DDK 3-body system in Lattice QCD

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University of Science and Technology in China (中科大)

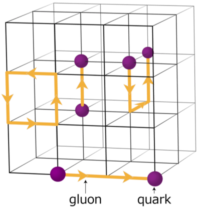
Outline

- *$D-K$ interaction*
  - $D^*_{s0}(2317)(c\bar{s})$, $I(J^P)=0(0^+)$ B. Aubert et.al. (BaBar) PRL 90,242001(2003)
  - *$D-K$ molecular state* F.-K. Guo, EPJ Web Conf. 202,02001(2019)
  - *$D-K$ strongly attractive interaction in S-wave*
- *DDK 3-body system*
  - *$D-D^*_{s0}$ interaction* M. Sanchez et al. PRD 98(2018),054001
  - *DDK $(cc\bar{s}\bar{q})$ $\frac{1}{2}(0^-)$, $DDDK$ $1(0^+)$ bound states* L.-S. Geng et al. PRD 100(2019)3,034029

DDK 3-body system in Lattice QCD

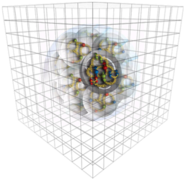
- *Lattice Quantum ChromoDynamics*

$$\mathscr{L}_{\text{QCD}} = \sum_i \bar{\psi}_i (i\not{D}_\mu - m_i)\psi_i - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}$$



Unphysical: lattice spacing a, lattice size L

Build operators for DDK system



$$D^0 = \bar{u}(x)\gamma_5 c(x), \; K^+ = \bar{s}(x)\gamma_5 u(x)$$

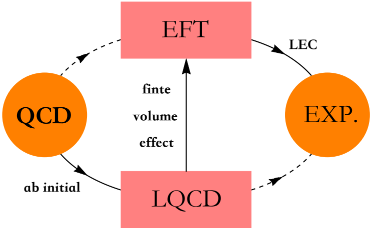
$$\mathscr{O}(\tau) \sim \left[D^0(\tau) D^0(\tau) K^+(\tau)\right]$$

$$C(\tau) = \langle \mathscr{O}(\tau)\mathscr{O}(0)\rangle \sim e^{-E\tau}$$

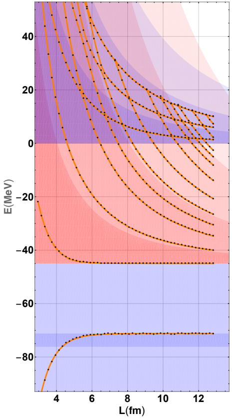
Eigen-energy of 3-body system

$E(L)$ is function of lattice size

- *DDK effective field theory*



Hammer, JYP and Rusetsky. JHEP 10(2017)



- *DDK lattice spectrum and physical quantities*
  - *DDK 3-body bound state*
  - $D - D^*_{s0}(2317)$ *scattering length*
  - *DDK 3-body decay Dalitz plot*

- $D-K$ *potential* L.-S. Geng et al. PRD 100(2019)3,034029

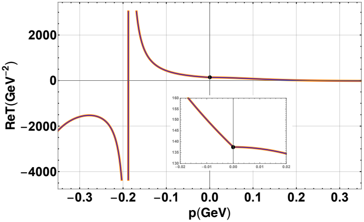
$$\tilde{V}_{DK}(r) = C_L e^{-(r/R_c)^2} + C_S e^{-(r/R_s)^2}, \quad R_c = 1\text{fm}, \ R_s = 0.5\text{fm} \tag{1}$$

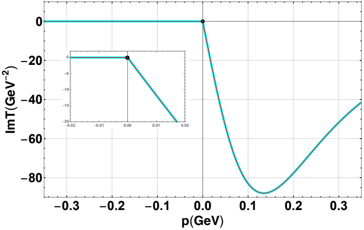
- $D-K$ *scattering equation*

$$T(\mathbf{p},\mathbf{q}) = V(\mathbf{p},\mathbf{q}) + \int \frac{d^3k}{(2\pi)^3} V(\mathbf{p},\mathbf{k})\, G(\mathbf{k})\, T(\mathbf{k},\mathbf{q}) \tag{2}$$

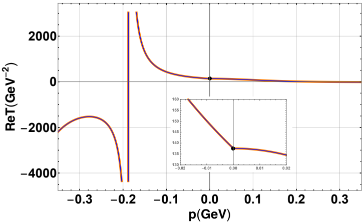
$$\text{where } \ V(\mathbf{p},\mathbf{k}) = \int d^3x \tilde{V}(\mathbf{x}) e^{-i(\mathbf{p}-\mathbf{k})\mathbf{x}} \tag{3}$$

