Charm meson decay constants from lattice QCD

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Outline

- **Motivation**
- **Pseudoscalar mesons**
- **Vector mesons**
- **Renormalization**
- **Summary**

χQCD, arXiv:1312.7628, 1410.3343, 1710.08678, 2008.05208

Charm physics and LQCD

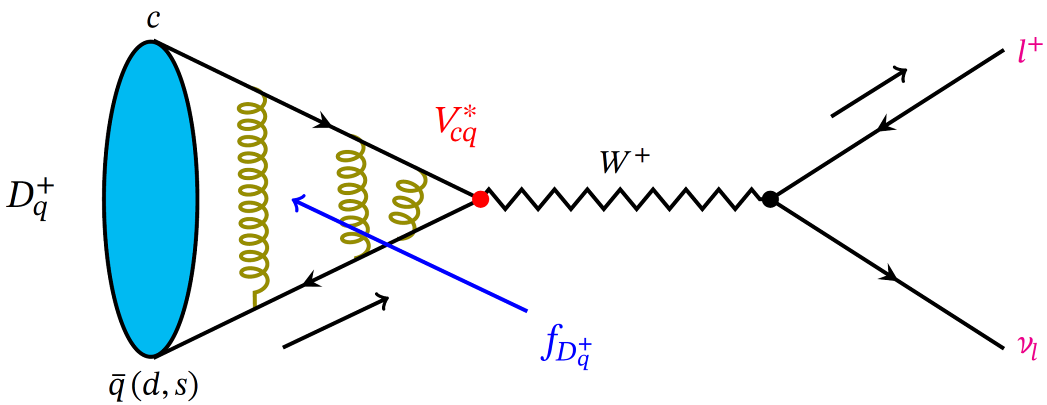
- **LQCD can calculate form factors and meson decay constants appearing in weak decays of hadrons**
- **Combined with experiments, they can give us CKM matrix elements**
- **Test the SM (is CKM unitary?)**
- **Or use V_{ab} from elsewhere to compare QCD/SM results with experiments**

$$\begin{pmatrix} \mathbf{V_{ud}} & \mathbf{V_{us}} & \mathbf{V_{ub}} \\ \pi \to \ell \nu & K \to \ell \nu & B \to \pi \ell \nu \\ & K \to \pi \ell \nu & \\ \mathbf{V_{cd}} & \mathbf{V_{cs}} & \mathbf{V_{cb}} \\ D \to \ell \nu & D_s \to \ell \nu & B \to D \ell \nu \\ D \to \pi \ell \nu & D \to K \ell \nu & B \to D^* \ell \nu \\ \mathbf{V_{td}} & \mathbf{V_{ts}} & \mathbf{V_{tb}} \\ B_d \leftrightarrow \overline{B}_d & B_s \leftrightarrow \overline{B}_s & \end{pmatrix}$$

from LQCD

For example: $\Gamma(P \to \ell \nu) = \frac{G_F^2 |V_{q_1 q_2}|^2}{8\pi} f_P^2 m_\ell^2 M_P (1 - \frac{m_\ell^2}{M_P^2})^2$

赝标粲介子衰变常数



图片摘自
arXiv:2103.00908

- **定义**

$f_{D_{(s)}}$

$$\langle 0|\bar{q}(0)\gamma_\mu\gamma_5 c(0)|P(p)\rangle = i f_P p_\mu, \quad q = d, s$$

- **利用部分守恒轴矢量流关系（PCAC）$\partial_\mu A^\mu = (m_q + m_c)P$，可得：**

$$(m_q + m_c)\langle 0|\bar{q}(0)\gamma_5 c(0)|P(p)\rangle = f_P m_{PS}^2$$

使用手征格点费米子可避免重正化常数的计算

矢量粲介子衰变常数

- **Definition**

f_V $$\langle 0|\bar{q}(0)\gamma^\mu q'(0)|V(p,\lambda)\rangle = f_V m_V e_\lambda^\mu$$

f_V^T $$\langle 0|\Big(\bar{q}(0)\sigma^{\mu\nu}q'(0)\Big)(\mu)|V(p,\lambda)\rangle = i f_V^T(\mu)\big(e_\lambda^\mu p^\nu - e_\lambda^\nu p^\mu\big)$$

- **It is not easy to measure f_V**
  - **Leptonic decay BRs are much smaller than those of strong decays**
- **Test the accuracy of HQET,**

$$f_V/f_{PS} = 1 + O(1/m_Q)$$

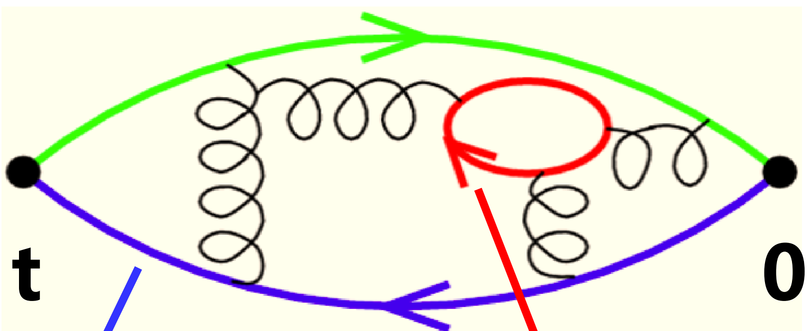
- f_V^T/f_V **for D^* and D_s^* can be used as inputs for LCSR in calculations of $B \to V$ form factors at low q^2**
- **Input parameters for QCD factorization in studies of nonleptonic B decays, e.g., $B \to D^{(*)}M$**

