# Triangle singularity appearing as an X(3872)-like peak in $B \to (J/\psi\ \pi^+\pi^-)\ K\ \pi$

arXiv: 1912.11830

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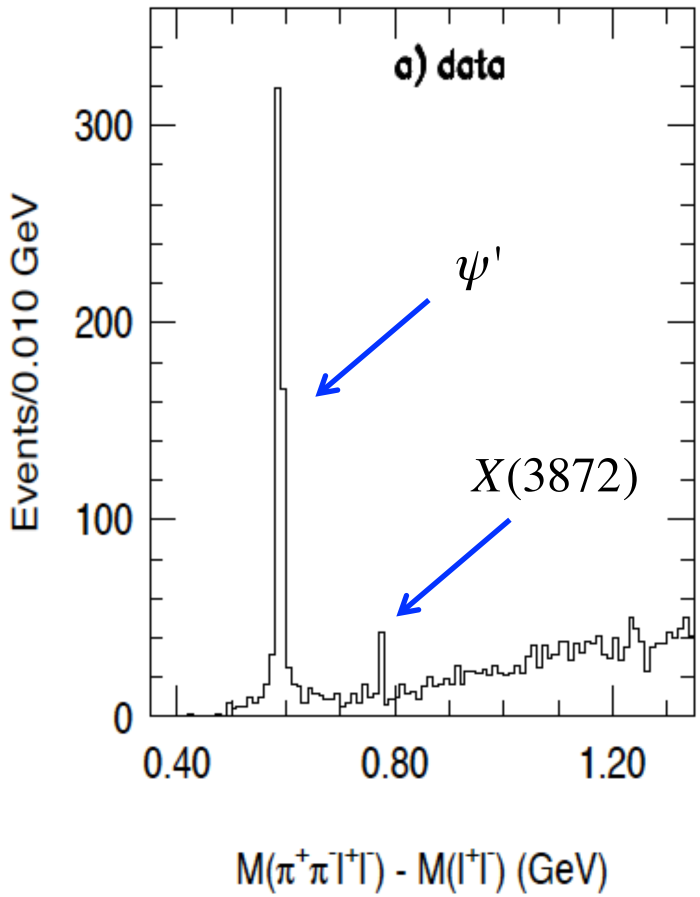
# Introduction

# Discovery of X(3872)

$$B^+ \to J/\psi \pi^+ \pi^- K^+$$




Belle, PRL 91 262001 (2003)

X(3872) is outstanding exotic hadron candidate

Mass different from quark model prediction

$$M(X(3872)) = 3871.69 \pm 0.17 \text{ MeV}$$

Almost exactly on $D^0\bar{D}^{*0}$ threshold

$$M(X(3872)) - M(D^0) - M(D^{*0}) = 0.00 \pm 0.18 \text{ MeV}$$

Very narrow width

$$\Gamma(X(3872)) < 1.2 \text{ MeV}$$

Large isospin violation

$$\frac{B(X(3872) \to J/\psi \rho^0 \to J/\psi \pi^+ \pi^-)}{B(X(3872) \to J/\psi\, \omega \to J/\psi \pi^+ \pi^- \pi^0)} \sim 1$$

# Interpretations of X(3872) (very incomplete list)

from several hundreds papers

## Phenomenological model

$D^0\bar{D}^{*0}$ molecule : Swanson PLB 588 (2004); Zhao et al. PRD 89 (2014)

$D^0\bar{D}^{*0} + c\bar{c}$ : Suzuki PRD 72 (2005); Kalashnikova PRD 72 (2005);
Takizawa et al. PTEP 2013 (2013)

Diquark-antidiquark : Maiani et al. PRD 71 (2005); Chen et al. PRD 83 (2011)

## (semi) First principle

Lattice QCD: Prelovsek *et al.* PRL 111 (2013); Padmanath *et al.* PRD 92 (2015)

X(3872)-like state found below $D^0\bar{D}^{*0}$ threshold; diquark-antidiquark disfavored

Dyson-Schwinger Eq. : Wallbott et al. PRD 100 (2019)

X(3872)-like state found at 3916(74) MeV

## Kinematical effect

Threshold cusp : Bugg PRD 71 (2005); JPG 35 (2008) ← already ruled out

# Most interpretations of X(3872) still survive !

**Main reason** Lots of uncertainties in relevant hadron dynamics

ex. coupling strengths of $D^0\bar{D}^{*0} \to D^0\bar{D}^{*0}, J/\psi\rho^0, J/\psi\omega, c\bar{c}$ ???

Common situation in hadron physics

Uncertain dynamical contents are fitted to data

→ Many models look ok and then more data are needed to discriminate them

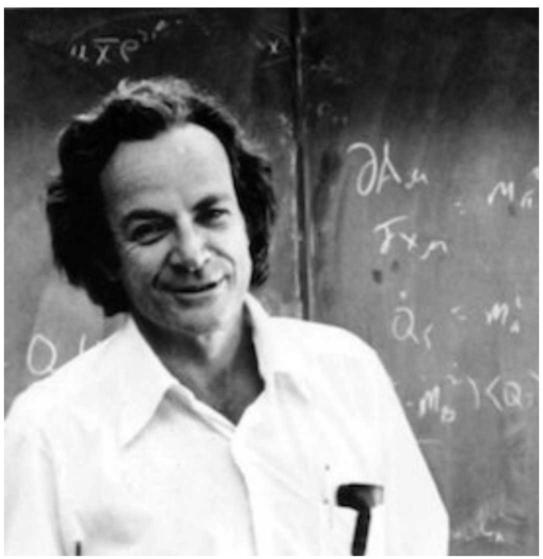
Seems difficult to reach a consensus within the so-far proposed models of X(3872)

(maybe except for LQCD)

## Q: How to stand out from the crowd ? Ans : Show a great predictive power !

**Problem**: Without fine-tuning model parameters (parameter-free calculation), get X(3872) mass, 3871.69 ± 0.17 MeV, within the error (0.01% precision)

Solve it !

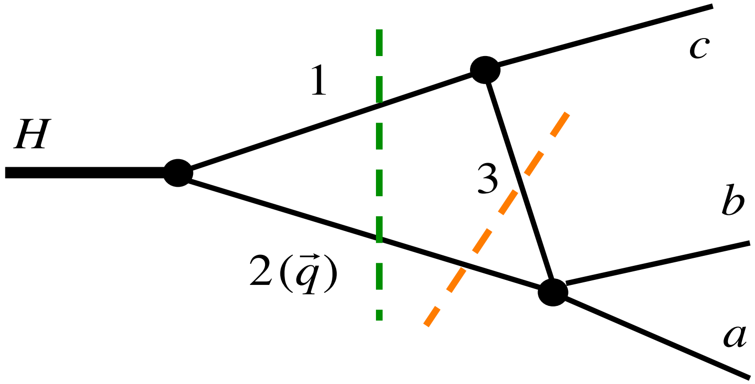


Surely you're joking,

Mr. Nakamura !

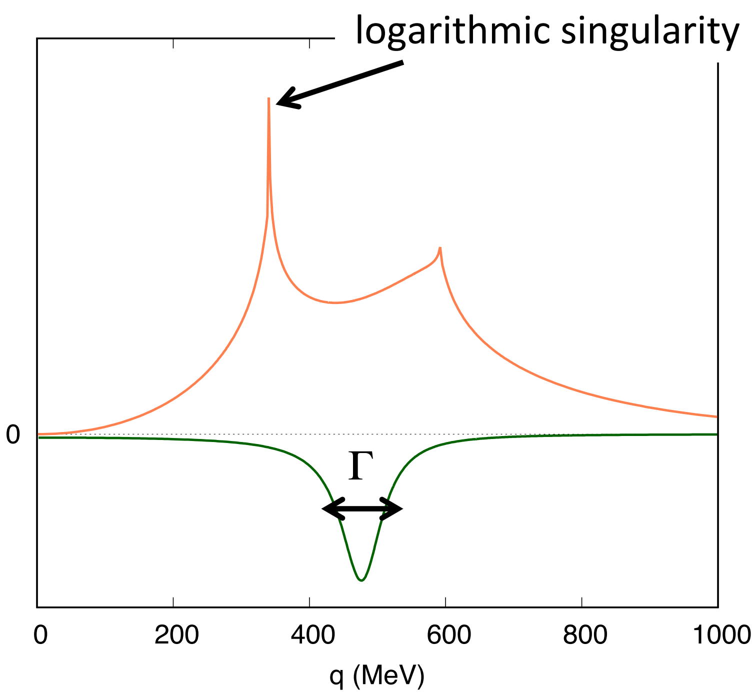
Now, I'll show you how (*I'm not kidding !*)

