Lattice calculation powered by factorization theory

Yan-Qing Ma
Peking University

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Outline

I. Hadron structure from experimental data

II. Hadron structure from lattice: ideas

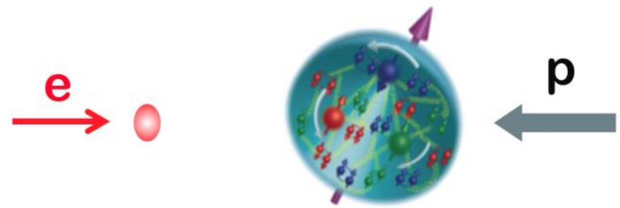
III. Hadron structure from lattice: details

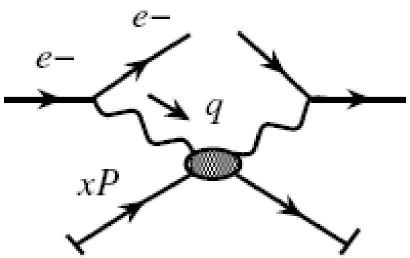
IV. Summary and outlook

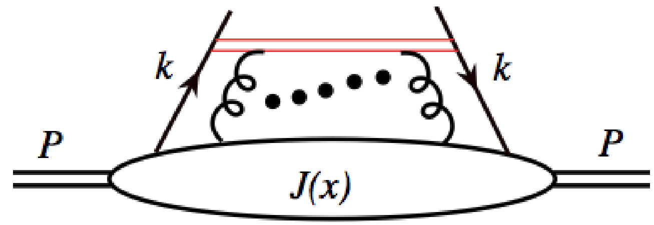
QCD factorization

- The key and a first principle method to relate experimental data to QCD theory

- Electron-hadron:



$\sigma_{\text{tot}}^{\text{DIS}} =$ 

$\otimes$ 

$+ O(\frac{1}{QR})$

Hard-part Probe	Parton-distribution Structure	Correction Approximation

- PDFs/TMDs/…: encoding nonperturbative information in hadrons

Properties of PDFs

- **Well defined in QCD; process independent**

Spin-averaged quark distribution

$$f_{q/p}(x,\mu^2) = \int \frac{d\xi_-}{4\pi} e^{-ix\xi_- P_+} \langle P|\overline{\psi}(\xi_-)\,\gamma_+ \exp\left\{-ig\int_0^{\xi_-} d\eta_- A_+(\eta_-)\right\}\psi(0)|P\rangle$$