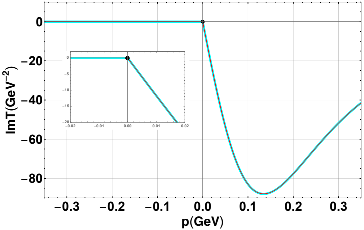
- $D-K$ *scattering amplitude*





- $D-K$ *scattering amplitude*





- $D-K$ *scattering phase shift*

$$T(p) = -\frac{2\pi}{\mu}\frac{1}{p\cot\delta - ip} \tag{4}$$

$$\text{where } p\cot\delta = -a_{DK}^{-1} + \frac{1}{2}r_{DK}p^2, \quad a_{DK} = 1.683\text{fm},\ r_{DK} = 0.792\text{fm} \tag{5}$$

- *Analytic continuation below threshold* $(\kappa = -ip)$ $D_{s0}^*(2317)$

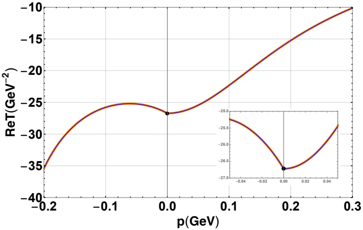
$$T(\kappa) = -\frac{2\pi}{\mu}\left[\frac{C_{DK}}{\kappa - \kappa_{DK}} + R_{DK} + R_{DK}'^2\kappa\right], \quad \kappa_{DK} = 187.795\text{MeV},\ C_{DK} = 3.881 \tag{6}$$

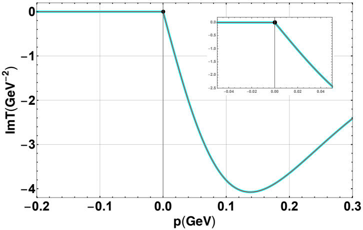
- $D-D$ *potential* L.-S. Geng et al. PRD 100(2019)3,034029

$$\tilde{V}_{DD}(\mathbf{p},\mathbf{k}) = \sum_{V=\sigma,\rho,\omega} C_{\text{iso.}}(V)\frac{g_V^2}{(\mathbf{p}-\mathbf{k})^2+m_V^2}\left(\frac{\Lambda^2-m_V^2}{(\mathbf{p},\mathbf{k})^2+\Lambda^2-q_0^2}\right)^2 \tag{7}$$

where $C_{\text{iso.}}(\sigma)=-1, C_{\text{iso.}}(\rho)=1, C_{\text{iso.}}(\omega)=+1$ and $\Lambda=1$GeV, $g_\sigma=3.4, g_\rho=g_\omega=2.6.$

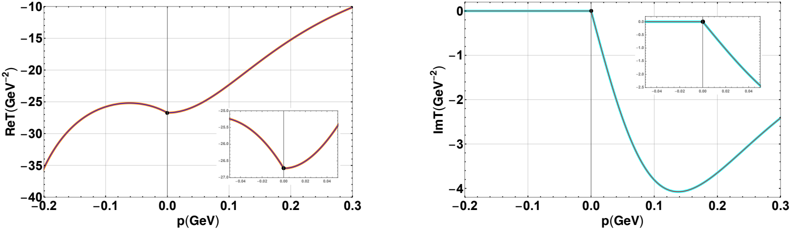
- $D-D$ *scattering amplitude*





$D-D$ interaction

- *$D-D$ scattering amplitude*



- *$D-D$ scattering phase shift*

$$T(p) = -\frac{8\pi}{m_D}\frac{1}{p\cot\delta - ip} \tag{8}$$

$$\text{where } p\cot\delta = -a_{DD}^{-1} + \frac{1}{2}r_{DD}p^2, \quad a_{DD} = -0.392\text{fm},\ r_{DD} = 3.236\text{fm} \tag{9}$$

- *Analytic continuation below threshold* $(\kappa = -ip)$

$$T(\kappa) = -\frac{8\pi}{m_D}\left[\frac{C_{DD}}{\kappa - \kappa_{DD}} + R_{DD} + R'^2_{DD}\kappa\right], \quad \kappa_{DD} = -195.166\text{MeV},\ C_{DD} = 0.243 \tag{10}$$

- *Lagrangian*

$$\mathscr{L} = \mathscr{L}_0 + \mathscr{L}_2 + \mathscr{L}_3 \tag{11}$$

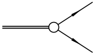
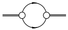
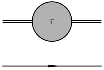
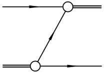
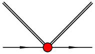
where

$$\mathscr{L}_0 = D^\dagger(i\partial_0 + \frac{\nabla^2}{2m_D})D + K^\dagger(i\partial_0 + \frac{\nabla^2}{2m_K})K + T^\dagger_{DK}\sigma_{DK}T_{DK} + T^\dagger_{DD}\sigma_{DD}T_{DD} \tag{12}$$

$$\mathscr{L}_2 = T^\dagger_{DK}[D\mathscr{F}_{DK}K] + T^\dagger_{DD}[D\mathscr{F}_{DD}D] + \text{h.c.} \tag{13}$$

$$\mathscr{L}_3 = \left[T^\dagger_{DK}D^\dagger\right][T_{DK}\mathscr{H}_D D] + \left[T^\dagger_{DD}K^\dagger\right][T_{DD}\mathscr{H}_K K] + \text{h.c.} \tag{14}$$

