,n}] = i\hbar (\delta_{km}L_{j,n} + \delta_{nj}L_{k,m} - \delta_{kn}L_{j,m} - \delta_{jm}L_{k,n}).$$

SO(3,1)

$$(M_{12}, M_{23}, M_{31}) = (J_3, J_1, J_2), \quad \frac{1}{2}M^{\mu\nu}M_{\mu\nu} = \mathbf{J}^2 - \mathbf{K}^2,$$

$$(M_{01}, M_{02}, M_{03}) = (K_1, K_2, K_3), \quad \frac{1}{2}\epsilon^{\mu\nu\sigma\tau}M_{\mu\nu}M_{\sigma\tau} = -\mathbf{J} \cdot \mathbf{K},$$

$$[M_{\lambda\rho}, M_{\mu\nu}] = -i(g_{\lambda\mu}M_{\rho\nu} + g_{\rho\nu}M_{\lambda\mu} - g_{\lambda\nu}M_{\rho\mu} - g_{\rho\mu}M_{\lambda\nu})$$

$$M_i = \frac{1}{2}(J_i + iK_i),$$

$$N_i = \frac{1}{2}(J_i - iK_i),$$

$$[M_i, M_j] = i\epsilon_{ijk}M_k,$$

$$[N_i, N_j] = i\epsilon_{ijk}N_k,$$

$$[M_i, N_j] = 0.$$

SO(3,1) Irreps

SO(3,1) not simply connected, find irrep by its covering group SL(2,C)

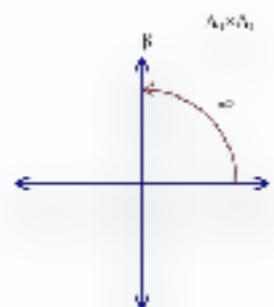
$$x^\mu = (x^0, x^1, x^2, x^3)$$

complexification

$$X = \sigma_\mu x^\mu = \begin{pmatrix} x^0 + x^3 & x^1 - ix^2 \\ x^1 + ix^2 & x^0 - x^3 \end{pmatrix}$$

$$x^\mu = \frac{1}{2}\text{Tr}(\underline{\sigma}^\mu X)$$

Spinor rep under SU(2) x SU(2)



The $(\frac{1}{2}, 0)$ Representation

$$N_i^+ = \frac{1}{2}(J_i + iK_i) := iK_i = \frac{1}{2}\sigma_i,$$

$$N_i^- = \frac{1}{2}(J_i - iK_i) = 0$$

$$K_i = -i\frac{1}{2}\sigma_i, \quad J_i = iK_i = \frac{1}{2}\sigma_i.$$

Complex conjugate

The $(0, \frac{1}{2})$ Representation

$$N_i^+ = \frac{1}{2}(J_i + iK_i) = 0$$

$$N_i^- = \frac{1}{2}(J_i - iK_i) = -iK_i = \frac{1}{2}\sigma_i,$$

$$J_i = \frac{1}{2}\sigma_i, \quad K_i = i\frac{1}{2}\sigma_i.$$

The $(\frac{1}{2}, \frac{1}{2})$ Representation

$$v^{\dot{a}\dot{b}} \rightarrow v^{\dot{c}\dot{d}} = \left(e^{\frac{i}{2}\theta\cdot\sigma - \frac{1}{2}\phi\cdot\sigma}\right)^{\dot{c}}_{\dot{a}} \left(e^{\frac{i}{2}\theta\cdot\sigma + \frac{1}{2}\phi\cdot\sigma}\right)^{\dot{d}}_{\dot{b}} v^{\dot{a}\dot{b}}$$

$$v^{\dot{a}\dot{b}} = v^\mu \sigma_{\mu}^{\dot{a}\dot{b}} = (v^0 \ v^1 \ v^2 \ v^3) \begin{pmatrix} \sigma_0 \\ \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix}$$

tensor rep

$$(v^0 \ v^1 \ v^2 \ v^3) = v^\mu$$

SM Fields Under $\text{SL}(2, \mathbb{C})$

$$\psi_\alpha \in (1/2, 0), \quad \psi_{\dot{\alpha}}^\dagger \in (0, 1/2), \quad H_i \in (0, 0), \quad H^{\dagger i} \in (0, 0),$$

$$F_{L\alpha\beta} = \frac{i}{2} F_{\mu\nu} \sigma_{\alpha\beta}^{\mu\nu} \in (1, 0), \quad F_{R\dot{\alpha}\dot{\beta}} = -\frac{i}{2} F_{\mu\nu} \bar{\sigma}_{\dot{\alpha}\dot{\beta}}^{\mu\nu} \in (0, 1). \quad X_{L,R}^{\mu\nu} = \frac{1}{2}(X^{\mu\nu} \mp i\tilde{X}^{\mu\nu})$$

$$h = j_r - j_l$$

Fields	$SU(2)_l \times SU(2)_r$	h	$SU(3)_C$	$SU(2)_W$	$U(1)_Y$	Flavor
$G_{L\alpha\beta}^A$	(1, 0)	-1	8	1	0	1
$W_{L\alpha\beta}^I$	(1, 0)	-1	1	3	0	1
$B_{L\alpha\beta}$	(1, 0)	-1	1	1	0	1
$L_{\alpha i}$	$(\frac{1}{2}, 0)$	-1/2	1	2	-1/2	n_f
$e_{c\alpha}$	$(\frac{1}{2}, 0)$	-1/2	1	1	1	n_f
$Q_{\alpha ai}$	$(\frac{1}{2}, 0)$	-1/2	3	2	1/6	n_f
$u_{c\alpha}^a$	$(\frac{1}{2}, 0)$	-1/2	3	1	-2/3	n_f
$d_{c\alpha}^a$	$(\frac{1}{2}, 0)$	-1/2	3	1	1/3	n_f
H_i	(0, 0)	0	1	2	1/2	1

Hermitian conjugate

$$(F_{L\alpha\beta})^\dagger = F_{R\dot{\alpha}\dot{\beta}}$$

$$H^\dagger \text{ (and } L^\dagger, Q^\dagger \text{) as a 2 of } SU(2) \quad \epsilon^{ij} H_i^\dagger H_j, \quad H_2^\dagger = \epsilon H_2^\dagger \quad e_R = e_C^\dagger, u_R = u_C^\dagger, d_R = d_C^\dagger$$

SO(3,1) Spinor Rep

Clifford algebra

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij}, \quad i = 1, \dots, N$$

$$\Gamma^{(\textcolor{brown}{f})}(i_1, \dots, i_f) \equiv (-)^{\frac{f(f-1)}{2}} \Gamma_{i_1} \cdots \Gamma_{i_f}$$

$$\gamma_\mu \rightarrow S^{-1}(a)\gamma_\mu S(a) = \sum_{\nu=1}^N a_{\mu\nu} \gamma_\nu$$

S(a) rep of SO(N)

SO(2n)

SO(2n+1)

irreducible rep of SO(N)

Γ_A

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \sigma_\mu^\dagger & 0 \end{pmatrix} \quad S(a) = \begin{pmatrix} S_1(a) & 0 \\ 0 & S_2(a) \end{pmatrix}$$

Reducible rep of SO(N)

$1, \gamma_\mu, \gamma_\mu \gamma_\nu (\mu < \nu), \gamma_\mu \gamma_\nu \gamma_\lambda (\mu < \nu < \lambda),$

$\dots, \gamma_1 \gamma_2 \gamma_3 \cdots \gamma_N.$

$$\text{Tr} [\Gamma_a^A \Gamma_b^B] = \delta^{AB} g_{ab}, \quad A, B = S, V, T, A, P,$$

$$\Lambda = \gamma_1 \gamma_2 \gamma_3 \cdots \gamma_N.$$

$$\gamma_\mu \Lambda = \begin{cases} \Lambda \gamma_\mu, & \text{if } N = 2M + 1 (\text{odd}), \\ -\Lambda \gamma_\mu, & \text{if } N = 2M (\text{even}) \end{cases}$$

$$[\Lambda, \Gamma_A] = 0 \quad \text{For odd N only}$$

$$S_1^{-1}(a)\sigma_\mu S_1(a) = \sum_{\nu=1}^N a_{\mu\nu} \sigma_\nu, \quad S_2^{-1}(a)\sigma_\mu^\dagger S_2(a) = \sum_{\nu=1}^N a_{\mu\nu} \sigma_\nu^\dagger,$$

$$J_{\mu\nu}^{(+)} = \frac{1}{4} (\sigma_\mu \sigma_\nu^\dagger - \sigma_\nu \sigma_\mu^\dagger)$$

$$J_{\mu\nu}^{(-)} = \frac{1}{4} (\sigma_\mu^\dagger \sigma_\nu - \sigma_\nu^\dagger \sigma_\mu)$$

$$[J_{\mu\nu}^{(\pm)}, J_{\alpha\beta}^{(\pm)}] = -\delta_{\mu\alpha} J_{\nu\beta}^{(\pm)} - \delta_{\nu\beta} J_{\mu\alpha}^{(\pm)} + \delta_{\mu\beta} J_{\nu\alpha}^{(\pm)} + \delta_{\nu\alpha} J_{\mu\beta}^{(\pm)}$$

Spinor Notation

$$\Psi = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot{\alpha}} \end{pmatrix}, \quad \bar{\Psi} = \Psi^\dagger \gamma^0 = \left(\chi^\alpha, \xi_{\dot{\alpha}}^\dagger \right)$$

$$\begin{aligned}\bar{\Psi}_1 \Psi_2 &= \chi_1^\alpha \xi_{2\alpha} + \xi_{1\dot{\alpha}}^\dagger \chi_2^{\dagger\dot{\alpha}}, \\ \bar{\Psi}_1 \gamma^\mu \Psi_2 &= \chi_1^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \chi_2^{\dagger\dot{\alpha}} + \xi_{1\dot{\alpha}}^\dagger \bar{\sigma}^{\mu\dot{\alpha}\alpha} \xi_{2\alpha}, \\ \bar{\Psi}_1 \sigma^{\mu\nu} \Psi_2 &= \chi_1^\alpha (\sigma^{\mu\nu})_\alpha^\beta \xi_{2\beta} + \xi_{1\dot{\alpha}}^\dagger (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} \chi_2^{\dot{\beta}}, \\ \Psi_1^T C \Psi_2 &= \xi_1^\alpha \xi_{2\alpha} + \chi_{1\dot{\alpha}}^\dagger \chi_2^{\dagger\dot{\alpha}}, \\ \Psi_1^T C \gamma^\mu \Psi_2 &= \xi_1^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \chi_2^{\dagger\dot{\alpha}} + \chi_{1\dot{\alpha}}^\dagger \bar{\sigma}^{\mu\dot{\alpha}\alpha} \xi_{2\alpha}, \\ \Psi_1^T C \sigma^{\mu\nu} \Psi_2 &= \xi_1^\alpha (\sigma^{\mu\nu})_\alpha^\beta \xi_{2\beta} + \chi_{1\dot{\alpha}}^\dagger (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} \chi_2^{\dot{\beta}}, \\ \bar{\Psi}_1 C \bar{\Psi}_2^T &= \xi_{1\dot{\alpha}}^\dagger \xi_2^{\dagger\dot{\alpha}} + \chi_1^\alpha \chi_{2\alpha}, \\ \bar{\Psi}_1 \gamma^\mu C \bar{\Psi}_2^T &= \chi_1^\alpha \sigma_{\alpha\dot{\alpha}}^\mu \xi_2^{\dagger\dot{\alpha}} + \xi_{1\dot{\alpha}}^\dagger \bar{\sigma}^{\mu\dot{\alpha}\alpha} \chi_{2\alpha}, \\ \bar{\Psi}_1 \sigma^{\mu\nu} C \bar{\Psi}_2^T &= \xi_{1\dot{\alpha}}^\dagger (\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} \xi_2^{\dagger\dot{\beta}} + \chi_1^\alpha (\sigma^{\mu\nu})_\alpha^\beta \chi_{2\beta}.\end{aligned}$$

$$q_L = \begin{pmatrix} Q \\ 0 \end{pmatrix}, \quad u_R = \begin{pmatrix} 0 \\ u_c^\dagger \end{pmatrix}, \quad d_R = \begin{pmatrix} 0 \\ d_c^\dagger \end{pmatrix}, \quad l_L = \begin{pmatrix} L \\ 0 \end{pmatrix}, \quad e_R = \begin{pmatrix} 0 \\ e_c^\dagger \end{pmatrix}.$$

$$\bar{q}_L = (0, Q^\dagger), \quad \bar{u}_R = (u_c, 0), \quad \bar{d}_R = (d_c, 0), \quad \bar{l}_L = (0, L^\dagger), \quad \bar{e}_R = (e_c, 0).$$

$$u_c \sigma^\mu u_c^\dagger = \bar{u} \gamma^\mu u, \quad e_c L = \bar{e} l, \quad u_c^\dagger d_c^\dagger = u^T C d.$$

Fierz and Schouten Identities

$$\begin{pmatrix} \delta_{ij}\delta_{kl} \\ (\gamma^\mu)_{ij}(\gamma_\mu)_{kl} \\ \frac{1}{2}(\sigma^{\mu\nu})_{ij}(\sigma_{\mu\nu})_{kl} \\ (\gamma^\mu\gamma_5)_{ij}(\gamma_\mu\gamma_5)_{kl} \\ (\gamma_5)_{ij}(\gamma_5)_{kl} \end{pmatrix} = \begin{pmatrix} 1/4 & 1/4 & 1/4 & -1/4 & 1/4 \\ 1 & -1/2 & 0 & -1/2 & -1 \\ 3/2 & 0 & -1/2 & 0 & 3/2 \\ -1 & -1/2 & 0 & -1/2 & 1 \\ 1/4 & -1/4 & 1/4 & 1/4 & 1/4 \end{pmatrix} \begin{pmatrix} \delta_{il}\delta_{kj} \\ (\gamma^\mu)_{il}(\gamma_\mu)_{kj} \\ \frac{1}{2}(\sigma^{\mu\nu})_{il}(\sigma_{\mu\nu})_{kj} \\ (\gamma^\mu\gamma_5)_{il}(\gamma_\mu\gamma_5)_{kj} \\ (\gamma_5)_{il}(\gamma_5)_{kj} \end{pmatrix}$$



$$\begin{aligned} g_{\mu\nu}\sigma_{\alpha\dot{\alpha}}^\mu\sigma_{\beta\dot{\beta}}^\nu &= 2\epsilon_{\alpha\beta}\epsilon_{\dot{\alpha}\dot{\beta}}, \\ \epsilon^{\alpha\beta}\delta_\kappa^\gamma + \epsilon^{\beta\gamma}\delta_\kappa^\alpha + \epsilon^{\gamma\alpha}\delta_\kappa^\beta &= 0, \\ \tilde{\epsilon}_{\dot{\alpha}\dot{\beta}}\delta_{\dot{\gamma}}^{\dot{\kappa}} + \tilde{\epsilon}_{\dot{\beta}\dot{\gamma}}\delta_{\dot{\alpha}}^{\dot{\kappa}} + \tilde{\epsilon}_{\dot{\gamma}\dot{\alpha}}\delta_{\dot{\beta}}^{\dot{\kappa}} &= 0 \end{aligned}$$

$$\begin{aligned} (\bar{d}\bar{l})(\bar{l}d) &= -\frac{1}{4}(\bar{d}\bar{d})(\bar{l}l) - \frac{1}{4}(\bar{d}\gamma^\mu d)(\bar{l}\gamma_\mu l) - \frac{1}{8}(\bar{d}\sigma^{\mu\nu}d)(\bar{l}\sigma_{\mu\nu}l) + \frac{1}{4}(\bar{d}\gamma^\mu\gamma_5d)(\bar{l}\gamma_\mu\gamma_5l) - \frac{1}{4}(\bar{d}\gamma_5d)(\bar{l}\gamma_5l) \\ &= -\frac{1}{2}(\bar{d}\gamma^\mu d)(\bar{l}\gamma_\mu l), \end{aligned} \quad (6)$$

$$\begin{aligned} (\bar{l}C\bar{q})(lCq) &= -\frac{1}{4}(\bar{l}l)(\bar{q}q) + \frac{1}{4}(\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q) + \frac{1}{8}(\bar{l}\sigma^{\mu\nu}l)(\bar{q}\sigma_{\mu\nu}q) + \frac{1}{4}(\bar{l}\gamma^\mu\gamma_5l)(\bar{q}\gamma_\mu\gamma_5q) - \frac{1}{4}(\bar{l}\gamma_5l)(\bar{q}\gamma_5q) \\ &= \frac{1}{2}(\bar{l}\gamma^\mu l)(\bar{q}\gamma_\mu q). \end{aligned} \quad (6)$$

SU(N) Group

$$[A_{\nu}^{\mu}, A_{\beta}^{\alpha}] = \delta_{\nu}^{\alpha} A_{\beta}^{\mu} - \delta_{\beta}^{\mu} A_{\nu}^{\alpha}, \quad A_{\mu}^{\mu} = 0, \quad \mu, \nu = 1, 2, \dots, N.$$

Root-weight: obtain transformation matrices for any irrep

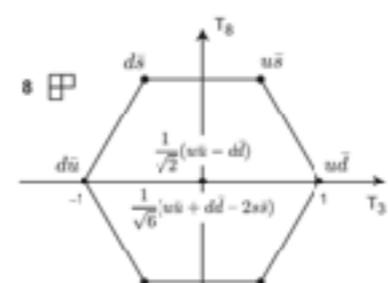
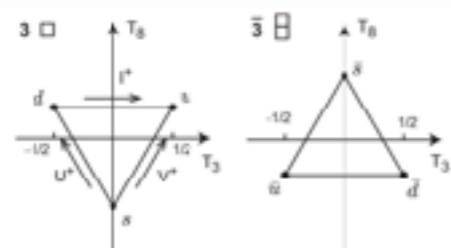
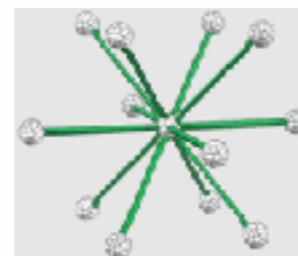
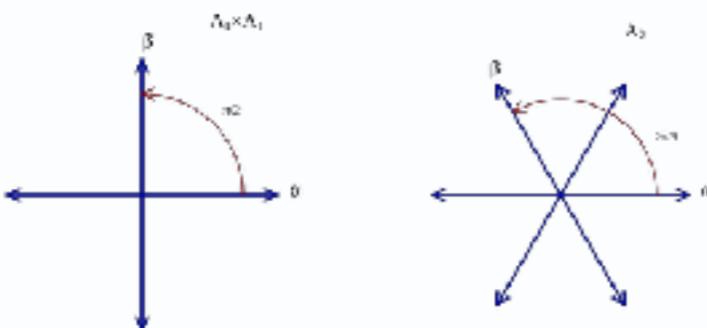
$$[H_i, H_j] = 0 \quad [H_i, E_{\alpha}] = a_i E_{\alpha} \quad [E_{\alpha}, E_{-\alpha}] = a^i H_i$$

Young tensor: obtain tensor basis for any irrep

$$\xi'^{\alpha} = U_{\beta}^{\alpha} \xi^{\beta} \quad \eta'_{\alpha} = \eta_{\beta} (U^{\dagger})^{\beta}_{\alpha} \quad (U^{\dagger})^{\alpha}_{\beta} U^{\beta}_{\gamma} = \delta^{\alpha}_{\gamma}.$$

$$\epsilon_{\beta_1 \dots \beta_n} U^{\beta_1}_{\alpha_1} \dots U^{\beta_n}_{\alpha_n} = \epsilon_{\alpha_1 \dots \alpha_n} \quad \det U = 1$$

A_n



$$\xi^a = 2 \text{ in } \text{SU}(2); 3 \text{ in } \text{SU}(3); \dots \quad \xi_a = \bar{2} \text{ in } \text{SU}(2); \bar{3} \text{ in } \text{SU}(3); \dots$$

$$\zeta^{ab} \equiv \xi^a \xi^b = \frac{1}{2} (\zeta^{ab} + \zeta^{ba}) + \frac{1}{2} (\zeta^{ab} - \zeta^{ba}) = \zeta^{\{ab\}} + \zeta^{\langle ab \rangle}$$

$$\boxed{a} \times \boxed{b} = \boxed{a \ b} + \begin{array}{|c|c|}\hline a \\ \hline b \\ \hline\end{array}.$$

$$2 \otimes 2 = 3 \oplus 1$$

$$3 \otimes 3 = \bar{3} \oplus 6$$

$$\zeta^{\alpha}_{\beta} = \xi^{\alpha} \xi_{\beta} = \hat{\zeta}^{\alpha}_{\beta} + \frac{1}{n} \delta^{\alpha}_{\beta} \xi^{\gamma} \xi_{\gamma} \quad \hat{\zeta}^{\alpha}_{\beta} = \zeta^{\alpha}_{\beta} - \frac{1}{n} \delta^{\alpha}_{\beta} \xi^{\gamma} \xi_{\gamma}$$

$$3 \otimes \bar{3} = 1 \oplus 8$$

$$n \otimes \bar{n} = 1 \oplus (n^2 - 1).$$

SU(N) Tensor Product

Tensor decomposition

$$\zeta^{\alpha_1 \alpha_2 \alpha_3} = \xi^{\alpha_1} \xi^{\alpha_2} \xi^{\alpha_3} = \zeta^{\{\alpha_1 \alpha_2 \alpha_3\}} \oplus \zeta^{\{\alpha_1 \alpha_2\} \alpha_3} \oplus \zeta^{\{\alpha_1 \alpha_3\} \alpha_2} \oplus \zeta^{[\alpha_1 \alpha_2 \alpha_3]}$$

$$2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$$

$$\begin{array}{c} [1] \otimes [2] \otimes [3] = [1|2|3] \oplus \begin{array}{c} [1|2] \\ |3| \end{array} \oplus \begin{array}{c} [1|3] \\ |2| \end{array} \oplus \begin{array}{c} [1] \\ |2| \\ |3| \end{array} \\ \frac{1}{6}n(n+1)(n+2) \quad \frac{1}{3}n(n^2-1) \quad \frac{1}{6}n(n-1)(n-2) \end{array}$$

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

Tensor Product

$$\begin{array}{c} \begin{array}{c} \square & \square \\ \square & \square \end{array} \otimes \begin{array}{c} \square & \square \\ \square & \square \end{array} = \bullet \oplus \begin{array}{c} \square & \square \\ \square & \square \end{array} \oplus \begin{array}{c} \square & \square \\ \square & \square \end{array} \oplus \begin{array}{c} \square & \square & \square \\ \square & \square & \square \end{array} \oplus \begin{array}{c} \square & \square & \square \\ \square & \square & \square \end{array} \oplus \begin{array}{c} \square & \square & \square \\ \square & \square & \square \end{array} \end{array}$$

$$8 \otimes 8 = 1 \oplus 8 \oplus 8 \oplus 10 \oplus \overline{10} \oplus 27$$

$$\begin{array}{c} \begin{array}{c} \square & \square \\ \square & \square \end{array} \otimes \begin{array}{c} 1|1 \\ |1 \end{array} = \begin{array}{c} \square & \square & 1 \\ |1 & |1 & |1 \end{array} \oplus \begin{array}{c} \square & \square & 1 \\ |1 & |1 & |1 \end{array} \oplus \begin{array}{c} \square & \square & 1 \\ |1 & |2 & |1 \end{array} \oplus \begin{array}{c} \square & \square & 1 \\ |1 & |2 & |1 \end{array} \oplus \begin{array}{c} \square & \square & 1 \\ |1 & |2 & |1 \end{array} \oplus \begin{array}{c} \square & \square & 1 \\ |1 & |2 & |1 \end{array} \end{array}$$

LieART 2.0 – A Mathematica Application for Lie Algebras and Representation Theory

Robert Feger^{a,*}, Thomas W. Kephart^b, Robert J. Saakowski^{1b}

<http://lieart.hepforge.org>

In[15]:= DecomposeProduct[Irrep[SU3][8], Irrep[SU3][8]]

Out[15]:= 1 + 2(8) + 10 + $\overline{10}$ + 27

In[16]:= DecomposeProduct[Irrep[SU4][4], Irrep[SU4][4], Irrep[SU4][6], Irrep[SU4][15]]

Out[20]:= 2(1) + 7(15) + 4(20') + 35 + 5(45) + 3($\overline{45}$) + 3(84) + 2(175) + 256

GroupMath: A Mathematica package for group theory calculations

Renato M. Fonseca

renatofonseca.net/groupmath

```
In[1]:= ReduceRepProduct[SU3, {3, 3, 8, 8, 8, -3, 8}]
Out[1]= {{(6, 5), 1}, {(7, 3), 4}, {(5, 4), 26}, {(4, 6), 5}, {(3, 5), 64}, {(6, 2), 48},
          ((4, 3), 166), ((2, 7), 9), ((1, 6), 66), ((2, 4), 260), ((5, 1), 176),
          ((3, 2), 334), ((8, 8), 5), ((8, 5), 137), ((1, 3), 448), ((8, 1), 5),
          ((7, 0), 27), ((4, 0), 235), ((2, 1), 510), ((0, 2), 297), ((1, 0), 217)}
```

SM Lagrangian

Particle(s)	Field(s)	Content	Charge	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Quarks <i>(Three generations)</i>	Q_i	$(u, d)_L$	$(\frac{2}{3}, -\frac{1}{3})$	$\frac{1}{2}$	$\mathbf{3}$	$\mathbf{2}$	$\frac{1}{3}$
	$u_R i$	u_R	$\frac{2}{3}$	$\frac{1}{2}$	$\bar{\mathbf{3}}$	1	$\frac{4}{3}$
	$d_R i$	d_R	$-\frac{1}{3}$	$\frac{1}{2}$	$\bar{\mathbf{3}}$	1	$-\frac{2}{3}$
Leptons <i>(Three generations)</i>	L_i	$(\nu_e, e)_L$	$(0, -1)$	$\frac{1}{2}$	1	$\mathbf{2}$	-1
	e_R		-1	$\frac{1}{2}$	1	1	-2
Gluons	G_μ^a	g	0	1	$\mathbf{8}$	1	0
W bosons	$W_\mu^{1,2}$	W^\pm	± 1	1	1	3	0
Photon, Z boson	W_μ^3, B_μ	γ, Z^0	0	1	1	$\mathbf{3, 1}$	0
Higgs boson	ϕ	H	0	0	1	2	1

$$\begin{aligned} \mathbf{2} \times \mathbf{2} &= \mathbf{1} + \mathbf{3} \\ \mathbf{3} \times \mathbf{2} &= \mathbf{2} + \mathbf{4} \\ \mathbf{3} \times \mathbf{3} &= \mathbf{1} + \mathbf{3} + \mathbf{5} \\ \mathbf{4} \times \mathbf{2} &= \mathbf{3} + \mathbf{5} \\ \mathbf{4} \times \mathbf{3} &= \mathbf{2} + \mathbf{4} + \mathbf{6} \\ \mathbf{4} \times \mathbf{4} &= \mathbf{1} + \mathbf{3} + \mathbf{5} + \mathbf{7} \end{aligned}$$

$$\begin{aligned} \bar{\mathbf{3}} \times \mathbf{3} &= \mathbf{1} + \mathbf{8} \\ \mathbf{3} \times \bar{\mathbf{3}} &= \bar{\mathbf{3}} + \mathbf{6} \\ \bar{\mathbf{6}} \times \mathbf{3} &= \bar{\mathbf{3}} + \mathbf{15} \\ \mathbf{6} \times \bar{\mathbf{3}} &= \mathbf{8} + \mathbf{10} \\ \bar{\mathbf{6}} \times \bar{\mathbf{6}} &= \mathbf{6} + \mathbf{15} + \mathbf{15}' \\ \mathbf{6} \times \bar{\mathbf{6}} &= \mathbf{1} + \mathbf{8} + \mathbf{27} \\ \mathbf{8} \times \mathbf{3} &= \mathbf{3} + \bar{\mathbf{6}} + \mathbf{15} \\ \mathbf{8} \times \bar{\mathbf{6}} &= \mathbf{3} + \bar{\mathbf{6}} + \mathbf{15} + \mathbf{24} \\ \mathbf{8} \times \mathbf{8} &= \mathbf{1} + 2(\mathbf{8}) + \mathbf{10} + \bar{\mathbf{10}} + \mathbf{27} \\ \bar{\mathbf{10}} \times \mathbf{3} &= \bar{\mathbf{6}} + \mathbf{24} \\ \mathbf{10} \times \mathbf{3} &= \mathbf{15} + \mathbf{15}' \\ \bar{\mathbf{10}} \times \bar{\mathbf{6}} &= \mathbf{15} + \mathbf{21} + \mathbf{24} \\ \mathbf{10} \times \bar{\mathbf{6}} &= \mathbf{3} + \mathbf{15} + \mathbf{42} \\ \mathbf{10} \times \mathbf{8} &= \mathbf{8} + \mathbf{10} + \mathbf{27} + \mathbf{35} \\ \bar{\mathbf{10}} \times \mathbf{10} &= \mathbf{1} + \mathbf{8} + \mathbf{27} + \mathbf{64} \\ \mathbf{10} \times \mathbf{10} &= \bar{\mathbf{10}} + \mathbf{27} + \mathbf{28} + \mathbf{35} \end{aligned}$$

