中国格点QCD第一届年会
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Finite-volume and finite-temperature effects in chiral effective field theory

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Outline:

1. Introduction

2. Chiral EFT in finite box and its application in lattice

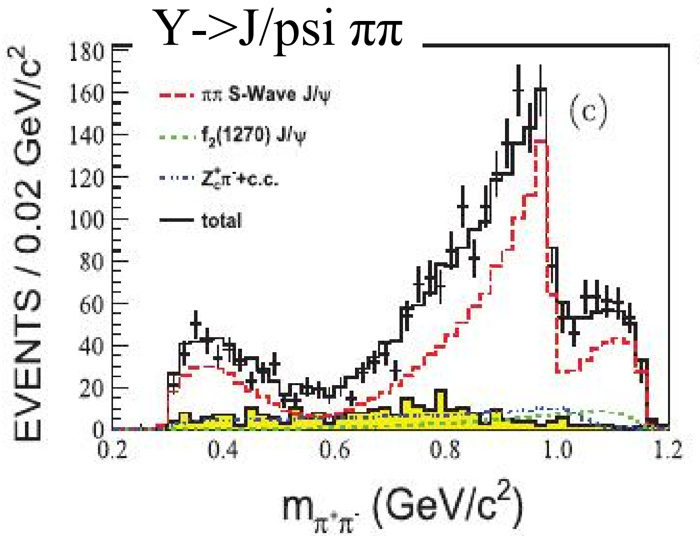
3. Hints of chiral symmetry restoration: scalar resonances at finite temperatures

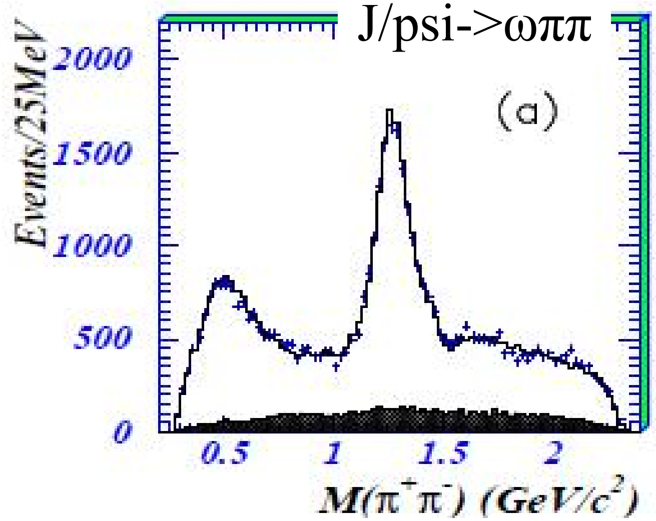
4. Conclusions

Introduction

Scattering: an important approach to study hadron resonances

Line shapes for the same resonance can be quite different in different channels.

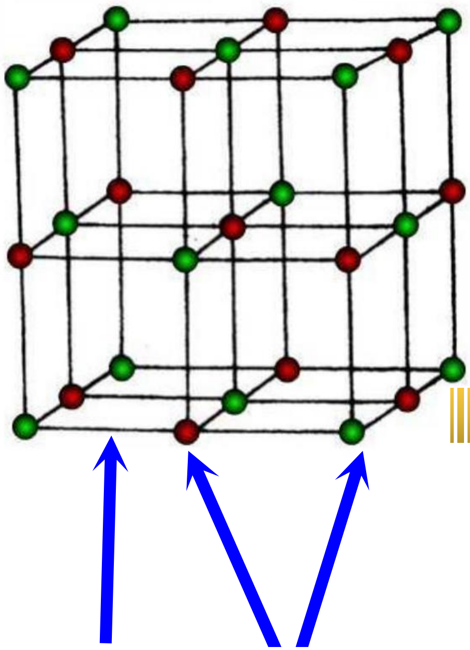




In contrast, resonance poles of scattering amplitudes are universal in all the channels. Different line shaples can be explained by different residues/couplings.

However it is rather challenging to extract scattering observables directly from Exp.

Lattice QCD provides a unique way for this problem !

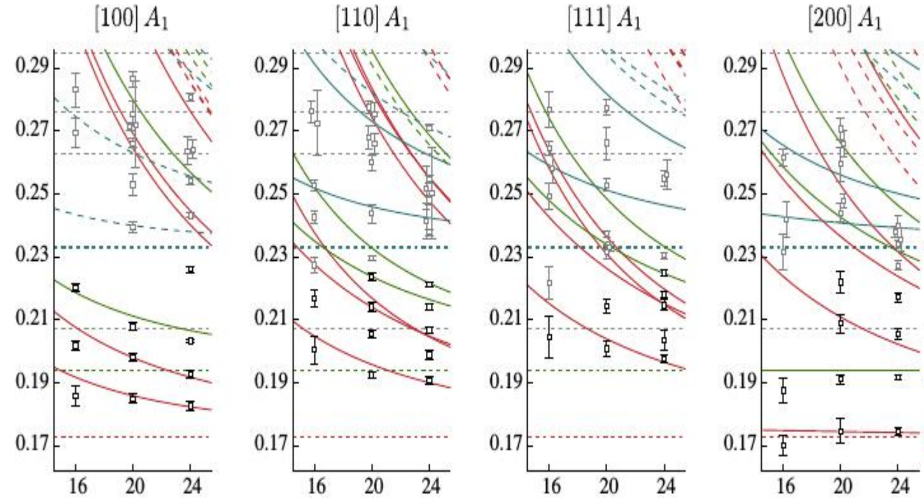


Gluon Quark

Significant progress on meson-meson scattering in lattice simulation

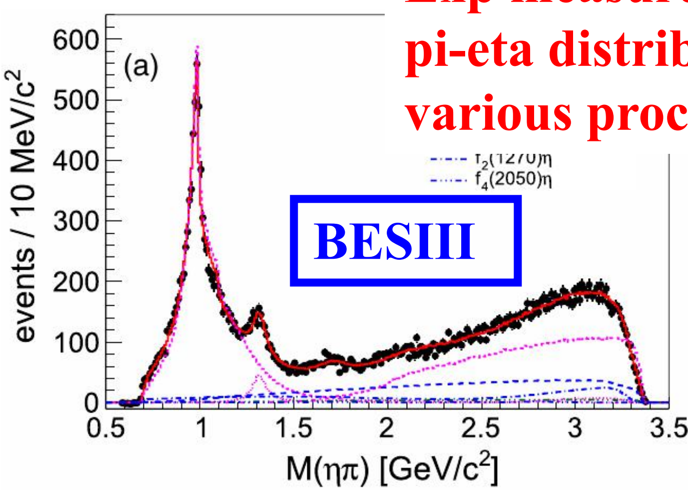
E.g. Pi-eta, KKbar, Pi-eta' scattering

[Dudek,Edwards,Wilson, PRD'16]



Data "measured" by lattice: finite-volume energy levels

Exp measurement of pi-eta distributions in various processes



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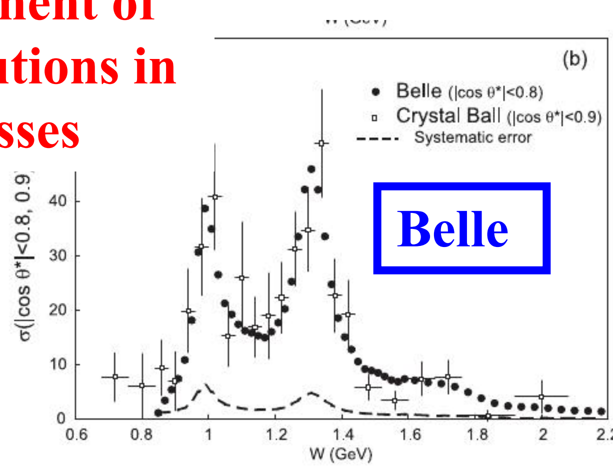


Belle

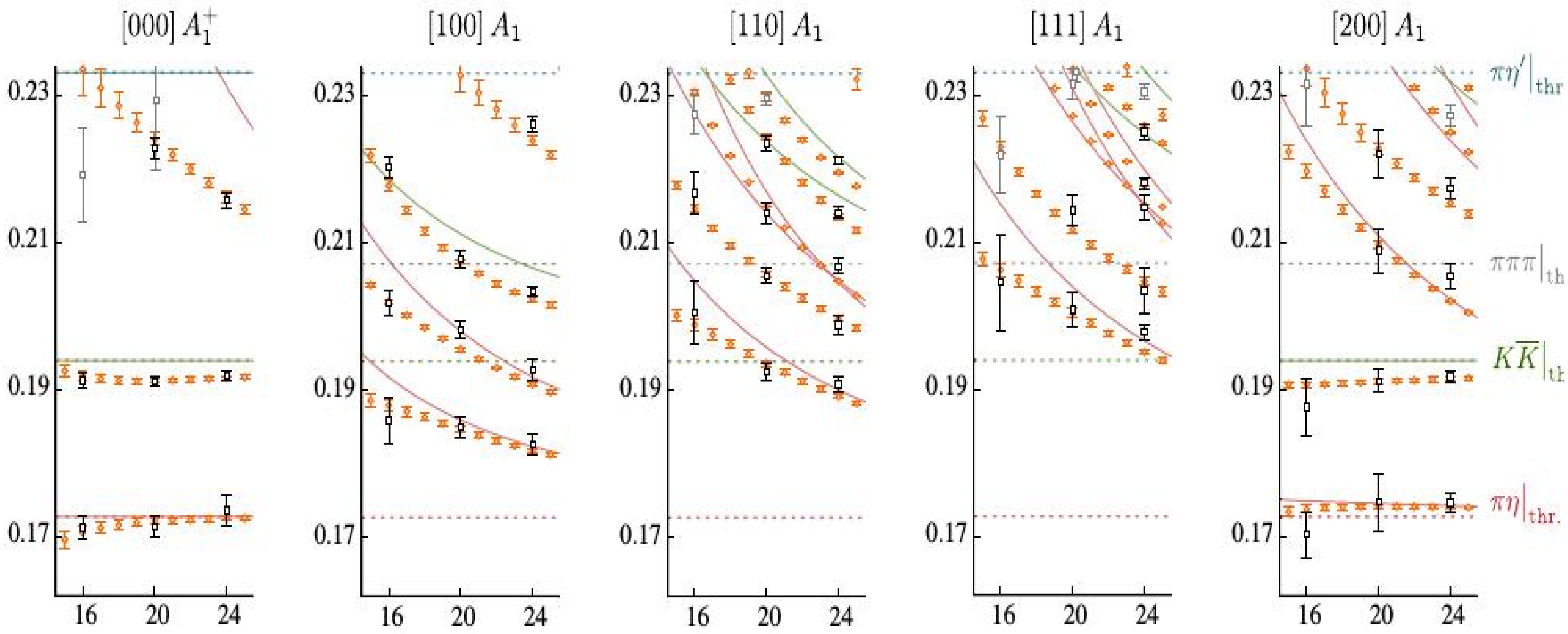
Physical Scattering Amplitudes

First Lattice calculation on coupled-channel pi-eta, KKbar

[Dudek,Edwards,Wilson, PRD'16]

$m_\pi = 390\ \text{MeV}$

Eigenenergies in finite box



Length of finite box

[He,Feng,Liu,JHEP'05] [Wilson,Briceno,Dudek,Edwards,Thomas, PRD '15]
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn, PRD'14] [Fu,PRD'12]
[Gockeler,Horsley,Lage,Meissner,Rakow,Rusetksy,Schierholz,Zanotti,PRD'12] ……

A widely used approach in the inelastic scattering case:

Luscher function + K matrix

$$\det[\mathbf{1} + i\rho \cdot \mathbf{t} \cdot (\mathbf{1} + i\mathcal{M})] = 0$$

Kinematical factor

K matrix: polynomial + pole terms

Luscher's finite-volume functions (complex objects)

• Free parameters in K matrix are determined by the finite-volume spectra. Then one can determine amplitudes in infinite volume.

