Collins-Soper kernel from transverse momentum-dependent wave functions in LaMET

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(Lattice Parton Collaboration)

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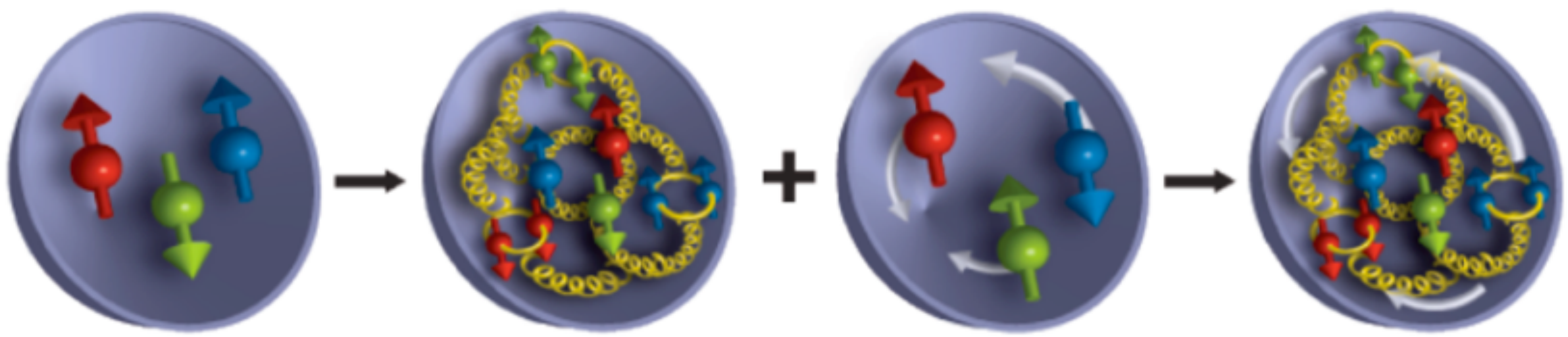
中国格点QCD第一届年会

Outline

- Motivation: Collins-Soper kernel
- Collins-Soper kernel from lattice QCD
- Numerical results
- Summary and outlook

Motivation: Collins-Soper kernel

Our understanding of the internal structure of nucleon

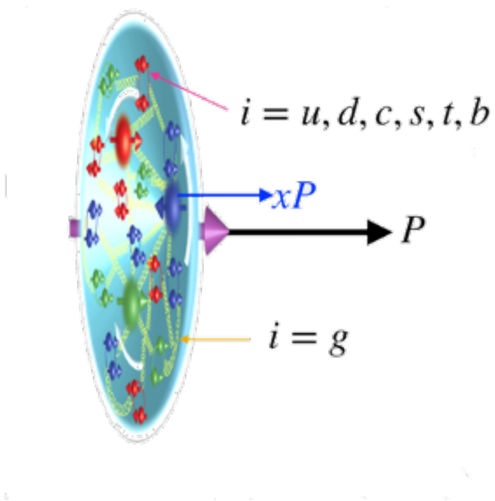


1970s　　1980s/2000s　　Now

Strong interaction in boundary state of quarks and gluons.

How to probe nucleon structure without "seeing" quarks and gluons?

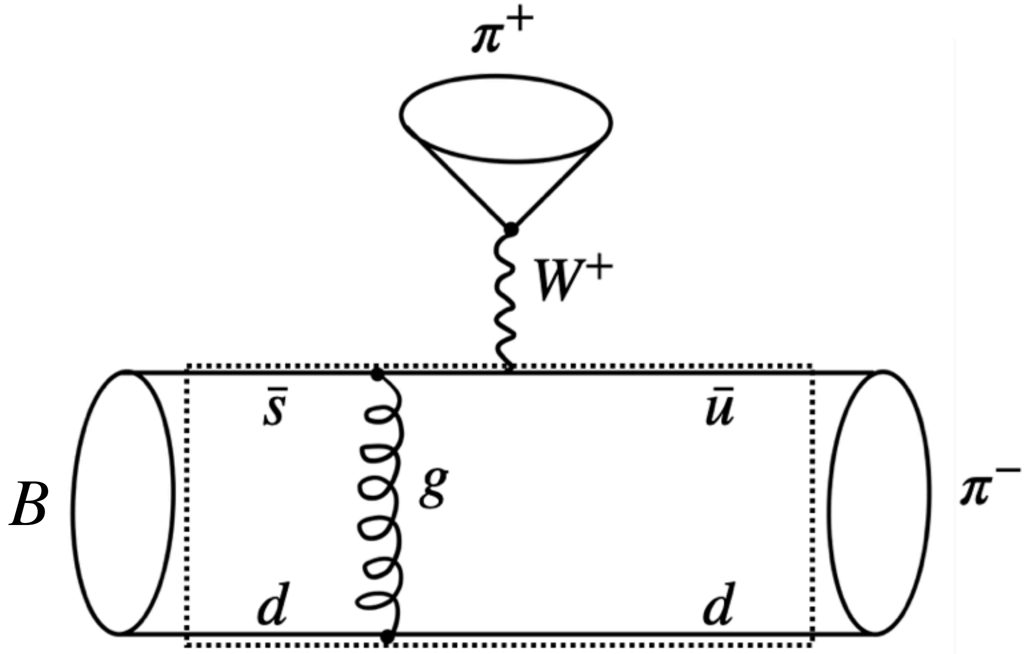
Motivation: Collins-Soper kernel



Parton distributions
with momentum fraction x
in longitudinal direction.

