Lattice QCD study of multi-quark states

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Outline

- Introduction
  - Spectroscopy on lattice
  - Scattering on lattice
- Preliminary studies:
  - Tetraquark: T_{cc}^{+}
  - Pentaquarks: P_c
  - Sexaquark: H-dibaryon
- Summary

Introduction

Spectroscopy on lattice:

- Write down an interpolating operator $\mathcal{O}$, e.g. pion operator $\bar{u}\gamma_5 d$
- Compute the correlation function

$$< 0|\mathcal{O}(t)\mathcal{O}(0)^\dagger|0 > = \sum_n \frac{< 0|\mathcal{O}|n > < n|\mathcal{O}|0 >}{2E_n} e^{-E_n t} \longrightarrow \propto e^{-E_0 t}$$

- At large t, fit the correlation function ot an exponential.
- Usually only the ground state can be obtained.

Introduction

Excited state:

✦ build large basis of operators $\{\mathcal{O}_1, \mathcal{O}_2, \cdots\}$ with desired quantum numbers, construct the matrix of correlation function:

$$C_{ij} = < 0 | \mathcal{O}_i \mathcal{O}_j^\dagger | 0 > = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}$$

✦Solve the eigenvalue problem:

$$C_{ij} v_j^n(t) = \lambda_n(t) C_{ij}^0 v_j^n(t)$$

✦Eigenvalues: $\lambda_n(t) \sim e^{-E_n t}(1 + e^{-\Delta E\, t})$

✦Optimal linear combinations of the operators to overlap on the n'th state:

$$\Omega_n = \sum_i v_i^n \mathcal{O}_i$$

Introduction

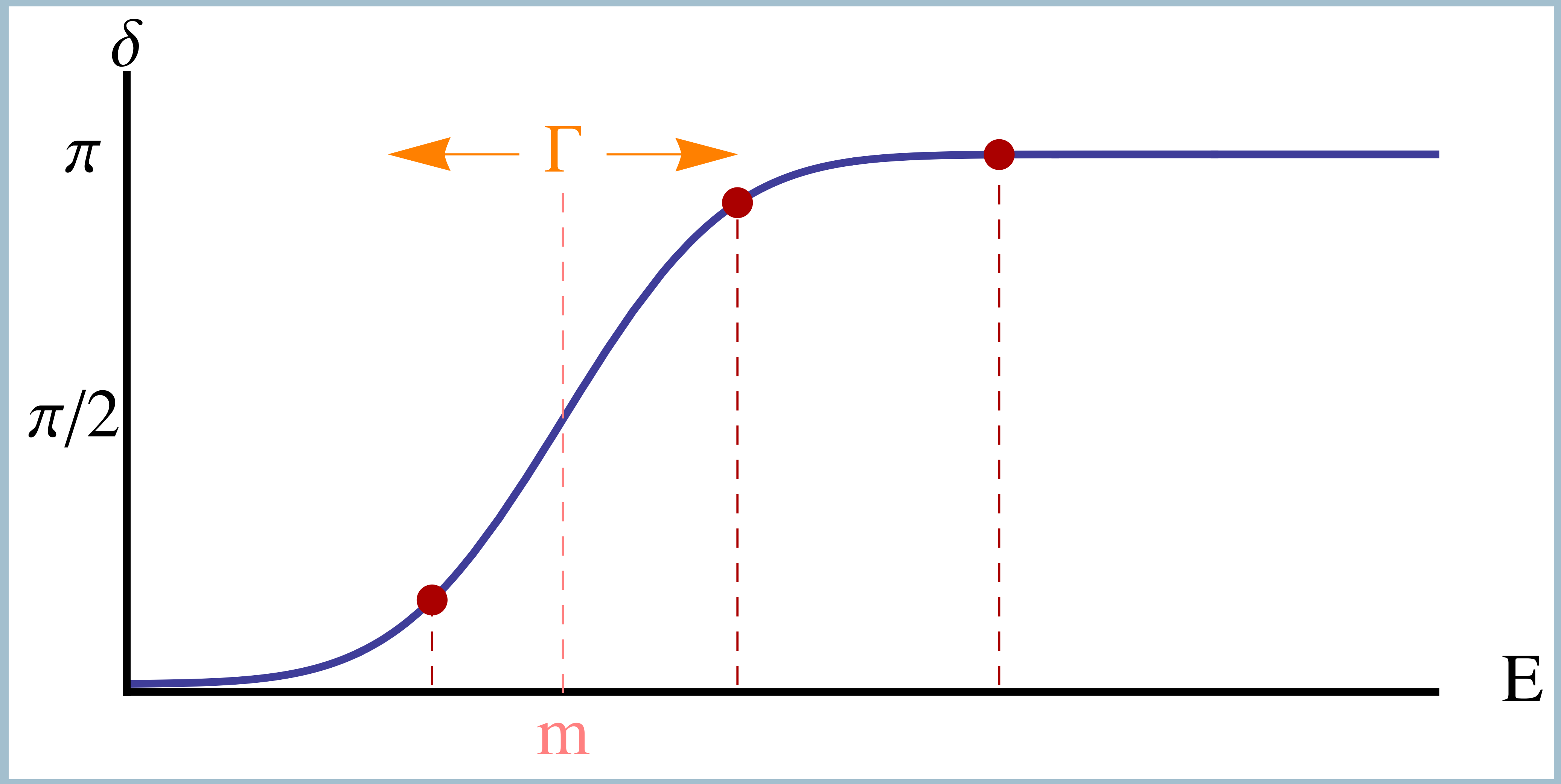
Scattering on lattice: Luscher's finite volume method

Finite volume energies ⟷ Luscher formula ⟷ Infinite volume scattering paremeters

S-wave elastic scattering:

$$p\cot\delta(p) = \frac{2Z_{00}(1;(\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$

$$E = \sqrt{m_1^2 + p^2} + \sqrt{m_2^2 + p^2}$$



Introduction

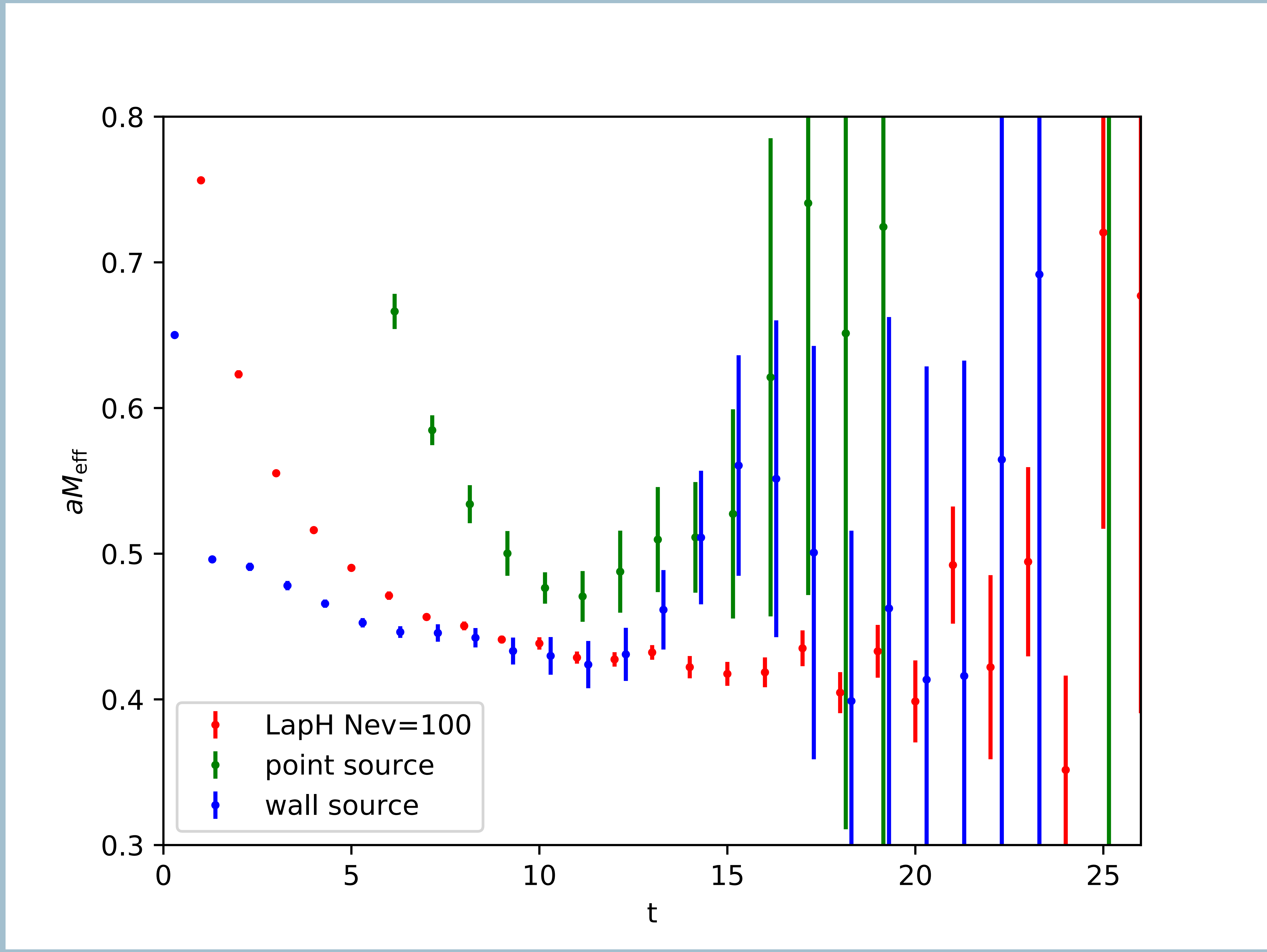
Distillation(LapH) quark smearing:

$$\square = \sum_{k=1}^{M} v_k \otimes v_k^*$$

v_k is the k-th eigenvector of the laplacian operator defined in terms of the gauge configuration.
Dimension $3 \times V \to M$