两点函数

- Let $O = \bar{q}\gamma_0\gamma_5 c$, its matrix element between the vacuum and a ground state hadron can be obtained from

$$C(t) = \langle\Omega|O(t)O^{\dagger}(0)|\Omega\rangle \xrightarrow{t\to\infty} |\langle\Omega|O|P\rangle|^2 e^{-m_P t} \equiv A e^{-m_P t}$$

强子矩阵元



Heavier hadrons with same quantum numbers as O are suppressed at large t.

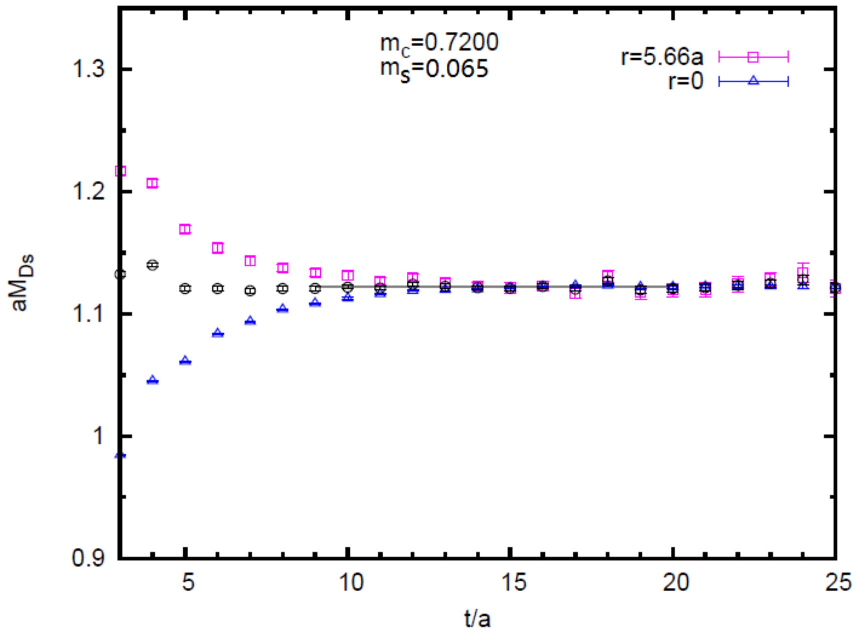
价夸克传播子

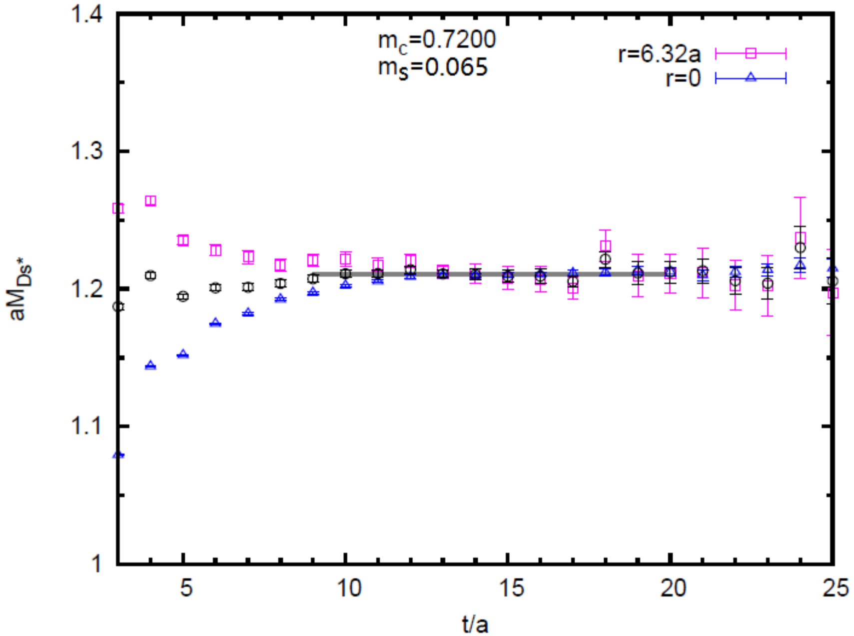
海夸克效应

- 定义有效质量$M_{\text{eff}}(t) = \ln[C(t)/C(t+1)]$，$t$较大时预期看到一个质量平台

Mass plateau, $M_{\text{eff}}(t) = \ln[C(t)/C(t+1)]$

例如: $C(t) = \langle \Omega | O(t) O^\dagger(0) | \Omega \rangle \xrightarrow{t\to\infty} |\langle \Omega | O | P \rangle|^2 e^{-m_P t}$





arXiv:2008.05208, CPC45,023109(2021)