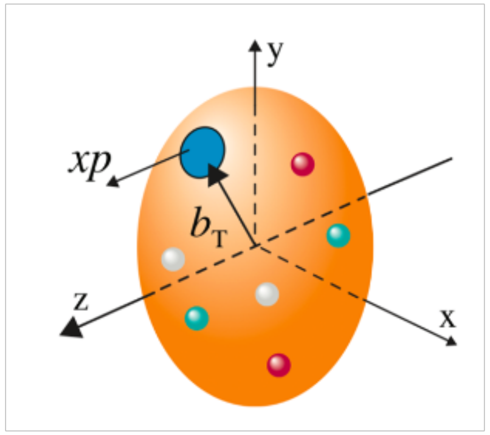
→ PDF/DA: $f_{q/P}(x)$

Example: $B \to \pi^+ + \pi^-$



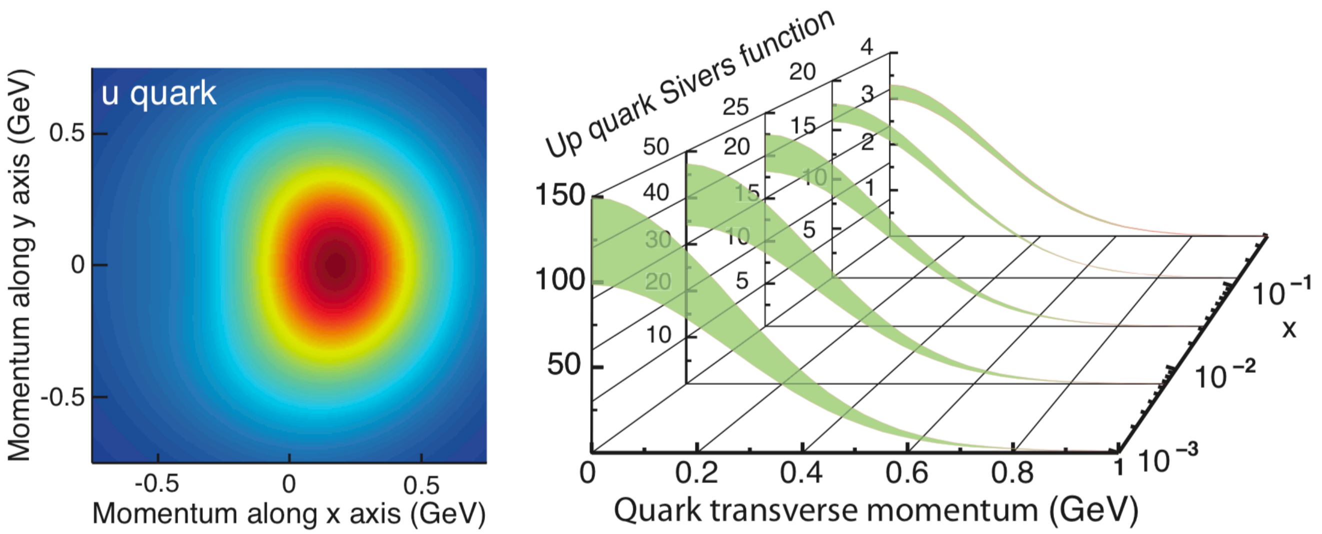
$$\mathscr{A} = \langle \pi^+\pi^- | H | B \rangle \sim \int d^4k_1 d^4k_2 d^4k_3 Tr[C(t)\psi_B(p_1)\psi_{\pi^+}(p_2)\psi_{\pi^-}(p_3)H(k_1, k_2, k_3, t)]$$

Motivation: Collins-Soper kernel



3-dimensional structures of hadrons. → TMD PDF/DA: $f_{q/P}^{TMD}(x, k_{\perp})$

- Three dimensional nuclear imaging
- DIS/Drell Yan process
- …



N. G. Stefanis, Constantia Aleandrolu et.al, arxiv 1612.03077(2016)

Lattice calculation of TMDWF is the calculation in first principle!

Motivation: Collins-Soper kernel

The Collins-Soper kernel relates the evolution of transverse momentum-dependent parton distributions and hadron wave function.

- TMD Evolution:

P. Shanahan, M. Wangman and Y. Zhao, arxiv 2003.06063(2020)

$$f^{TMD}(x, b_\perp, \mu, \zeta) = f^{TMD}(x, b_\perp, \mu_0, \zeta_0)\ \exp\left[\int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'}\gamma(\mu', \zeta_0)\right] \exp\left[\frac{1}{2}K(\mu, b_\perp)\ln\frac{\zeta}{\zeta_0}\right]$$

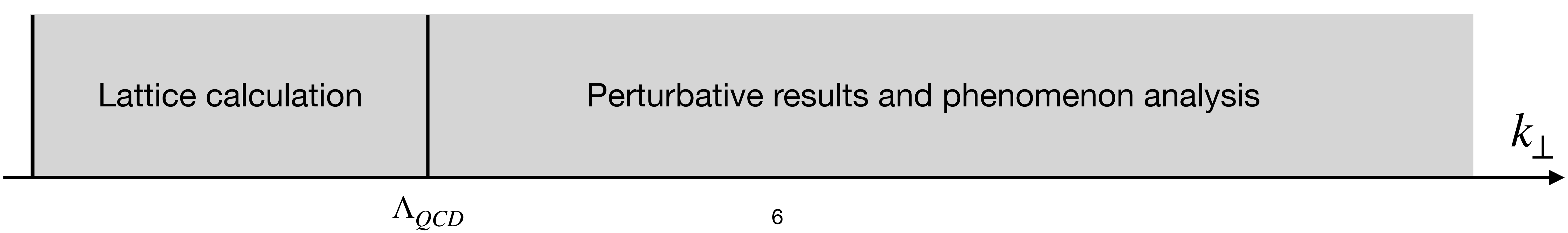
- LaMET matching for TMD:

X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)

$$\psi(b, x, \zeta) = H_1^{-1}\left(4x^2P_z^2, 4(1-x)^2P_z^2\right) e^{-\frac{1}{2}\ln\left(\frac{4x^2P_z^2}{\zeta}\right)K(b)} S_r^{\frac{1}{2}}(b)\tilde{\psi}(b, x, P_z)$$

Transverse momentum of parton $k_\perp$ represents the scale of TMD physics.

Energy scale of parton in hadron



Collins-Soper kernel from lattice QCD

The P_z dependence of quasi-TMDWF is related to CS kernel:

X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)

$$\psi_N^{\pm}(b,x,\zeta) = H_1^{-1}\left(4x^2P_z^2, 4(1-x)^2P_z^2\right) e^{-\frac{1}{2}\ln\left(\frac{4x^2P_z^2}{\zeta}\right)K(b)} S_r^{\frac{1}{2}}(b)\tilde{\psi}_N^{\pm}(b,x,P_z)$$

$$K^{\pm}(b) = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{H_N^{\pm}(4x^2P_1^2, 4(1-x)^2P_1^2)\tilde{\psi}_N^{\pm}(b,x,P_2)}{H_N^{\pm}(4x^2P_2^2, 4(1-x)^2P_2^2)\tilde{\psi}_N^{\pm}(b,x,P_1)}\right]$$

$$K^{\pm}(b)_{tree} = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{\tilde{\psi}_N^{\pm}(b,z=0,P_2)}{\tilde{\psi}_N^{\pm}(b,z=0,P_1)}\right]$$

Tree level: $H_1 = 1$

Fourier transformation: $\int_0^1 \tilde{\psi}(b,x)dx = \tilde{\psi}(b,z=0)$

Matching kernel up to 1-loop level:

X.D. Ji and Y.Z Liu, arxiv 2106.05310 (2021)