✦High precision
✦Effective all-to-all propagator
✦Disconnected diagrams
✦Efficient for large basis of operators
✦……

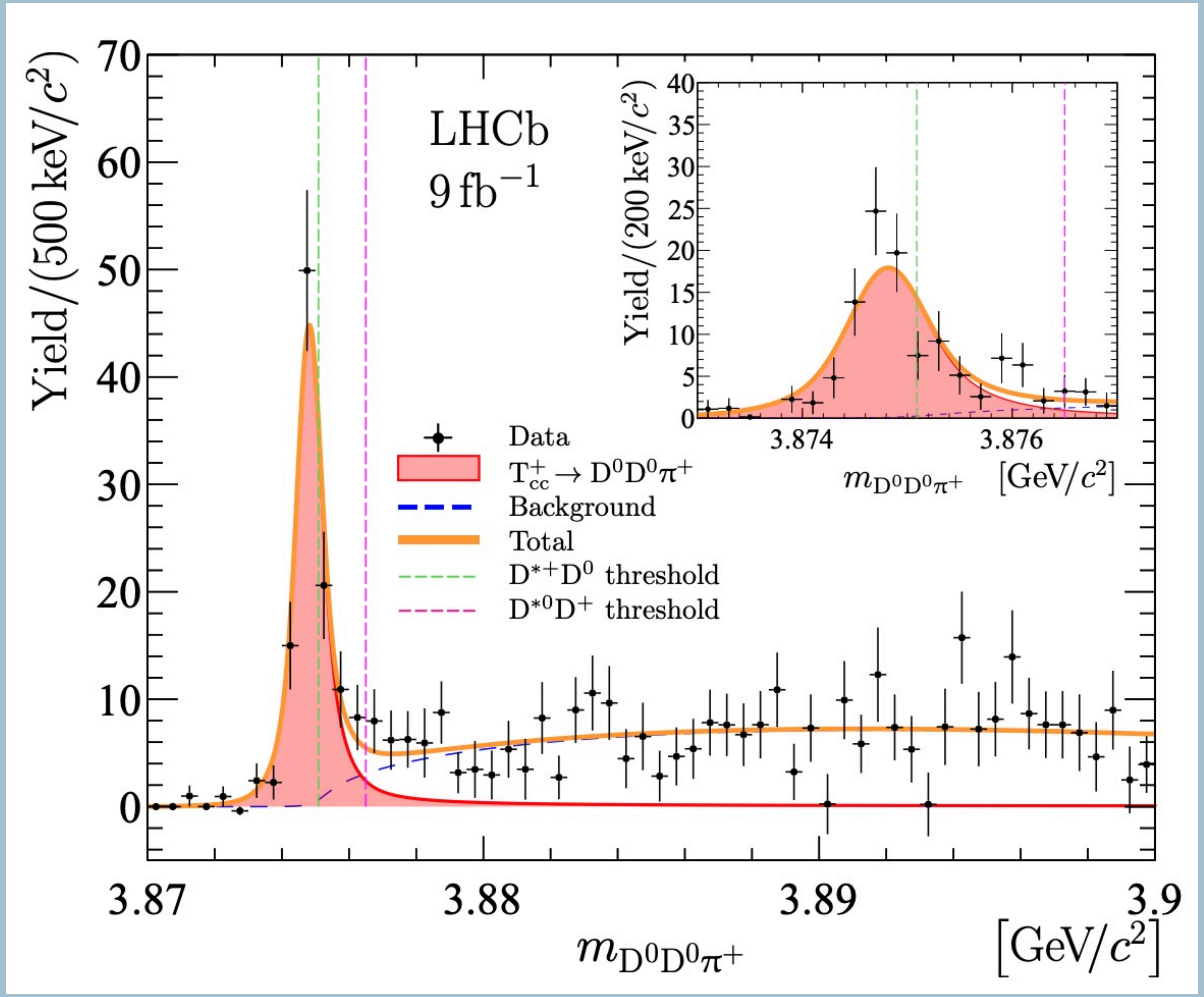
Nucelon effective mass



Lattice setup

- $N_f = 2 + 1$
- Quark action: Wilson clover
- Lattice spacing: 0.08fm
- Volume: $32^3 \times 96$
- $M_\pi = 300\text{MeV}$
- Number of configs: 240

T_{cc}^{+}



$$\delta m = M_{T_{cc}^{+}} - (M_{D^{*+}} + M_{D^0})$$
$$= -361 \pm 40(\text{keV})$$

$$\Gamma = 47.8 \pm 1.9(\text{keV})$$

LHCb collaboration, R. Aaij et al., "Observation of an exotic narrow doubly charmed tetraquark", arXiv:2109.01038.

LHCb collaboration, R. Aaij et al., "Study of the doubly charmed tetraquark T_{cc}^{+}", arXiv:2109.01056.

T_{cc}^{+}

Compact four quark operator:

$$\mathcal{O}_{4\text{quarks}}(\Gamma_1,\Gamma_2)=\epsilon^{ijk}\epsilon^{imn}[c_j^T(C\Gamma_1)c_k][\bar{u}_m(\Gamma_2 C)\bar{d}_n^T]$$

$$\boxed{I(J^P)=0(1^+)}$$

$$\mathcal{O}_1=\mathcal{O}_{4\text{quarks}}(\Gamma_1=\gamma_i,\Gamma_2=\gamma_5)$$

$$\mathcal{O}_2=\mathcal{O}_{4\text{quarks}}(\Gamma_1=\gamma_i\gamma_4,\Gamma_2=\gamma_4\gamma_5)$$

$$\mathcal{O}_3=D^0_{(p=0)}D^{*+}_{(p=0)}-D^+_{(p=0)}D^{*0}_{(p=0)}$$

$$\mathcal{O}_4=D^0_{(p=1)}D^{*+}_{(p=-1)}-D^+_{(p=1)}D^{*0}_{(p=-1)}$$

$$\boxed{I(J^P)=1(1^+)}$$

$$\mathcal{O}_1=\mathcal{O}_{4\text{quarks}}(\Gamma_1=\gamma_4,\Gamma_2=\epsilon^{ijk}\gamma_j\gamma_k)$$

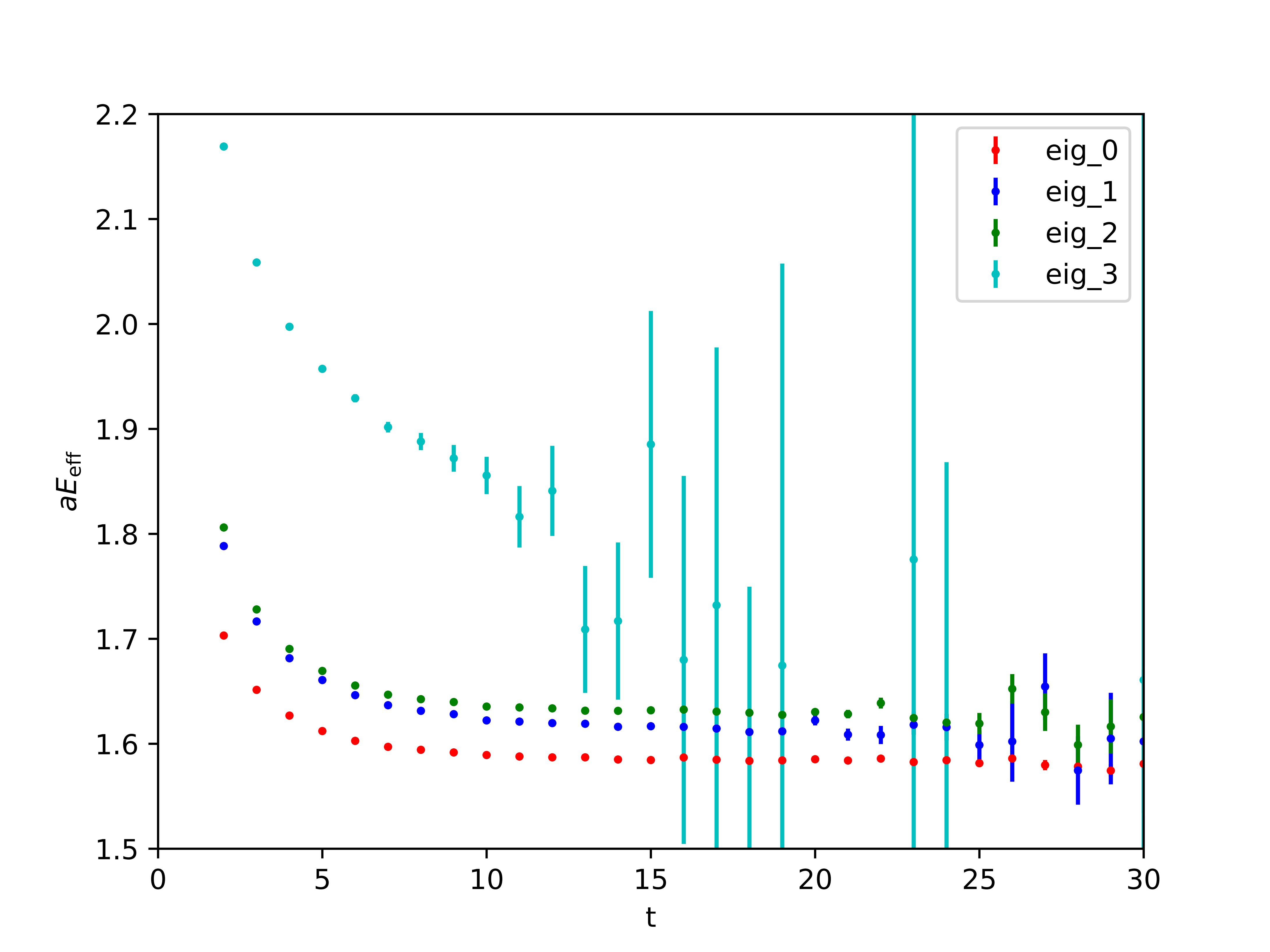
$$\mathcal{O}_1=\mathcal{O}_{4\text{quarks}}(\Gamma_1=\epsilon^{ijk}\gamma_j\gamma_k,\Gamma_2=\gamma_4)$$