• K matrix does not automatically respect the QCD symmetries, such as the chiral symmetry. It could be problematic for chiral extrapolation.

Chiral approach + Luscher formula:

Step 1: Put chiral perturbation theory (ChPT) in finite volume.

Step 2: The free parameters in ChPT, which are indepdent of quark masses and volumes, are fitted to the finite-volume energy levels obtained at (un)physical quark masses.

Step 3: Perform the chiral extrapolation and give the predictions in infinite volume with physical quark masses, including phase shifts, inelasticities, resonance poles, etc.

Step 4: Proceed with the standard routine of hadron phenomenologies by taking the physical amplitudes determined from Lattice QCD.

Chiral amplitudes in finite box

Finite-volume effects in scattering process

Two types of finite volume dependence of scattering amplitudes:

- **Exponentially suppressed type $\propto$ *exp*(-m_pL) : *s, t, u* channels**
- **Power suppressed type $\propto$ $1/L^3$: only *s* channel**

Current lattice calculations usually take big enough volume L, and therefore one could neglect the exponentially suppressed terms, indicating that finite-volume effects only enter through s channel.

Prescription: Algebraic approximation of N/D (a variant version of K-matrix)

$$T_J(s) = \frac{N(s)}{1 + G(s)\ N(s)}$$

- **The s-channel unitariy is exact. The crossed-channel dyanmics is included in a perturbative manner.**
- **Unitarity condition:** $\mathrm{Im}G(s) = -\rho(s)$

$$G(s) = \boxed{a^{SL}(s_0)} - \frac{s - s_0}{\pi} \int_{4m^2}^{\infty} \frac{\rho(s')}{(s' - s)(s' - s_0)} ds'$$

Finite-volume effects enter via $G(s)$

- **$N(s)$: crossed-channel contributions of partial-wave amplitudes from chiral EFT. Finite-volume effects are suppressed for $N(s)$.**

Finite-volume effects in s-channel unitarity function G(s)

$$G(s) = i\int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_1^2 + i\epsilon)[(P-q)^2 - m_2^2 + i\epsilon]}\,, \qquad s \equiv P^2$$

Sharp momentum cutoff to regularize G(s)

$$G(s)^{\mathrm{cutoff}} = \int^{|\vec{q}|<q_{\mathrm{max}}} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3} I(|\vec{q}|)\,, \qquad \begin{aligned} I(|\vec{q}|) &= \frac{w_1 + w_2}{2w_1 w_2\left[E^2 - (w_1+w_2)^2\right]}\,, \\ w_i &= \sqrt{|\vec{q}|^2 + m_i^2}\,, \quad s = E^2 \end{aligned}$$

G(s) in a finite box of length L with periodic boundary condition

$$\widetilde{G} = \frac{1}{L^3}\sum_{\vec{n}}^{|\vec{q}|<q_{\mathrm{max}}} I(|\vec{q}|)\,, \qquad \vec{q} = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3$$

Finite-volume correction ΔG [Doring, Meissner, Oset, Rusetsky, EPJA11]

$$\begin{aligned} \Delta G &= \widetilde{G} - G^{\mathrm{cutoff}} \\ &= \left\{\frac{1}{L^3}\sum_{\boldsymbol{q}}^{|\boldsymbol{q}|<q_{\mathrm{max}}} - \int^{|\boldsymbol{q}|<q_{\mathrm{max}}} \frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^3}\right\} \frac{1}{2\omega_1(\boldsymbol{q})\,\omega_2(\boldsymbol{q})} \frac{\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q})}{E^2 - (\omega_1(\boldsymbol{q}) + \omega_2(\boldsymbol{q}))^2} \end{aligned}$$

Finite-volume effects in the moving frames

Lorentz invariance is lost in finite box. One needs to work out the explicit form of the loops when boosting from one frame to another.

transforming $\vec{q}_{i=1,2}$ to $\boxed{\vec{q}^{\,*}_{i=1,2}}$ $\longrightarrow$ **CM quantities**

$$\vec{q}_i^{\,*} = \vec{q}_i + \left[\left(\frac{P^0}{E} - 1\right)\frac{\vec{q}_i \cdot \vec{P}}{|\vec{P}|^2} - \frac{q_i^0}{E}\right]\vec{P}$$

moving frame with total four-momentum $P^\mu = (P^0, \vec{P})$ $\quad s = E^2 = (P^0)^2 - |\vec{P}|^2$

Impose on-shell condition $\quad q_i^{*\,0} = \sqrt{|\vec{q}_i^{\,*}|^2 + m_i^2}$

$$q_i^0 = \frac{q_i^{*\,0} E + \vec{q}_i \cdot \vec{P}}{P^0} \longrightarrow q_i^0 = \sqrt{|\vec{q}_i|^2 + m_i^2}$$

G function in the moving frame

$$\int^{|\vec{q}_1|^* < q_{\max}} \frac{d^3 \vec{q}_1^{\,*}}{(2\pi)^3} I(|\vec{q}_1^{\,*}|) \Longrightarrow \widetilde{G}^{\rm MV} = \frac{E}{P^0 L^3} \sum_{\vec{q}_1}^{|\vec{q}_1^{\,*}| < q_{\max}} I(|\vec{q}_1^{\,*}(\vec{q}_1)|) \qquad \vec{q}_1 = \frac{2\pi}{L}\vec{n}, \quad \vec{n} \in \mathbb{Z}^3, \qquad \vec{P} = \frac{2\pi}{L}\vec{N}, \quad \vec{N} \in \mathbb{Z}^3$$

Finite-volume correction ΔG^{MV}: $\quad \Delta G^{\rm MV} = \widetilde{G}^{\rm MV} - G^{\rm cutoff}$

[Doring, Meissner, Oset, Rusetsky, EPJA12]

Mixing of different partial waves in finite volume

The mixing between different partial waves is absent in the infinite volume:

$$\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta d\theta Y_{\ell m}(\theta,\phi) Y^*_{\ell' m'}(\theta,\phi) = \delta_{\ell\ell'}\delta_{mm'}$$

The mixing appears in finite-volume case, due to the absence of the general orthogonal conditions of spherical harmonic functions.

The mixing patterns vary in different irreducible representations and different moving frames.

$$\det[\mathbf{1} + i\boldsymbol{\rho}\cdot\mathbf{t}\cdot(\mathbf{1} + i\boldsymbol{\mathcal{M}})] = 0$$

[He,Feng,Liu,JHEP'05] [Wilson,Briceno,Dudek,Edwards,Thomas, PRD '15]
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn, PRD'14] [Fu,PRD'12]
[Gockeler,Horsley,Lage,Meissner,Rakow,Rusetksy,Schierholz,Zanotti,PRD'12] ......

We adapt the following approach to proceed the study of unitarized ChpT in finite volume.

[Gockeler,Horsley,Lage,Meissner,Rakow,Rusetksy,Schierholz,Zanotti,PRD'12] ......

Finite-volume correction to G function:

$$\Delta G^{\mathrm{MV}}_{\ell m} = \widetilde{G}^{\mathrm{MV}}_{\ell m} - G^{\mathrm{cutoff}} \delta_{\ell 0} \delta_{m 0}$$

$$\widetilde{G}^{\mathrm{MV}}_{\ell m} = \sqrt{\frac{4\pi}{2\ell+1}} \frac{1}{L^3} \frac{E}{P^0} \sum_{\vec{n}}^{|\vec{q}^*|<q_{\max}} \left(\frac{|\vec{q}^*|}{|\vec{q}^{\mathrm{on}*}|} \right)^{\ell} Y_{\ell m}(\hat{q}^*)\, I(|\vec{q}^*|)$$

It is equavelent to the w_{lm} function, up to exponentially suppressed terms [Gockeler,Horsley,Lage,et al.,PRD'12]

$$w_{lm} = \frac{1}{\pi^{3/2}\sqrt{2l+1}} \gamma^{-1} q^{-l-1} Z^{\mathbf{\Delta}}_{lm}(1, q^2)$$

$$Z^{\mathbf{\Delta}}_{js}(\delta, q^2) = \sum_{z \in P_{\Delta}} \frac{\mathcal{Y}_{js}(z)}{(z^2 - q^2)^{\delta}}$$

$$\mathcal{Y}_{lm}(\boldsymbol{r}) = |\boldsymbol{r}|^l Y_{lm}(\hat{\boldsymbol{r}}), \qquad \hat{\boldsymbol{r}} = \frac{\boldsymbol{r}}{|\boldsymbol{r}|}$$

$$\widetilde{G}^{\mathrm{MV}}_{\ell m} = -\frac{|\vec{q}^{\,\mathrm{on}*}|}{8\pi E} w_{\ell m}$$

Final expression for the G function:

$$\widetilde{G}^{\mathrm{MV}}_{\ell m} = G^{\mathrm{V}\infty} \delta_{\ell 0} \delta_{m 0} + \Delta G^{\mathrm{MV}}_{\ell m}$$

Example: **when only S and P waves are included.**

A_1^+ (0,0,0) : $$\det[1+N_0(s).\widetilde{G}_{00}]=0$$

T_1^- (0,0,0): $$\det[1+N_1(s).\widetilde{G}_{00}]=0$$

A_1 (0,0,1): $$\det[I+N_{0,1}\cdot\mathcal{M}_{0,1}^{A_1}]=0\,,$$

$$N_{0,1}=\begin{pmatrix}N_0 & 0\\ 0 & N_1\end{pmatrix}\,,\qquad \mathcal{M}_{0,1}^{A_1}=\begin{pmatrix}\widetilde{G}_{00} & i\sqrt{3}\widetilde{G}_{10}\\ -i\sqrt{3}\widetilde{G}_{10} & \widetilde{G}_{00}+2\widetilde{G}_{20}\end{pmatrix}$$

$$N_{0,1}\cdot\mathcal{M}_{0,1}^{A_1}=\begin{pmatrix}N_{0,11}\widetilde{G}_{00,1} & N_{0,12}\widetilde{G}_{00,2} & N_{0,13}\widetilde{G}_{00,3} & i\sqrt{3}N_{0,11}\widetilde{G}_{10,1}\\ N_{0,21}\widetilde{G}_{00,1} & N_{0,22}\widetilde{G}_{00,2} & N_{0,23}\widetilde{G}_{00,3} & i\sqrt{3}N_{0,21}\widetilde{G}_{10,1}\\ N_{0,31}\widetilde{G}_{00,1} & N_{0,32}\widetilde{G}_{00,2} & N_{0,33}\widetilde{G}_{00,3} & i\sqrt{3}N_{0,31}\widetilde{G}_{10,1}\\ -i\sqrt{3}N_1\widetilde{G}_{00,1} & 0 & 0 & N_1(\widetilde{G}_{00,1}+2\widetilde{G}_{20,1})\end{pmatrix}$$