  - *Non-relativistic kinematics of D-meson and K-menson*
  - *2-body sub-system(dimer):* T_{DK}, T_{DD}
  - *operator and LEC:* $\mathscr{F} = f_0 + f_2\overleftrightarrow{\nabla}^2$, $\mathscr{H} = h_0 + h_2\nabla^2$

EFT	2-body Physics	3-body Physics	
2-body Operators with 2-body LEC 	Dimer Self-Energy 	Dimer-Spectator Propagation 	Particle-Dimer Potential  
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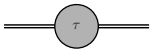
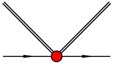
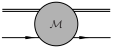
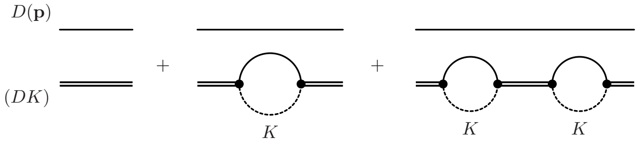
	Dimer Propagation 		
3-body Operators with 3-body LEC (3-body Force) 		Particle-Dimer Scattering Equation	
		Particle-Dimer Scattering Amplitude 	

2-body matching

- *(DK)-dimer propagator*

$\tau_{DK}(p;E) =$



$+ \cdots$

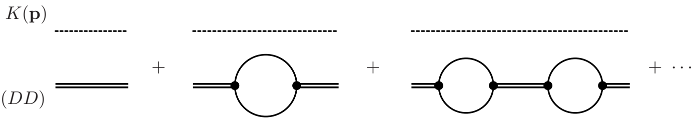
$$= \frac{1}{p_* \cot \delta_{DK}(p_*) - ip_*}, \quad \text{with } p_*^2 = 2\mu E - (1 - m_D^2/M^2)p^2 + i\epsilon$$

- *$D - K$ scattering phase shift*
  - *on-shell relative momentum in (DK) sub-system:* $p_*^2 = 2\mu E - (1 - m_D^2/M^2)p^2 + i\varepsilon$
  - *phase shift above threshold:* $p_* \cot \delta_{DK}(p_*) = -a_{DK}^{-1} + \frac{1}{2} r_{DK} p_*^2, (p_*^2 \geq 0)$
  - *analytic continuation:* $\tau_{DK}(p;E) = \frac{C_{DK}}{\kappa - \kappa_{DK}} + R_{DK} + R'^2_{DK}\kappa, (p_*^2 = -\kappa^2 < 0)$

2-body matching

- *(DD)-dimer propagator*

$\tau_{DD}(p;E) \;=$

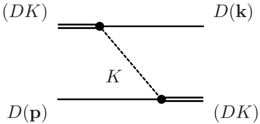


$$= \frac{4(1-m_D/M)}{p_*\cot\delta_{DD}(p_*) - ip_*}, \quad \text{with } p_*^2 = m_D E - \frac{m_K + 2m_D}{4m_K}p^2 + i\epsilon$$

- *$D-D$ scattering phase shift*
  - *on-shell relative momentum in (DD) sub-system:* $p_*^2 = m_D E - \frac{m_K+2m_D}{4m_K}p^2 + i\varepsilon$
  - *phase shift above threshold:* $p_*\cot\delta_{DD}(p_*) = -a_{DD}^{-1} + \frac{1}{2}r_{DD}p_*^2, \quad (p_*^2 \geq 0)$
  - *analytic continuation:* $\tau_{DD}(p;E) = 4(1-m_D/M)\left[\frac{C_{DD}}{\kappa-\kappa_{DD}} + R_{DD} + R_{DD}'^2\kappa\right], \quad (p_*^2 = -\kappa^2 < 0)$

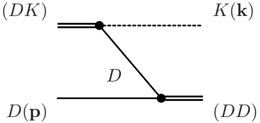
Particle-dimer scattering potential

- *K-hopping* $(DK)D \to (DK)D$



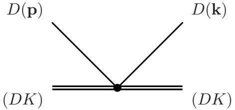
$$= \frac{1}{p^2 + k^2 + 2(m_D/M)\mathbf{pk} - 2\mu E}$$

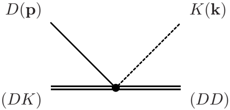
- *D-hopping* $(DK)D \to (DD)K$

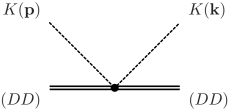


$$= \frac{M/(2m_K)}{p^2 + Mk^2/(2m_K) + \mathbf{pk} - m_D E}$$

- *Contact potential:* $D^\dagger D^\dagger K^\dagger DDK$ *encoding high-momentum physics*







- *Particle-dimer scattering equation*

$$\begin{pmatrix} \mathscr{M}_1 & \mathscr{M}_{12} \\ \mathscr{M}_{21} & \mathscr{M}_2 \end{pmatrix}(\mathbf{p},\mathbf{q}) = \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix}(\mathbf{p},\mathbf{q}) + 4\pi \int^{\Lambda} \frac{d^3k}{(2\pi)^3} \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix}(\mathbf{p},\mathbf{k}) \times \begin{pmatrix} \tau_1(\mathbf{k}) & \\ & \tau_2(\mathbf{k}) \end{pmatrix} \begin{pmatrix} \mathscr{M}_1 & \mathscr{M}_{12} \\ \mathscr{M}_{21} & \mathscr{M}_2 \end{pmatrix}(\mathbf{k},\mathbf{q}). \quad (15)$$

