Quarkonia and heavy quark diffusion in the hot gluonic medium

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arXiv: 2109.11303

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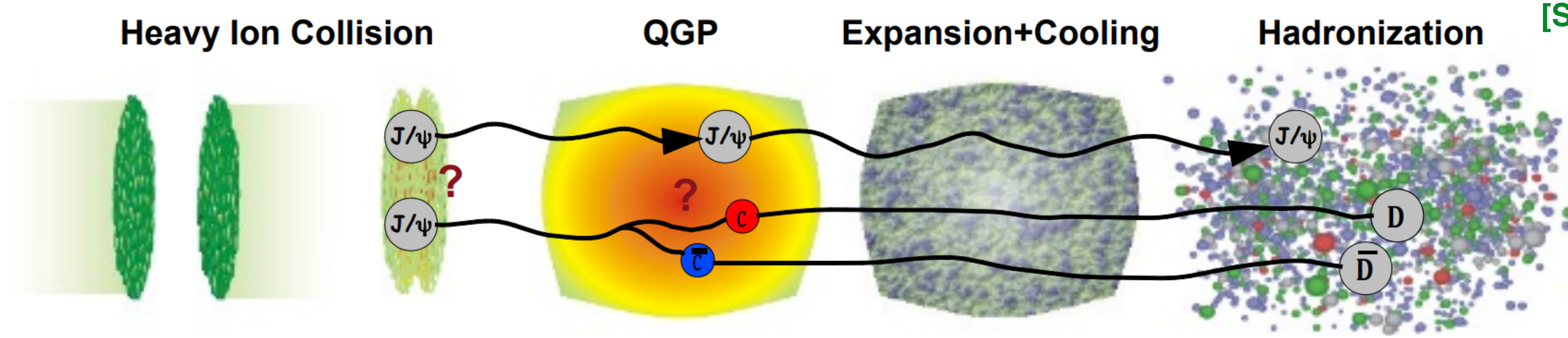
Universität Regensburg

Outline

- Background
- Quarkonia in the hot medium
- Charm quark and bottom quark diffusion
- Infinity heavy quark diffusion
- Conclusion

Heavy quarks in heavy ion collisions

[SA.Bass]

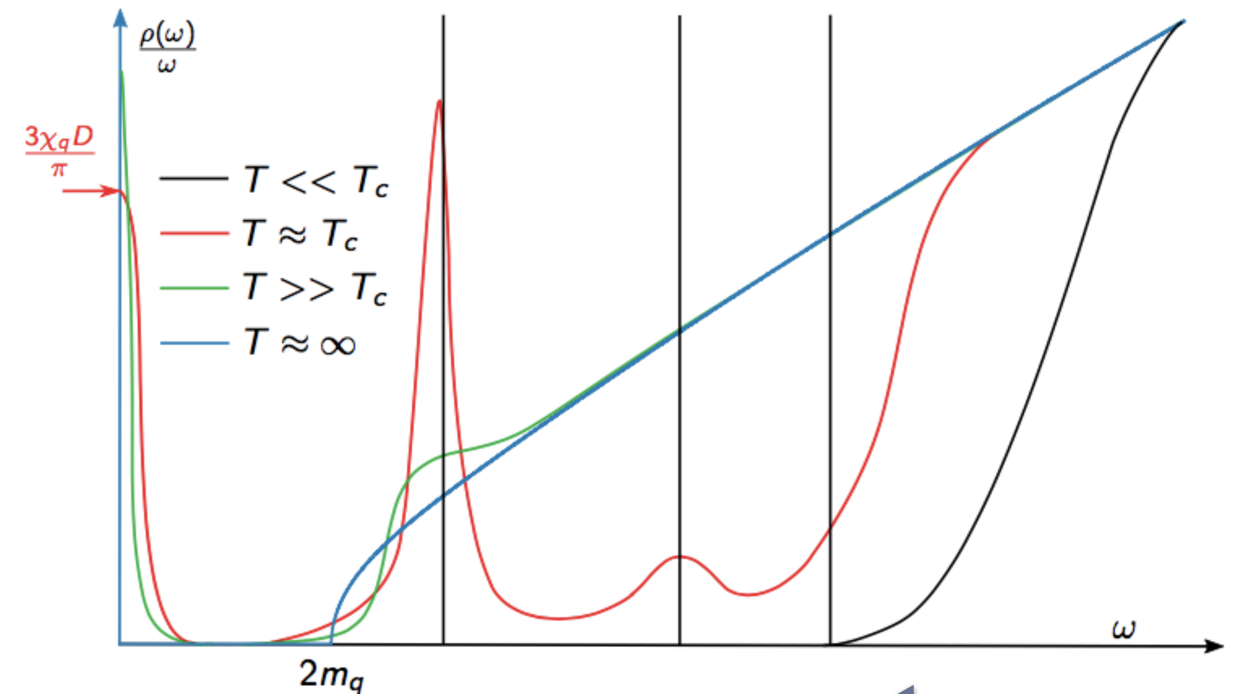


Heavy quarkonia are produced only in the early stage of collisions

- Some remain as bound state in the whole evolution
- Some **dissociate** in the hot medium and release constituents which travel through QGP, thermalize via **diffusion**
- Form open charm/bottom mesons during hadronization
  - ➡ Heavy quarkonia as thermometer of QGP
  - ➡ Heavy quark diffusion coefficient as crucial input for hydro/transport models to describe the experimental data

Hadron spectral functions

- Carry all information about the in-medium properties of quarkonia

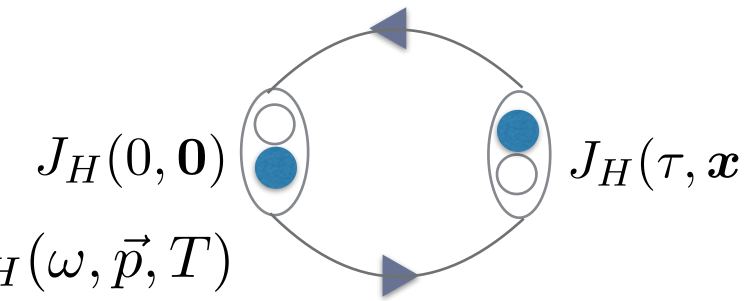


✳ Deformation of SPF
—> dissociation temperature

✳ Transport peak of SPF:
—> heavy quark diffusion coefficient

$$D = \frac{1}{3\chi_q} \lim_{\omega\to 0} \frac{\rho_{ii}^{trans}(\omega)}{\omega}$$

- Analytic continuation and spectral reconstruction



$$G_H(\tau,\vec{p}) = \sum_{x,y,z} \exp(-i\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^\dagger(\tau,\vec{x})\rangle = \int \frac{d\omega}{\pi} K(\omega,\tau,T)\rho_H(\omega,\vec{p},T)$$

✳ Backus-Gilbert Method B. B. Brandt, et al., PRD93, 054510(2016)
✳ Maximum Entropy Method M. Asakawa, et al., PPNP. 46(2001) 445-508
✳ New Bayesian Method Y. Burnier and A. Rothkopf, PRL 111,18,182003
✳ Stochastic Approaches H.-T. Ding, et al., PRD97, 094503
✳ …
✳ Fit with theoretically inspired ansatz

Lattice setup

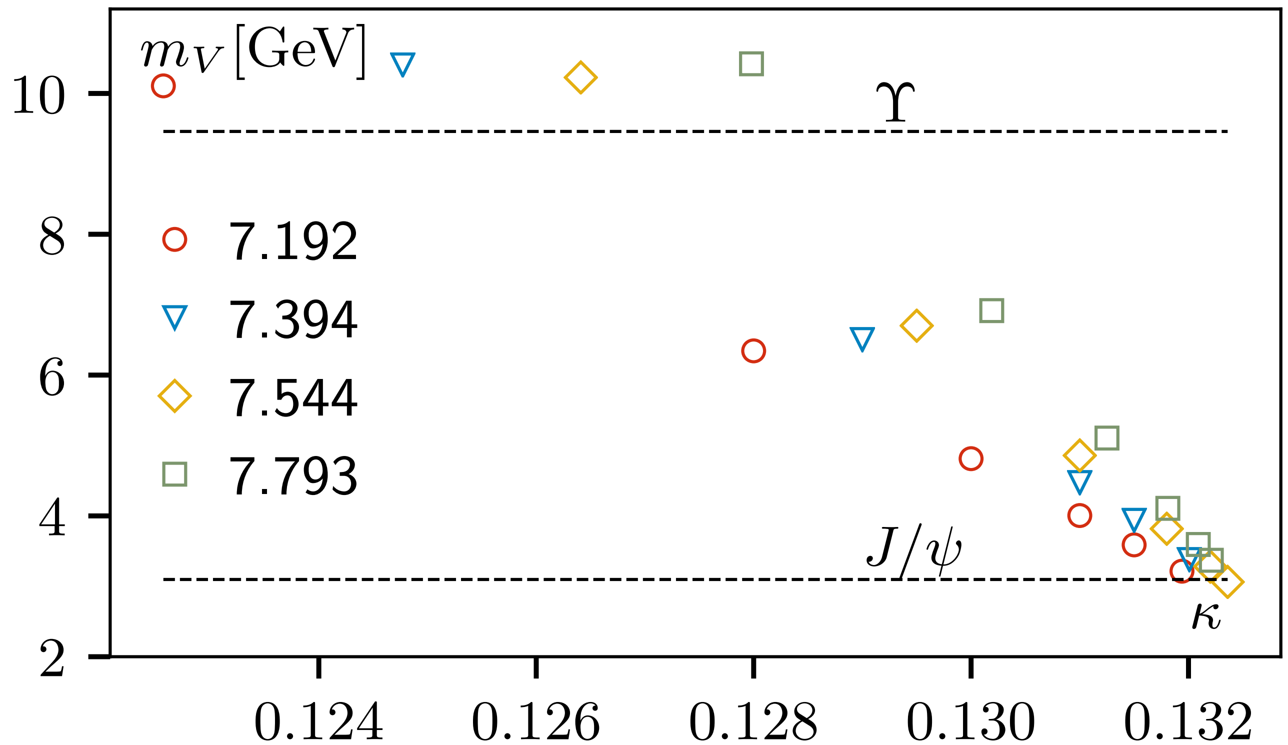
β	r_0/a	a[fm](a^{-1}[GeV])	N_σ	N_τ	T/T_c	# confs
7.192	26.6	0.018(11.19)	96	48	0.75	237
				32	1.1	476
				28	1.3	336
				24	1.5	336
				16	2.25	237
7.394	33.8	0.014(14.24)	120	60	0.75	171
				40	1.1	141
				30	1.5	247
				20	2.25	226
7.544	40.4	0.012(17.01)	144	72	0.75	221
				48	1.1	462
				42	1.3	660
				36	1.5	288
				24	2.25	237
7.793	54.1	0.009(22.78)	192	96	0.75	224
				64	1.1	291
				56	1.3	291
				48	1.5	348
				32	2.25	235

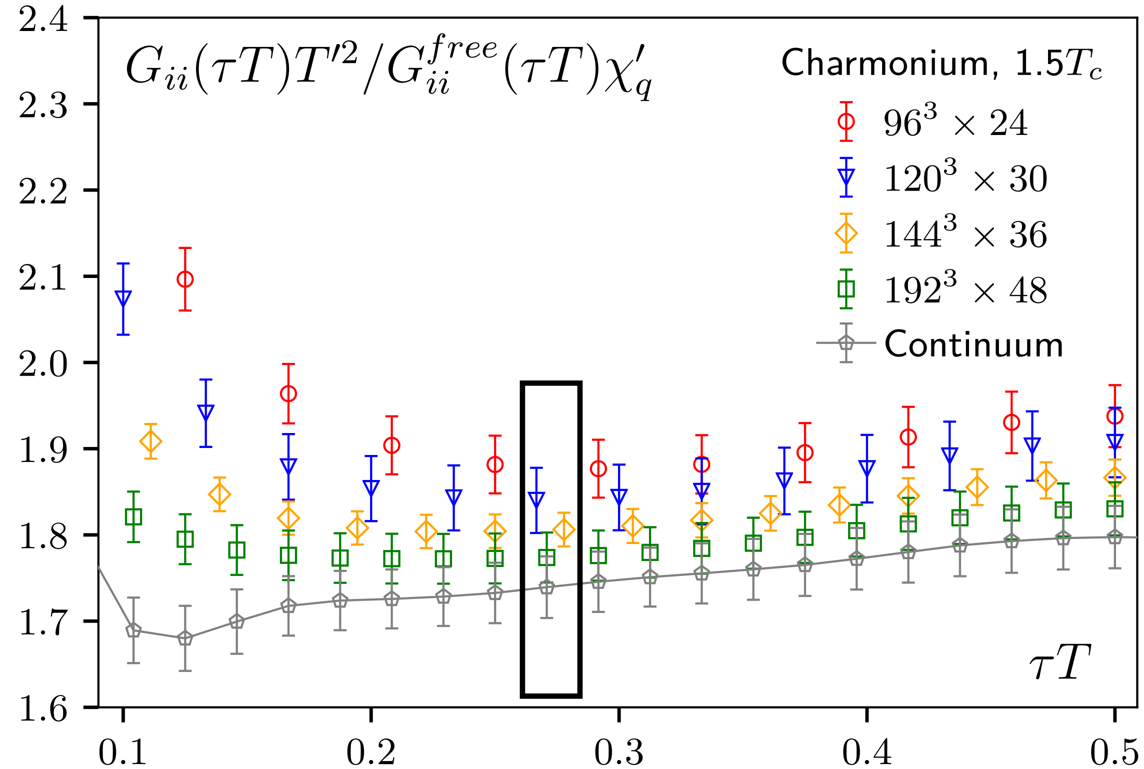
β	κ	m_V[GeV]	β	κ	m_V[GeV]
7.192	0.13194	3.21(1)	7.394	0.132008	3.38(2)
	0.1315	3.59(1)		0.1315	3.94(2)
	0.131	4.01(1)		0.131	4.47(2)
	0.13	4.81(1)		0.129	6.50(2)
	0.128	6.34(1)		0.124772	10.04(1)
	0.12257	10.11(1)			
7.544	0.13236	3.06(2)	7.793	0.13221	3.37(1)
	0.1322	3.28(1)		0.13209	3.59(1)
	0.1318	3.82(2)		0.13181	4.11(1)
	0.131	4.86(2)		0.13125	5.11(1)
	0.1295	6.70(2)		0.13019	6.92(1)
	0.12641	10.23(2)		0.12798	10.42(1)

- Large, fine, isotropic lattices in the quenched approximation (for large N_t)
- Five different temperatures
- Clover improved Wilson fermions
- Wide kappa (quark mass) range

Quark mass interp. and continuum extrap.