Decompose tensor products of product irreps $(\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1}) \otimes (\bar{\mathbf{3}}, \mathbf{3}, \mathbf{1})$ of $SU(3) \otimes SU(3) \otimes SU(3)$:

```
n[23]:= DecomposeProduct[
  ProductIrrep[Irrep[SU3][3], Irrep[SU3][Bar[3]], Irrep[SU3][1]],
  ProductIrrep[Irrep[SU3][Bar[3]], Irrep[SU3][3], Irrep[SU3][1]]]
out[23]:= {1,1,1}+(8,1,1)+(1,8,1)+(8,8,1)
```

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{8}\text{tr}(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) \\ & - \underbrace{\frac{1}{2}\text{tr}(G_{\mu\nu}G^{\mu\nu})}_{\text{U(1), SU(2) and SU(3) gauge terms}} \\ & + (\bar{\nu}_L, \bar{e}_L) \bar{\sigma}^\mu iD_\mu \left(\begin{array}{c} \nu_L \\ e_L \end{array} \right) + \bar{e}_R \sigma^\mu iD_\mu e_R \\ & + \bar{\nu}_R \sigma^\mu iD_\mu \nu_R + \text{Hermitian conjugate} \\ & \underbrace{- \frac{\sqrt{2}}{v} \left[(\bar{\nu}_L, \bar{e}_L) \phi M^e \nu_R + \bar{e}_R \bar{M}^e \bar{\phi} \left(\begin{array}{c} \nu_L \\ e_L \end{array} \right) \right]}_{\text{lepton dynamical term}} \\ & - \frac{\sqrt{2}}{v} \left[(-\bar{e}_L, \bar{\nu}_L) \phi^* M^e \nu_R + \bar{\nu}_R \bar{M}^e \phi^T \left(\begin{array}{c} -e_L \\ \nu_L \end{array} \right) \right] \\ & \underbrace{+ (\bar{u}_L, \bar{d}_L) \bar{\sigma}^\mu iD_\mu \left(\begin{array}{c} u_L \\ d_L \end{array} \right) + \bar{u}_R \sigma^\mu iD_\mu u_R}_{\text{neutrino mass term}} \\ & + \bar{d}_R \sigma^\mu iD_\mu d_R + \text{Hermitian conjugate} \\ & \underbrace{- \frac{\sqrt{2}}{v} \left[(\bar{u}_L, \bar{d}_L) \phi M^d d_R + \bar{d}_R \bar{M}^d \bar{\phi} \left(\begin{array}{c} v_L \\ d_L \end{array} \right) \right]}_{\text{quark dynamical term}} \\ & - \frac{\sqrt{2}}{v} \left[(-\bar{d}_L, \bar{u}_L) \phi^* M^u u_R + \bar{u}_R \bar{M}^u \phi^T \left(\begin{array}{c} -d_L \\ u_L \end{array} \right) \right] \\ & \underbrace{+ (D_\mu \phi) D^\mu \phi - m_h^2 [\bar{\phi} \phi - v^2/2]^2 / 2v^2}_{\text{Higgs dynamical and mass term}} \end{aligned}$$

Covariant Derivative

SMEFT should be gauge invariant: all derivatives should be covariant derivatives

$$D_\mu = \partial_\mu - ig_s \frac{1}{2} \lambda^A G_\mu^A - ig \frac{1}{2} T^I W^I - ig' Y B$$

$$\varphi^\dagger i \overleftrightarrow{D}_\mu \varphi \equiv i \varphi^\dagger \left(D_\mu - \overleftarrow{D}_\mu \right) \varphi \quad \text{and} \quad \varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi \equiv i \varphi^\dagger \left(\tau^I D_\mu - \overleftarrow{D}_\mu \tau^I \right) \varphi. \quad \varphi^\dagger \overleftarrow{D}_\mu \varphi \equiv (D_\mu \varphi)^\dagger \varphi$$

Gauge field should be gauge field tensor:

$$X_{\mu\nu} \in \{G_{\mu\nu}^A, W_{\mu\nu}^I, B_{\mu\nu}\} \quad \tilde{X}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\rho\sigma} X^{\rho\sigma}$$

$$\begin{aligned} G_{\mu\nu}^A &= \partial_\mu G_\nu^A - \partial_\nu G_\mu^A - g_s f^{ABC} G_\mu^B G_\nu^C, & (D_\rho G_{\mu\nu})^A &= \partial_\rho G_{\mu\nu}^A - g_s f^{ABC} G_\rho^B G_{\mu\nu}^C, \\ W_{\mu\nu}^I &= \partial_\mu W_\nu^I - \partial_\nu W_\mu^I - g \varepsilon^{IJK} W_\mu^J W_\nu^K, & (D_\rho W_{\mu\nu})^I &= \partial_\rho W_{\mu\nu}^I - g \varepsilon^{IJK} W_\rho^J W_{\mu\nu}^K, \\ B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, & D_\rho B_{\mu\nu} &= \partial_\rho B_{\mu\nu}. \end{aligned}$$

Bianchi identity (BI):

$$D_\rho X_{\mu\nu}] = 0 \quad D^\rho D_\rho X_{\mu\nu} = - (D^\rho D_\mu X_{\nu\rho} + D^\rho D_\nu X_{\rho\mu})$$

Covariant Derivative Commutator (CDC):

$$[D_\rho, D_\alpha] \sim X_{\rho\alpha} \quad \bar{\psi} \sigma^{\mu\nu} \psi D_\mu D_\nu \varphi - \varphi \bar{\psi} \sigma^{\mu\nu} D_\mu D_\nu \psi \longrightarrow \psi^2 X \varphi$$

The Dim-5 Operators

The only operator at dim-5 is Weinberg operator (1979)

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \mathcal{L}_{D=5} + \frac{1}{\Lambda^2} \mathcal{L}_{D=6} + \frac{1}{\Lambda^3} \mathcal{L}_{D=7} + \frac{1}{\Lambda^4} \mathcal{L}_{D=8} + \dots$$

$c_{ij} \frac{v^2}{\Lambda} (L_i H)(L_j H) + \text{h.c.} \rightarrow c_{ij} \frac{v^2}{\Lambda} \nu_i \nu_j + \text{h.c.}$

$H \rightarrow \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$
 $L_i \rightarrow \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$

How to analyse?

$N - h.$	1	3	5
$N + h.$			
1		$\psi^2 D^2, F\phi D^2$	$F^2 \phi, F\psi^2$
3	$\bar{\psi}^2 D^2, \bar{F}\phi D^2$	$\phi^3 D^2, \bar{\psi}\psi\phi D$	$\psi^2 \phi^2$
5	$\bar{F}^2 \phi, \bar{F}\bar{\psi}^2$	$\bar{\psi}^2 \phi^2$	ϕ^5

For SM fields, $F^2 \phi$, $F\psi^2$ and ϕ^5 vanish after taking account of their gauge structures. For example, ϕ^5 does not preserve $U(1)_Y$ charge conservation since Higgs has $U(1)_Y$ charge 1/2, and can not form a $SU(2)$ singlet since Higgs is a $SU(2)$ doublet. $F^2 \phi$ and $F\psi^2$ do not preserve $U(1)_Y$ charge conservation. The only complex type of $\psi^2 \phi^2$ that survives after taking account of $U(1)_Y$ symmetry is $LLHH$, which can preserve $SU(2)_W$ and $SU(3)_C$ symmetries.

Odd power of scalar, and $SU(2)L$ transformation $\bar{\psi}_L \sigma^{\mu\nu} \psi_R$

Operator with Derivatives

New types of operators beyond SM Lagrangian: operator with more derivatives or more fields than just kinetic term in SM

Standard Model

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \bar{\psi} D^\mu \psi + h.c. \\ + \bar{\psi}_i Y_j \psi_j \phi + h.c. + |D_\mu \phi|^2 - V(\phi)$$

Dim-5 & dim-6 operators contain D

$\bar{\psi}^2 D^2, \bar{F}\phi D^2$	$\psi^2 D^2, F\phi D^2$
$\bar{\phi}^3 D^2, \bar{\psi}\psi\phi D$	$\phi^3 D^2, \bar{\psi}\psi\phi D$

		$F^2 D^2$
	$\bar{\psi}\psi D^3, \bar{F}FD^2, \phi^2 D^4$	$F\bar{\psi}\psi D, \psi^2\phi D^2, F\phi^2 D^2$
$\bar{F}^2 D^2$	$\bar{F}\bar{\psi}\psi D, \bar{\psi}^2\phi D^2, \bar{F}\phi^2 D^2$	$\bar{\psi}\psi\phi^2 D, \bar{\psi}^2\psi^2, \phi^4 D^2$

Which fields Ds should act to?

$$[\bar{\psi}\psi D_\mu D^\mu \varphi, \varphi \bar{\psi} D_\mu D^\mu \psi, (D_\mu \varphi) \bar{\psi} \sigma^{\mu\nu} D_\nu \psi \text{ and } (D^\mu \varphi) \bar{\psi} D_\mu \psi] \quad \bar{\psi} \sigma^{\mu\nu} \psi D_\mu D_\nu \varphi \quad \varphi \bar{\psi} \sigma^{\mu\nu} D_\mu D_\nu \psi$$

$$X^{\mu\nu} \bar{\psi} \gamma_\mu D_\nu \psi \quad \bar{\psi} \gamma_\rho \gamma_\mu \gamma_\nu \psi D^\rho X^{\mu\nu} \quad \bar{\psi} \gamma^\mu \psi D^\rho X_{\rho\mu}$$

$$(\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D_\mu \varphi) \quad (\varphi^\dagger \varphi) [(D_\mu \varphi)^\dagger (D^\mu \varphi)] \quad (\varphi^\dagger \varphi) \square (\varphi^\dagger \varphi) \\ (\varphi^\dagger \tau^I \varphi) [(D_\mu \varphi)^\dagger \tau^I (D^\mu \varphi)]$$

Field Redefinition

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi_i \exp \left(i \int d^4x \left[\mathcal{L}_0 + \eta \mathcal{L}_1 + \sum_i j_i \varphi_i + \mathcal{O}(\eta^2) \right] \right)$$

\downarrow

$$\phi^\dagger = (\phi')^\dagger + \eta T[\varphi] \quad T[\varphi] \text{ is any local function of any of the fields } \varphi$$

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi'_i \left| \frac{\delta\phi^\dagger}{\delta(\phi')^\dagger} \right| \exp \left(i \int d^4x \left[\mathcal{L}'_0 + \delta\mathcal{L}'_0 + \eta \mathcal{L}'_1 + \eta \delta\mathcal{L}'_1 + \sum_i j_i \varphi_i + j_{\phi^\dagger} \eta T + \mathcal{O}(\eta^2) \right] \right)$$

\downarrow

$$\begin{aligned} \mathcal{L}'_i &\equiv \mathcal{L}_i \left((\phi')^\dagger, \partial_\mu (\phi')^\dagger \right) & \delta\phi^\dagger &\equiv \phi^\dagger - (\phi')^\dagger = \eta T[\varphi] \\ \delta\mathcal{L}'_i &\equiv \frac{\delta\mathcal{L}'_i}{\delta(\phi')^\dagger} \delta\phi^\dagger - \frac{\delta\mathcal{L}'_i}{\delta\partial_\mu(\phi')^\dagger} \delta\partial_\mu\phi^\dagger & &= \left(\frac{\delta\mathcal{L}'_i}{\delta(\phi')^\dagger} - \partial_\mu \frac{\delta\mathcal{L}'_i}{\delta\partial_\mu(\phi')^\dagger} \right) \delta\phi^\dagger \\ &&&= \left(\frac{\delta\mathcal{L}'_i}{\delta(\phi')^\dagger} - \partial_\mu \frac{\delta\mathcal{L}'_i}{\delta\partial_\mu(\phi')^\dagger} \right) \eta T[\varphi] \end{aligned}$$

$$Z[j_i] = \int \prod_i \mathcal{D}\varphi'_i \left| \frac{\delta\phi^\dagger}{\delta(\phi')^\dagger} \right| \exp \left(i \int d^4x \left[\mathcal{L}'_0 + \left(\frac{\delta\mathcal{L}'_0}{\delta(\phi')^\dagger} - \partial_\mu \frac{\delta\mathcal{L}'_0}{\delta\partial_\mu(\phi')^\dagger} \right) \eta T[\varphi] + \eta \mathcal{L}'_1 + \sum_i j_i \varphi_i + j_{\phi^\dagger} \eta T + \mathcal{O}(\eta^2) \right] \right)$$

the source term and the Jacobian can be neglected
[hep-ph/9304230].

Gaussian theorem on action

Equation of Motion (EOM)

Two equivalent operators related by EOM

Integration by part (IBP)

$\partial_\mu \mathcal{O}^\mu$

Total derivatives are removed

EOM and IBP

SM Equations of motion:

$$(D^\mu D_\mu \varphi)^j = m^2 \varphi^j - \lambda (\varphi^\dagger \varphi) \varphi^j - \bar{e} \Gamma_e^\dagger l^j + \varepsilon_{jk} \bar{q}^k \Gamma_u u - \bar{d} \Gamma_d^\dagger q^j$$

$$i\cancel{D}l = \Gamma_e e \varphi, \quad i\cancel{D}e = \Gamma_e^\dagger \varphi^\dagger l, \quad i\cancel{D}q = \Gamma_u u \tilde{\varphi} + \Gamma_d d \varphi, \quad i\cancel{D}u = \Gamma_u^\dagger \tilde{\varphi}^\dagger q, \quad i\cancel{D}d = \Gamma_d^\dagger \varphi^\dagger q.$$

$$\begin{aligned} (D^\rho G_{\rho\mu})^A &= g_s (\bar{q} \gamma_\mu T^A q + \bar{u} \gamma_\mu T^A u + \bar{d} \gamma_\mu T^A d), \\ (D^\rho W_{\rho\mu})^I &= \frac{g}{2} (\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu^I \varphi + \bar{l} \gamma_\mu \tau^I l + \bar{q} \gamma_\mu \tau^I q), \quad \partial^\rho B_{\rho\mu} = g' Y_\varphi \varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi + g' \sum_{\psi \in \{l, e, q, u, d\}} Y_\psi \bar{\psi} \gamma_\mu \psi. \end{aligned}$$

Total derivative examples:

$$\begin{aligned} (D_\mu \varphi)^\dagger \varphi + \varphi^\dagger D_\mu \varphi &= (\partial_\mu \varphi^\dagger) \varphi + i(A_\mu \varphi)^\dagger \varphi + \varphi^\dagger \partial_\mu \varphi - i \varphi^\dagger A_\mu \varphi \\ &= (\partial_\mu \varphi^\dagger) \varphi + \varphi^\dagger \partial_\mu \varphi = \partial_\mu (\varphi^\dagger \varphi) \end{aligned}$$

$$(D^n \varphi)^\dagger (D^m \varphi) = -(D^{n+1} \varphi)^\dagger (D^{m-1} \varphi) + \partial \left[(D^n \varphi)^\dagger (D^{m-1} \varphi) \right]$$

The Dim-6 Operators

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \mathcal{L}_{D=5} + \frac{1}{\Lambda^2} \mathcal{L}_{D=6} + \frac{1}{\Lambda^3} \mathcal{L}_{D=7} + \frac{1}{\Lambda^4} \mathcal{L}_{D=8} + \dots$$

Class of operators:

$N - h$	0	2	4	6
$N + h$				
0			$F^2 D^2$	F^3
2		$\bar{\psi}\psi D^3, \bar{F}FD^2, \phi^2 D^4$	$F\bar{\psi}\psi D, \psi^2\phi D^2, F\phi^2 D^2$	$F^2\phi^2, F\psi^2\phi, \psi^4$
4	$\bar{F}^2 D^2$	$\bar{F}\bar{\psi}\psi D, \bar{\psi}^2\phi D^2, \bar{F}\phi^2 D^2$	$\bar{\psi}\psi\phi^2 D, \bar{\psi}^2\psi^2, \phi^4 D^2$	$\psi^2\phi^3$
6	\bar{F}^3	$\bar{F}^2\phi^2, \bar{F}\bar{\psi}^2\phi, \bar{\psi}^4$	$\bar{\psi}^2\phi^3$	ϕ^6

Similar to dim-5, but surprisingly difficult!

Complete dim-6 Operators

It took 30 years to find the full (independent and complete) dim-6 operator basis

tedious and prone-to-error

$$\begin{aligned}
 O_\varphi &= [(\varphi^\dagger \varphi)^3], & O_{\alpha} &= f_{ABC} G_A^{\alpha A} G_B^{\beta B} G_C^{\gamma C}, \\
 O_{\alpha\varphi} &= [\partial_\alpha (\varphi^\dagger \varphi) \partial^\mu (\varphi^\dagger \varphi)], & O_{\alpha\bar{\varphi}} &= f_{ABC} \tilde{G}_A^{\alpha A} G_B^{\beta B} G_C^{\gamma C}, \\
 O_{\alpha\bar{\varphi}} &= (\varphi^\dagger \varphi)(\bar{\ell}\ell\varphi), & O_W &= \varepsilon_{ijk} W_\mu^{ik} W_\nu^{jk} W_\lambda^{ki}, \\
 O_{\alpha\bar{\varphi}} &= (\varphi^\dagger \varphi)(\bar{q}q\bar{\varphi}), & O_{\bar{W}} &= \varepsilon_{ijk} \bar{W}_\mu^{ik} W_\nu^{jk} W_\lambda^{ki}, \\
 O_{\alpha\varphi} &= (\varphi^\dagger \varphi)(\bar{q}d\varphi), & O_{\alpha\bar{d}} &= (\varphi^\dagger \varphi) G_{\mu i}^A G^{A\mu i}, \\
 O_{\alpha\bar{d}} &= [(\varphi^\dagger \varphi) G_{\mu i}^A] G^{A\mu i}, & O_{\alpha\bar{d}} &= (\varphi^\dagger \varphi) \tilde{G}_{\mu i}^A G^{A\mu i}, \\
 O_{\alpha W} &= [(\varphi^\dagger \varphi) W_{\mu i}^j] W_i^{jk} W_{\lambda}^{ki}, & O_{\alpha\bar{W}} &= (\varphi^\dagger \varphi) \bar{W}_{\mu i}^j W_i^{jk} W_{\lambda}^{ki}, \\
 O_{\alpha\bar{W}} &= [(\varphi^\dagger \varphi) B_{\mu i}^j] B_i^{jk} B_{\lambda}^{ki}, & O_{\alpha\bar{B}} &= (\varphi^\dagger \varphi) \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}, \\
 O_{\alpha\bar{B}} &= (\varphi^\dagger \tau^i \varphi) W_{\mu i}^j B_{\lambda}^{ki}, & O_{\alpha\bar{B}} &= (\varphi^\dagger \tau^i \varphi) \bar{W}_{\mu i}^j B_{\lambda}^{ki}, \\
 O_\varphi^{(1)} &= (\varphi^\dagger \varphi) (D_\mu \varphi^\dagger D^\mu \varphi), & O_\varphi^{(1)} &= (\varphi^\dagger D^\mu \varphi) (D_\mu \varphi^\dagger \varphi). \\
 \end{aligned}$$

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[Buchmuller, Wyler, 1986]

Equation of motion (Field redefinition)

$$\begin{aligned}
 (D^\mu D_\mu \varphi)^j &= m^2 \varphi^j - \lambda (\varphi^\dagger \varphi) \varphi^j - \bar{e} \Gamma_e^\dagger l^j + \varepsilon_{jk} \bar{q}^k \Gamma_u u - \bar{d} \Gamma_d^\dagger q^j \\
 i\cancel{D}l &= \Gamma_e e \varphi, \quad i\cancel{D}e = \Gamma_e^\dagger \varphi^\dagger l, \quad i\cancel{D}q = \Gamma_u u \tilde{\varphi} + \Gamma_d d \varphi, \quad i\cancel{D}u = \Gamma_u^\dagger \tilde{\varphi}^\dagger q, \\
 (D^\mu W_{\mu i})^I &= \frac{g}{2} \left(\varphi^\dagger i \tilde{D}_\mu^I \varphi + \bar{l} \gamma_\mu \tau^I l + \bar{q} \gamma_\mu \tau^I q \right),
 \end{aligned}$$

Covariant derivative commutator

$$[D_\rho, D_\alpha] \sim X_{\rho\alpha}$$

$$\text{Bianchi identity } D_\rho X_{\mu\nu} = 0$$

Integration by part (total derivatives)

$$(D^n \varphi)^\dagger (D^m \varphi) = -(D^{n+1} \varphi)^\dagger (D^{m-1} \varphi) + \partial \left[(D^n \varphi)^\dagger (D^{m-1} \varphi) \right]$$

$$\text{Fierz identity } T_{\alpha\beta}^A T_{\kappa\lambda}^A = \frac{1}{2} \delta_{\alpha\lambda} \delta_{\kappa\beta} - \frac{1}{6} \delta_{\alpha\beta} \delta_{\kappa\lambda}$$

$$\tau_{jk}^I \tau_{mn}^J = 2\delta_{jn} \delta_{mk} - \delta_{jk} \delta_{mn}$$

$$\begin{aligned}
 O_{\alpha\bar{\varphi}}^{(1)} &= \frac{1}{2} (\bar{\ell} \gamma_\mu \ell) (\bar{\ell} \gamma^\mu \ell), & O_{\alpha\bar{\varphi}}^{(2)} &= \frac{1}{2} (\bar{\ell} \gamma_\mu \tau^i \ell) (\bar{\ell} \gamma^\mu \tau^i \ell), \\
 O_{\alpha\bar{\varphi}}^{(1,1)} &= \frac{1}{2} (\bar{q} \gamma_\mu q) (\bar{q} \gamma^\mu q), & O_{\alpha\bar{\varphi}}^{(1,1)} &= \frac{1}{2} (\bar{q} \gamma_\mu \lambda^\mu q) (\bar{q} \gamma^\mu \lambda^\mu q), \\
 O_{\alpha\bar{\varphi}}^{(1,2)} &= \frac{1}{2} (\bar{q} \gamma_\mu \tau^i q) (\bar{q} \gamma^\mu \tau^i q), & O_{\alpha\bar{\varphi}}^{(1,2)} &= \frac{1}{2} (\bar{q} \gamma_\mu \lambda^\mu \tau^i q) (\bar{q} \gamma^\mu \lambda^\mu \tau^i q), \\
 O_{\alpha\bar{\varphi}}^{(2)} &= (\bar{\ell} \gamma_\mu \ell) (\bar{q} \gamma^\mu q), & O_{\alpha\bar{\varphi}}^{(2)} &= (\bar{\ell} \gamma_\mu \tau^i \ell) (\bar{q} \gamma^\mu \tau^i q).
 \end{aligned}$$

$$\begin{aligned}
 O_{\alpha\bar{d}} &= \frac{1}{2} (\bar{e} \gamma_\mu e) (\bar{e} \gamma^\mu e), & O_{\alpha\bar{d}} &= (\bar{d} u) (\bar{d} \bar{e}), & O_{\alpha\bar{d}}^{(1)} &= (\bar{q} u) (\bar{q} d), \\
 O_{\alpha\bar{d}}^{(1)} &= \frac{1}{2} (\bar{e} \gamma_\mu e) (\bar{e} \gamma^\mu e), & O_{\alpha\bar{d}}^{(1)} &= (\bar{d} u) (\bar{d} \gamma^\mu e), & O_{\alpha\bar{d}}^{(1)} &= (\bar{q} u) (\bar{q} d), \\
 O_{\alpha\bar{d}}^{(2)} &= \frac{1}{2} (\bar{d} \gamma_\mu d) (\bar{d} \gamma^\mu d), & O_{\alpha\bar{d}}^{(2)} &= \frac{1}{2} (\bar{d} \gamma_\mu \lambda^\mu d) (\bar{d} \gamma^\mu \lambda^\mu d), & O_{\alpha\bar{d}}^{(2)} &= (\bar{q} \gamma_\mu \lambda^\mu d) (\bar{q} \gamma^\mu \lambda^\mu d), \\
 O_{\alpha\bar{d}} &= (\bar{e} \gamma_\mu e) (\bar{d} \gamma^\mu d), & O_{\alpha\bar{d}} &= (\bar{d} \gamma_\mu d) (\bar{d} \gamma^\mu d), & O_{\alpha\bar{d}} &= (\bar{q} \gamma_\mu \lambda^\mu d) (\bar{q} \gamma^\mu \lambda^\mu d), \\
 O_{\alpha\bar{d}}^{(2)} &= (\bar{d} \gamma_\mu d) (\bar{d} \gamma^\mu d), & O_{\alpha\bar{d}}^{(2)} &= (\bar{d} \gamma_\mu \lambda^\mu d) (\bar{d} \gamma^\mu \lambda^\mu d), & O_{\alpha\bar{d}}^{(2)} &= (\bar{q} \gamma_\mu \lambda^\mu d) (\bar{q} \gamma^\mu \lambda^\mu d), \\
 O_{\alpha\bar{d}} &= (\bar{e} \gamma_\mu e) (\bar{d} \gamma^\mu d), & O_{\alpha\bar{d}} &= (\bar{d} \gamma_\mu d) (\bar{d} \gamma^\mu d), & O_{\alpha\bar{d}} &= (\bar{q} \gamma_\mu \lambda^\mu d) (\bar{q} \gamma^\mu \lambda^\mu d),
 \end{aligned}$$

	X^5	$\varphi^i \text{ and } \varphi^\dagger D^2$	$\varphi^2 \varphi^2$
Q_G	$f^{AB\bar{C}} G_A^{\alpha A} G_B^{\beta B} G_C^{\gamma C}$	Q_G	$(\varphi^\dagger \varphi)^3$
$Q_{\bar{G}}$	$f^{AB\bar{C}} \tilde{G}_A^{\alpha A} G_B^{\beta B} G_C^{\gamma C}$	$Q_{\bar{G}}$	$(\varphi^\dagger \varphi) \square (\varphi^\dagger \varphi)$
Q_W	$\varepsilon^{ijk} W_\mu^{ik} W_\nu^{jk} W_\lambda^{ki}$	Q_W	$(\varphi^\dagger D^\mu \varphi)^\dagger (\varphi^\dagger D_\mu \varphi)$
$Q_{\bar{W}}$	$\varepsilon^{ijk} \bar{W}_\mu^{ik} W_\nu^{jk} W_\lambda^{ki}$	$Q_{\bar{W}}$	$(\varphi^\dagger \varphi) (\bar{q} \mu^\mu \bar{\varphi})$