- **Defined in infinite frame: not direct physical observable**

Determined by using factorization

- **Logarithmic UV divergent, renormalizable**

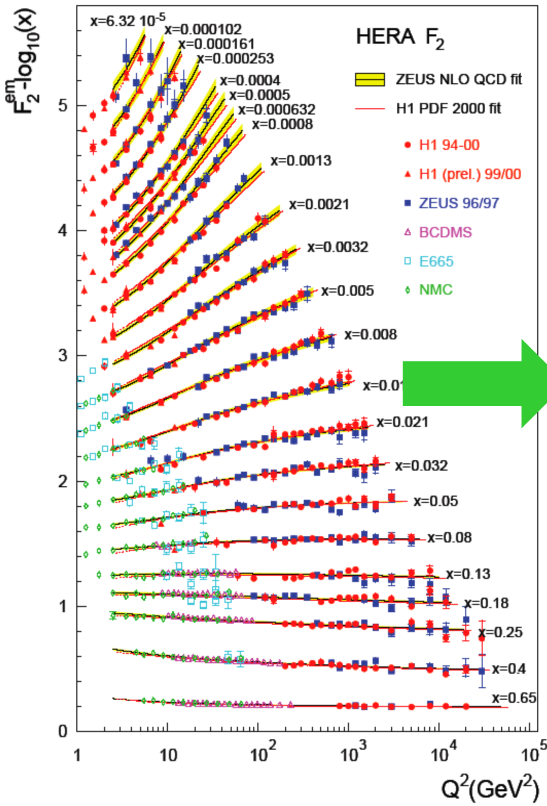
DGLAP evolution equations

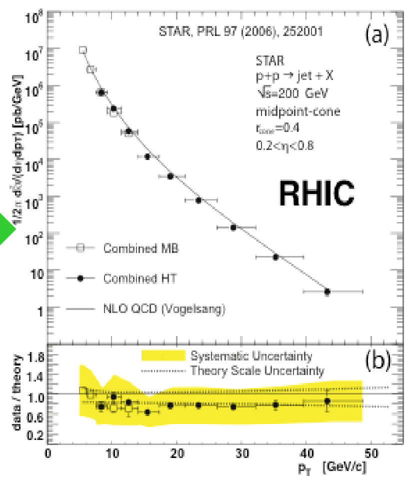
- **Operator defining PDFs: time dependent!**