[ZHG,Liu,Meissner,Oller,Rusetsky, EPJC'19]

Several applications in analyzing the lattice finite-volume energy levels

$\pi\eta$, KKbar, $\pi\eta'$ coupled-channel system and $a_0(980)$

Leading order amplitudes from U(3) ChPT:

$$T_{J=0}^{I=1,\pi\eta\to\pi\eta}(s)^{(2)} = \frac{(c_\theta - \sqrt{2}s_\theta)^2 m_\pi^2}{3F_\pi^2}\,,$$

$$T_{J=0}^{I=1,\pi\eta\to K\bar{K}}(s)^{(2)} = \frac{c_\theta(3m_\eta^2 + 8m_K^2 + m_\pi^2 - 9s) + 2\sqrt{2}s_\theta(2m_K^2 + m_\pi^2)}{6\sqrt{6}F_\pi^2}\,,$$

$$T_{J=0}^{I=1,\pi\eta\to\pi\eta'}(s)^{(2)} = \frac{(\sqrt{2}c_\theta^2 - c_\theta s_\theta - \sqrt{2}s_\theta^2)m_\pi^2}{3F_\pi^2}\,,$$

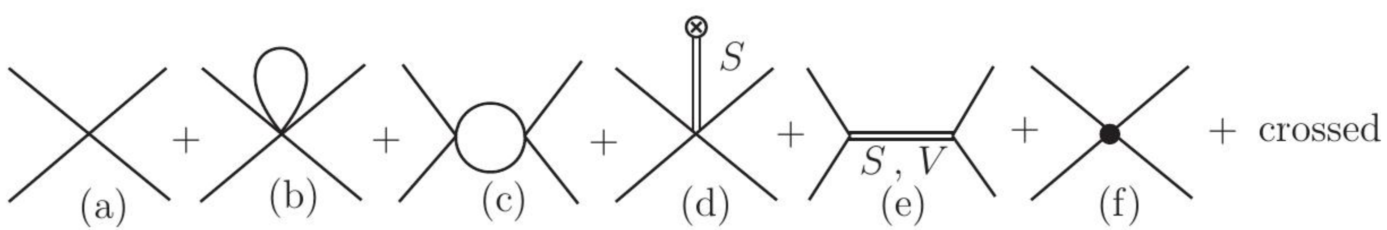
$$T_{J=0}^{I=1,K\bar{K}\to K\bar{K}}(s)^{(2)} = \frac{s}{4F_\pi^2}\,,$$

$$T_{J=0}^{I=1,K\bar{K}\to\pi\eta'}(s)^{(2)} = \frac{s_\theta(3m_{\eta'}^2 + 8m_K^2 + m_\pi^2 - 9s) - 2\sqrt{2}c_\theta(2m_K^2 + m_\pi^2)}{6\sqrt{6}F_\pi^2}\,,$$

$$T_{J=0}^{I=1,\pi\eta'\to\pi\eta'}(s)^{(2)} = \frac{(\sqrt{2}c_\theta + s_\theta)^2 m_\pi^2}{3F_\pi^2}\,,$$

Higher-order amplitudes are also explored

Scattering amplitude :



$\pi\eta$, KKbar, $\pi\eta'$ coupled-channel system and $a_0(980)$

Other inputs to fit the lattice data:

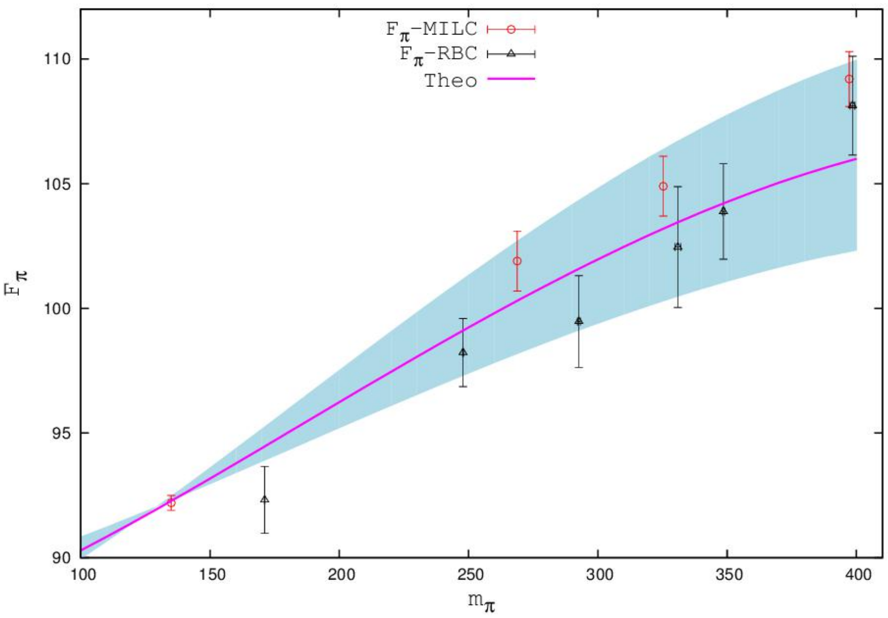
$$m_\pi = 391.3\pm0.7\,\text{MeV}\,,\ m_K = 549.5\pm0.5\,\text{MeV}\,,\ m_\eta = 587.2\pm1.1\,\text{MeV}\,,\ m_{\eta'} = 929.8\pm5.7\,\text{MeV}$$

Our estimate of the leading order η-η' mixing angle at unphysical masses

$$\theta = (-10.0 \pm 0.1)^\circ \quad (\ \theta^{\text{phys}} = -16.2^\circ\ )$$

We also need to estimate F_π at the unphysical meson masses.

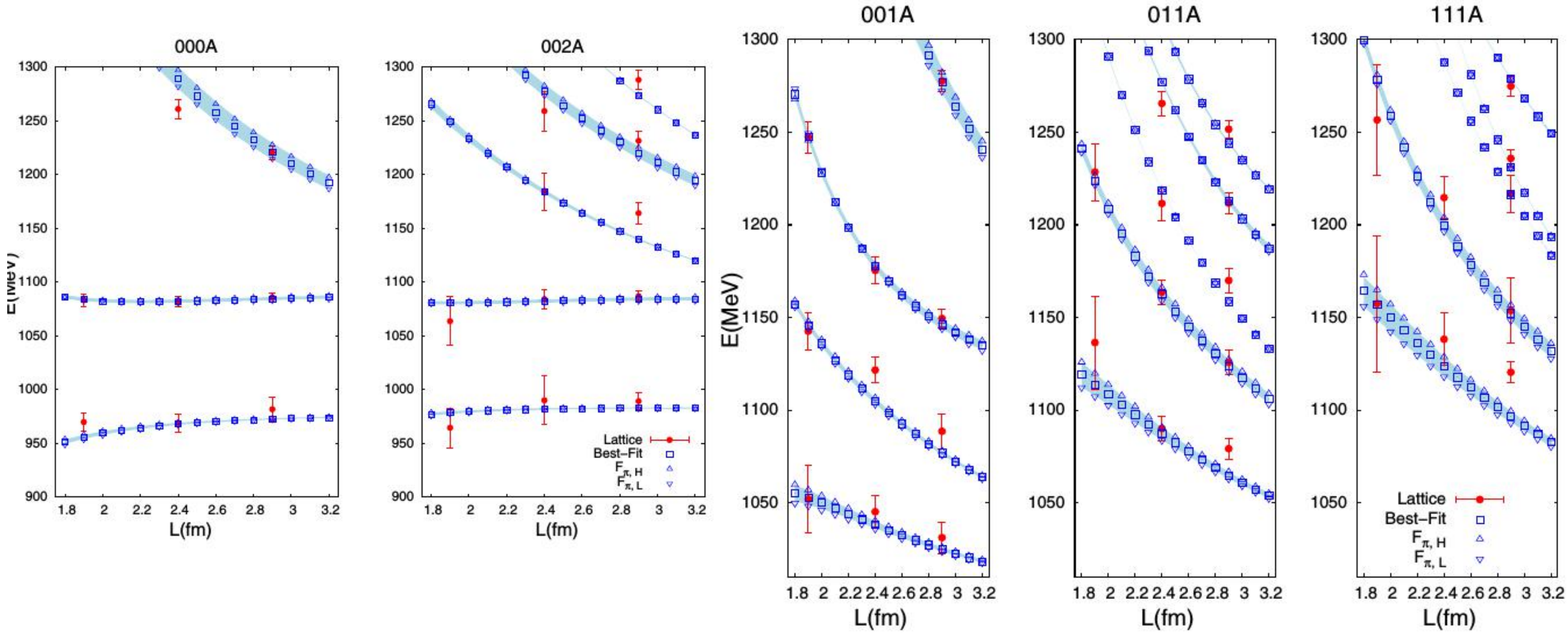
$$F_\pi = F\left\{1 - \frac{1}{16\pi^2 F^2}\left[m_\pi^2 \ln\frac{m_\pi^2}{\mu^2} + \frac{m_K^2}{2}\ln\frac{m_K^2}{\mu^2}\right] + \left[\frac{4\widetilde{c}_d\,\widetilde{c}_m(m_\pi^2+2m_K^2)}{F^2 M_{S_1}^2} - \frac{8c_d\,c_m\,(m_K^2-m_\pi^2)}{3F^2 M_{S_8}^2}\right]\right\}$$



πη, KKbar, πη' coupled-channel system and $a_0(980)$

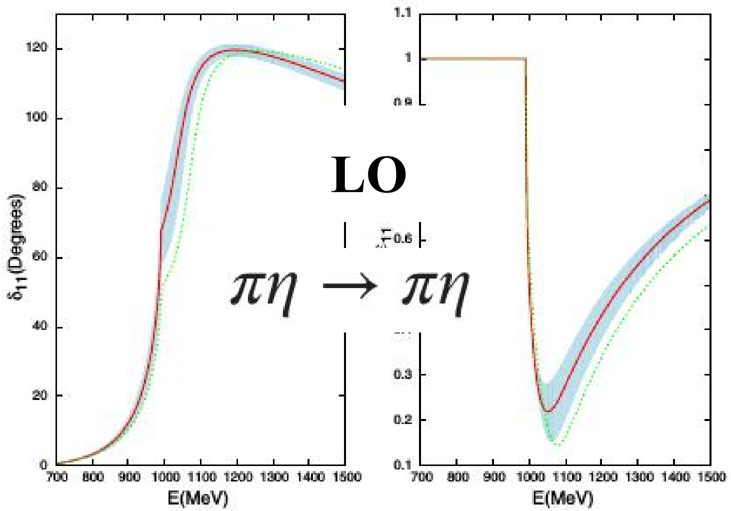
Leading order Fit (Only LO amplitudes are included in the N(s) function.)