- Several heavy quark masses (κ) at each lattice spacing
- Inter/extra-polate correlators to physical J/ψ, Υ mass

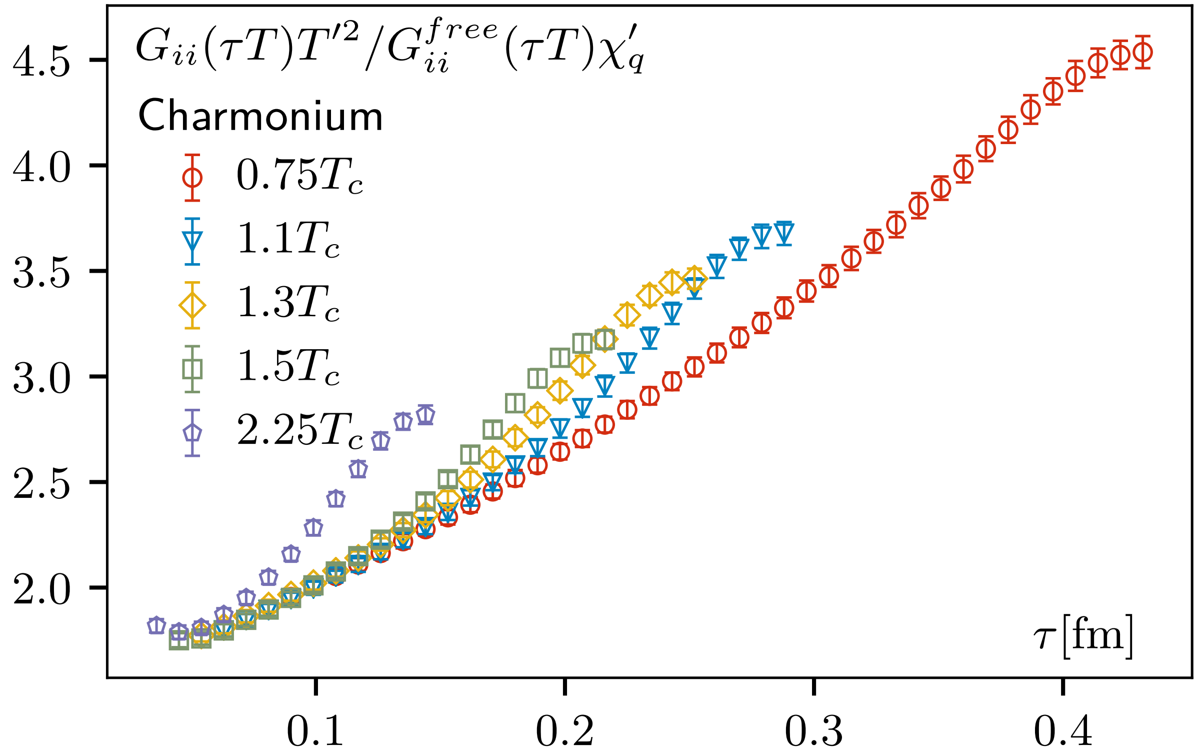


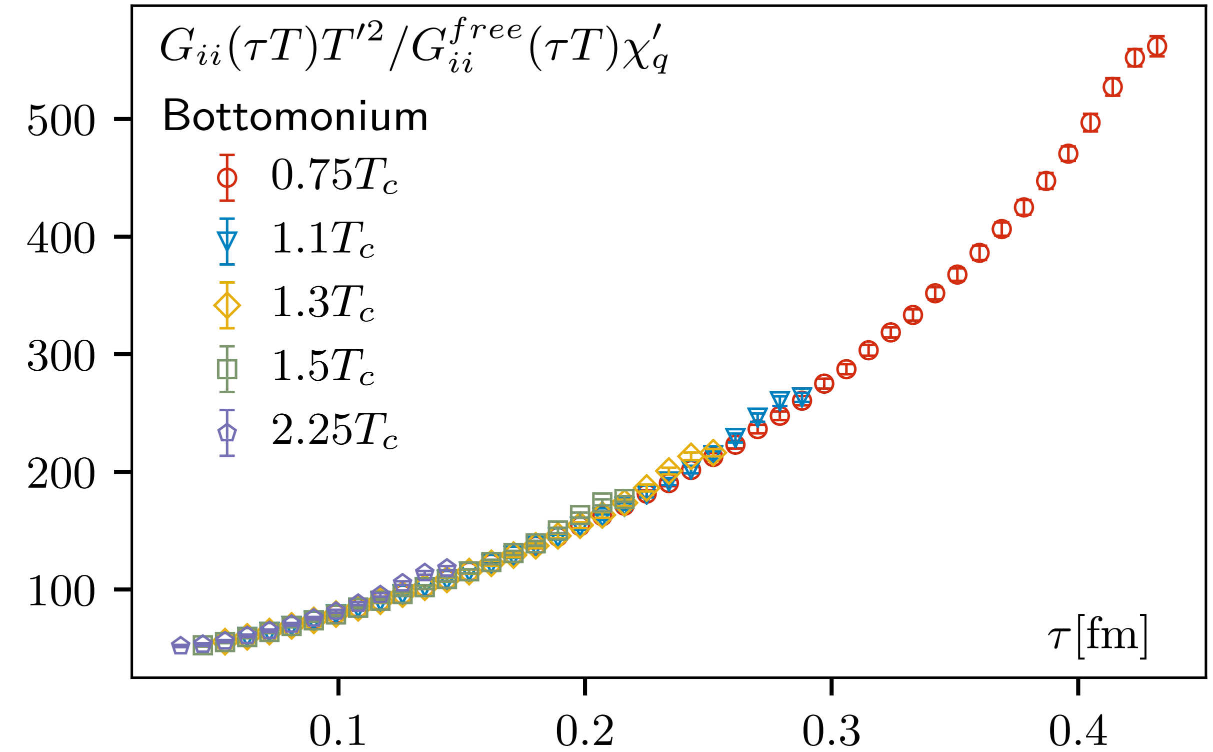


- Extrapolate the correlators to continuum limit with ansatz:

$$a_i = \frac{m}{N_\tau^2} + b$$

Temperature dependence of correlators

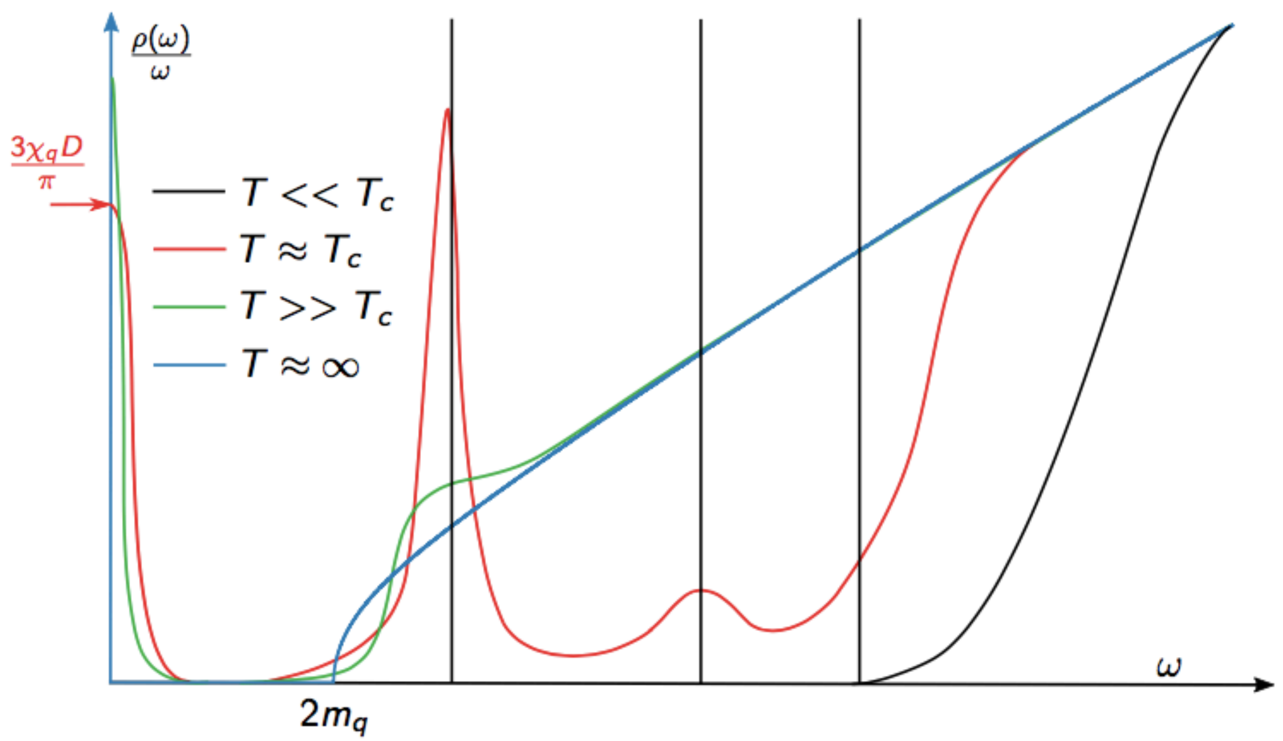




- Extract spectral functions from correlators via:

$$G_H(\tau) = \int \frac{d\omega}{\pi} K(\omega,\tau,T)\rho_H(\omega,T)$$

- Use a perturbatively inspired model spectral function

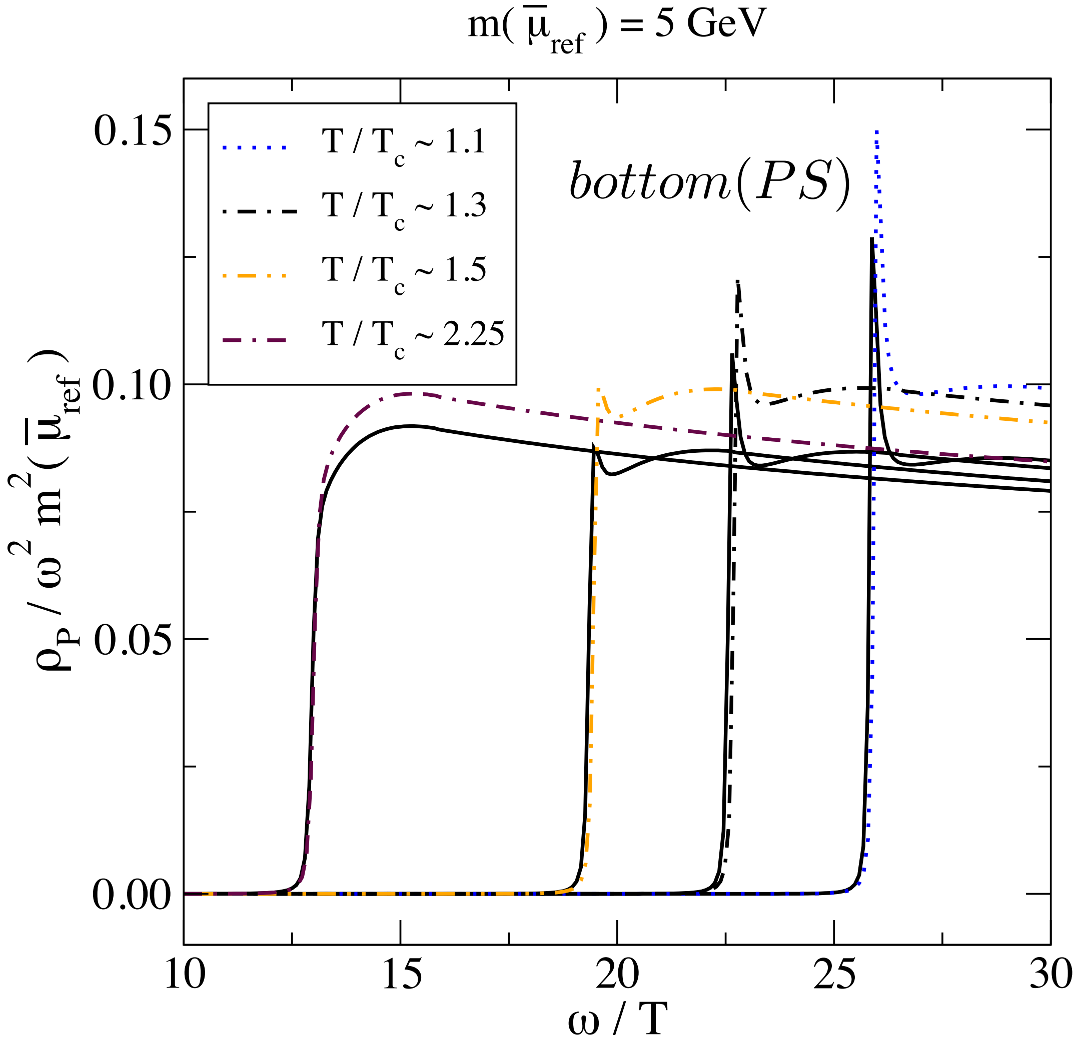


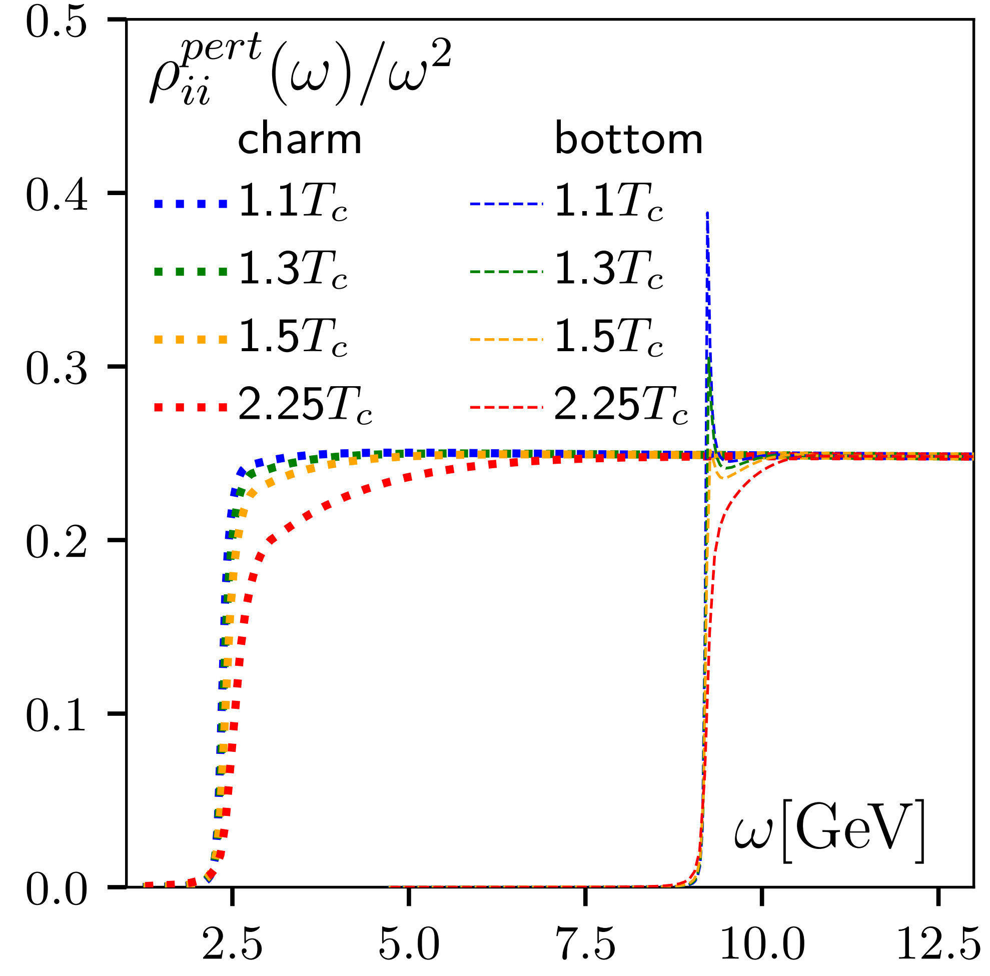
- transport peak
- bound state region
- continuum part

Perturbative spectral functions

- pNRQCD calculations applicable around the threshold [M. Laine, JHEP05(2007)028]
- Ultraviolet asymptotics valid well above the threshold [Y. Burnier and M. Laine, EPJC72, 1902(2012)]
- Combine two parts by interpolation: [Y. Burnier et al., JHEP11(2017)206]

$$\rho_V^{pert}(\omega) = A^{match}\Phi(\omega)\rho_V^{\mathrm{pNRQCD}}(\omega)\theta(\omega^{match}-\omega) + \rho_V^{vac}(\omega)\theta(\omega-\omega^{match})$$





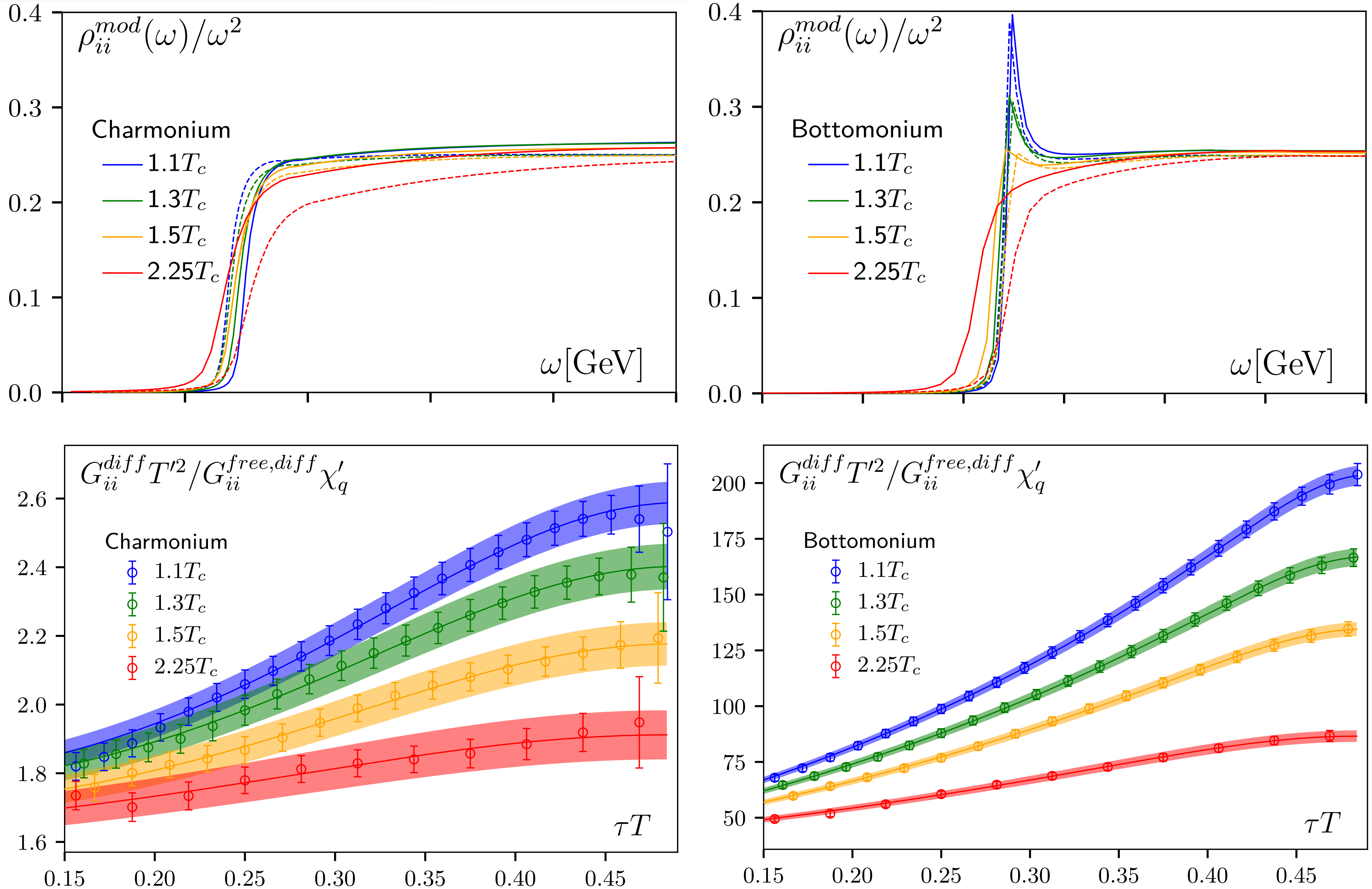
- No transport peak in this channel:

$$\rho_{PS}^{mod}(\omega) = A\rho_{PS}^{pert}(\omega - B)$$

- Separate from the (sharp) transport peak:

$$\rho_{ii}(\omega) = \rho_{ii}^{trans}(\omega) + \rho_{ii}^{mod}(\omega)$$

Dissociation temperatures



- Fit model to the difference of adjacent correlators

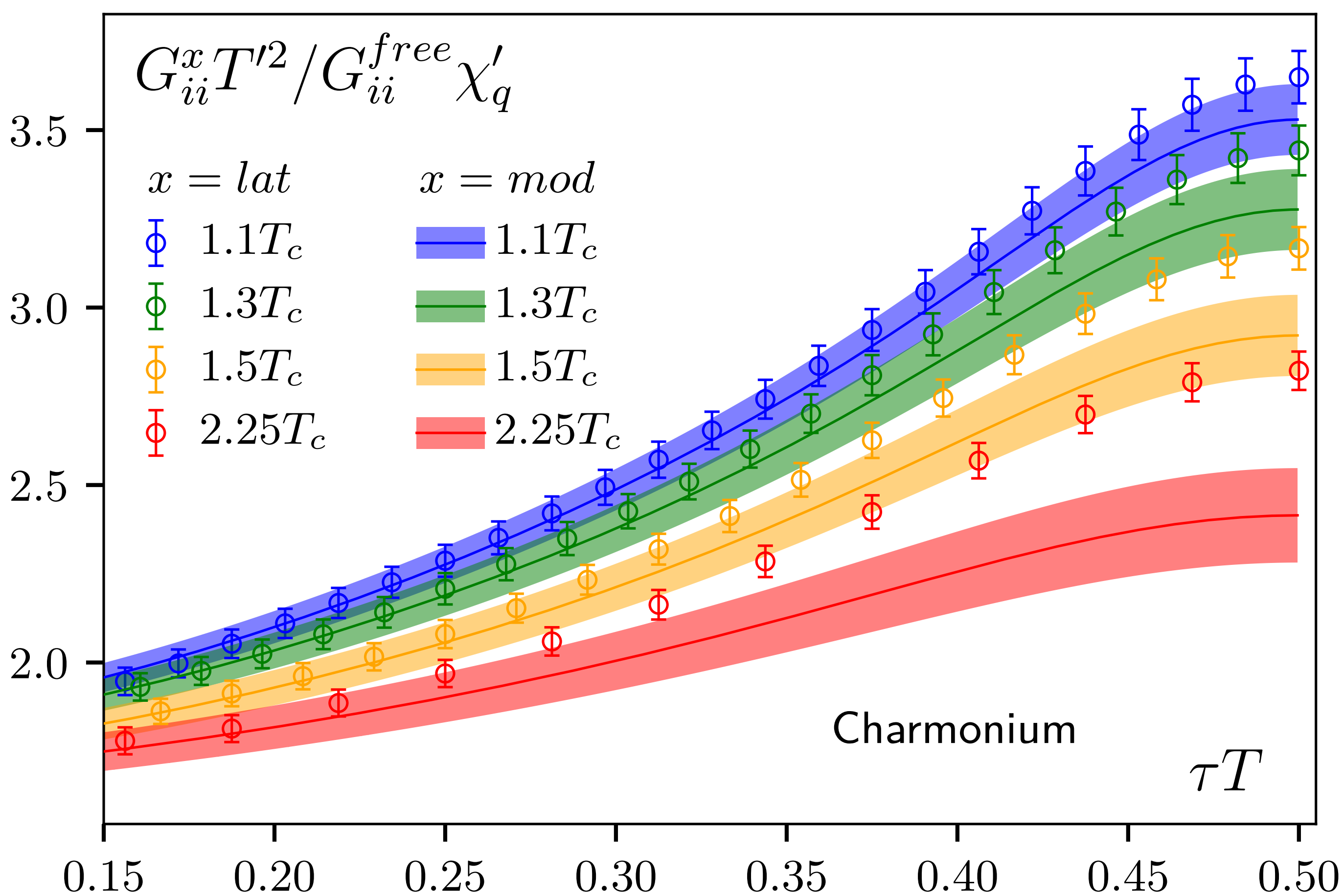
$$G_{ii}^{diff}(\tau/a) = G_{ii}(\tau/a+1) - G_{ii}(\tau/a) \approx G_{ii}^{mod}(\tau/a)$$