	$X^5 \varphi^2$	$\varphi^2 X \varphi$	$\varphi^2 \varphi^2 D$
$Q_{\alpha G}$	$\bar{e}^\dagger \varphi G_{\mu i}^A G^{A\mu i}$	$Q_{\alpha W}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{G}}$	$\bar{e}^\dagger \varphi \tilde{G}_{\mu i}^A G^{A\mu i}$	$Q_{\alpha\bar{W}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha W}$	$\bar{e}^\dagger \varphi W_{\mu i}^j W_i^{jk} W_{\lambda}^{ki}$	$Q_{\alpha\bar{W}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{W}}$	$\bar{e}^\dagger \varphi \bar{W}_{\mu i}^j W_i^{jk} W_{\lambda}^{ki}$	$Q_{\alpha\bar{G}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi B_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha\bar{B}}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \varphi \bar{B}_{\mu i}^j B_i^{jk} B_{\lambda}^{ki}$	$Q_{\alpha B}$	$(\varphi^\dagger \bar{D}_\mu \varphi) (\bar{q}_\mu \gamma^\mu \bar{\varphi})$
$Q_{\alpha\bar{B}}$	$\bar{e}^\dagger \$		

Redundancies

Main difficulties:

1. Group relations (e.g. Fierz identities)

$$5 - 1 = 4$$

2. Equation of motion (EOM)

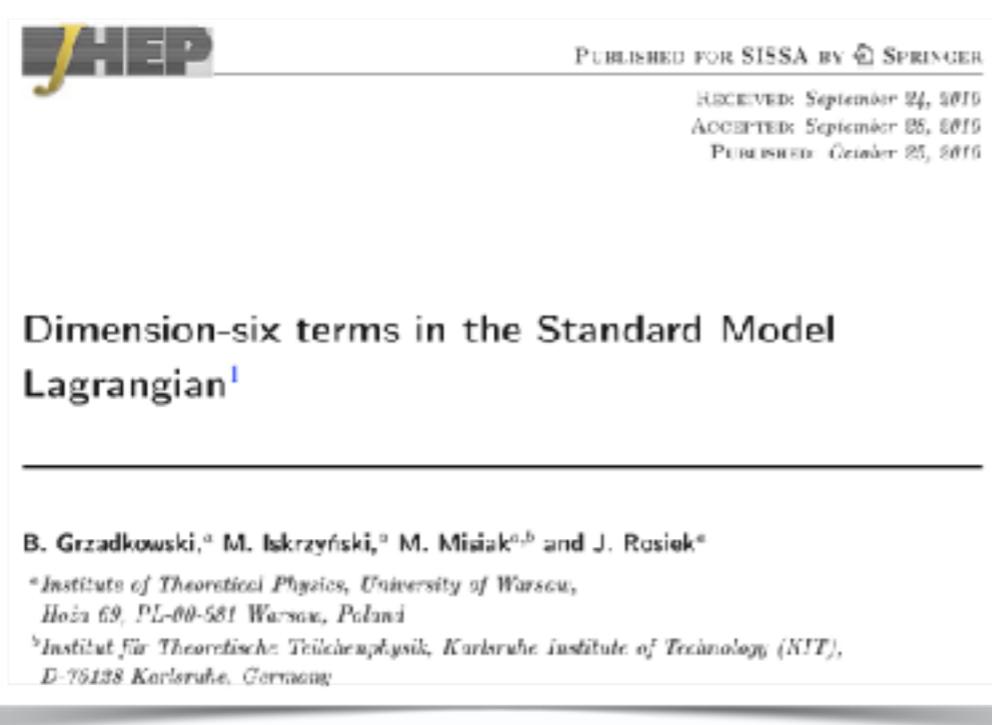
$$10 + 6 = 16$$

3. Integration by parts (IBP)

$$1$$

$$\partial^2 \phi = -m^2 \phi + \dots$$

redundant building block



Standard Model Lagrangian

#3

[Warsaw U.), M. Misiak (Warsaw U. and KIT, Karlsruhe, TTP),

: 1008.4884 [hep-ph]

1,403 citations

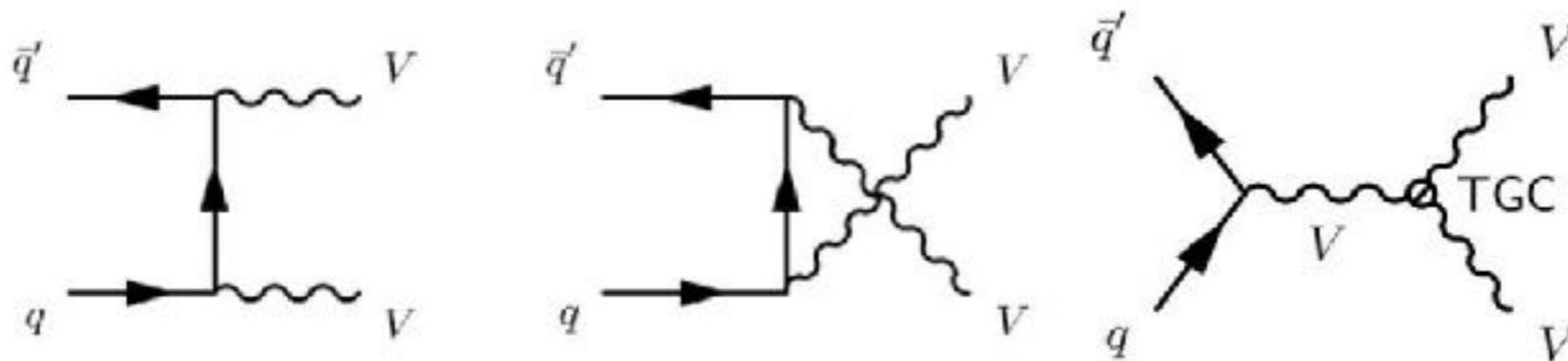
4. Repeated fields (spin-statistics)

Higher Dim Operator?

According to power counting, the lower dim, more dominant operators

$$\begin{aligned} |\mathcal{A}|^2 &\sim \left| A_{\text{SM}} + \frac{A_{\text{dim-6}}}{\Lambda^2} + \frac{A_{\text{dim-8}}}{\Lambda^4} + \dots \right|^2 \\ &\sim |A_{\text{SM}}|^2 + \frac{2}{\Lambda^2} A_{\text{dim-6}} A_{\text{SM}}^* + \frac{1}{\Lambda^4} |A_{\text{dim-6}}|^2 + \frac{2}{\Lambda^4} A_{\text{dim-8}} A_{\text{SM}}^* \end{aligned}$$

Dim-8 operator could give rise to the leading contributions



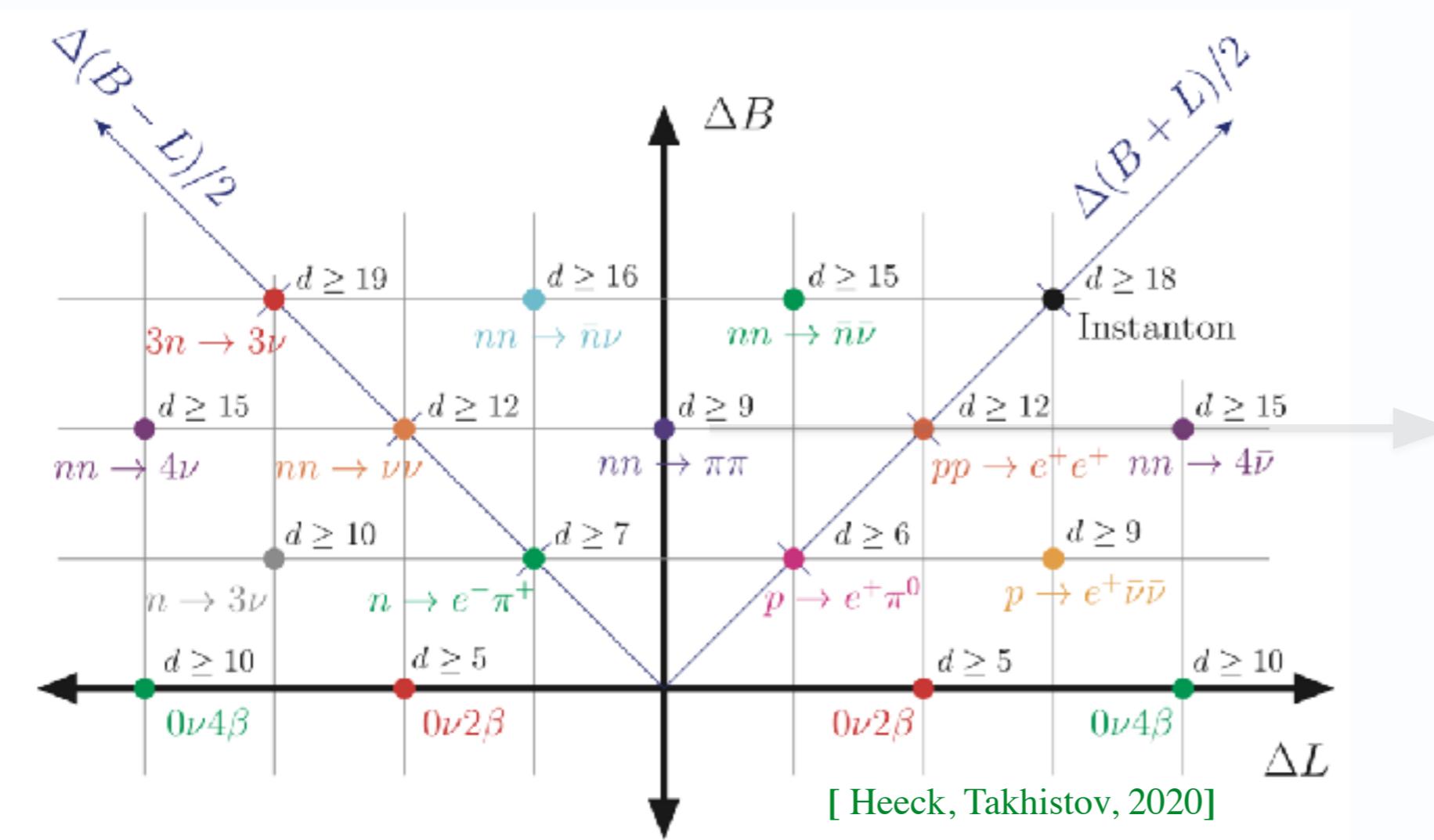
Neutral triple gauge
boson ZZZ, ZZA, ZAA

dim-8 neutral neutrino NSI > dim-6 neutral NSI

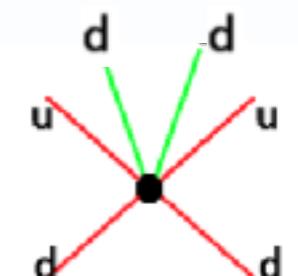
Higher Dim Operators

new physics without new particle: neutrino masses and baryon asymmetry

B and L violation

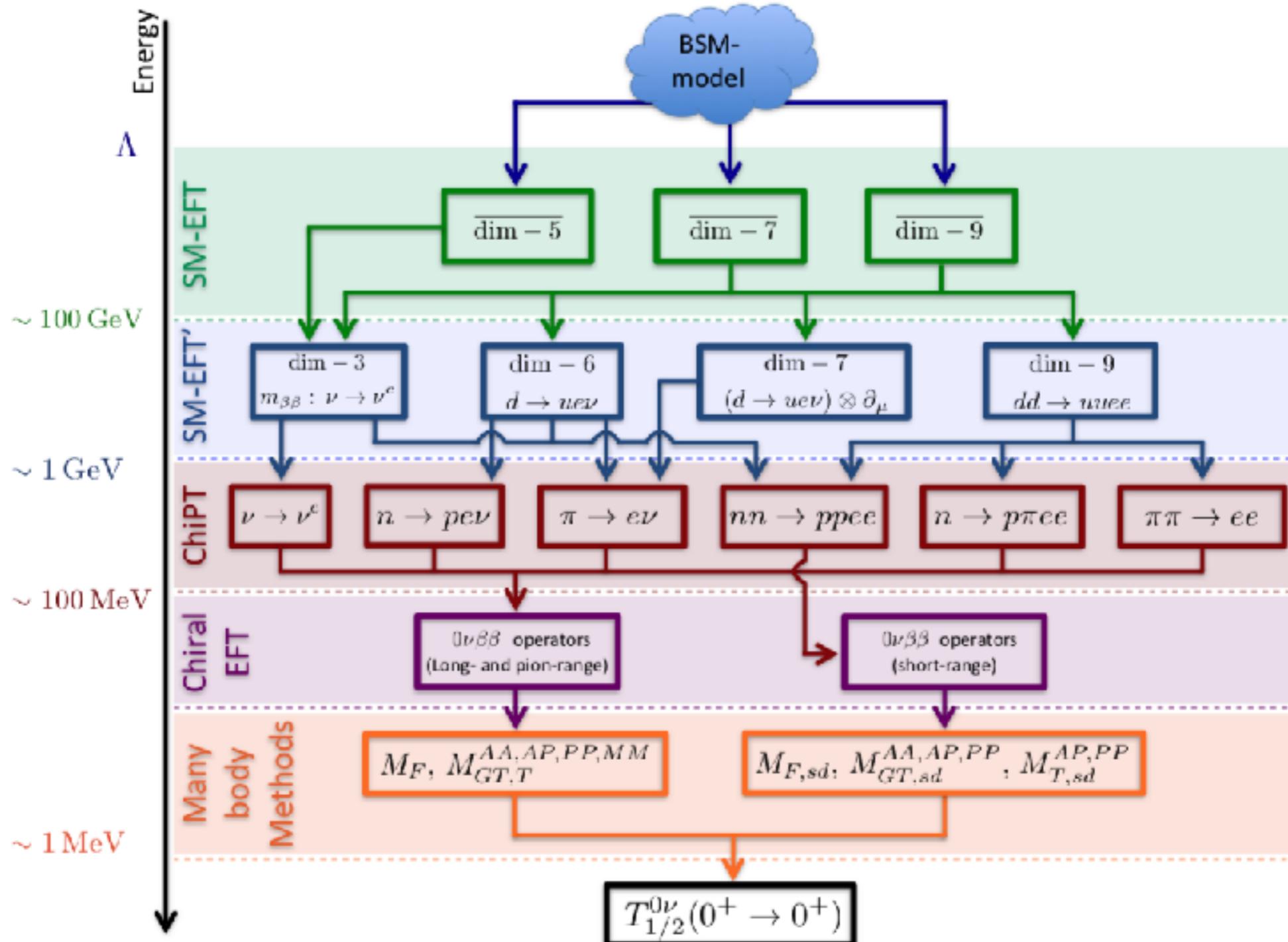


n-nbar oscillation



Dim-9

Higher Dim Operators

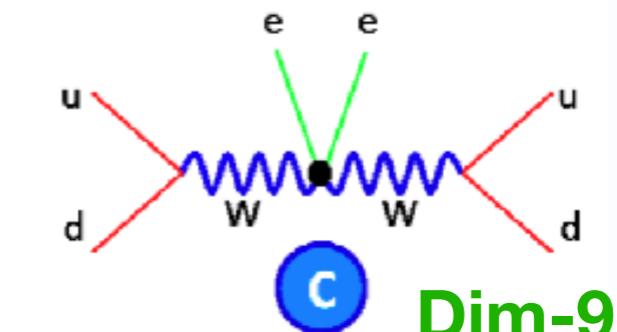
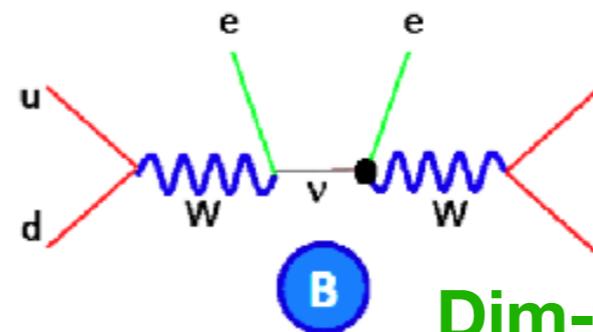
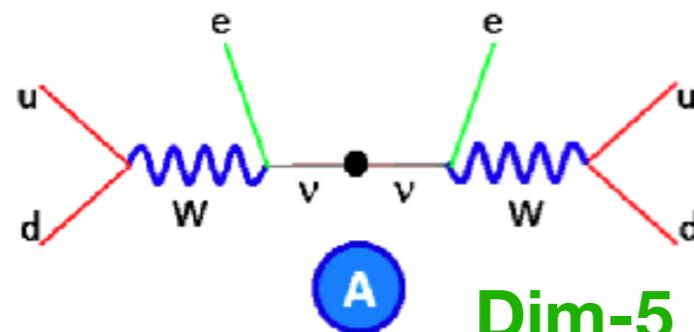


[Cirigliano, Dekens, de Vries, Graesser, 2018]

0vbb Operators

Very different types of operators contribute to the same process

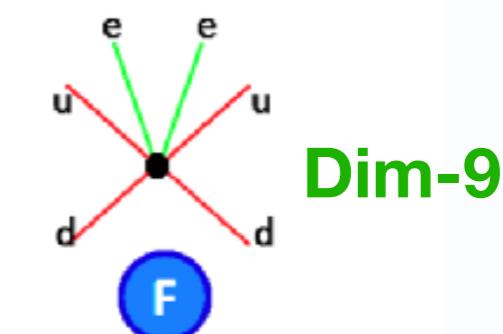
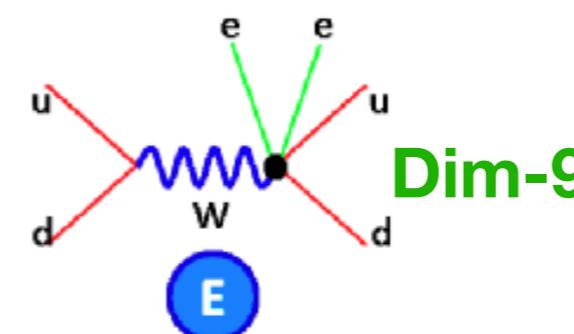
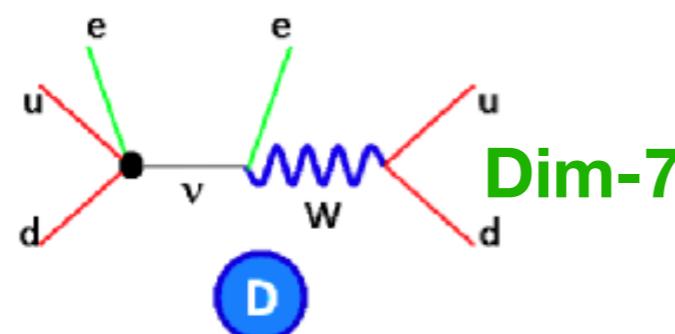
$$e_R = e_C^\dagger, u_R = u_C^\dagger, d_R = d_C^\dagger$$



$$H^2 L^2 \quad H^3 H^\dagger L^2$$

$$D ec H^\dagger{}^3 L^\dagger \quad H^\dagger{}^2 L^\dagger{}^2 WR$$

$$D^2 H^\dagger{}^2 L^\dagger{}^2 \quad D^2 H^\dagger{}^2 L^\dagger{}^2 WL$$



$$dc^\dagger H^\dagger L^\dagger{}^2 Q^\dagger$$

$$D dc^\dagger L^\dagger{}^2 uc$$

$$D dc^\dagger L^\dagger{}^2 uc$$

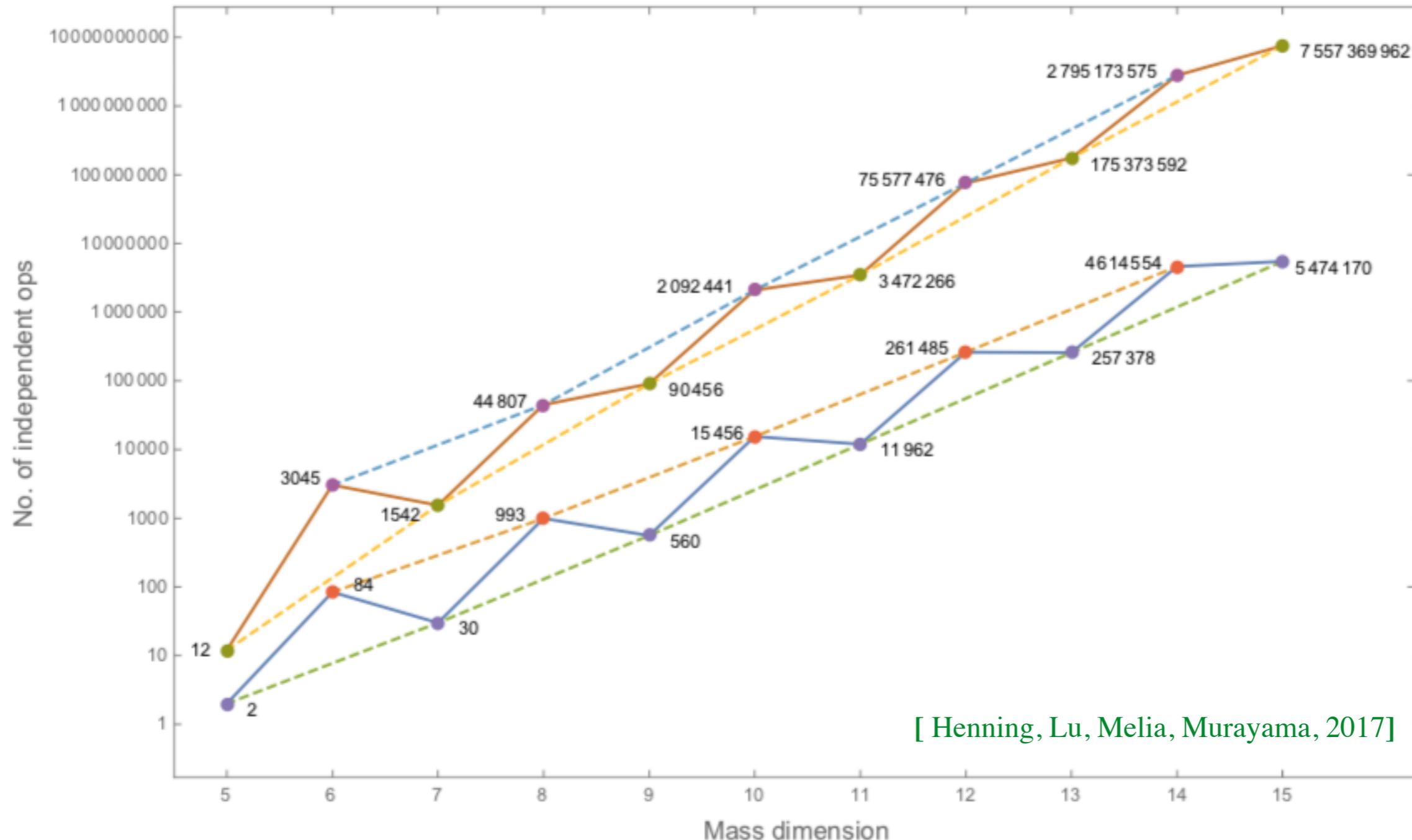
$$dc^\dagger ec H^\dagger L^\dagger uc WL$$

$$dc^2 L^2 Q^2, dc^2 dc^\dagger L^2 uc^\dagger, dc L^2 uc uc^\dagger, dc^2 ec^\dagger L Q uc^\dagger$$

$$dc^\dagger{}^2 ec^2 uc^2, dc L^2 QQ^\dagger uc^\dagger, dc^\dagger ec L^\dagger Q uc^2, L^{-2} Q^2 uc^2$$

Need write down complete set of operators up to dim-9

Hilbert Series Counting



Also [Lehman, Martin 2015]

Higher Dim Operators!?

Given numbers of independent operators

Still difficult to write down explicit form of operators!

Derivatives

$$BWHH^\dagger D^2$$

2

Repeated fields

$$QQQL$$

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$$\begin{aligned}
 & (D^2 H^\dagger) H B_{L\mu\nu} W_L^{v\mu}, (D^\mu D_\nu H^\dagger) H B_{L\mu\nu} W_L^{v\rho}, (D_\nu D^\mu H^\dagger) H B_{L\mu\nu} W_L^{v\rho}, (D_\mu H^\dagger) (D^\mu H^\dagger) B_{L\mu\nu} W_L^{v\rho}, \\
 & (D_\mu H^\dagger) (D^\mu H) B_{L\mu\nu} W_L^{v\mu}, (D^\nu H^\dagger) (D_\mu H) B_{L\mu\nu} W_L^{v\mu}, (D_\mu H^\dagger) H [D^\mu B_{L\mu\nu}] W_L^{v\rho}, (D_\mu H^\dagger) H (D^\mu B_{L\mu\nu}) W_L^{v\rho}, \\
 & (D^\nu H^\dagger) H (D_\mu B_{L\mu\nu}) W_L^{v\mu}, (D_\mu H^\dagger) H B_{L\mu\nu} (D^\nu W_L^{v\rho}), (D_\mu H^\dagger) H B_{L\mu\nu} (D^\nu W_L^{v\rho}), (D^\nu H^\dagger) H B_{L\mu\nu} (D_\mu W_L^{v\rho}), \\
 & H^\dagger (D^2 H) B_{L\mu\nu} W_L^{v\rho}, H^\dagger (D^\mu D_\nu H) B_{L\mu\nu} W_L^{v\rho}, H^\dagger (D_\nu D^\mu H) B_{L\mu\nu} W_L^{v\rho}, H^\dagger (D^\mu H) (D_\mu B_{L\mu\nu}) W_L^{v\rho}, \\
 & H^\dagger (D^\nu H) (D_\mu B_{L\mu\nu}) W_L^{v\mu}, H^\dagger (D_\mu H) (D^\nu B_{L\mu\nu}) W_L^{v\mu}, H^\dagger (D^\mu H) B_{L\mu\nu} (D_\mu W_L^{v\rho}), H^\dagger (D^\mu H) B_{L\mu\nu} (D_\mu W_L^{v\rho}), \\
 & H^\dagger (D_\mu H) B_{L\mu\nu} (D^\rho W_L^{v\rho}), H^\dagger H (D^2 B_{L\mu\nu}) W_L^{v\rho}, H^\dagger H (D^2 D_\nu B_{L\mu\nu}) W_L^{v\rho}, H^\dagger H (D_\mu D^\nu B_{L\mu\nu}) W_L^{v\rho}, \\
 & H^\dagger H (D^\mu B_{L\mu\nu}) (D_\mu W_L^{v\rho}), H^\dagger H (D^\mu B_{L\mu\nu}) (D_\mu W_L^{v\rho}), H^\dagger H (D_\mu B_{L\mu\nu}) (D^\rho W_L^{v\rho}), H^\dagger H B_{L\mu\nu} (D^2 W_L^{v\rho}), \\
 & H^\dagger H B_{L\mu\nu} (D^\mu D_\nu W_L^{v\rho}), H^\dagger H B_{L\mu\nu} (D_\mu D^\nu W_L^{v\rho})
 \end{aligned} \tag{14}$$

Which 2 should be picked up?