$$\mathcal{O}_3=D^0_{(p=0)}D^{*+}_{(p=0)}+D^+_{(p=0)}D^{*0}_{(p=0)}$$

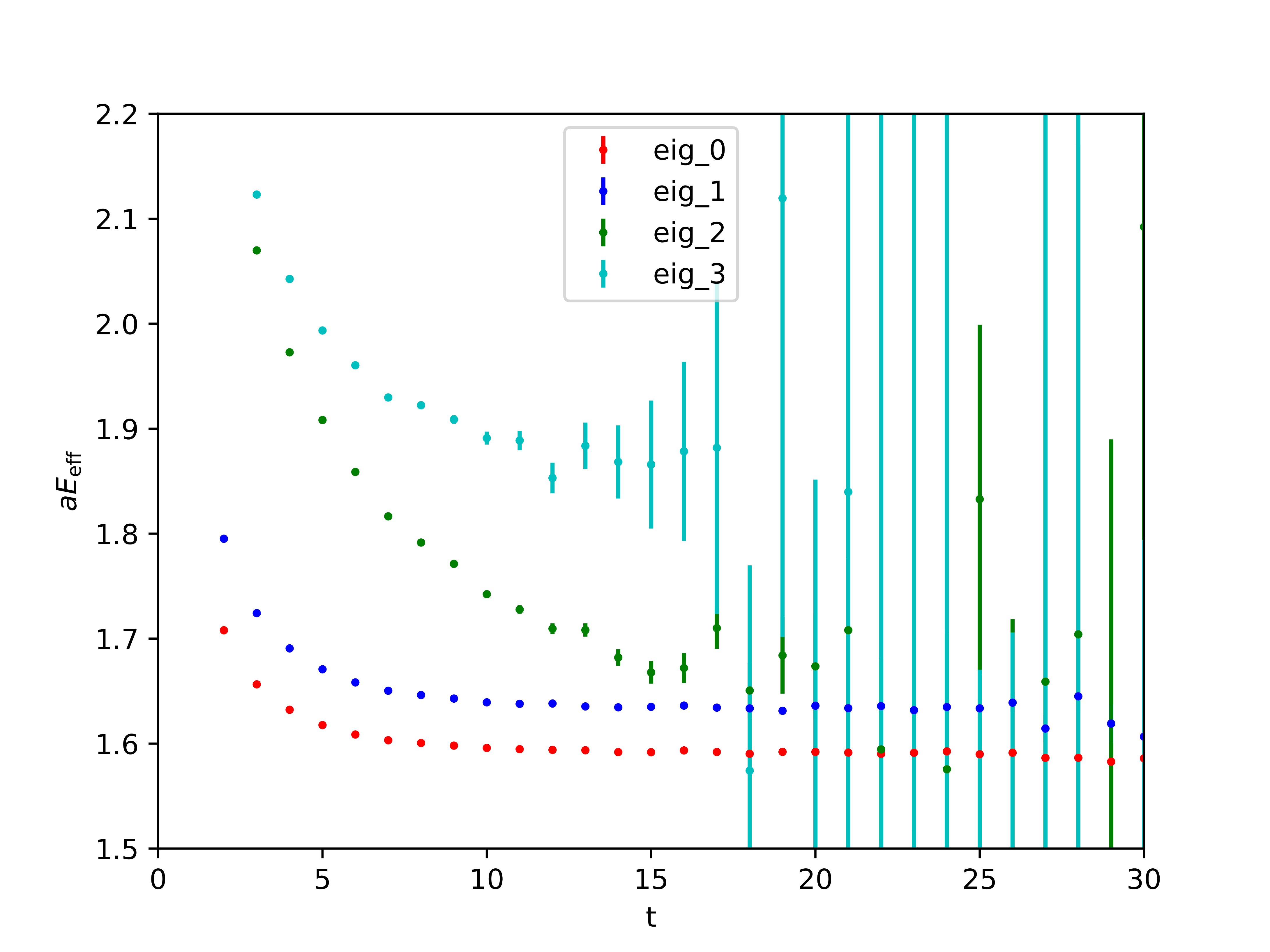
$$\mathcal{O}_4=D^0_{(p=1)}D^{*+}_{(p=-1)}+D^+_{(p=1)}D^{*0}_{(p=-1)}$$

