Proton momentum and angular momentum decomposition with overlap fermions

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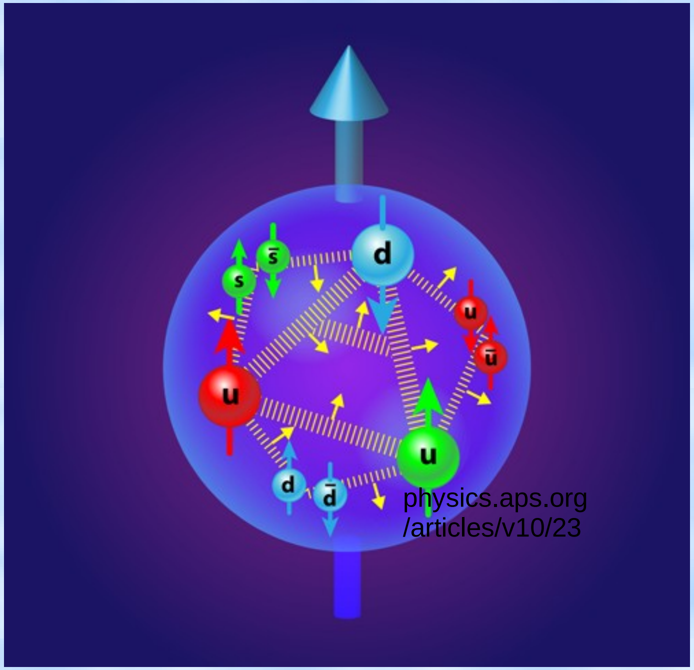
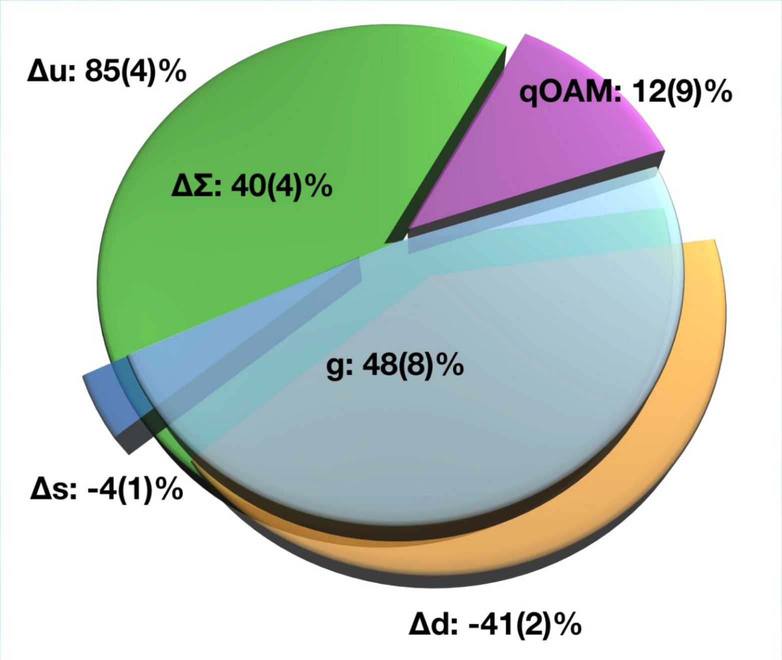
χQCD Collaboration

Motivation

Only 30% of the proton's spin is carried by the spin of quark constituents [1]





J. Liang, et al. Phys. Rev. D 98, 074505 (2018)
Y.B. Yang, et al. Phys. Rev. Lett. 121, 212001 (2018)

[1] C. A. Aidala, et al. Rev. Mod. Phys. 85, 655 (2013)

Nucleon energy-momentum tensor

Energy-momentum tensor (EMT) between two nucleon state to T1, T2, $\overline{\mathrm{C}}$ and D form factors

$$\langle p', s' | \, \mathcal{T}^{\{\mu\nu\}q,g} \, | p, s \rangle = \frac{1}{2} \bar{u}(p', s') \Big[T_1(q^2)(\gamma^\mu \bar{p}^\nu + \gamma^\nu \bar{p}^\mu)$$

$$+ \frac{1}{2m} T_2(q^2) \left(i q_\alpha (\bar{p}^\mu \sigma^{\nu\alpha} + \bar{p}^\nu \sigma^{\mu\alpha}) \right) + D(q^2) \frac{q^\mu q^\nu - \eta^{\mu\nu} q^2}{M} + \bar{C}(q^2) M \eta^{\mu\nu} \Big]^{q,g} u(p, s),$$

T1(0) and [T1+T2](0) to momentum and angular momentum fractions

$$\mathcal{T}^{\{4i\}q,g} \qquad \Longrightarrow \qquad \langle x \rangle^{q,g} = T_1(0)^{q,g} \qquad \langle J \rangle^{q,g} = \frac{1}{2} \left[T_1(0) + T_2(0) \right]^{q,g}$$