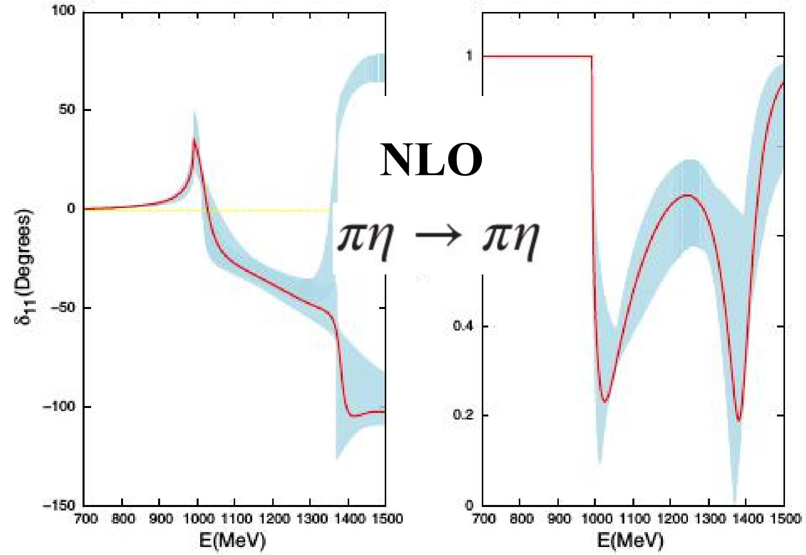
[ZHG,Liu,Meissner,Oller,Rusetsky, PRD'17]



- Remark: there is only one free parameter in the fits, i.e. the common subtraction constant !
- The quality of NLO fit is quite similar to the LO case.

Phase shifts and inelasticities at physical meson masses



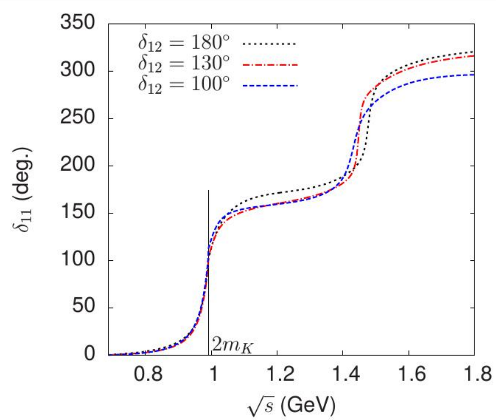


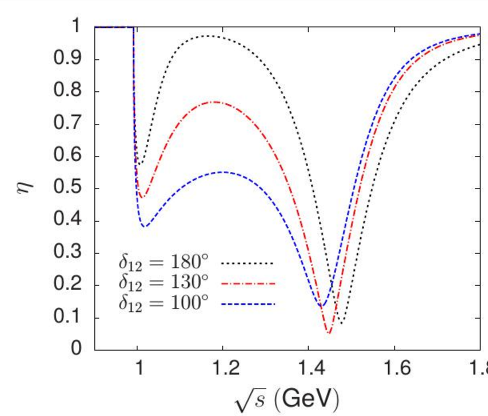
Pole positions and residues of $a_0(980)$ at physical meson masses

Resonance	RS	Mass (MeV)	Width/2 (MeV)	$\lvert\text{Residue}\rvert_{\pi\eta}^{1/2}$ (GeV)	Ratios	
LO						
$a_0(980)$	II	1037^{+17}_{-14}	44^{+6}_{-9}	$3.8^{+0.3}_{-0.2}$	$1.43^{+0.03}_{-0.03}$ $(K\bar{K}/\pi\eta)$	$0.05^{+0.01}_{-0.01}$ $(\pi\eta'/\pi\eta)$
NLO						
$a_0(980)$	IV	1019^{+22}_{-8}	24^{+57}_{-17}	$2.8^{+1.4}_{-0.6}$	$1.8^{+0.1}_{-0.3}$ $(K\bar{K}/\pi\eta)$	$0.01^{+0.06}_{-0.01}$ $(\pi\eta'/\pi\eta)$
$a_0(1450)$	V	1397^{+40}_{-27}	62^{+79}_{-8}	$1.7^{+0.3}_{-0.4}$	$1.4^{+2.4}_{-0.6}$ $(K\bar{K}/\pi\eta)$	$0.9^{+0.8}_{-0.2}$ $(\pi\eta'/\pi\eta)$

Comparison with the results from another group

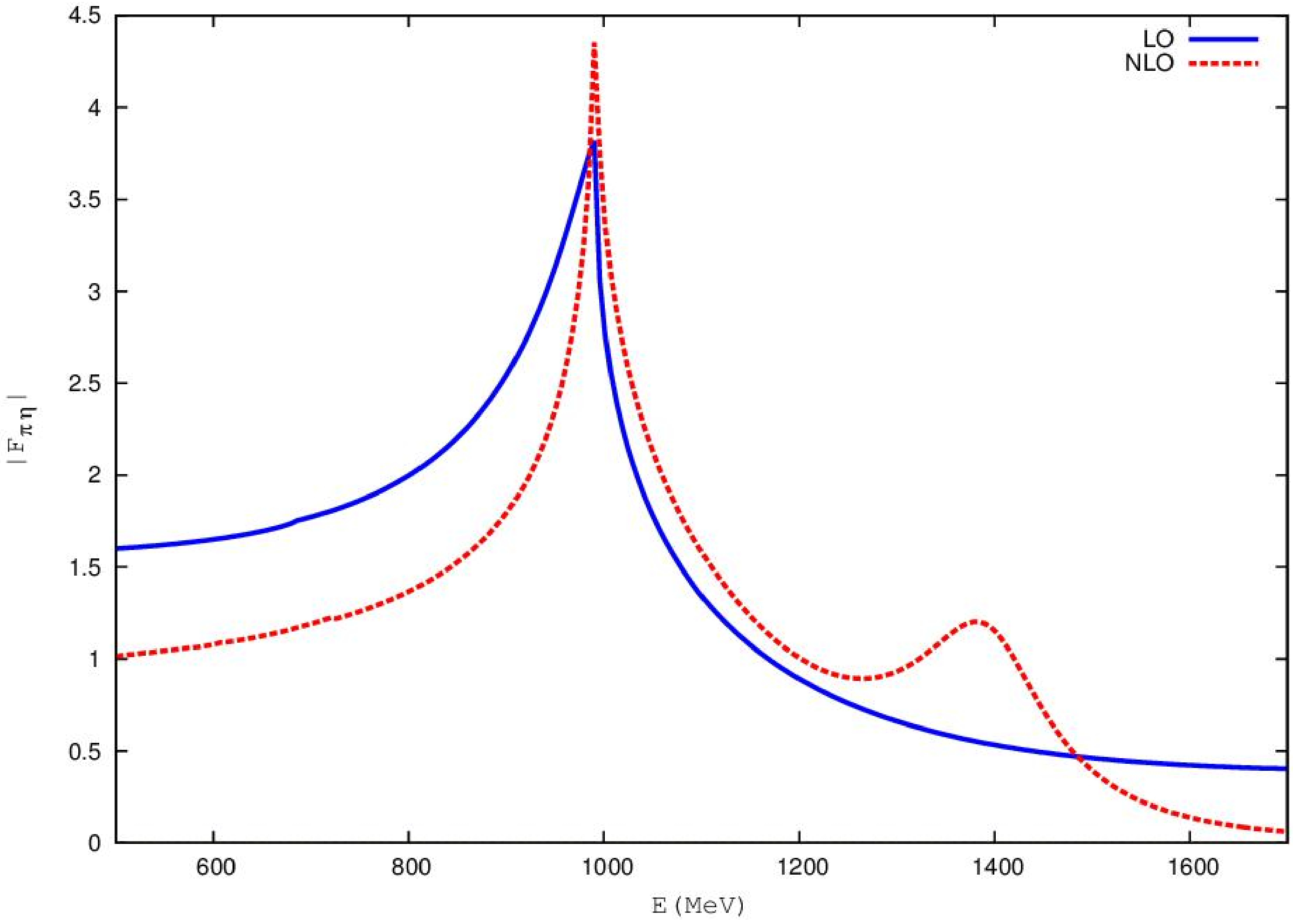
[Albaladejo,Moussallam, EPJC'17]





Prediction of the pi-eta scalar form factors

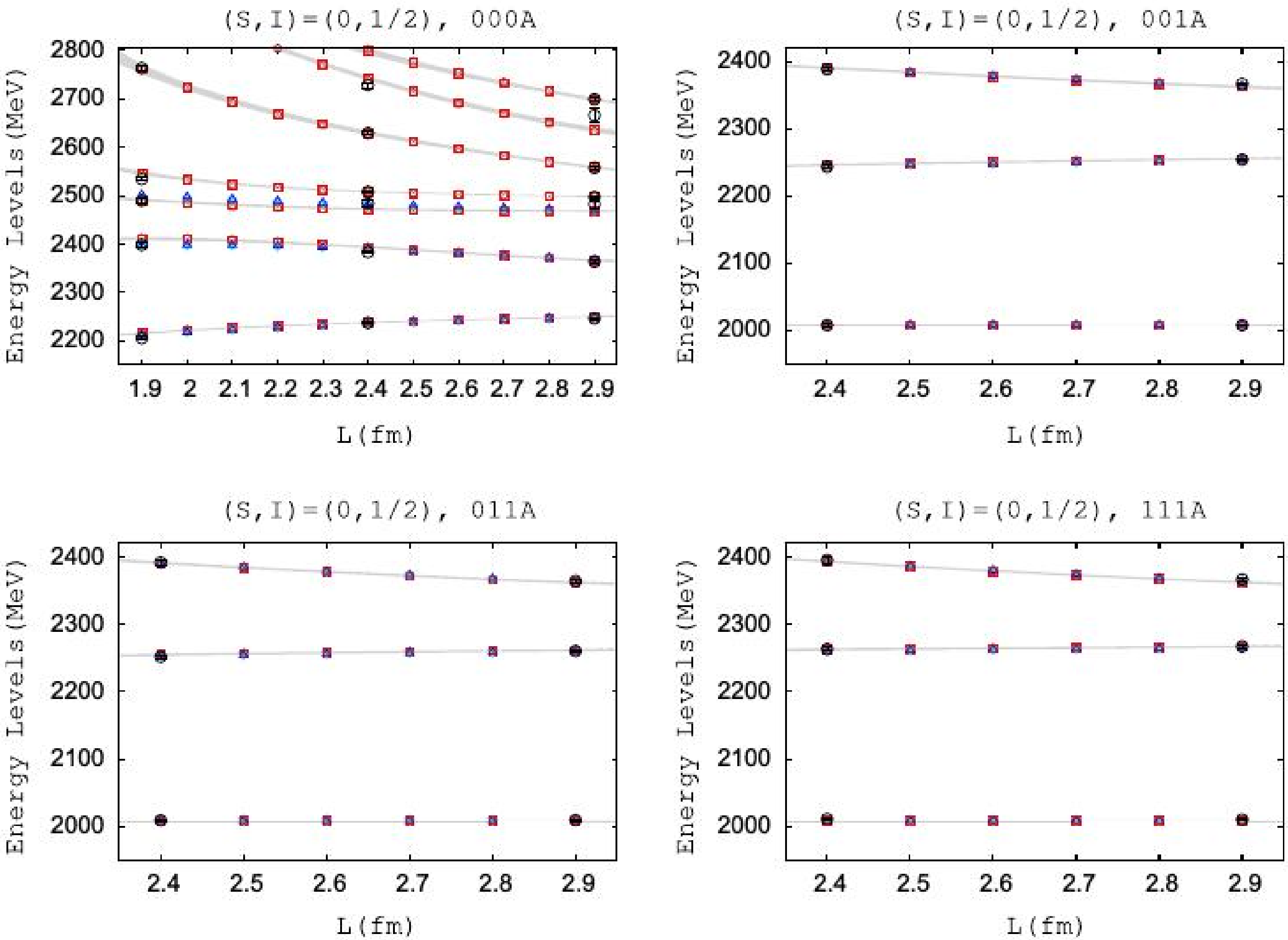
$$F^I(s) = [1 + N^{IJ}(s)g^{IJ}(s)]^{-1}R^I(s)$$