What flavor relations should be imposed?

$$\epsilon^{abc} \epsilon^{ik} \epsilon^{jl} (L_{pi} Q_{rak}) (Q_{sbk} Q_{tcl})$$

$$\epsilon^{abc} \epsilon^{ik} \epsilon^{jl} (L_{pi} Q_{slk}) (Q_{raj} Q_{tcl})$$

$$\epsilon^{abc} \epsilon^{ij} \epsilon^{kl} (L_{pi} Q_{slk}) (Q_{raj} Q_{tcl})$$

$$\epsilon^{abc} \epsilon^{ij} \epsilon^{kl} (L_{pi} Q_{raj}) (Q_{sik} Q_{tcl})$$

$p, r, s, t = 1, 2, 3$

$$Q_{prst}^{qqq\ell} = C^{prst}$$

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Operator as Spinor Tensor

Any operator can be written with totally symmetric spinor indices:

$$\mathcal{O} = (\epsilon^{\alpha_i \alpha_j})^n (\tilde{\epsilon}_{\dot{\alpha}_i \dot{\alpha}_j})^{\tilde{n}} \prod_{i=1}^N (D^{r_i - |h_i|} \Phi_i)^{\dot{\alpha}_i^{r_i + h_i}}_{\alpha_i^{r_i - h_i}}$$

$$\epsilon_{\alpha_1 \alpha_3} \epsilon_{\alpha_1 \alpha_3} \epsilon_{\alpha_2 \alpha_4} \tilde{\epsilon}^{\dot{\alpha}_3 \dot{\alpha}_4} F_1^{\alpha_1^2} \psi_2^{\alpha_2} (D\psi_3)_{\dot{\alpha}_3}^{\alpha_3^2} (D\phi_4)_{\dot{\alpha}_4}^{\alpha_4}$$

$$F_1^{\alpha \beta} \psi_2^\gamma (D\psi_3)_{\alpha \beta \dot{\alpha}} (D\phi_4)_\gamma^{\dot{\alpha}}$$

Epsilon tensor transformations under $\text{SL}(2,\mathbb{C}) \times \text{SU}(N)$

$$\epsilon^{\alpha_i \alpha_j} \rightarrow \sum_{k,l} \mathcal{U}_k^i \mathcal{U}_l^j \epsilon^{\alpha_k \alpha_l}, \quad \tilde{\epsilon}_{\dot{\alpha}_i \dot{\alpha}_j} \rightarrow \sum_{k,l} \mathcal{U}_i^{\dagger k} \mathcal{U}_j^{\dagger l} \tilde{\epsilon}_{\dot{\alpha}_k \dot{\alpha}_l}.$$

$i, j, k, l = 1 \text{ to } N$

$$\boxed{} = [1^2]$$

$$\boxed{} = [1^{N-2}]$$

$$\epsilon^{\alpha_i \alpha_j}$$

$$\mathcal{E}^{ijk_1, \dots, k_{N-2}} \tilde{\epsilon}_{\dot{\alpha}_i \dot{\alpha}_j}$$

$$\underbrace{\boxed{} \otimes \dots \otimes \boxed{}}_{n}$$

$$\underbrace{\boxed{} \otimes \dots \otimes \boxed{}}_{\tilde{n}}$$

= Irrep $\oplus \dots \oplus$ Irrep

Total Derivatives

$$\begin{array}{c}
 \text{Diagram showing } n \text{ boxes} \otimes \cdots \otimes n \text{ boxes} \otimes \tilde{n} \text{ boxes} \otimes \cdots \otimes \tilde{n} \text{ boxes} \\
 \xrightarrow{\quad \epsilon^{\otimes 2} = \boxed{} \otimes \boxed{} = \boxed{} \oplus \boxed{} \oplus \boxed{} \quad} \\
 \text{Schouten identity} \\
 \frac{i \mid l}{j \mid k} \sim \epsilon^{\alpha_i \alpha_j} \epsilon^{\alpha_k \alpha_l} + \epsilon^{\alpha_k \alpha_i} \epsilon^{\alpha_j \alpha_l} + \epsilon^{\alpha_j \alpha_k} \epsilon^{\alpha_i \alpha_l} = 0
 \end{array}$$

$$\begin{array}{c}
 = N - 2 \left\{ \begin{array}{c} \text{Diagram with } \tilde{n} \text{ boxes, one } n \text{ box above it, and a sum over } i} \\ \text{total derivatives (integration by part)} \end{array} \right. \\
 \left. \sum_i \epsilon^{\alpha_i \alpha_j} \tilde{\epsilon}_{\dot{\alpha}_i \dot{\alpha}_k} \right) \\
 \text{the sum over } i \text{ means a total derivative}
 \end{array}$$

[Such Young diagram also obtained from conformal K harmonics]

[Henning, Melia, 2019]

Differently we obtain Young diagram using epsilon tensor transformation
No need conformal symmetry!

Independent Lorentz Structure

To obtain Lorentz structure, we invent a **new** Young diagram **filling** procedure!

Filling rules on semi-standard Young tableau (SSYT)

with given class

$$\{ \underbrace{1, \dots, 1}_{\#1}, \underbrace{2, \dots, 2}_{\#2}, \dots \}$$

$$\#i = \tilde{n} - 2h_i$$

$$Y_{N,n,\tilde{n}} = \begin{matrix} N \\ \vdots \\ \tilde{n} \end{matrix} \left\{ \begin{matrix} \boxed{\text{ }} & \cdots & \boxed{\text{ }} & \cdots & \boxed{\text{ }} \\ \vdots & & \vdots & & \vdots \\ \boxed{\text{ }} & \cdots & \boxed{\text{ }} & & \end{matrix} \right\}^n$$

Fock's condition removes redundancy

$$\left(\tau^I \right)_j^i W_{\mu\lambda}^I \left(e_{cp} \sigma^{\nu\lambda} L_{ri} \right) D^\mu D_\nu H^{\dagger j}$$

$$(\tilde{n} = 1, n = 3)$$

$$\#1 = 3, \#2 = \#3 = 2, \#4 = 1.$$

Basis YT method guarantees independence!
Filling all SSYT guarantees completeness!

New filling: any operator could be converted to this basis

1	1	1	2
2	3	3	4

$$\epsilon_{\alpha_1\alpha_3} \epsilon_{\alpha_1\alpha_3} \epsilon_{\alpha_2\alpha_4} \tilde{\epsilon}^{\dot{\alpha}_3\dot{\alpha}_4} \quad F_{L1}^{\alpha\beta} \psi_2^\gamma (D\psi_3)_{\alpha\beta\dot{\alpha}} (D\phi_4)_{\gamma}{}^{\dot{\alpha}} \quad \langle 13 \rangle \langle 13 \rangle \langle 24 \rangle [34]$$

1	1	1	3
2	2	3	4

$$\epsilon_{\alpha_1\alpha_2} \epsilon_{\alpha_1\alpha_3} \epsilon_{\alpha_3\alpha_4} \tilde{\epsilon}^{\dot{\alpha}_3\dot{\alpha}_4} \quad F_{L1}^{\alpha\beta} \psi_{2\alpha} (D\psi_3)_{\beta}{}^{\gamma\dot{\alpha}} (D\phi_4)_{\gamma}{}^{\dot{\alpha}} \quad \langle 12 \rangle \langle 13 \rangle \langle 34 \rangle [34]$$

Repeated Field: QQQL

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

B-violating	
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk} \left[(d_p^\alpha)^T C u_r^\beta \right] \left[(q_s^\gamma)^T C l_t^k \right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk} \left[(q_p^\alpha)^T C q_r^\beta \right] \left[(u_s^\gamma)^T C e_t \right]$
$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn} \left[(q_p^\alpha)^T C q_r^\beta \right] \left[(q_s^\gamma)^T C l_t^n \right]$
$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn} \left[(q_p^\alpha)^T C q_r^\beta \right] \left[(q_s^\gamma)^T C l_t^n \right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma} \left[(d_p^\alpha)^T C u_r^\beta \right] \left[(u_s^\gamma)^T C e_t \right]$

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$$Q_{prst}^{qqq\ell(1)} = -(Q_{prst}^{qqq\ell} + Q_{rpst}^{qqq\ell})$$

$$Q_{prst}^{qqq\ell(3)} = -(Q_{prst}^{qqq\ell} - Q_{rpst}^{qqq\ell})$$

[Grzadkowski, et.al. v3 2017]

B-violating	
Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk} \left[(d_p^\alpha)^T C u_r^\beta \right] \left[(q_s^\gamma)^T C l_t^k \right]$
Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk} \left[(q_p^\alpha)^T C q_r^\beta \right] \left[(u_s^\gamma)^T C e_t \right]$
Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km} \left[(q_p^\alpha)^T C q_r^\beta \right] \left[(q_s^\gamma)^T C l_t^n \right]$
Q_{duu}	$\varepsilon^{\alpha\beta\gamma} \left[(d_p^\alpha)^T C u_r^\beta \right] \left[(u_s^\gamma)^T C e_t \right]$

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[Alonso, Chang, Jenkins, Manohar, Shotwell 2014]

$$Q_{prst}^{qqq\ell} + Q_{rpst}^{qqq\ell} = Q_{sprt}^{qqq\ell} + Q_{srpt}^{qqq\ell}$$

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Flavor relations not easy task!

Dim-5 Weinberg Operator

Repeated fields leads to symmetries in the coupling parameters

$$2 \times 2 = 1_A + 3_S \quad SU(2) \quad \text{in QM}$$

Only one Higgs doublet in SM

$$\frac{L \times L}{\cancel{1_A} + 3_S} \times \frac{H \times H}{\cancel{1_A} + 3_S}$$

$$\kappa_{ij} L_i L_j H H$$

κ_{ij} is symmetric: $\kappa_{ij} = \kappa_{ji}$

3 generation = 6 parameters!

If more than one Higgs doublet

$$(1_A + 3_S) \times (1_A + 3_S) = 1_{SS} + 1_{AA} + \dots$$

$$\kappa_{ijkl}^{SS} (L_i L_j H_k H_l)_{SS} + \kappa_{ijkl}^{AA} (L_i L_j H_k H_l)_{AA}$$

4-fermion operators

$$L^* L^* L L$$

Higgs self couplings

$$g_{ijkl}^{(\alpha)} H_i^* H_j^* H_k H_l$$

$$\{\{\textcolor{orange}{S}, \textcolor{orange}{S}\}, \frac{1}{4} n^2 (1+n)^2, 1\} + \{\{\textcolor{blue}{A}, \textcolor{blue}{A}\}, \frac{1}{4} (-1+n)^2 n^2, 1\}$$

2HDM = 10 parameters

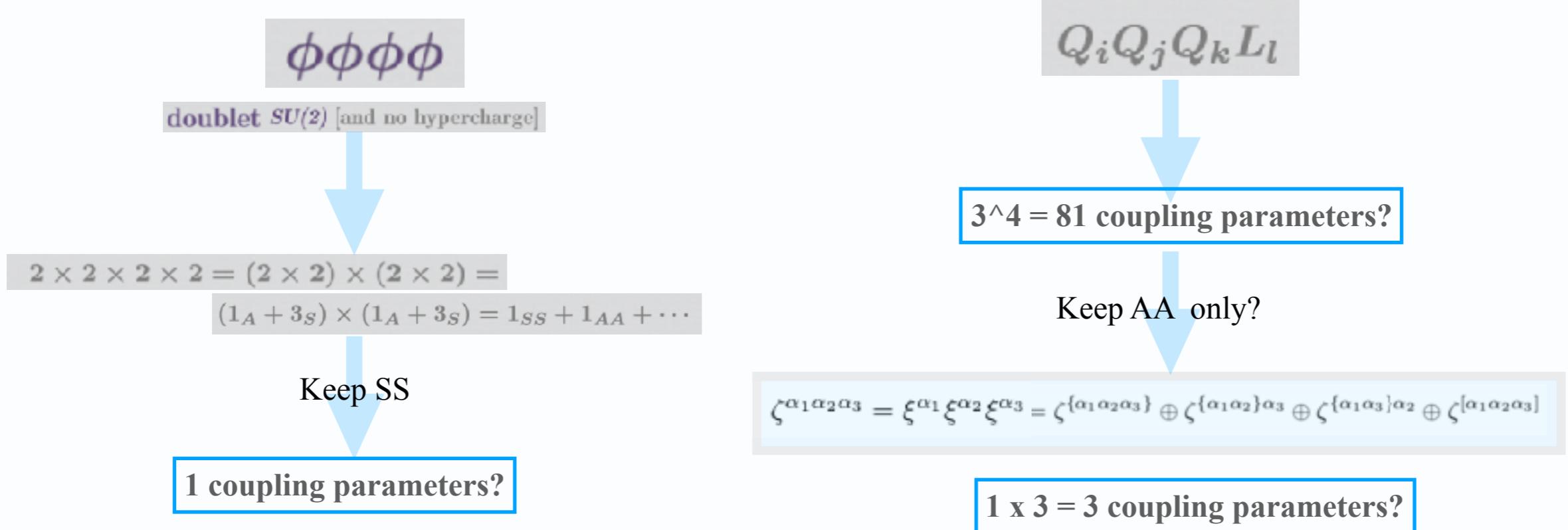
$$2^* \times 2^* \times 2 \times 2 = (1_A + 3_S) \times (1_A + 3_S)$$

$$g_{ijkl}^{(SS)} \left(L_i^* L_j^* L_k L_l \right)_{(SS)} + g_{ijkl}^{(AA)} \left(L_i^* L_j^* L_k L_l \right)_{(AA)}$$

45 parameters in 4-fermion operator

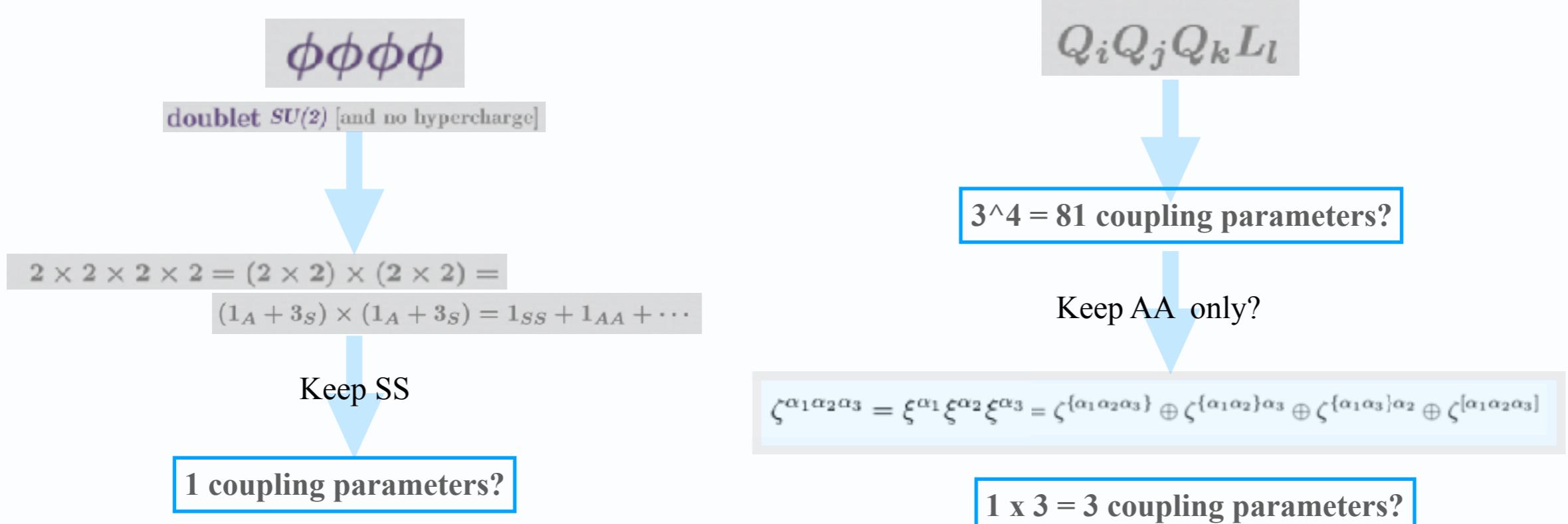
More Repeated Fields

So we need to keep track of S and A symmetries for repeated field



More Repeated Fields

So we need to keep track of S and A symmetries for repeated field



If the case of $2 \times 2 \times 2 \times 2$, the code would be the following:

```

In[1]:= PermutationSymmetry[SU2, {2,2,2,2}, UseLabel -> True]
Out[1]= {{1, 2, 3, 4}}, {{(3, (1, 1, 1)), 1}, {{3, {1, 1, 1}}, 1}, {{1, {1, 1, 1}}, 1}}
  
```

It says that, under the $SU(2)_L$ group, the product breaks as $5c + 3a + 1s$ (using the notation $\{1\} = S$, $\{3, 1\} = 3$, $\{2, 2\} = 2$ for the S_6 representations). So, for example, neither of the two

Answer: 0!!!

Answer: $19 \times 3 = 57$ parameters!!!

It is not enough by keeping only S and A symmetries for repeated field!

Plethysm

N Identical non-fundamental particles with nf flavors

$$\mathbf{6} \times \mathbf{6} \times \mathbf{6} = \mathbf{1}_{\{3\}} + \mathbf{27}_{\{3\}} + \overline{\mathbf{28}}_{\{3\}} + \mathbf{8}_{\{2,1\}} + \mathbf{27}_{\{2,1\}} + \overline{\mathbf{35}}_{\{2,1\}} + \mathbf{10}_{\{1,1,1\}} + \overline{\mathbf{10}}_{\{1,1,1\}}.$$

```
In[=]:= Plethysms[SU3, 6, {3}, UserName → True]
```

```
Out[=]= {{\overline{\mathbf{28}}, 1}, {\mathbf{1}, 1}, {\mathbf{27}, 1}}
```

```
In[=]:= Plethysms[SU3, 6, {2, 1}, UserName → True]
```

```
Out[=]= {{\overline{\mathbf{35}}, 1}, {\mathbf{8}, 1}, {\mathbf{27}, 1}}
```

```
In[=]:= Plethysms[SU3, 6, {1, 1, 1}, UserName → True]
```

```
Out[=]= {{\overline{\mathbf{10}}, 1}, {\mathbf{10}, 1}}
```

[Fonseca, GroupMath]

```
In[=]:= PermutationSymmetry[SU2, {2, 2, 3, 3}, UserName → True]
```

```
Out[=]= {{{{1, 2}, {3, 4}}, {{{7, {□, □}}, 1}, {{3, {□, □}}, 2}, {{3, {□, □}}, 1}, {{3, {□, □}}, 1}, {{5, {□, □}}, 1}, {{5, {□, □}}, 1}, {{5, {□, □}}, 1}, {{1, {□, □}}, 1}, {{1, {□, □}}, 1}}}}
```

```
In[=]:= PermutationSymmetry[SU2, {2, 2, 2, 3}, UserName → True]
```

```
Out[=]= {{{{1, 2, 3}, {4}}, {{{6, {□□□, □}}, 1}, {{2, {□□□, □}}, 1}, {{2, {□□□, □}}, 1}, {{4, {□□□, □}}, 1}, {{4, {□□□, □}}, 1}}}}
```

$S_n \times SU(m)$ Group

Three spin-1/2 particle wave function:

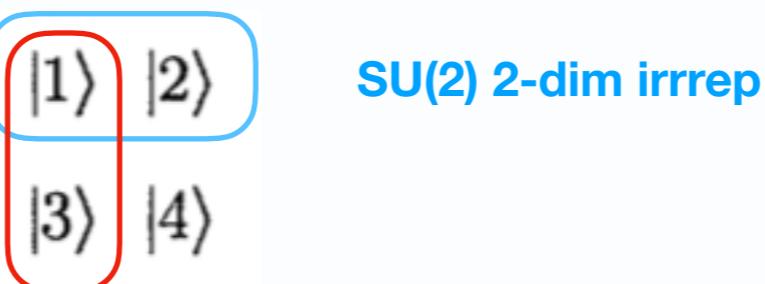
$$2 \otimes 2 \otimes 2 = 4 \oplus 2 \oplus 2$$

$$\begin{aligned} & \psi_+(1)\psi_+(2)\psi_+(3) \\ & \psi_+(1)\psi_+(2)\psi_-(3) + \psi_+(1)\psi_-(2)\psi_+(3) + \psi_-(1)\psi_+(2)\psi_+(3) \\ & \psi_+(1)\psi_-(2)\psi_-(3) + \psi_-(1)\psi_+(2)\psi_-(3) + \psi_-(1)\psi_-(2)\psi_+(3) \\ & \psi_-(1)\psi_-(2)\psi_-(3) \end{aligned}$$

SU(2) 4-dim irrep [4]

S3 1-dim irrep [3]

$$\begin{aligned} |1\rangle &= 2\psi_+(1)\psi_+(2)\psi_-(3) - \psi_+(1)\psi_-(2)\psi_+(3) - \psi_-(1)\psi_+(2)\psi_+(3) \\ |2\rangle &= 2\psi_-(1)\psi_-(2)\psi_+(3) - \psi_+(1)\psi_-(2)\psi_-(3) - \psi_-(1)\psi_+(2)\psi_-(3) \\ |3\rangle &= 2\psi_+(1)\psi_-(2)\psi_+(3) - \psi_+(1)\psi_+(2)\psi_-(3) - \psi_-(1)\psi_+(2)\psi_+(3) \\ |4\rangle &= 2\psi_-(1)\psi_+(2)\psi_-(3) - \psi_+(1)\psi_-(2)\psi_-(3) - \psi_-(1)\psi_-(2)\psi_+(3) \end{aligned}$$



S3 2-dim irrep [2 1]

Flavor Tensor

Three 1/2-particle with 3 flavors:

$$3 \otimes 3 \otimes 3 = 10 \oplus 8 \oplus 8 \oplus 1$$

SU(3) 1-d irrep

<i>r</i>	1
<i>s</i>	2
<i>t</i>	3

SU(3) 8-d irrep

<i>r s</i>	1 1, 1 1, 2 2, 1 2, 1 3, 2 3, 1 2, 1 3,
<i>t</i>	2 3, 3 3, 3 2, 2 2,

SU(3) 10-d irrep

<i>r s t</i>	1 1 1, 1 1 2, 1 1 3, 1 2 2, 1 2 3, 1 3 3, 2 2 2, 2 2 3, 2 3 3, 3 3 3,
--------------	---

S3 1-dim irrep [111]

S3 2-dim irrep [21]

S3 1-dim irrep [3]

1>	2>	3>	4>	5>	6>	7>	8>
1'>	2'>	3'>	4'>	5'>	6'>	7'>	8'>

SU(3) 8-dim irrep

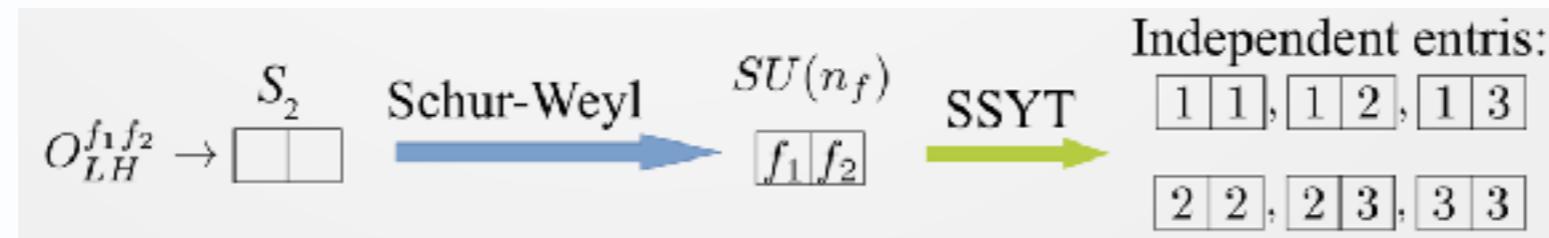
S3 2-dim irrep [2 1]

Schur-Weyl theorem:

$$\underbrace{(\{1\} \otimes (1)) \otimes (\{1\} \otimes (1)) \otimes \cdots \otimes (\{1\} \otimes (1))}_{N \text{ factors}} = \sum_{\lambda \vdash N} \{\lambda\} \otimes (\lambda)$$

Flavor Symmetry

According to Schur-Weyl theorem, flavor tensor decomposed via $S(n_f)$ symmetry



$$O_{qqql}^{p,rst} \epsilon^{abc} \epsilon_{jji} \epsilon_{km} [(q_r^{aj})^T C q_s^{bk}] [(q_t^{cm})^T C l_p^i] \quad S_3 : \begin{array}{|c|c|c|} \hline \end{array} + \begin{array}{|c|c|} \hline \end{array} + \begin{array}{|c|c|} \hline \end{array} + \begin{array}{|c|} \hline \end{array}$$

$$SU(n_f) : \begin{array}{|c|c|c|} \hline \end{array} + \begin{array}{|c|c|} \hline \end{array} + \begin{array}{|c|c|} \hline \end{array}$$

$$19 \times 3 = 57$$

Each span's an irreducible $SU(n_f)$ subspace	
$\begin{array}{ c } \hline r \\ \hline s \\ \hline t \\ \hline \end{array}$:	$\begin{array}{ c } \hline 1 \\ \hline 2 \\ \hline 3 \\ \hline \end{array}$,
$\begin{array}{ c c } \hline r & s \\ \hline t \\ \hline \end{array}$:	$\begin{array}{ c c } \hline 1 & 1 \\ \hline 2 & 3 \\ \hline 1 & 1 \\ \hline 3 & 3 \\ \hline 2 & 2 \\ \hline 1 & 2 \\ \hline 3 & 2 \\ \hline 1 & 3 \\ \hline 2 & 3 \\ \hline 1 & 2 \\ \hline 3 & 2 \\ \hline 1 & 3 \\ \hline 2 & 2 \\ \hline \end{array}$,
$\begin{array}{ c c c } \hline r & s & t \\ \hline \end{array}$:	$\begin{array}{ c c c } \hline 1 & 1 & 1 \\ \hline 1 & 1 & 2 \\ \hline 1 & 1 & 3 \\ \hline 1 & 2 & 2 \\ \hline 1 & 2 & 3 \\ \hline 1 & 3 & 3 \\ \hline 2 & 2 & 2 \\ \hline 2 & 2 & 3 \\ \hline 2 & 3 & 3 \\ \hline 3 & 3 & 3 \\ \hline \end{array}$,

How to recognize $SU(n_f)$ symmetry according to S_n ?