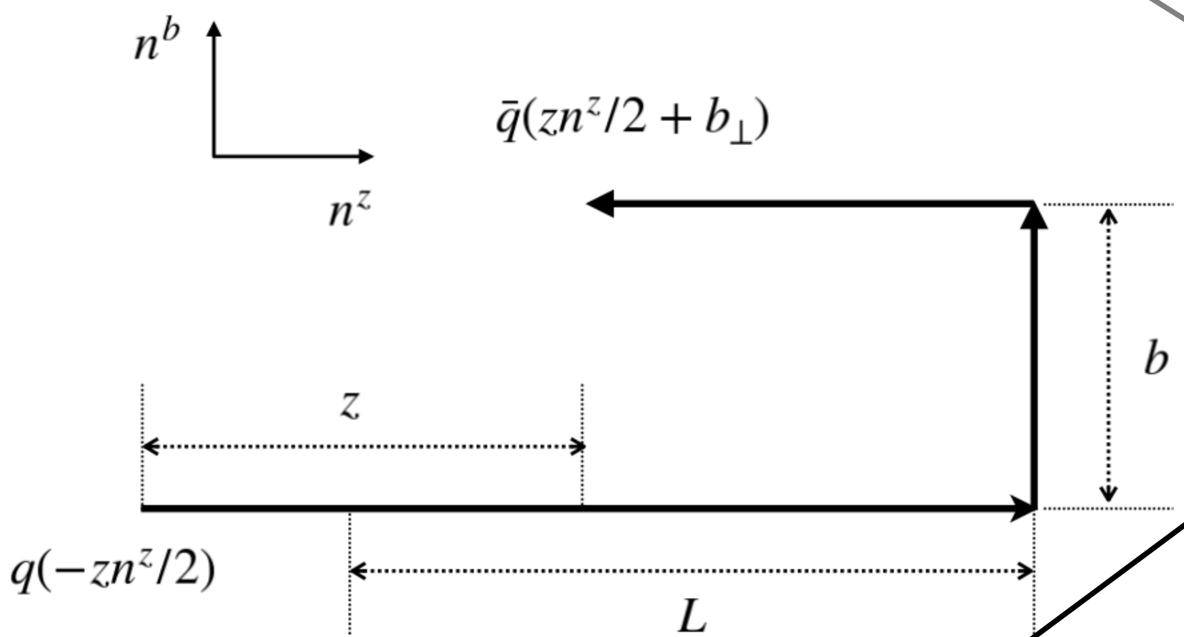
$$H_N^{\pm}(l,\bar{l}) = e^h, \quad h = \frac{\alpha_s C_F}{4\pi}\left[-\frac{5\pi^2}{6} - 4 + \ln\frac{-l \pm i0}{\mu^2} + \ln\frac{-\bar{l} \pm i0}{\mu^2} - \frac{1}{2}\left(\ln^2\frac{-l \pm i0}{\mu^2} + \ln^2\frac{-\bar{l} \pm i0}{\mu^2}\right)\right] + O(\alpha_s^2)$$

$$l = (2xP^z)^2, \quad \bar{l} = (2(1-x)P^z)^2$$

Collins-Soper kernel from lattice QCD

leading twist for pion: $\Gamma_1 = \gamma^t\gamma_5,\ \ \gamma^z\gamma_5$

TMDWF: $\tilde{\psi}(b, z, \overrightarrow{P}) = \lim_{l\to\infty} \langle 0 | \bar{q}_1(bn^b + \frac{z}{2}n^z)\Gamma_1 U_c q_2(-\frac{z}{2}n^z) | \pi(\overrightarrow{P})\rangle$



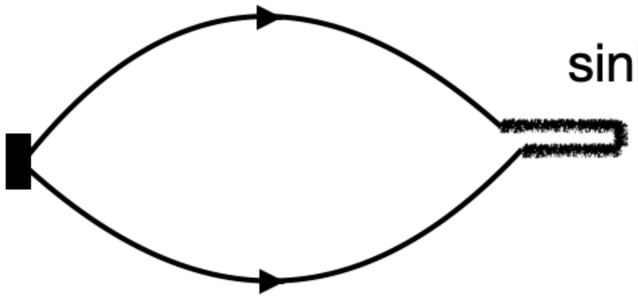
$$C_2(0,0,0,t) = C_0 e^{Et}(1 + c_0(0)e^{-\Delta Et})$$

$$\frac{C_2(L, b, z, t)}{C_2(0,0,0,t)} = \frac{\tilde{\psi}_l(L, b, z)(1 + c_1(L, b, z)e^{-\Delta Et})}{1 + c_0(0)e^{-\Delta Et}}$$

Two point correlation function:

$$C_2(b, z, t, \overrightarrow{P}) = \int d^3x e^{-i\overrightarrow{P}\overrightarrow{x}} \langle 0 | \chi(\overrightarrow{x}, t, b, z, \Gamma_1)\bar{\chi}(0,0,0,0,\gamma_5) | 0\rangle$$

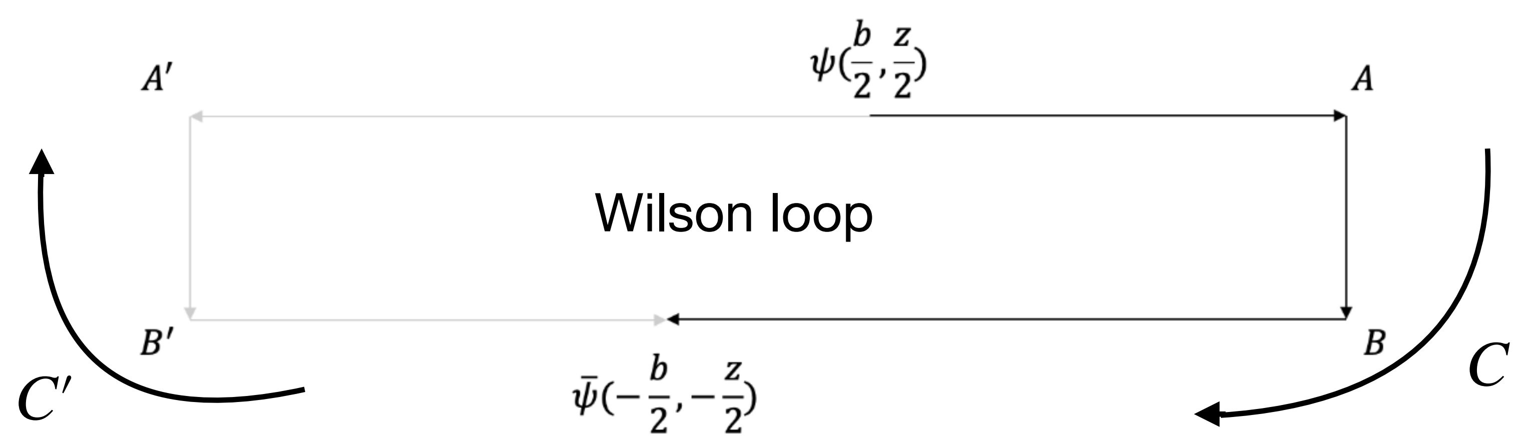
$$\chi(\overrightarrow{x}, t, b, z) = \bar{\psi}_1(\overrightarrow{x} - \frac{b}{2}\hat{x} - \frac{z}{2}\hat{z})\Gamma_1 U_c \psi_2(\overrightarrow{x} + \frac{b}{2}\hat{x} + \frac{z}{2}\hat{z})$$



Numerical result: Lattice setup

Subtracted quasi-TMDWF:

$$\tilde{\psi}(z,b,P_z)_{sub} = \lim_{l\to\infty} \frac{\tilde{\psi}(l,z,b,P_z)}{\sqrt{U_{loop}}} = \lim_{l\to\infty} \frac{\langle 0|\bar{q}_1(bn^b+\frac{z}{2}n^z)\Gamma_1 U_c q_2(-\frac{z}{2}n^z)|\pi(\vec{P})\rangle}{\langle 0|\bar{q}_1(0)\Gamma_1 q_2(0)|\pi(\vec{P})\rangle\sqrt{\langle 0|U_{c+c'}|0\rangle}}$$