  - *2 coupled channels:* $(DK)D$ *and* $(DD)K$
  - $\mathscr{M}$*: particle-dimer scattering amplitude*
  - *Z: particle-dimer potential*

$$Z_1(\mathbf{p},\mathbf{q}) = \frac{1}{p^2 + q^2 + 2(m_D/M)\mathbf{pq} - 2\mu E} + \frac{H_0}{\Lambda^2}, \quad (16)$$

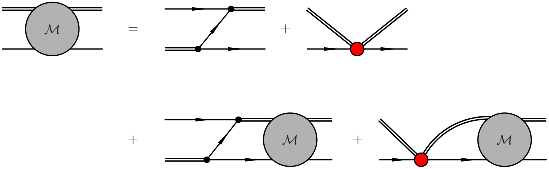
$$Z_{12}(\mathbf{p},\mathbf{q}) = \frac{M/(2m_K)}{p^2 + Mq^2/(2m_K) + \mathbf{pq} - m_D E} \quad (17)$$

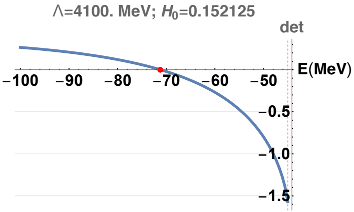
  - τ*: particle-dimer propagator*

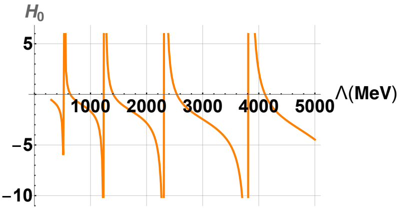
- *DDK 3-body bound state* L.-S. Geng et al. PRD 100(2019)3,034029

C_S (MeV)	C_L (MeV)	E_2 (MeV)	E_3 (only DK)	E_3 (both DK and DD)
$R_c = 1\text{fm}, R_s = 0.5\text{fm}$				
0	−320.1	−45.0	−65.8	−71.2
500	−455.4	−45.0	−65.8	−70.4

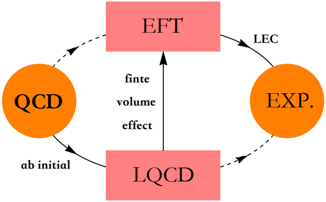
- *DDK 3-body force*

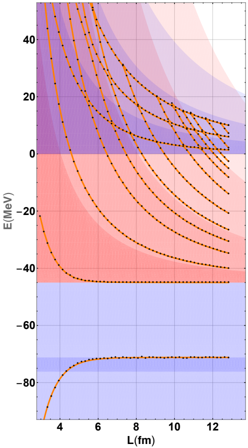




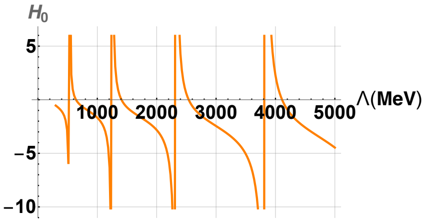


- *Lattice QCD and effective field theory* Hammer, JYP and Rusetsky. JHEP 10(2017)

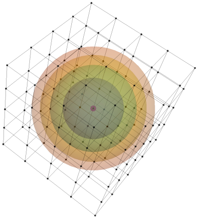


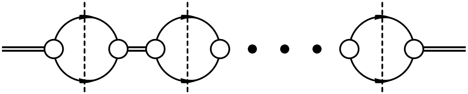


- *Lattice spectrum and 3-body force*



- *Finite Volume Correction* (*Discrete momentum and Invariant 3-body force*)





- *Loop summation*

$$\Sigma(\mathbf{p};E) = \frac{i}{2}\int \frac{d^4q}{(2\pi)^4} \frac{f^*(\mathbf{p},\mathbf{q})f(\mathbf{p},\mathbf{q})}{\left[q_0-\omega_{D,\mathbf{q}}+i\varepsilon\right]\left[(E-\omega_{K,\mathbf{p}}-q_0)-\omega_{D,-\mathbf{p}-\mathbf{q}}+i\varepsilon\right]}$$

$$\to \quad \frac{i}{2}\int \frac{dq_0}{2\pi}\frac{1}{L^3}\sum_{\mathbf{q}} \frac{f^*(\mathbf{p},\mathbf{q})f(\mathbf{p},\mathbf{q})}{\left[q_0-\omega_{D,\mathbf{q}}+i\varepsilon\right]\left[(E-\omega_{K,\mathbf{p}}-q_0)-\omega_{D,-\mathbf{p}-\mathbf{q}}+i\varepsilon\right]} \qquad (18)$$