D-pi, D-eta, Ds-Kbar scattering and $D^*_0(2400)$

Reproduction of the finite-volume energy levels

[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]



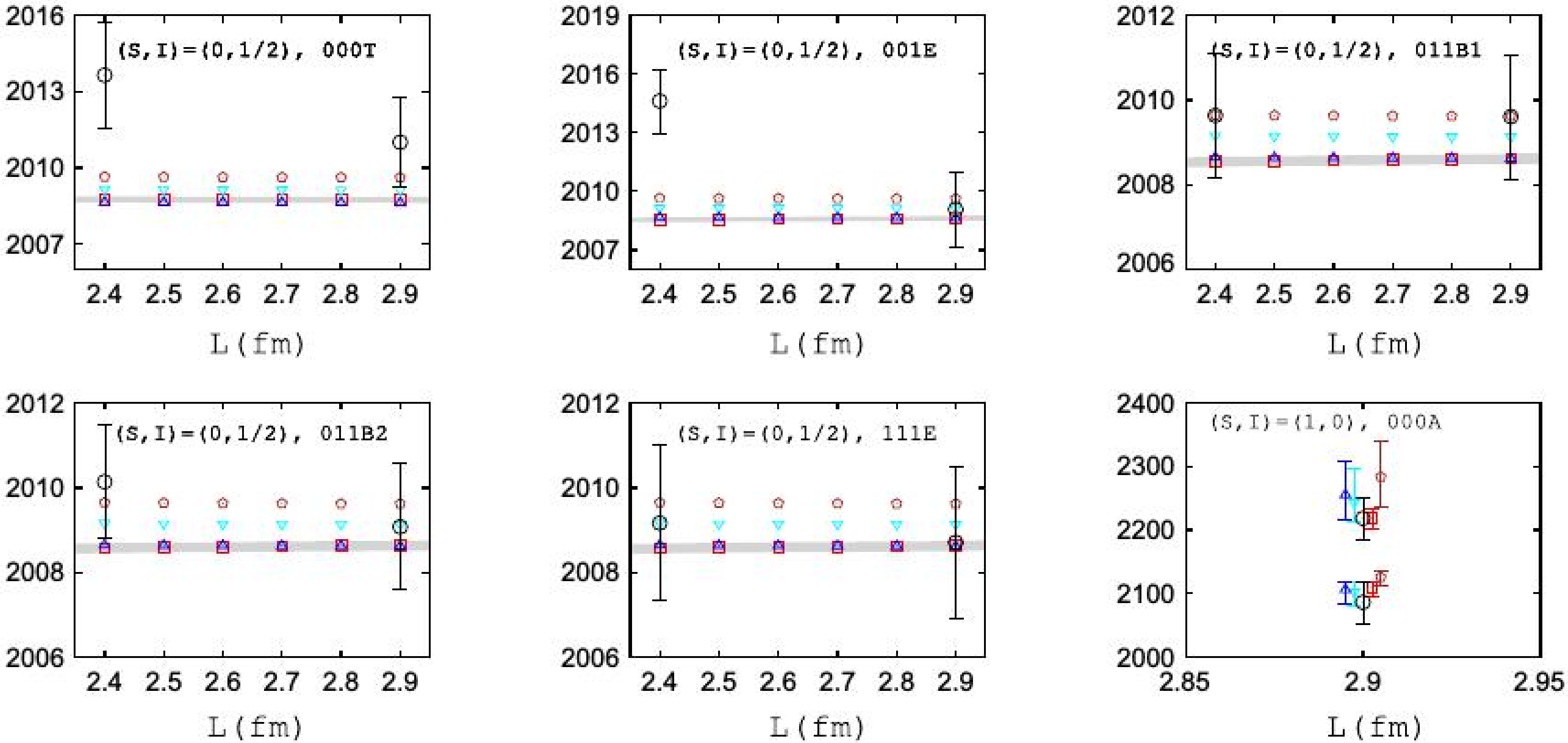
[ZHG,Liu,Meissner,Oller,Rusetsky, EPJC'19]

D-pi, D-eta, Ds-Kbar scattering and $D^*_0(2400)$

Reproduction of the finite-volume energy levels

[Moir,Peardon,Ryan,Thomas, Wilson, JHEP'16]

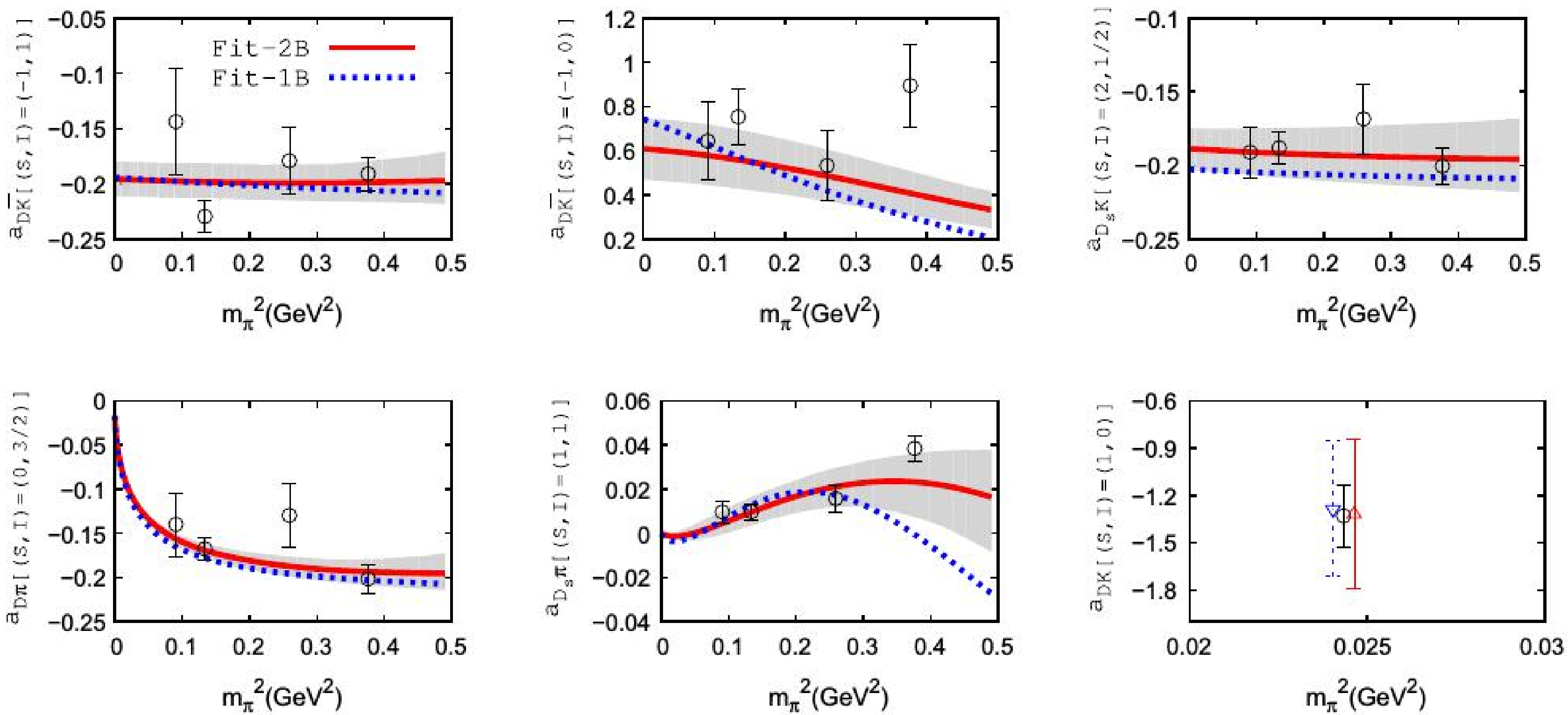
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]



D-pi, D-eta, Ds-Kbar scattering and $D^*_0(2400)$

Reproduction of lattice scattering lengths

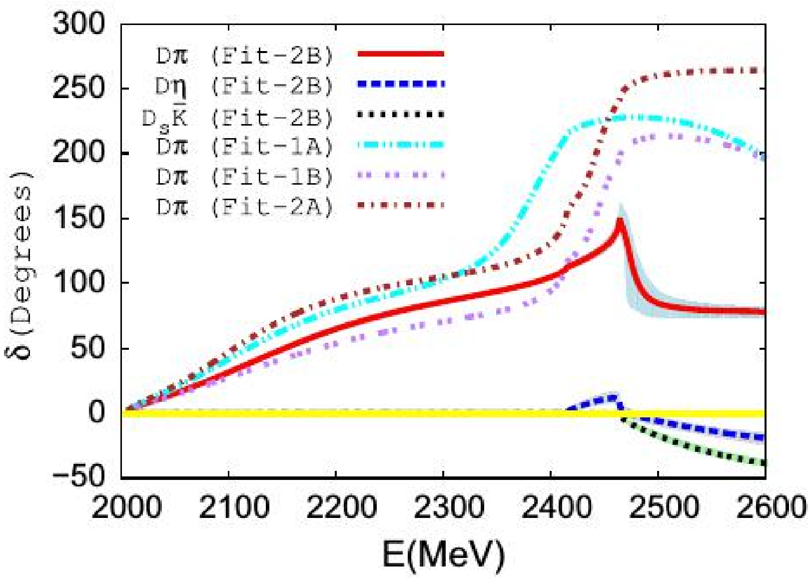
[L.Liu,Orginos,F.K.Guo,Meissner, PRD'13]
[Lang,Leskovec,Mohler,Prelovsek,Woloshyn,PRD'14]