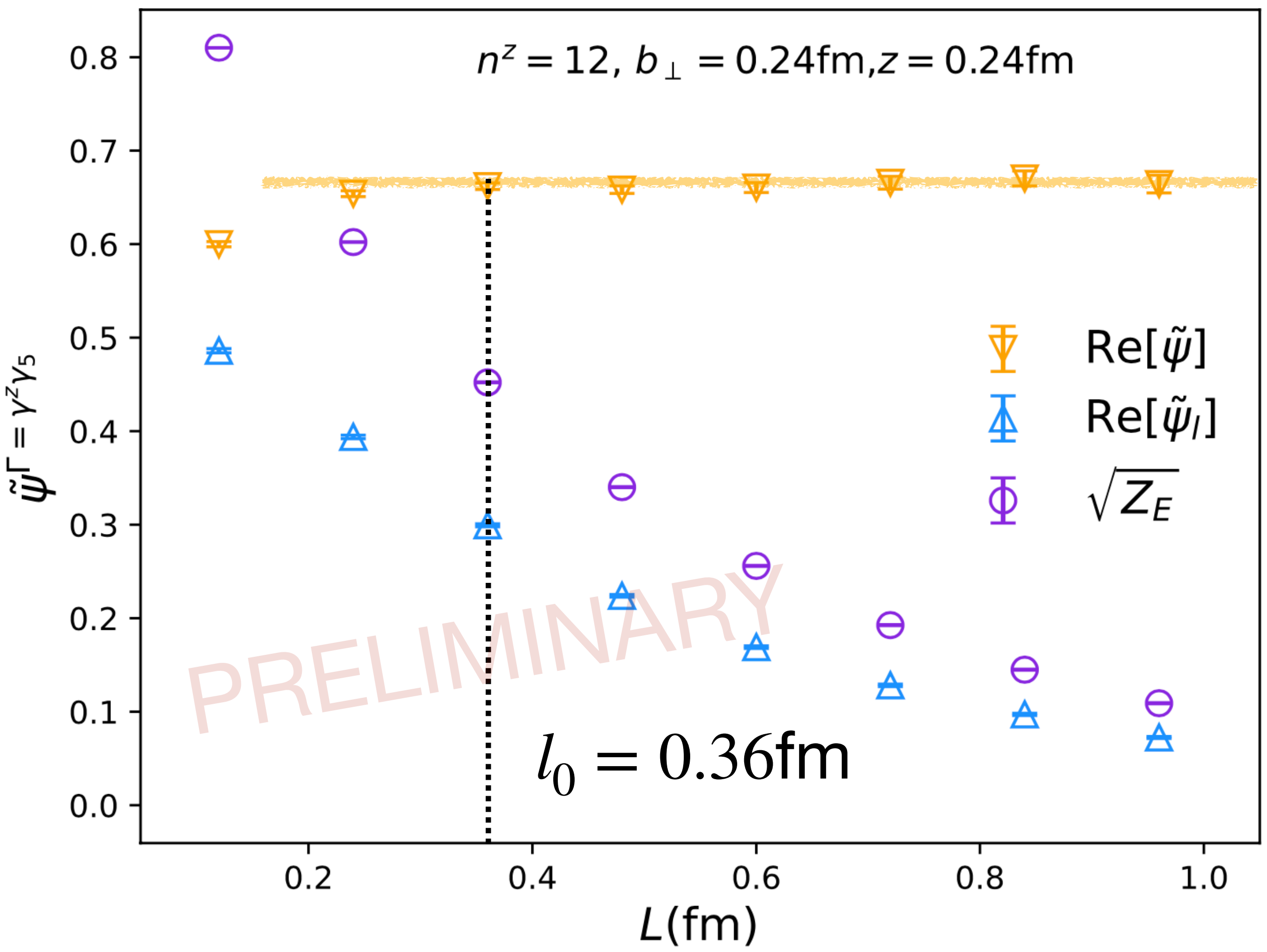


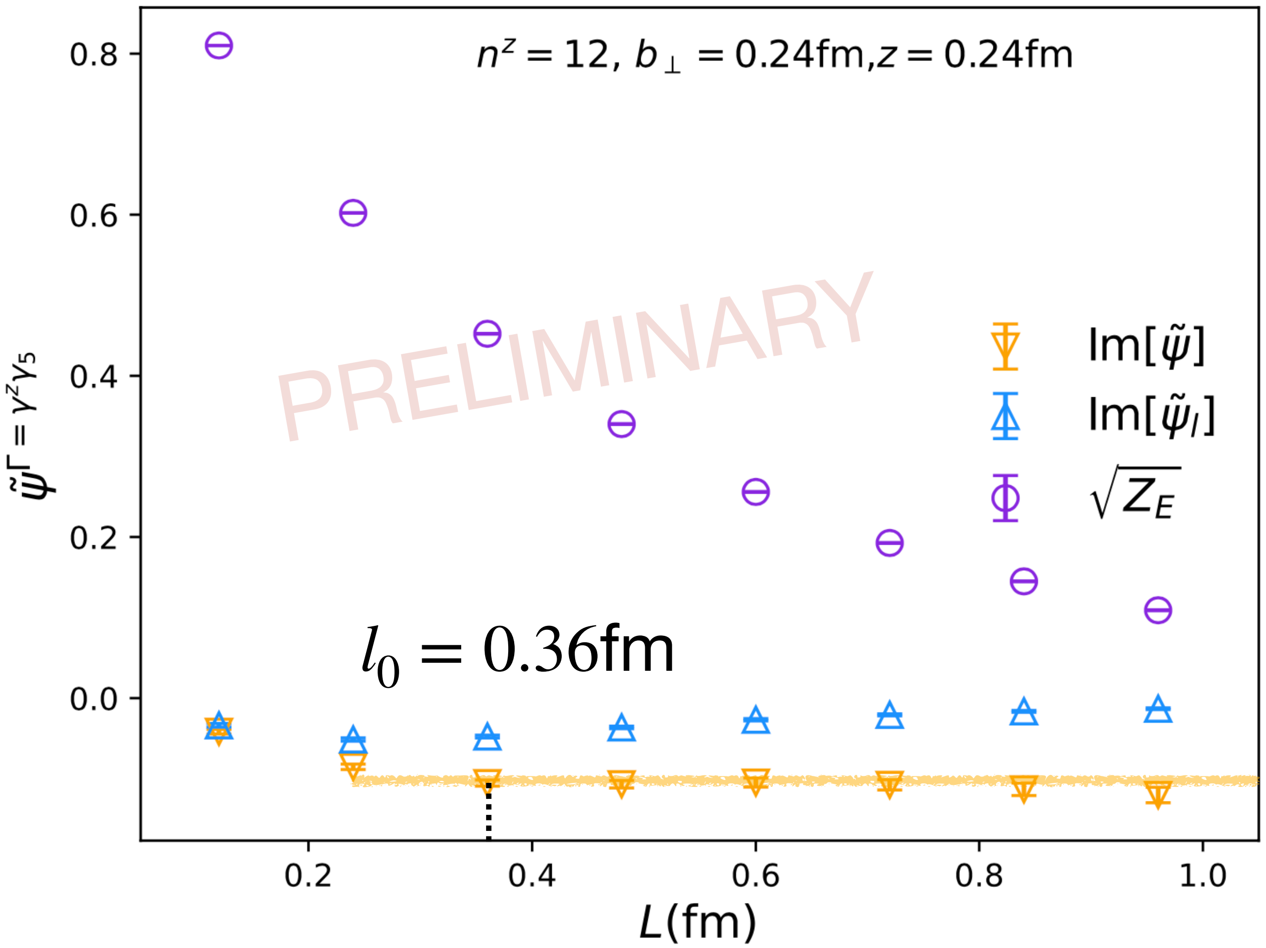
Lattice setup: MILC configurations with $a = 0.12$fm and choose three hadron momenta $P_z = \{1.72, 2.15, 2.58\}$ GeV. We used 382 configurations and in each we measured 4 times, so the total measurement is $4 \times 382 = 1528$