# New interpretation: Triangle Singularity

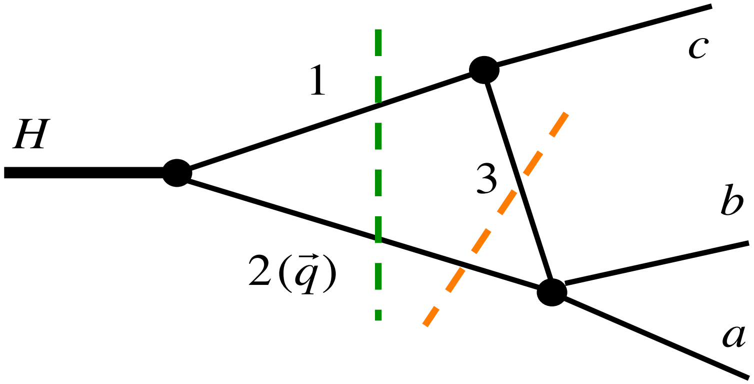



$$A \sim \int dq \, \frac{1}{E - E_1 - E_2 + i\frac{\Gamma}{2}}$$

$$\times \int d\Omega_q \, \frac{q^2}{E - E_2 - E_3 - E_c + i\varepsilon}$$



# New interpretation: Triangle Singularity




$$A \sim \int dq \, \frac{1}{E - E_1 - E_2 + i\frac{\Gamma}{2}}$$

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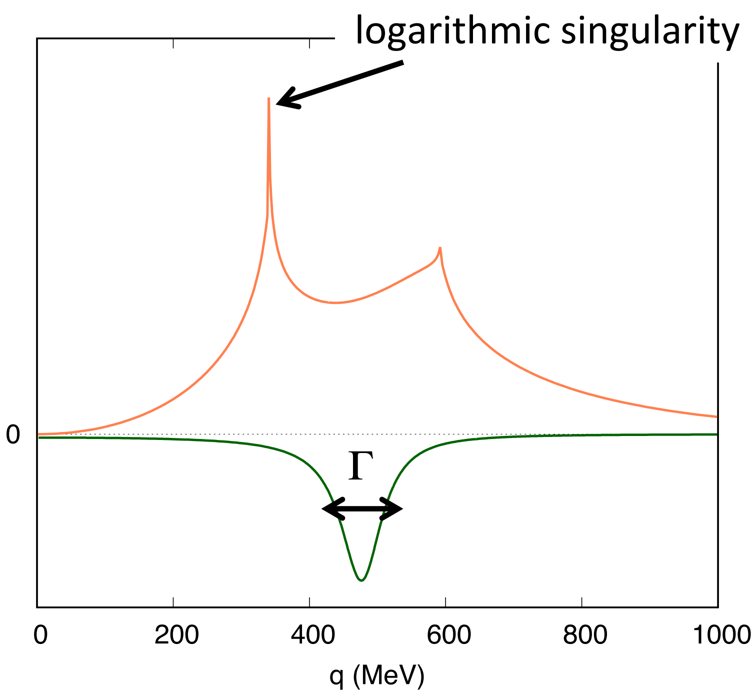
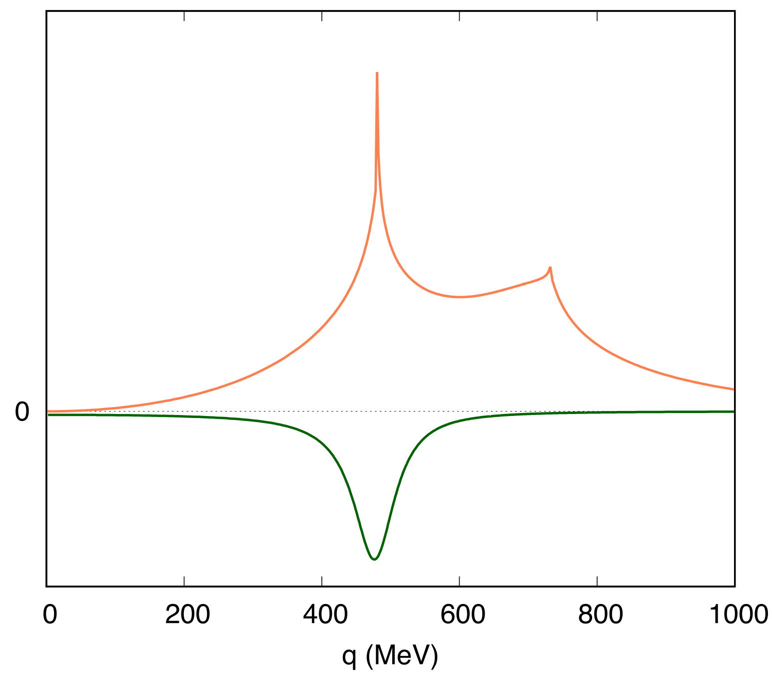
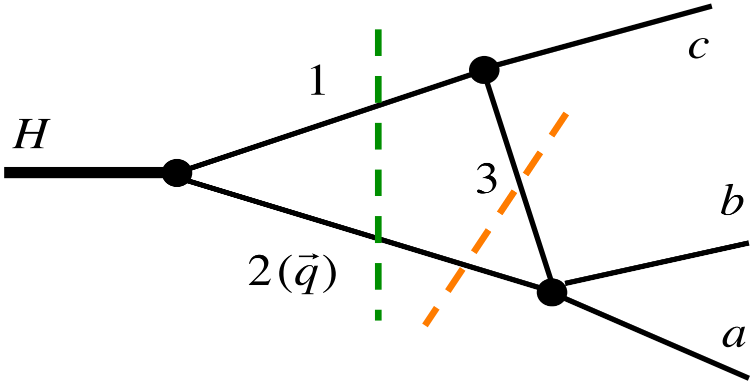


# New interpretation: Triangle Singularity




$$A \sim \int dq \, \frac{1}{E - E_1 - E_2 + i\frac{\Gamma}{2}}$$

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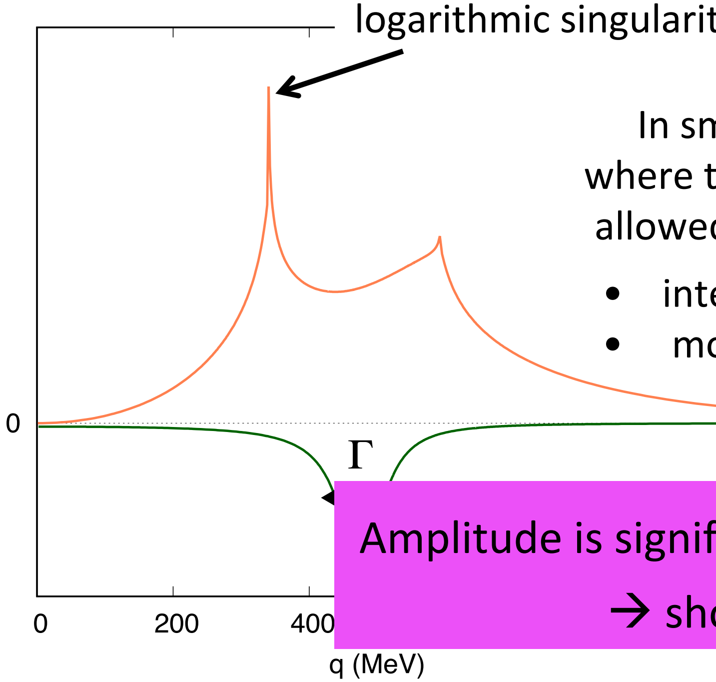
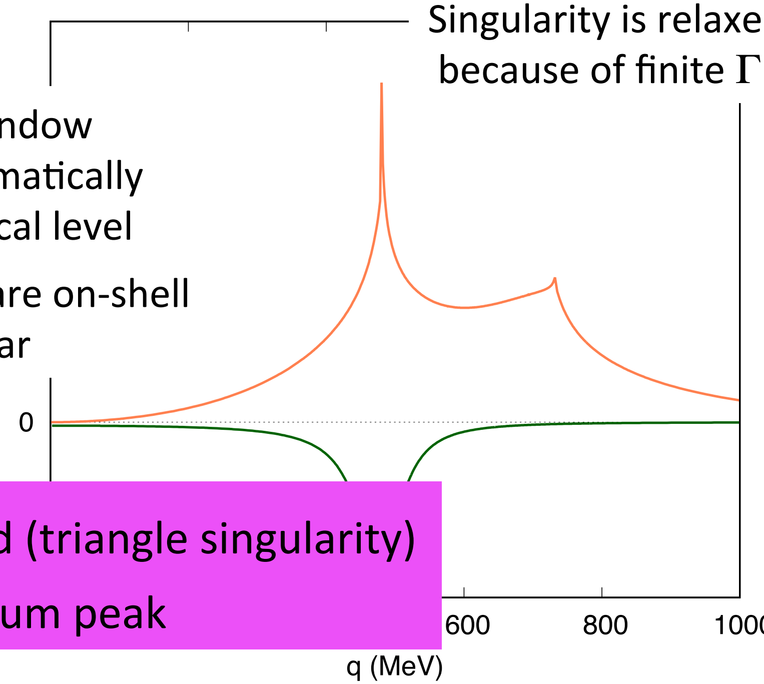



In small kinematical window where the process is kinematically allowed to occur at classical level

- intermediate states are on-shell
- momenta are collinear

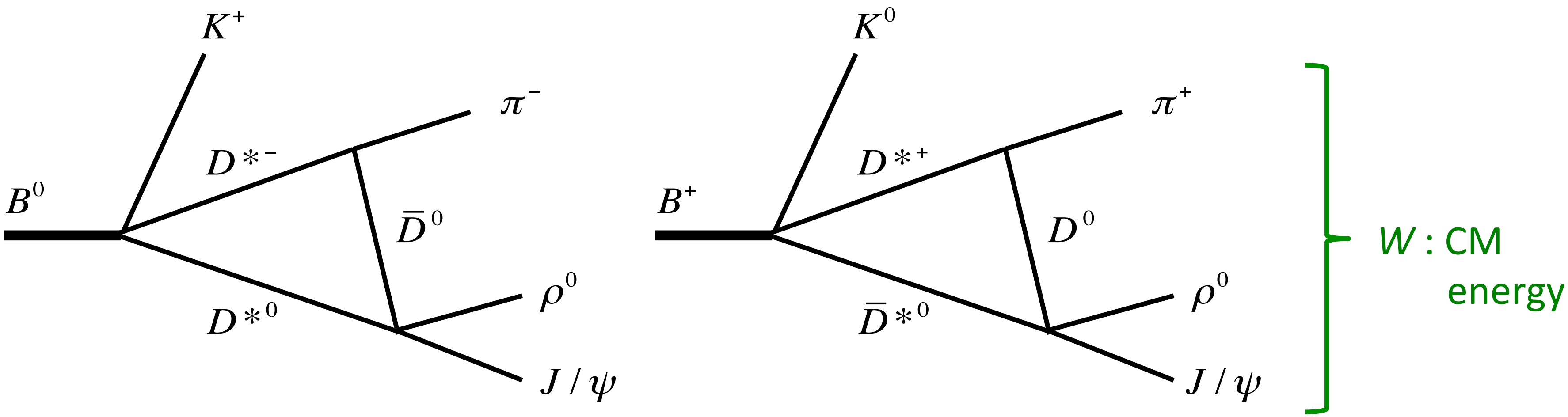



Amplitude is significantly enhanced (triangle singularity)

→ shows up as spectrum peak

# Triangle singularity for X(3872) in B → (J/ψ π⁺π⁻) K π

X(3872) observed by Belle, PRD 91 051101(R) (2015)




At zero-width limit, diagrams exactly hit triangle singularity at (using PDG masses)

$M_{J/\psi\,\rho^0} = 3871.6 - 3871.9\,\text{MeV}$ when $m_{D^{*\pm}} + m_{D^{*0}} < W < m_{D^{*\pm}} + m_{D^{*0}} + 0.9$ MeV

cf. $M(X(3872)) = 3871.69 \pm 0.17$ MeV

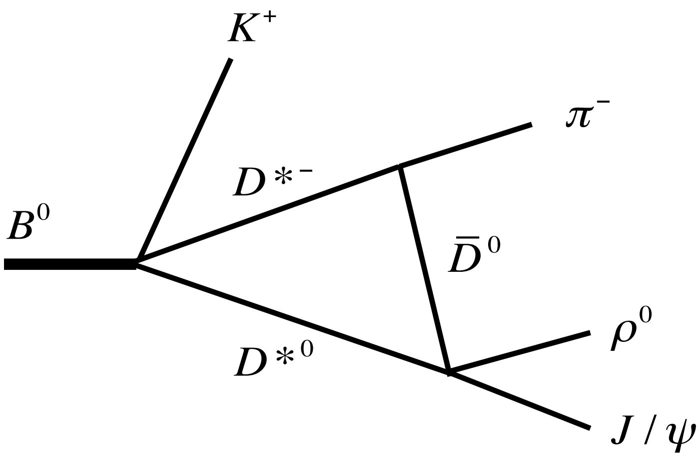
For finite widths, triangle singularities are relaxed but $\Gamma_{D^{*\pm}} \approx 83$ keV and $\Gamma_{D^{*0}} \approx 55$ keV

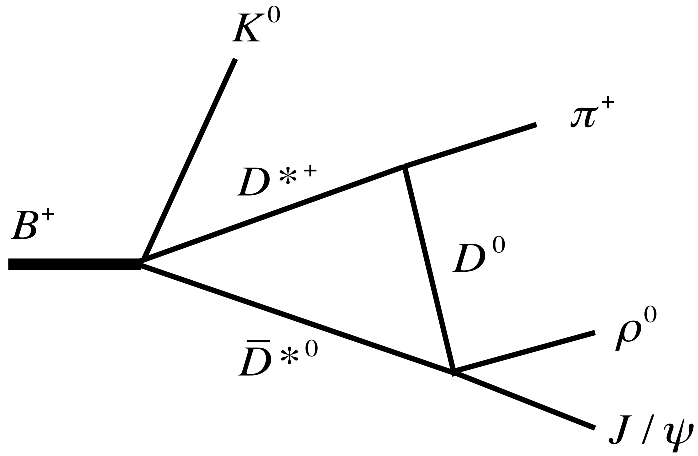
→ sharp spectrum peak expected at $M_{J/\psi\,\rho^0} \sim 3871.7\,\text{MeV}$ → Numerical demonstration

# Triangle singularity for X(3872) in B → (J/ψ π⁺π⁻) K π

Triangle singularity (TS) is very sensitive to hadron masses

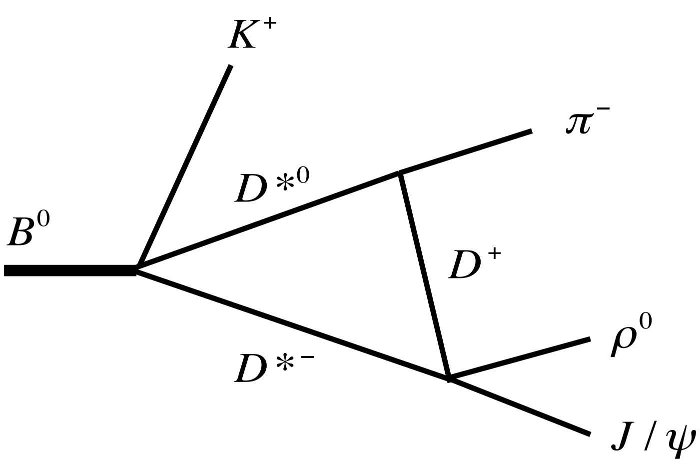
TS exists ?