- (DK)*-dimer in a box*

$$\tau_{DK,L}(\mathbf{p};E) = \frac{1}{p_* \cot\delta_{DK}(p_*) - 4\pi S_{DK,L}(\mathbf{p};E)} \tag{19}$$

where $p_*^2 = 2\mu E - (1-m_D^2/M^2)p^2$ and finite volume correction is

$$S_{DK,L}(\mathbf{p};E) = \left(\frac{1}{L^3}\sum_{\mathbf{q}} - \mathrm{PV}\int\frac{d^3q}{(2\pi)^3}\right)\frac{1}{(\mathbf{q}+(m_D/M)\mathbf{p})^2 - p_*^2} \tag{20}$$

2-body input: $p_*\cot\delta_{DK}(p_*) = -a_{DK}^{-1} + \frac{1}{2}r_{DK}p_*^2, \quad (p_*^2 \geq 0)$

Analytic continuation: $p_*\cot\delta_{DK}(p_*) = \frac{\kappa-\kappa_{DK}}{C_{DK}+[R_{DK}+R_{DK}'^2\kappa](\kappa-\kappa_{DK})} - \kappa, \quad (p_*^2 = -\kappa^2 < 0)$

- *(DD)-dimer in a box*

$$\tau_{DD,L}(\mathbf{p};E) = \frac{4(1-m_D/M)}{p_*\cot\delta_{DD}(p_*) - 4\pi S_{DD,L}(\mathbf{p};E)} \tag{21}$$

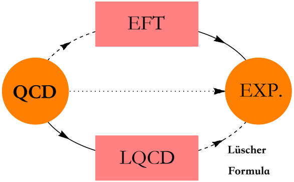
where $p_*^2 = m_D E - \frac{m_K+2m_D}{4m_K}p^2$ and finite volume correction is

$$S_{DD,L}(\mathbf{p};E) = \left(\frac{1}{L^3}\sum_{\mathbf{q}} - \text{PV}\int\frac{d^3q}{(2\pi)^3}\right)\frac{1}{(\mathbf{q}+\mathbf{p}/2)^2 - p_*^2} \tag{22}$$

2-body input: $p_*\cot\delta_{DD}(p_*) = -a_{DD}^{-1} + \frac{1}{2}r_{DD}p_*^2, \quad (p_*^2 \geq 0)$

Analytic continuation: $p_*\cot\delta_{DD}(p_*) = \frac{\kappa-\kappa_{DD}}{C_{DD}+[R_{DD}+R'^2_{DD}\kappa](\kappa-\kappa_{DD})} - \kappa, \quad (p_*^2 = -\kappa^2 < 0)$

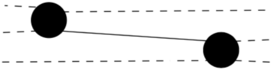
- *Lüscher Formula* M. Lüscher, NPB 354(1991) 531



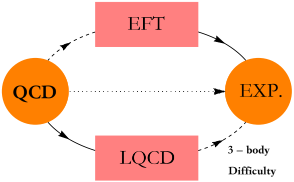
$$\rho \cot\delta(E) = \mathscr{Z}_{00,00}(E, L)$$

- *3-body problem*

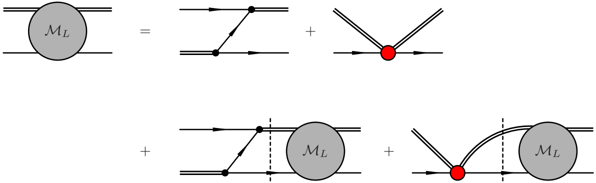
$3 \to 3$ scattering process



M. Hansen et al. , Phys. Rev. D90(2014) 116003



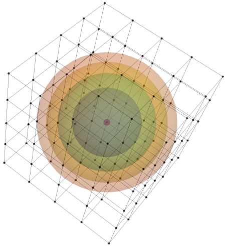
- *Particle-dimer scattering equation in a box*

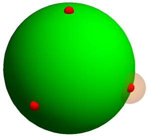


$$\begin{pmatrix} \mathcal{M}_{1,L} & \mathcal{M}_{12,L} \\ \mathcal{M}_{21,L} & \mathcal{M}_{2,L} \end{pmatrix}(\mathbf{p},\mathbf{q}) = \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix}(\mathbf{p},\mathbf{q}) + \frac{4\pi}{L^3}\sum_{\mathbf{k}}^{\Lambda}\begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix}(\mathbf{p},\mathbf{k}) \times \begin{pmatrix} \tau_{1,L}(\mathbf{k}) & \\ & \tau_{2,L}(\mathbf{k}) \end{pmatrix}\begin{pmatrix} \mathcal{M}_{1,L} & \mathcal{M}_{12,L} \\ \mathcal{M}_{21,L} & \mathcal{M}_{2,L} \end{pmatrix}(\mathbf{k},\mathbf{q}) \quad (23)$$

- *3-body force $H_0(\Lambda)$ keeps invariant*
- *loop integral $\to$ loop summation*
- *finite volume dimer τ_L*