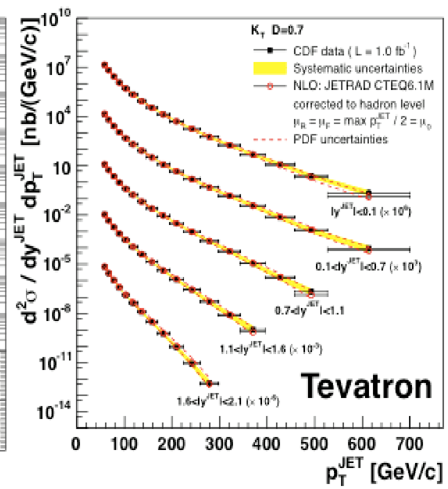
Extract PDFs by fitting data

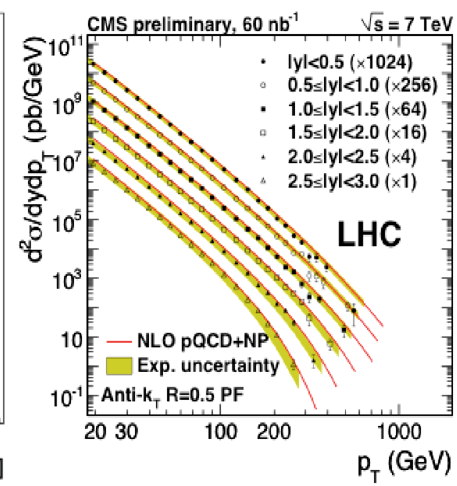
- **Successful**

Measure e-p at 0.3 TeV (HERA)
Predict p-p at 0.2, 1.96, and 7 TeV

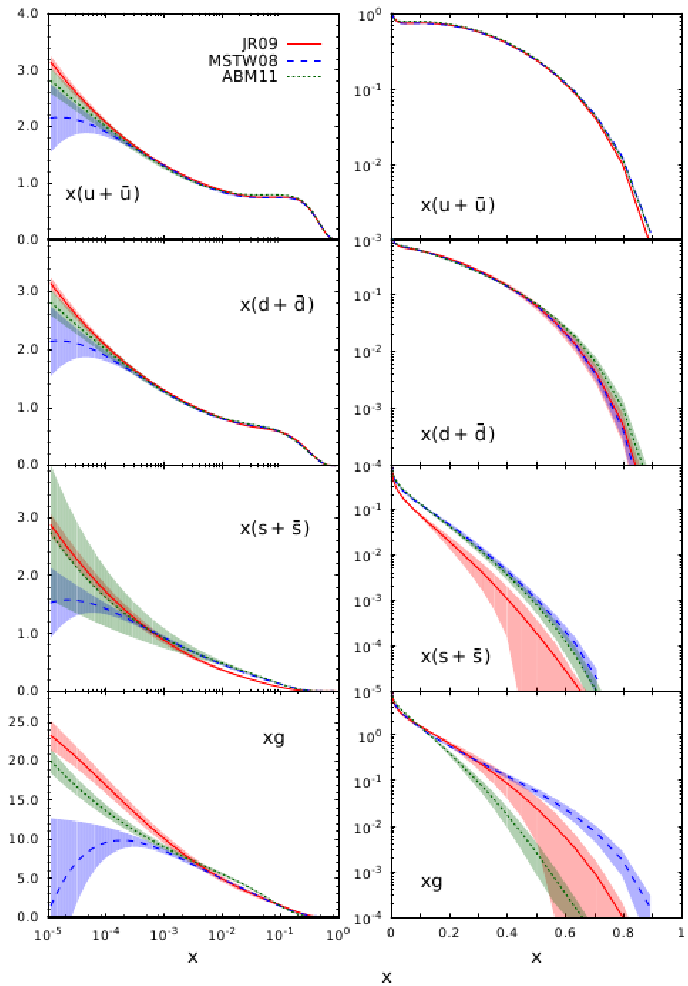








Uncertainty of PDFs



Large uncertainty in both small-x and large-x region

Questions

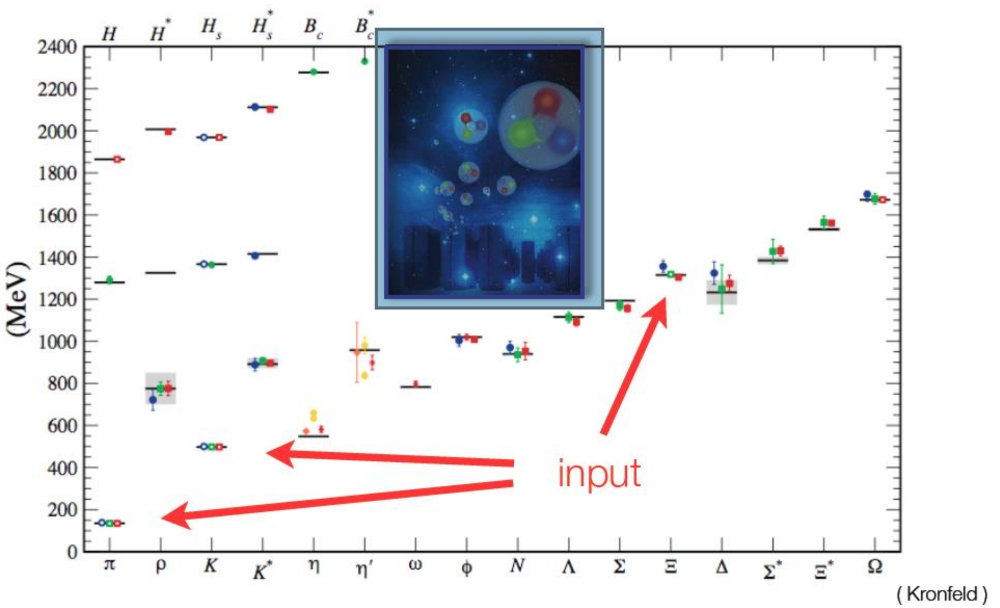
- Can theoretical calculation verify the extracted values?
- Can theoretical calculation provide more information for hadron structure?

Need to calculate PDFs/TMDs/…
nonperturbatively from first principle!

Lattice QCD

- **The main nonperturbative approach to solve QCD**

- **Predict the hadron mass**

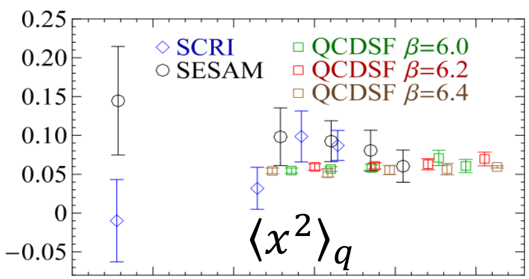


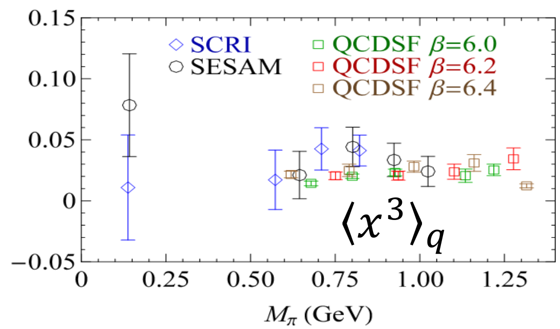
- **Intrinsically Euclidean time:** $\tau = i\,t$