Spin-statistics

N-identical particle operator obeys spin-statistics:

$$\pi \circ \mathcal{O}(1, 2, \dots, m, \dots, N) = \mathcal{O}(\pi(1), \pi(2), \dots, \pi(m), \dots, N) = \pm \mathcal{O}(1, 2, \dots, m, \dots, N)$$

Amplitude view:

$$Amp(1, \dots, N) \sim C_{f_1, \dots, f_N} T_G^{a_1, \dots, a_N} B^{(d)}(h_1, \dots, h_N)$$

$$\lambda_{Amp} = [m] \text{ or } [1^m] \in \lambda_C \odot \lambda_G \odot \lambda_B$$
$$\lambda_C^{(T)} = \lambda_C \odot \lambda_B$$

[Li, Ren, Xiao, Yu, Zheng, 2020]

Operator view:

$$\underbrace{\pi \circ \mathcal{O}(f_k, \dots)}_{\text{permute flavor}} = T_{SU3}^{\{g_k, \dots\}} T_{SU2}^{\{h_k, \dots\}} \mathcal{M}_{\{g_k, \dots\}, \{h_k, \dots\}}^{\{f_{\pi(k)}, \dots\}}$$

$$= T_{SU3}^{\{g_{\pi(k)}, \dots\}} T_{SU2}^{\{h_{\pi(k)}, \dots\}} \mathcal{M}_{\{g_{\pi(k)}, \dots\}, \{h_{\pi(k)}, \dots\}}^{\{f_{\pi(k)}, \dots\}}$$

$$= \underbrace{\left(\pi \circ T_{SU3}^{\{g_k, \dots\}} \right)}_{\text{permute gauge}} \underbrace{\left(\pi \circ T_{SU2}^{\{h_k, \dots\}} \right)}_{\text{permute Lorentz}} \underbrace{\left(\pi \circ \mathcal{M}_{\{g_k, \dots\}, \{h_k, \dots\}}^{\{f_k, \dots\}} \right)}_{\text{permute Lorentz}}$$

[Fonseca, 2020]

	LL	HH
$SU(3)_C$	\	\
$SU(2)_W$		
$SU(2)_L$		\
$SU(2)_R$	\	\
Grassmann		\
Flavor	\times \times =	

	Q^3	L
$SU(3)_C$		\
$SU(2)_W$		
$SU(2)_L$		
$SU(2)_R$	\	\
Grassmann		\
Flavor	\times \times \times = + + $10 + 8 + 1$	$\times 3 = 57$

SMEFT

Dimension-5

$$\epsilon_{ij}\epsilon_{mn}(L^iCL^m)H^jH^n$$

[Weinberg, 1979]

2

Dimension-6

A^{μ}	$\text{ad } O^{1,2}$	$\phi^{(1)}$	X
$O_{\mu 1} = (\Phi^1 \Phi^2)^T$	$O_{\mu 1} = (\Phi^1 \Phi^2) \tilde{O}_1 (I_2, 4)$	$O_{\mu 1} = -J^{123} O_1^{12} O_2^{32} O_3^{23}$	
$O_{\mu 2} = (\Phi^1 \Phi^2) \tilde{O}_2 (\Phi^1 \Phi^2)$	$O_{\mu 2} = (\Phi^1 \Phi^2) \tilde{O}_2 (\Phi^1 \Phi^2)$	$O_{\mu 2} = -J^{123} O_2^{12} O_3^{23} O_1^{32}$	
$O_{\mu 3} = (\Phi^1 \Phi^2) \tilde{O}_3 (I_2, 4)$	$O_{\mu 3} = (\Phi^1 \Phi^2) \tilde{O}_3 (I_2, 4)$	$O_{\mu 3} = -e^{i\theta} W_1^{12} W_2^{23} W_3^{31}$	
$O_{\mu 4} = -(\Phi^1 \Phi^2) \tilde{O}_4 (I_2, 4)$	$O_{\mu 4} = -(\Phi^1 \Phi^2) \tilde{O}_4 (I_2, 4)$	$O_{\mu 4} = -e^{i\theta} \tilde{W}_1^{12} \tilde{W}_2^{23} \tilde{W}_3^{31}$	
$\mathcal{L}^2 O^2$	$\phi^2 X$		
		$(\bar{L} L)(\bar{L} L)$	$(\bar{L} R)(\bar{R} R)$
$O_{LL} = (\Phi^1 \Phi^2) \tilde{O}_1^1 \tilde{O}_2^1 O^{12}$	$O_{LL} = (\bar{L}_1 \bar{L}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$	$O_{LR} = (\bar{L}_1 \bar{L}_2) (\bar{L}_3)^2 L_4$	$O_{RL} = (\bar{L}_1 \bar{L}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$
$O_{LR} = (\Phi^1 \Phi^2) \tilde{O}_2^1 \tilde{O}_3^1 O^{12}$	$O_{LR}^1 = (\bar{Q}_1 \bar{L}_2 \bar{L}_3) (\bar{Q}_2)^2 Q_2$	$O_{RR} = (\bar{Q}_1 \bar{Q}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$	$O_{RR} = (\bar{L}_1 \bar{L}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$
$O_{RR} = (\Phi^1 \Phi^2) \tilde{O}_3^1 \tilde{O}_4^1 O^{12}$	$O_{RR}^1 = (\bar{Q}_1 \bar{Q}_2 \bar{L}_3) (\bar{Q}_3)^2 Q_3$	$O_{RR} = (\bar{L}_1 \bar{L}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$	$O_{RR} = (\bar{Q}_1 \bar{Q}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$
$O_{RL} = (\Phi^1 \Phi^2) \tilde{O}_1^1 \tilde{O}_4^1 O^{12}$	$O_{RL}^1 = (\bar{L}_1 \bar{L}_2 \bar{L}_3) (\bar{Q}_1)^2 Q_1$	$O_{RR} = (\bar{L}_1 \bar{L}_2) (\bar{L}_3)^2 L_4$	$O_{RR} = (\bar{Q}_1 \bar{Q}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$
$O_{RR} = (\Phi^1 \Phi^2) \tilde{O}_2^1 \tilde{O}_4^1 O^{12}$	$O_{RR}^2 = (\bar{L}_1 \bar{L}_2 \bar{L}_3) (\bar{Q}_2)^2 Q_2$	$O_{RR} = (\bar{L}_1 \bar{L}_2) (\bar{L}_3)^2 L_4$	$O_{RR} = (\bar{Q}_1 \bar{Q}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$
$O_{RR} = (\Phi^1 \Phi^2) \tilde{O}_3^1 \tilde{O}_4^1 O^{12}$	$O_{RR}^3 = (\bar{L}_1 \bar{L}_2 \bar{L}_3) (\bar{Q}_3)^2 Q_3$	$O_{RR} = (\bar{L}_1 \bar{L}_2) (\bar{L}_3)^2 L_4$	$O_{RR} = (\bar{Q}_1 \bar{Q}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$
$O_{RR} = (\Phi^1 \Phi^2) \tilde{O}_1^1 \tilde{O}_4^1 O^{12}$	$O_{RR}^4 = (\bar{L}_1 \bar{L}_2 \bar{L}_3) (\bar{Q}_4)^2 Q_4$	$O_{RR} = (\bar{L}_1 \bar{L}_2) (\bar{L}_3)^2 L_4$	$O_{RR} = (\bar{Q}_1 \bar{Q}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$
$O_{RR} = -(\Phi^1 \Phi^2) \tilde{O}_2^1 \tilde{O}_3^1 O^{12}$	$O_{RR}^5 = (\bar{L}_1 \bar{L}_2 \bar{L}_3) (\bar{Q}_5)^2 Q_5$	$O_{RR} = (\bar{L}_1 \bar{L}_2) (\bar{L}_3)^2 L_4$	$O_{RR} = (\bar{Q}_1 \bar{Q}_2 \bar{L}_3) (\bar{L}_4)^2 L_4$
$O_{RRR} = -(\Phi^1 \Phi^2) \tilde{O}_1^1 \tilde{O}_2^1 O^{12}$	$O_{RRR} = (\bar{L}_1 \bar{L}_2 \bar{L}_3) (\bar{Q}_6)^2 Q_6$	$O_{RRR} = -e^{i\theta} \bar{L}_1 \bar{L}_2 \bar{L}_3 \bar{L}_4 [(\bar{Q}_1^1)^2 C_1^1] / [(\bar{Q}_2^1)^2 C_1^1]$	
$O_{RRR} = -(\Phi^1 \Phi^2) \tilde{O}_1^1 \tilde{O}_2^1 O^{12}$	$O_{RRR}^1 = (\bar{L}_1^1 \bar{L}_2 \bar{L}_3) (\bar{Q}_6)^2 Q_6$	$O_{RRR} = -e^{i\theta} \bar{L}_1 \bar{L}_2 \bar{L}_3 \bar{L}_4 [(\bar{Q}_2^1)^2 C_1^1] / [(\bar{Q}_1^1)^2 C_1^1]$	
	$(\bar{L} R)(\bar{R} L)$ and $(\bar{L} R)(\bar{R} R)$	B-violating	
		$O_{RRR} = -e^{i\theta} \bar{L}_1 \bar{L}_2 \bar{L}_3 \bar{L}_4 [(\bar{Q}_1^1)^2 C_1^1] / [(\bar{Q}_2^1)^2 C_1^1]$	
		$O_{RRR} = -e^{i\theta} \bar{L}_1 \bar{L}_2 \bar{L}_3 \bar{L}_4 [(\bar{Q}_2^1)^2 C_1^1] / [(\bar{Q}_1^1)^2 C_1^1]$	
		$O_{RRR} = -e^{i\theta} \bar{L}_1 \bar{L}_2 \bar{L}_3 \bar{L}_4 [(\bar{Q}_1^1)^2 C_1^1] / [(\bar{Q}_2^1)^2 C_1^1]$	
		$O_{RRR} = -e^{i\theta} \bar{L}_1 \bar{L}_2 \bar{L}_3 \bar{L}_4 [(\bar{Q}_1^1)^2 C_1^1] / [(\bar{Q}_2^1)^2 C_1^1]$	

[Buchmuller, Wyler, 1986]

[Grzadkowski, Iskrzynski, Misiak, Rosiek, 2010]

Dimension-7

$1 : \psi^2 X H^2 + \text{h.c.}$	$2 : \psi^2 H^4 + \text{h.c.}$
$Q_{DWH^2} : e_{mn} (t^T \epsilon)_{jk} (\bar{l}_p^m C i \sigma^{\mu\nu} l_j^l) H^n H^k W_{\mu\nu}^l$	$Q_{D^2 H^2} : e_{mn} e_{jk} (\bar{l}_p^m C i \sigma^{\mu\nu} l_j^l) H^n H^k (H^\dagger H)$
$Q_{DHH^2} : e_{mn} e_{jk} (\bar{l}_p^m C i \sigma^{\mu\nu} l_j^l) H^n H^k B_{\mu\nu}$	
$3(B) : \psi^4 H + \text{h.c.}$	$3(B) : \psi^4 H + \text{h.c.}$
$Q_{V_{\mu}H} : e_{jk} e_{mn} (g_p l_j^l) (\bar{l}_s^k C l_m^l) H^n$	$Q_{B_{\mu\nu}H} : e_{\alpha\beta\gamma} (\bar{l}_p d_\nu^{\alpha}) (g_s^2 C l_\gamma^l) \tilde{H}$
$Q_{B_{\mu\nu}H} : e_{jk} (\bar{d}_p l_j^l) (g_s^2 C e_\nu) H^k$	$Q_{B_{\mu\nu}H} : e_{\alpha\beta\gamma} e_{jk} (\bar{l}_p^m d_\nu^{\alpha}) (g_s^2 C g_\gamma^{(1)}) \tilde{H}^k$
$Q_{J/\psi HH}^{(1)} : e_{jk} e_{lm} (\bar{d}_p l_j^l) (g_s^2 C l_m^l) H^n$	$Q_{B_{\mu\nu}H} : e_{\alpha\beta\gamma} (\bar{l}_p d_\nu^{\alpha}) (g_s^2 C C_l^\gamma) H$
$Q_{J/\psi HH}^{(2)} : e_{jm} e_{lm} (\bar{d}_p l_j^l) (g_s^2 C l_m^l) H^n$	$Q_{B_{\mu\nu}H} : e_{\alpha\beta\gamma} e_{jk} (\bar{e}_p q_\nu^{\alpha}) (g_s^2 C d_l^\gamma) \tilde{H}^k$
$Q_{B_{\mu\nu}H} : e_{jk} (\bar{q}_p^m u_s) (l_m C l_j^l) H^k$	
$4 : \psi^2 H^2 D + \text{h.c.}$	$5(B) : \psi^4 D + \text{h.c.}$
$Q_{D_{\mu}H^2 D} : e_{mn} e_{jk} (\bar{l}_p^m C \gamma^\mu e_\nu) H^n H^j i D_\mu H^k$	$Q_{D_{\mu}H^2 D} : e_{jk} (\bar{d}_p \gamma^\mu u_s) (l_1^j C (D_\mu d_1^k))$
$6 : \psi^2 H^2 D^2 + \text{h.c.}$	$5(B) : \psi^4 D + \text{h.c.}$
$Q_{D_{\mu}H^2 D}^{(1)} : e_{jk} e_{mn} (\bar{l}_p^j C D^\mu \bar{l}_v^k) H^m (D_\mu H^n)$	$Q_{D_{\mu}H^2 D} : e_{\alpha\beta\gamma} (\bar{l}_p \gamma^\mu q_\nu^{\alpha}) (d_1^j C (D_\mu d_1^k))$
$Q_{D_{\mu}H^2 D}^{(2)} : e_{jk} e_{mn} (\bar{l}_p^j C D^\mu \bar{l}_v^k) H^m (D_\mu H^n)$	$Q_{D_{\mu}D} : e_{\alpha\beta\gamma} (\bar{e}_p q_\nu^{\alpha}) (d_1^j C (D_\mu d_1^k))$

[Lehman, 2014]

Dimension-8

[Li, Ren, Shu, Xiao, Yu, Zheng, 2020]

T	(n, m)	Satisfiability	N_{sys}	N_{vars}	$N_{\text{equations}}$	Equation
4	(1, 9)	$\beta_0^2 + \text{h.c.}$	16	20	26	[4.18]
	(3, 1)	$E_0^2 \psi^2 D = h.c.$	22	22	22n ₁	[4.20]
		$\psi^2 D^2 + h.c.$	4-1	14-14	$132n_1^2 + n_1(2n_1 - 1)$	[4.75, 4.76, 4.89]
		$E_0 \psi^2 \phi D^2 + h.c.$	16	32	32n ₁ ²	[4.40]
		$E_0^2 \phi^2 D^2 + h.c.$	8	12	12	[4.14]
	(3, 2)	$\beta_0^2 \psi^2$	16	17	17	[4.18]
5		$E_0 E_0 \psi^2 D$	27	39	39n ₁	[4.50, 4.51]
		$\psi^2 \phi^2 D^2$	17-11	54-18	$n_1^2(17n_1^2 + 31)16n_1$	[4.71, 4.72-4.89]
		$E_0 \psi^2 \phi D^2 + h.c.$	16	16	16n ₁ ²	[4.40]
		$E_0 E_0 \psi^2 D^2$	5	6	6	[4.14]
		$\psi^2 \phi^2 D^2$	7	16	16n ₁ ²	[4.31, 4.32]
		$\psi^2 D^4$	1	2	2	[4.30]
6	(3, 6)	$E_0 \psi^2 + h.c.$	13-13	66-54	$13n_1^2(42n_1^2 + 3n_1 - 1)$	[4.86, 4.88, 4.89, 4.91]
		$E_0^2 \psi^2 \phi + h.c.$	32	90	90n ₁ ²	[4.47, 4.48]
		$E_0 (\phi^2 + h.c.)$	6	6	6	[4.14]
	(3, 1)	$E_0 \psi^2 \phi^2 D + h.c.$	84-78	178-170	$18n_1^2(9n_1^2 + 2)14n_1$	[4.33-4.39, 4.88-4.91]
		$E_0^2 \psi^2 \phi + h.c.$	32	30	30n ₁ ²	[4.47, 4.48]
		$\psi^2 \phi^2 \phi D + h.c.$	23-18	140-120	$n_1^2(135n_1^2 + 1) + n_1^2(28n_1 + 3)$	[4.66, 4.69-4.72]
7		$E_0 \psi^2 \phi^2 D + h.c.$	38	32	32n ₁ ²	[4.33, 4.40]
		$\psi^2 \phi^2 D^2 + h.c.$	6	36	36n ₁ ²	[4.28]
		$E_0 \phi^2 \psi^2 + h.c.$	4	6	6	[4.14]
	(2, 6)	$\beta_0^2 \psi^2 + h.c.$	12-12	46-16	$5(5n_1 + n_1^2) - 3(2n_1^2 + \bar{\eta})$	[4.52, 4.53, 4.62, 4.64]
		$E_0 \psi^2 \phi^2 + h.c.$	16	22	22n ₁ ²	[4.28]
		$E_0 \phi^2 \psi^2 + h.c.$	8	10	10	[4.14]
8	(3, 1)	$\psi^2 \phi^2 \phi^2$	23-13	22-14	$n_1^2(14n_1^2 + n_1 + 2) + 2n_1(3n_1 - 1)$	[4.51, 4.55, 4.58-4.61]
		$\psi^2 \phi^2 D^2$	7	13	13n ₁ ²	[4.28, 4.31]
		$\phi^2 D^2$	1	2	2	[4.14]
9	(1, 9)	$\psi^2 \phi^2 + h.c.$	8	8	9n ₁ ²	[4.20]
9	(0, 0)	ϕ^2	1	1	1	[4.14]
Total		48	177-170	1070-1056	99-20(n ₁ = 3), 1108-1117(n ₁ = 2)	

[Murphy, 2020]

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Dimension-9

[Li, Ren, Xiao, Yu, Zheng, 2020]

N	(n, \bar{n})	Class	N_{type}	N_{van}	N_{spur}	Equation
4	$\langle 3, 2 \rangle$	$\psi^1 \bar{\psi}^2 D^3 + h.c.$ $\psi^1 \bar{\psi}^2 D^4 + h.c.$	$0+4+2=0$ $0+0+2=0$	10 6	$\frac{1}{2} n_f^2 (7n_f^2 - 1)$ $3n_f(n_f + 1)$	[5.50]-[5.54] [5.21]
5	$\langle 3, 1 \rangle$	$F_L \psi^2 \bar{\psi}^2 D + h.c.$ $\psi^1 \bar{\psi}^2 D^2 + h.c.$ $F_R \psi^2 \bar{\psi}^2 D^2 + h.c.$	$0+12+6=0$ $0+4+4=0$ $0+0+4=0$	22 10 24	$32n_f^4$ $2n_f$ $17n_f^2 - n_f$	[5.58]-[5.59] [5.45]-[5.48] [5.28]-[5.29]
	$\langle 2, 2 \rangle$	$F_{LR} \psi^2 \bar{\psi}^2 D + h.c.$ $\psi^1 \bar{\psi}^{12} \bar{\psi}^2 D^2$ $F_R \psi^2 \bar{\psi}^2 L D^2 + h.c.$ $\psi^1 \bar{\psi}^2 L D^2$	$0+12+6=0$ $0+4+4=0$ $0+0+4=0$ $0+0+2=0$	14 64 10 6	$4n_f^3 (5n_f + 1)$ $n_f^2 (4n_f + 1)$ $2n_f (5n_f - 1)$ $8n_f^2$	[5.59]-[5.63] [5.45]-[5.46] [5.28]-[5.29] [5.11]
6	$\langle 2, 0 \rangle$	$\psi^6 + h.c.$ $F_L \psi^3 \bar{\psi} + h.c.$ $F_R \psi^2 \bar{\psi}^2 + h.c.$	$2+4+5=0$ $0+12+13=0$ $0+0+8=0$	116 102 20	$\frac{1}{2} n_f^2 (410n_f^4 + 33n_f^2 - 59n_f^2 + 139n_f + 6)$ $2n_f^2 (21n_f + 1)$ $2n_f (5n_f + 2)$	[5.54]-[5.59] [5.34]-[5.46] [5.20]
	$\langle 3, 1 \rangle$	$\psi^1 \bar{\psi}^{12} + h.c.$ $F_L \psi^2 \bar{\psi}^{12} \bar{\psi} + h.c.$ $F_R^2 \psi^1 \bar{\psi}^2 + h.c.$ $\psi^2 \bar{\psi}^2 D^2 + h.c.$ $F_L \psi^1 \bar{\psi}^2 D^2 + h.c.$ $\psi^1 \bar{\psi}^2 D^3 + h.c.$	$4+26+29+4$ $0+24+24+0$ $0+0+8=0$ $0+12+15=0$ $0+0+8=0$ $0+0+4=0$	244 152 32 186 15 24	$\frac{1}{2} n_f^2 (382n_f^4 - 9n_f^2 + 2n_f + 21)$ $52n_f$ $2n_f (3n_f + 2)$ $\frac{3}{2} n_f^2 (486n_f^2 + 1)$ $12n_f^2$ $2n_f (5n_f + 1)$	[5.53]-[5.69] [5.54]-[5.66] [5.36] [5.38]-[5.42] [5.26] [5.11]
7	$\langle 2, 0 \rangle$	$\psi^4 \bar{\psi}^4 + h.c.$ $F_{LR} \psi^2 \bar{\psi}^4 + h.c.$	$0+6+5=0$ $0+0+4=0$	22 8	$\frac{5}{2} n_f^4 (10n_f^2 - 1)$ $2n_f (2n_f - 1)$	[5.35]-[5.27] [5.26]
	$\langle 1, 1 \rangle$	$\psi^1 \bar{\psi}^1 \bar{\psi}^3$ $\psi^1 \bar{\psi}^1 \bar{\psi}^5 D$	$0+6-10+0$ $0+0+2=0$	24 2	$14n_f^4$ $2n_f^2$	[5.35]-[5.27] [5.15]
8	$\langle 1, 0 \rangle$	$\psi^2 \bar{\psi}^2 + h.c.$	$0+0+2=0$	2	$n_f^2 + c_f$	[5.9]
Total		42	$6(12) + 6(4)$	1262	$8 + 234 + 345 - 0 (n_f = 1)$ $2342 + 42234 - 4(874 + 486) (n_f = 2)$	

Low Energy EFT

Dimension-5

Dim-5 operators		
N	(n, \bar{n})	Classes
3	(2,0)	$F_L^3 \psi_L^2 + h.c.$

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[Jenkins, Manohar, Stoffer, 2017]

Dim-6 operators				
N	(n, \bar{n})	Classes	N_{typo}	N_{term}
3	(3,0)	$F_L^3 + h.c.$	$2+0+0+0$	2
4	(2,0)	$\psi_L^4 + h.c.$	$14+12+8+2$	78
	(1,1)	$\psi_L^2 \psi_R^2$	$40+20+12+0$	84
	Total		5	164
			$56+32+20+2$	

Dim-6 operators

Dimension-7

Dim-7 operators				
N	(n, \bar{n})	Classes	N_{typo}	N_{term}
4	(3,0)	$F_L^2 \psi_L^2 + h.c.$	$16+0+4+0$	32
	(2,1)	$F_L^2 \psi_R^2 + h.c.$	$16+0+4+0$	24
		$\psi_L^3 \psi_R D + h.c.$	$50+32+22+$	
	Total		6	$82+32+30+$
				166

[Liao, Ma, Wang, 2020]

Dimension-8

[Li, Ren, Xiao, Yu, Zheng, 2020]

N	(n, \bar{n})	Subclasses	N_{typo}	N_{term}	N_{operator}	Equation
4	(1,0)	$F_L^4 + h.c.$	14	25	26	(4.16)
	(3,1)	$F_L^3 \psi_L^2 D + h.c.$ $\psi^2 \psi^2 D + h.c.$ $\psi^1 \psi^3 D + h.c.$ $F_L \psi^2 \psi^2 D^2 + h.c.$ $F_L^2 \psi^2 D^2 + h.c.$	22 18 16 16	22 18 32 12	226 $12n_f^3 + 12(n_f - 1)$ $3n_f^2$ 12	(4.21) (4.22), (4.23) (4.24)
	(2,2)	$F_L^2 F_R^2$ $F_L F_R \psi^2 D$ $\psi^2 \psi^2 D^2$ $F_R \psi^2 \psi^2 D^2 + h.c.$ $F_L F_R \psi^2 D^2$ $\psi^1 \psi^2 D^2$ $\psi^2 D^4$	14 27 17 16 5 7 1	17 35 $3n_f^2$ $\frac{1}{2}n_f(2n_f^2 + 10) + 6n_f^2$ 16n_f 4 $16n_f^2$ 2	(4.25) (4.26), (4.27) (4.28) (4.29) (4.30) (4.31), (4.32)	
5	(1,0)	$\psi_L^4 + h.c.$ $F_L^2 \psi_L^2 \phi + h.c.$ $F_L^2 \phi^2 + h.c.$	12 32 4	18 53 6	26 $32n_f^2$ 6	(4.33), (4.34), (4.35), (4.36)
	(2,1)	$F_L \psi^2 \psi^2 D + h.c.$ $\psi_L^2 \psi^2 \phi + h.c.$ $\psi^2 \psi^2 \phi^2 + h.c.$ $F_L \psi^2 \psi^2 D^2 + h.c.$ $\psi^2 \psi^2 D^2 + h.c.$ $F_L \psi^2 D^2 + h.c.$	80 32 32 108 38 6	172 32 $32n_f^2 - 2 + 24n_f^2$ $n_f^2(13n_f - 1) + n_f^2(29n_f + 3)$ $30n_f^2$ $30n_f^2$	(4.37)-(4.43), (4.48)-(4.52)	
	(3,0)	$\psi^2 \psi^2 \phi^2 + h.c.$ $F_L \psi^2 \phi^2 + h.c.$ $F_L^2 \phi^2 + h.c.$	12 16 8	18 22 10	32 $22n_f^2$ 10	(4.44), (4.45), (4.46)
	(1,1)	$\psi^2 \psi^2 \phi^2$ $\psi^2 \psi^2 D$ $\psi^2 D^2$	20 7 1	20 15 2	$2n_f^2(2n_f + 1)$ $11n_f^2$ 2	(4.53), (4.54)-(4.56)
6	(1,0)	$\psi^2 \phi^2 + h.c.$ $F_L \psi^2 \phi^2 + h.c.$ $F_L^2 \phi^2 + h.c.$	12 12 8	12 12 8	$32n_f^2(12n_f^2 + 32n_f^2 - 58n_f^2 + 129n_f + 6)$ $2n_f^2(2n_f + 1)$ $2n_f(2n_f + 2)$	(4.57)-(4.60)
	(2,1)	$\psi^2 \phi^2 D + h.c.$ $F_L \psi^2 \phi^2 D + h.c.$ $F_L^2 \psi^2 \phi^2 D + h.c.$ $\psi^2 \phi^2 D^2 + h.c.$ $F_L \psi^2 \phi^2 D^2 + h.c.$ $\psi^2 \phi^2 D^4 + h.c.$	44 12 12 12 12 4	4+26+29+4 12 $4 + 24 + 24 + 0$ $n_f^2(2n_f + 2)$ $n_f^2(3n_f + 2)$ $2n_f(3n_f + 1)$ $12n_f^2$	(4.61)-(4.69)	
	(3,0)	$\psi^2 \phi^2 + h.c.$ $F_L \psi^2 \phi^2 + h.c.$ $F_L^2 \phi^2 + h.c.$	12 12 8	12 12 8	$32n_f^2(12n_f^2 + 1)$ $2n_f^2(2n_f + 1)$ $2n_f(2n_f + 2)$	(4.74)-(4.76)
	(1,1)	$\psi^2 \phi^2 \phi^2 + h.c.$ $\psi^2 \phi^2 D$ $\psi^2 D^2$	12 8 2	12 8 2	$2n_f^2(12n_f^2 + 1)$ $12n_f^2$ $2n_f^2$	(4.77)-(4.82)
7	(2,0)	$\phi^4 \phi^4 + h.c.$ $F_L \phi^2 \phi^4 + h.c.$	8 8	8 8	$8 + 16 + 8 + 0$ $2n_f(2n_f - 1)$	(4.83)-(4.87)
	(1,1)	$\psi^2 \phi^2 \phi^2$ $\psi^2 \phi^2 D$	8 8	8 8	$14n_f^2$ $2n_f^2$	(4.88)-(4.91)
8	(1,0)	$\psi^2 \phi^2 + h.c.$	8	8	$n_f^2 + c_f$	(4.92)
	Total		42	6+12+16+4	126	$8 + 234 + 345 - 8(n_f - 1)$ $2942 + 42254 - 4(874 + 486)(n_f - 3)$
						80

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Jiang-Hao Yu

vSMEFT with Sterile Neutrino

Dimension-5

Dim-5 operators			
N (n, \bar{n})	Classes	N_{sp}	N_{sw}
3 (2, 0)	$F_L \psi^2 + \text{h.c.}$	0 + 0 + 2 + 0	2
4 (1, 0)	$\phi^2 \phi^2 + \text{h.c.}$	0 + 0 + 2 + 0	2
Total		0 + 0 + 4 + 0	4

2

[Aguila, Bar-Shalom, Soni, Wudka, 2009]

Dimension-6

Dim-6 operators			
N (n, \bar{n})	Classes	N_{sp}	N_{sw}
4 (2, 0)	$\psi^4 + \text{h.c.}$	4 + 2 + 0 + 2	14
	$F_L \psi^2 \phi + \text{h.c.}$	4 + 0 + 0 + 0	4
5 (1, 1)	$\psi^2 \psi^{12}$	10 + 2 + 0 + 0	12
	$\psi \psi^\dagger \psi^2 D$	3 + 0 + 0 + 0	3
5 (1, 0)	$\psi^2 \phi^3 + \text{h.c.}$	2 + 0 + 0 + 0	2
	Total	8	23 + 4 + 0 + 2

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Dimension-7

N (n, \bar{n})	Classes	N_{sp}	N_{sw}
4 (2, 1)	$F_L^2 \psi^2 + \text{h.c.}$	0 + 0 + 5 + 0	6
	$F_L^2 \phi^2 + \text{h.c.}$	0 + 5 + 5 + 0	6
5 (1, 1)	$\phi^2 \psi^2 D + \text{h.c.}$	0 + 4 + 20 + 0	24
	$F_L \psi \psi^\dagger \phi D + \text{h.c.}$	0 + 0 + 5 + 0	8
5 (1, 0)	$\psi^2 \phi^2 D^2 + \text{h.c.}$	0 + 0 + 4 + 0	6
	Total	18	0 + 10 + 56 + 0

80

[Bhattacharya, Wudka, 2016]

[Liao, Ma, 2017]

Dimension-8

[Li, Ren, Xiao, Yu, Zheng, 2021]