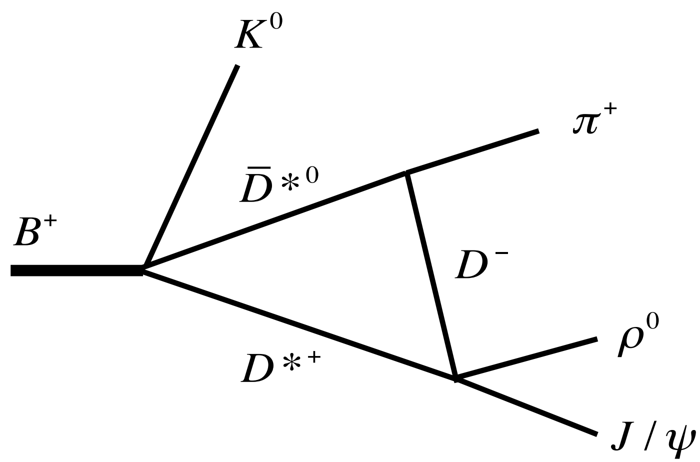



← YES

Isospin partner diagrams



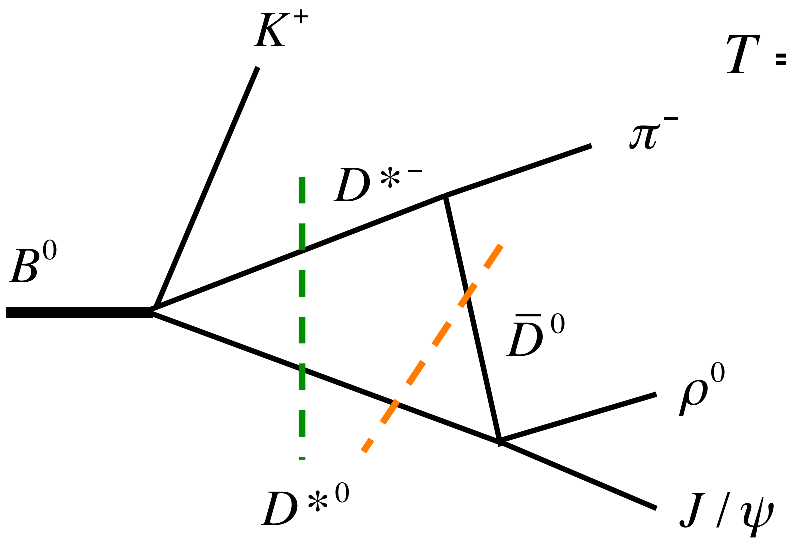

← NO

$D^{*0} \not\to D^+\pi^-$

at on-shell

# Model

# Triangle amplitude for X(3872) in B → (J/ψ π⁺π⁻) K π




$$T = \int d^3q\, v_{\bar{D}^0 D^{*0} \to J/\psi \rho^0} \frac{1}{W - E_{D^{*0}} - E_{\bar{D}^0} - E_\pi + i\frac{\Gamma_{D^{*0}}}{2}}$$

$$\times\ \Gamma_{D^{*-} \to \bar{D}^0 \pi^-} \frac{1}{W - E_{D^{*-}} - E_{D^{*0}} + i\frac{\Gamma_{D^{*-}} + \Gamma_{D^{*0}}}{2}}$$

$$\times\ \Gamma_{B^0 \to D^{*-} D^{*0} K^+}$$

$$v_{\bar{D}^0 D^{*0} \to J/\psi \rho^0} = f(p_{J/\psi \rho^0}) f(p_{\bar{D}^0 D^{*0}})\, \vec{\varepsilon}_{J/\psi} \times \vec{\varepsilon}_{\rho^0} \cdot \vec{\varepsilon}_{D^{*0}}$$

: s-wave interaction

consistent with $J^P$=1⁺ of X(3872)

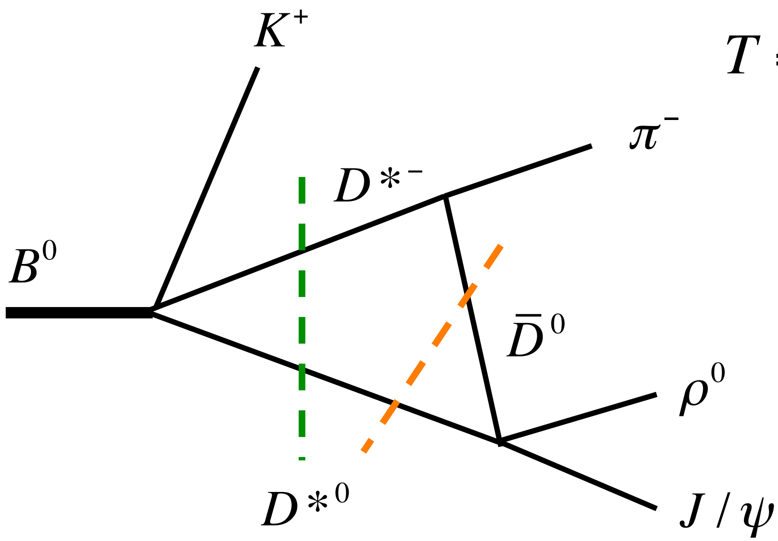
Microscopically, $D_1$-meson exchange or quark exchange mechanism (No X(3872) pole assumed)

Arbitrary coupling strength (no experimental and LQCD inputs) : irrelevant to spectrum shape

$f(p_{ij})$ : dipole form factor with cutoff 1 GeV

Spectrum shape is determined by kinematical effect → insensitive to cutoff → check

# Triangle amplitude for X(3872) in B → (J/ψ π⁺π⁻) K π




$$T = \int d^3q \; v_{\bar{D}^0 D^{*0} \to J/\psi \rho^0} \frac{1}{W - E_{D^{*0}} - E_{\bar{D}^0} - E_\pi + i\frac{\Gamma_{D^{*0}}}{2}}$$

$$\times \; \Gamma_{D^{*-} \to \bar{D}^0 \pi^-} \frac{1}{W - E_{D^{*-}} - E_{D^{*0}} + i\frac{\Gamma_{D^{*-}} + \Gamma_{D^{*0}}}{2}}$$

$$\times \; \Gamma_{B^0 \to D^{*-} D^{*0} K^+}$$

$$\Gamma_{D^{*-} \to \bar{D}^0 \pi^-} = f(p_{\bar{D}^0 \pi^-}) \; \vec{p}_{\pi^-} \cdot \vec{\varepsilon}_{D^{*-}}$$ ← determined by partial decay width data

$$\Gamma_{B^0 \to D^{*-} D^{*0} K^+} = f(p_{K^+}) f(p_{D^{*-} D^{*0}}) \; \vec{\varepsilon}_{D^{*-}} \cdot \vec{\varepsilon}_{D^{*0}}$$ ← strength left arbitrary

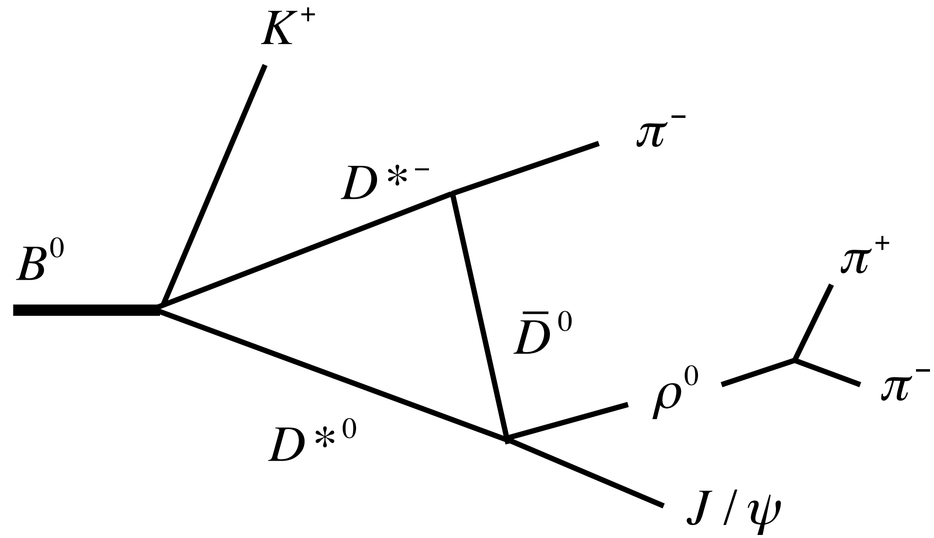
Fairly large branching reported: $\mathrm{Br}(B^0 \to D^{*-} D^{*0} K^+) = 1.06 \pm 0.03 \text{ (stat.)} \pm 0.086 \text{ (syst.)}\%$

BaBar, PRD 83 032004 (2011)

Not useful information to determine the relevant amplitude at $M_{D^{*\pm} D^{*0}} \sim m_{D^{*\pm}} + m_{D^{*0}}$

# J/ψ $\pi^+\pi^-$ invariant mass distribution for

# B → (J/ψ $\pi^+\pi^-$) K π




$$\frac{d\Gamma_{B\to\psi\pi^+\pi^- K\pi}}{dW\, dM_{\psi\pi^+\pi^-}} = \int \frac{dM_{\pi\pi}}{2\pi} \frac{d\Gamma_{B\to\psi\rho^0 K\pi}}{dW\, dM_{\psi\rho^0}}\bigg|_{m_{\rho^0}=M_{\pi\pi}} \times \frac{[M_{\pi\pi}/E_{\rho^0}]^2\, \Gamma_{\rho^0\to\pi^+\pi^-}(M_{\pi\pi})}{|\tilde{W} - E_{\rho^0} + \frac{i}{2}\Gamma_{\rho^0}(M_{\pi\pi})|^2}$$

$$E_{\rho^0} = \sqrt{\bar{m}^2_{\rho^0} + \boldsymbol{p}^2_{\rho^0}} \quad (\bar{m}_{\rho^0} = 775 \text{ MeV})$$