Difficulty of PDFs calculation

- PDFs have (Minkowski) time dependence, lattice QCD cannot calculate PDFs directly

- Very limited moments of PDFs





Dolgov et al., hep-lat/0201021

Gockeler et al., hep-ph/0410187

$$\langle x^n(\mu^2) \rangle_q \equiv \int_0^1 dx\, x^n f_{q/p}(x, \mu^2)$$

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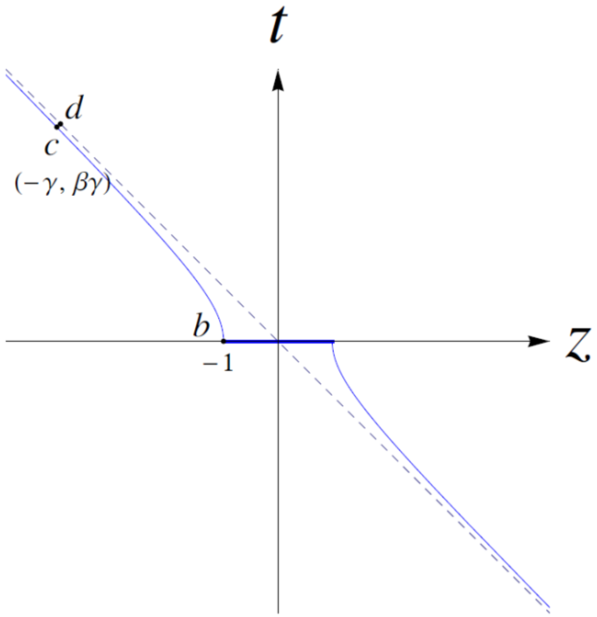
Quasi-PDFs

- What if quark bilinear is slightly off light cone?
  - Exist a frame where quark bilinear is equal time, but proton is moving fast



- Quasi-PDFs Ji, 1305.1539

$$\tilde{f}_{q/p}(x,\mu^2,P_z) = \int \frac{d\xi_z}{4\pi} e^{-ix\xi_z P_z} \langle P|\overline{\psi}(\xi_z)\,\gamma_z \exp\left\{-ig\int_0^{\xi_z} d\eta_z A_z(\eta_z)\right\}\psi(0)|P\rangle$$

- Classical picture: When proton is moving fast enough, i.e. $P_z \to \infty$, there exists a frame so that quark bilinear in quasi-PDFs are only slightly off light cone, approach PDFs

Quasi-PDFs

➢ Advantage

- Fields separated along the z-direction, no time dependence, calculable using standard lattice method

➢ Difficulty

- Quantum fluctuation, UV divergences, does the simple classical picture still hold in QFT?
- P_z on lattice is very limited
- In fact, relies on the existence of factorization

$$q(x,\mu^2,P^z)=\int_x^1\frac{dy}{y}Z\left(\frac{x}{y},\frac{\mu}{P^z}\right)q(y,\mu^2)+\mathcal{O}\left(\Lambda^2/(P^z)^2,M^2/(P^z)^2\right)$$

Ji, 1305.1539

$$\sigma(P\cdot\xi,\xi^2,\tilde{\mu}^2)\approx\sum_{i=q,\bar{q},g}\int_0^1\frac{dx}{x}f_i(x,\mu^2)\mathcal{C}_i(xP\cdot\xi,\xi^2,\tilde{\mu}^2,\mu^2)+O(\xi^2)$$

$$\tilde{f}^{\text{ren}}_{q/h}=f_{i/h}\otimes\mathcal{C}_{q/i}+O((\tilde{x}P_z)^{-2})$$

YQM, Qiu, 1404.6860

quasi $\mathcal{Q}(x,p)=q(x)\Big\{1+\mathcal{O}\Big(\frac{\Lambda^2}{p^2x^2(1-x)}\Big)\Big\},$

pseudo $\mathcal{P}(x,z)=q(x)\Big\{1+\mathcal{O}\Big(z^2\Lambda^2(1-x)\Big)\Big\},$

Braun, Vladimirov, Zhang, 1810.00048

Generalization

- Why quasi-PDFs work? Is there a more general way to calculate PDFs?