X.-D. Ji, Phys. Rev. D 52, 271 (1995), arXiv:hep-ph/9502213

Mixing and Renormalization

T1, T2 and T3 form factors are renormalized by

$$\begin{aligned}
T^{u,d}(\mathrm{CI})^R &= Z_{QQ}^{\overline{\mathrm{MS}}}(\mu) T^{u,d}(\mathrm{CI}), \\
T^{u,d,s}(\mathrm{DI})^R &= Z_{QQ}^{\overline{\mathrm{MS}}}(\mu) T^{u,d,s}(\mathrm{DI}) + \delta Z_{QQ}^{\overline{\mathrm{MS}}}(\mu) \sum_{q=u,d,s} [T^q(\mathrm{CI}) + T^q(\mathrm{DI})] \\
&\quad + Z_{QG}^{\overline{\mathrm{MS}}}(\mu) T^g(\mathrm{DI}), \\
T^g(\mathrm{DI})^R &= Z_{GQ}^{\overline{\mathrm{MS}}}(\mu) \sum_{q=u,d,s} [T^q(\mathrm{CI}) + T^q(\mathrm{DI})] + Z_{GG}^{\overline{\mathrm{MS}}} T^g(\mathrm{DI}),
\end{aligned}$$

Non-perturbative renormalization constants include mixing calculated in [1]

Lattice	Z_{QQ}	δZ_{QQ}	Z_{QG}	Z_{GQ}	Z_{GG}
32ID	1.25(0)(2)	0.018(2)(2)	0.017(17)	0.57(3)(6)	1.29(5)(9)

[1] Y-B Yang, J. Liang, et al., χQCD Collaboration, Phys. Rev. Lett. 121, 212001 (2018)

Normalization

With momentum and angular momentum conservation, the momentum and angular momentum sum rules are

$$\langle x\rangle^q + \langle x\rangle^g = T_1(0)^q + T_1(0)^g = 1$$

$$J^q + J^g = \frac{1}{2}\{[T_1(0)^q + T_2(0)^q] + [T_1(0) + T_2(0)]^g\} = \frac{1}{2}$$

Normalization conditions for the local current

$$N_q \langle x\rangle_R^q + N_g \langle x\rangle_R^g = 1$$

$$N_q J_R^q + N_g J_R^g = \frac{1}{2}$$

As T2 is too small and noisy

$$N_q = N_g = N$$

M. Deka, T. Doi, Y-B Yang, et. al., χQCD collaboration, PRD91, 014505 (2015)

Matrix element and form factors

Nucleon three point functions on the Lattice

$$G_{\alpha\beta}^{N\mathcal{T}_{\mu\nu}N}(\tau, t_f, \vec{p}_i, \vec{p}_f) = \sum_{\vec{x}_1, \vec{x}_2} e^{-i\vec{p}_i\cdot\vec{x}_f} e^{-\vec{q}\cdot\vec{z}} \times \langle \chi_\alpha(\vec{x}_f, t_f) \mathcal{T}_{\mu\nu}(z, \tau) \chi_\beta(0) \rangle$$

T1(0) from current momentum to be equal

$$\mathrm{Tr}\left[\Gamma_e G_{\alpha\beta}^{\mathcal{T}_{4j}}(\tau, t_f, \vec{p}, \vec{p})\right] \to \epsilon_{i,j,k} p_k (T_1)(0)$$

T1+T2 form factor from nucleon initial/final momentum to be zero

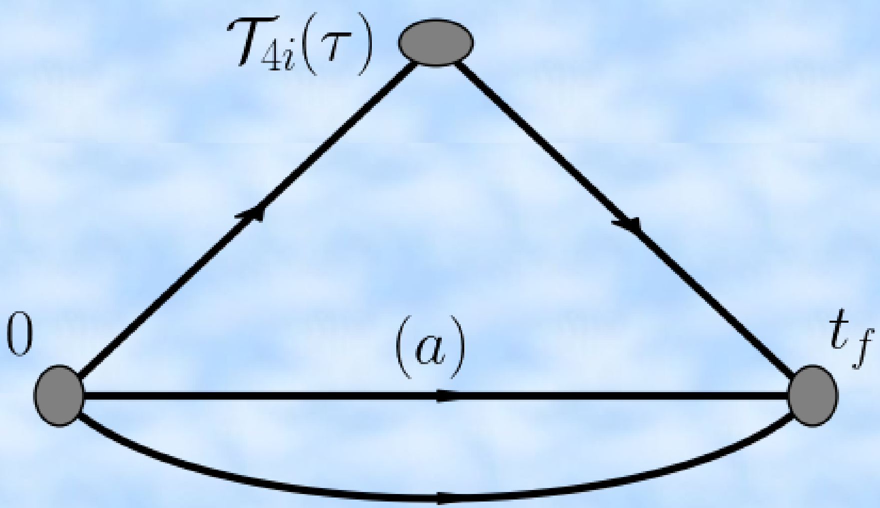
$$\mathrm{Tr}\left[\Gamma_i G_{\alpha\beta}^{\mathcal{T}_{4j}}(\tau, t_f, 0, \vec{p})\right] \to \epsilon_{i,j,k} p_k [T_1 + T_2](Q^2)$$

$$\mathrm{Tr}\left[\Gamma_i G_{\alpha\beta}^{\mathcal{T}_{4j}}(\tau, t_f, \vec{p}, 0)\right] \to \epsilon_{i,j,k} p_k [T_1 + T_2](Q^2)$$