- *Cubic periodical boundary condition* $\rightarrow$ *Octahedral group*
  - *Lattice spectrum in irreps.* $\Gamma = A_1^{\pm}, A_2^{\pm}, E^{\pm}, T_1^{\pm}, T_2^{\pm}$
  - *Shell structure of discrete momenta*





- *Projection scheme*

$$\phi(\mathbf{p}) = \phi(g\,\mathbf{p}_0) = \sum_{\Gamma} \mathrm{tr}\left(T^{\Gamma}(g)\phi^{\Gamma}(\mathbf{p}_0)\right) \tag{24}$$

  - g: *symmetry transformation on momentum*
  - $\mathbf{p}_0$: *reference momentum on each shell*
  - $T^{\Gamma}(g)$: *the corresponding transformation matrix in irreps.* Γ
  - $\phi^{\Gamma}(\mathbf{p}_0)$: *matrix component on specific shell in irreps.* Γ

- *Symmetry of equation*

$$\begin{pmatrix} \mathscr{M}_{1,L} & \mathscr{M}_{12,L} \\ \mathscr{M}_{21,L} & \mathscr{M}_{2,L} \end{pmatrix} (\mathbf{p},\mathbf{q}) = \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix} (\mathbf{p},\mathbf{q}) + \frac{4\pi}{L^3} \sum_{\mathbf{k}}^{\Lambda} \begin{pmatrix} Z_1 & Z_{12} \\ Z_{21} & 0 \end{pmatrix} (\mathbf{p},\mathbf{k}) \times \begin{pmatrix} \tau_{1,L}(\mathbf{k}) & \\ & \tau_{2,L}(\mathbf{k}) \end{pmatrix} \begin{pmatrix} \mathscr{M}_{1,L} & \mathscr{M}_{12,L} \\ \mathscr{M}_{21,L} & \mathscr{M}_{2,L} \end{pmatrix} (\mathbf{k},\mathbf{q})$$

  - *loop summation is invariant under symmetry transformation*
  - $\tau_L(g\mathbf{k}) = \tau_L(\mathbf{k}) \quad \Rightarrow \quad \tau_L(\mathbf{k}) = \tau_L(g\mathbf{k}_0^{(s)}) = \tau_L(\mathbf{k}_0^{(s)}) = \tau_L(s)$
  - $Z(g\mathbf{p}, g\mathbf{k}) = Z(\mathbf{p},\mathbf{k}) \quad \Rightarrow \quad Z(\mathbf{p},\mathbf{k}) = Z(g'\mathbf{p}_0^{(r)}, g\mathbf{k}_0^{(s)}) = Z(h\mathbf{p}_0^{(r)}, \mathbf{k}_0^{(s)}) = \sum_{\Gamma} s_{\Gamma} tr\left[T^{\Gamma}(h) Z^{\Gamma}(r,s)\right]$

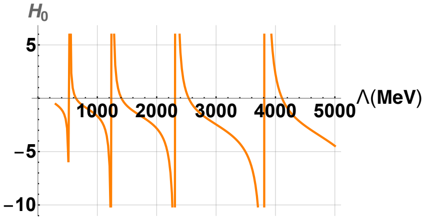
- *``Angular momentum" conservation*

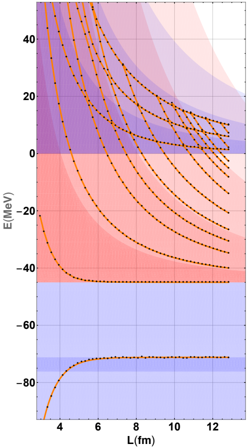
$$\begin{pmatrix} \mathscr{M}_1^{\Gamma} & \mathscr{M}_{12}^{\Gamma} \\ \mathscr{M}_{21}^{\Gamma} & \mathscr{M}_2^{\Gamma} \end{pmatrix} (r,t) = \begin{pmatrix} Z_1^{\Gamma} & Z_{12}^{\Gamma} \\ Z_{21}^{\Gamma} & 0 \end{pmatrix} (r,t) + \frac{4\pi}{L^3} \sum_{s}^{s_\Lambda} \vartheta_s \begin{pmatrix} Z_1^{\Gamma} & Z_{12}^{\Gamma} \\ Z_{21}^{\Gamma} & 0 \end{pmatrix} (r,s) \times \begin{pmatrix} \tau_{1,L}(s) & \\ & \tau_{2,L}(s) \end{pmatrix} \begin{pmatrix} \mathscr{M}_1^{\Gamma} & \mathscr{M}_{12}^{\Gamma} \\ \mathscr{M}_{21}^{\Gamma} & \mathscr{M}_2^{\Gamma} \end{pmatrix} (s,t) \tag{25}$$

- A_1^+ *Quantization Condition*

$$\det\left[\delta_{rs}\begin{pmatrix}\tau_{1,L}^{-1}(s) & \\ & \tau_{2,L}^{-1}(s)\end{pmatrix} - \frac{4\pi}{L^3}\vartheta_s\begin{pmatrix}Z_{1}^{\Gamma}(r,s) & Z_{12}^{\Gamma}(r,s)\\ Z_{21}^{\Gamma}(r,s) & 0\end{pmatrix}\right]=0$$