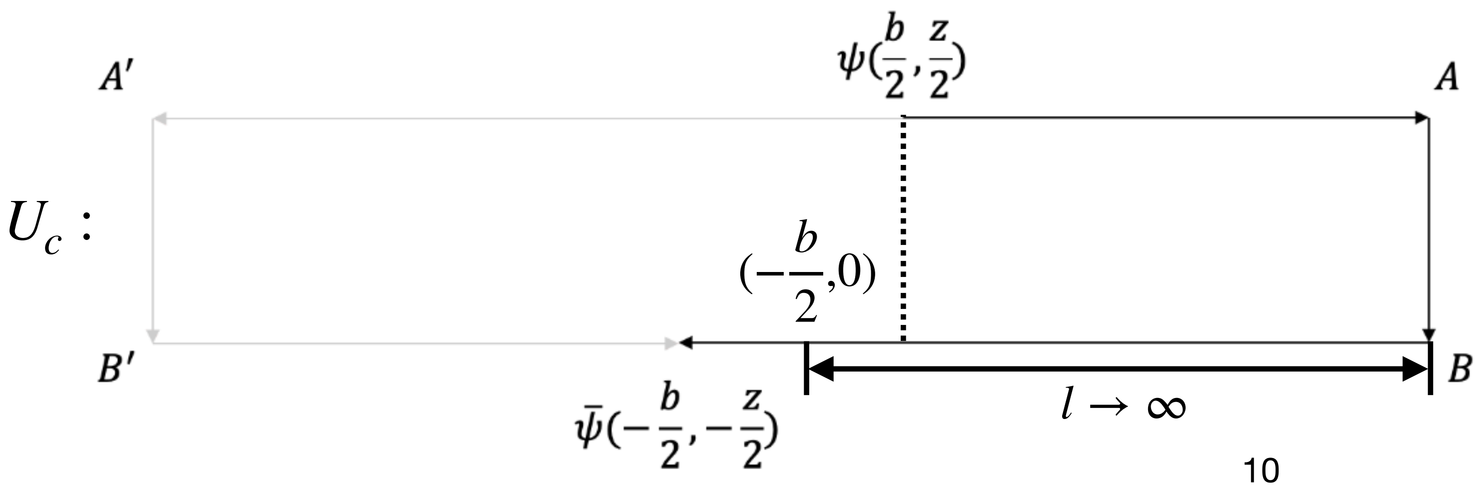
Ensemble	a(fm)	$L^3 \times T$	$m_{\pi,sea}$(MeV)	$m_{\pi,val}$(MeV)	momentum(γ)
a12m130	0.12	48×64	140	670	$1.72GeV(2.57)$, $2.15GeV(3.21)$, $2.58GeV(3.85)$
a12m130	0.12	48×64	140	310	$1.72GeV(5.55)$, $2.15GeV(6.93)$, $2.58GeV(8.32)$