- **设法让质量平台早出现**
  - **Combination of correlators:** $C(\omega, t) = C(r=1, t) + \omega C(r, t)$
  - **Smearing**
  - **……**

Flavor Lattice Averaging Group (FLAG) 味物理格点计算世界平均

http://flag.unibe.ch/

约三年综述一次格点结果

- 2010 Eur. Phys. J. C (2011) 71, 1695 (arXiv: 1011.4408)
- 2013 Eur. Phys. J. C (2014) 74, 2890 (arXiv: 1310.8555)
- 2016 Eur. Phys. J. C (2017) 77, 112 (arXiv: 1607.00299)
- 2019 Eur. Phys. J. C (2020) 80, 113 (arXiv: 1902.08191)

- LECs
- α_s
- quark masses
- decay constants
- form factors
- nucleon matrix elements
- ……

对正式发表结果的系统误差做评估

- 连续极限
- 手征外推
- 有限体积
- 重正化常数
- 重夸克作用量

Color-coding of systematic errors:
★ has been estimated in a satisfactory manner.
○ reasonable, could be improved.
■ no estimation, or unsatisfactory.

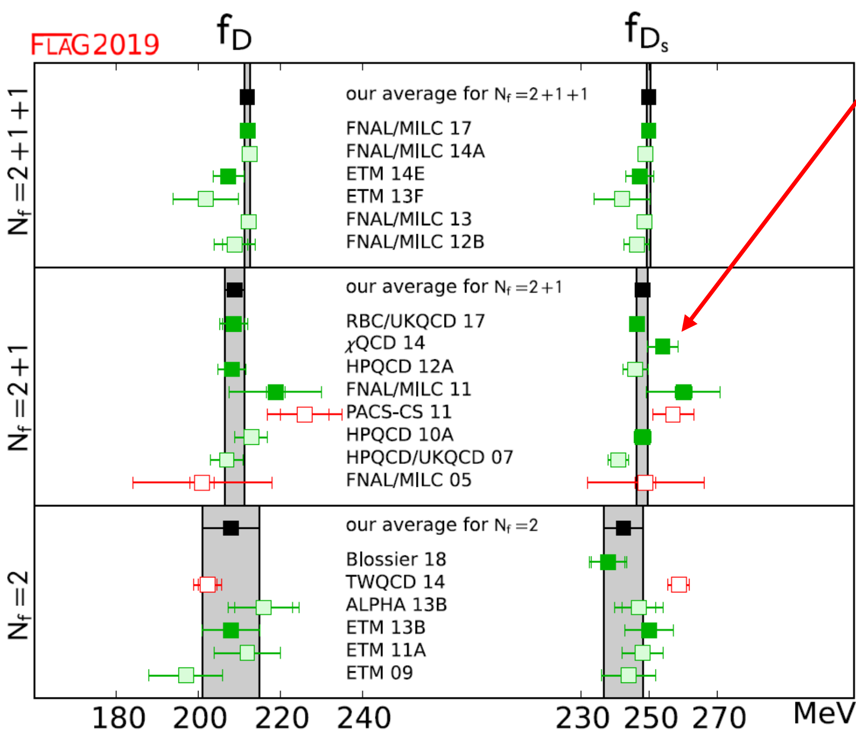
获得绿色评分的结果做世界平均

f_D、f_{D_s}格点QCD结果

- 赝标粲介子衰变常数的LQCD计算精度已达到1%或更高, 例如f_{D_s}
- 矢量介子衰变常数的计算相对匮乏

Collaboration	Ref.	N_f	pu	co	ch	fin	re	he	f_D	f_{D_s}
χQCD 14	[22]	2+1	A	○	○	○	★	✓		254(2)(4)

1410.3343, PRD92.034517.2015



FLAG Review 2019, EPJC (2020) 80:113 [arXiv:1902.08191]

- $N_f = 2+1$:
  $f_{D_s} = 248.0(1.6)$ MeV
  $f_D = 209.0(2.4)$ MeV
- $N_f = 2+1+1$:
  $f_{D_s} = 249.9(0.5)$ MeV
  $f_D = 212.0(0.7)$ MeV
- PDG2016 (CPC40):
  $f_{D_s^+}^{exp} = 257.8(4.1)$ MeV
- 格点结果与实验在2σ之内一致
- PDG2018:
  $f_{D^+}|V_{cd}| = 45.91(1.05)$ MeV
  $f_{D_s^+}|V_{cs}| = 250.9(4.0)$ MeV

BESIII最新结果：$f_{D_s} = 251.1(2.4)(3.0)$ MeV [PRL127.171801.2021]