T_{cc}^{+}

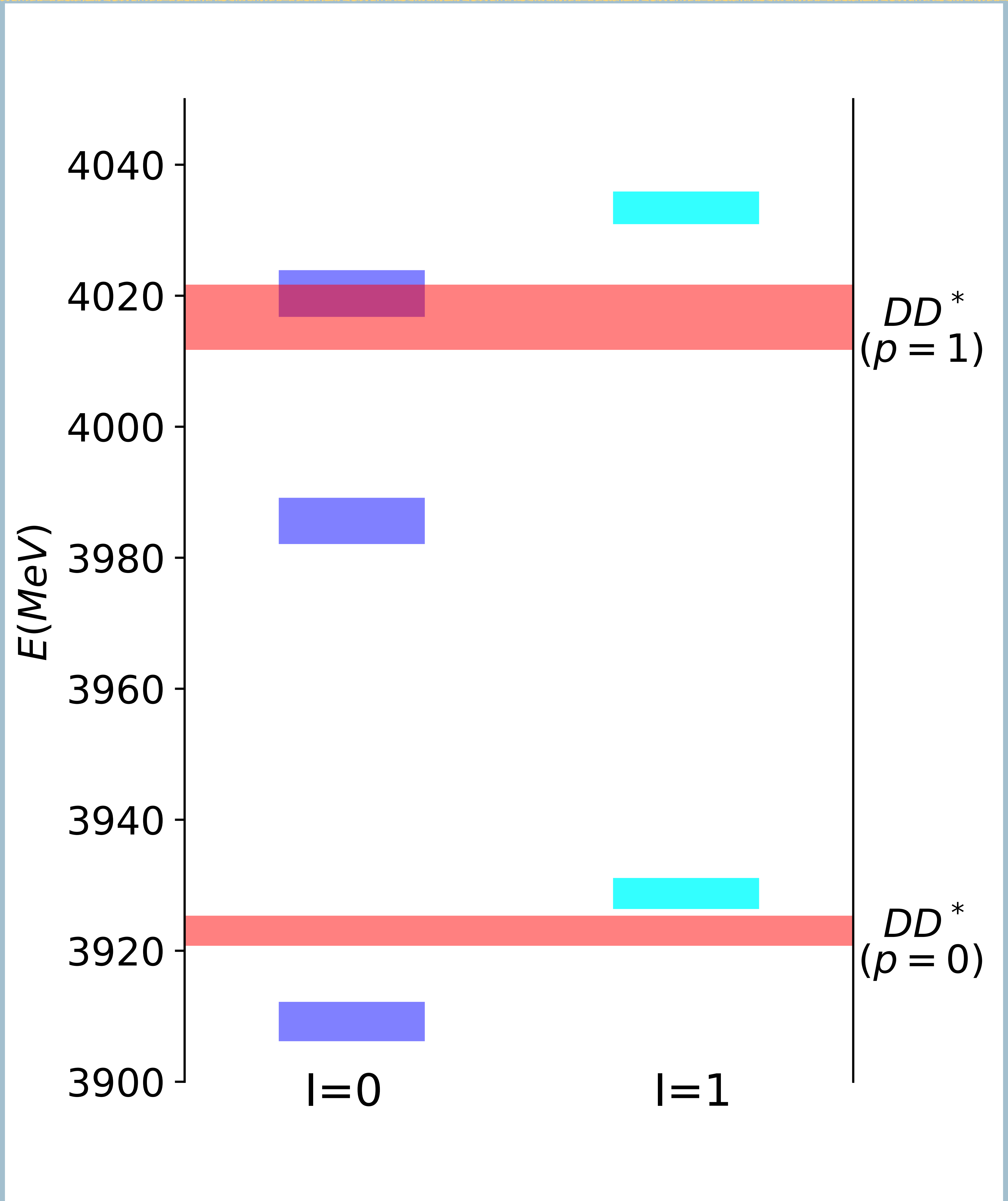
$I(J^P) = 0(1^+)$



$I(J^P) = 1(1^+)$



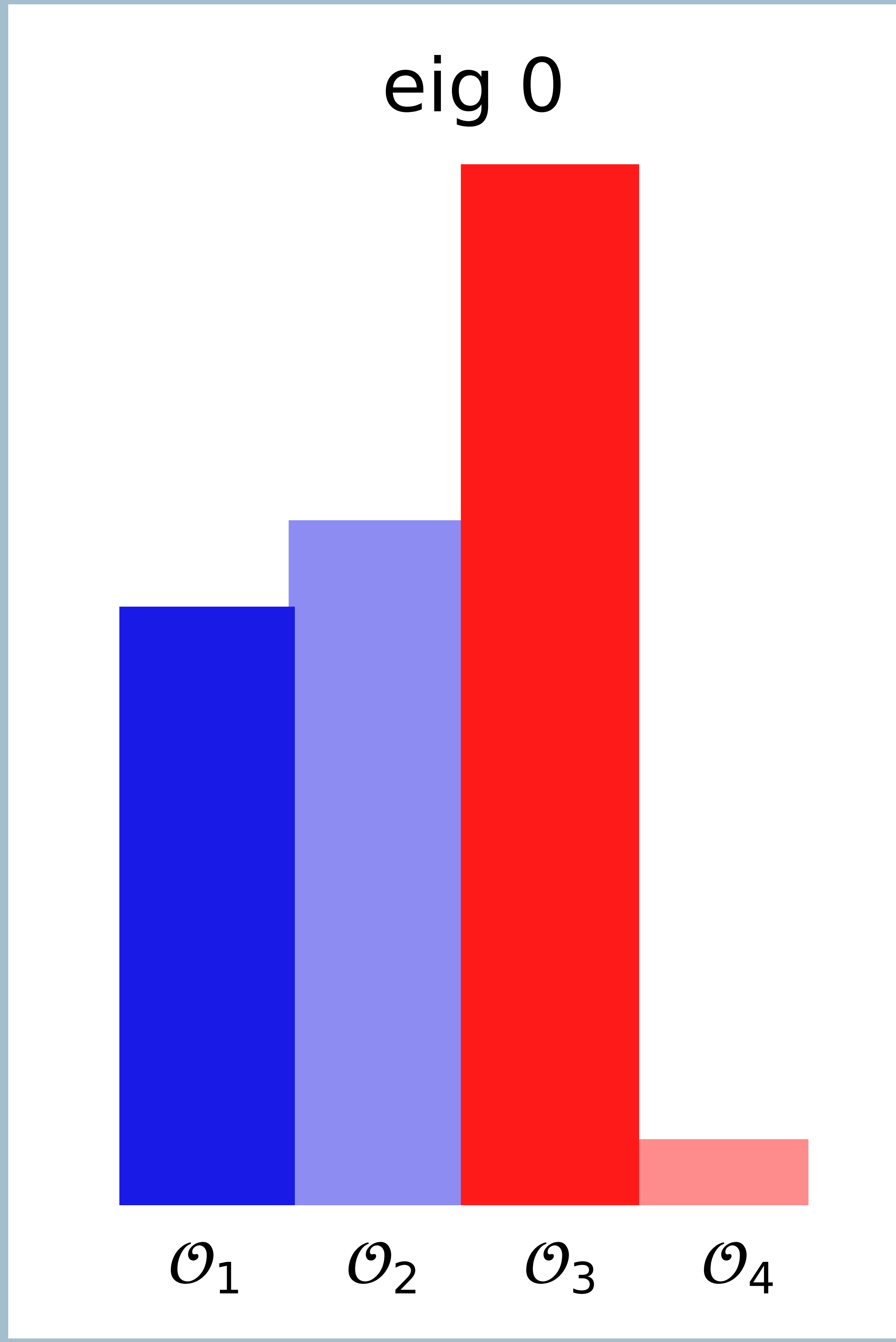
T_{cc}^{+}

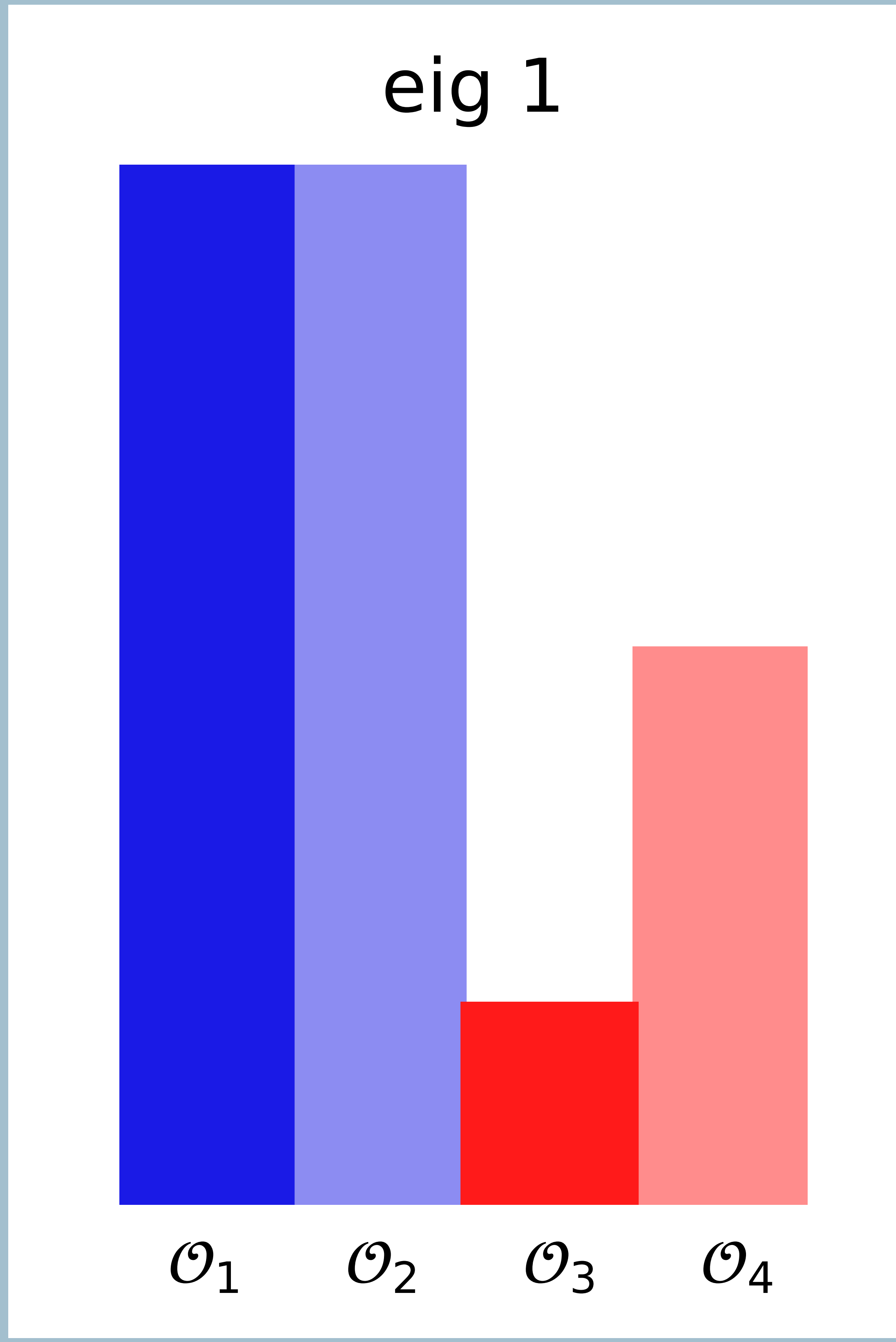


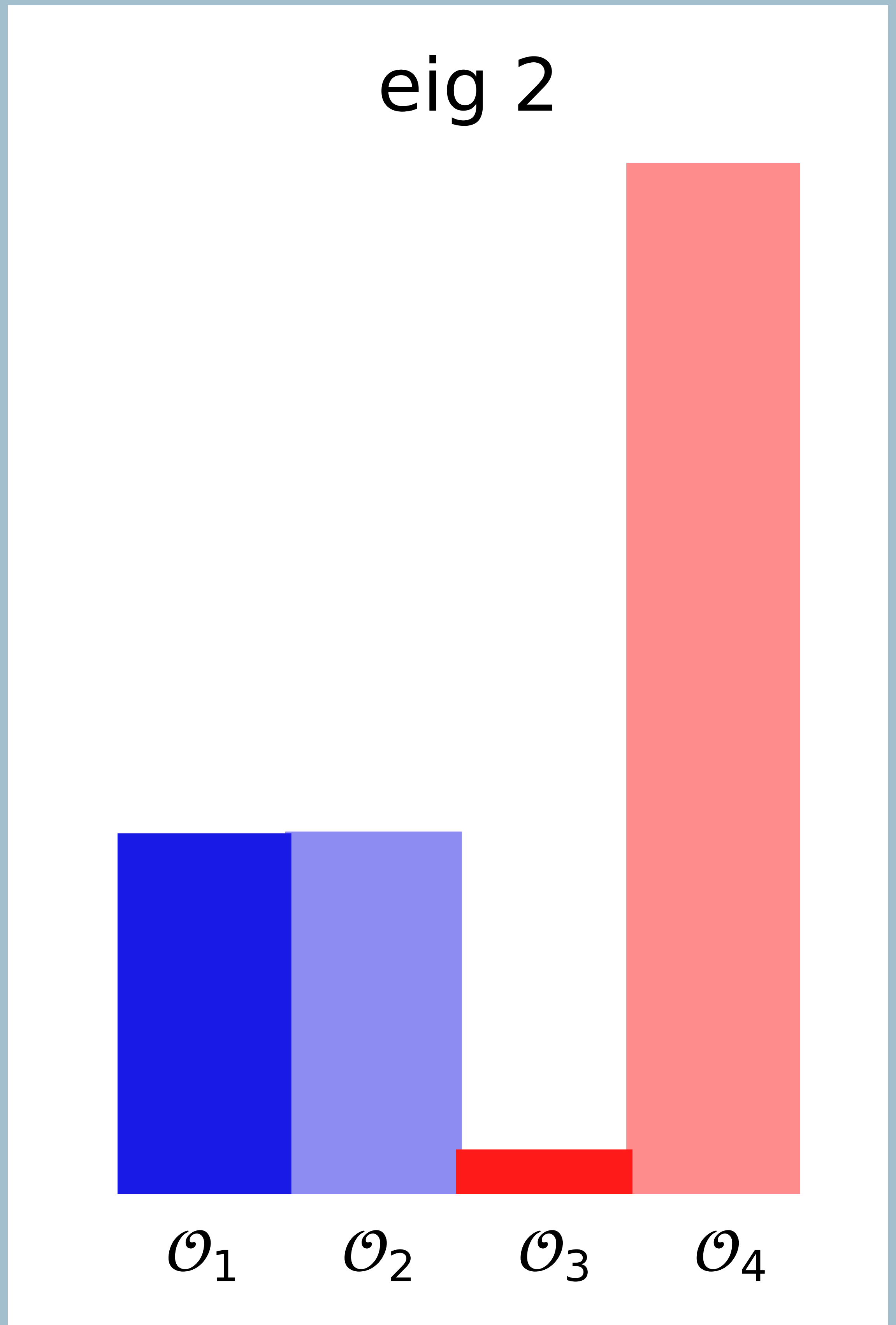
T_{cc}^{+}

$$I(J^P) = 0(1^+)$$

Overlap of the energy eigenstates to the interpolating operators:







T_{cc}^{+}

Luscher's formula:

$$p cot\delta(p) = \frac{2Z_{00}(1;(\frac{pL}{2\pi})^2)}{L\sqrt{\pi}}$$

Effective range expansion:

$$p cot\delta(p) = \frac{1}{a_0} + \frac{1}{2}r_0 p^2 + \cdots$$

Scattering length:

$$a_0 = -4(2)\text{fm}$$

Scattering amplitude(above threshold):

$$T^{-1} \propto p cot\delta(p) - ip = \frac{2Z_{00}}{L\sqrt{\pi}} - ip$$

Analytically extrapolate to below threshold:

$$T^{-1} \propto \frac{2Z_{00}}{L\sqrt{\pi}} + |p|$$

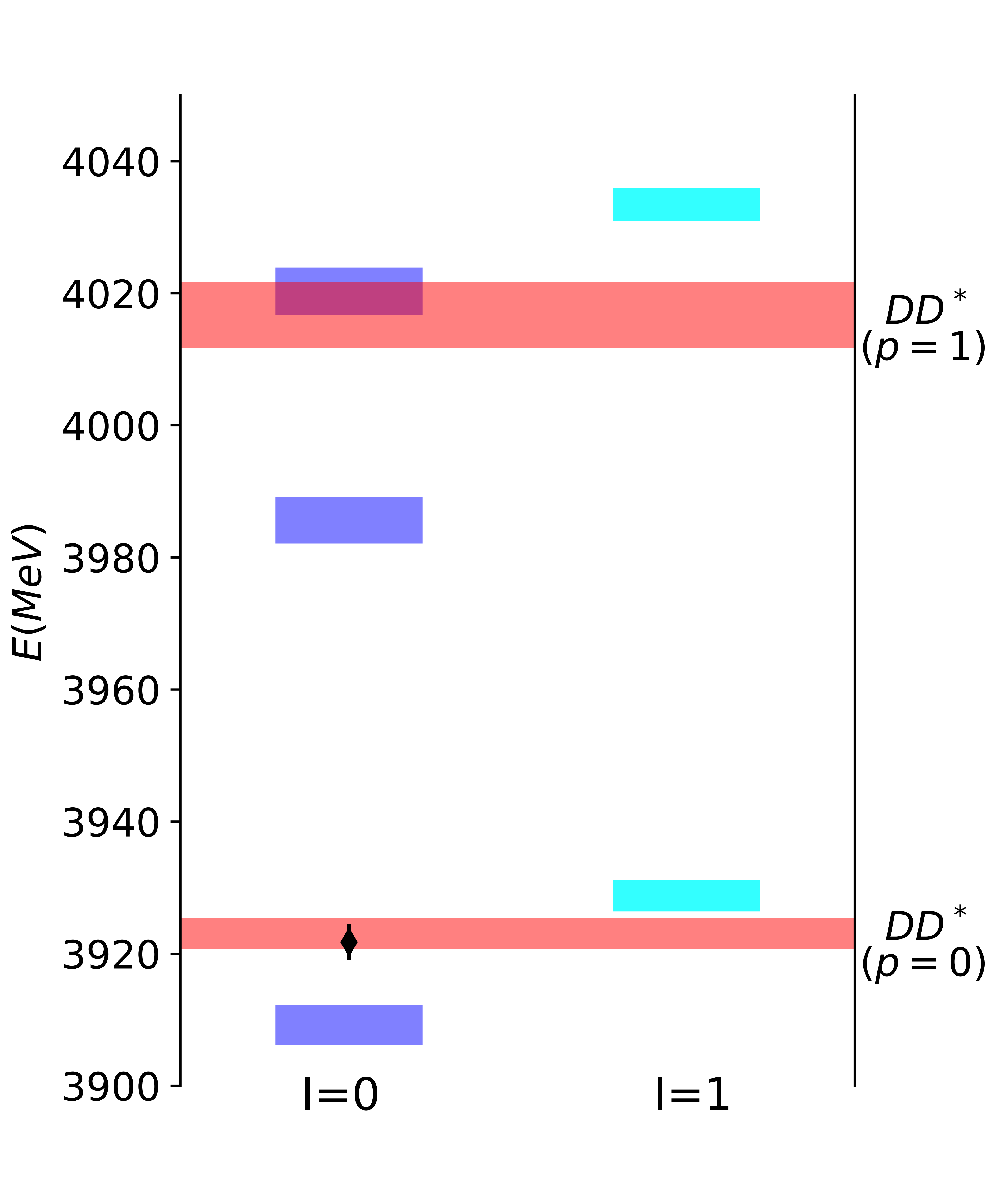
Bound state pole condition:

$$\frac{1}{a_0} + \frac{1}{2}r_0 p_B^2 = -|p_B|$$

Pole position at infinite volume:

$$E_B = 3922(3)\text{MeV}$$

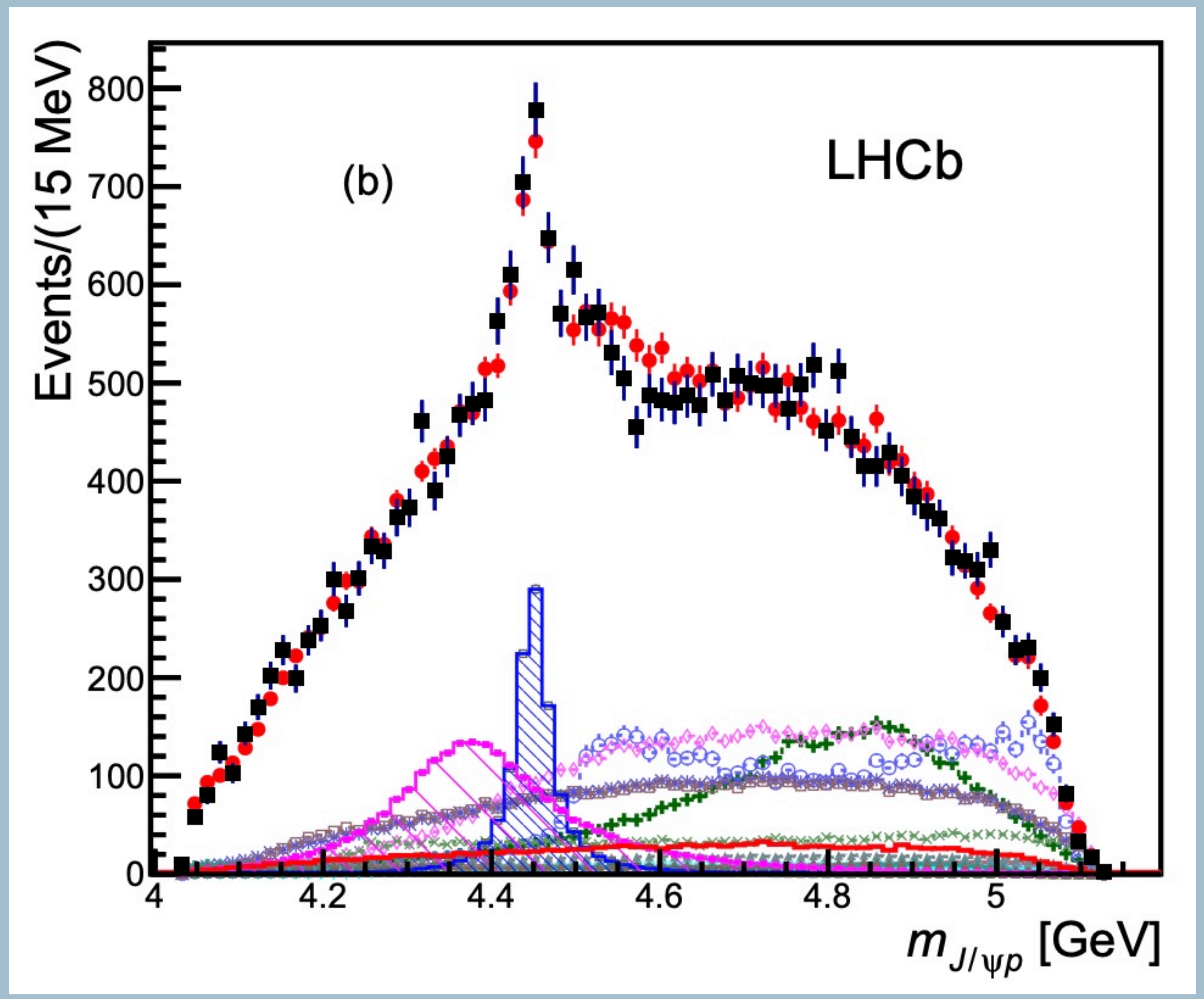
T_{cc}^+

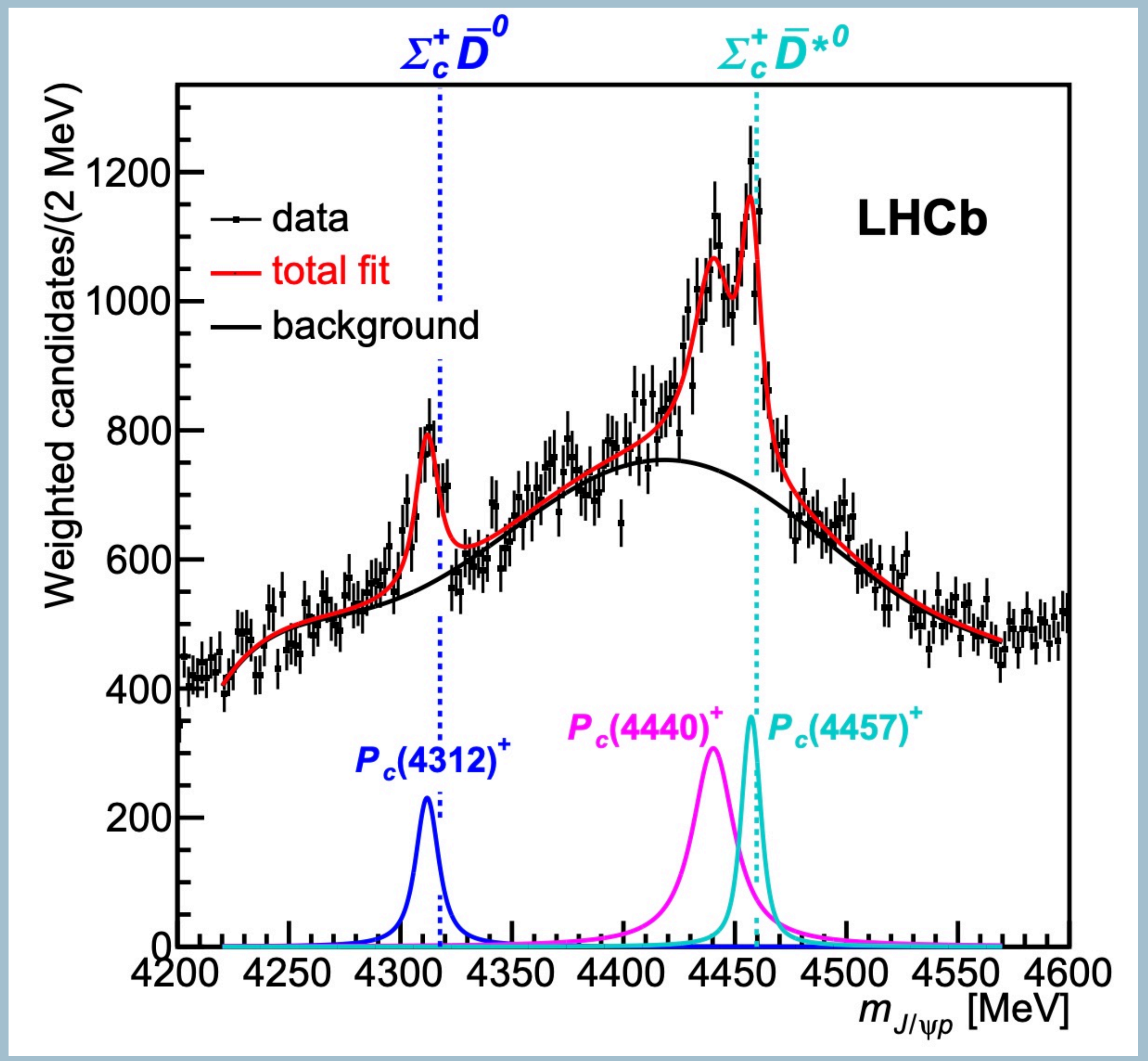


Pentaquarks

LHCb, Phys. Rev. Lett. 115, 072001 (2015)

LHCb, Phys. Rev. Lett. 122, 222001 (2019)





$P_c(4380)$ $\quad$ $P_c(4312)$

$P_c(4450) \rightarrow P_c(4440)$

$P_c(4450) \rightarrow P_c(4457)$

$P_c(4312)$:

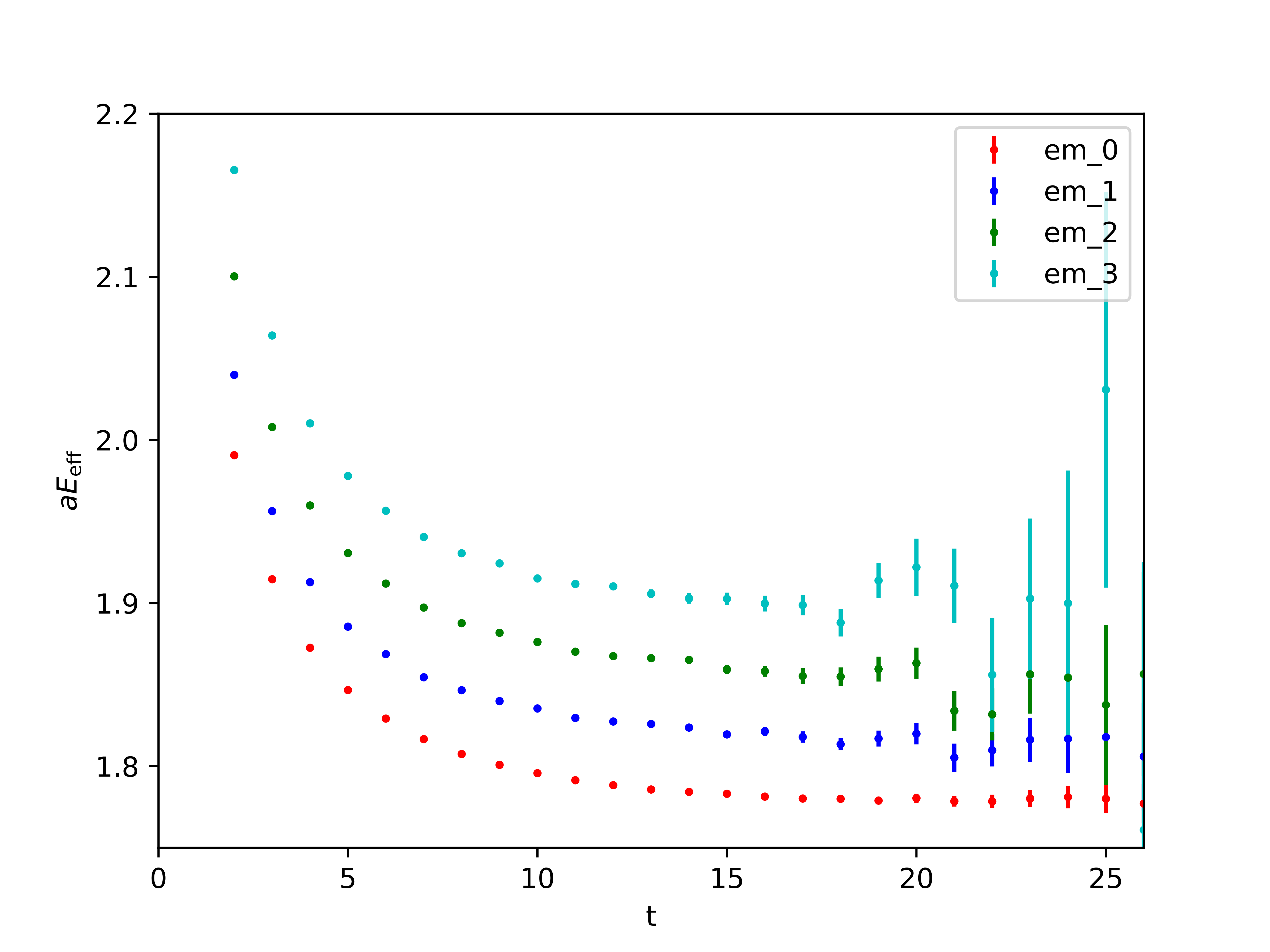
$\Sigma_c \bar{D}$ bound state

$P_c(4440)$ and $P_c(4457)$:

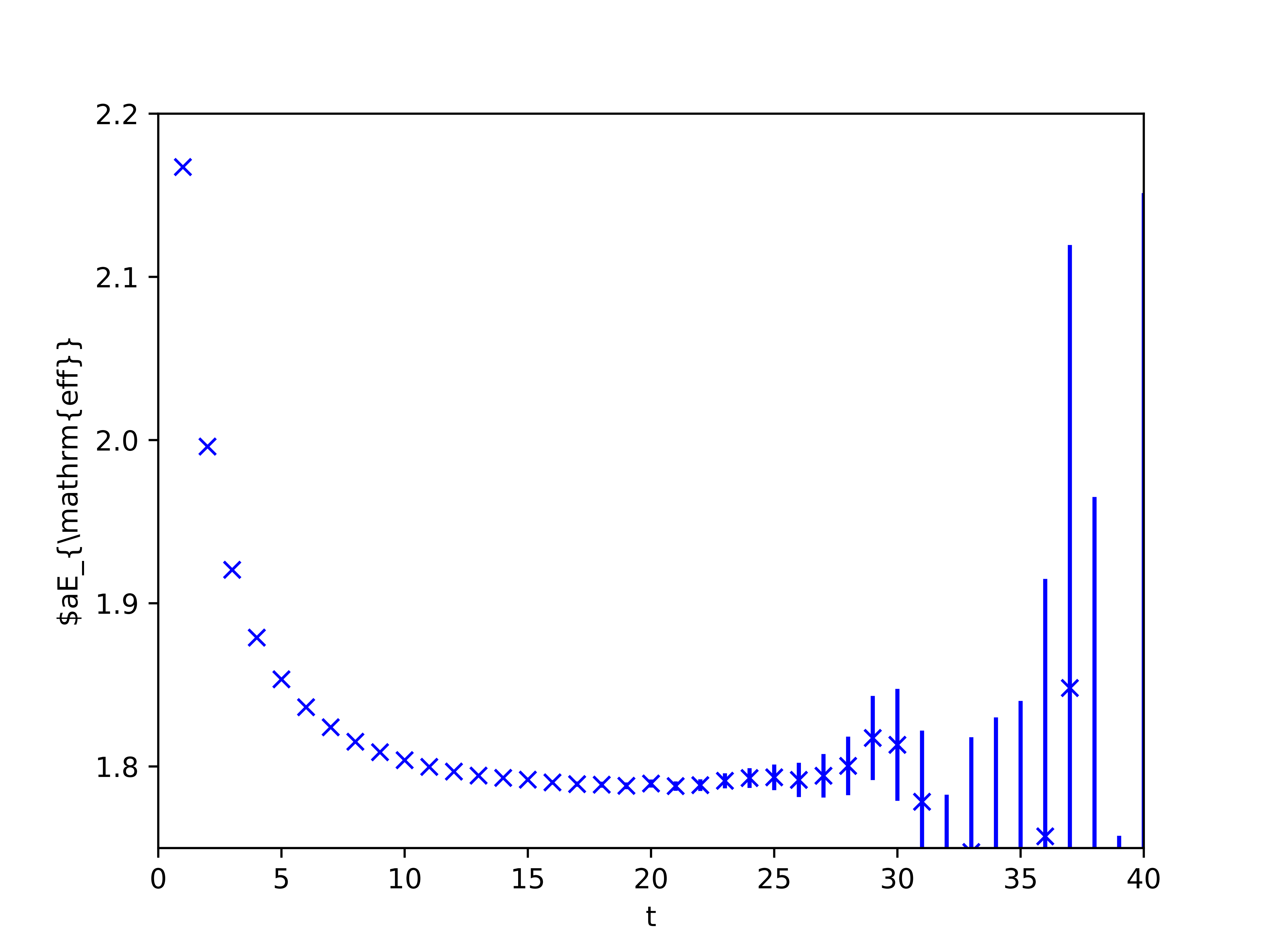
$\Sigma_c \bar{D}^*$ bound state

Pentaquarks - $\Sigma_c \bar{D}$

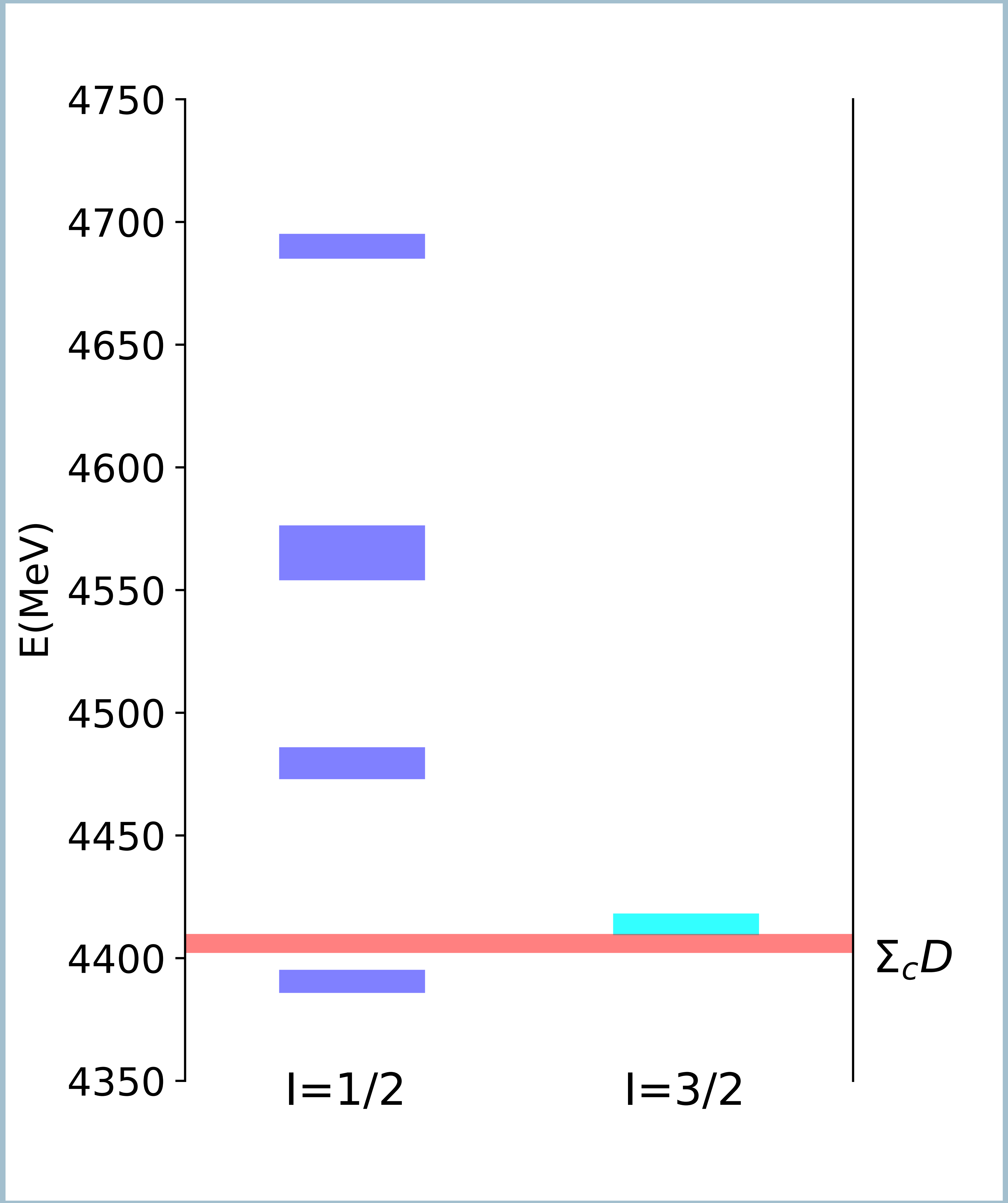
$$I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$$