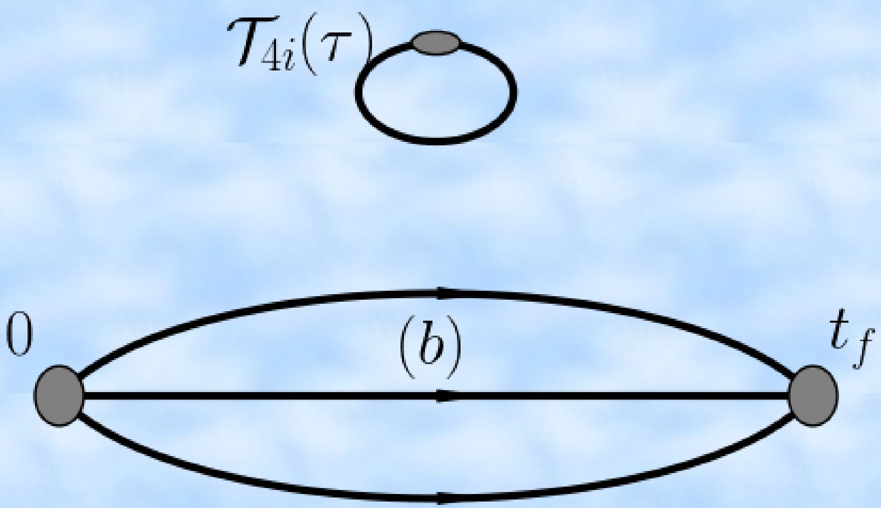
$$\mathrm{Tr}\left[\Gamma_i G_{\alpha\beta}^{\mathcal{T}_{4j}}(\tau, t_f, \vec{p}, -\vec{p})\right] \to \epsilon_{i,j,k} p_k [T_1 + T_2](Q^2)$$

Lattice Simulations

- Lattice
  - 32ID--Domain Wall 2+1 Lattice, $32^3 * 64$, a=0.143 fm, Pion 172 MeV
  - Overlap Fermions with six different valence quark masses



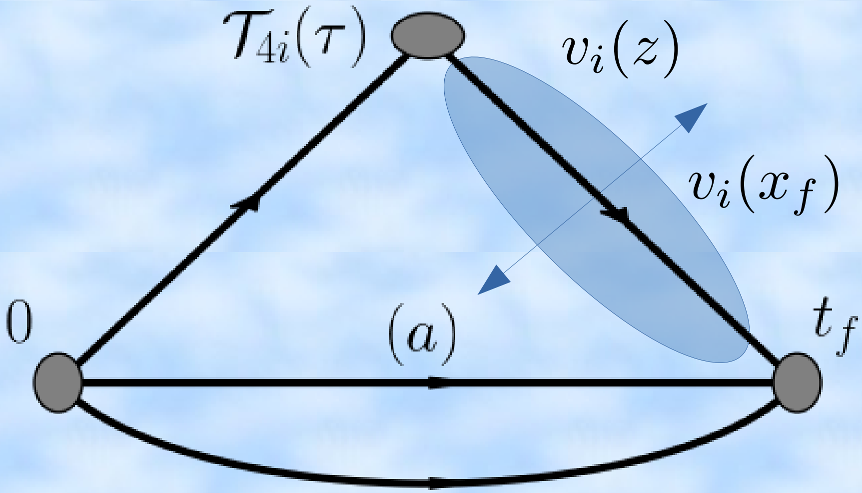
Connected insertions (CI)
for light quarks



Disconnected insertions (DI)
quarks and glue

CI 3pt with FFT

The stochastic-sandwich method combined with low mode substitution and FFT



$$S(z|x_{\rm f}) = S^L(z|x_{\rm f}) + S^H(z|x_{\rm f}),$$
$$S^L(z|x_{\rm f}) = \sum_{\lambda_i \le \lambda_c} \frac{1}{\lambda_i + m} v_i(z) v_i^\dagger(x_{\rm f})$$

"Low mode" part of the 3pt

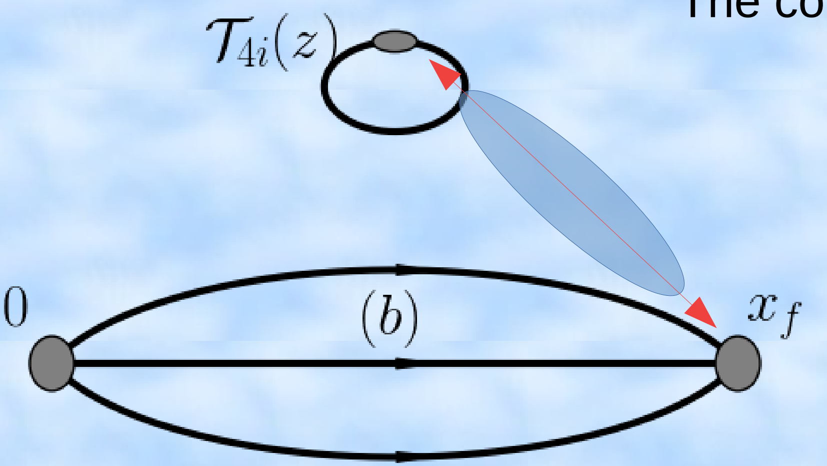
$$C^{u/d,}_{\rm CI,3pt} = \langle \sum_{\lambda_i \le \lambda_c} {\rm Tr}[\frac{1}{\lambda_i + m} G_i^L(\vec{q},\tau) F_i^{L,u/d}(\vec{p}_f, t_f)] \rangle$$