- *From 3-body force to lattice spectrum*





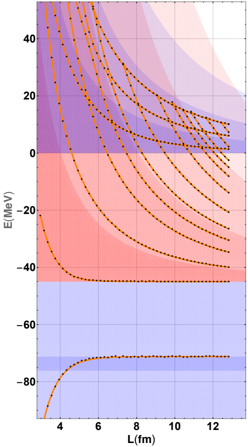
- *Spectrum structure*
  - *DDK-threshold:* $2m_D + m_K \to 0$*MeV*
  - *$DD^*_{s0}(2317)$-threshold:* $m_D + m_{D^*} \to -44.5$*MeV*
  - *DDK-bound state:* -71.2*MeV*

- *Free lines*
  - *3-body (DDK) free lines*

$$E_3 = \frac{1}{2m_D}\left(\frac{2\pi\mathbf{n}_1}{L}\right)^2 + \frac{1}{2m_D}\left(\frac{2\pi\mathbf{n}_2}{L}\right)^2 + \frac{1}{2m_K}\left(\frac{2\pi(\mathbf{n}_1+\mathbf{n}_2)}{L}\right)^2$$
$$= \left(\frac{2\pi}{L}\right)^2 \frac{1}{2\mu}\left[n_1^2 + n_2^2 + 2(m_D/M)\mathbf{n}_1\cdot\mathbf{n}_2\right]$$

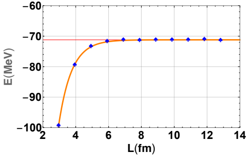
  - *2-body ($DD^*_{s0}(2317)$) free lines*

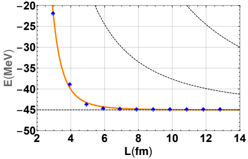
$$\tau^{-1}_{DK}(\frac{2\pi n}{L}; E) = 0 \quad \Rightarrow \quad E_{1+2} = -\frac{\kappa^2_{DK}}{2\mu} + \frac{1}{2\mu}(1 - \frac{m_D^2}{M^2})\left(\frac{2\pi n}{L}\right)^2$$

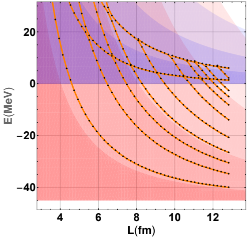


Energy shift

- *DDK 3-body bound state (2-body scattering length a ; 3-body binding momentum κ)*
  - *DDK 3-body picture:* $\kappa a \gg 1 \Rightarrow \Delta E \sim (\kappa L)^{-3/2}\exp(-\#\kappa L)$
    U.-G. Meißner, PRL 114(9) (2015),091602
  - *$DD^*_{s0}(2317)$ 2-body picture:* $\kappa^2 - a^{-2} \ll \kappa^2$
    $\Rightarrow \Delta E \sim (\kappa L)^{-1}\exp(-\#\sqrt{\kappa^2 - a^{-2}}L)$ M. Lüscher, NPB 354(1991) 531
  - *linear combination of 2 pictures*
- *$DD^*_{s0}(2317)$ 2-body scattering state*
  - *check the scatting length between $D - D^*_{s0}(2317)$*
  - *multiple spectra between two free lines*
- *DDK 3-body scattering state*
  - *avoided level crossing*
  - *perturbative calculation* JYP, J.-J Wu et. al. PRD 99(2019),074513
    $$\Delta E \sim \frac{g_0}{L}\left(1 + \frac{g_1}{L} + \frac{g_2}{L^2} + \frac{g_3}{L^3}\ln\frac{mL}{2\pi} + \frac{g_4}{L^3} + \cdots\right)$$

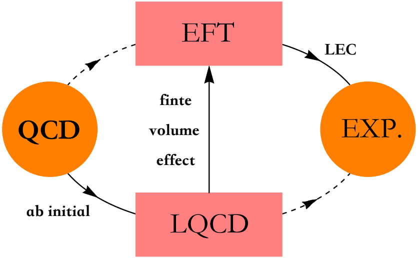






Outline

Summary

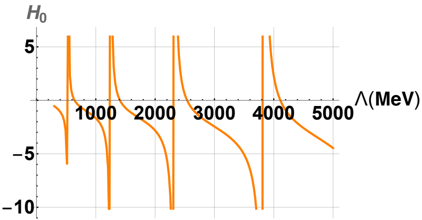


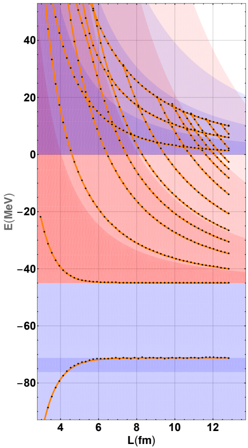
- *Build 3-body EFT for DDK 3-body system* (based on L.-S. Geng et al. PRD 100(2019)3,034029)
- *Construct DDK 3-body quantization condition in A_1^+*

  (based on Hammer, JYP and Rusetsky. JHEP 10(2017))
- *Calculate A_1^+ lattice spectrum for DDK 3-body system*

Outlook

- *Consider 3-body force in* $O(p^2)$
- *Resolve the energy shift*
- *Calculate lattice spectrum in irreps.* $A_1^-, E^{\pm}, \cdots$

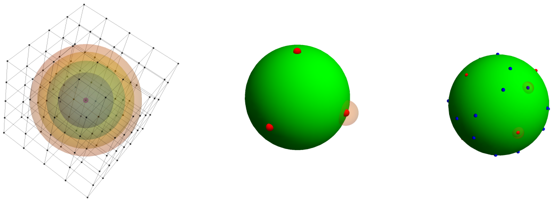




Thank you for your attention!