N (n, \bar{n})	Classes	N_{sp}	N_{sw}
4 (3, 1)	$\psi^4 D^2 + \text{h.c.}$	4 + 0 + 2 + 2	22
	$F_L \psi^2 \phi D^2 + \text{h.c.}$	4 + 0 + 0 + 0	8
(2, 2)	$F_L F_R \psi \psi^\dagger D$	3 + 0 + 0 + 0	3
	$\psi^2 \psi^{12} D^2$	10 + 2 + 0 + 0	24
5 (3, 0)	$F_L \psi^2 \phi^2 + \text{h.c.}$	4 + 0 + 0 + 0	4
	$\psi \psi^\dagger \phi^2 D^2$	3 + 0 + 0 + 0	4
5 (2, 1)	$F_L \psi^4 + \text{h.c.}$	10 + 4 + 0 + 2	50
	$F_L^2 \psi^2 \phi + \text{h.c.}$	8 + 0 + 0 + 0	12
(2, 1)	$F_L \psi^2 \psi^{12} + \text{h.c.}$	42 + 12 + 0 + 0	58
	$F_L^2 \psi^{12} \phi + \text{h.c.}$	8 + 0 + 0 + 0	8
(1, 1)	$\psi^3 \psi^\dagger \phi D + \text{h.c.}$	24 + 6 + 0 + 2	108
	$F_L \psi \psi^\dagger \phi^2 D + \text{h.c.}$	12 + 0 + 0 + 0	16
6 (2, 0)	$\psi^4 \phi^2 + \text{h.c.}$	8 + 2 + 0 + 2	30
	$F_L \psi^2 \phi^3 + \text{h.c.}$	4 + 0 + 0 + 0	6
(1, 1)	$\psi^2 \psi^{12} \phi^2$	16 + 4 + 0 + 2	28
	$\psi \psi^\dagger \phi^2 D$	3 + 0 + 0 + 0	3
7 (1, 0)	$\psi^2 \phi^3 + \text{h.c.}$	2 + 0 + 0 + 0	2
	Total	31	167 + 30 + 2 + 10

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Dimension-9

[Li, Ren, Xiao, Yu, Zheng, 2021]

N (n, \bar{n})	Classes	N_{sp}	N_{sw}
4 (4, 1)	$F_L^2 \psi^2 \phi^2 + \text{h.c.}$	0 + 6 + 0 + 0	12
	$F_L^2 F_R \psi^2 D^2 + \text{h.c.}$	0 + 6 + 0 + 0	6
(3, 1)	$F_L^2 \psi^{12} D^2 + \text{h.c.}$	0 + 6 + 0 + 0	6
	$F_L^2 \psi^{12} \phi^2 + \text{h.c.}$	0 + 6 + 0 + 0	6
(2, 2)	$\psi^2 \psi^{12} D^3 + \text{h.c.}$	4 + 20 + 0 + 3	48
	$F_L \psi^2 \psi^2 D^2 + \text{h.c.}$	0 + 8 + 0 + 0	15
(2, 1)	$\psi^2 \psi^2 D^2 + \text{h.c.}$	0 + 1 + 0 + 0	8
	Total	13	0 + 10 + 0 + 0
(3, 1)	$F_L^2 \psi^2 + \text{h.c.}$	0 + 4 + 0 + 0	4
	$F_L^2 \psi^2 D + \text{h.c.}$	10 + 42 + 0 + 0	222
(2, 2)	$F_L^2 \psi \psi^\dagger \phi D + \text{h.c.}$	0 + 10 + 0 + 3	30
	$\psi^2 \psi^2 D^2 + \text{h.c.}$	9 + 10 + 0 + 3	190
(2, 1)	$F_L \psi^2 \psi^2 D^2 + \text{h.c.}$	0 + 8 + 0 + 0	42
	$F_L F_R \psi^2 + \text{h.c.}$	0 + 12 + 0 + 3	33
(1, 1)	$F_L \psi^2 \psi^2 D + \text{h.c.}$	10 + 12 + 0 + 0	166
	$F_L \psi \psi^\dagger \phi D + \text{h.c.}$	0 + 10 + 0 + 3	30
(1, 0)	$\psi^2 \psi^2 \phi D^2 + \text{h.c.}$	4 + 22 + 0 + 3	210
	$F_L \psi^2 \psi^2 D^2 + \text{h.c.}$	0 + 8 + 0 + 0	24
(0, 1)	$\psi \psi^\dagger \phi^2 D^2 + \text{h.c.}$	0 + 2 + 0 + 0	20
	$\psi^6 + \text{h.c.}$	0 + 10 + 0 + 2	100
(0, 0)	$F_L \psi^2 \phi + \text{h.c.}$	6 + 26 + 0 + 3	110
	$F_L^2 \psi^2 \phi^2 + \text{h.c.}$	0 + 12 + 0 + 3	35
(2, 1)	$\psi^4 \phi^2 + \text{h.c.}$	0 + 106 + 0 + 0	474
	$F_L \psi^2 \psi^{12} \phi + \text{h.c.}$	24 + 116 + 0 + 0	176
(1, 1)	$F_L \psi^2 \psi^{12} \phi^2 + \text{h.c.}$	0 + 10 + 0 + 3	33
	$\psi^2 \psi^2 \phi^2 D + \text{h.c.}$	10 + 14 + 0 + 0	268
(1, 0)	$F_L \psi^2 \psi^{12} D + \text{h.c.}$	0 + 6 + 0 + 0	32
	$\psi^2 \psi^2 D^2 + \text{h.c.}$	0 + 4 + 0 + 0	20
(0, 1)	$\psi^4 \phi^2 + \text{h.c.}$	2 + 12 + 0 + 3	28
	$F_L \psi^2 \phi^4 + \text{h.c.}$	0 + 6 + 0 + 0	6
(0, 0)	$\psi^2 \psi^{12} \phi^2 + \text{h.c.}$	4 + 22 + 0 + 3	36
	$\psi \psi^\dagger \phi^2 D + \text{h.c.}$	0 + 2 + 0 + 0	3

1358

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Exercise: Real Scalar EFT

Question: how many independent operators at dim-6

$$\mathcal{L}_{\text{EFT}} = \frac{1}{2} \left[(\partial_\mu \phi)^2 - m^2 \phi^2 \right] - C_4 \frac{\phi^4}{4!} - \frac{C_6}{M^2} \frac{\phi^6}{6!} + \mathcal{O}(M^{-4})$$

$$O_6 \equiv \phi^6, \quad \hat{O}_6 \equiv (\square \phi)^2, \quad \tilde{O}_6 \equiv \phi^3 \square \phi, \quad \tilde{O}'_6 \equiv \phi^2 \square \phi^2, \quad \tilde{O}''_6 \equiv \phi^2 \partial_\mu \phi \partial_\mu \phi, \quad \dots$$

Use Leibniz rule + integration by parts:

$$\phi^2 \partial_\mu \phi \partial_\mu \phi = -2\phi \partial_\mu \phi \partial_\mu \phi \phi - \phi^3 \square \phi \quad \Rightarrow \quad \tilde{O}''_6 = -\frac{1}{3} \phi^3 \square \phi = -\frac{1}{3} \tilde{O}_6$$

$$\phi^2 \square \phi^2 = 2\phi^2 \partial_\mu (\phi \partial_\mu \phi) = 2\phi^3 \square \phi + 2\phi^2 (\partial_\mu \phi)^2 \quad \Rightarrow \quad \tilde{O}'_6 = 2\tilde{O}_6 + 2\tilde{O}''_6 = \frac{4}{3} \tilde{O}_6$$

Use equations of motion: $\square \phi = -m^2 \phi - \frac{C_4}{6} \phi^3 + \mathcal{O}(M^{-2})$

$$\tilde{O}_6 \equiv \phi^3 \square \phi = -m^2 \phi^4 - \frac{C_4}{6} \phi^6 = -m^2 O_4 - \frac{C_4}{6} O_6$$

This is relevant only if
we want to keep track
of dimension-8 operators

$$\hat{O}_6 \equiv (\square \phi)^2 = m^4 \phi^2 + \frac{m^2 C_4}{3} \phi^4 + \frac{C_4^2}{36} \phi^6 = m^4 O_2 + \frac{m^2 C_4}{3} O_4 + \frac{C_4^2}{36} O_6$$

$O_2 \equiv \phi^2$
 $O_4 \equiv \phi^4$

Exercise: Real Scalar EFT

How many independent operators of the form $\partial^{2n}\phi^4$?

$$\partial^{2n}\phi^2, \partial^{2n}\phi^3, \partial^{2n}\phi^4, \dots$$

$2n$	0	2	4	6	8	10	12	14	16
# independent $\partial^{2n}\phi^4$ operators	1	0	1	1	1	1	2	1	2

1

$$s+t+u=0$$

$$s^2+t^2+u^2$$

$$s^3+t^3+u^3 \sim stu$$

$$(s^2+t^2+u^2)^2$$

$$stu (s^2+t^2+u^2)$$

$$(s^2+t^2+u^2)^3 \& (stu)^2$$

$$stu (s^2+t^2+u^2)^2$$

$$(s^2+t^2+u^2)^4 \& (stu)^2(s^2+t^2+u^2)$$

Magic things happen?!

Application: Dim-6 SMEFT

Disclaimer for this section: all materials not original

Warsaw Basis Operators

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2 \varphi^3$		$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger \varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger \varphi)(\bar{l}_p e_r \varphi)$	Q_{ll}	$(\bar{l}_p \gamma_\mu l_r)(\bar{l}_s \gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p \gamma_\mu e_r)(\bar{e}_s \gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p \gamma_\mu l_r)(\bar{e}_s \gamma^\mu e_t)$
$Q_{\bar{G}}$	$f^{ABC} \tilde{G}_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\square}$	$(\varphi^\dagger \varphi) \square (\varphi^\dagger \varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p u_r \tilde{\varphi})$	$Q_{qq}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p \gamma_\mu u_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p \gamma_\mu l_r)(\bar{u}_s \gamma^\mu u_t)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$(\varphi^\dagger D^\mu \varphi)^*$ $(\varphi^\dagger D_\mu \varphi)$	$Q_{d\varphi}$	$(\varphi^\dagger \varphi)(\bar{q}_p d_r \varphi)$	$Q_{qq}^{(3)}$	$(\bar{q}_p \gamma_\mu \tau^I q_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar{d}_p \gamma_\mu d_r)(\bar{d}_s \gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{\bar{W}}$	$\varepsilon^{IJK} \tilde{W}_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$					$Q_{lq}^{(1)}$	$(\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p \gamma_\mu e_r)(\bar{u}_s \gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p \gamma_\mu q_r)(\bar{e}_s \gamma^\mu e_t)$
$X^2 \varphi^2$		$\psi^2 X \varphi$		$\psi^2 \varphi^2 D$		$Q_{lq}^{(3)}$	$(\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{u}_s \gamma^\mu u_t)$
$Q_{\varphi G}$	$\varphi^\dagger \varphi G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi)(\bar{l}_p \gamma^\mu l_r)$					$Q_{qu}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A q_r)(\bar{u}_s \gamma^\mu T^A u_t)$
$Q_{\varphi \bar{G}}$	$\varphi^\dagger \varphi \tilde{G}_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar{l}_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu^I \varphi)(\bar{l}_p \tau^I \gamma^\mu l_r)$					$Q_{qd}^{(1)}$	$(\bar{q}_p \gamma_\mu q_r)(\bar{d}_s \gamma^\mu d_t)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar{q}_p \sigma^{\mu\nu} T^A u_r) \tilde{\varphi} G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi)(\bar{e}_p \gamma^\mu e_r)$					$Q_{qd}^{(8)}$	$(\bar{q}_p \gamma_\mu T^A u_r)(\bar{d}_s \gamma^\mu T^A d_t)$
$Q_{\varphi \bar{W}}$	$\varphi^\dagger \varphi \tilde{W}_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tau^I \tilde{\varphi} W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi)(\bar{q}_p \gamma^\mu q_r)$						
$Q_{\varphi B}$	$\varphi^\dagger \varphi B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar{q}_p \sigma^{\mu\nu} u_r) \tilde{\varphi} B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu^I \varphi)(\bar{q}_p \tau^I \gamma^\mu q_r)$						
$Q_{\varphi \bar{B}}$	$\varphi^\dagger \varphi \tilde{B}_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar{q}_p \sigma^{\mu\nu} T^A d_r) \varphi G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi)(\bar{u}_p \gamma^\mu u_r)$						
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i \overset{\leftrightarrow}{D}_\mu \varphi)(\bar{d}_p \gamma^\mu d_r)$						
$Q_{\varphi \bar{W}B}$	$\varphi^\dagger \tau^I \varphi \tilde{W}_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar{q}_p \sigma^{\mu\nu} d_r) \varphi B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\tilde{\varphi}^\dagger D_\mu \varphi)(\bar{u}_p \gamma^\mu d_r)$						
$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$						B-violating					
Q_{ledq}						Q_{duq}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(d_p^\alpha)^T C u_r^\beta] [(q_s^\gamma)^T C l_t^k]$				
$Q_{quqd}^{(1)}$						Q_{qqu}	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(u_s^\gamma)^T C e_t]$				
$Q_{quqd}^{(8)}$						$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma} \varepsilon_{jk} \varepsilon_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$				
$Q_{lequ}^{(1)}$						$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma} (\tau^I \varepsilon)_{jk} (\tau^I \varepsilon)_{mn} [(q_p^{\alpha j})^T C q_r^{\beta k}] [(q_s^{\gamma m})^T C l_t^n]$				
$Q_{lequ}^{(3)}$						Q_{duu}	$\varepsilon^{\alpha\beta\gamma} [(d_p^\alpha)^T C u_r^\beta] [(u_s^\gamma)^T C e_t]$				

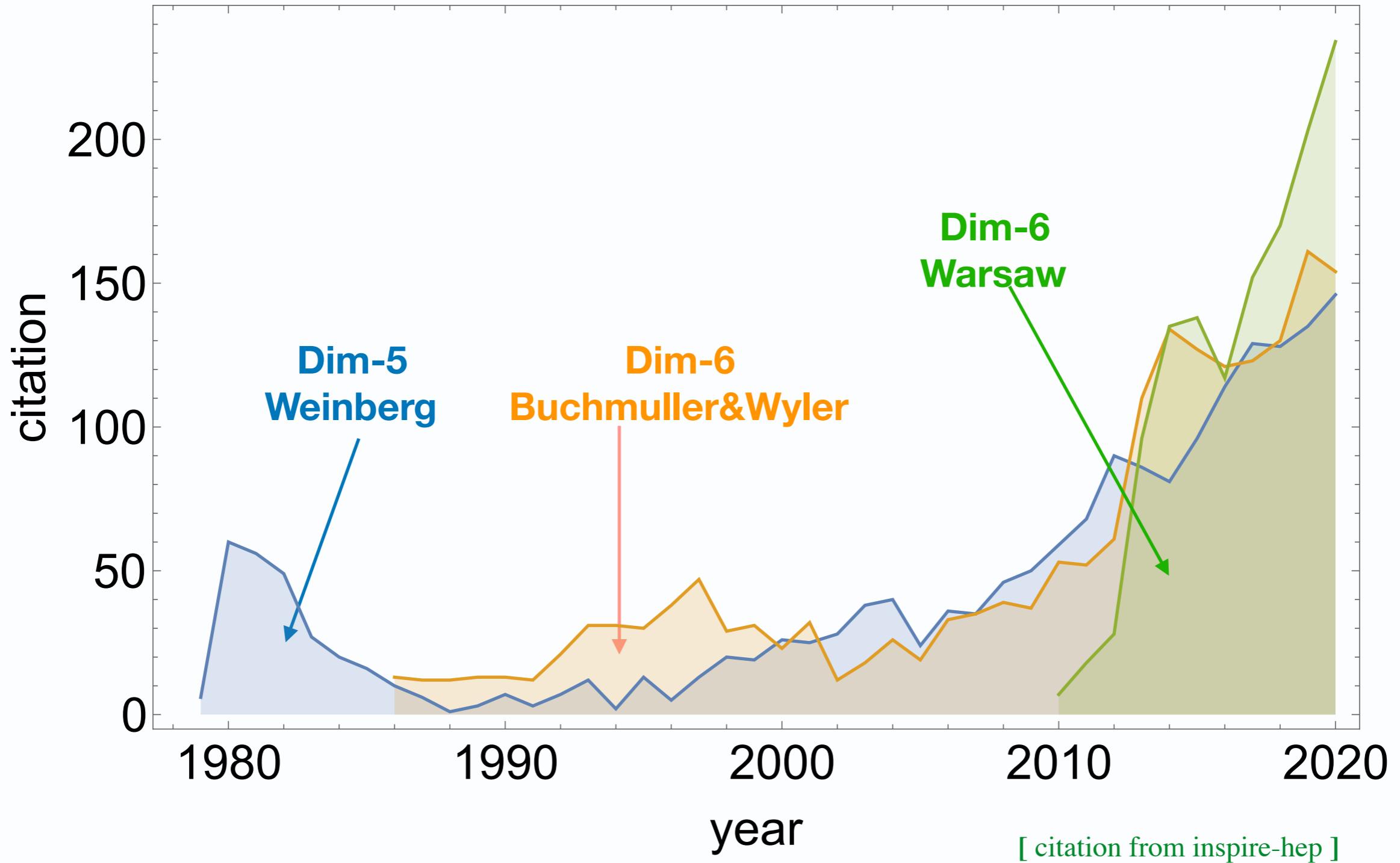
59 independent operators

real parameters (degrees of freedom)

76: flavor universal All fermion generations have the same coefficient

2499: flavor general Independent coefficient for all flavor combinations

Many Applications



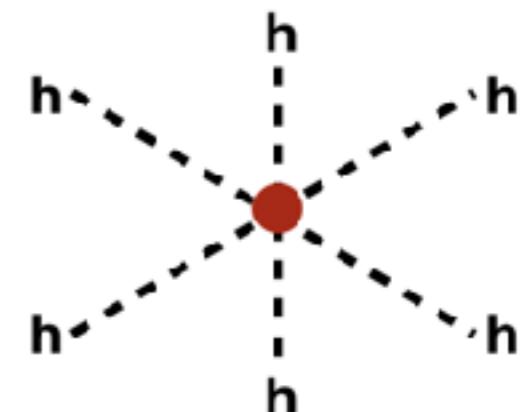
Operator Classes

Dimension-6 operators of the SMEFT:	Interaction	Impact
$X^3 : \epsilon_{IJK} W_{\mu\nu}^I W^{J,\nu\rho} W_{\rho}^{K,\mu}$	gauge boson self-coupling	diboson
$H^6 : (\varphi^\dagger \varphi)^3$	Higgs potential, self-coupling	di-Higgs
$\psi^2 H^3 : (\varphi^\dagger \varphi) (\bar{q}_i u_j \tilde{\varphi})$	Higgs-fermion (Yukawa)	ttH, H → bb
$\psi^2 H^2 D : (\varphi^\dagger \overset{\leftrightarrow}{D}_\mu \varphi)(\bar{q}_i \gamma^\mu q_j)$	gauge-fermion (Z,W)	Z,W prod.
$X^2 H^2 : (\varphi^\dagger \varphi) G_{\mu\nu}^a G_a^{\mu\nu}$	gauge-Higgs	ggH, H → VV
$H^4 D^2 : (\varphi^\dagger D^\mu \varphi)^* (\varphi^\dagger D^\mu \varphi)$	Higgs-Z	m_Z (LEP)
$\psi^2 X H : (\bar{q}_i \sigma^{\mu\nu} u_j \tilde{\varphi}) B_{\mu\nu}$	dipole	ffV, ffVH
$\psi^4 : (\bar{q}_i \gamma^\mu q_j)(\bar{q}_k \gamma_\mu q_l)$	SM gauge group singlets	ffff scattering

New Interactions

1. New vertices

e.g. $\frac{1}{\Lambda^2} |H|^6 \rightarrow \frac{h^6}{8\Lambda^2} + \frac{3vh^5}{4\Lambda^2} + \dots$



in particular, violation of
global symmetries of the SM

e.g. $\frac{1}{\Lambda^2} u^c d^c u^c e^c$

2. New Lorentz structures

e.g. $\frac{1}{\Lambda^2} |H|^2 W_{\mu\nu}^i W_{\mu\nu}^i \rightarrow \frac{2v}{\Lambda^2} h W_{\mu\nu}^+ W_{\mu\nu}^- + \dots$

in addition to

$$\frac{h}{v} 2m_W^2 W_\mu^+ W_\mu^- \quad \text{present in the SM}$$

in particular, violation of CP

e.g. $\frac{1}{\Lambda^2} \epsilon_{ijk} W_{\mu\nu}^i W_{\nu\rho}^j \tilde{W}_{\rho\mu}^k \rightarrow -\frac{3i \sin \theta_W}{2\Lambda^2} W_{\mu\nu}^+ W_{\nu\rho}^- \tilde{F}_{\rho\mu} + \dots$

Modified Interaction Strength

There are 3 ways higher-dimensional operators may modify SM interaction strength

1. Directly: after electroweak symmetry breaking, an operator contributes to a gauge or Yukawa interaction already present in the SM
2. Indirectly: after electroweak symmetry breaking, an operator contributes to the kinetic term of a SM field, thus effectively shifting the strength of all interactions of that field
3. Stealthily: after electroweak symmetry breaking, an operator contributes to an experimental observable from which some SM parameter is extracted

Modified Interaction: Directly

Example: $\frac{i}{\Lambda^2} \bar{e}_R \gamma^\mu e_R (H^\dagger D_\mu H - D_\mu H^\dagger H)$

After electroweak symmetry breaking $i(H^\dagger D_\mu H - D_\mu H^\dagger H) \rightarrow -\frac{v^2}{2} \sqrt{g_L^2 + g_Y^2} Z_\mu + \dots$

$$\frac{ic_{He}}{\Lambda^2} \bar{e}_R \gamma^\mu e_R (H^\dagger D_\mu H - D_\mu H^\dagger H) \rightarrow -c_{He} \frac{v^2 \sqrt{g_L^2 + g_Y^2}}{2\Lambda^2} \bar{e}_R \gamma^\mu e_R Z_\mu$$

This adds up to the weak interaction in the SM

$$\sqrt{g_L^2 + g_Y^2} (T_f^3 - \sin^2 \theta_W Q_f + \delta g^{Zf}) \bar{f} \gamma^\mu f Z_\mu$$

$$\boxed{\delta g_R^{Ze} = -c_{He} \frac{v^2}{2\Lambda^2}}$$

Thus c_{He} can be constrained, e.g., from LEP-1 Z-pole data

Modified Interaction: Indirectly

Example: $(H^\dagger H) \square (H^\dagger H)$

This contributes to the kinetic term of the Higgs boson

$$\frac{c_{H\square}}{\Lambda^2} (H^\dagger H) \square (H^\dagger H) \rightarrow -\frac{c_{H\square} v^2}{\Lambda^2} (\partial_\mu h)^2$$

Together with the SM kinetic term:

$$\mathcal{L}_{\text{SMEFT}} \supset \frac{1}{2} (\partial_\mu h)^2 \left(1 - \frac{2c_{H\square} v^2}{\Lambda^2} \right)$$

To restore canonical normalization, we need to rescale the Higgs boson field:

$$h \rightarrow h \left(1 + \frac{c_{H\square} v^2}{\Lambda^2} \right)$$

This restores canonical normalization of the Higgs boson field,
up to terms of order $1/\Lambda^4$, which we ignore here

Modified Interaction: Indirectly

$$h \rightarrow h \left(1 + \frac{c_H \square v^2}{\Lambda^2} \right)$$

After this rescaling, the dimension-6 contribution vanishes from the Higgs boson kinetic term

However, it resurfaces in all Higgs boson couplings present in the SM !

$$\frac{h}{v} [2m_W^2 W_\mu^+ W_\mu^- + m_Z^2 Z_\mu Z_\mu] \rightarrow \frac{h}{v} \left(1 + \frac{c_H \square v^2}{\Lambda^2} \right) [2m_W^2 W_\mu^+ W_\mu^- + m_Z^2 Z_\mu Z_\mu]$$

$$\frac{h}{v} m_f \bar{f} f \rightarrow \frac{h}{v} \left(1 + \frac{c_H \square v^2}{\Lambda^2} \right) m_f \bar{f} f$$

Hence, the Higgs boson interaction strength predicted by the SM is universally shifted

LHC measurements of the Higgs signal strength provide a bound on the Wilson coefficient

$$\mu = 1.09 \pm 0.11$$



$$\frac{c_H \square v^2}{\Lambda^2} = 0.09 \pm 0.11$$

or, equivalently

$$\frac{c_H \square}{\Lambda^2} = \frac{1}{(820 \text{GeV})^2} \pm \frac{1}{(740 \text{GeV})^2}$$

Higgs measurements only probe new physics scale of order a TeV

Modified Interaction: Stealth

Consider the dimension-6 operator

$$|H^\dagger D_\mu H|^2$$

After electroweak symmetry breaking:

$$\frac{c_{HD}}{\Lambda^2} |H^\dagger D_\mu H|^2 \rightarrow \frac{c_{HD} v^2}{2\Lambda^2} \frac{(g_L^2 + g_Y^2)v^2}{8} Z_\mu Z_\mu + \dots$$

Thus it modifies the Z boson mass:

$$m_Z^2 = \frac{(g_L^2 + g_Y^2)v^2}{4} \left(1 + \frac{c_{HD}v^2}{2\Lambda^2} \right)$$

We have this very precise $O(10^{-4})$ measurement of the Z boson mass

$$m_Z = (91.1876 \pm 0.0021) \text{ GeV}$$

From which we find the very stringent constraint

$$\frac{|c_{HD}|}{\Lambda^2} \leq \frac{1}{(26 \text{ TeV})^2}$$

Modified Interaction: Stealth

Consider the dimension-6 operator

$$|H^\dagger D_\mu H|^2$$

After electroweak symmetry breaking:

$$\frac{c_{HD}}{\Lambda^2} |H^\dagger D_\mu H|^2 \rightarrow \frac{c_{HD} v^2}{2\Lambda^2} \frac{(g_L^2 + g_Y^2)v^2}{8} Z_\mu Z_\mu + \dots$$

Thus it modifies the Z boson mass:

$$m_Z^2 = \frac{(g_L^2 + g_Y^2)v^2}{4} \left(1 + \frac{c_{HD}v^2}{2\Lambda^2} \right)$$

We cannot use the Z-boson mass measurement to constrain new physics because, it is one of the inputs to determine the electroweak parameters of the SM

In the SM: $G_F = \frac{1}{\sqrt{2}v^2}$

$$\alpha = \frac{g_L^2 g_Y^2}{4\pi(g_L^2 + g_Y^2)}$$

$$m_Z^2 = \frac{(g_L^2 + g_Y^2)v^2}{4}$$



$$g_L = 0.6485$$

$$g_Y = 0.3580$$

$$v = 246.22 \text{ GeV}$$

with very small errors

Modified Interaction: Stealth

$|H^\dagger D_\mu H|^2$ In the presence of our dimension-6 operators, the relation between electroweak couplings and observables is disrupted

$$G_F = \frac{1}{\sqrt{2}v^2} \quad \alpha = \frac{g_L^2 g_Y^2}{4\pi(g_L^2 + g_Y^2)} \quad m_Z^2 = \frac{(g_L^2 + g_Y^2)v^2}{4} \left(1 + \frac{c_{HD}v^2}{2\Lambda^2}\right)$$

Now we cannot assign numerical values to the electroweak parameters, because they depend on c_{HD}

A useful trick is to get rid of the dimension-6 pollution in the input equations by redefining the SM electroweak parameters

$$g_L \rightarrow \tilde{g}_L \left(1 - \frac{c_{HD}g_L^2v^2}{4(g_L^2 - g_Y^2)\Lambda^2}\right) \quad g_Y \rightarrow \tilde{g}_Y \left(1 + \frac{c_{HD}g_Y^2v^2}{4(g_L^2 - g_Y^2)\Lambda^2}\right)$$

For the twiddle electroweak parameter, we can now assign numerical values

$$G_F = \frac{1}{\sqrt{2}v^2}$$

$$\alpha = \frac{\tilde{g}_L^2 \tilde{g}_Y^2}{4\pi(\tilde{g}_L^2 + \tilde{g}_Y^2)}$$

$$m_Z^2 = \frac{(\tilde{g}_L^2 + \tilde{g}_Y^2)v^2}{4}$$



$$\tilde{g}_L = 0.6485$$

$$\tilde{g}_Y = 0.3580$$

$$v = 246.22 \text{ GeV}$$

same as in the SM

Modified Interaction: Stealth

Z mass cannot be used to constrain new physics, because it was already used to set numerical values for the twiddle electroweak parameter

But new physics emerges now in other observables, e.g. in the W mass

$$m_W = \frac{g_L v}{2} = \frac{\tilde{g}_L v}{2} \left(1 - \frac{c_{HD} g_L^2 v^2}{4(g_L^2 - g_Y^2) \Lambda^2} \right) = \frac{\tilde{g}_L v}{2} \left(1 - \frac{c_{HD} \tilde{g}_L^2 v^2}{4(\tilde{g}_L^2 - \tilde{g}_Y^2) \Lambda^2} \right)$$

We can now use the experimental measurement of the W mass

$$m_W = (80.379 \pm 0.012) \text{ GeV}$$

to constrain the Wilson coefficients

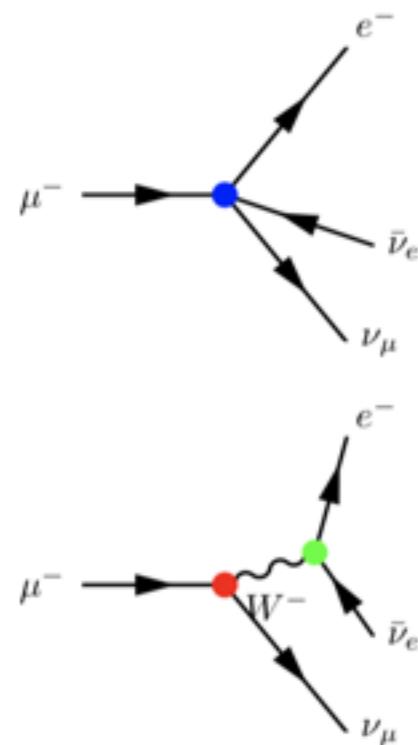
$$-\frac{1}{(7 \text{ TeV})^2} \leq \frac{c_{HD}}{\Lambda^2} \leq -\frac{1}{(12 \text{ TeV})^2} \quad \text{at 1 sigma}$$

Numerically very different constraint than what one would (incorrectly) obtain from Z mass!