Similar separations could also be done with the stochastic "high mode" part

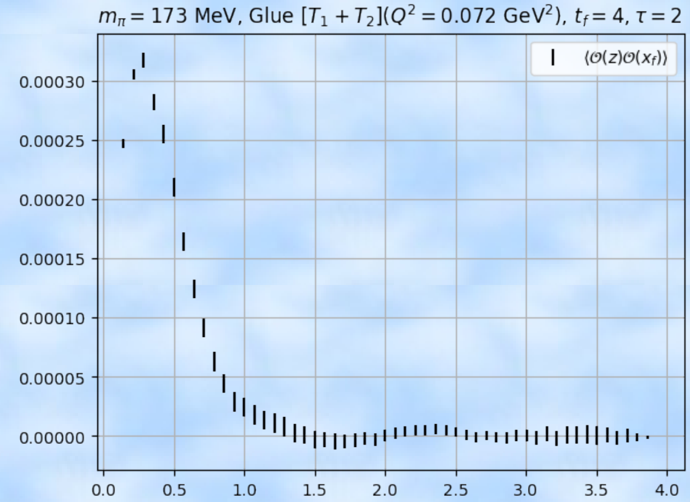
G. Wang, J. Liang, T. Draper, K.-F. Liu, and Y.-B. Yang (chiQCD), arXiv:2006.05431

DI parts

Cluster-decomposition error (CDER) [1] technique are used for DI parts

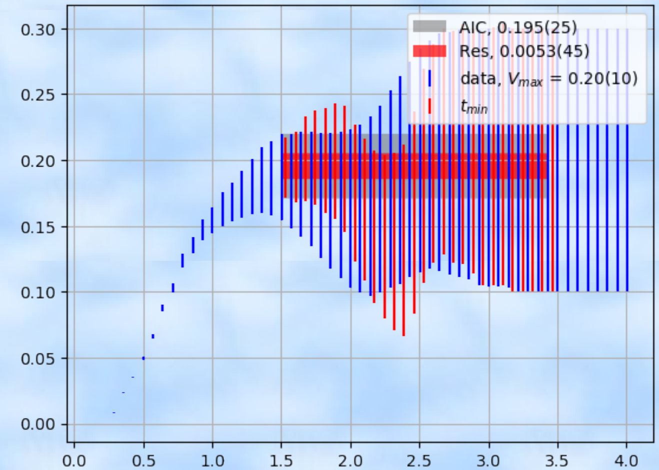


The correlations satisfy [2] $\langle \mathcal{O}(z)\mathcal{O}(x_f)\rangle_s \leq A r^{-\frac{2}{3}} e^{-Mr}$



Use Akaike information criterion (AIC) weight factors to average different fit results

$$AIC = exp\left[-\frac{1}{2}(\chi^2 - 2n_{\mathrm{dof}})\right]$$



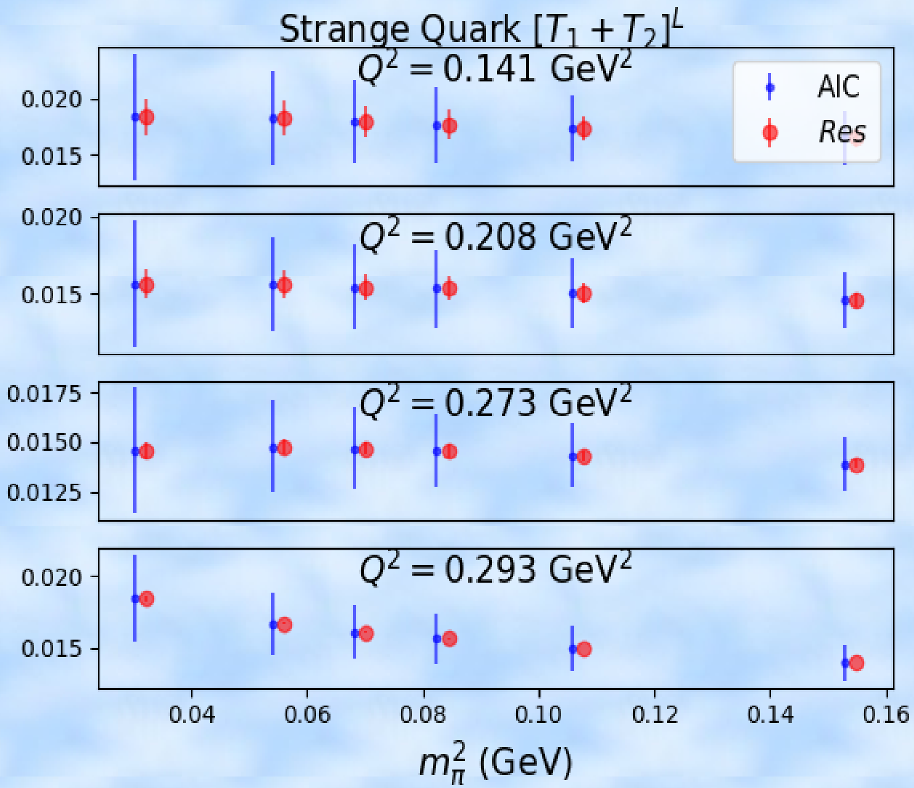
The residue of the correlator from 1.5 fm

$$Res = \sum_{r>r_{\mathrm{cut}}}^{r<r_{\mathrm{max}}} A r^{-\frac{3}{2}} e^{-Mr}$$