Computation: Presetting

TMDWF: $\tilde{\psi}(b,z) = \lim_{l\to\infty} \tilde{\psi}(l,b,z)$





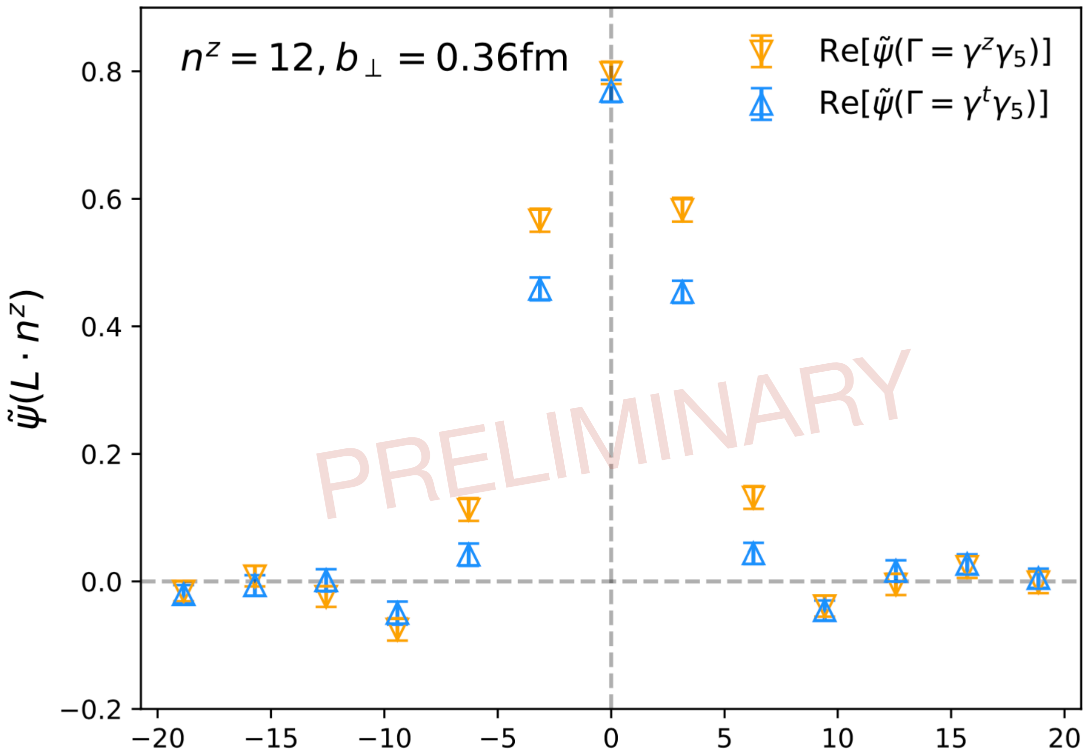


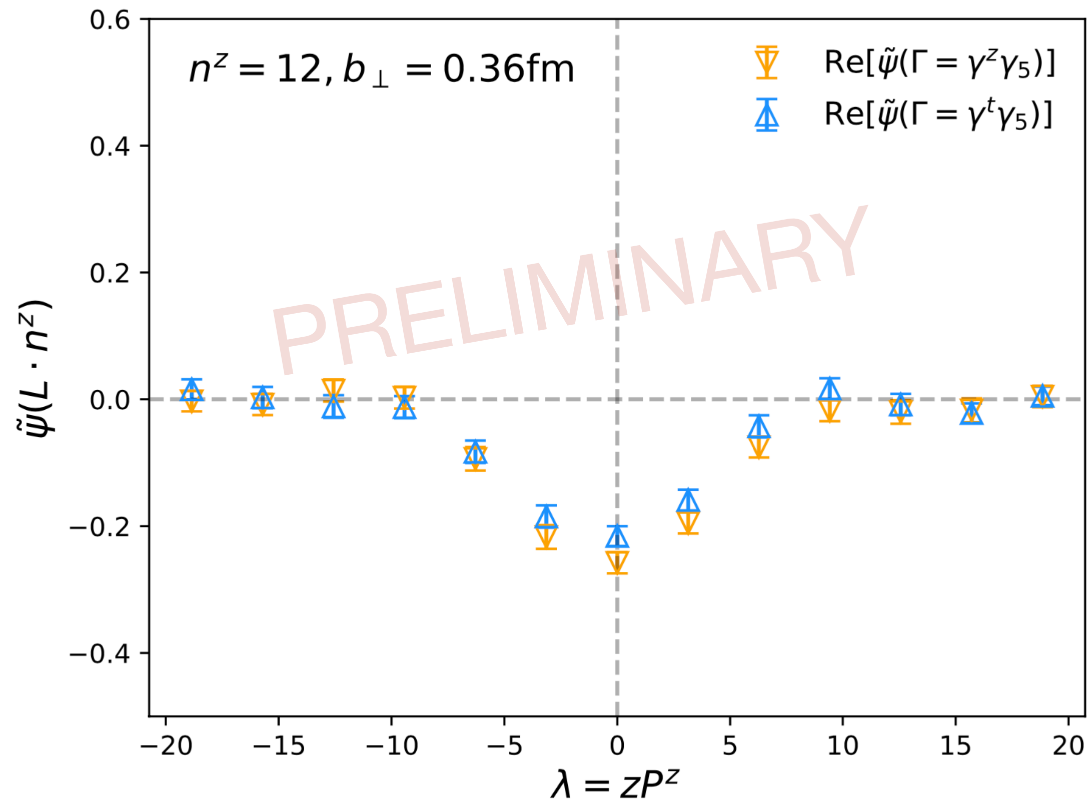
Due to $l_0 \geq \frac{z}{2}$, we choose $l_0 = \frac{z_{max}}{2}$

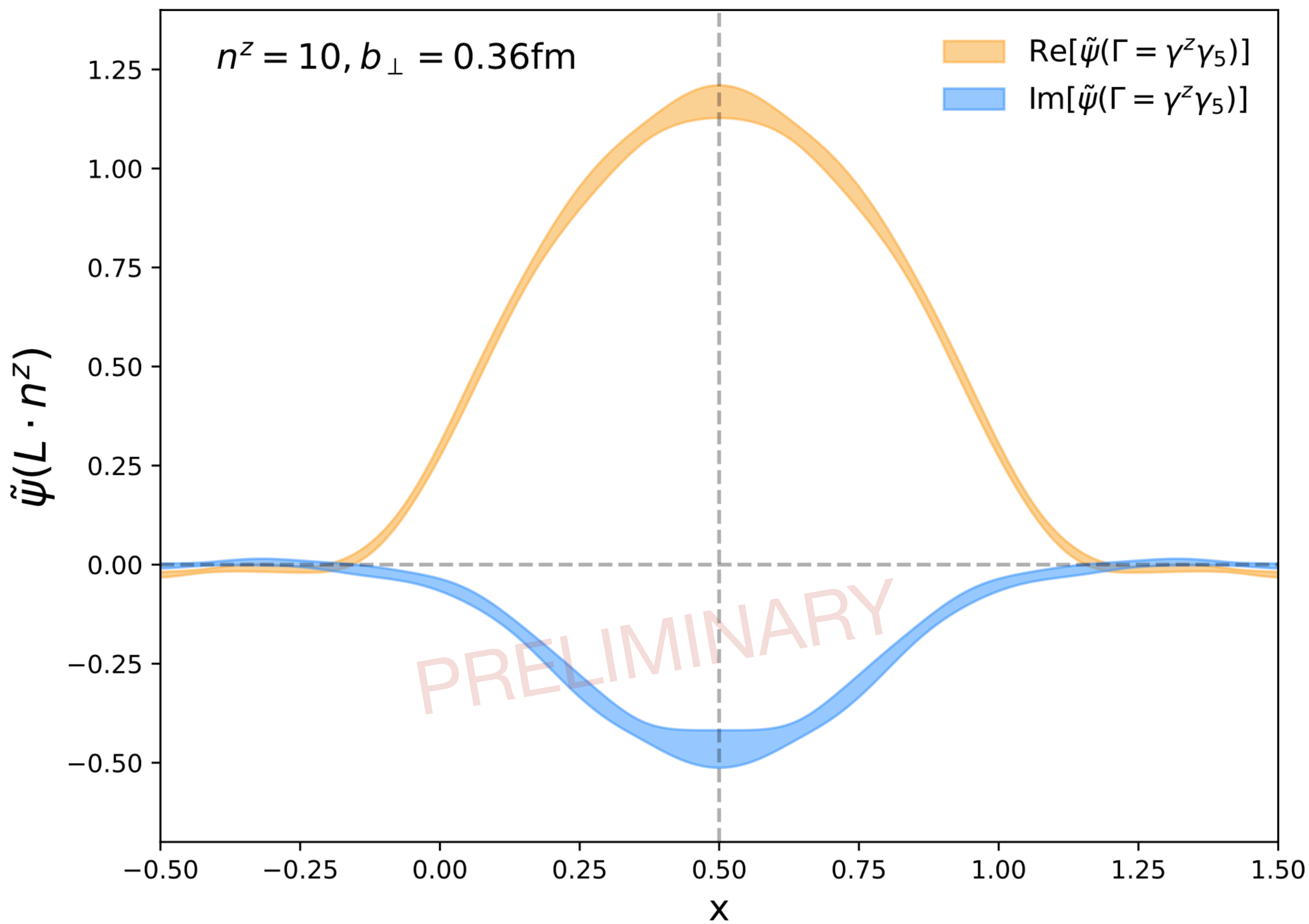
In our data, it is $l_0 = 6a = 0.72fm$

Numerical result: Fourier Transformation

TMDWF: $\tilde{\psi}(l = l_0, b, z) \rightarrow \tilde{\psi}(b, \boxed{z}) \xrightarrow{\text{Fourier transformation}} \tilde{\psi}(b, \boxed{x})$