Energy-dependent ρ width

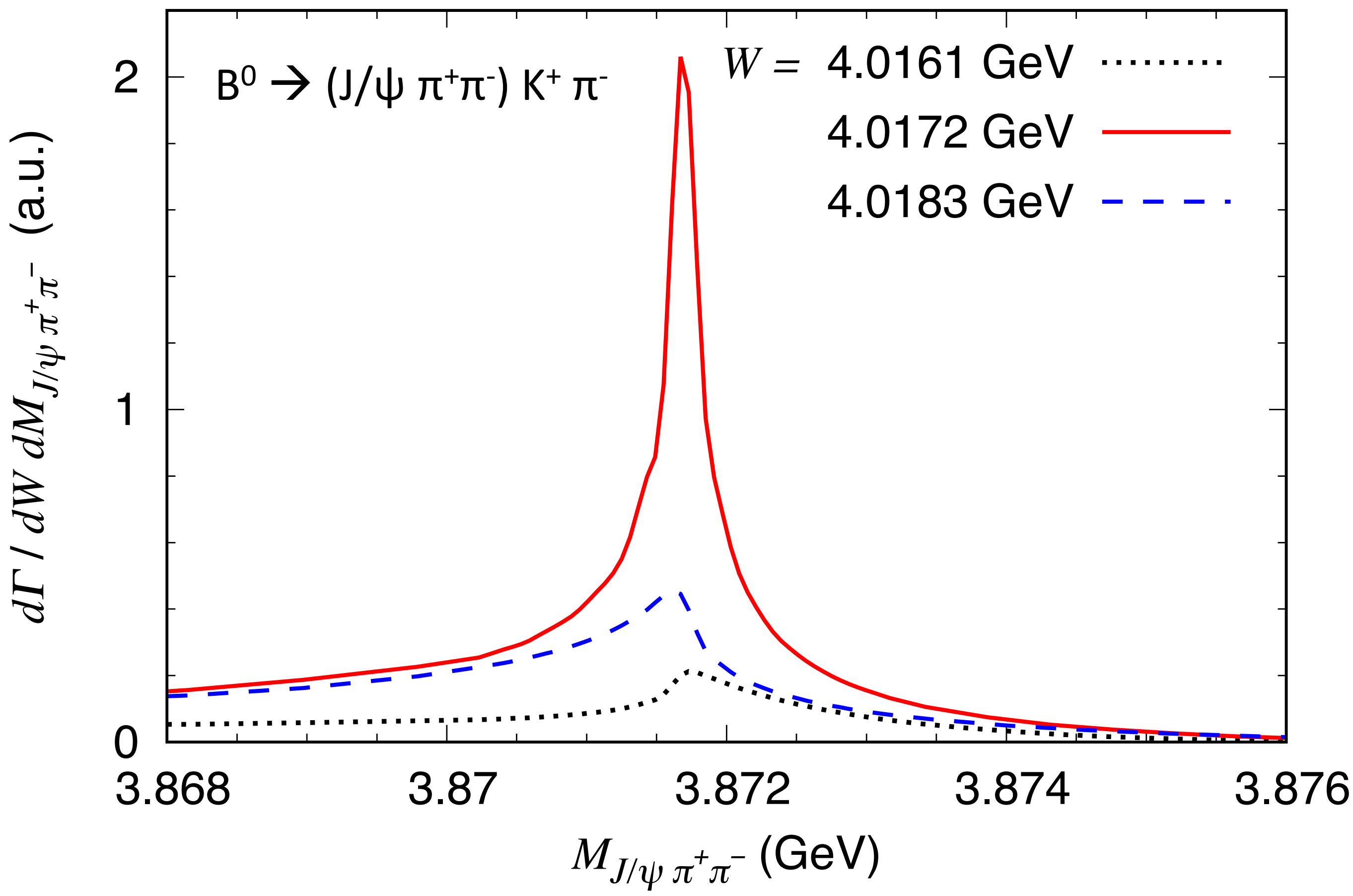
$$\Gamma_{\rho^0}/\bar{\Gamma}_{\rho^0} = (q/\bar{q})^3 (\bar{m}_{\rho^0}/M_{\pi\pi})[f^{10}_{\pi\pi}(q)/f^{10}_{\pi\pi}(\bar{q})]^2 \;,$$

$q$: pion momentum

$$\bar{\Gamma}_{\rho^0} = 150 \text{ MeV}$$

# Results

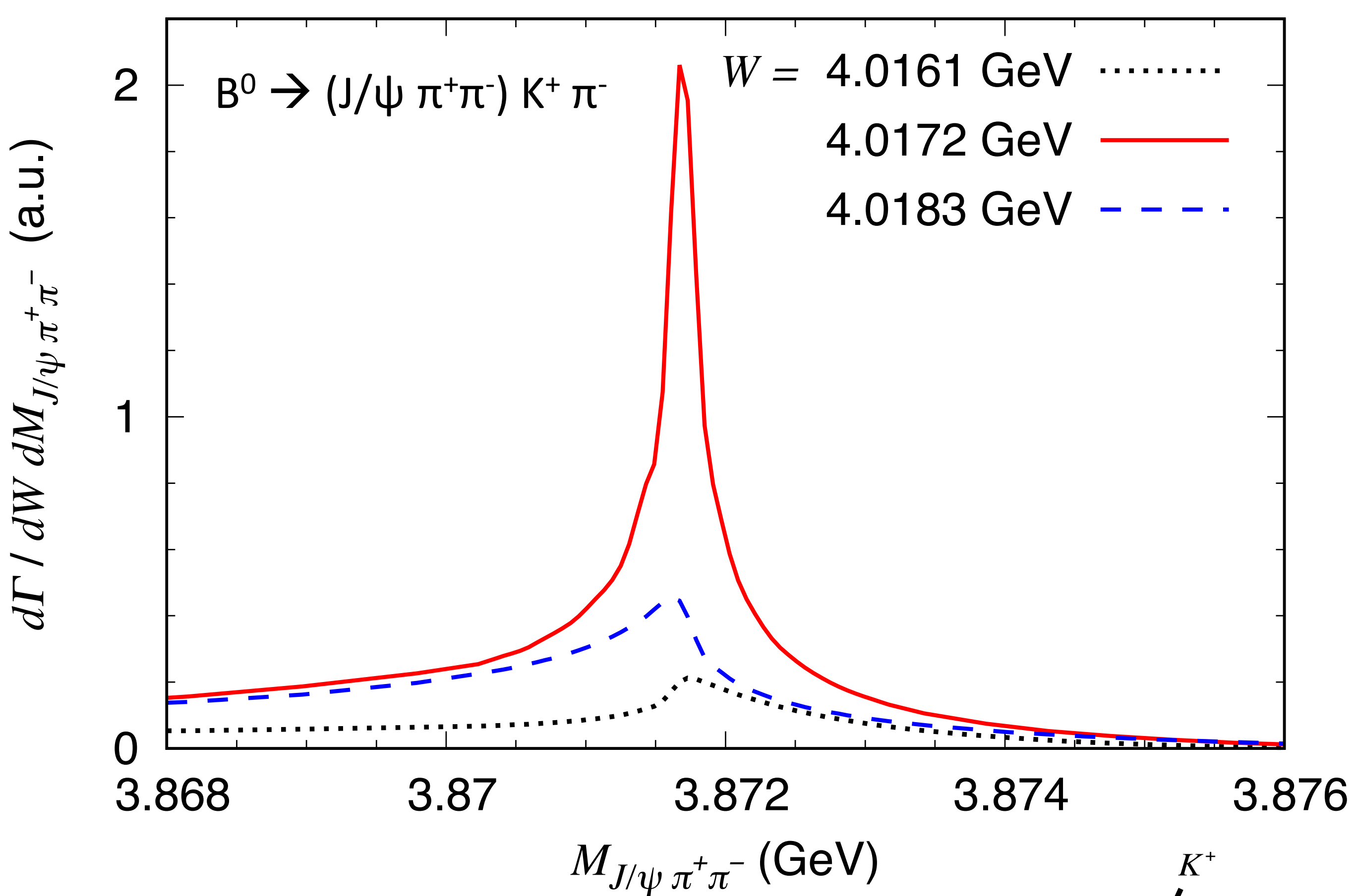
# $J/\psi\ \pi^+\pi^-$ invariant mass spectrum from triangle diagram

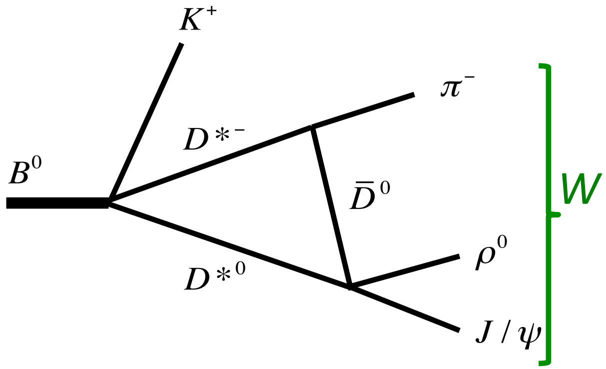



- Clear resonance-like peaks generated by the triangle singularity (TS) at ~3871.7 MeV
- Cutoff dependence does not change the peak position and shape

← TS dominates due to very small D* width, and TS does not depend on dynamical details

# $J/\psi\ \pi^+\pi^-$ invariant mass spectrum from triangle diagram

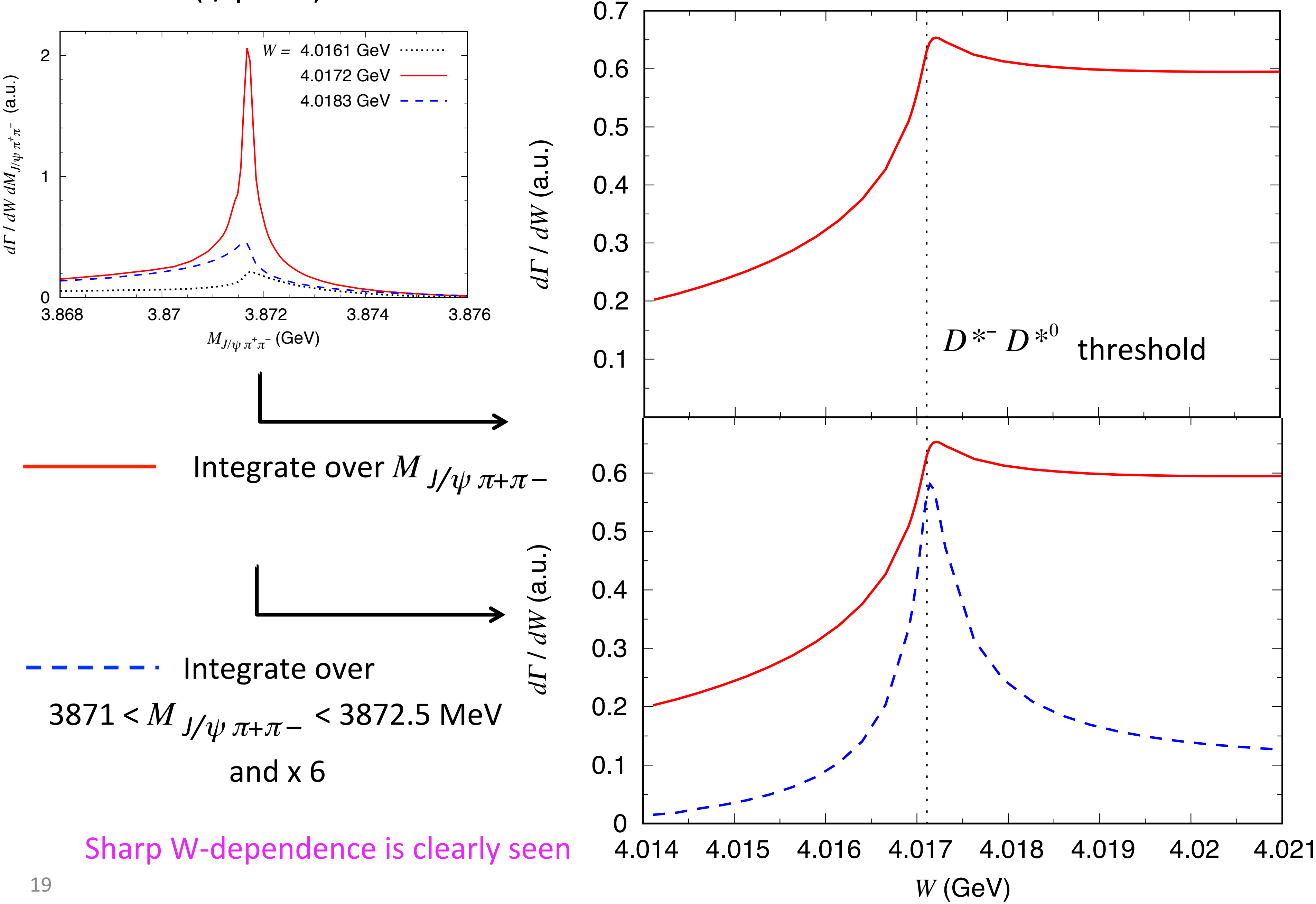



- Acute sensitivity of peak height to $W$ because TS happens only when $m_{D^{*\pm}} + m_{D^{*0}} < W < m_{D^{*\pm}} + m_{D^{*0}} + 0.9\ \mathrm{MeV}$