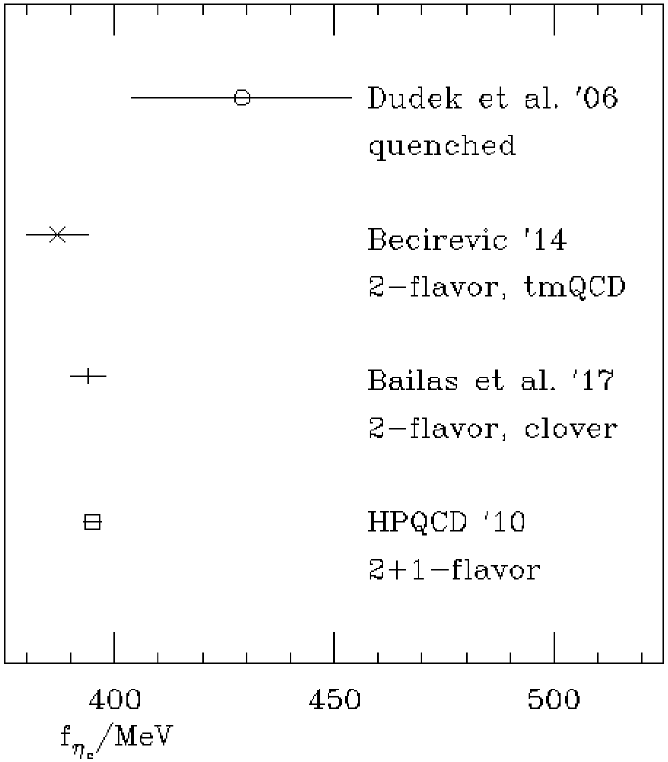
Isospin-breaking effects

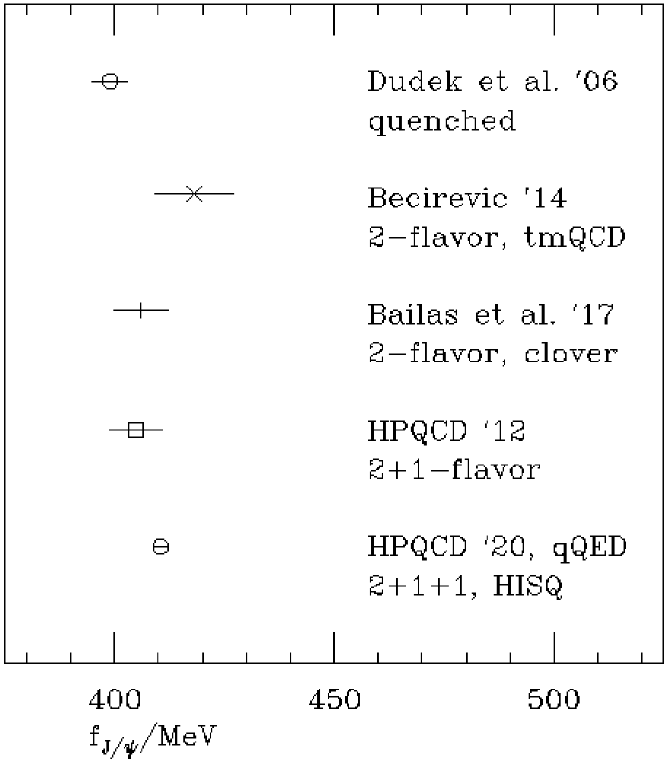
- 精度达到约1%时，需开始考虑同位旋破缺效应
  - 上下夸克质量差：$\mathcal{O}((m_d - m_u)/\Lambda_{\mathrm{QCD}}) \sim 1\%$
  - QED效应：$\mathcal{O}(\alpha_{\mathrm{em}}) \sim 1\%$
- 1+1+1+1味QED+QCD模拟 [Borsanyi et al., Science 2015]
  - 质量差：$n - p$, $D^{\pm} - D^{0}$, ……
  - Wilson-clover, 4 $a \sim [0.06 - 0.10]$ fm
  - $\alpha_{\mathrm{em}} = 0, 1/137, 1/10, 1/6$
- 3味QED+QCD模拟 [Horsley et al., J. Phys. G 2016]
  - 质量差：$n - p$, $\pi^{+} - \pi^{0}$
- Quenched QED
  - 海夸克不带电荷
  - 价夸克带电

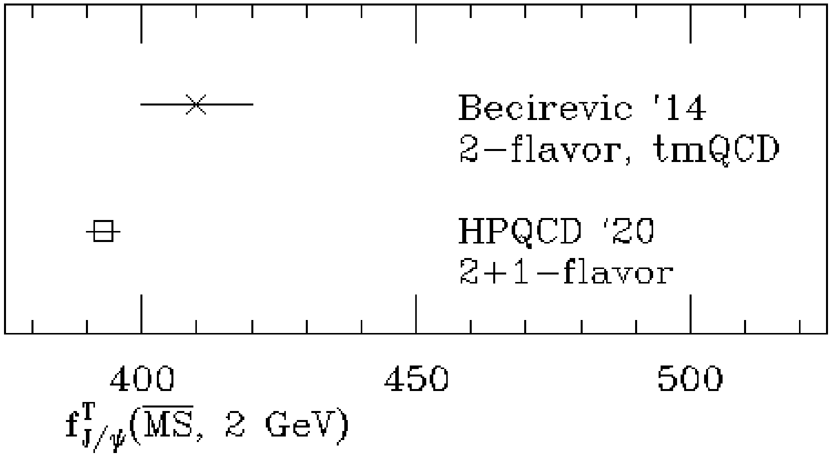
Quenched QED

- de Divitiis et al., PRD87.114505.2013
  - 质量差：$n-p$, $\pi^{+}-\pi^{0}$
  - 2味ETMC组态
- Giusti et al., Lattice2017
  - 质量差：$\pi^{+}-\pi^{0}$, $K^{+}-K^{0}$, $D^{\pm}-D^{0}$
  - $\delta M_{D_s^+}$
  - 2+1+1味ETMC组态
- Di Carlo et al., PRD100.034514.2019
  - π^{+}, K^{+}的纯轻衰变，2+1+1味ETMC组态
- HPQCD, PRD102.054511.2020
  - $f_{J/\psi}=410.4(1.7)$ MeV
  - HISQ作用量
  - 2+1+1味MILC组态