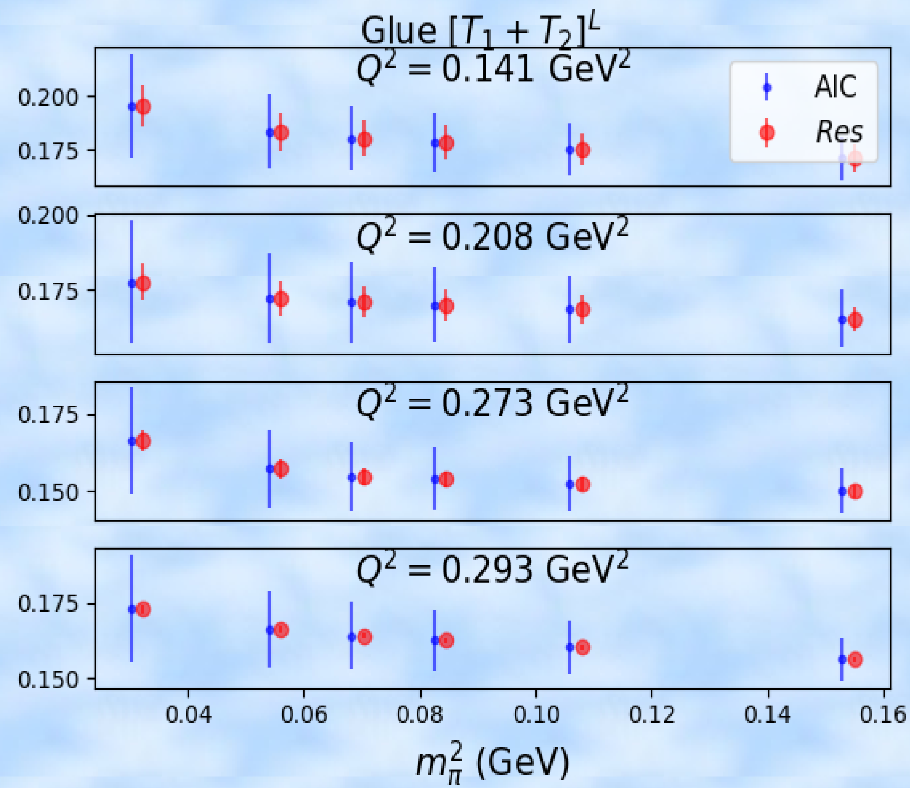
[1] K.-F. Liu, J. Liang, and Y.-B. Yang, Phys. Rev. D 97, 034507 (2018), arXiv:1705.06358
[2] H. Araki, K. Hepp, and D. Ruelle, Helv. Phys. Acta 35, 164 (1962)