# *W* distribution

$B^0 \rightarrow (J/\psi\, \pi^+\pi^-)\, K^+\, \pi^-$




Integrate over $M_{J/\psi\,\pi+\pi-}$

Integrate over

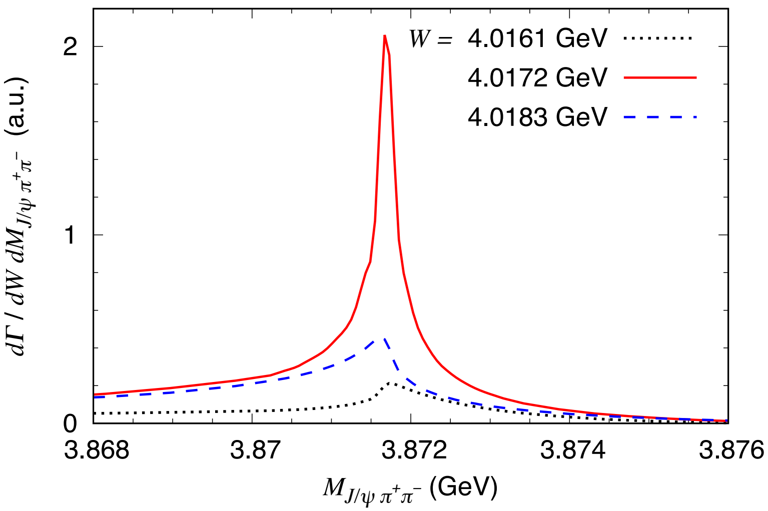
$3871 < M_{J/\psi\,\pi+\pi-} < 3872.5$ MeV

and x 6

Sharp W-dependence is clearly seen

# $J/\psi\ \pi^+\pi^-$ distribution

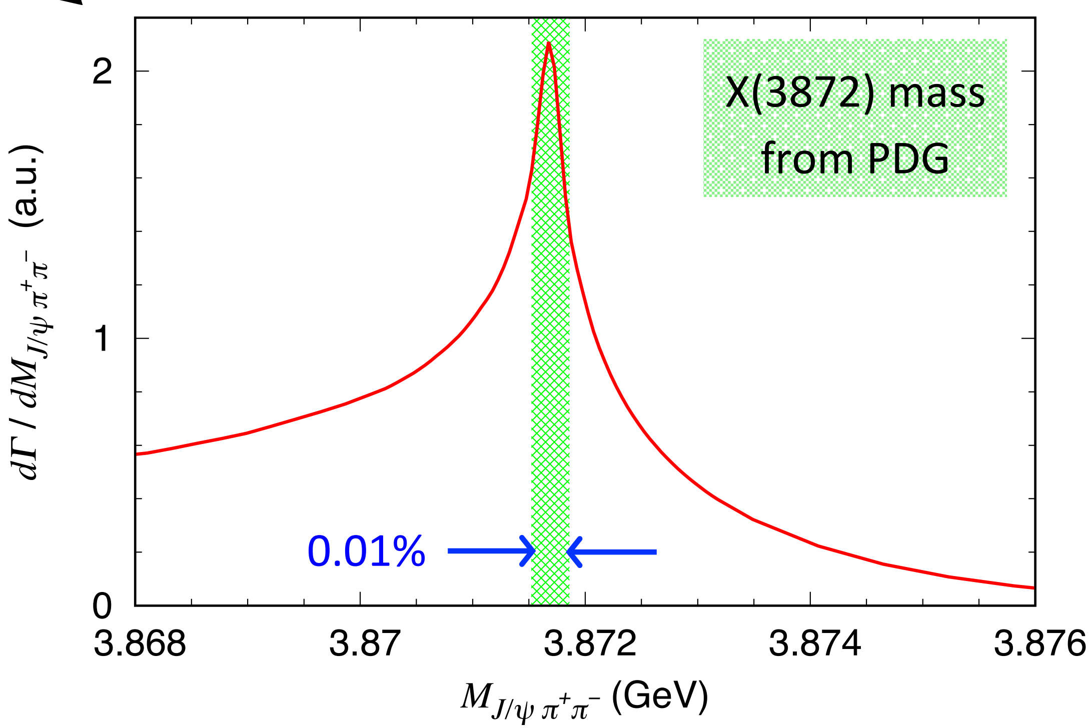
$B^0 \to (J/\psi\ \pi^+\pi^-)\ K^+\ \pi^-$




Belle studied $d\Gamma / d\,M_{J/\psi\ \pi+\pi-}$ of $B \to (J/\psi\ \pi^+\pi^-)\ K\pi$ and found X(3872) in $J/\psi\ \pi^+\pi^-$ distribution

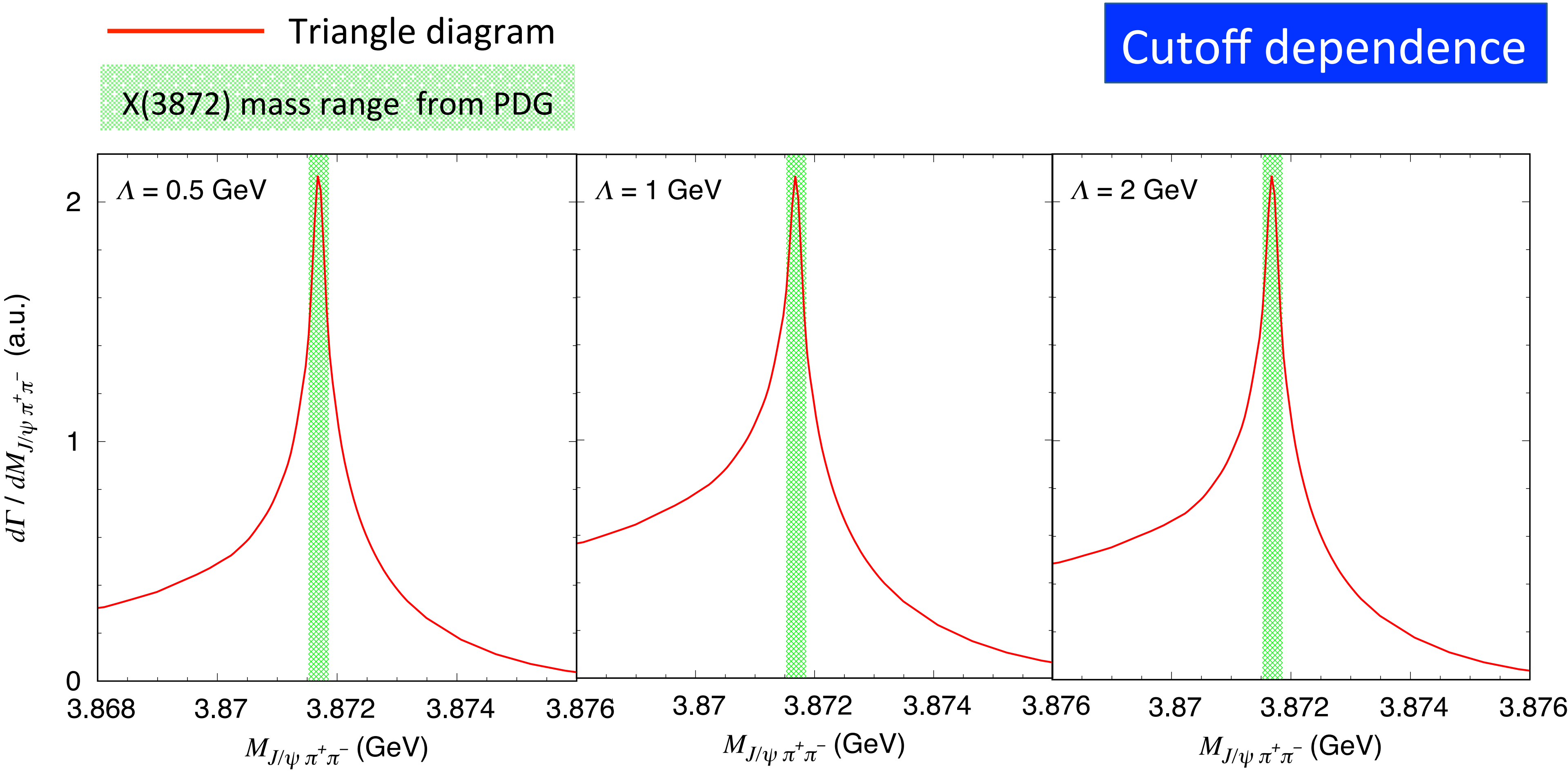
Integrate over $4014 < W < 4021$ MeV

($W$ region where the peak is visible)