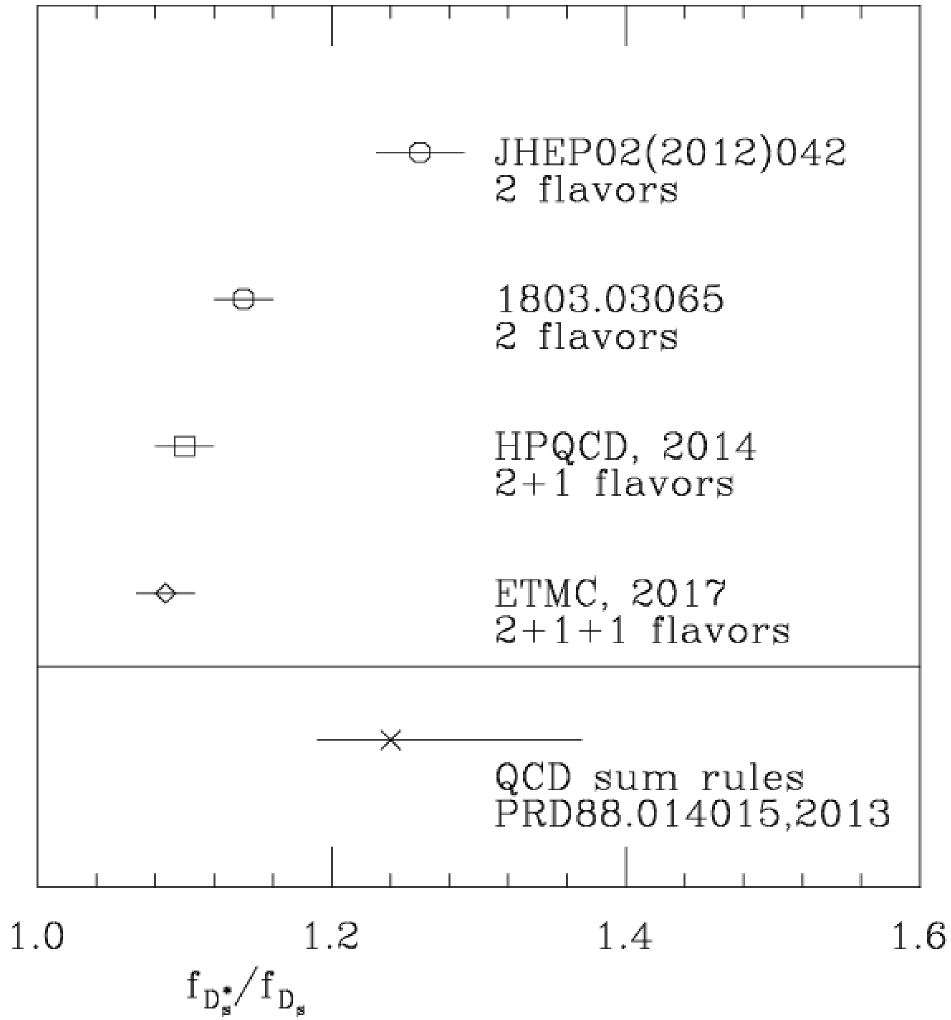
粲偶素衰变常数：f_{η_c}，$f_{J/\psi}$，$f^T_{J/\psi}$