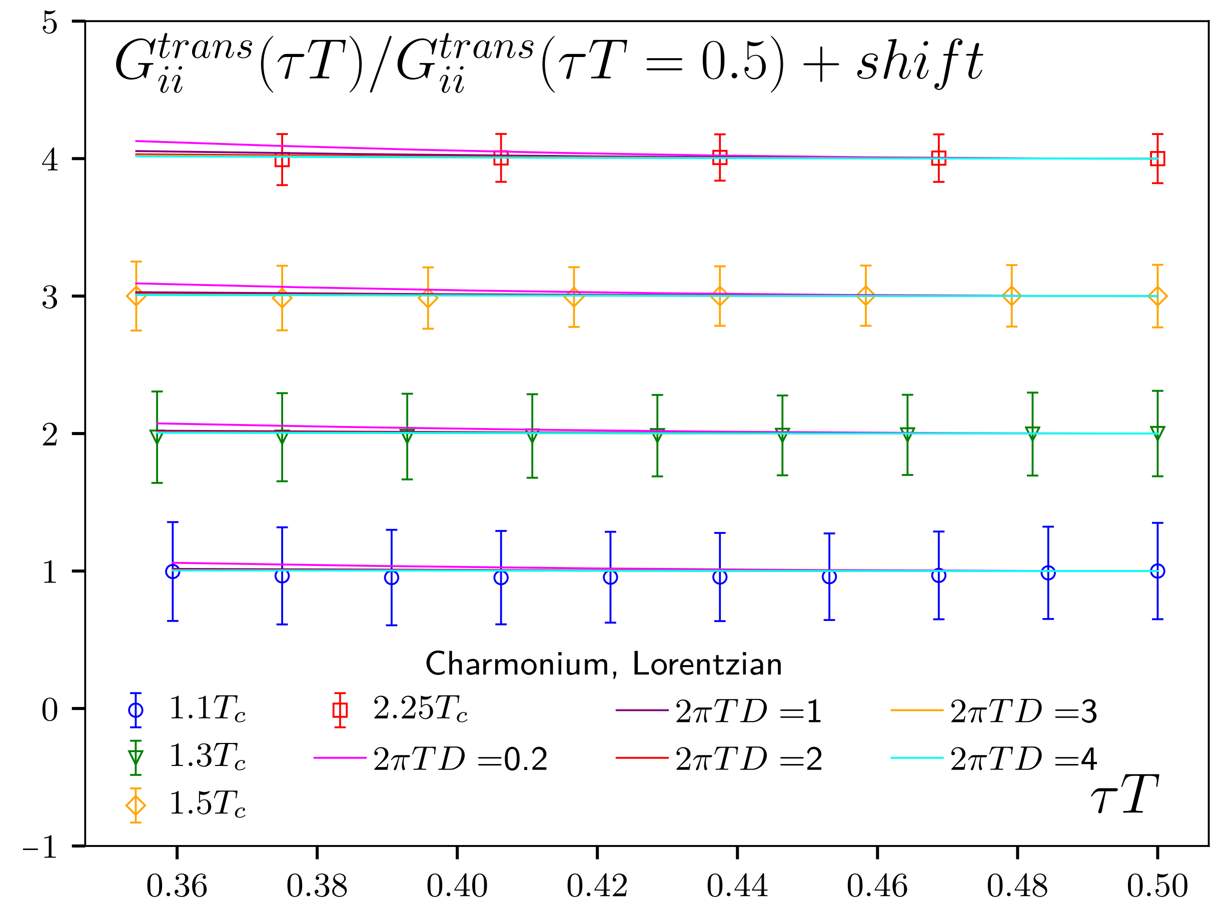
- For J/psi no resonance peak is needed to describe the lattice data even at 1.1T$_c$
- For Upsilon the resonance peak persists to 1.5T$_c$

Transport contribution

- Reconstruct the transport contribution: $G_{ii}^{trans}(\tau T) = G_{ii}(\tau T) - G_{ii}^{mod}(\tau T)$
- Solving transport peak using Lorentzian ansatz:

$$\rho_{ii}^{trans}(\omega) = 3\chi_q \frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2} \quad \longrightarrow \quad D = \frac{1}{3\chi_q}\lim_{\omega\to 0}\frac{\rho_{ii}^{trans}(\omega)}{\omega} \qquad \eta = \frac{T}{MD}$$



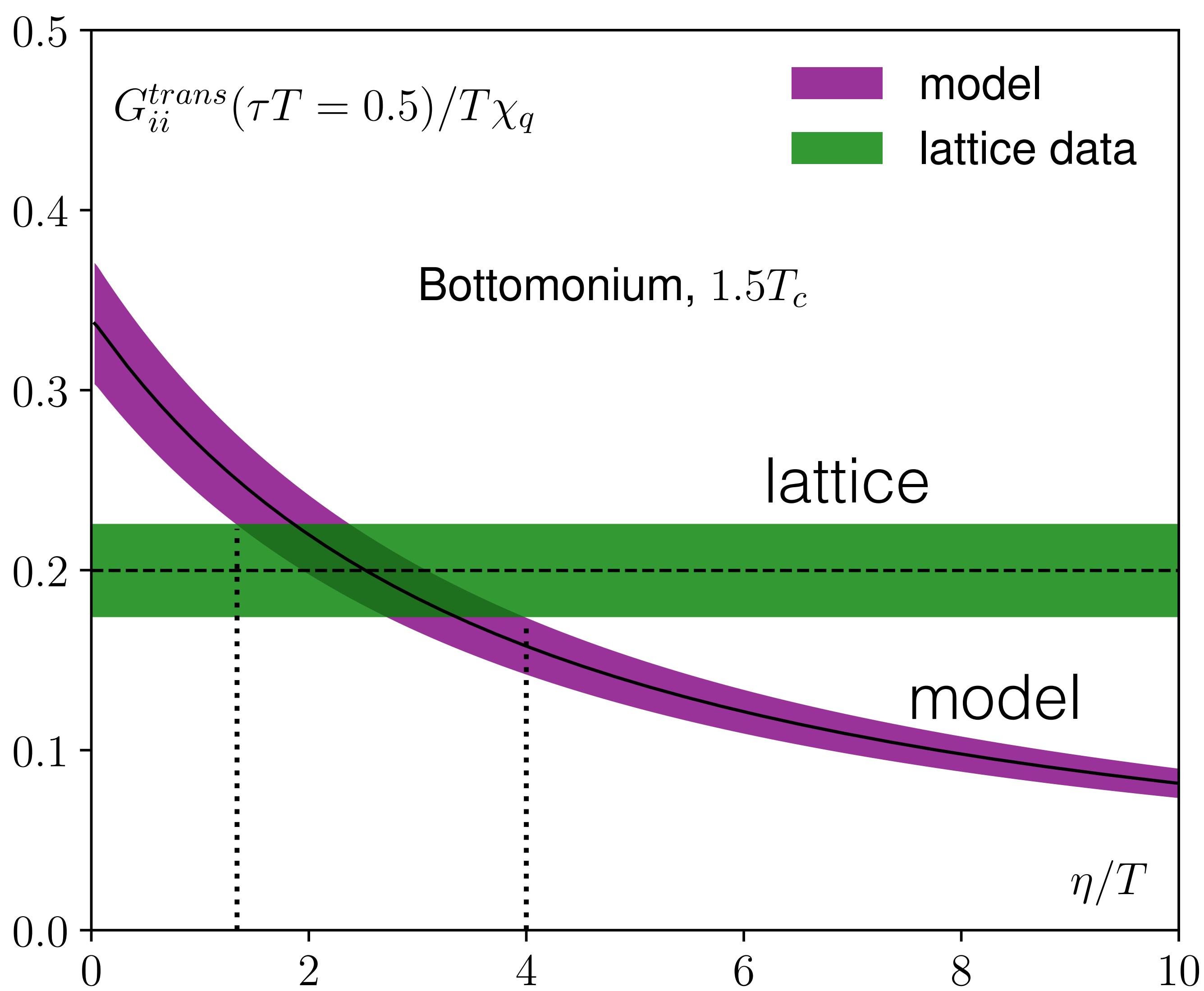


✳ Difficult to resolve D from the tiny curvature of G^{trans}

✳ Not shown here but similar for bottomonium

Estimate transport peak from midpoint correlators

- Transport peak plays its most significant role at midpoint τT=0.5
- Calculate midpoint correlator by integrating Lorentzian ansatz with varying eta at physical charm&bottom quark mass
- Compare with lattice data and find range for eta from intersections



$$\rho_{ii}^{trans}(\omega) = 3\chi_q \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2}$$

	Charmonium		Bottomonium	
T/T_c	η/T	$2\pi TD$	η/T	$2\pi TD$
1.1	7.37-21.38	0.08-0.24	<0.81	>0.66
1.3	7.75-20.28	0.10-0.26	0.30-2.76	0.22-2.04
1.5	7.93-17.08	0.14-0.29	1.40-4.02	0.18-0.51
2.25	4.98-10.45	0.33-0.70	0.62-3.20	0.33-1.73

✳ Analysis of the upper integration limit suggests the results for charmonium not trustable

Estimate transport peak from thermal moments (I)

- Expand the correlation/kernel function around the midpoint

$$G_H(\tau T) = \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \rho_H(\omega) \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$$

$$= \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \frac{\rho_H(\omega)}{\sinh(\frac{\omega}{2T})} \left(1 + \frac{1}{2!}\left(\frac{\omega}{T}\right)^2 (\tau T - 0.5)^2 + \frac{1}{4!}\left(\frac{\omega}{T}\right)^4 (\tau T - 0.5)^4 + \dots\right)$$

$$\approx \underline{G_H^{(0)}} + \underline{G_H^{(2)}}(\tau T - 0.5)^2 + \underline{G_H^{(4)}}(\tau T - 0.5)^4$$

- Thermal moments defined as Taylor coefficients $G_H^{(n)} = \frac{1}{n!} \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \left(\frac{\omega}{T}\right)^n \frac{\rho_H(\omega)}{\sinh(\frac{\omega}{2T})}$

- Fit correlators to get the ratio of thermal moments $R_H^{n,m} = G_H^{(n)}/G_H^{(m)}$

$$\frac{\Delta_H(\tau T)}{G_H(\tau T = 0.5)} \approx R_H^{2,0} \left(1 + \sum_{n=1}^{N} R_H^{2n+2,2n} (\tau T - 0.5)^{2n}\right) \qquad \Delta_H(\tau T) = \frac{G_H(\tau T) - G_H(\tau T = 0.5)}{(\tau T - 0.5)^2}$$

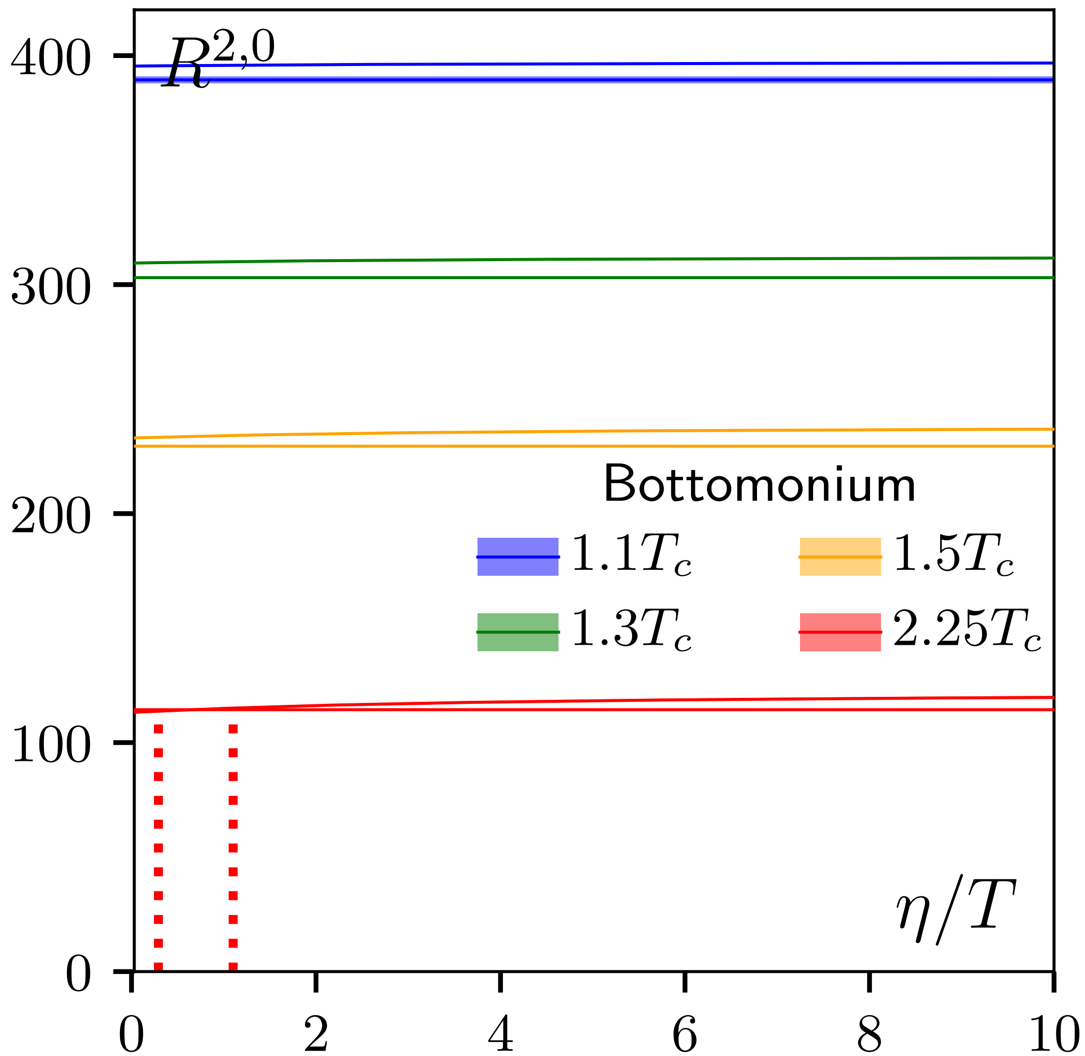
- Calculate the ratios from spectral function