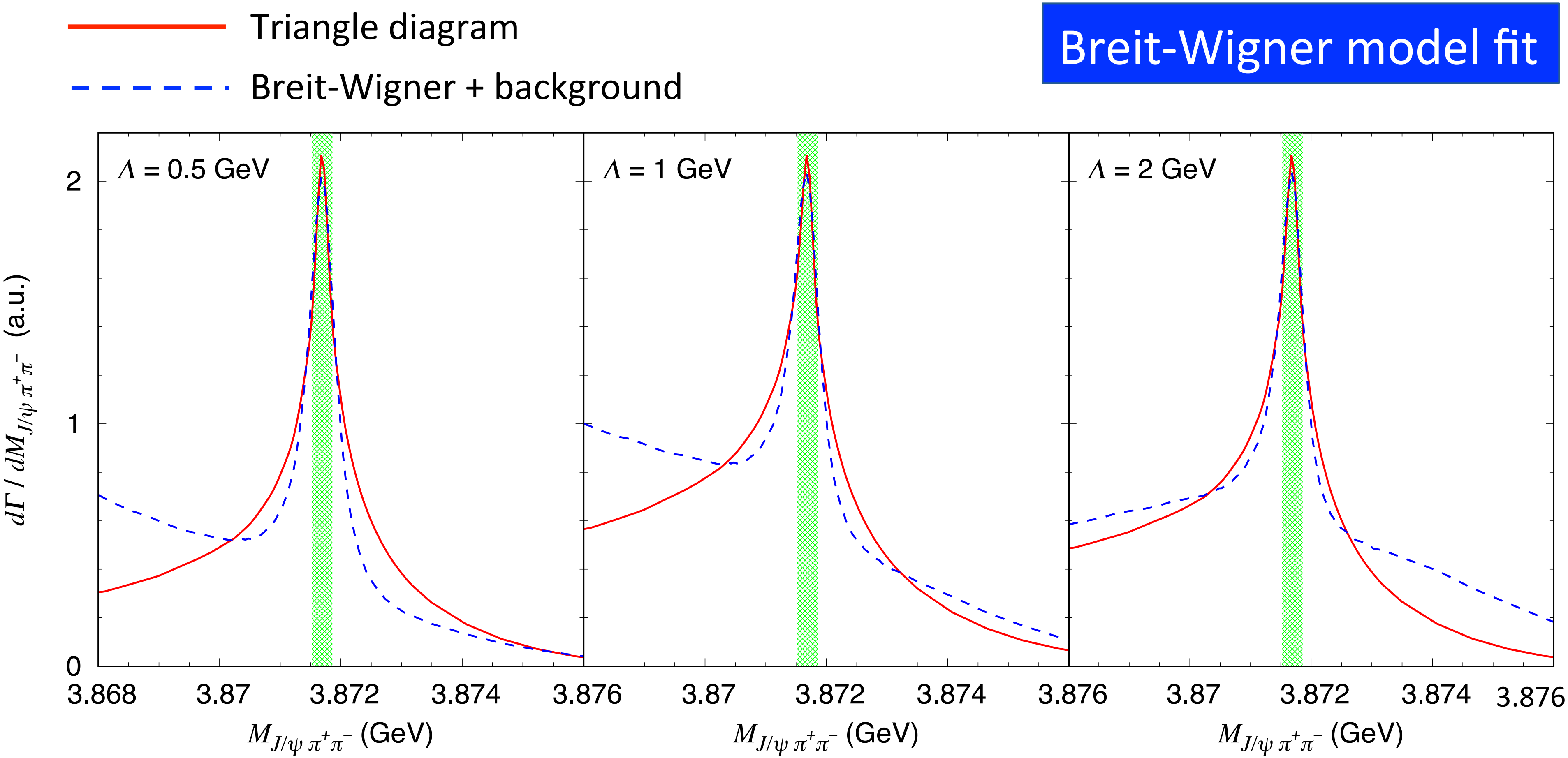

- Clear peak at $\sim 3872$ MeV remains
- Peak position in perfect agreement with X(3872) mass range of PDG value

# Cutoff dependence

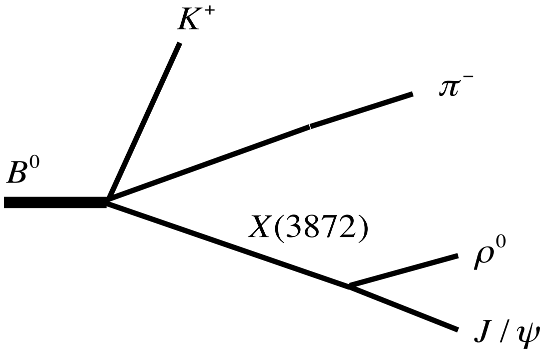



- In the model, cutoff is only parameter that can change the spectrum shape
- Peak position and width hardly depend on the cutoff values

# Breit-Wigner model fit




Breit-Wigner model



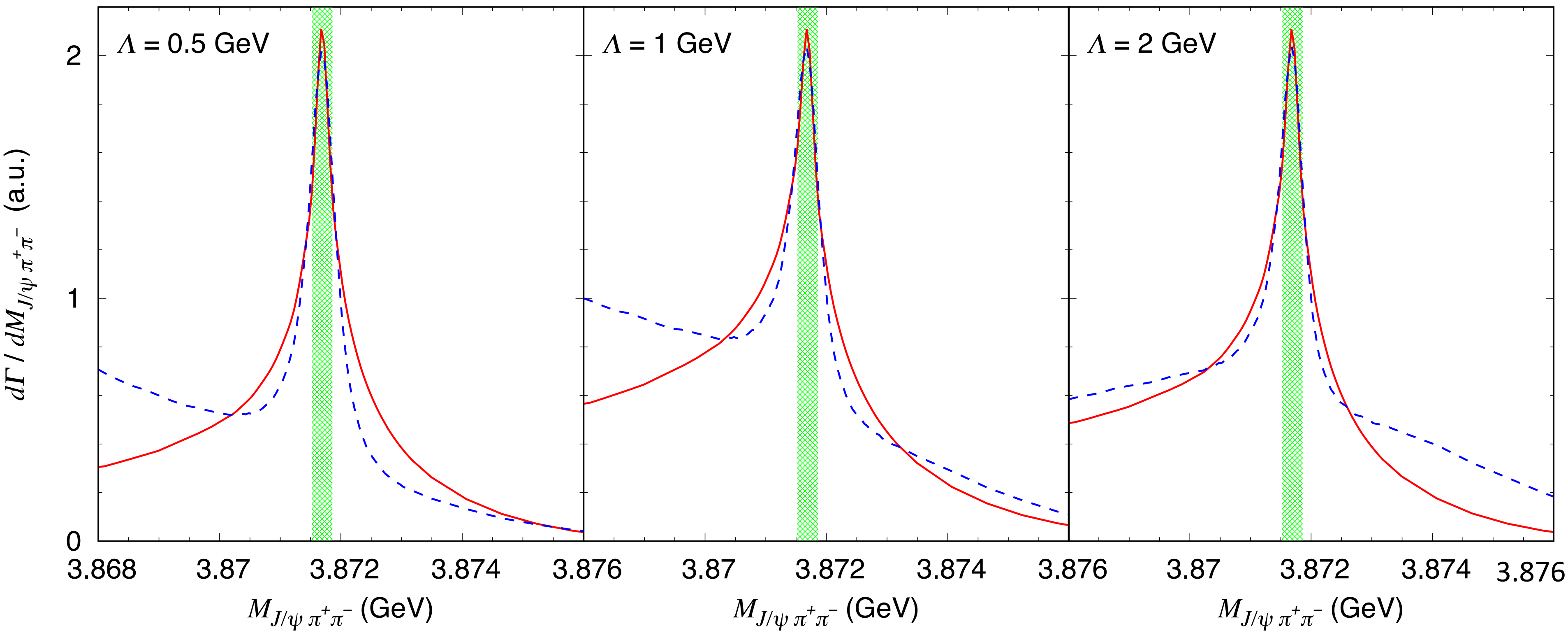

- Spectrum shape is different from Breit-Wigner

  + background (quadratic polynomial of $M_{J/\psi\ \pi+\pi-}$)

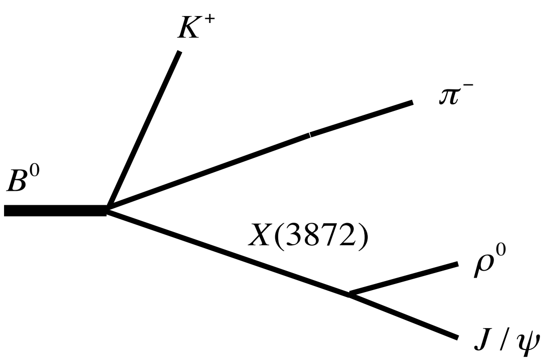
  → tail region is not well fitted

# Breit-Wigner model fit

Triangle diagram
Breit-Wigner + background




Breit-Wigner model




# Breit-Wigner parameters

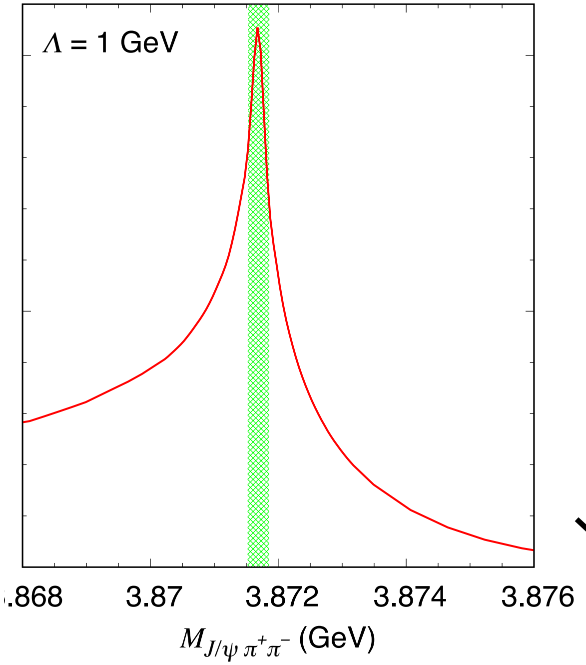
~30 keV above $D^0\bar{D}^{*0}$ threshold

| | Triangle model | PDG: X(3872) |
|---|---|---|
| Mass (MeV) | 3871.71 ± 0.00 | 3871.69 ± 0.17 |
| Width (MeV) | 0.51 ± 0.03 | < 1.2 |

Parameter ranges are cutoff-dependence ($\Lambda$=0.5−2 GeV)

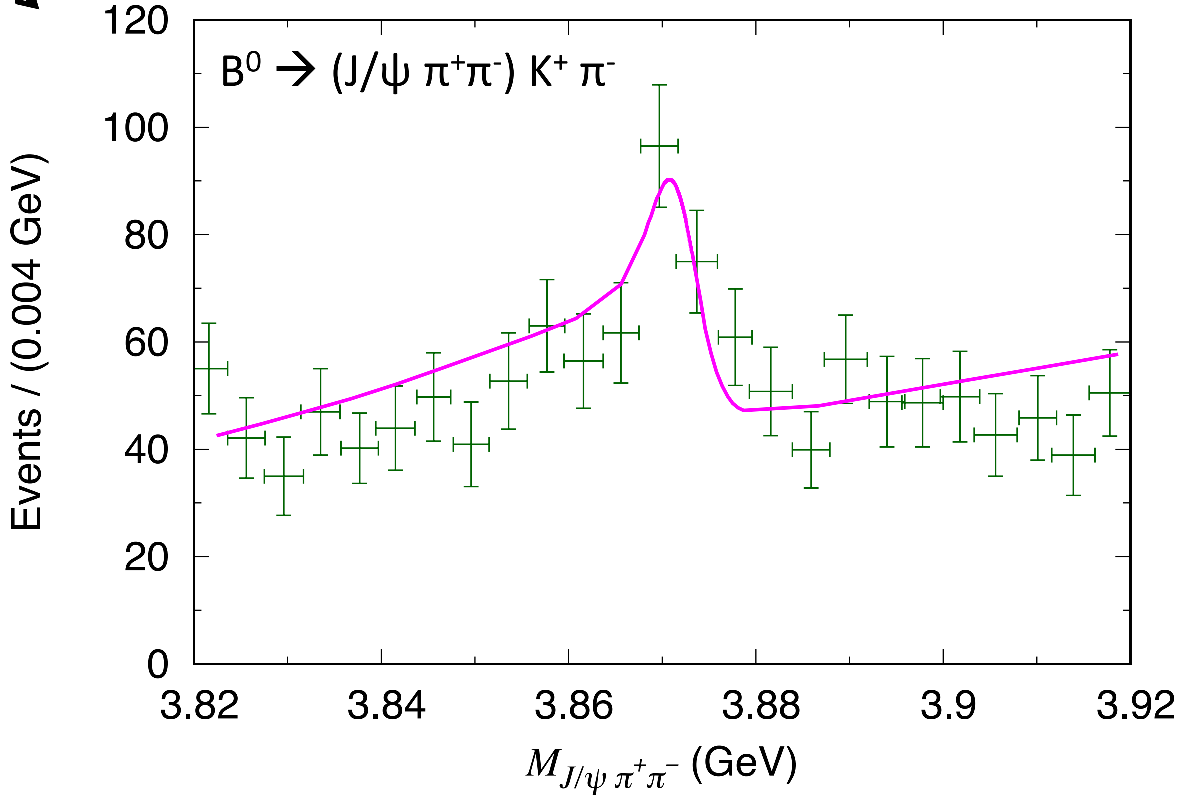
# Comparison with Belle data

Belle, PRD 91 051101(R) (2015)