CDER systematics

Comparisons of the AIC average and residue estimations

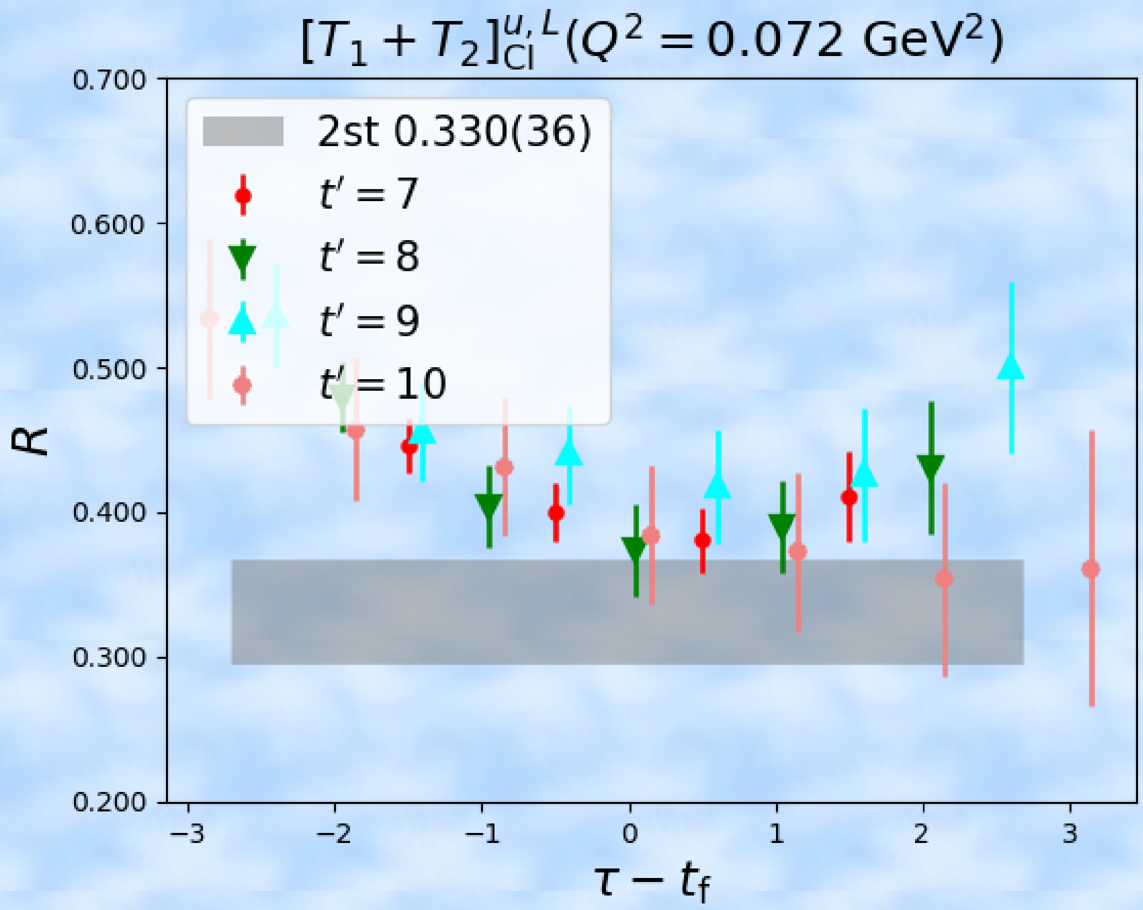


T1+T2 form factors of up quark



T1+T2 form factors of gluon

CI two-state fit

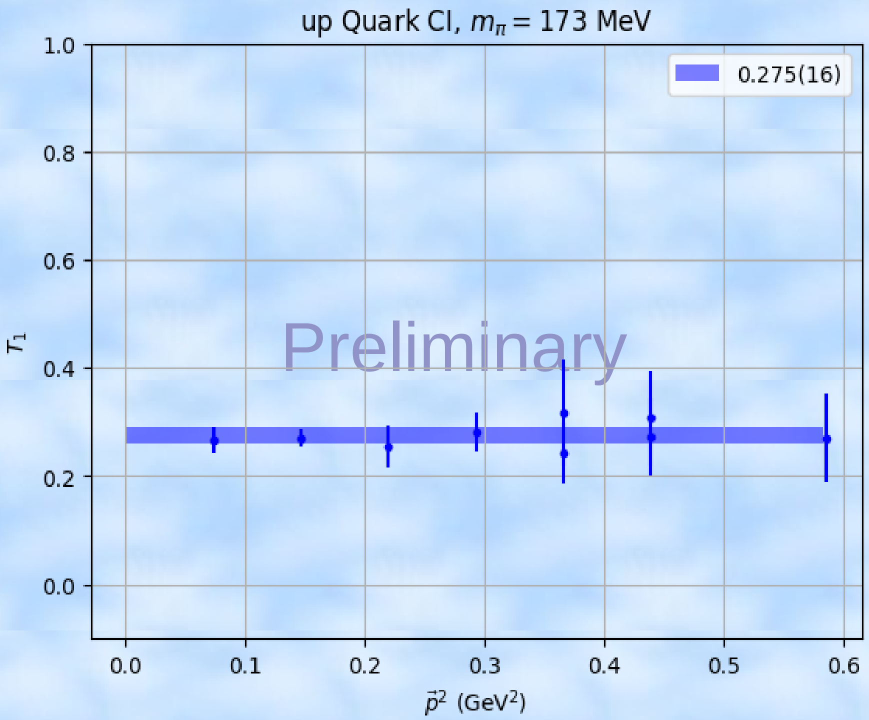


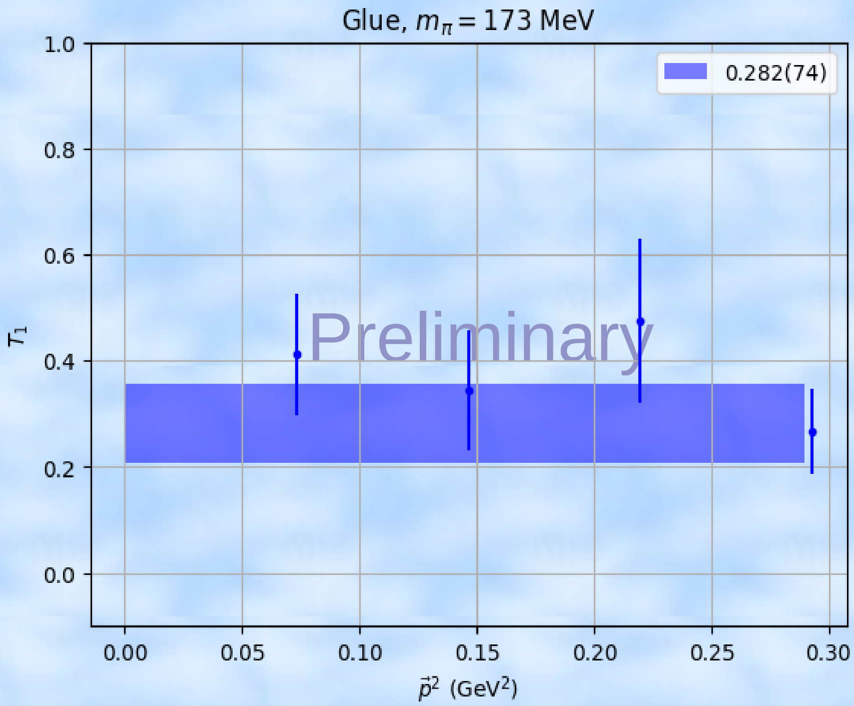
Example fit of up quark T1+T2 form factor at valence pion mass 173 MeV

$$\frac{C_{3pt}}{C_{2pt}} = A + B_1 e^{-\Delta E_{p_f}(t_f-\tau)} + B_2 e^{-\Delta E_{p_i}(\tau)} + B_3 e^{-\Delta E_{p_i}(\tau)-\Delta E_{p_f}(t_f-\tau)}$$

Currently, the energy gap are constrained by using results from two-point functions as a prior