- From quasi-PDFs to LaMET and LCS

Quasi-PDFs

Ji, 1305.1539

Large momentum EFT: expansion of $1/P_z$

Ji, 1404.6680
Ji, Liu, Liu, Zhang, Zhao, 2004.03543

Lattice cross section: factorization

YQM, Qiu, 1404.6860, 1709.03018

Appeared on the same day!

The idea of lattice cross sections

YQM, Qiu, 1404.6860, 1709.03018

➢ Direct calculation is hard

- Lattice: time-dependence of operators defining PDFs
- LaMET: P_z cannot be too large on lattice

➢ The most general way of indirect calculation

- Relating PDFs, not calculable on lattice QCD, to some quantities calculable on lattice QCD
- The relation is nothing but a factorization!

$$\sigma_n(\xi^2,\omega,P^2)=\sum_a\int_{-1}^{1}\frac{dx}{x}\,f_a(x,\mu^2)\,K_n^a(\xi^2,x\omega,x^2P^2,\mu^2)+O(\xi^2\Lambda_{\rm QCD}^2)$$

Parton-distribution Structure

Hard-part Probe

Correction Approximation

- Analogy: measured experimental cross section!
- We call these quantities: "lattice cross sections"

What kinds of LCSs are useful?

➢ Conditions for a good LCS

① Calculable on Euclidean lattice QCD
② Renormalizable for UV divergences
③ Factorizable for CO divergence with IR safe coefficients

$$\sigma_n(\xi^2,\omega,P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x,\mu^2) K_n^a(\xi^2, x\omega, x^2P^2, \mu^2) + O(\xi^2\Lambda_{\rm QCD}^2)$$

- The last condition relates LCSs to, e.g., PDFs

➢ First condition: at least two possibilities

$$\sigma_n(\xi^2,\omega,P^2) = \langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle$$ with $\xi_0 = 0$, e.g., quasi-PDFs

Lin, et. al. , 1402.1462

$$\widetilde{\sigma}_n(q^2,\widetilde{\omega},P^2) = \int \frac{d^4\xi}{\xi^4} e^{iq\cdot\xi} \sigma_n(\xi^2, P\cdot\xi, P^2)$$ with $q_0 = 0$, e.g., OPE w/o OPE

Chambers, et. al. , 1703.01153

➢ Second and third conditions: need to prove

LCSs to determine PDFs

➢ LCSs in coordinate space or in momentum space

$$\sigma_n(\xi^2,\omega,P^2)=\langle P|T\{\mathcal{O}_n(\xi)\}|P\rangle \qquad \widetilde{\sigma}_n(q^2,\widetilde{\omega},P^2)=\int\frac{d^4\xi}{\xi^4}e^{iq\cdot\xi}\sigma_n(\xi^2,P\cdot\xi,P^2)$$

- $1/\xi^2$ and q^2: hard scales to enable factorization

➢ Possible choices of nonlocal operators

Parton correlators

$$\mathcal{O}_q(\xi)=Z_q(\xi^2)\overline{\psi}_q(\xi)\,\not{\xi}\,\Phi(\xi,0)\,\psi_q(0)$$