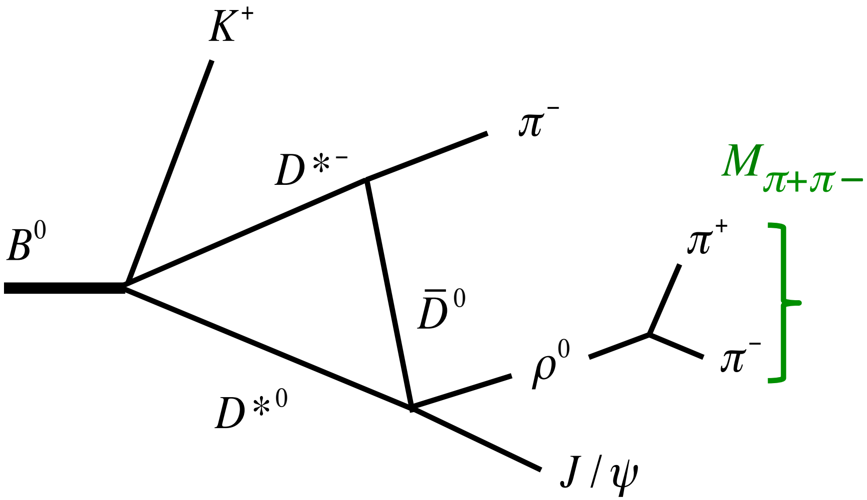

Convolute with Gaussian (experimental resolution)

\+ linear background (incoherent)



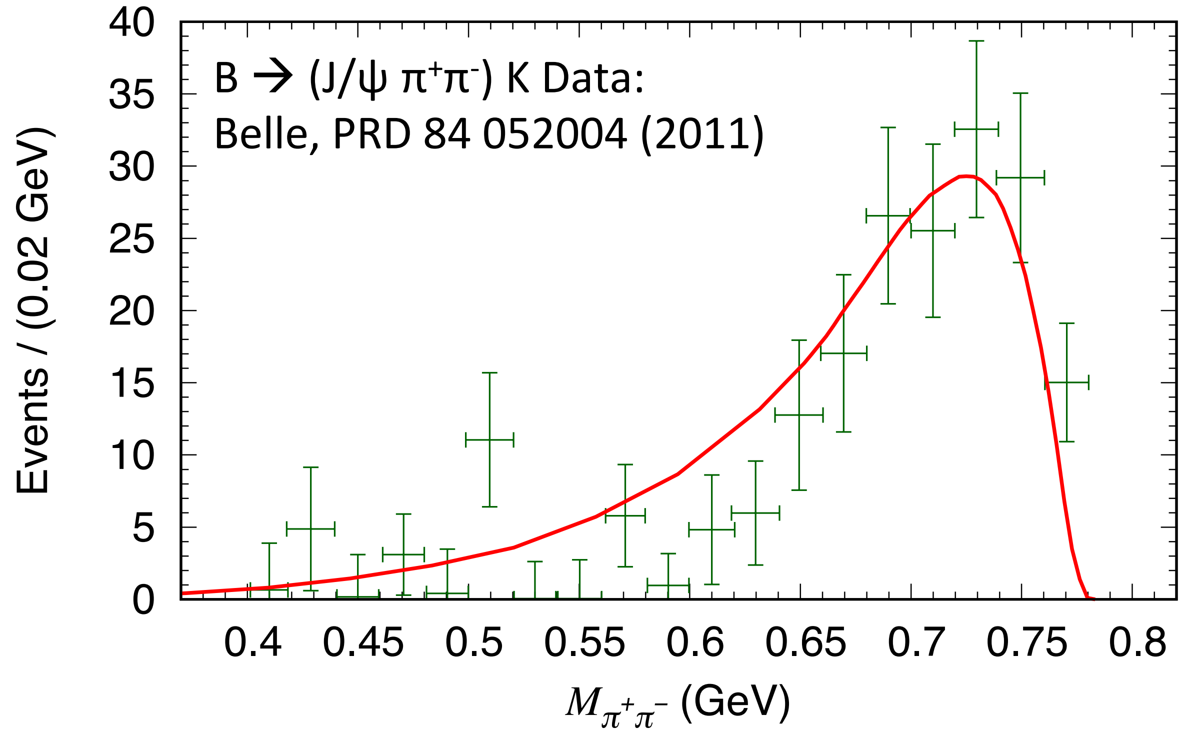

Reasonable agreement

# Dipion spectrum



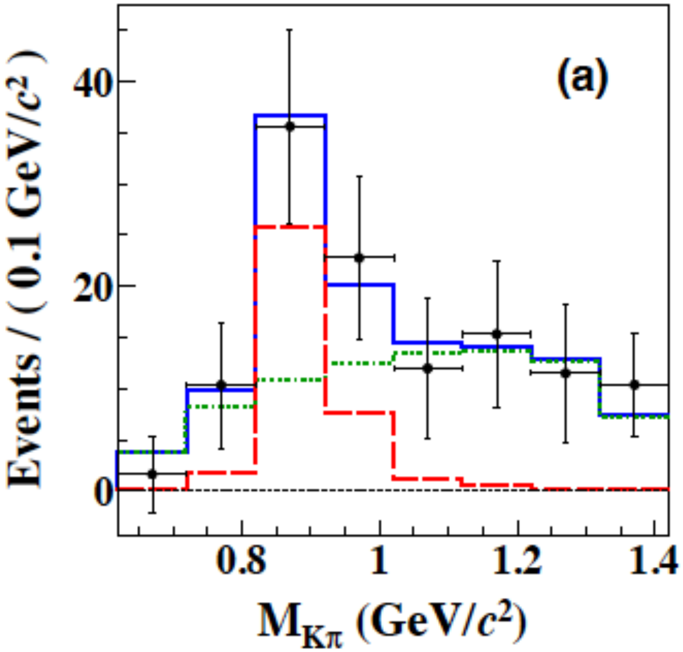

- Include $3871 < M_{J/\psi\,\pi+\pi-} < 3872.5$ MeV and 4014 < W < 4021 MeV (peak region)
- Data is from different process: B → ($J/\psi\ \pi^+\pi^-$) K Data
- If X(3872) peaks are from common origin, comparison makes sense

Reasonable agreement

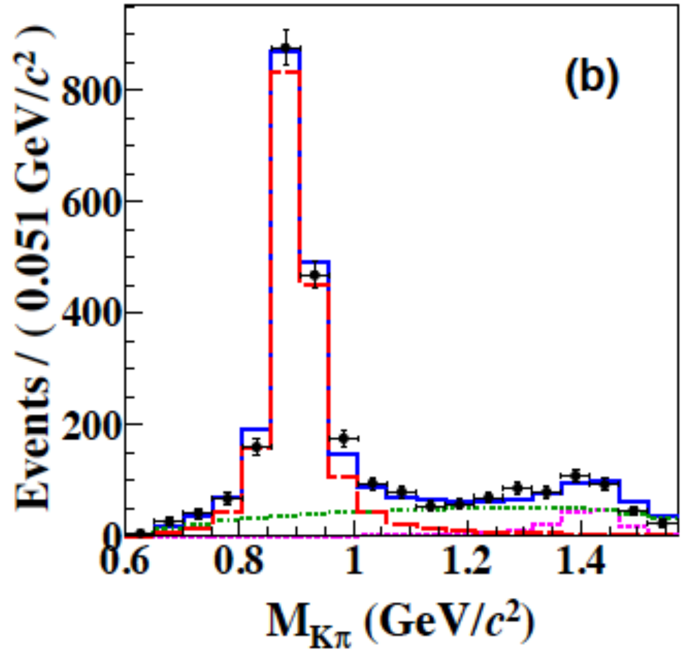


# K π spectrum

$B^0 \rightarrow X(3872)\ (K^+ \pi^-)$




$B^0 \rightarrow \psi'\ (K^+ \pi^-)$




K*(892) contribution

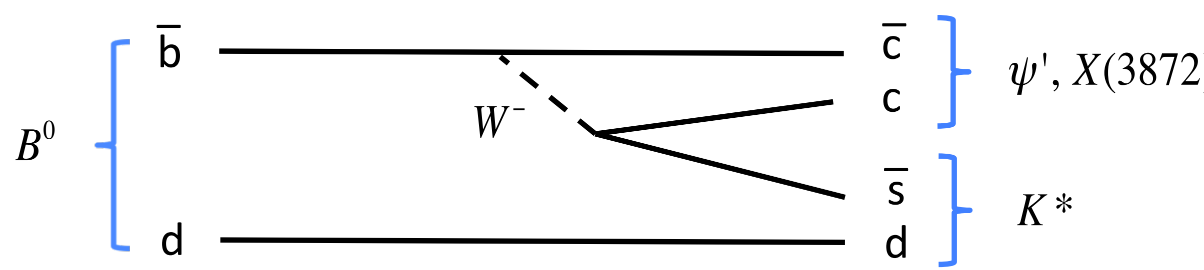
Belle data: PRD 91 051101(R) (2015)

$$\mathcal{B}\,(B^0 \rightarrow \psi'\,(K^*(892))\ \gg\ \mathcal{B}\,(B^0 \rightarrow \psi'\,(K^+\pi^-)_{NR})$$

Why $\mathcal{B}\,(B^0 \rightarrow X(3872)\ (K^*(892))\ \sim \frac{1}{2}\,\mathcal{B}\,(B^0 \rightarrow X(3872)\,(K^+\pi^-)_{NR})$ ?