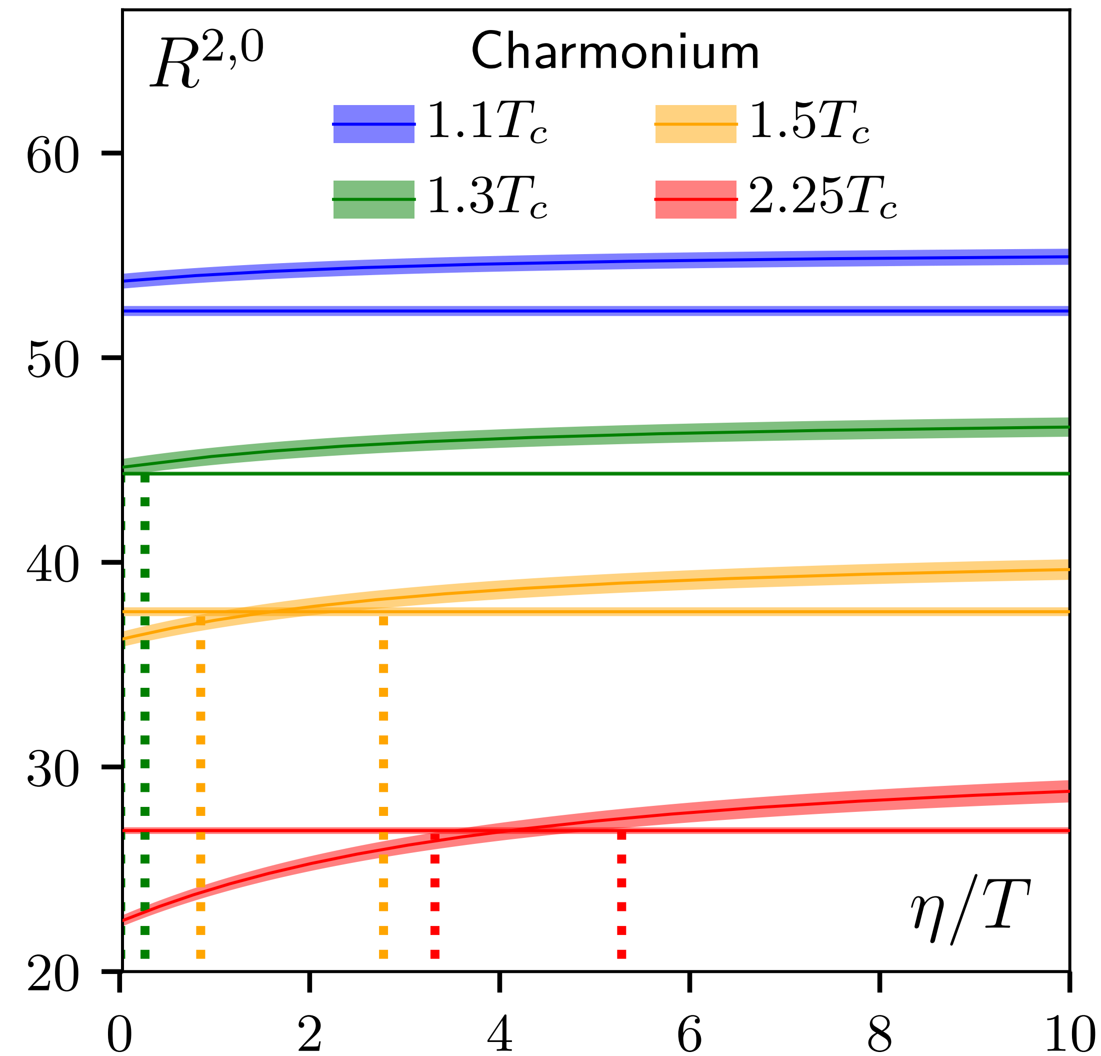
$$R^{2,0}(A, B, \eta) = \frac{G^{(2)}_{mod}(A, B) + G^{(2)}_{trans}(\eta)}{G^{mod}_{ii}(\tau T = 0.5) + G^{trans}_{ii}(\tau T = 0.5)}$$

$$G^{(2)}_{mod}(A, B) = \frac{1}{2} \int_0^\infty \frac{\mathrm{d}\omega}{\pi} \left(\frac{\omega}{T}\right)^2 A\rho^{pert}_{ii}(\omega - B) \frac{1}{\sinh\left(\frac{\omega}{2T}\right)}$$

$$G^{(2)}_{trans}(\eta) = \frac{1}{2} \int_0^\infty \frac{\mathrm{d}\omega}{T} \left(\frac{\omega}{T}\right)^2 3\chi_q \frac{T}{M} \frac{\omega\eta}{\omega^2 + \eta^2} \frac{1}{\cosh\left(\frac{\omega}{2\pi T}\right) \sinh\left(\frac{\omega}{2T}\right)}$$

Estimate transport peak from thermal moments (II)

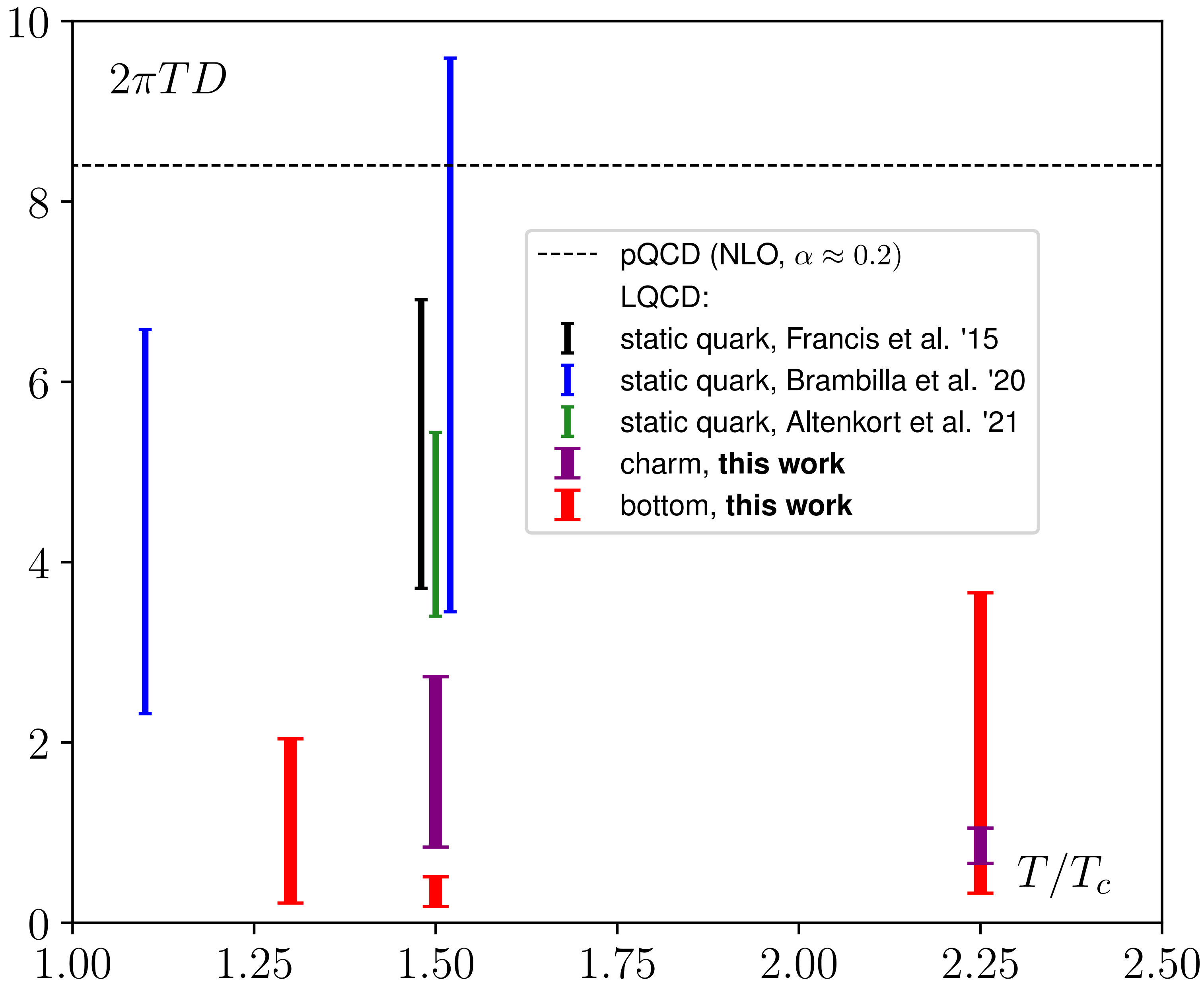




	Charmonium		Bottomonium	
T/T_c	η/T	$2\pi TD$	η/T	$2\pi TD$
1.1	-	-	-	-
1.3	<0.27	>7.48	-	-
1.5	0.85-2.78	0.84-2.73	-	-
2.25	3.32-5.28	0.66-1.05	0.29-1.10	0.97-3.66

- The intersections of different methods determine the range of eta

Combine the results and compare with literature



- The most reliable results from different methods:

	Charm		Bottom	
T/T_c	η/T	$2\pi TD$	η/T	$2\pi TD$
1.1	-	-	-	-
1.3	-	-	0.30-2.76	0.22-2.04
1.5	0.85-2.78	0.84-2.73	1.40-4.02	0.18-0.51
2.25	3.32-5.28	0.66-1.05	0.29-3.20	0.33-3.66

✳Smaller than results from LQCD at heavy quark mass limit (high order corrections?)

[A. Bouttefeux and M. Laine, JHEP12(2020)150]

A simpler case: heavy quark mass limit

- Construct a kinetic mass dependent momentum diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_q}\sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Big|^{\eta\ll|\omega|\lesssim\omega_{\rm UV}} \quad + \quad D = T/(\eta M) \quad \Longrightarrow \quad D = \frac{2T^2}{\kappa^{(M)}}$$

- Large quark mass limit in effective field theory [S. Caron-Huot et al., JHEP 0904 (2009) 053]

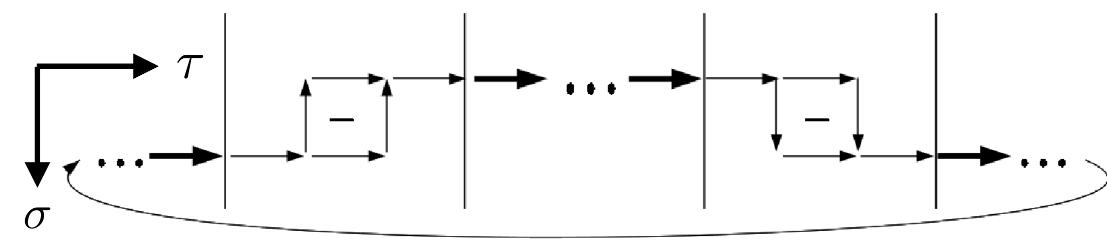
$$\kappa \;\equiv\; \frac{\beta}{3}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_q}\int_{-\infty}^{\infty}\mathrm{d}t\; e^{i\omega(t-t')}\int \mathrm{d}^3\vec{x}\,\left\langle\frac{1}{2}\left\{\frac{\mathrm{d}\mathcal{J}^i(t,\vec{x})}{\mathrm{d}t},\frac{\mathrm{d}\mathcal{J}^i(0,\vec{0})}{\mathrm{d}t'}\right\}\right\rangle\right]$$

- Carry out large quark mass limit for the operators

$$\kappa \;\equiv\; \frac{\beta}{3}\sum_{i=1}^{3}\lim_{\omega\to 0}\left[\lim_{M\to\infty}\frac{M^2}{\chi_q}\int_{-\infty}^{\infty}\mathrm{d}t\;\int\;\mathrm{d}^3\vec{x}\,\left\langle\frac{1}{2}\left\{\left[\hat{\phi}^\dagger gE^i\hat{\phi}-\hat{\theta}^\dagger gE^i\hat{\theta}\right](t,\vec{x})\,,\left[\hat{\phi}^\dagger gE^i\hat{\phi}-\hat{\theta}^\dagger gE^i\hat{\theta}\right](0,\vec{0})\right\}\right\rangle\right]$$

- Perform analytic continuation and discretize the operator on the lattice

$$G_{EE}(\tau) = -\frac{1}{3}\sum_{ii=1}^{3}\frac{\langle \mathrm{Re\ Tr}[U(\beta,\tau)gE_i(\tau,\vec{0})U(\tau,0)gE_i(0,\vec{0})]\rangle}{\langle \mathrm{Re\ Tr}[U(\beta,0)]\rangle}$$



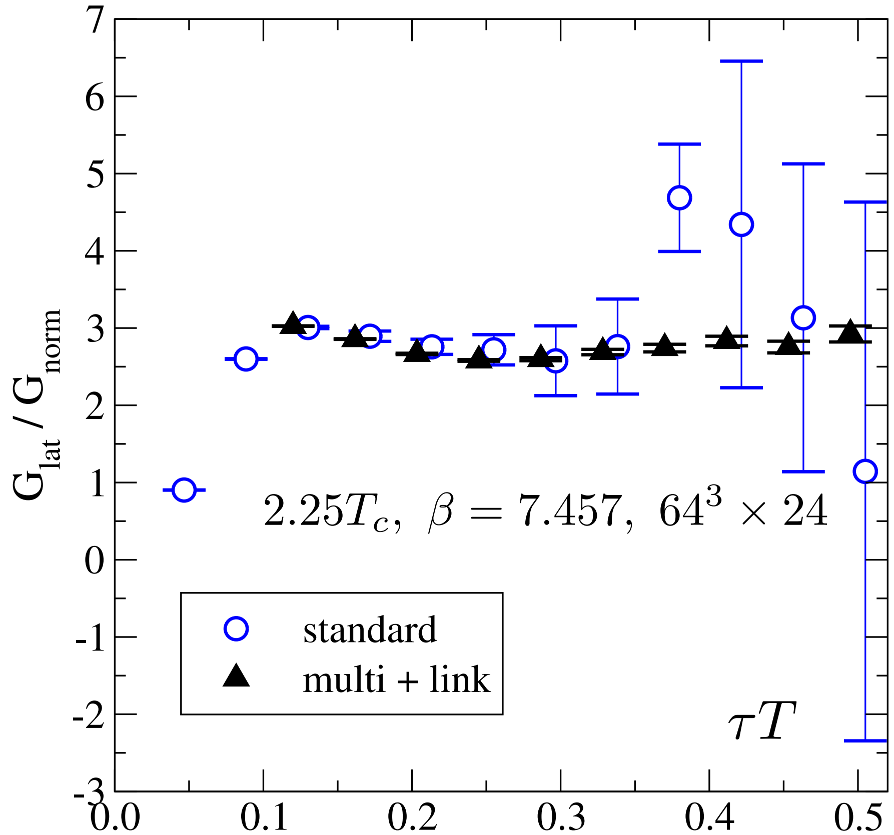
$$\Rightarrow \quad \kappa = \lim_{\omega\to 0} 2T\frac{\rho(\omega)}{\omega}$$

$$D = 2T^2/\kappa \;\; (M \gg \pi T)$$

- Correlators cheap to measure on the lattice
- Less structure in spectral functions (no transport peak and resonance peak)