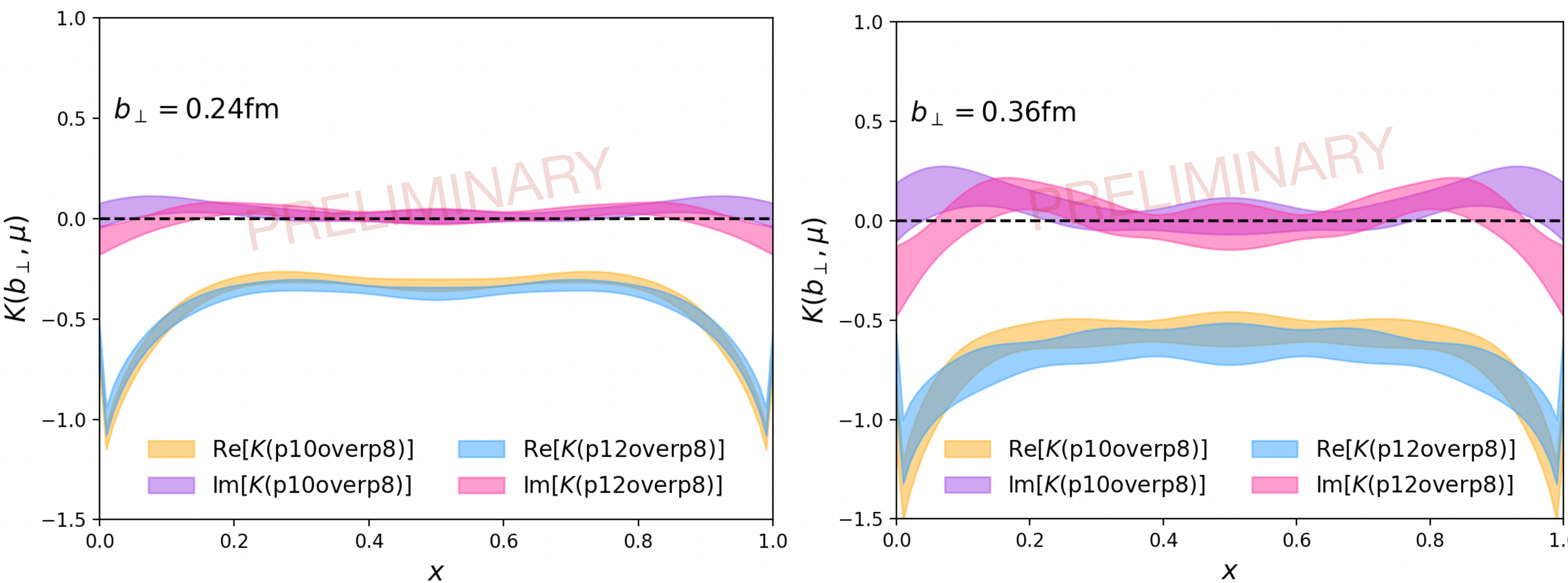






Collins-Soper kernel

$$K(b_\perp,\mu)=\frac{1}{2\ln(P_2/P_1)}\left[\ln\left(\frac{H_N^+(4x^2P_1^2,4(1-x)^2P_1^2)\tilde{\psi}_N^+(b,x,P_2)}{H_N^+(4x^2P_2^2,4(1-x)^2P_2^2)\tilde{\psi}_N^+(b,x,P_1)}\right)+\ln\left(\frac{H_{\bar{N}}^-(4x^2P_1^2,4(1-x)^2P_1^2)\tilde{\psi}_{\bar{N}}^-(b,x,P_2)}{H_{\bar{N}}^-(4x^2P_2^2,4(1-x)^2P_2^2)\tilde{\psi}_{\bar{N}}^-(b,x,P_1)}\right)\right]$$



$K(b_\perp,\mu)$ does not have imaginary part, as we expected.

Numerical Results: Collins-Soper kernel

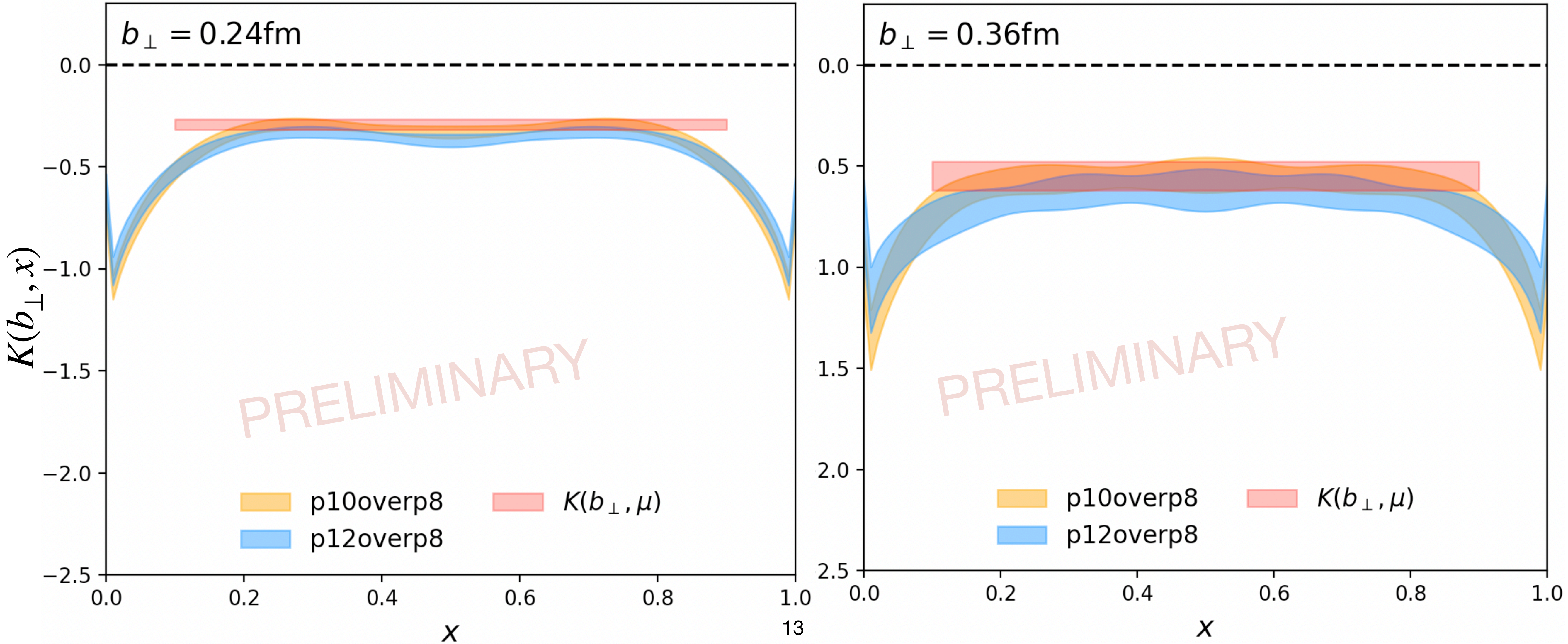
Collins-Soper kernel

$$K(b_\perp,\mu) = \frac{1}{2\ln(P_2/P_1)}\left[\ln\left(\frac{H_N^+(4x^2P_1^2, 4(1-x)^2P_1^2)\tilde{\psi}_N^+(b,x,P_2)}{H_N^+(4x^2P_2^2, 4(1-x)^2P_2^2)\tilde{\psi}_N^+(b,x,P_1)}\right) + \ln\left(\frac{H_N^-(4x^2P_1^2, 4(1-x)^2P_1^2)\tilde{\psi}_N^-(b,x,P_2)}{H_N^-(4x^2P_2^2, 4(1-x)^2P_2^2)\tilde{\psi}_N^-(b,x,P_1)}\right)\right]$$