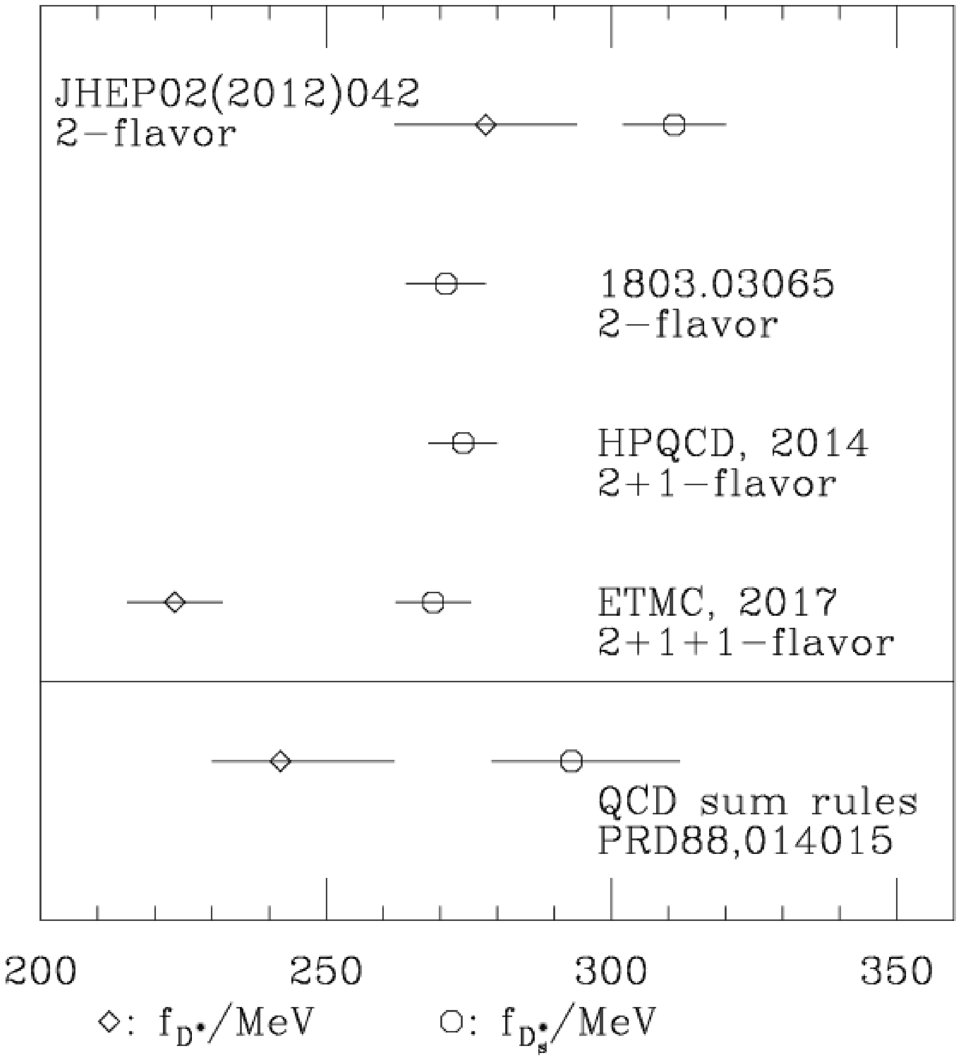


$f_{D_s^*}/f_{D_s}$



- Becirevic et al., JHEP02 (2012) 042
  - 4*a*, 2-flavor, tmQCD
- Blossier, Heitger, Post, PRD98.054506 (1803.03065)
  - 2*a*, 2-flavor, Clover fermions
- HPQCD, PRL112.212002 (2014)
  - 2*a*, 2+1-flavor, HISQ+asqtad
- ETMC, PRD96.034524 (2017)
  - 3*a*, 2+1+1-flavor, tmQCD
- Sea quark effects from the strange quark?

f_{D^*} and $f_{D_s^*}$



- Becirevic et al., JHEP02 (2012) 042
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- Sea quark effects from the strange quark?

chiQCD, Chin. Phys. C45, no.4 (2021), arXiv:2008.05208

Lattice setup

- 2+1-flavor ensemble [RBC/UKQCD Collab., PRD93.074505, 2016]
- Physical sea quark mass: $m_\pi^{sea} = 139.2(4)$ MeV
- 45 configurations

$L^3 \times T$	$48^3 \times 96$
a^{-1}(GeV)	1.730(4)
$N_{\rm conf}$	45
$am_l^{\rm (val)}$	0.0017, 0.0024, 0.0030, 0.0060
m_π/MeV	114(2), 135(2), 149(2), 208(2)
$am_s^{\rm (val)}$	0.0580, 0.0650
$am_c^{\rm (val)}$	0.6800, 0.7000, 0.7200, 0.7400

- Overlap valence and domain wall fermion sea
- Partial quenching effects are small: $\Delta_{\rm mix} = 0.030(6)(5)\ {\rm GeV}^4$ [chiQCD, PRD86.014501, 2012]
- 4 light val. quark masses: $m_\pi \sim 114 - 208$ MeV
- $Lm_\pi = 3.2/3.7/4.1/5.8$
- 2 strange val. quark masses, slightly $< m_s^{\rm phy.}$