In $B^0 \rightarrow c\bar{c}\,(K\pi)$,

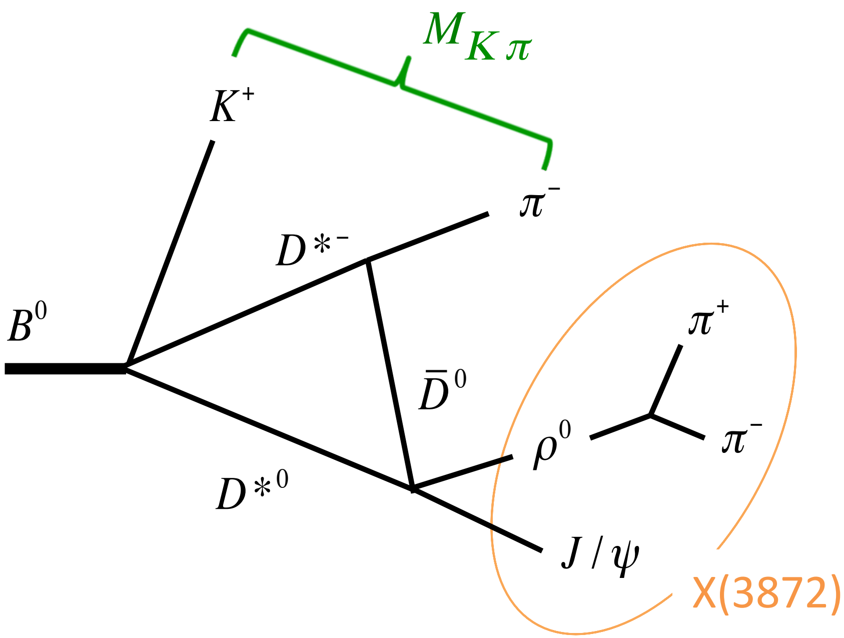
$K^*$ dominance is expected :

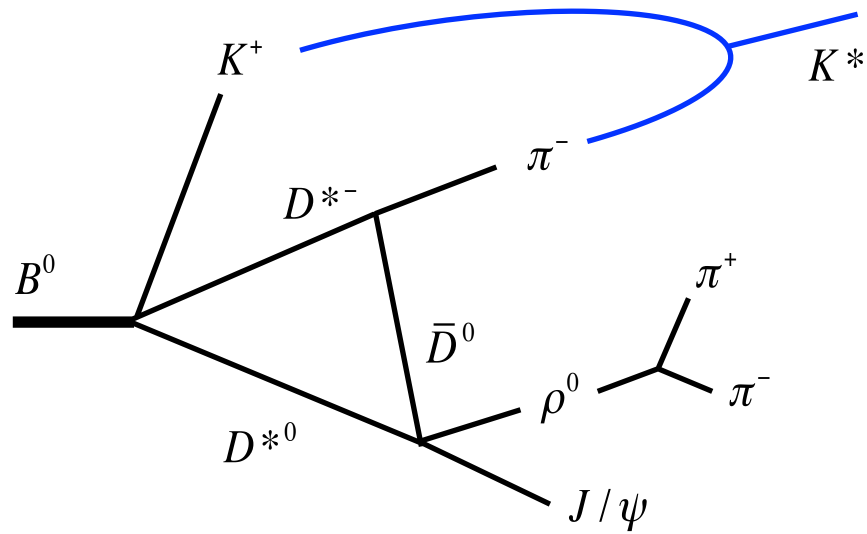


Why $\boldsymbol{B}$ ($B^0 \to$ X(3872) (K*(892)) $\sim \frac{1}{2}$ $\boldsymbol{B}$ ( $B^0 \to$ X(3872) ($K^+ \pi^-$)$_{NR}$) ?

## K π spectrum

We can infer a plausible reason from TS-based interpretation of X(3872) !



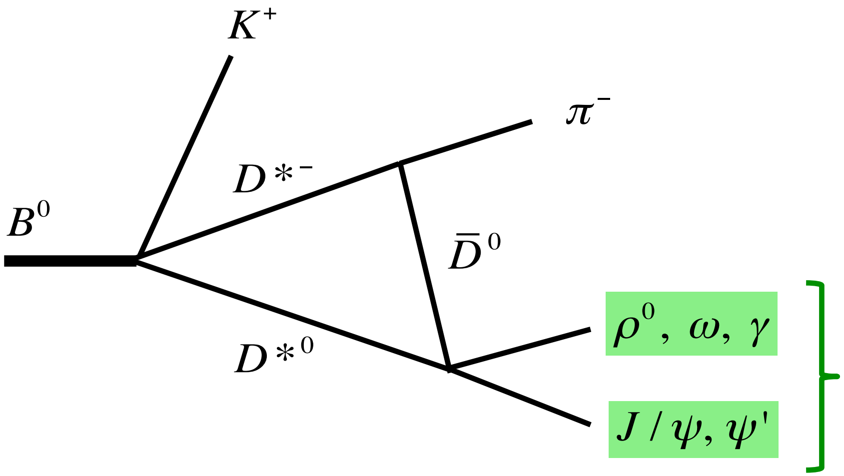

$K\pi$ pair is mostly non-resonant (NR)

$K^*$ formation with smaller probability

To be studied quantitatively in future

Experiments observed X(3872) → J/ψ $\rho^0$, J/ψ ω, J/ψ γ, ψ' γ

## Other final states




Similar X(3872)-like peaks for all final states from the triangle model, due to TS

How about relative branchings ? such as $\dfrac{B(X(3872)\to J/\psi\,\rho^0 \to J/\psi\,\pi^+\pi^-)}{B(X(3872)\to J/\psi\,\omega \to J/\psi\,\pi^+\pi^-\pi^0)} \sim 1$

- TS is from $D^0\bar{D}^{*0}$ loop (no $D^+D^{*-}$) where isospin 0 and 1 are maximally mixed

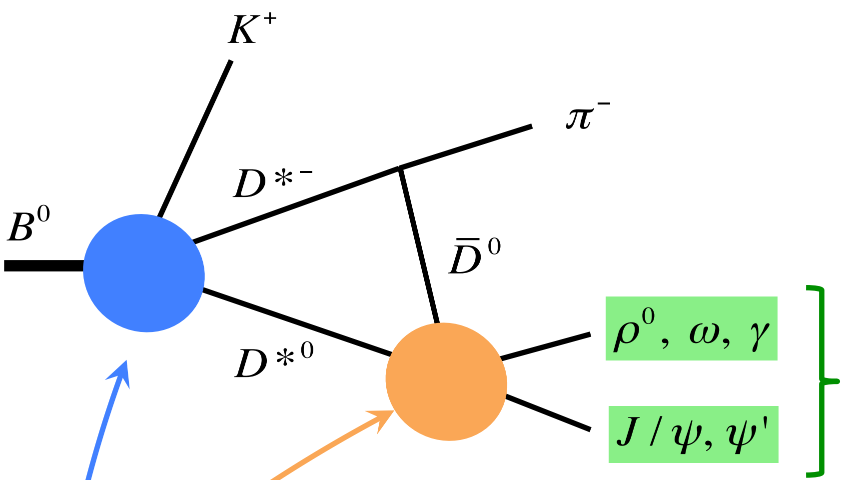
  → This ratio does not seem large isospin violation

- Considering the available phase space, enhancement (and/or suppression) of isospin 0 J/ψ ω (isospin 1 J/ψ $\rho^0$) contribution needs to be understood

  → Question about complicated dynamics; TS cannot answer

# Other final states

Experiments observed X(3872) → $J/\psi\ \rho^0$, $J/\psi\ \omega$, $J/\psi\ \gamma$, $\psi'\ \gamma$




Similar X(3872)-like peaks for all final states from the triangle model, due to TS

What is needed to understand relative and absolute branchings from X (3872) peaks ?

→ More experimental and Lattice QCD inputs in future would be crucial

- Need to be replaced by non-perturbative scattering amplitude (from LQCD) involving coupled-channels such as $D^0\bar{D}^{*0}$, $J/\psi\ \rho^0$, $J/\psi\ \omega$

- Need to be constrained by data for $B \rightarrow D^*\bar{D}^*K$ at $M_{D^*D^*} \simeq 2\ m_{D^*}$

# Conclusion

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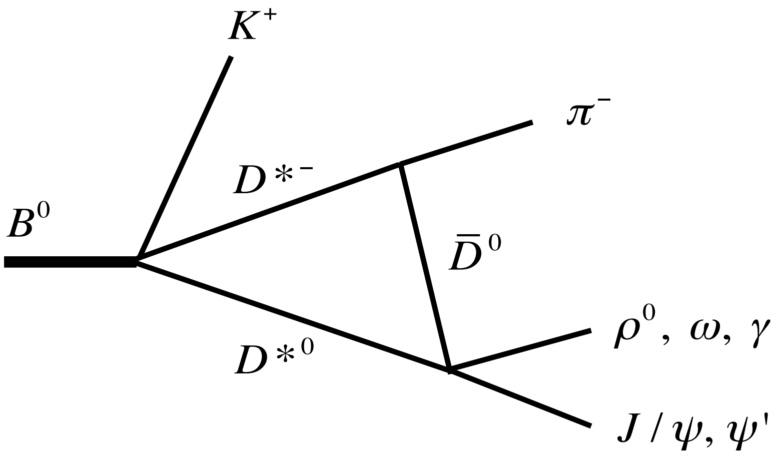



| | Triangle model | PDG: X(3872) |
| --- | --- | --- |
| Mass (MeV) | 3871.71 ± 0.00 | 3871.69 ± 0.17 |
| Width (MeV) | 0.51 ± 0.03 | < 1.2 |
| | ($\Lambda$=0.5−2 GeV) | |

- Identified triangle diagrams (singularities) generating spectrum bumps similar to X(3872) in B → ($J/\psi\ \pi^+\pi^-$) K π , ($J/\psi\ \omega$) K π , ($J/\psi\ \gamma$) K π , ($\psi'\ \gamma$) K π
- Breit-Wigner mass and width fitted to the spectrum bump are in perfect agreement with precisely measured values
- Cutoff dependence is extremely small; virtually parameter-free result

  ← triangle singularity (TS) does not depend on dynamical details

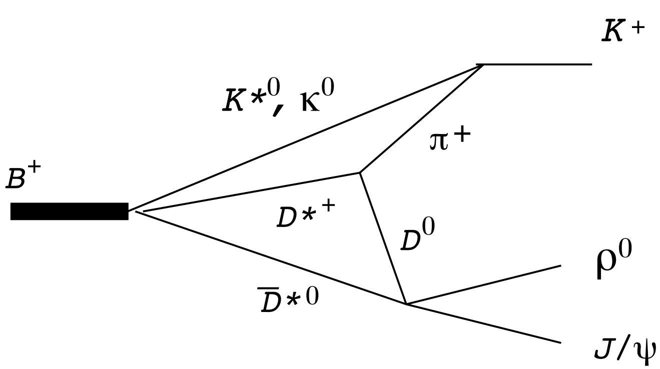
  TS dominance due to very small D* width (< 100 keV)

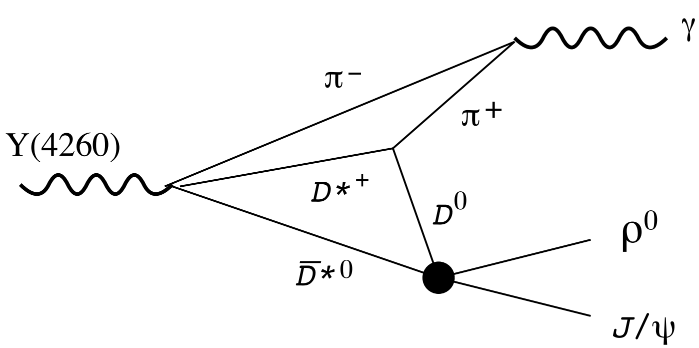
# This work might hint:

X(3872) is manifestations of triangle singularities ?

Too early to conclude: Yet to identify triangle mechanisms for other processes such as $B \rightarrow (J/\psi\ \pi^+\pi^-)\ K$, $e^+e^- \rightarrow (J/\psi\ \pi^+\pi^-)\ \gamma$ , etc. where X(3872) peaks have been observed

Possible (?) mechanisms including the same TS as studied in this work




Need quantitative analysis (future work)

## At least,

The TS mechanism (non-pole) studied in this work should be taken into account

when studying $B \rightarrow X(3872)\, K\, \pi$ ,  even if  X(3872)-pole exists

Thank you for your attention !

# Backup