T1 form factors



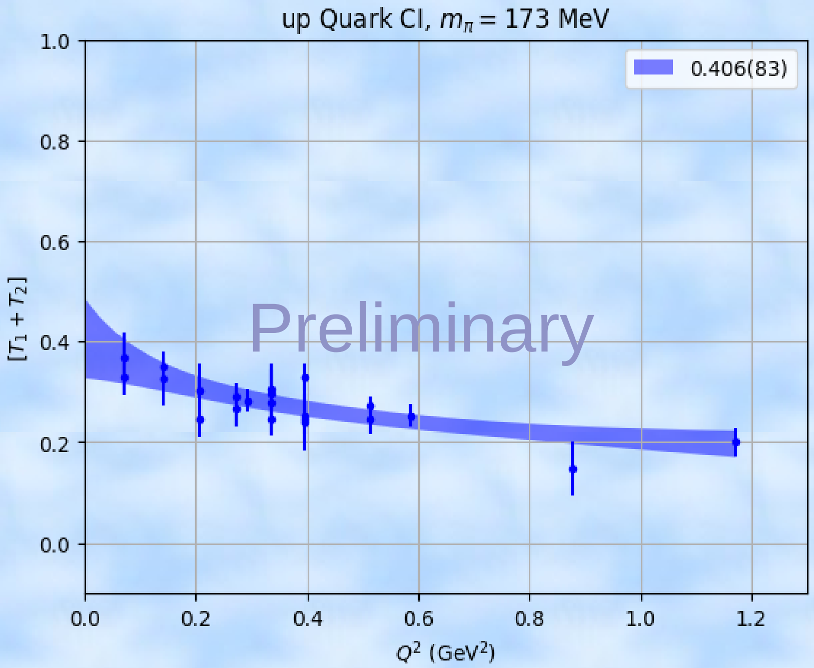


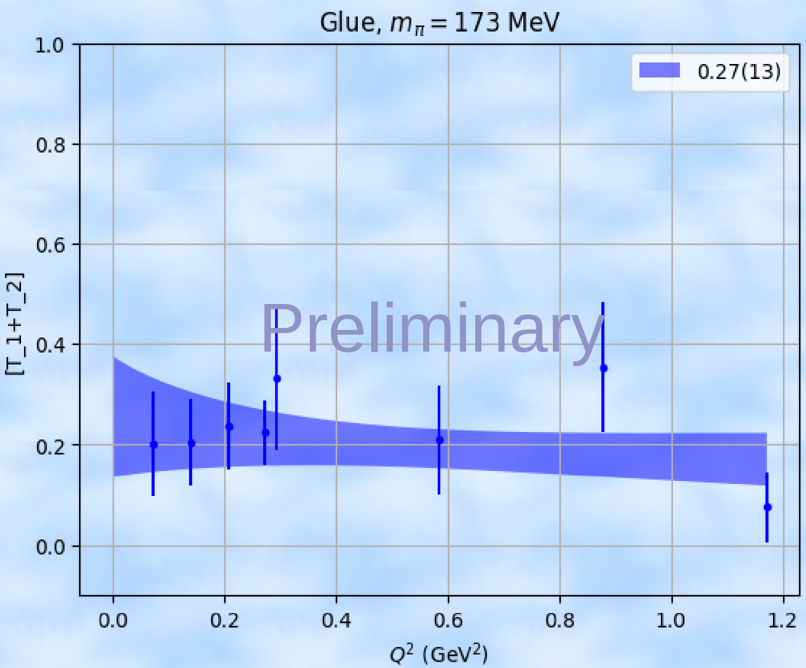
T1(0) of quark CI

T1(0) of glue CI

Averaged over results from different nucleon initial momenta

T1+T2 form factors





z-expansion fit to extrapolate to zero momentum transfer

$$f_{\pi\pi}(Q^2) = \sum_{k=0}^{k_{max}} a_k z^k$$

$$k_{max} = 3$$

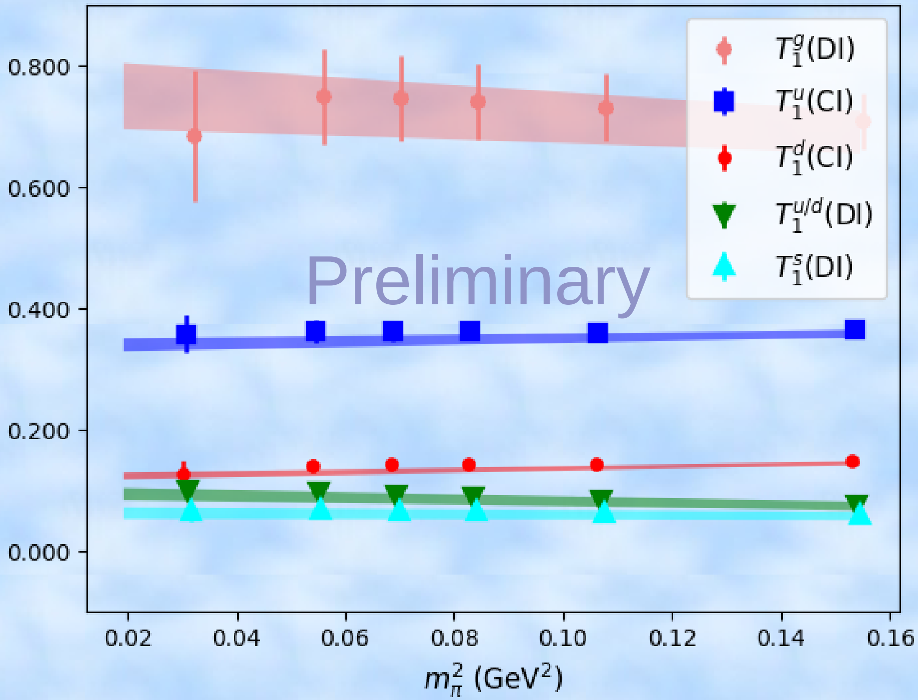
$$\mathrm{t}_{cut} = 4m_\pi^2$$

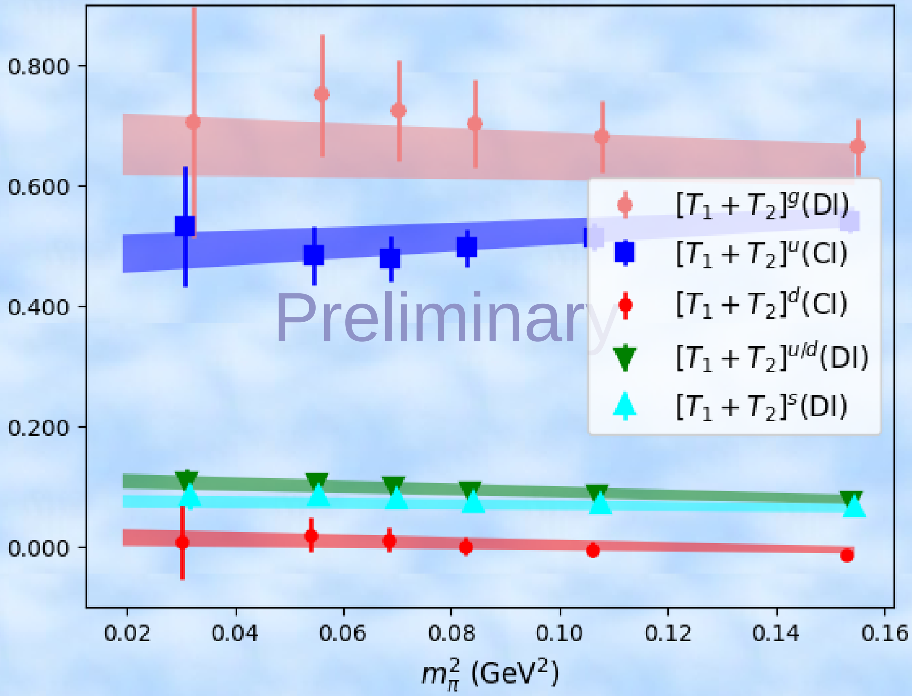
$$z(t, t_{cut}, t_0) = \frac{\sqrt{t_{cut} - t} - \sqrt{t_{cut} - t_0}}{\sqrt{t_{cut} - t} + \sqrt{t_{cut} - t_0}}$$

$$\mathrm{t}_0^{opt}(Q^2_{max}) = t_{cut}(1 - \sqrt{1 + Q^2_{max}/t_{cut}})$$

T1 and T1+T2 form factors Summary

Simple linear extrapolations of each constituents under current statistics

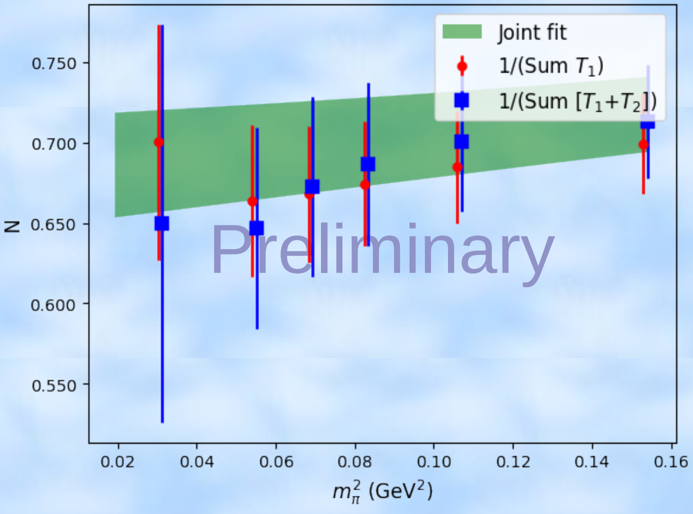




Sum of renormalized T1(0) and [T1+T2](0)

$$N \langle x \rangle_R^q + N \langle x \rangle_R^g = 1$$

$$N\, J_R^q + N\, J_R^g = \frac{1}{2}$$



Momentum and Angular momentum fractions

Summary table of the CI and DI parts for quark and gluon constituents

Preliminary

	u(CI)	d(CI)	u/d(DI)	s(DI)	glue
$\langle x \rangle$	0.233(29)	0.085(06)	0.065(06)	0.043(07)	0.509(31)