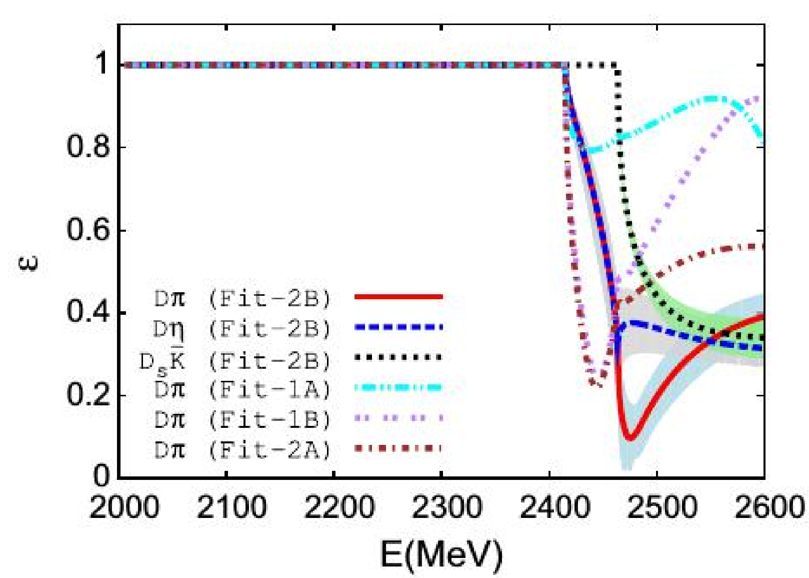


D-pi, D-eta, Ds-Kbar scattering and $D^*_0(2400)$

Prediction of the D-pi phase shifts and inelasticities at physical masses

[ZHG,Liu,Meissner,Oller,Rusetsky, EPJC'19]

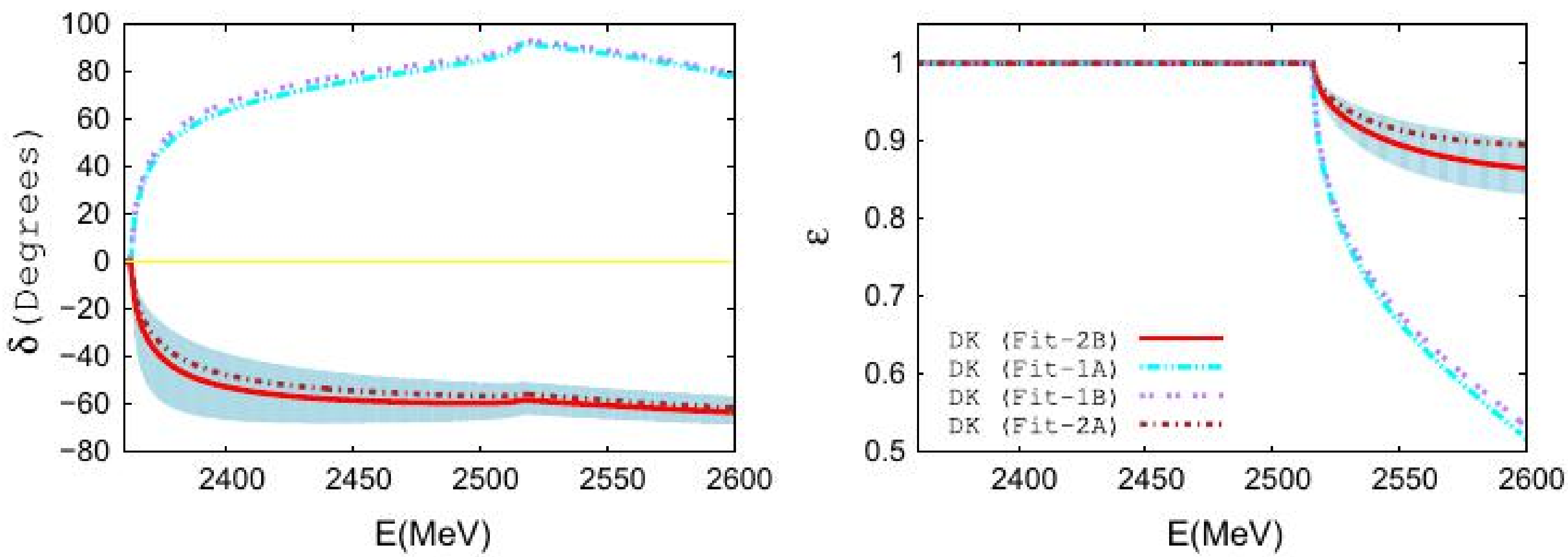




Poles and residues

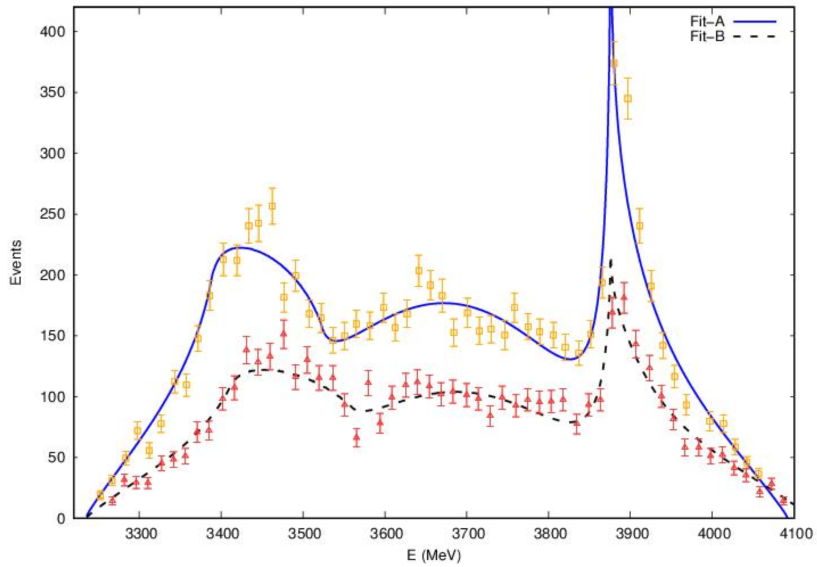
Fit	RS	M	$\Gamma/2$ (MeV)	$\lvert\gamma_1\rvert$ (GeV)	$\lvert\gamma_2/\gamma_1\rvert$	$\lvert\gamma_3/\gamma_1\rvert$
Fit-1A	II	$2097.7^{+6.8}_{-6.1}$	$112.2^{+16.5}_{-14.2}$	$9.6^{+0.3}_{-0.3}$	$0.10^{+0.05}_{-0.04}$	$0.78^{+0.08}_{-0.08}$
Fit-1A	II	$2384.4^{+26.4}_{-23.6}$	$36.0^{+9.9}_{-10.0}$	$4.8^{+0.5}_{-0.6}$	$1.51^{+0.15}_{-0.16}$	$2.09^{+0.18}_{-0.18}$
Fit-1B	II	$2106.4^{+5.1}_{-5.0}$	$170.6^{+12.5}_{-13.0}$	$10.1^{+0.3}_{-0.2}$	$0.11^{+0.07}_{-0.07}$	$0.79^{+0.07}_{-0.07}$
Fit-1B	III	$2409.0^{+22.7}_{-24.5}$	$78.6^{+20.5}_{-15.2}$	$6.1^{+0.7}_{-0.6}$	$1.22^{+0.19}_{-0.19}$	$2.72^{+0.48}_{-0.49}$
Fit-2A	II	$2095.7^{+5.2}_{-6.8}$	$97.1^{+10.3}_{-10.7}$	$9.4^{+0.2}_{-0.2}$	$0.10^{+0.02}_{-0.02}$	$0.63^{+0.03}_{-0.03}$
Fit-2A	III	$2401.3^{+20.4}_{-19.6}$	$55.0^{+14.5}_{-10.8}$	$5.1^{+0.5}_{-0.5}$	$1.31^{+0.19}_{-0.15}$	$2.50^{+0.31}_{-0.28}$
Fit-2B	II	$2117.7^{+3.8}_{-3.4}$	$145.0^{+8.0}_{-6.8}$	$10.2^{+0.2}_{-0.1}$	$0.09^{+0.03}_{-0.03}$	$0.58^{+0.04}_{-0.03}$
Fit-2B	III	$2470.5^{+25.1}_{-24.9}$	$104.1^{+16.0}_{-12.5}$	$6.7^{+0.7}_{-0.6}$	$1.14^{+0.12}_{-0.12}$	$2.06^{+0.16}_{-0.16}$

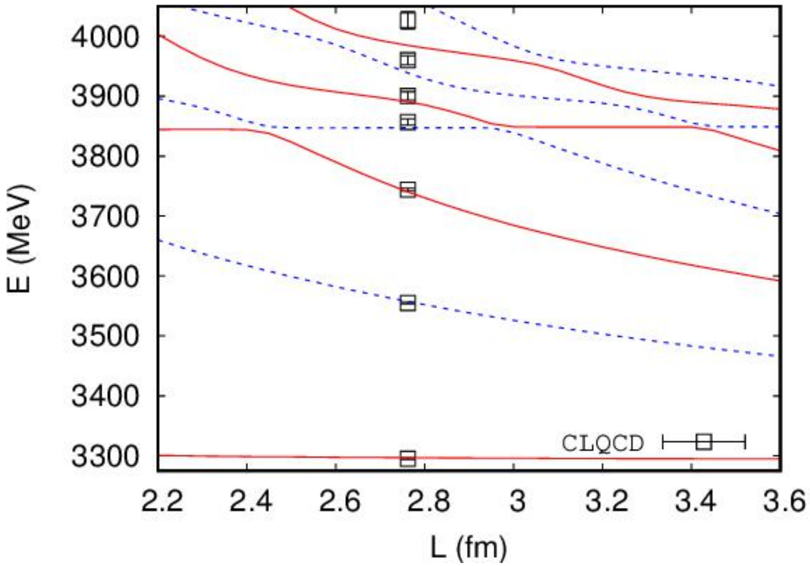
D-K、Ds-eta scattering and $D^*_{s0}(2317)$

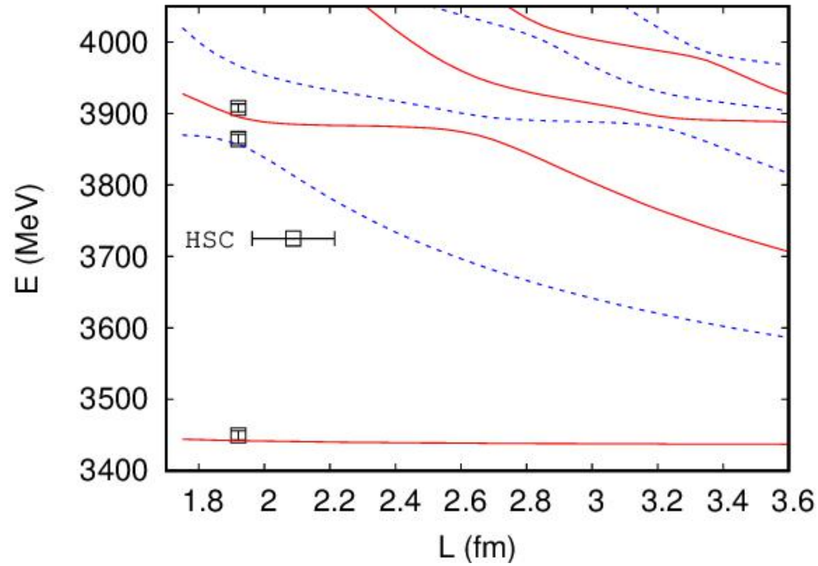


Fit	RS	M (MeV)	$\Gamma/2$ (MeV)	$\lvert\gamma_1\rvert$ (GeV)	$\lvert\gamma_2/\gamma_1\rvert$
Fit-1A	I	2356.7−2362.8	0	1.3−6.9	1.03−1.20
Fit-1A	II	2316.7−2362.8	0	0.4−10.1	1.14−1.50
Fit-1B	I	2357.1−2362.8	0	0.5−6.7	1.05−1.22
Fit-1B	II	2316.0−2362.8	0	0.6−10.3	1.12−1.56
Fit-2A	I	$2345.1^{+14.7}_{-41.5}$	0	$8.3^{+2.3}_{-2.6}$	$0.96^{+0.06}_{-0.08}$
Fit-2B	I	$2350.7^{+9.0}_{-25.7}$	0	$7.7^{+2.1}_{-2.0}$	$0.83^{+0.08}_{-0.06}$