$$\Phi(\xi,0)\;=\;\mathcal{P}e^{-ig\int_0^1\xi\cdot A(\lambda\xi)\,d\lambda}$$

Ji, 1305.1539

- Path ordered gauge link needed
- Renormalization is complicated

Current-current operators

$$\mathcal{O}_S(\xi)=\xi^4Z_S^2[\overline{\psi}_q\psi_q](\xi)\,[\overline{\psi}_q\psi_q](0)\,,$$
$$\mathcal{O}_V(\xi)=\xi^2Z_V^2[\overline{\psi}_q\not{\xi}\psi_q](\xi)\,[\overline{\psi}_q\not{\xi}\psi_q](0)\,,$$
$$\mathcal{O}_{\widetilde{V}}(\xi)=-\frac{\xi^4}{2}Z_V^2[\overline{\psi}_q\gamma_\nu\psi_q](\xi)\,[\overline{\psi}_q\gamma^\nu\psi_q](0)\,,$$
$$\mathcal{O}_{V'}(\xi)=\xi^2Z_{V'}^2[\overline{\psi}_q\not{\xi}\psi_{q'}](\xi)\,[\overline{\psi}_{q'}\not{\xi}\psi_q](0)\,,\;\ldots\,,$$

YQM, Qiu, 1709.03018

- Renormalization is very simple

Straight forward to construct much more operators with both quark fields and gluon fields

Different methods

- Inverse Laplace transform for structure functions

  Liu, Dong, 9306299, Liu, 9910306, 1603.07352

- Coordinate-space method for LDA

  Aglietti, et. al., 9806277; Abada, et.al., 0105221; Braun, Mueller, 0709.1348

- Quasi-PDFs

  Ji, 1305.1539, 1404.6680

- Pseudo-DFs

  Orginos, Radyushkin, Karpie, Zafeiropoulos, 1706.05373

- OPE without OPE

  Chambers, et. al. , 1703.01153

- Current-current correlators

  YQM, Qiu, 1709.03018

All existed methods rely on factorization, can be interpreted as constructing LCSs

Analogy: meat, vegetables, fruits are all food

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Renormalization

➢ Why proof is important?

- Need to take continuum limit for lattice calculation
- All-order proof of factorization needs multiplicative renormalization
- Find out all operators mixing under renormalization YQM, Qiu, 1404.6860

Lessons from gluon-gluon correlator: proof is crucial!

➢ Current-current operators: known

➢ Parton correlators: difficult

- Because of z-direction dependence, Lorentz symmetry is broken, hard to exhaust all possible UV divergences
- Renormalization of nonlocal composite operator

All-order proofs for renormalization

- Diagrammatic method: key is to show that UV divergences are local in space-time
  - Nontrivial conclusion! E.g. UV divergences for normal PDFs are non-local in "–" direction
  - The most difficult part in this proof

Ishikawa YQM, Qiu, Yoshida, 1707.03107
Li, YQM, Qiu, 1809.01836

- Auxiliary field method: gauge link obtained by integration out an auxiliary field in HQET
  - Taking advantage of the renormalizability of heavy-light current in HQET

Ji, Zhang, Zhao, 1706.08962
Green, Jansen, Steffens, 1707.07152
Zhang, et. al., 1808.10824

All-order proofs for factorization

➢ Diagrammatic method:

YQM, Qiu, 1404.6860

- Rigorous, but hard to resum target mass corrections

➢ OPE method:

YQM, Qiu, 1709.03018
Izubuchi, et. al., 1801.03917

- For quasi-PDFs, no proof of OPE: only a derivation of factorization

$$\sigma_n(\xi^2, \omega, P^2) = \sum_a \int_{-1}^{1} \frac{dx}{x} f_a(x, \mu^2) K_n^a(\xi^2, x\omega, x^2P^2, \mu^2) + O(\xi^2 \Lambda_{\rm QCD}^2)$$

$$\widetilde{\sigma}_n = \sum_a f_a \otimes \widetilde{K}_n^a + O(\Lambda_{\rm QCD}^2/q^2)$$

$$f_{\bar{a}/h}(x, \mu^2) = -f_{a/h}(-x, \mu^2)$$

where

$$K_n^a = \sum_J 2W_n^{(J,a)}(\xi^2, \mu^2)\, \Sigma_J(x\omega, x^2P^2\xi^2)$$

$$\widetilde{K}_n^a = \int \frac{d^4\xi}{\xi^4} e^{iq\cdot\xi} K_n^a(\xi^2, xP\cdot\xi, x^2P^2, \mu)$$

Theoretical status

➢ Conditions satisfied up to now

- With $\xi_0 = 0$ or $q_0 = 0$, calculable on Euclidean lattice
- Renormalizable
- Factorizable to PDFs

With these conditions, σ_n and $\tilde{\sigma}_n$ are good LCSs to extract PDFs

Questions still need to answer:

1) OPE for parton-parton correlators?