Weak Decay, Again

Muon decay data gives: $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2} \rightarrow v = 246 \text{ GeV}$

- EFT contribution: A $[\mathcal{O}_u]^{ijkl} = [\bar{l}^{(i)} \gamma^\mu l^{(j)}] [\bar{l}^{(k)} \gamma_\mu l^{(l)}]$
B,C $[\mathcal{O}_{\varphi l}^{(3)}]^{ij} = [\varphi^\dagger \tau_k \overleftrightarrow{D}_\mu \varphi] [\bar{l}^{(i)} \tau^k \gamma^\mu l^{(j)}], \quad i = 1, 2.$
- A contains the O_F up to a Fierz transformation $[\mathcal{O}_u]^{1212} = \frac{1}{2} (\bar{e} \gamma^\mu (1 - \gamma_5) \nu_e) (\bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu) + \dots$
- B,C shifts W couplings to leptons after EWSB $[\mathcal{O}_u]^{1221} = \frac{1}{4} (\bar{e} \gamma^\mu (1 - \gamma_5) \nu_\mu) (\bar{\nu}_e \gamma_\mu (1 - \gamma_5) \mu) + \dots$
 $-i \frac{g}{\sqrt{2}} \rightarrow -i \frac{g}{\sqrt{2}} \left[1 + [C_{\varphi l}^{(3)}]^{ii} \frac{v^2}{\Lambda^2} \right]$
 $[\mathcal{O}_u]^{2112} = \frac{1}{4} (\bar{e} \gamma^\mu (1 - \gamma_5) \nu_\mu) (\bar{\nu}_e \gamma_\mu (1 - \gamma_5) \mu) + \dots$



$$\frac{G_F}{\sqrt{2}} = \frac{1}{2v^2} \left[1 + \frac{v^2}{\Lambda^2} [C_{\varphi l}^{(3)}]^{11} \right] \left[1 + \frac{v^2}{\Lambda^2} [C_{\varphi l}^{(3)}]^{22} \right] - \frac{1}{4} \frac{1}{\Lambda^2} (2 [c_u]^{1212} + [c_u]^{1221} + [c_u]^{2112})$$

$$v_0^2 = \frac{1}{\sqrt{2} G_f} \quad \text{New relation between Higgs vev and Fermi constant}$$

$$v = v_0 \left(1 + \left([C_{\varphi l}^{(3)}]^{11} + [C_{\varphi l}^{(3)}]^{22} - [c_u]^{1212} - [c_u]^{1221} \right) \frac{v_0^2}{\Lambda^2} \right)^{\frac{1}{2}}$$

Propagate to all observables that depend on v!
(expanded to order $1/\Lambda^2$)

Theory Input Parameters

The SMEFT Wilson coefficients are **theory input parameters**

- In addition & analogous to SM input parameters $\{\alpha_s, \alpha_{EW}, G_F, m_Z, m_H, m_{f_i}\}$

$$\bar{g}' = g' \left(1 + C_{\Phi B} \frac{v_T^2}{\Lambda^2} \right) \quad , \quad \bar{g} = g \left(1 + C_{\Phi W} \frac{v_T^2}{\Lambda^2} \right) \quad , \quad \bar{g}_s = g_s \left(1 + C_{\Phi G} \frac{v_T^2}{\Lambda^2} \right)$$

$$\tan \bar{\theta}_W = \frac{\bar{g}'}{\bar{g}} + \frac{v_T^2}{2\Lambda^2} \left(1 - \frac{\bar{g}'^2}{\bar{g}^2} \right) C_{\Phi WB}$$

$$M_W^2 = \frac{v_T^2}{4} \bar{g}^2 \quad , \quad M_Z^2 = \frac{v_T^2}{4} (\bar{g}^2 + \bar{g}'^2) + \frac{\bar{v}_T^4}{8\Lambda^2} \{ (\bar{g}^2 + \bar{g}'^2) C_{\Phi D} + 4 \bar{g} \bar{g}' C_{\Phi WB} \}$$

$$\bar{e} = \bar{g} \sin \bar{\theta}_W \left\{ 1 - \cot \bar{\theta}_W \frac{\bar{v}_T^2}{2\Lambda^2} C_{\Phi WB} \right\} \quad , \quad \bar{g}_Z = \frac{\bar{e}}{\sin \bar{\theta}_W \cos \bar{\theta}_W} \left\{ 1 + \frac{\bar{g}^2 + \bar{g}'^2}{2\bar{g} \bar{g}'} \frac{\bar{v}_T^2}{\Lambda^2} C_{\Phi WB} \right\}$$

$$\rho \equiv \frac{\bar{g}^2 M_Z^2}{\bar{g}_Z^2 M_W^2} = 1 + \frac{\bar{v}_T^2}{2\Lambda^2} C_{\Phi D}$$

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- $V(H) = \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 - \frac{C_\Phi}{\Lambda^2} \left(\Phi^\dagger \Phi \right)^3 \quad \rightarrow \quad v_T = \left(1 + \frac{3C_\Phi}{8\lambda} \frac{v^2}{\Lambda^2} \right) v$

- Normalization:** $H \rightarrow \left(1 + c_H^{\text{kin}} \right) H$, $c_H^{\text{kin}} = \left(C_{\Phi\square} - \frac{1}{4} C_{\Phi D} \right) \frac{v^2}{\Lambda^2}$

$$\rightarrow M_H^2 = 2\lambda v_T^2 \left(1 - \frac{3C_\Phi}{2\lambda} \frac{v^2}{\Lambda^2} + 2c_H^{\text{kin}} \right)$$

- Flavour:** $\mathcal{Y}_f \propto M_f$

$$\mathcal{L} = -\bar{Q}_L^r [Y_d]_{rs} \Phi d_R^s + \dots \quad \rightarrow \quad [M_f]_{rs} = \frac{v_T}{\sqrt{2}} \left([Y_\psi]_{rs} - \frac{v^2}{2\Lambda^2} [C_{f\Phi}]_{rs} \right)$$

$$[\mathcal{Y}_f]_{rs} = \frac{1}{v_T} [M_f]_{rs} \left(1 + c_H^{\text{kin}} \right) - \frac{v^2}{\Lambda^2} [C_{f\Phi}]_{rs}$$

- Fermi:** $\frac{4G_F}{\sqrt{2}} = \frac{2}{v_T^2} + \frac{1}{\Lambda^2} \left(2 [C_{\Phi I}^{(3)}]_{ee} + 2 [C_{\Phi I}^{(3)}]_{\mu\mu} - [C_{ll}]_{\mu e, e\mu} - [C_{ll}]_{e\mu, \mu e} \right)$

Theory Input Parameters

The SMEFT Wilson coefficients are **theory input parameters**

- In addition & analogous to SM input parameters $\{\alpha_s, \alpha_{EW}, G_F, m_Z, m_H, m_{f_i}\}$

- $V(H) = \lambda \left(\Phi^\dagger \Phi - \frac{v^2}{2} \right)^2 - \frac{C_\Phi}{\Lambda^2} \left(\Phi^\dagger \Phi \right)^3 \quad \rightarrow \quad v_T = \left(1 + \frac{3C_\Phi}{8\lambda} \frac{v^2}{\Lambda^2} \right) v$

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$$\rightarrow M_H^2 = 2\lambda v_T^2 \left(1 - \frac{3C_\Phi}{2\lambda} \frac{v^2}{\Lambda^2} + 2c_H^{\text{kin}} \right)$$

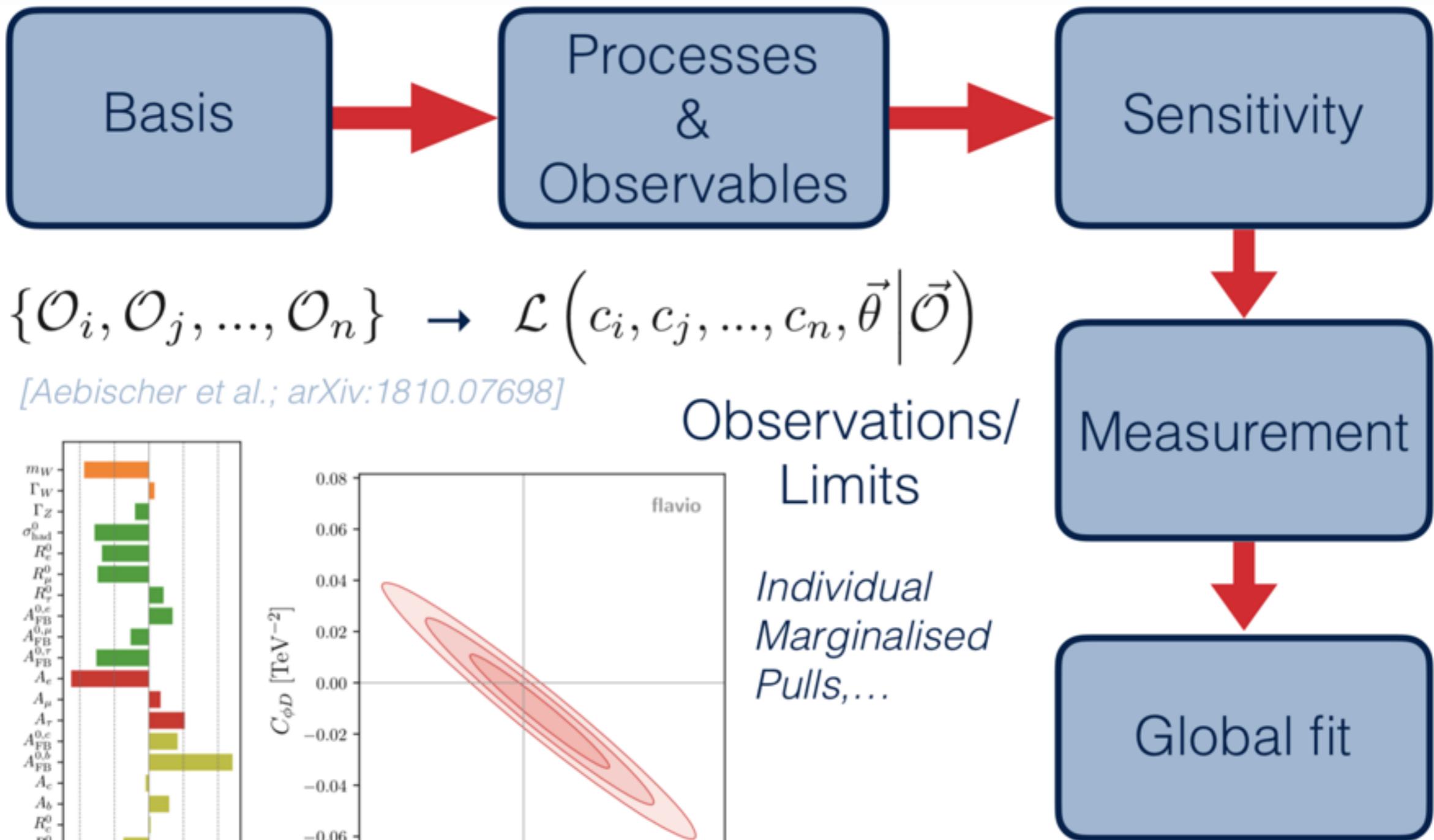
- Flavour:** $\mathcal{Y}_f \approx M_f$

$$\mathcal{L} = -\bar{Q}_L^r [Y_d]_{rs} \Phi d_R^s + \dots \quad \rightarrow \quad [M_f]_{rs} = \frac{v_T}{\sqrt{2}} \left([Y_\psi]_{rs} - \frac{v^2}{2\Lambda^2} [C_{f\Phi}]_{rs} \right)$$

$$[\mathcal{Y}_f]_{rs} = \frac{1}{v_T} [M_f]_{rs} \left(1 + c_H^{\text{kin}} \right) - \frac{v^2}{\Lambda^2} [C_{f\Phi}]_{rs}$$

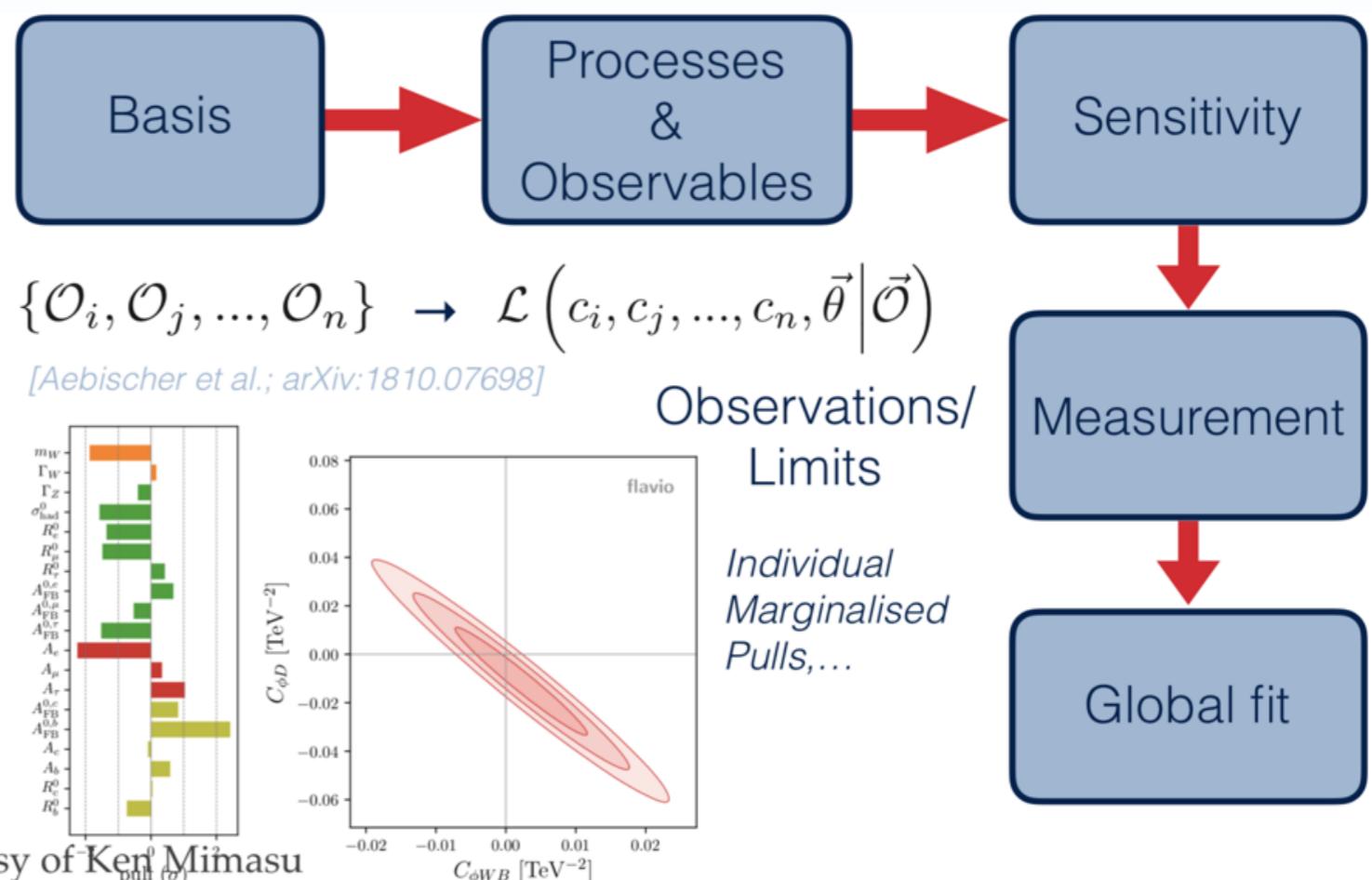
- Fermi:** $\frac{4G_F}{\sqrt{2}} = \frac{2}{v_T^2} + \frac{1}{\Lambda^2} \left(2 [C_{\Phi I}^{(3)}]_{ee} + 2 [C_{\Phi I}^{(3)}]_{\mu\mu} - [C_{ll}]_{\mu e, e\mu} - [C_{ll}]_{e\mu, \mu e} \right)$

Global Fitting of SMEFT



tesy of Ken Mimasu

Global Fitting of SMEFT



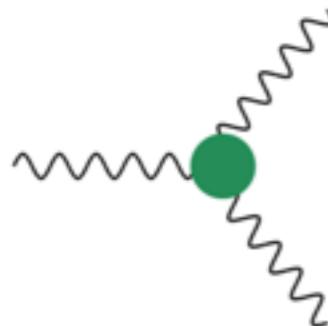
Cen Zhang

Madgraph school lectures

Example: Triple Gauge Coupling

Lorentz, \mathcal{P} , \mathcal{C} and e.m. gauge invariance:

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$$



$$\begin{aligned} \mathcal{L}_{WWV} = & -i g_{WWV} \left\{ g_1^V (W_{\mu\nu}^+ W^{-\mu} V^\nu - W_\mu^+ W^{-\mu\nu} V_\nu) \right. \\ & \left. + \kappa_V W_\mu^+ W_\nu^- V^{\mu\nu} + \frac{\lambda_V}{M_W^2} W_{\mu\nu}^+ W^{-\nu\rho} V_\rho^\mu \right\} \end{aligned}$$

$$g_{WW\gamma} = e = g \sin \theta_W \quad , \quad g_{WWZ} = g \cos \theta_W \quad , \quad g_1^\gamma = 1 \quad , \quad g_1^Z = 1 + \delta g_1^Z \quad , \quad \kappa_V = 1 + \delta \kappa_V$$

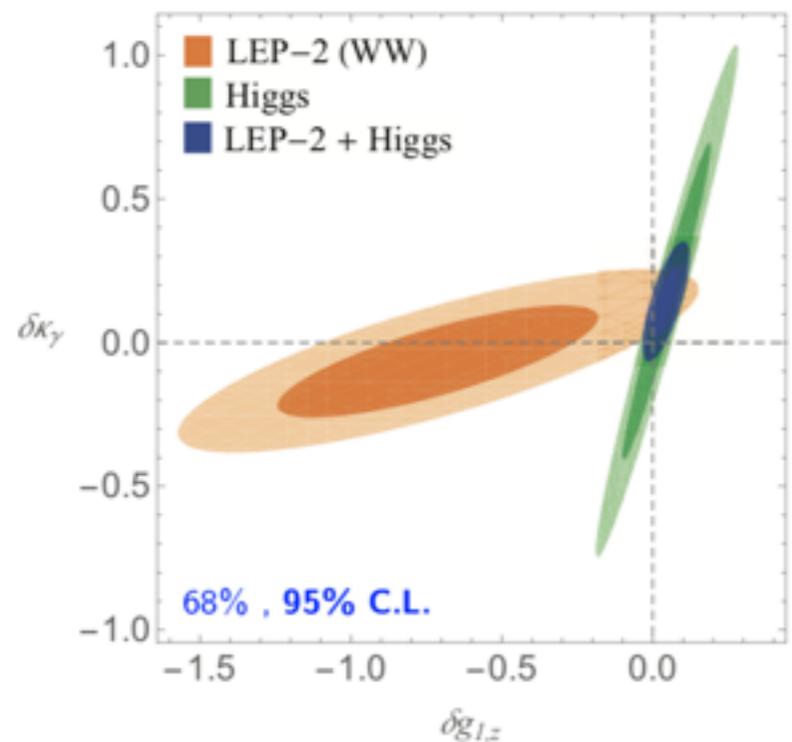
- D=6 relations: $\lambda_\gamma = \lambda_z$, $\delta \kappa_z = \delta g_1^Z - \tan^2 \theta_W \delta \kappa_\gamma$

→ 3 free parameters: δg_1^Z , $\delta \kappa_\gamma$, λ_z (SM: $\delta g_1^Z = \delta \kappa_\gamma = \lambda_z = 0$)

- SMEFT:

$$\delta g_1^Z = \frac{1}{\sqrt{2} G_F \sin 2\theta_W} \frac{C_{\Phi WB}}{\Lambda^2} , \quad \delta \kappa_\gamma = \frac{\cot \theta_W}{\sqrt{2} G_F} \frac{C_{\Phi WB}}{\Lambda^2} , \quad \lambda_z = \frac{6 \cos \theta_W M_W^2}{g_{WWZ}} \frac{C_W}{\Lambda^2}$$

Example: Triple Gauge Coupling

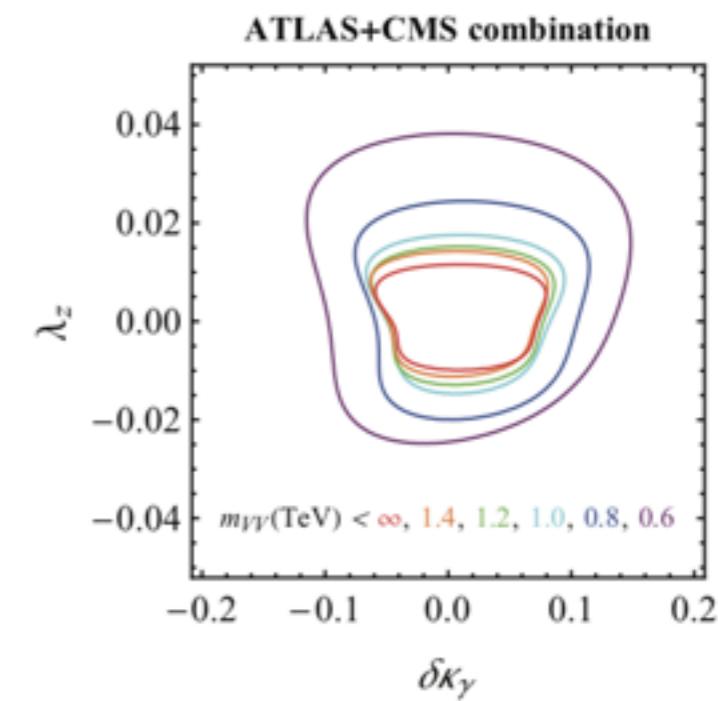
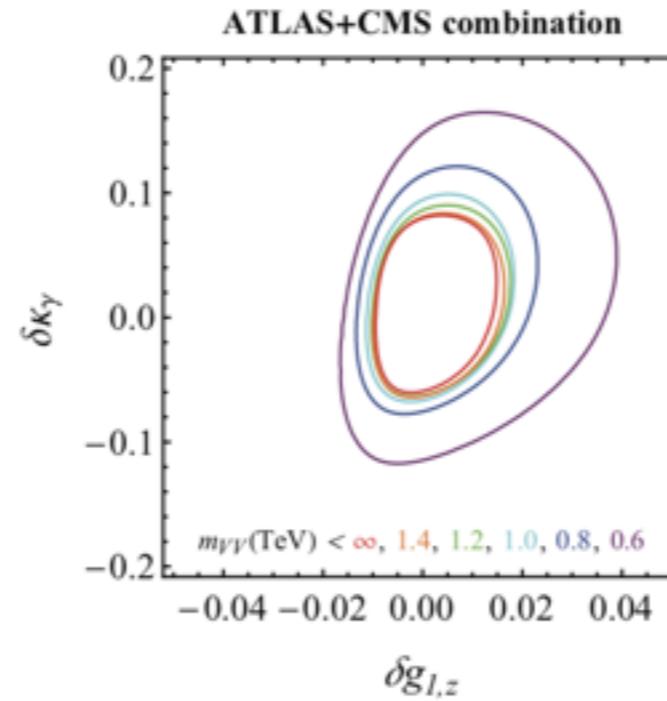
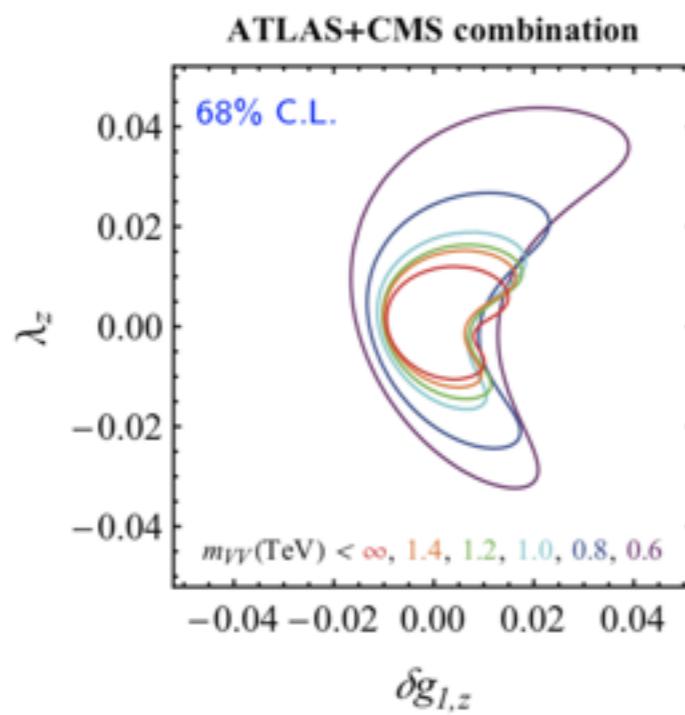


Falkowski, González-Alonso, Greljo, Marzocca, 1508.00581

$$\begin{pmatrix} \delta g_{1,z} \\ \delta \kappa_\gamma \\ \lambda_z \end{pmatrix} = \begin{pmatrix} 0.043 \pm 0.031 \\ 0.142 \pm 0.085 \\ -0.162 \pm 0.073 \end{pmatrix}$$

$$\rho = \begin{pmatrix} 1 & 0.74 & -0.85 \\ 0.74 & 1 & -0.88 \\ -0.85 & -0.88 & 1 \end{pmatrix}, \quad \text{MFV assumed}$$