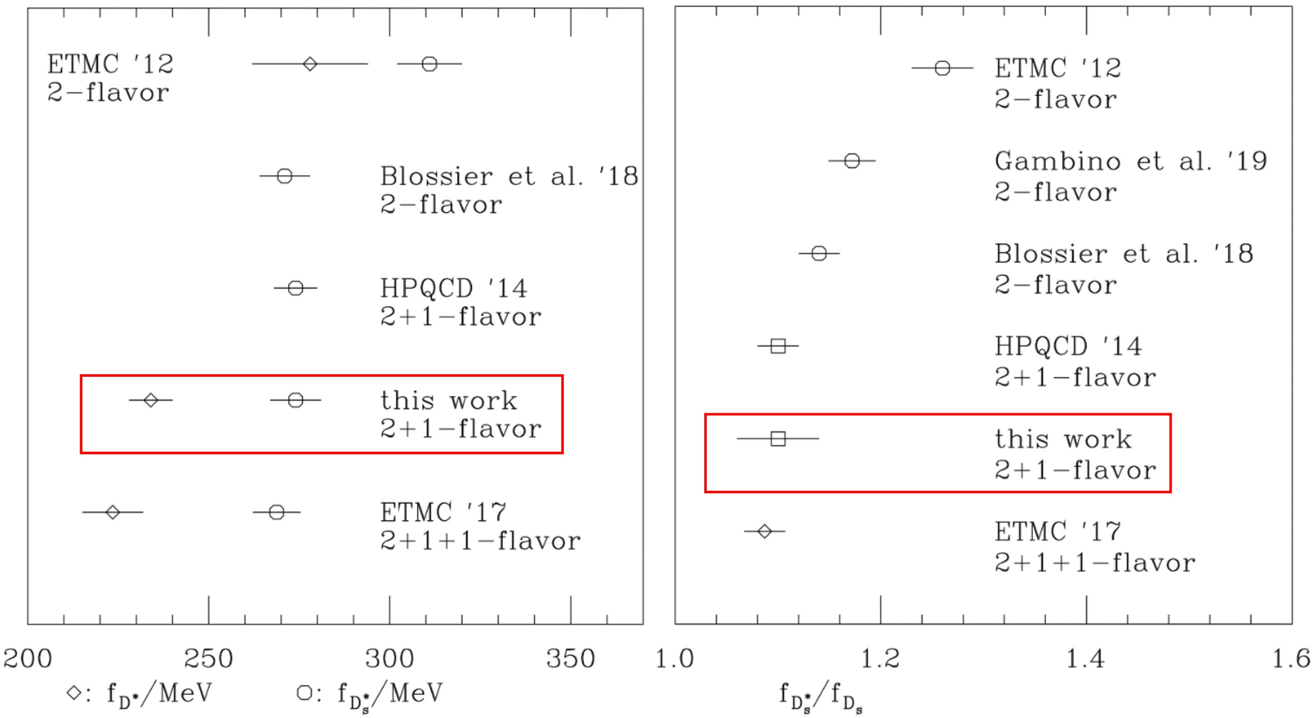
Results

	D	D^*	D_s	D_s^*	ϕ
Mass/MeV	1873(5)	2026(5)	-	2116(6)	1018(17)
M^{exp}/MeV	1869.6	2010.3	1968.3	2112.2	1019.5
f_M/MeV	213(2)(4)	234(3)(5)	249(5)(5)	274(5)(5)	241(9)(2)
f_V^T/f_V		0.91(3)(2)		0.92(3)(2)	

- M_{D^*} is ~1% higher than experiments
- $f_{D_s} = 249(5)$ MeV vs. 254(2)(4) MeV [chiQCD, 1410.3343, PRD92.034517]
- Errors from stat., interp./extrap., Z-factors and finite a
- f_D agrees with FLAG2019 (2+1-flavor): 209.0(2.4) MeV
  - 利用 $f_{D^+}|V_{cd}| = 45.91(1.05)$ MeV，得 $|V_{cd}| = 0.2155(51)(49)$ (lat.)(exp.)
- $f^T_{D^*_{(s)}}/f_{D^*_{(s)}}$ are the first lattice QCD results

Results

- **Heavy quark symmetry breaking (~10%)**
  - $f_V/f_{PS} = 1 + O(1/m_Q)$
  - $f_{D^*}/f_D = 1.10(2)(2), \quad f_{D_s^*}/f_{D_s} = 1.10(3)(2)$
- **SU(3) flavor symmetry breaking (~17%)**
  - $f_{D_s}/f_D = 1.163(14)(23), \quad f_{D_s^*}/f_{D^*} = 1.17(2)(2)$



- Becirevic et al., JHEP02 (2012) 042
- Blossier, Heitger, Post PRD98.054506 (2018)
- Gambino et al., J. Phys. Conf. Ser. 1137, 012005 (2019)
- HPQCD, PRL112.212002 (2014)
- ETMC, PRD96.034524 (2017)

Renormalization

Y. Bi et al. (χQCD), PRD97, 094501 (2018)

$\overline{\text{MS}}$, 2 GeV	D^*	D_s^*
f_V^T/f_V	0.91(3)(2)	0.92(3)(2)

$$Z_T/Z_A\,(2\text{ GeV}) = 1.055(31)$$

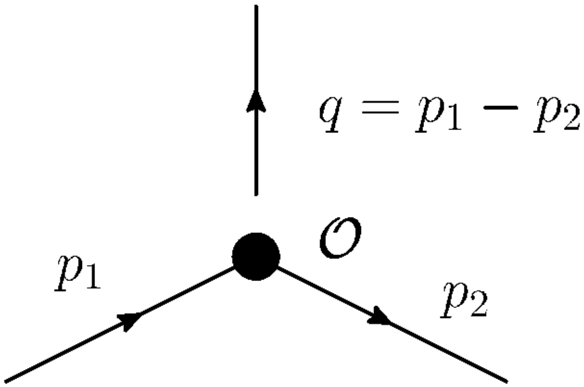
Uncertainties: stat., interp./extrap., Z-factors (~3%), finite a