$2J$	0.319(67)	0.017(25)	0.075(18)	0.052(12)	0.461(49)

Compare with previous calculation and phenomenological global fit results

Preliminary

	u	d	$[u-d]$	s	glue
$\langle x \rangle$	0.298(27)	0.150(08)	0.148(31)	0.043(07)	0.509(31)
$\langle x \rangle_{[1]}$	0.307(35)	0.160(48)	0.151(40)	0.051(26)	0.482(84)
$\langle x \rangle_{\mathrm{CT14}}$	0.348(05)	0.190(05)	0.158(06)	0.035(09)	0.416(09)

[1] Y-B Yang, J. Liang, et al., χQCD Collaboration, Phys. Rev. Lett. 121, 212001 (2018)
[2] S. Dulat, et al., Phys. Rev. D, 93(3):033006, (2016)
[3] M. Deka, T. Doi, Y-B Yang, et. al., χQCD collaboration, PRD91, 014505 (2015)

Further Calculations

- FFT and low mode substitution has been successfully used with stochastic-sandwich method for CI to reach better statistics
- CDER technique greatly increase DI statistics with systematic errors under control
- A complete calculation of proton momentum and angular momentum fractions at several overlap valence pion masses has been done on one Lattice
- Extend the calculation to other lattice spacing and volumes to extrapolate to physical limit
- Extend to all kinematics to obtain T1, T2, $\bar{C}$ and D form factors at different Q^2

Thank You