2) Renormalization using lattice regulator?

➢ What next?

- Calculate matching coefficients perturbatively
- Calculate LCSs nonperturbatively (including renormalization methods)

Matching coefficients

- **Obtained by calculating Feynman diagrams**
  - Most quantities: calculated to one-loop order
  - Flavor nonsinglet quark-quark correlator (qPDF, pPDF): two-loop order

Li, YQM, Qiu, 2006.12370
Chen, Wang, Zhu, 2006.14825

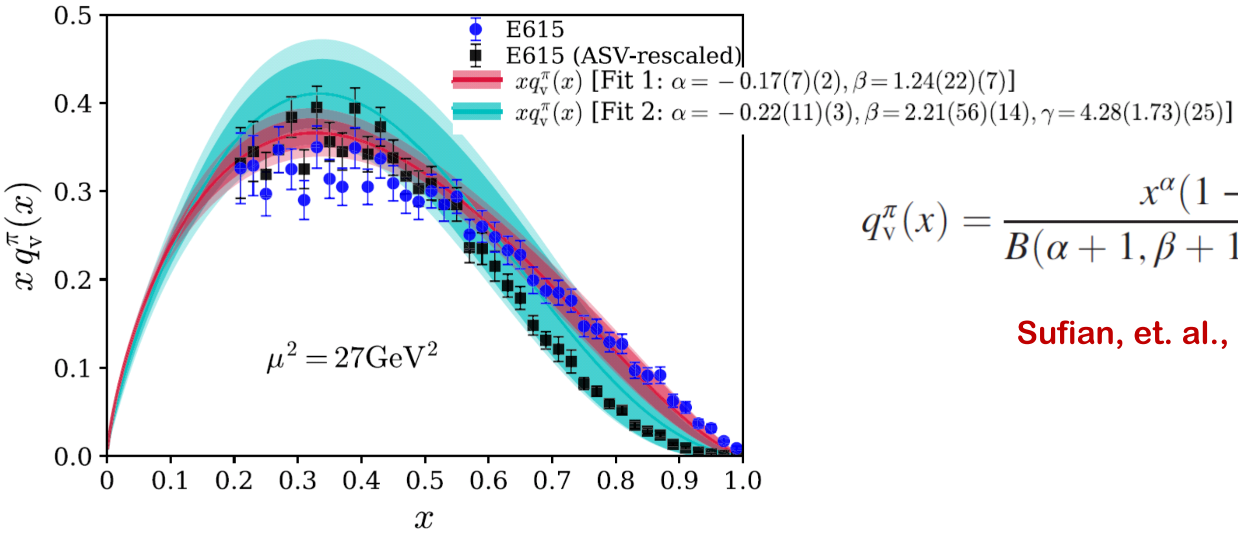
High-order calculations for more quantities are needed,
make lattice data more useful

- **Renormalization schemes**

See Jian-Hui Zhang's talk

Lattice results

- LaMET: see many talks in this meeting
- Many interesting results from pseudo-PDFs
- An example using current-current correlator



$$q_v^\pi(x) = \frac{x^\alpha(1-x)^\beta(1+\gamma x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+2,\beta+1)}$$

Sufian, et. al., 2001.04960

Summary

- Hadronic matrix elements that are calculable + renormalizable + factorizable
  - Like experimental cross section, can constrain hadron structure
- Quasi-PDFs, pseudo-PDFs, OPE w/o OPE, current-current correlators: good LCSs
- Construct better operators?
- Many (non-)perturbative works to do

With world (lattice and experimental) data, a global fit to determine PDFs/TMDs/…

Thank you!