Gradient flow

- Color-electric correlators from multi-level and link-integration: [PRD92(2015)116003]



❖Multi-level [Luscher & Weisz, JHEP09 (2001)010]
❖Link-integration [Forcrand &Roiesnel, PLB151(1985)77]

However, only works in quenched approximation
Need Gradient Flow in full QCD

[Luscher & Weisz, JHEP1102(2011)051]
[Narayanan & Neuberger, JHEP0603(2006)064]

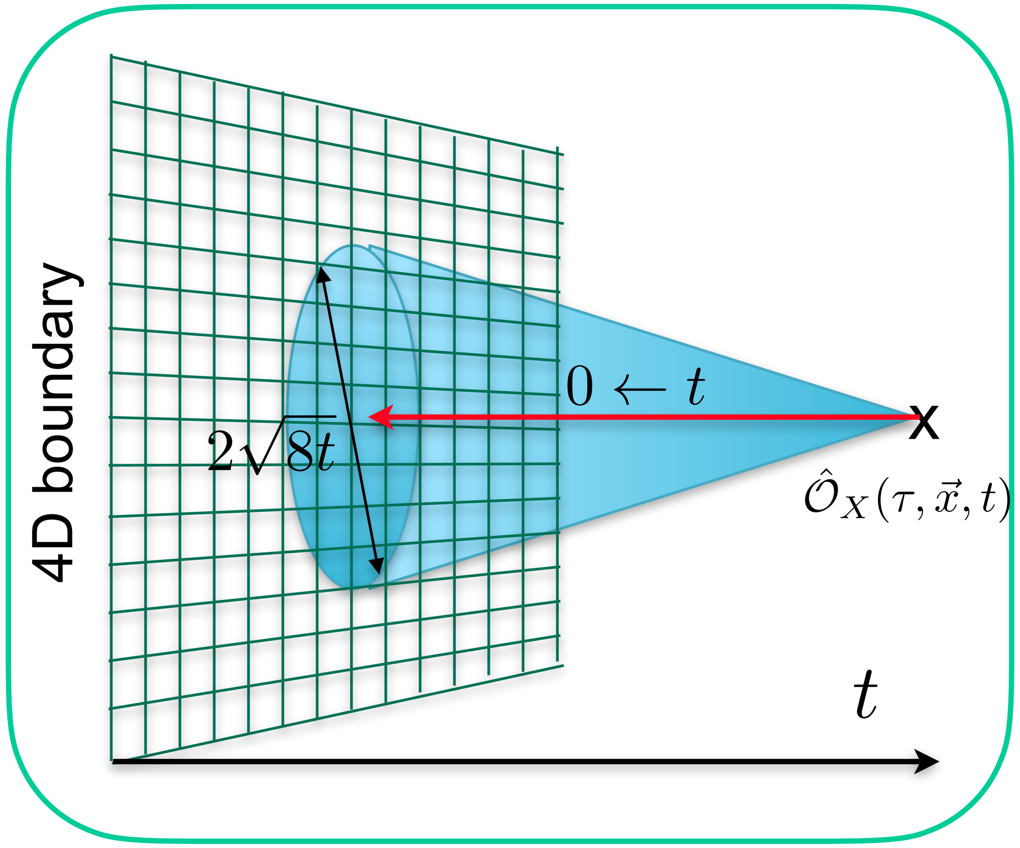
- Gradient flow as a "diffusion" equation along *t* :

$$\partial_t B(x,t) = D_\nu G_{\nu\mu}(x,t) \qquad B_\nu(x,t)|_{t=0} = A_\nu(x)$$

- Small *t* expansion: $\mathcal{O}(x,t) \xrightarrow{t \to 0} \sum_k c_k(t) \mathcal{O}_k^R(x)$

- Applications:

running coupling / topo. charge / scale setting
defining operators / noise reduction / ...



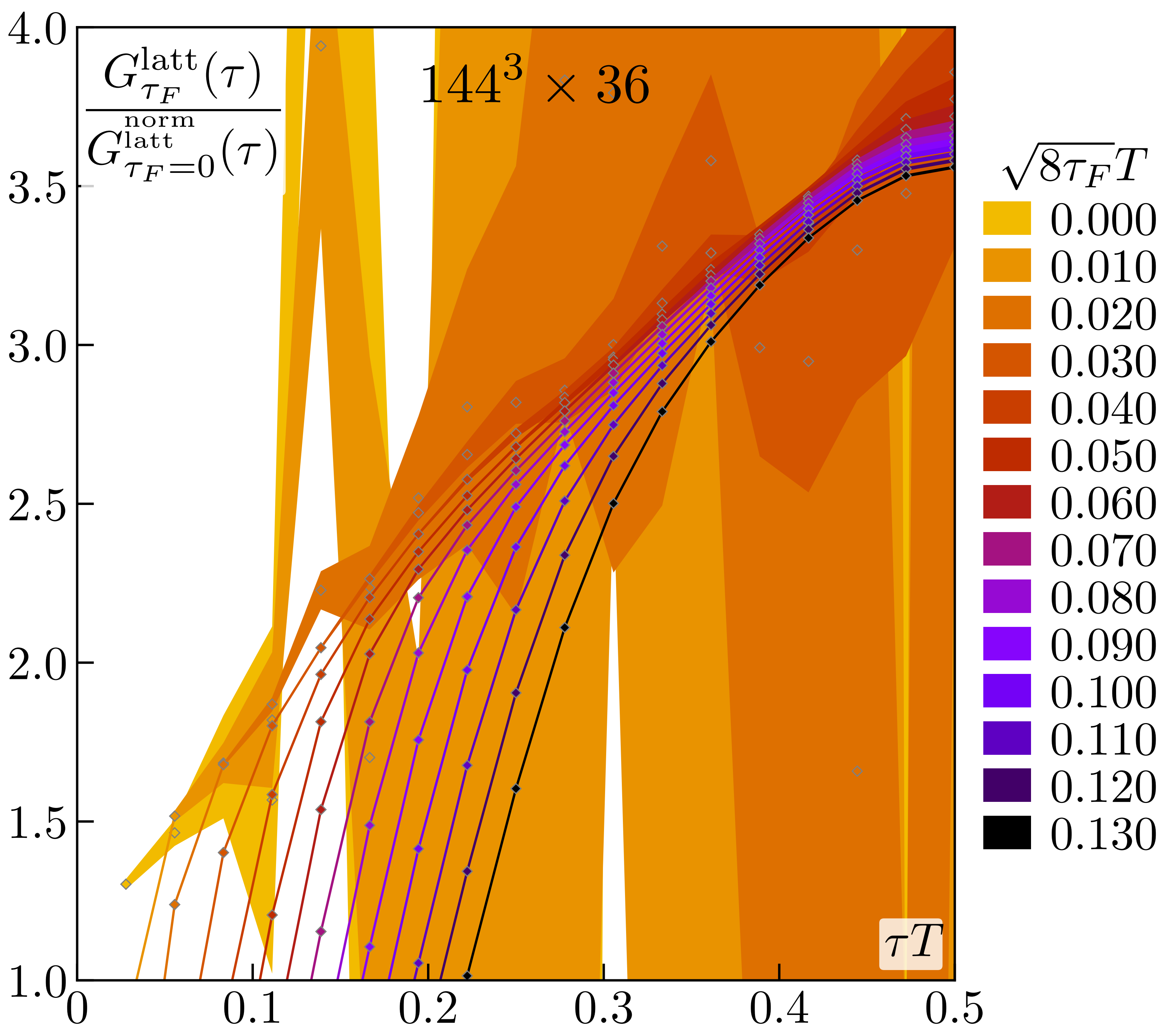
Lattice setup

β	a[fm](a^{-1}[GeV])	N_σ	N_τ	T/T_c	#confs.
6.8736	0.026 (7.496)	64	16	1.50	10000
7.0350	0.022 (9.119)	80	20	1.50	10000
7.1920	0.018 (11.19)	96	24	1.50	10000
7.3940	0.014 (14.21)	120	30	1.50	10000
7.5440	0.012 (17.01)	144	36	1.50	10000

- Large quenched isotropic lattice
- Five different lattices (and beta)
- Enough statistics
- Intensive discrete flow times

Data good enough for reliable continuum extrapolation and flow time extrapolation !

Flow effects on the correlators

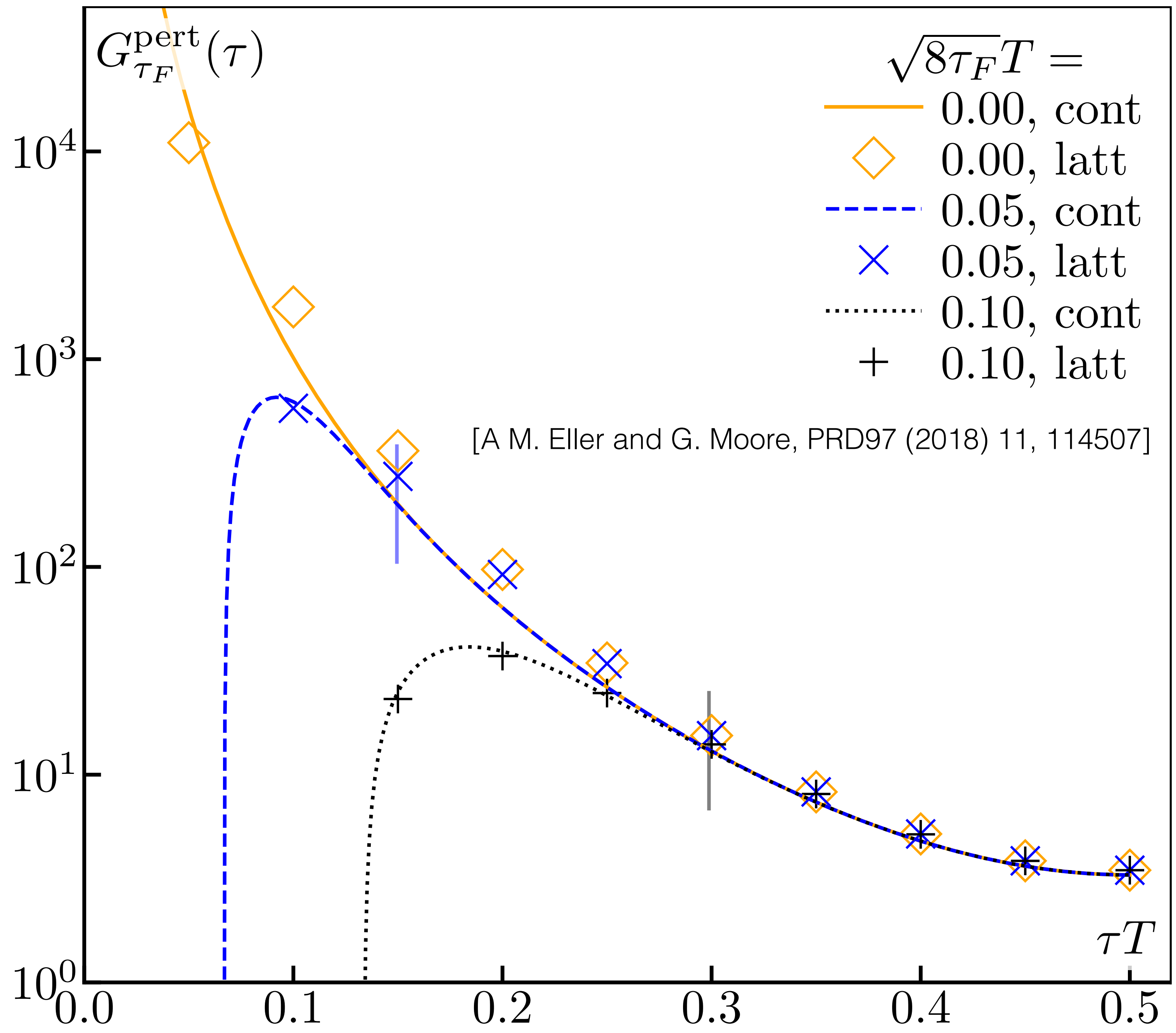


$$G_{\rm norm}(\tau T) = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)}\right]$$

[S. Caron-Huot & M. Laine & G.D. Moore]