$$I(J^P) = \frac{3}{2}(\frac{1}{2}^-)$$



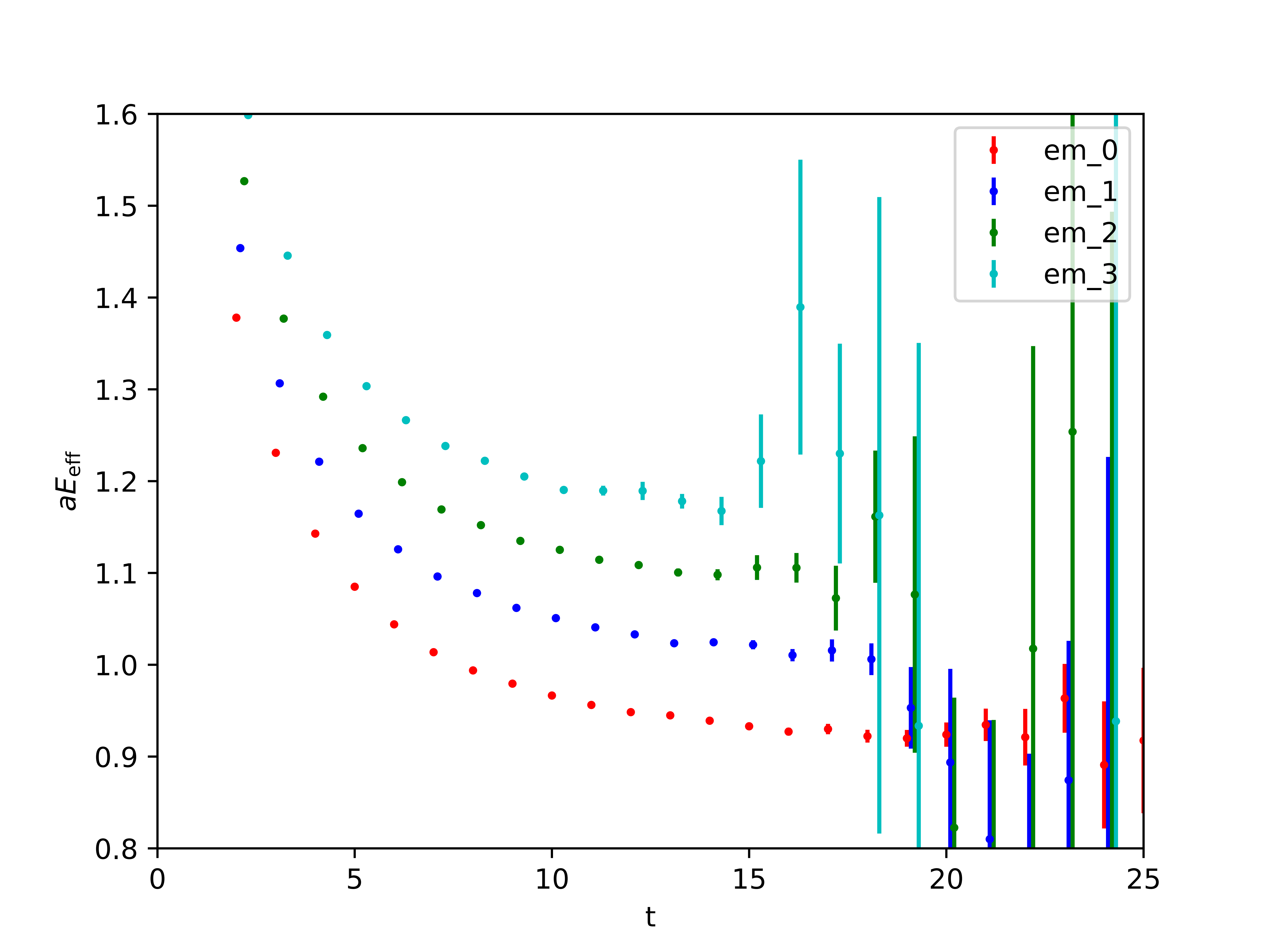
Pentaquarks - $\Sigma_c\bar{D}$



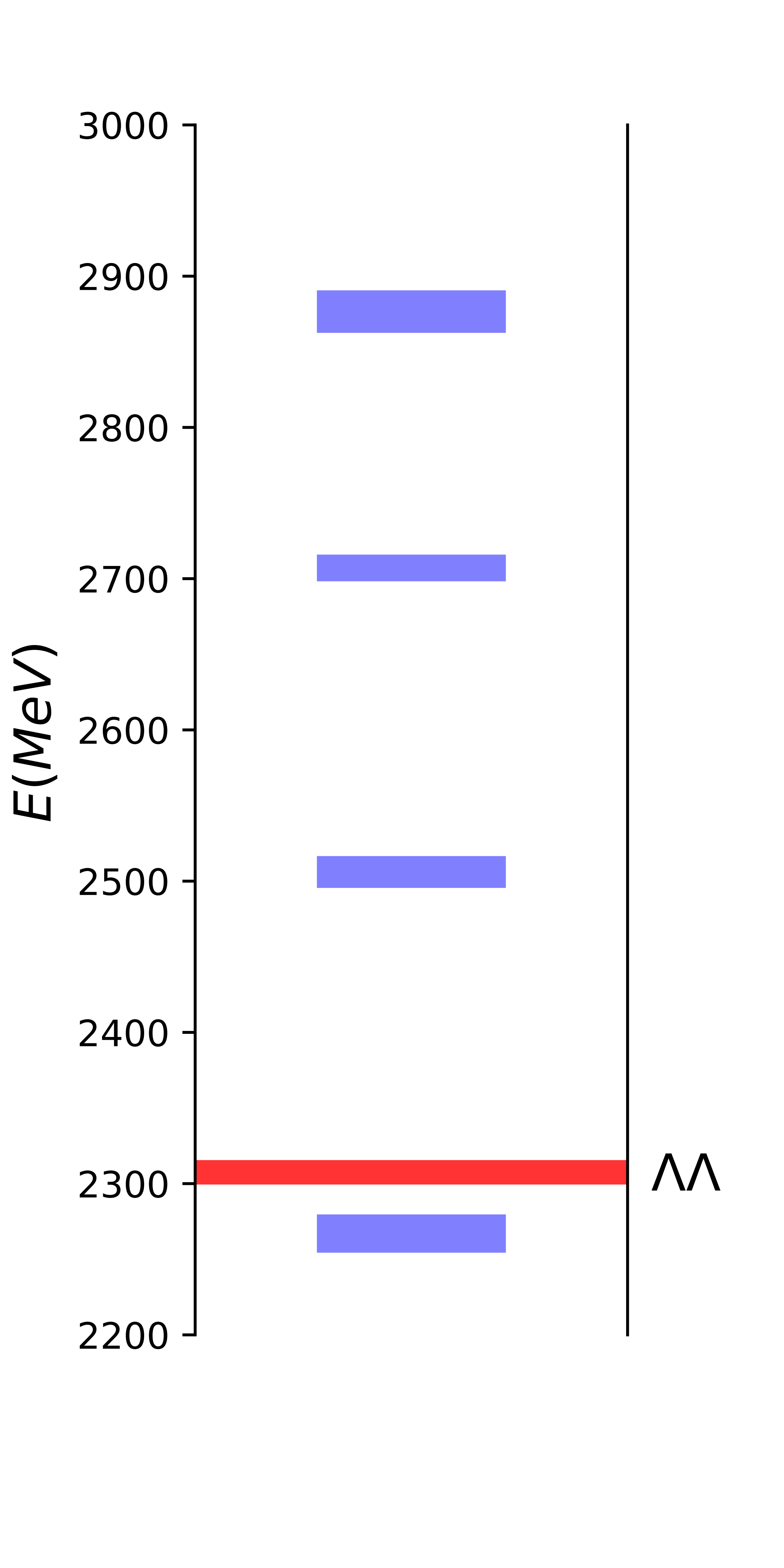
H-dibaryon

- Theoretical prediction of a deeply bound di-baryon with quark content uuddss.
- No solid experimental evidence.
- Controvertial lattice results.
  - arXiv: 2108.09644(HALQCD), weekly attractive without a bound state.
  - arXiv: 1912.08630(HALQCD), virtual state.
  - arXiv:1805.03966, bound state with binding energy 19(10)MeV
  - arXiv:1109.2889(NPLQCD), bound state with binding energy 13.2(1.8)(4.0)MeV
  - arXiv:1012.5968(HALQCD), bound state with binding energy 30-40MeV
  - ……

H-dibaryon

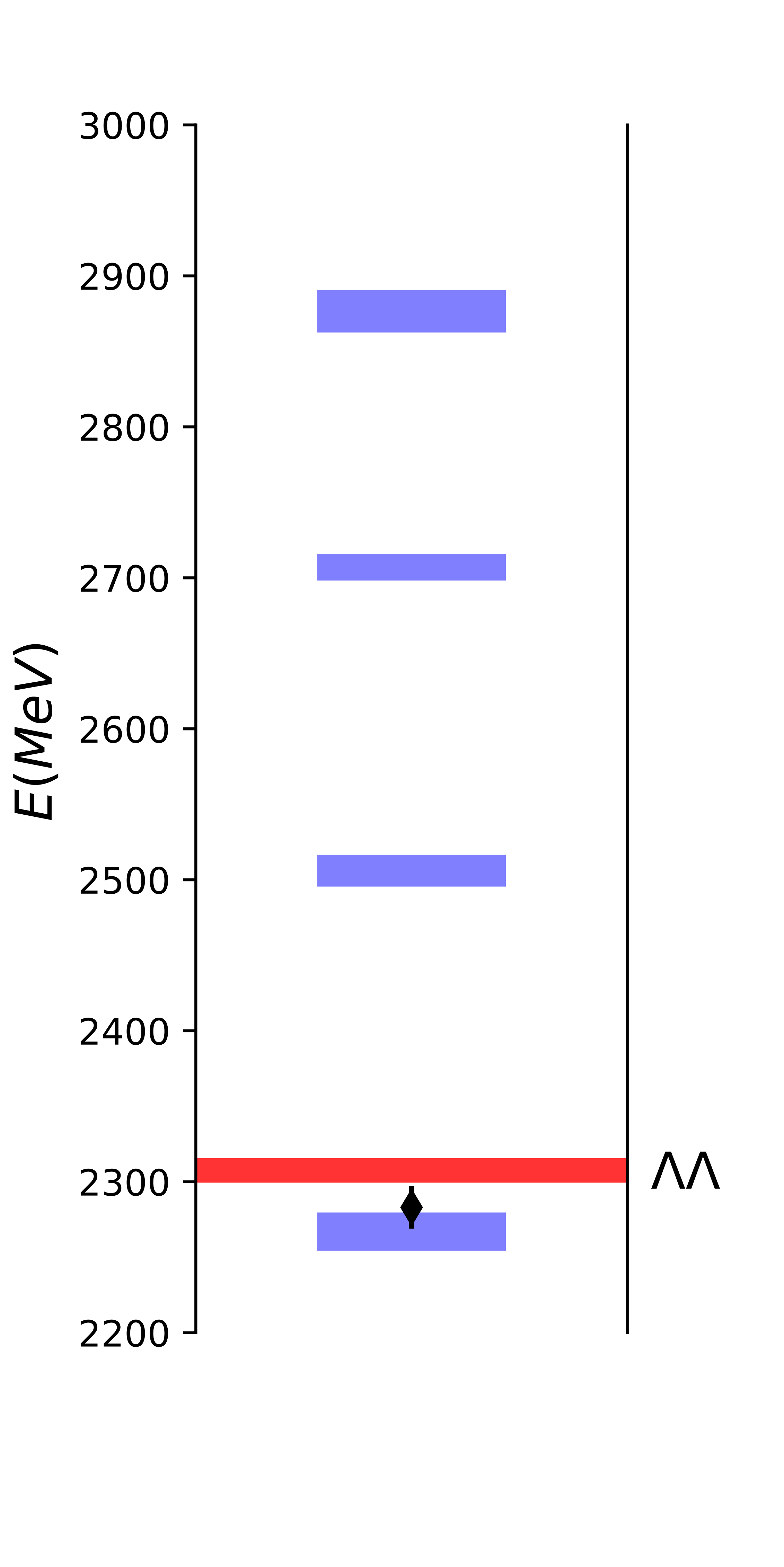


1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
aE_{eff}
0
5
10
15
20
25
t
em_0
em_1
em_2
em_3



3000
2900
2800
2700
2600
2500
2400
2300
2200
$E(MeV)$
ΛΛ

H-dibaryon



Scattering length: -1.2(0.3)fm

Binding energy: 24(12)MeV

Configurations

Lattice spacing	Volume($L^3 \times T$)	M_π (MeV)	# of trajectories
~0.108fm	$24^3 \times 72$	290	25000
	$32^3 \times 64$	290	15000
	$32^3 \times 64$	220	15000
	$48^3 \times 96$	220	1000
	$48^3 \times 96$	120	1000
	$48^3 \times 96$	145	700
~0.080fm	$32^3 \times 96$	300	11000
	$48^3 \times 96$	210	1000
~0.055fm	$48^3 \times 144$	360	650
	$48^3 \times 144$	300	900

孙鹏，11月1日下午报告

南方核科学计算中心（华南师大+近物所）
理论所计算集群

Summary

- Preliminary results of T_{cc}^{+}, $\Sigma_c\bar{D}$, and $\Lambda\Lambda$ are presented.

- We are ready for general extensive study of hadron spectrum and exotic states.

致谢:

梁剑、桂龙成、邢瀚洋、刘航、王伟 ......