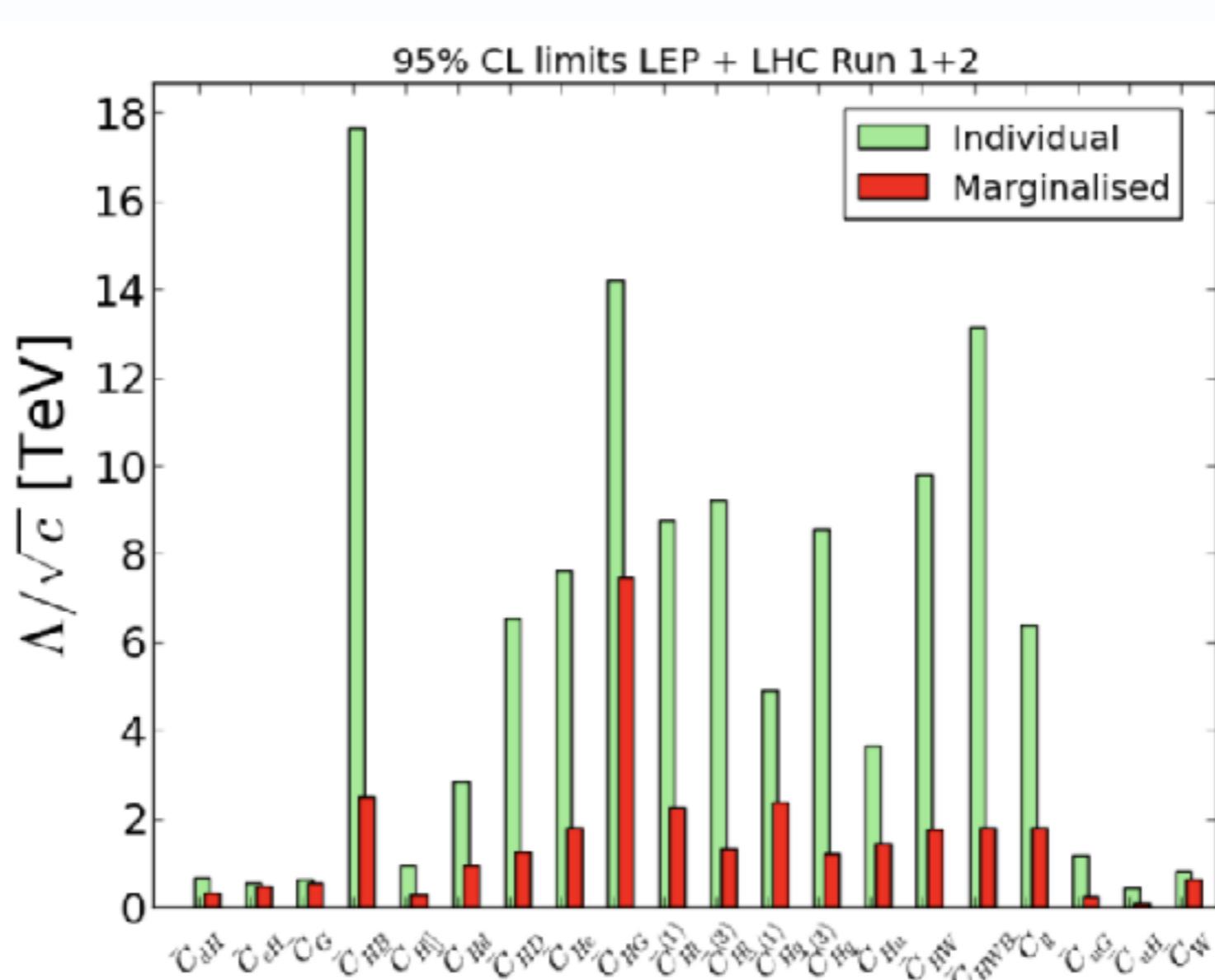
$pp \rightarrow WZ (WW) \rightarrow \ell\nu\ell^+\ell^- (\ell\nu\ell\nu)$



Example: Higgs Sector

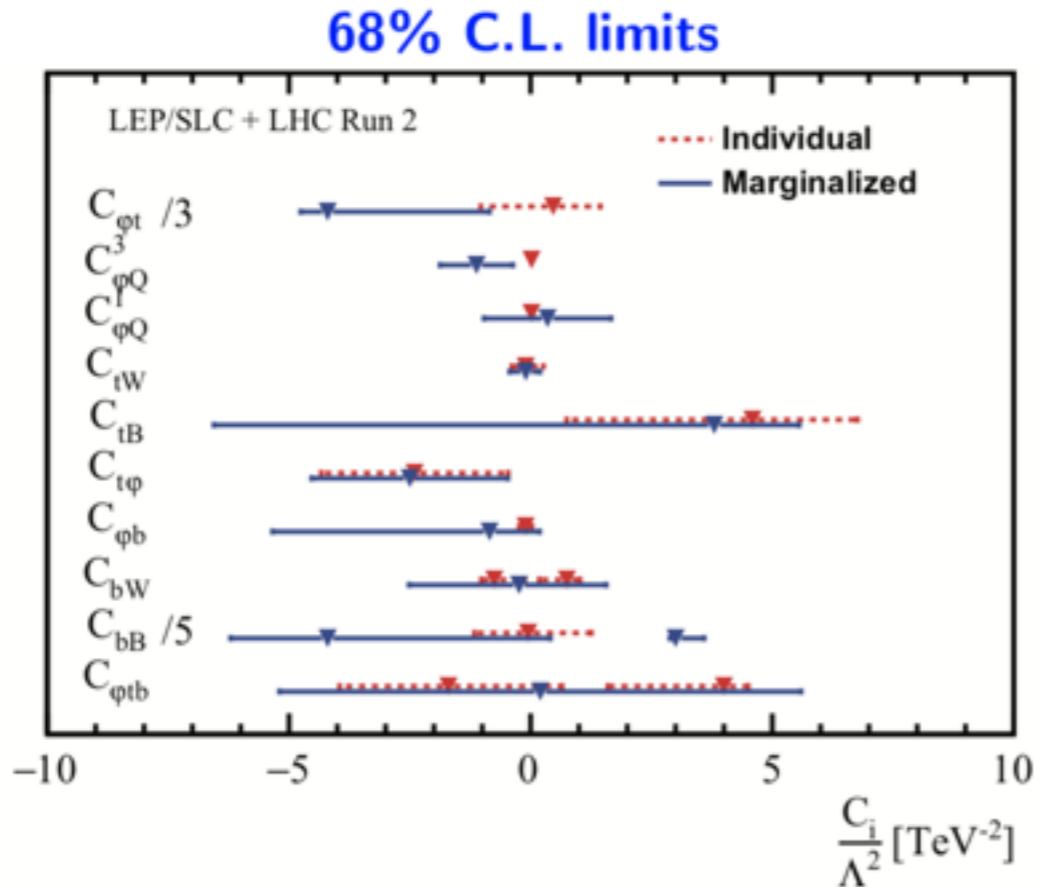
[Ellis, Murphy, Sanz, You, 2018]

$$\begin{aligned} \mathcal{L}_{\text{SMEFT}}^{\text{Warsaw}} \supset & \frac{\bar{C}_{eH}}{v^2} y_e (H^\dagger H) (\bar{l} e H) + \frac{\bar{C}_{dH}}{v^2} y_d (H^\dagger H) (\bar{q} d H) + \frac{\bar{C}_{uH}}{v^2} y_u (H^\dagger H) (\bar{q} u \tilde{H}) \\ & + \frac{\bar{C}_G}{v^2} f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu} + \frac{\bar{C}_{H\square}}{v^2} (H^\dagger H) \square (H^\dagger H) + \frac{\bar{C}_{uG}}{v^2} y_u (\bar{q} \sigma^{\mu\nu} T^A u) \tilde{H} G_{\mu\nu}^A \\ & + \frac{\bar{C}_{HW}}{v^2} H^\dagger H W_{\mu\nu}^I W^{I\mu\nu} + \frac{\bar{C}_{HB}}{v^2} H^\dagger H B_{\mu\nu} B^{\mu\nu} + \frac{\bar{C}_{HG}}{v^2} H^\dagger H G_{\mu\nu}^A G^{A\mu\nu}. \end{aligned}$$



Example: Top Sector

Durieux et al., 1907.10619



	Λ^{-2} and Λ^{-4} terms	Λ^{-2} term only
$C_{\varphi t}/\Lambda^2$	(-16, -2.4)	(-2.1, +4.5)
$C_{\varphi Q}^3/\Lambda^2$	(-1.9, -0.4)	(-0.7, +0.5)
$C_{\varphi Q}^1/\Lambda^2$	(-1, +1.7)	(-0.6, +0.7)
C_{tW}/Λ^2	(-0.4, +0.2)	(-0.42, +0.24)
C_{tB}/Λ^2	(-6.8, +5.6)	(-9.6, +38.4)
$C_{t\varphi}/\Lambda^2$	(-4.6, -0.4)	(-4.42, 0)
$C_{\varphi b}/\Lambda^2$	(-5.4, +0.2)	(-0.6, +0.2)
C_{bW}/Λ^2	(-2.6, +2.1)	—
C_{bB}/Λ^2	(-31.2, +2.4), (+14.4, +18)	—
$C_{\varphi tb}/\Lambda^2$	(-5.2, 5.6)	—

$$\begin{aligned}
 O_{\varphi Q}^1 &\equiv \frac{y_t^2}{2} \bar{q} \gamma^\mu q \quad \varphi^\dagger i \overleftrightarrow{D}_\mu \varphi, & O_{uW} &\equiv y_t g_W \bar{q} \tau^I \sigma^{\mu\nu} u \quad \epsilon \varphi^* W_{\mu\nu}^I, \\
 O_{\varphi Q}^3 &\equiv \frac{y_t^2}{2} \bar{q} \tau^I \gamma^\mu q \quad \varphi^\dagger i \overleftrightarrow{D}_\mu^I \varphi, & O_{dW} &\equiv y_t g_W \bar{q} \tau^I \sigma^{\mu\nu} d \quad \epsilon \varphi^* W_{\mu\nu}^I, & O_{u\varphi} &\equiv \bar{q} u \quad \epsilon \varphi^* \varphi^\dagger \varphi \\
 O_{\varphi u} &\equiv \frac{y_t^2}{2} \bar{u} \gamma^\mu u \quad \varphi^\dagger i \overleftrightarrow{D}_\mu \varphi, & O_{uB} &\equiv y_t g_Y \bar{q} \sigma^{\mu\nu} u \quad \epsilon \varphi^* B_{\mu\nu}, & O_{d\varphi} &\equiv \bar{q} d \quad \epsilon \varphi^* \varphi^\dagger \varphi \\
 O_{\varphi d} &\equiv \frac{y_t^2}{2} \bar{d} \gamma^\mu d \quad \varphi^\dagger i \overleftrightarrow{D}_\mu \varphi, & O_{dB} &\equiv y_t g_Y \bar{q} \sigma^{\mu\nu} d \quad \epsilon \varphi^* B_{\mu\nu}, \\
 O_{\varphi ud} &\equiv \frac{y_t^2}{2} \bar{u} \gamma^\mu d \quad \varphi^T \epsilon i D_\mu \varphi,
 \end{aligned}$$

See also Hartland et al., 1901.05965

Example: Z-pole Physics

[Dawson, Giardino, 2018]

\mathcal{O}_{ll}	$(\bar{l}\gamma_\mu l)(\bar{l}\gamma^\mu l)$	$\mathcal{O}_{\phi WB}$	$(\phi^\dagger \tau^a \phi) W_{\mu\nu}^a B^{\mu\nu}$	$\mathcal{O}_{\phi D}$	$(\phi^\dagger D^\mu \phi)^* (\phi^\dagger D_\mu \phi)$
$\mathcal{O}_{\phi e}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{e}_R \gamma^\mu e_R)$	$\mathcal{O}_{\phi u}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{u}_R \gamma^\mu u_R)$	$\mathcal{O}_{\phi d}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{d}_R \gamma^\mu d_R)$
$\mathcal{O}_{\phi q}^{(3)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu^a \phi)(\bar{q} \tau^a \gamma^\mu q)$	$\mathcal{O}_{\phi q}^{(1)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{q} \tau^a \gamma^\mu q)$	$\mathcal{O}_{\phi l}^{(3)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu^a \phi)(\bar{l} \tau^a \gamma^\mu l)$
$\mathcal{O}_{\phi l}^{(1)}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)(\bar{l} \tau^a \gamma^\mu l)$				

Coefficient	NLO	Coefficient	NLO	Coefficient	NLO
c_W	$[-4.8, 0.48]$	c_{uu}	$[-1.1, 0.99]$	c_{uW}	$[-0.78, 0.29]$
c_{uB}	$[-0.57, 0.11]$	$c_{qu}^{(1)}$	$[-2.2, 1.3]$	$c_{qq}^{(3)}$	$[-0.32, 0.29]$
$c_{qq}^{(1)}$	$[-0.93, 1.49]$	c_{qe}	$[-0.75, 0.48]$	$c_{qd}^{(1)}$	$[-9.8, 5.0]$
$c_{\phi \square}$	$[-22, 1.9]$	$c_{\phi W}$	$[-17, 2.2]$	$c_{\phi B}$	$[-19, 3.3]$
c_{ln}	$[-0.49, 0.19]$	$c_{lq}^{(3)}$	$[-0.32, 0.57]$	$c_{lq}^{(1)}$	$[-0.25, 0.66]$
c_{le}	$[-5.3, 11]$	c_{id}	$[-3.8, 8.7]$	c_{dd}	$[-51, 26]$
c_{ed}	$[-12, 6.7]$	c_{ee}	$[-3.9, 2.4]$	c_{eu}	$[-0.36, 0.58]$
$c_{ud}^{(1)}$	$[-3.0, 5.6]$				

Coefficient	LO	NLO
c_{ll}	$[-0.0039, 0.021]$	$[-0.0044, 0.019]$
$c_{\phi WB}$	$[-0.0088, 0.0013]$	$[-0.0079, 0.0016]$
$c_{\phi u}$	$[-0.072, 0.091]$	$[-0.035, 0.084]$
$c_{\phi q}^{(3)}$	$[-0.011, 0.014]$	$[-0.010, 0.014]$
$c_{\phi q}^{(1)}$	$[-0.027, 0.043]$	$[-0.031, 0.036]$
$c_{\phi l}^{(3)}$	$[-0.012, 0.0029]$	$[-0.010, 0.0028]$
$c_{\phi l}^{(1)}$	$[-0.0043, 0.012]$	$[-0.0047, 0.012]$
$c_{\phi e}$	$[-0.013, 0.0094]$	$[-0.013, 0.0080]$
$c_{\phi D}$	$[-0.025, 0.0019]$	$[-0.023, 0.0023]$
$c_{\phi d}$	$[-0.16, 0.060]$	$[-0.13, 0.063]$

Example: Precision EW

[Falkowski, et.al 2016]

- They constrain 61 combinations of Wilson Coefficients [Higgs / Warsaw basis]

$$\left(\begin{array}{c} \delta g_L^{W_e} \\ \delta g_L^{W_\mu} \\ \delta g_L^{W_\tau} \\ \delta g_L^{Z_e} \\ \delta g_L^{Z_\mu} \\ \delta g_L^{Z_\tau} \\ \delta g_R^{Z_e} \\ \delta g_R^{Z_\mu} \\ \delta g_R^{Z_\tau} \\ \delta g_L^{Z_u} \\ \delta g_L^{Z_c} \\ \delta g_L^{Z_t} \\ \delta g_R^{Z_u} \\ \delta g_R^{Z_c} \\ \delta g_R^{Z_t} \\ \delta g_L^{Z_d} \\ \delta g_L^{Z_s} \\ \delta g_L^{Z_b} \\ \delta g_R^{Z_d} \\ \delta g_R^{Z_s} \\ \delta g_R^{Z_b} \\ \delta g_R^{W_{q_1}} \end{array} \right) = \left(\begin{array}{c} -1.00 \pm 0.64 \\ -1.36 \pm 0.59 \\ 1.95 \pm 0.79 \\ -0.023 \pm 0.028 \\ 0.01 \pm 0.12 \\ 0.018 \pm 0.059 \\ -0.033 \pm 0.027 \\ 0.00 \pm 0.14 \\ 0.042 \pm 0.062 \\ -0.8 \pm 3.1 \\ -0.15 \pm 0.36 \\ -0.3 \pm 3.8 \\ 1.4 \pm 5.1 \\ -0.35 \pm 0.53 \\ -0.9 \pm 4.4 \\ 0.9 \pm 2.8 \\ 0.33 \pm 0.17 \\ 3 \pm 16 \\ 3.4 \pm 4.9 \\ 2.30 \pm 0.88 \\ -1.3 \pm 1.7 \end{array} \right) \times 10^{-2}.$$

$$\left(\begin{array}{c} [c_{\ell\ell}]_{1111} \\ [c_{\ell e}]_{1111} \\ [c_{ee}]_{1111} \\ [c_{\ell\ell}]_{1221} \\ [c_{\ell e}]_{1122} \\ [c_{\ell e}]_{1122} \\ [c_{ee}]_{1122} \\ [c_{\ell\ell}]_{1331} \\ [c_{\ell e}]_{1133} \\ [c_{\ell e}]_{1133} \\ [c_{ee}]_{1133} \\ [c_{\ell\ell}]_{2222} \\ [c_{\ell e}]_{2332} \end{array} \right) = \left(\begin{array}{c} 1.01 \pm 0.38 \\ -0.22 \pm 0.22 \\ 0.20 \pm 0.38 \\ -4.8 \pm 1.6 \\ 1.5 \pm 2.1 \\ 1.5 \pm 2.2 \\ -1.4 \pm 2.2 \\ 3.4 \pm 2.6 \\ 1.5 \pm 1.3 \\ 0 \pm 11 \\ -2.3 \pm 7.2 \\ 1.7 \pm 7.2 \\ -1 \pm 12 \\ -2 \pm 21 \\ 3.0 \pm 2.3 \end{array} \right) \times 10^{-2}.$$

$$\left(\begin{array}{c} [c_{\ell q}^{(3)}]_{1111} \\ [\hat{c}_{eq}]_{1111} \\ [\hat{c}_{\ell u}]_{1111} \\ [\hat{c}_{\ell d}]_{1111} \\ [\hat{c}_{eu}]_{1111} \\ [\hat{c}_{ed}]_{1111} \\ [\hat{c}_{\ell q}]_{1122} \\ [c_{\ell u}]_{1122} \\ [\hat{c}_{\ell d}]_{1122} \\ [c_{eu}]_{1122} \\ [c_{ed}]_{1122} \\ [\hat{c}_{\ell q}]_{1133} \\ [c_{\ell d}]_{1133} \\ [\hat{c}_{eq}]_{1133} \\ [\hat{c}_{ed}]_{1133} \\ [c_{\ell q}]_{2211} \\ [c_{\ell u}]_{2211} \\ [\hat{c}_{\ell d}]_{2211} \\ [\hat{c}_{eq}]_{2211} \\ [c_{\ell equ}]_{1111} \\ [c_{\ell edq}]_{1111} \\ [\hat{c}_{\ell equ}]_{1111} \\ \epsilon_F^d(2 \text{ GeV}) \end{array} \right) = \left(\begin{array}{c} -2.2 \pm 3.2 \\ 100 \pm 180 \\ -5 \pm 11 \\ -5 \pm 23 \\ -1 \pm 12 \\ -4 \pm 21 \\ -61 \pm 32 \\ 2.4 \pm 8.0 \\ -310 \pm 130 \\ -21 \pm 28 \\ -87 \pm 46 \\ 270 \pm 140 \\ -8.6 \pm 8.0 \\ -1.4 \pm 10 \\ -3.2 \pm 5.1 \\ 18 \pm 20 \\ -1.2 \pm 3.9 \\ 1.3 \pm 7.6 \\ 15 \pm 12 \\ 25 \pm 34 \\ 4 \pm 41 \\ -0.080 \pm 0.075 \\ -0.079 \pm 0.074 \\ -0.02 \pm 0.19 \\ -0.02 \pm 0.15 \end{array} \right) \times 10^{-2}.$$



EFT Operator Tools

- ❖ Project for global, public SMEFT likelihood

[Aebischer et al.; EPJ C79 (2019) no.6, 509]

<https://smelli.github.io>

- ❖ Wrapper around flavio tool



- ❖ Initially developed for flavour physics observables

[Straub; arXiv:1810:08132]

<https://flav-io.github.io>

- ❖ EFT predictions for flavour, EWPO, LFV decays, lepton MDM, neutron EDM

- ❖ Interface to Wilson tool



[Aebisher et al.; EPJ C78 (2018) no.12, 1026]

<https://wilson-eft.github.io>

- ❖ Running SMEFT coefficients down to EW scale

- ❖ Based on DsixTools implementation

[Celis et a.; EPJ C77 (2017) no.6, 405]

<https://dsixtools.github.io>

- ❖ Matching to Weak Effective Theory below EW scale + running

- ❖ “Full stack” suite of global EFT analysis software

- ❖ Common interface: Wilson coefficient exchange format (WCxf)

[Aebischer et al.; Comp. Phys. Comm. 232 (2018) 71-83]

<https://wcxf.github.io>

[Falkowski et al.; EPJ C75 (2015) no.12, 583]

<https://rosetta.hepforge.org>

EFT MC Tools

At LO:

- HEL, SMEFTsim, dim6top...
- Higgs Characterisation, BSMC, HiggsPO
- NLO QCD (FeynRules//NLOCT/UFO)
<http://feynrules.irmp.ucl.ac.be/wiki/SMEFTatNLO>

[Alloul et al.; JHEP 1404 (2014) 110]
[Brivio et al.; JHEP 1712 (2017) 070]
[Aguilar Saavedra et al.; arXiv:1802.07237]
[Artoisenet et al.; JHEP 1311 (2013) 043]
[Falkowski et al.; EPJC 75 (2015) 12, 583]
[Greljo et al.; EPJC 77 (2017) 12, 838]

Based on:

	[Zhang; PRL 116 (2016) 162002]	single-top
[Bylund, Maltoni, Tsinikos, Vryonidou, Zhang; JHEP 1605 (2016) 052]		ttZ & ttY
[Maltoni, Vryonidou, Zhang; JHEP 1610 (2016) 123]		ttH, ggH, H+J
[Degrande, Fuks, Mawatari, KM, Sanz; EPJC 77 (2017) 4, 262]		VH & VBF
[Degrande, Maltoni, KM, Zhang, Vryonidou; JHEP 1810 (2018) 005]		tZq & tHq

Single & double Higgs

- HiGlu, SusHi, HPAIR, HiggsPair

[Dawson, Dittmaier & Spira; Phys. Rev. D58:115012]
[Harlander, Liebler & Mantler; arXiv:1605.03190]
[Goertz et al.; JHEP 1504 (2015) 167]

eHDECAY for BR

[Contino et al.; Comp. Phys. Comm. 185 (2014) 3412-3423]
<https://www.ifp.kit.edu/~maggie/eHDECAY/>

HAWK

- VBF and VH @ NLO in QCD & EW for SM + 2 anomalous couplings

[Denner et al.; JHEP 1203 (2012) 075]
<http://omnibus.uni-freiburg.de/~sd565/programs/hawk/hawk>

VBFNLO

- General (FO) tool for Higgs/weak boson production @ NLO in QCD

[Baglio et al.; arXiv:1404.3940]
<https://www.ifp.kit.edu/vtfnk>

VH via POWHEG-BOX/MCFM

[KM, Sanz & Williams.; JHEP 1608 (2016) 039]
<http://powhegbox.mbl.infn.it>

NLO QCD + PS event generation for Higgs/EW operators (SILH)

HELaNLO

[Degrande, et al.; EPJC 77 (2017) 4, 262]
<http://feynRules.irmp.ucl.ac.be/wiki/HELaNLO>

- FeynRules/NLOCT/UFO implementation of Higgs/EW operators

- VH, VBF & any other process of interest (CPV operators also on the way)

Dilepton & EW Higgs in POWHEG-BOX

[Alloul et al.; JHEP 08 (2018) 205]
<http://powhegbox.mbl.infn.it>

- Larger set of operators considered

Exercise: Explore EFT Tools



SMEFT-Tools 2019

12-14 June 2019
IPPP Durham
Europe/Zurich timezone

Enter your search term

Overview

- Practical Information
- Timetable
- Registration
- Fee Payment
- Participant List
- List of Confirmed Speakers
- Getting to the IPPP
- Conference dinner
- SCAM ALERT

SMEFT-Tools Support
smefttools@gmail.com

This is the 1st Workshop on Tools for Low-Energy SMEFT Phenomenology, **SMEFT-Tools 2019**.
The conference will be held at the Institute for Particle Physics Phenomenology (IPPP) in Durham (see the announcement [here](#) and on [INSPIRE](#)).
The aim of this conference is to discuss the status of and future prospects for computing tools designed for phenomenological analyses of the Standard Model EFT (SMEFT) and the Weak EFT (WET). The related SMEFT phenomenology as well as other SMEFT/WET related topics will also be discussed.

 Starts Jun 12, 2019, 12:00 AM
Ends Jun 14, 2019, 11:45 PM
Europe/Zurich

 IPPP Durham
DC218

 Jason Aebischer
Matteo Fael
Alexander Lenz
Michael Spannowsky
Javier Virto

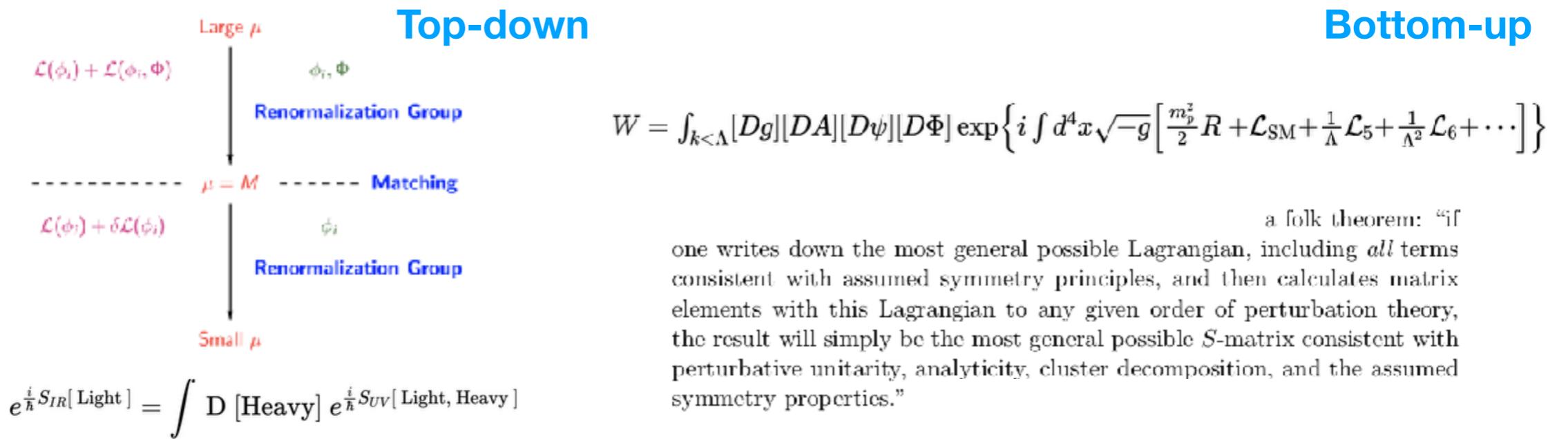
 creditcardform.doc
creditcardform.pdf

<https://indico.cern.ch/event/787665/>

Summary & Outlook

Summary

Take home message 1: Core of EFT are decoupling theorem, fields and symmetry



Take home message 2: SMEFT provides systematical parametrization of
... all possible Lorentz inv. new physics!

$$\mathcal{L}_{\text{SMEFT}} = \mathcal{L}_{\text{SM}} + \frac{1}{\Lambda} \mathcal{L}_{D=5} + \frac{1}{\Lambda^2} \mathcal{L}_{D=6} + \frac{1}{\Lambda^3} \mathcal{L}_{D=7} + \frac{1}{\Lambda^4} \mathcal{L}_{D=8} + \dots$$

Known SM Lagrangian Higher-dimensional $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant interactions added to the SM

Thanks!

Appreciate all materials from web, prepared for this school