Joint fit for Collins-Soper kernel as a function of momentum fraction x

$$K(b_\perp, x, P_3/P_1) = K + \frac{a}{x^2(1-x)^2} \qquad K(b_\perp, x, P_2/P_1) = K + \frac{b}{x^2(1-x)^2}$$

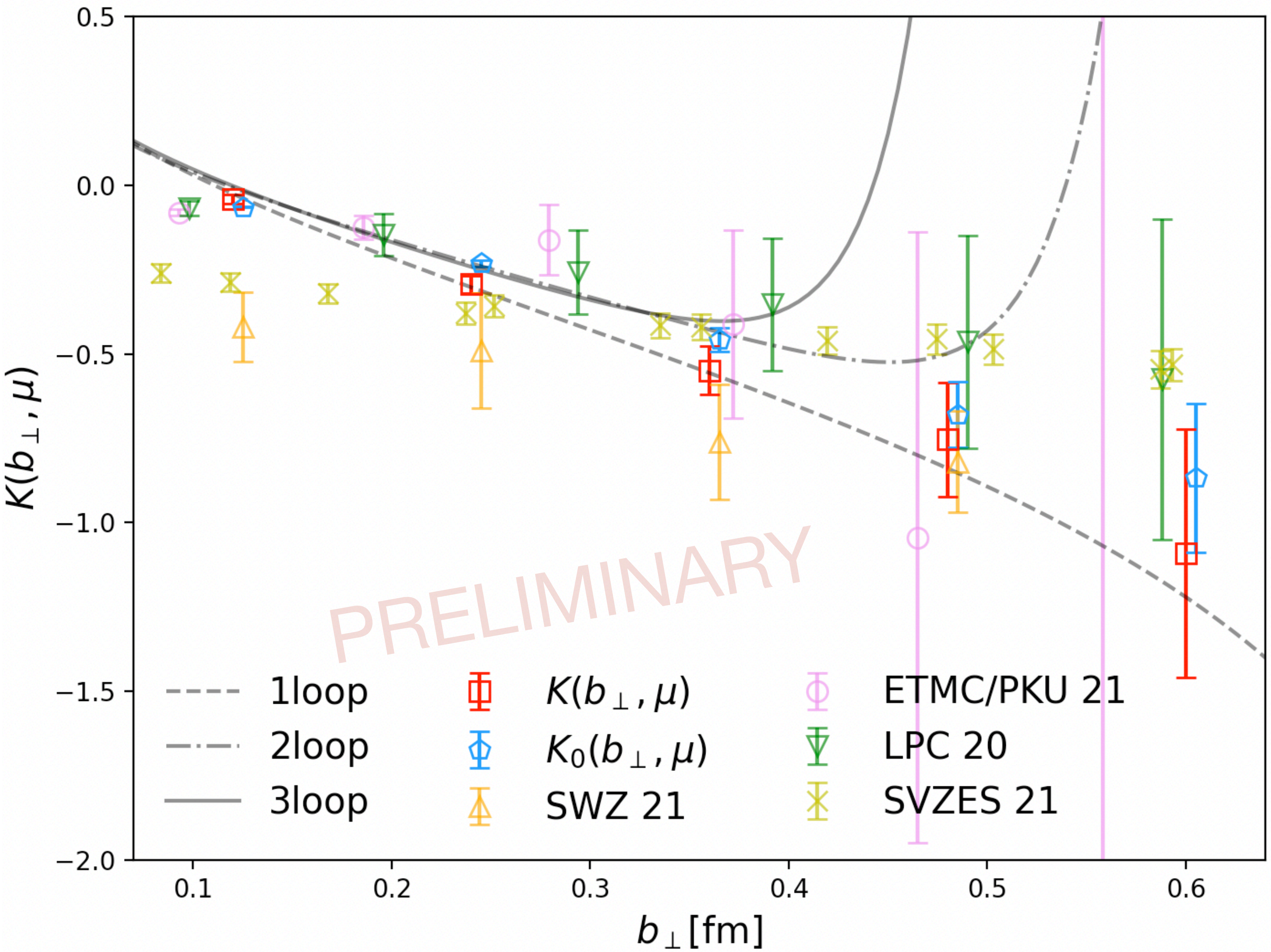
fit range: 0.1~0.9



Numerical Results: Final result

- Collins-Soper kenel: $K_0(b) = \frac{1}{2\ln(P_2/P_1)}\left[\ln\left(\frac{\tilde{\psi}^{+}_{\bar N}(z=0, b_\perp, P_1)}{\tilde{\psi}^{+}_{\bar N}(z=0, b_\perp, P_2)}\right) + \ln\left(\frac{\tilde{\psi}^{-}_{\bar N}(z=0, b_\perp, P_1)}{\tilde{\psi}^{-}_{\bar N}(z=0, b_\perp, P_2)}\right)\right]$

$$K(b_\perp,\mu) = \frac{1}{2\ln(P_2/P_1)}\left[\ln\left(\frac{H^{+}_{\bar N}(4x^2P_1^2, 4(1-x)^2P_1^2)\tilde{\psi}^{+}_{\bar N}(b,x,P_2)}{H^{+}_{\bar N}(4x^2P_2^2, 4(1-x)^2P_2^2)\tilde{\psi}^{+}_{\bar N}(b,x,P_1)}\right) + \ln\left(\frac{H^{-}_{\bar N}(4x^2P_1^2, 4(1-x)^2P_1^2)\tilde{\psi}^{-}_{\bar N}(b,x,P_2)}{H^{-}_{\bar N}(4x^2P_2^2, 4(1-x)^2P_2^2)\tilde{\psi}^{-}_{\bar N}(b,x,P_1)}\right)\right]$$



Extracting $K(b_\perp,\mu)$ from one loop kernel for TMDPDFs

SVZES 21: 2103.16991

SWZ 21: 2107.11930

ETMC/PKU 21: 2106.13027

LPC 20: 2005.14572

Extracting $K(b_\perp,\mu)$ from tree level kernel for TMDWFs

Summary and outlook

- **1.Collins-Soper kernel from the first principle can be used to the matching of the TMDPDFs and TMDWFs. It will reveal the internal structure of hadrons.**

- **2. This is the first attempt for extracting Collins-Soper kernel from TMDWFs up to one loop level matching.**

- **3. In future, TMDWFs can be determined through Collins-Soper kernel and one loop matching. (In progress)**

Thanks for your attention!