J/psi-pi、DD* scattering and $Z_c(3900)$ [ZHG, et al., in preparation]







Lattice data : [T.Chen, et al., (CLQCD), '19CPC] [Cheung, et al., (HSC), '17JHEP]

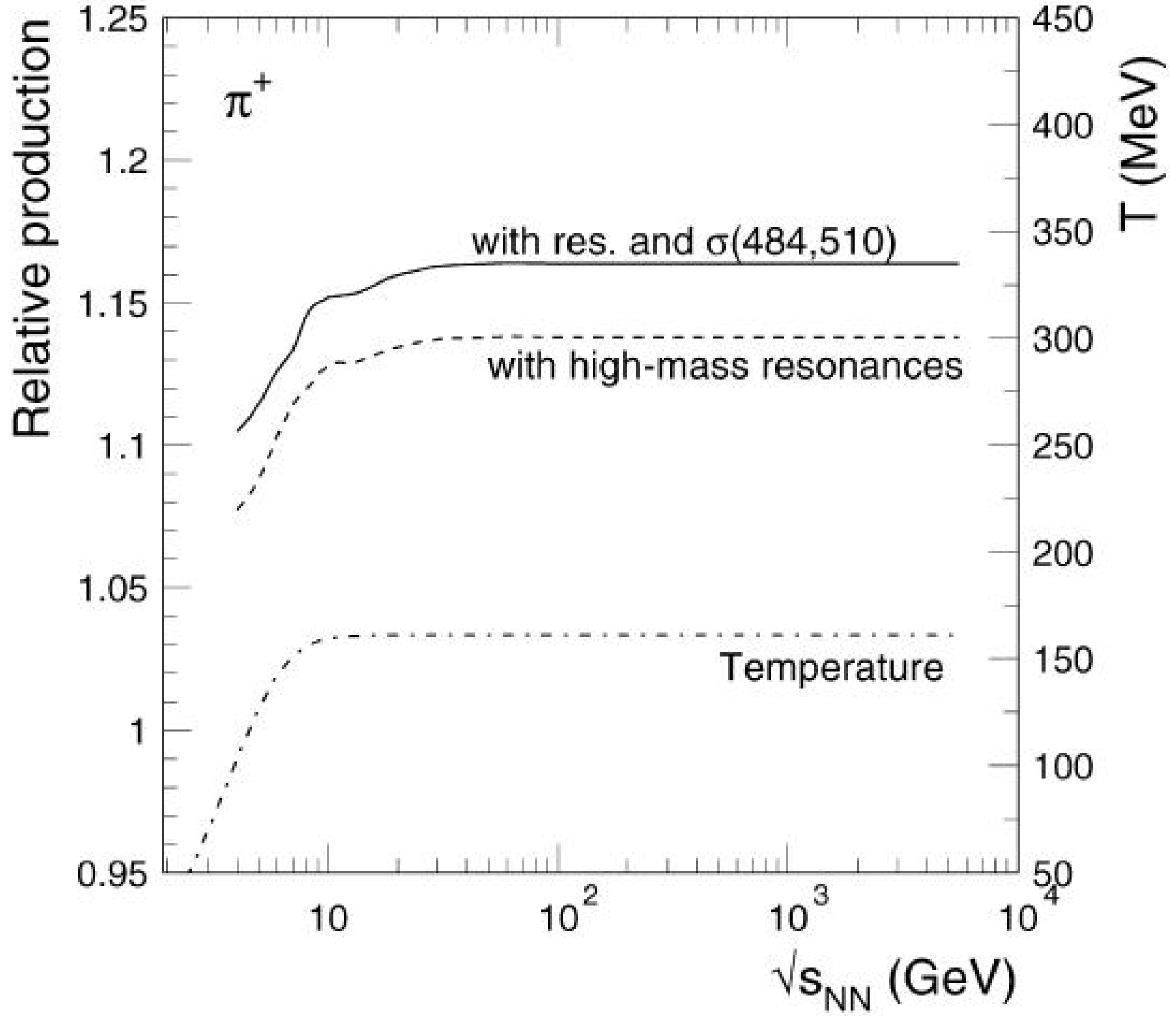
Scalar resonance at finite temeratures

Importance of the broad scalar sigma resonance:
to improve the description of the hadron yields

[Andronic, Braun-Munzinger, Stachel, PLB'09]

Problem:

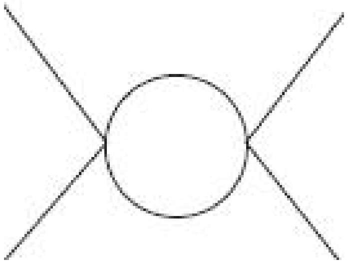
Breit-Wigner formula unsuitable for such a broad resonance !



Finite temperature effects in unitarized chiral amplitudes

$$T_J(s) = \frac{N(s)}{1 + G(s)\; N(s)}$$

- **Temperatures can enter via the G(s)**



$$G(s) = i \int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{1}{(q^2 - m_1^2)[(P-q)^2 - m_2^2]}$$

$$= i \int \frac{\mathrm{d}^3 \vec{q}}{(2\pi)^3} \int \frac{dq_0}{2\pi} \frac{1}{q_0^2 - E_1^2} \frac{1}{(P_0 - q_0)^2 - E_2^2}$$

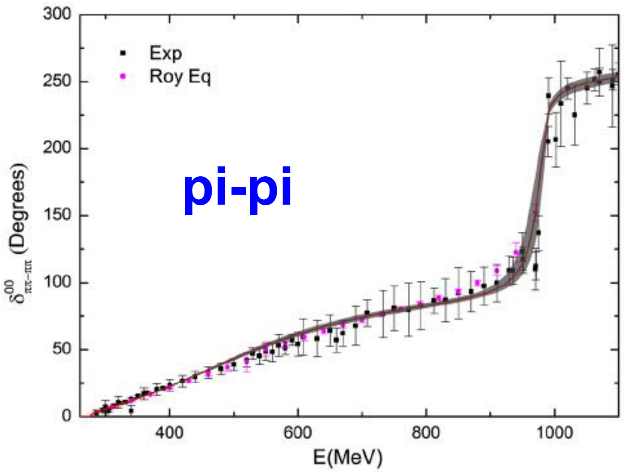
$$q_0 \to i\omega_n = i2\pi n T\,, \quad dq_0 \to i2\pi T$$

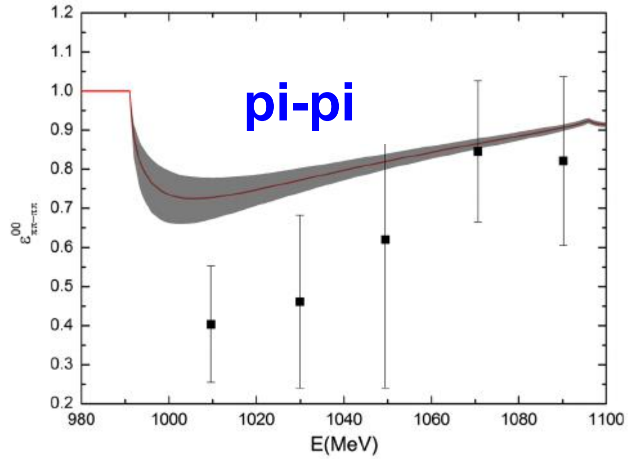
$$= \int \frac{\mathrm{d}^3 \vec{q}}{(2\pi)^3} \sum_{n=-\infty}^{n=+\infty} T \frac{1}{\omega_n^2 + E_1^2} \frac{1}{(P_0 - i\omega_n)^2 - E_2^2}$$

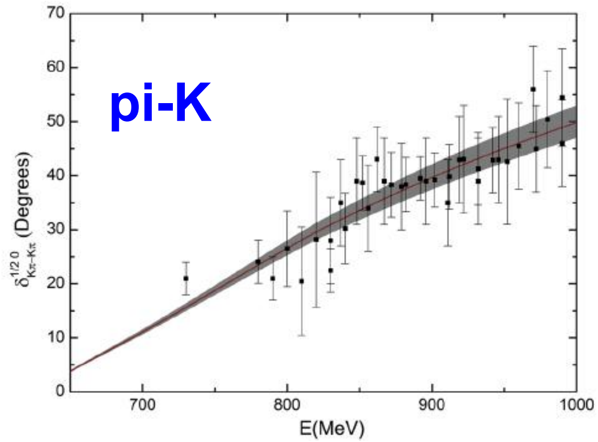
Standard Matsubara techniques to calculate G(s) .

Be careful about the complex extrapolation !