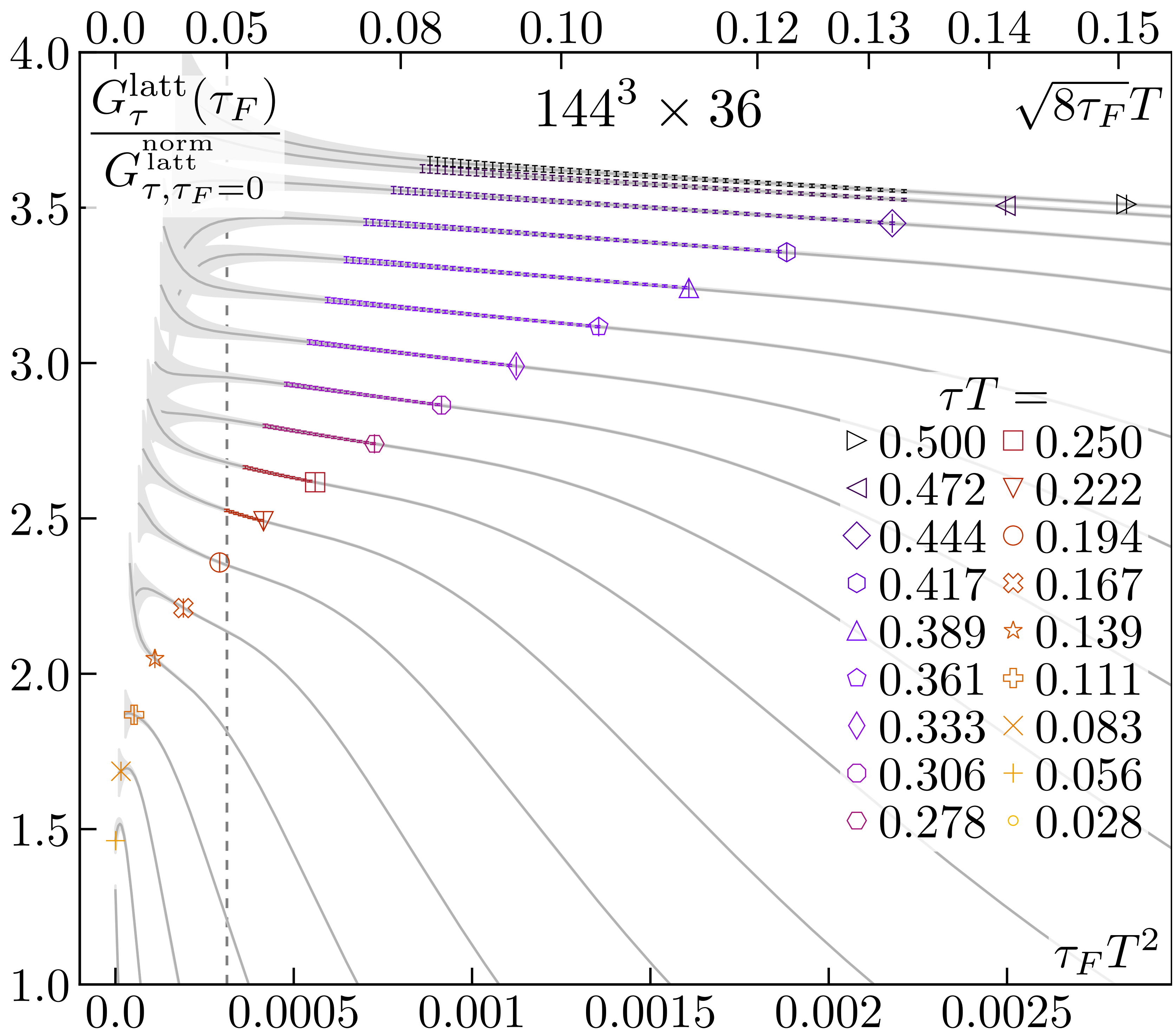
- Gradient flow reduces the error
- Destroys the signal at small distances

Perturbative flow time limits



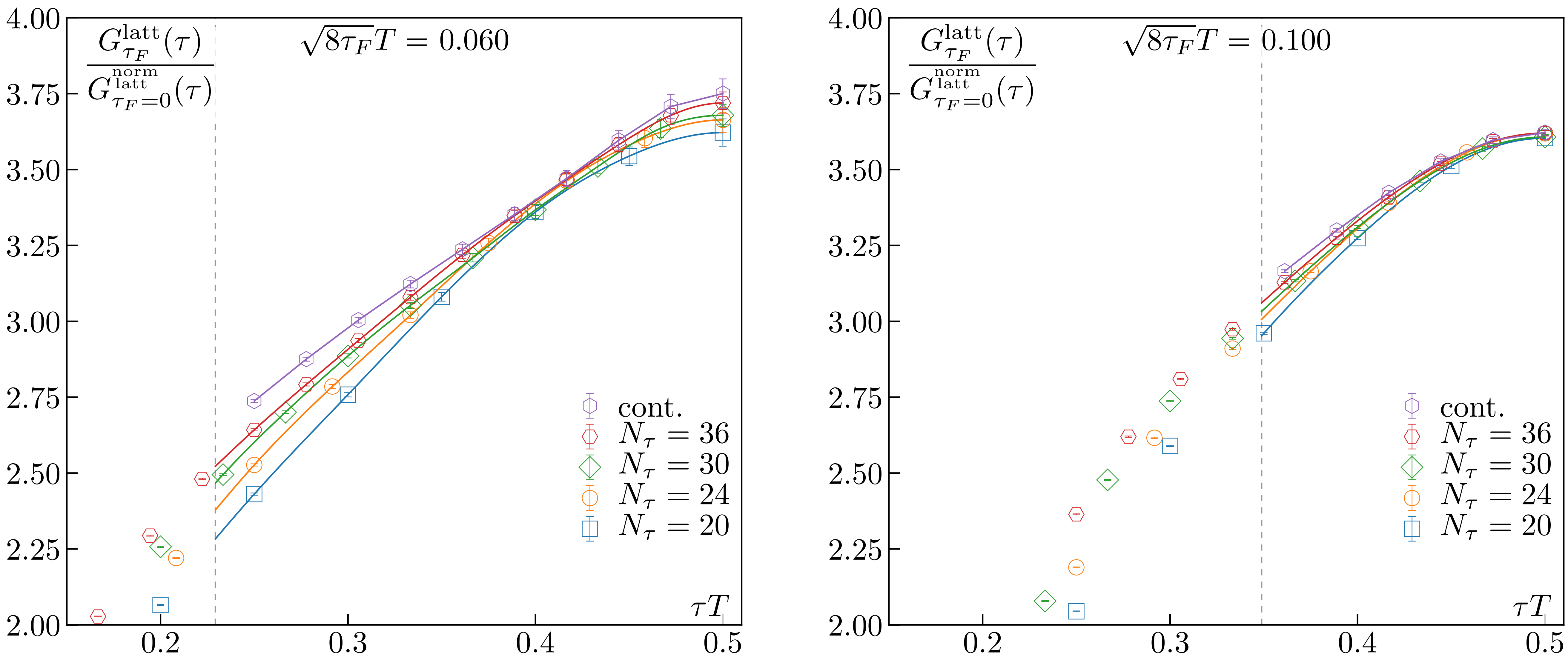
- G_{EE} at τT smaller than LO flow time limit are affected
- G_{EE} at τT larger than LO flow time limit are used in extrapolation

Correlators under gradient flow (1)



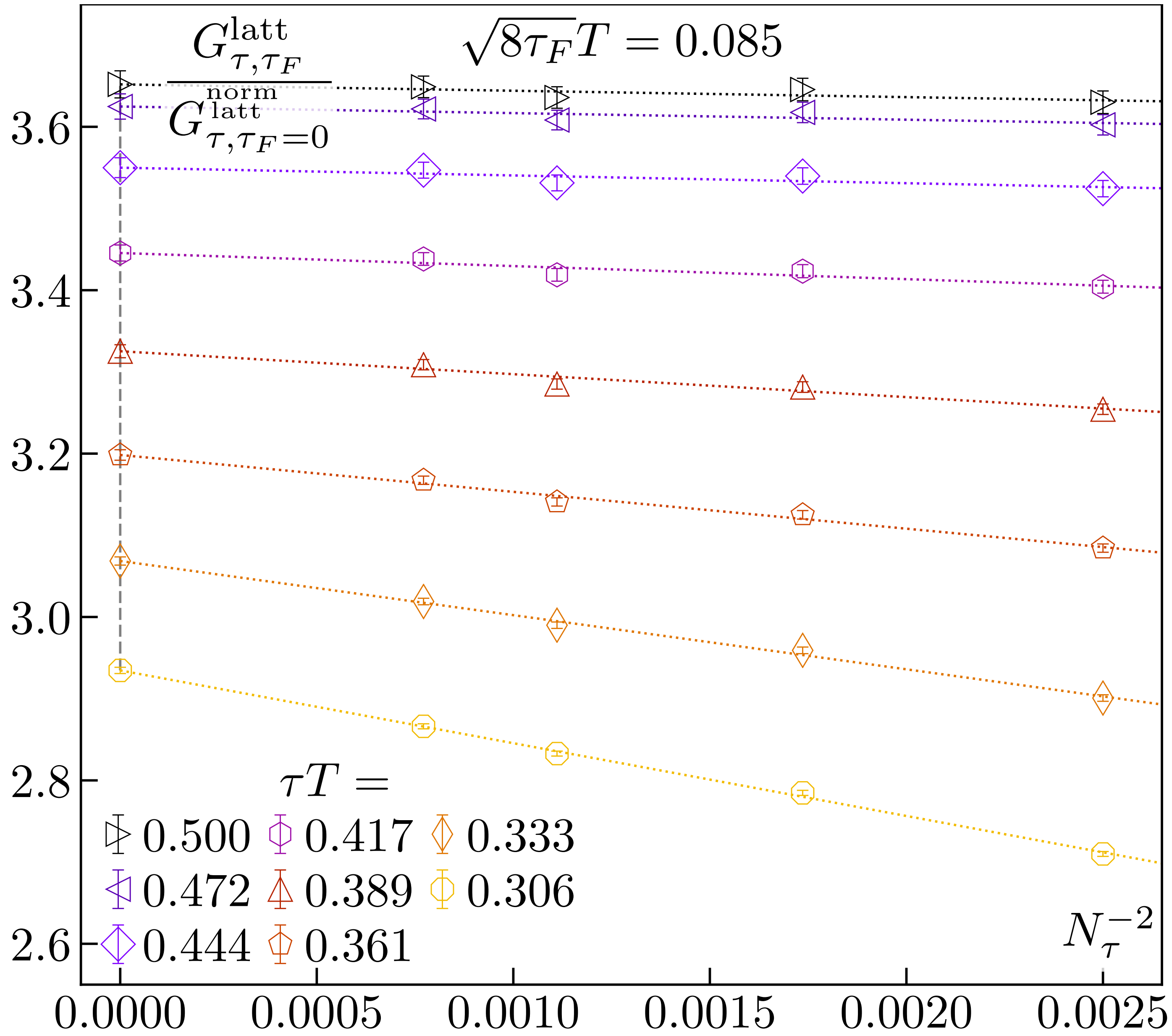
- Linear behavior can be seen in appropriate flow time range

Correlators under gradient flow (2)



- Less distances available for larger flow time
- Gradient flow removes the lattice effects

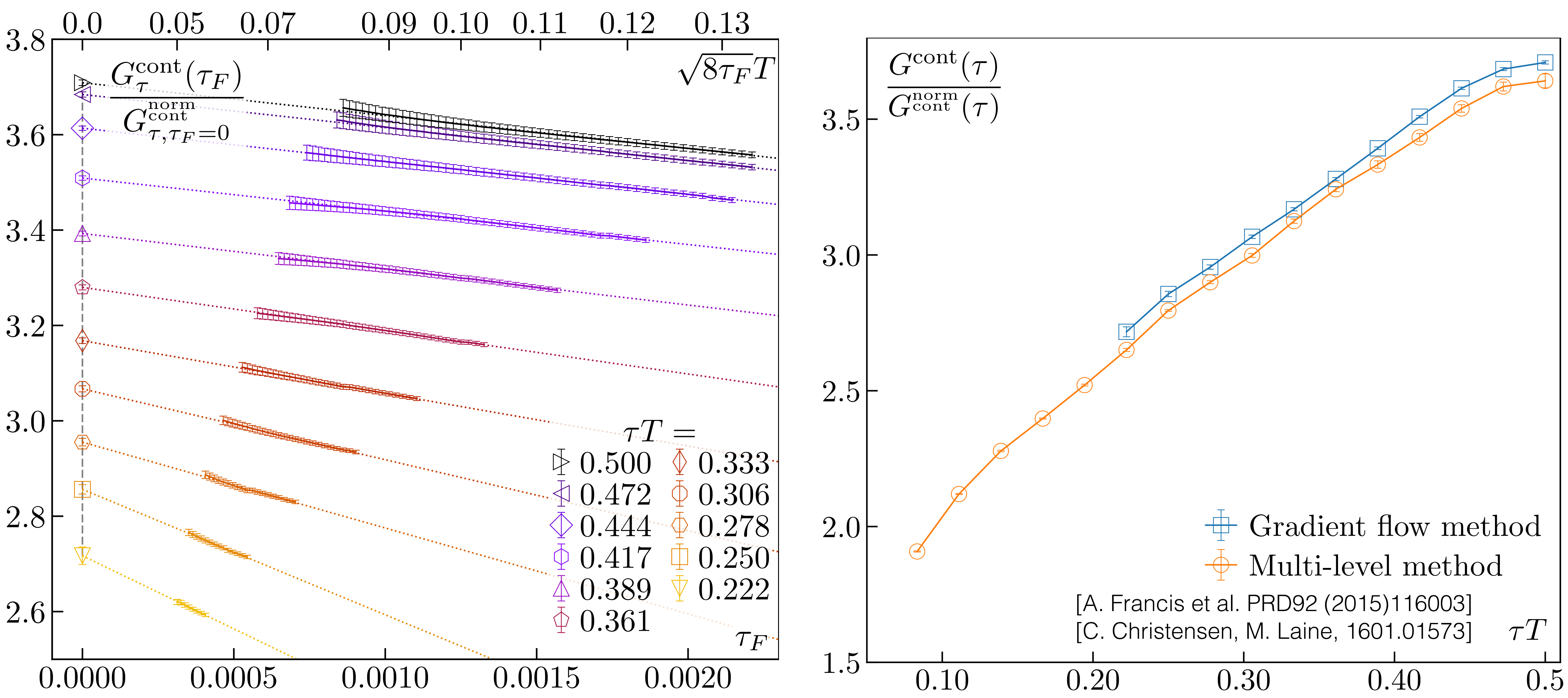
Continuum extrapolation



- Reliable and precise continuum can be achieved with ansatz:

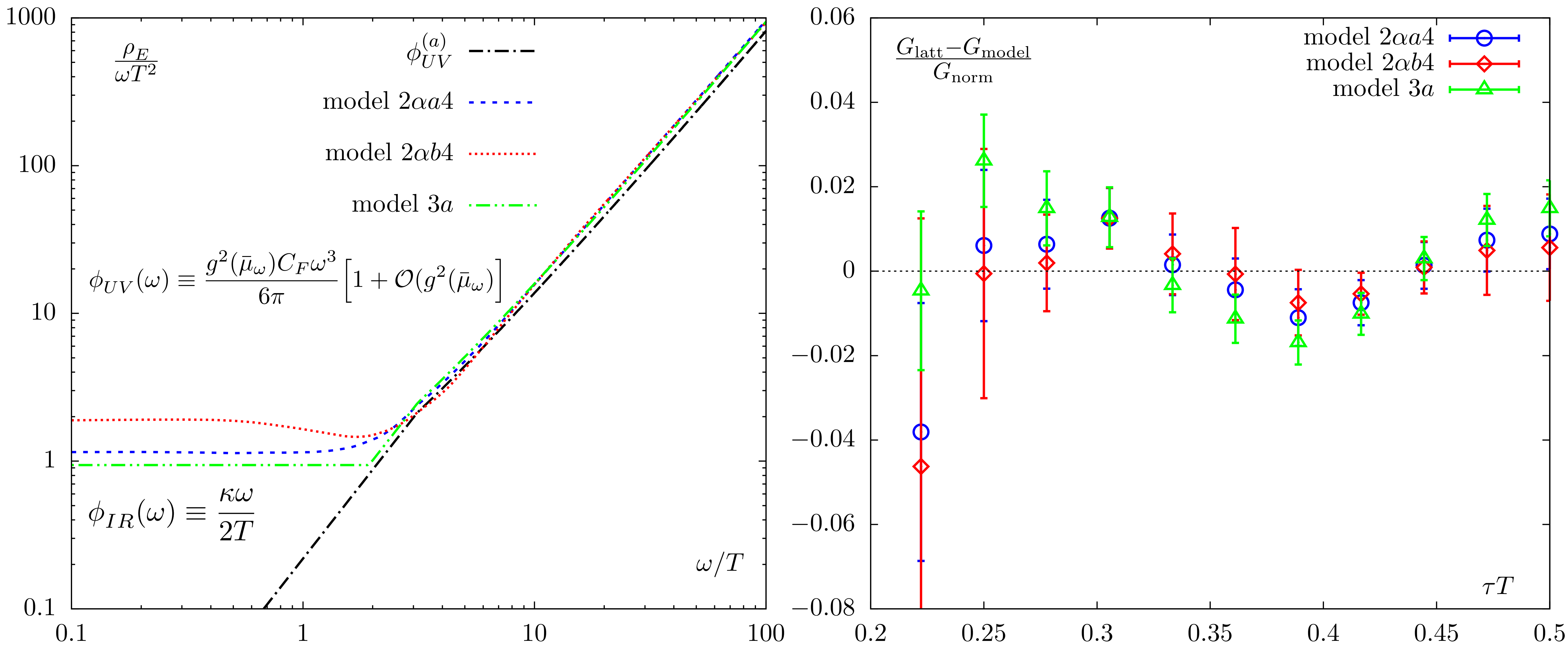
$$G_{\tau,\tau_F}(N_\tau) = \frac{m}{N_\tau^2} + G^{\text{cont}}_{\tau,\tau_F}$$

Flow time extrapolation



- Linear flow time behavior can be seen in continuum-extrapolated correlators
- An overall shift between correlators from GF method and ML method
- Non-pert. renormalization (GF) *v.s.* pert. renormalization (ML)

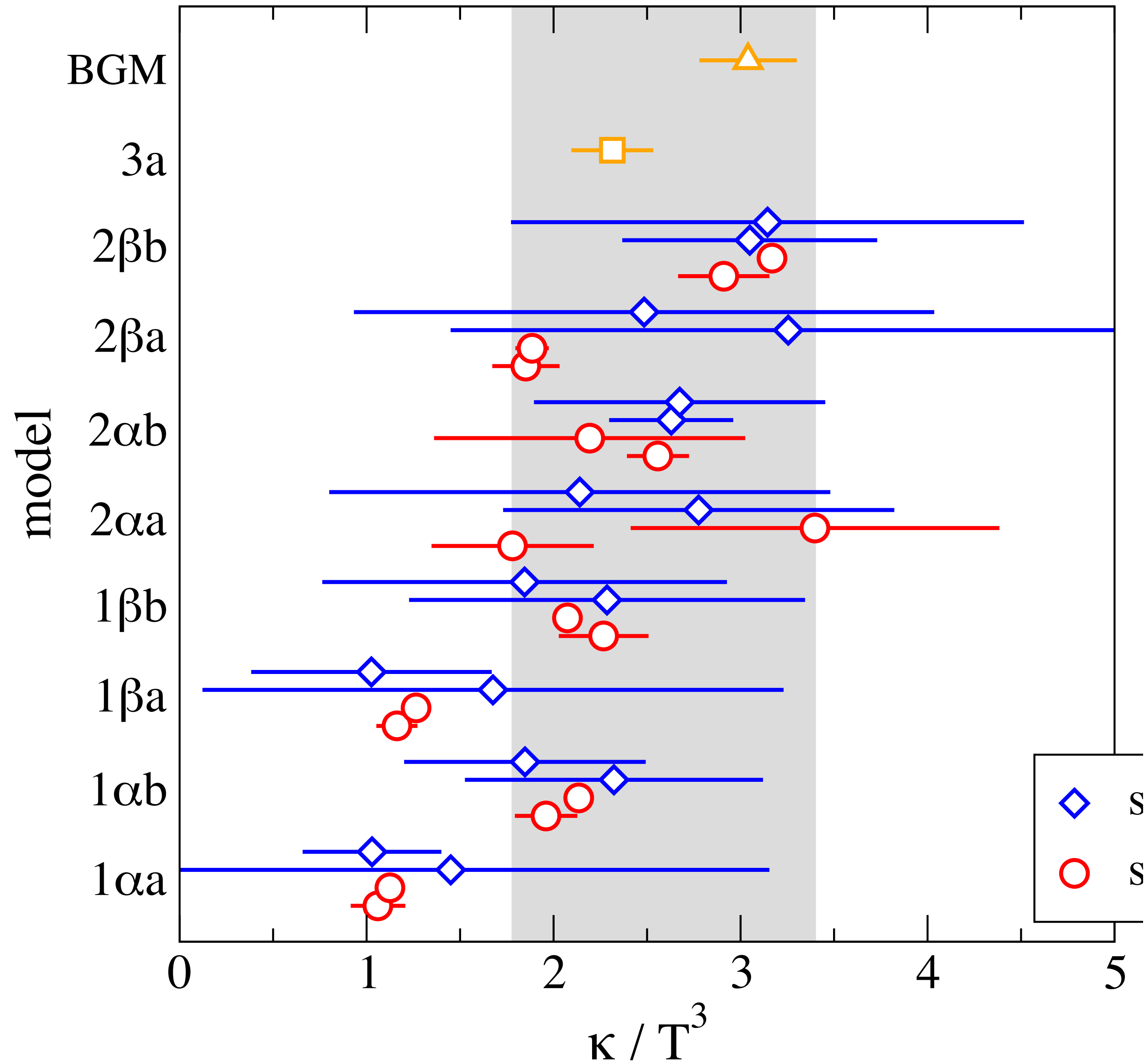
Spectral function reconstruction

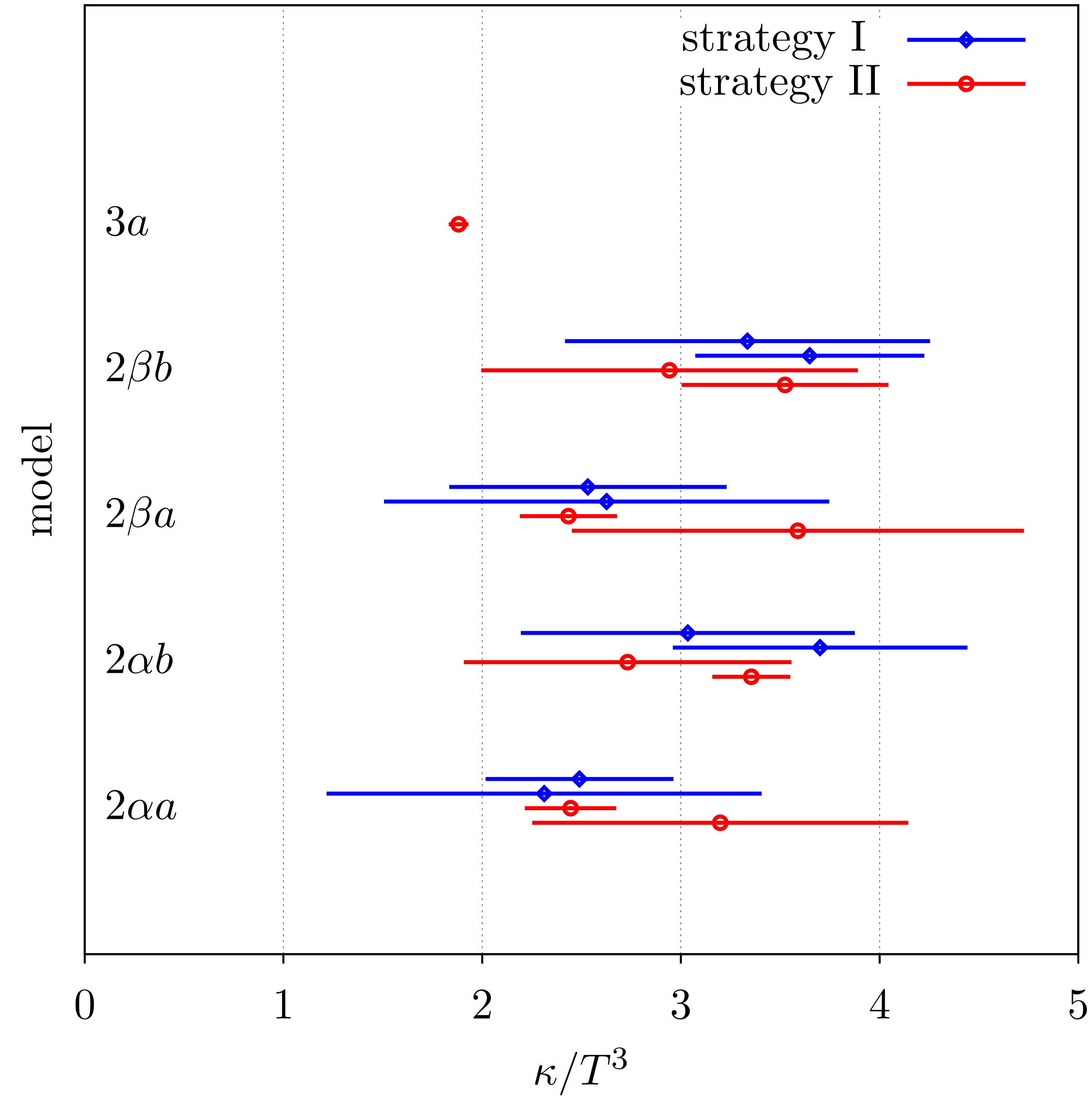


- Chi-square fitting with theoretically motivated models
- Interpolation between different regimes is needed

HQ momentum diffusion coefficient

[A. Francis et al. PRD92 (2015)116003]





$$\Rightarrow \quad \kappa/T^3 = 1.8 - 3.4$$

$$DT = 0.59 - 1.1$$

$$\Rightarrow \quad \kappa/T^3 = 1.80 - 3.70$$

$$DT = 0.54 - 1.11$$

(in agreement with TUMQCD Collaboration, 2007.10078 : $\kappa/T^3 = 1.31 - 3.64$)

Conclusion

- First principle calculations of charmonium and bottomonium correlation functions at physical masses
- Well described by perturbatively inspired models
- No resonance peak needed to describe charmonium down to $1.1T_c$ while resonance peak needed for bottomonium up to $1.5T_c$
- Consistent estimates on heavy quark diffusion coefficient
- High order mass correction to infinite heavy quark diffusion
- Extend to full QCD using large and fine 2+1-flavor HISQ lattices