Appendix: Shell Structure and Cubic Irreps. Expansion

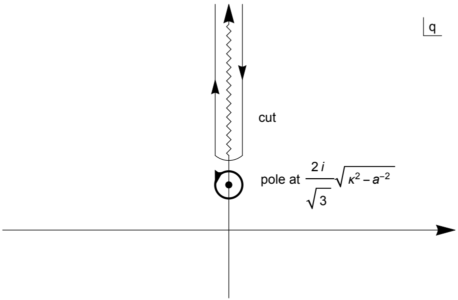
- ***Shell Structure*** Shell is a set of momenta with the same $|\mathbf{p}|$, which can be obtained from reference momentum $\mathbf{p}_0$, $\mathbf{p} = g\mathbf{p}_0, \quad g \in O_h$. The momenta unrelated by the O_h, but having $|\mathbf{p}| = |\mathbf{p}'|$, belong to the different shells.



- ***Cubic Irreps. Expansion***

$$f(\mathbf{p}) \quad = f(g\mathbf{p}_0(s)) = \sum_{\Gamma, ij} f_{ij}^{(\Gamma)}(s)\, T_{ij}^{(\Gamma)}(g). \tag{26}$$

$$\sum_{\mathbf{p}} f(\mathbf{p}) = \underbrace{\sum_{s}}_{\text{different shells}} \underbrace{\frac{\vartheta_s}{G} \sum_{g}}_{\text{orientations inside shell } s} f(g\mathbf{p}_0(s)). \tag{27}$$



- *Energy Shifts of Bound States*

$$\Delta E = 8\pi \int \frac{d^3 q}{(2\pi)^3} \phi^\dagger(\mathbf{q}) \sum_{\mathbf{n}\neq 0} e^{i\mathbf{n}\mathbf{q}L} \tau(\mathbf{q})\phi(\mathbf{q}) + \cdots \tag{28}$$

  - *Regular Wave Function* $\phi \sim$ *const.*
  - *Cut and Pole of* $\tau(\mathbf{q}; E) = \frac{1}{-a^{-1} + \sqrt{\frac{3}{4}\mathbf{q}^2 - mE - i\varepsilon}}$

Appendix: Energy Shifts

The energy of the scattering states vanishes in the infinite volume limit. We quote their finite volume energy E in terms of the quantity $\kappa^2 = L^2 mE/(2\pi)^2$.

The energy shift of the ground state (which resides in the A_1^+ irrep) is:

$$\kappa^2 = \frac{g_0}{L}\left(1 + \frac{g_1}{L} + \frac{g_2}{L^2} + \frac{g_3}{L^3}\ln\frac{mL}{2\pi} + \frac{g_4}{L^3} + \cdots\right), \tag{29}$$

with

$$\begin{aligned}
g_0 &= \frac{3}{\pi}a, \\
g_1 &= 2.837297480\, a, \\
g_2 &= 9.725330808\, a^2, \\
g_3 &= 8\pi\left(\frac{2\sqrt{3}}{\pi} - \frac{8}{3}\right)a^3, \\
g_4 &= \left(-5.159159617 + 6\pi\left(\frac{r}{a}\right) - 8\pi\left(\frac{\hat{\mathscr{M}}}{a^2}\right)\right)a^3.
\end{aligned} \tag{30}$$

The energy shift of the 1st excited state in the A_1^+ irrep is

$$\kappa^2 - 1 = \frac{h_0}{L}\left(1 + \frac{h_1}{L} + \frac{h_2}{L^2} + \frac{h_3}{L^3}\ln\frac{mL}{2\pi} + \frac{h_4}{L^3} + \cdots\right), \tag{31}$$

with

$$\begin{aligned} h_0 &= \frac{10}{\pi} a, \\ h_1 &= 0.279070\, a, \\ h_2 &= \left(8.494802 + \frac{7\pi^2}{5}\left(\frac{r}{a}\right)\right) a^2, \\ h_3 &= \frac{27}{5} \times 8\pi \left(\frac{2\sqrt{3}}{\pi} - \frac{8}{3}\right) a^3, \\ h_4 &= \left(-172.001650 + 83.745841\left(\frac{r}{a}\right) - \frac{27}{5} \times 8\pi\left(\frac{\hat{\mathscr{M}}}{a^2}\right)\right) a^3. \end{aligned} \tag{32}$$