Fits to Exp observables at zero temperature





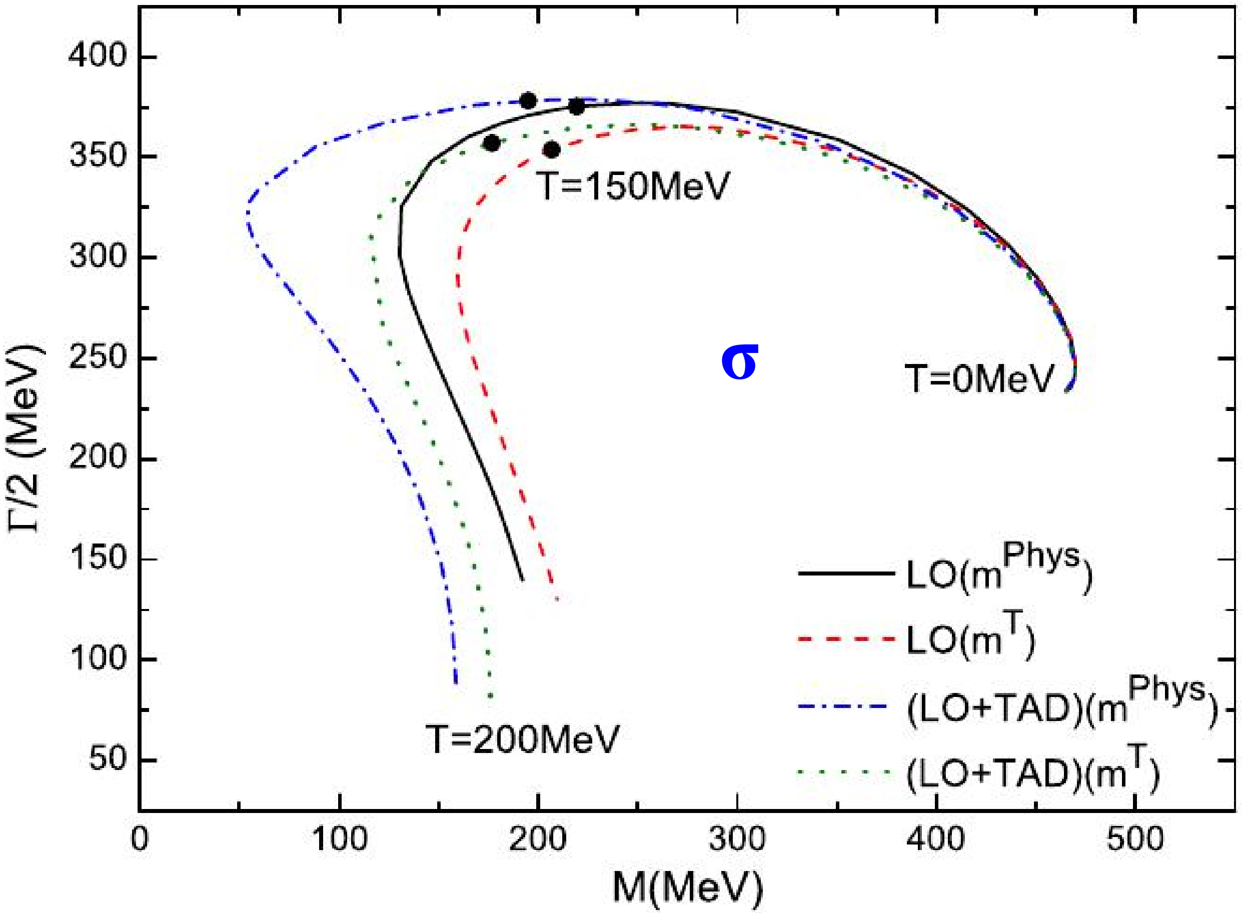


Poles and residues at zero temperature

R	M(MeV)	Width/2(MeV)	$\lvert\gamma_1\rvert$ (GeV)	Ratios	
σ	465^{+1}_{-2}	234^{+8}_{-8}	$3.14^{+0.03}_{-0.03}$	$0.45^{+0.01}_{-0.01}(K\bar{K}/\pi\pi)$	$0.02^{+0.02}_{-0.01}(\eta\eta/\pi\pi)$
				$0.067^{+0.007}_{-0.007}(\eta\eta'/\pi\pi)$	$0.06^{+0.01}_{-0.02}(\eta'\eta'/\pi\pi)$
$f_0(980)$	977^{+6}_{-9}	15^{+5}_{-3}	$1.29^{+0.19}_{-0.15}$	$3.05^{+0.64}_{-0.57}(K\bar{K}/\pi\pi)$	$2.23^{+0.56}_{-0.47}(\eta\eta/\pi\pi)$
				$1.06^{+0.20}_{-0.19}(\eta\eta'/\pi\pi)$	$1.10^{+0.24}_{-0.21}(\eta'\eta'/\pi\pi)$
κ	738^{+8}_{-9}	274^{+8}_{-9}	$4.22^{+0.06}_{-0.07}$	$0.46^{+0.02}_{-0.02}(K\eta/K\pi)$	$0.39^{+0.01}_{-0.02}(K\eta'/K\pi)$
$a_0(980)$	1037^{+17}_{-14}	44^{+6}_{-9}	$3.8^{+0.3}_{-0.2}$	$1.43^{+0.03}_{-0.03}(K\bar{K}/\pi\eta)$	$0.05^{+0.01}_{-0.01}(\pi\eta'/\pi\eta)$

Prediction of the trajectories of sigma resonance when increasing temperatures

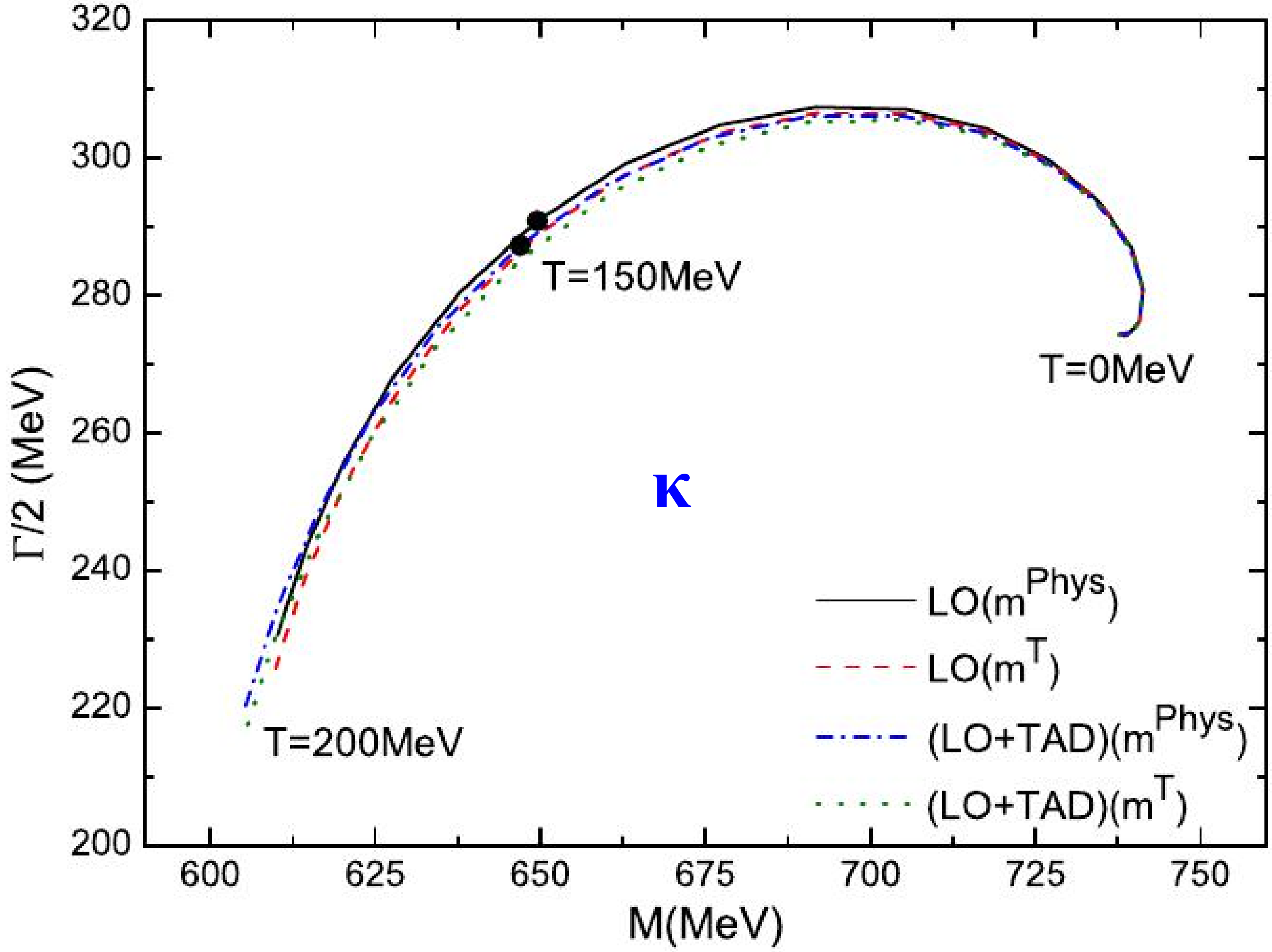
[Gao, ZHG, Pang, PRD'19]

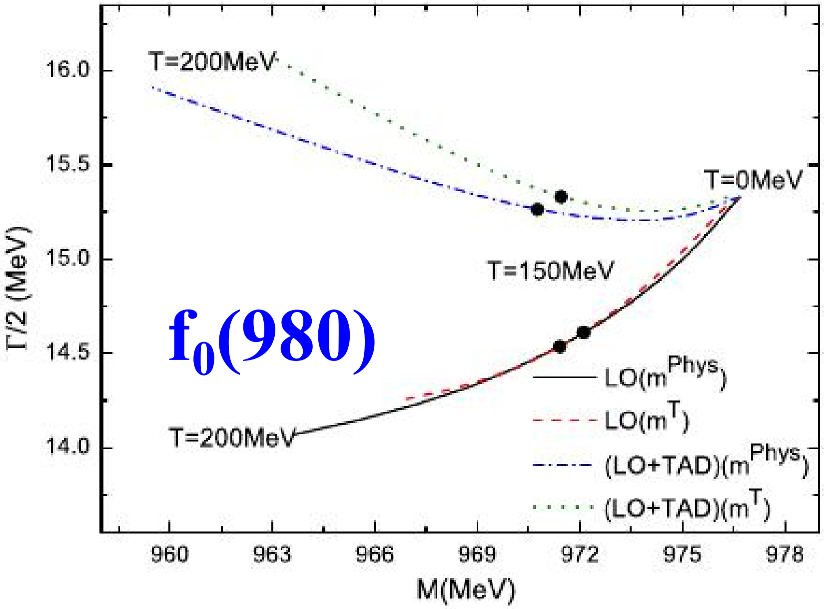


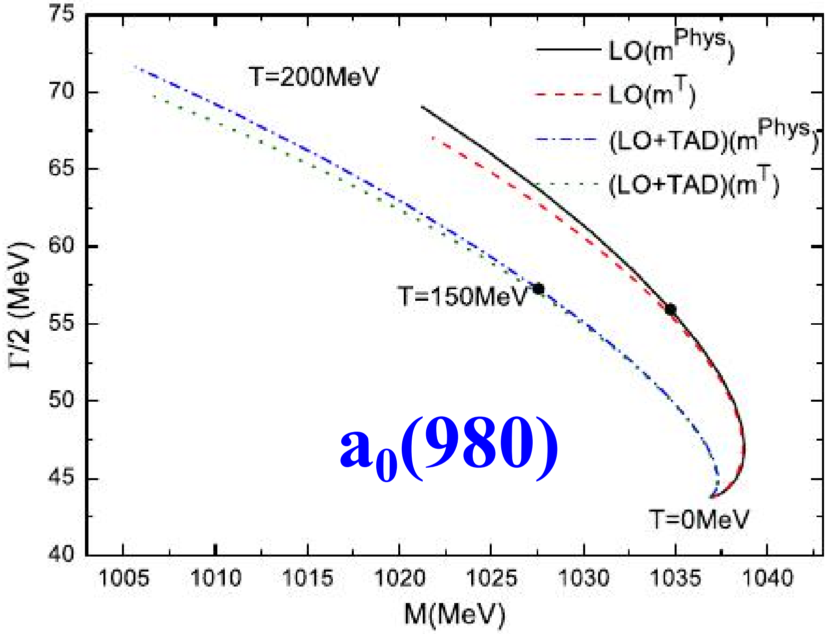
- **It would be very interesint for our lattice colleagues to calculate the pi-pi scattering at finite temperatures**

Kappa resonance at finite temperatures

[Gao, ZHG, Pang, PRD'19]







[Gao, ZHG, Pang, PRD'19]

Conclusions

- The chiral approach illustrated in this talk provides an efficient way to study the finite-volume energy levels.

- It can build a bridge to connect the lattice eigenenergies in finite box obtained at unphysical masses with the physical observables, such as phase shifts, inelasticities, at physical meson masses.

- It is rather encouraging for the lattice community to calculate the scattering processes at finite temperatures! So one can trace the finite-temperature trends of the physical resonances, instead of the toy ones!

Thanks for your attention!