- **Renormalization constants (RCs) are needed in determining**
  - **quark masses**
  - **quark chiral condensate**
  - **meson decay constants**
  - **form factors in hadron decays**
  - **……**
- **High precision needed to test the Standard Model**

RI/(S)MOM renormalization

G. Martinelli et al., NPB445, 81 (1995) C. Sturm et al., PRD80, 014501 (2009)

$$O(\mu) = Z(\mu, a) O(a)$$



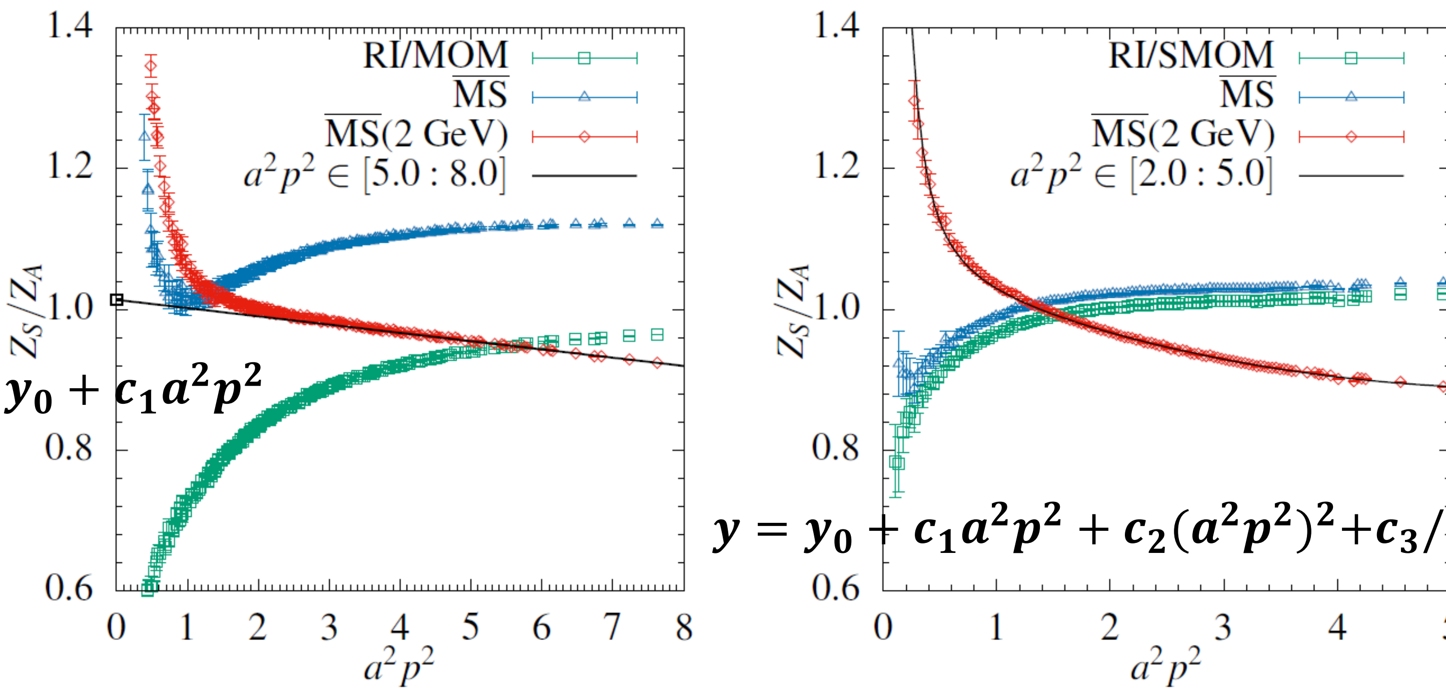
- Consider local bilinear operators
- $\overline{\mathrm{MS}}$ works with dimensional regularization
- RI/MOM with Ward Identity (WI)
  - $\partial_\mu A^\mu = 2m_q P$ gives Z_A for chiral fermions
  - Then Z_q and other Z-factors ($p_1 = p_2$, $p_1^2 = p_2^2 = \mu^2$)
- RI'/MOM
  - Z_q from quark propagator, then the other Z-factors
- RI/SMOM with WI
  - Use WI to get Z_A, then the other Z-factors
  - $p_1^2 = p_2^2 = (p_1 - p_2)^2 = q^2$ (Symmetric momentum)
  - Infrared effects suppressed by $1/p^6$ (VS. $1/p^2$ for MOM)
  - $Z_S^{\overline{\mathrm{MS}}}/Z_S^{\mathrm{SMOM}}$ converges faster than $Z_S^{\overline{\mathrm{MS}}}/Z_S^{\mathrm{MOM}}$
- (S)MOM schemes have different systematic uncertainties

working window:
$\Lambda_{\mathrm{QCD}} < \mu < \pi/a$

Z_S (MOM vs. SMOM)

Overlap fermion on Domain-Wall fermion configurations

Y. Bi et al. (χQCD), PRD97, 094501 (2018)



$y = y_0 + c_1 a^2p^2$

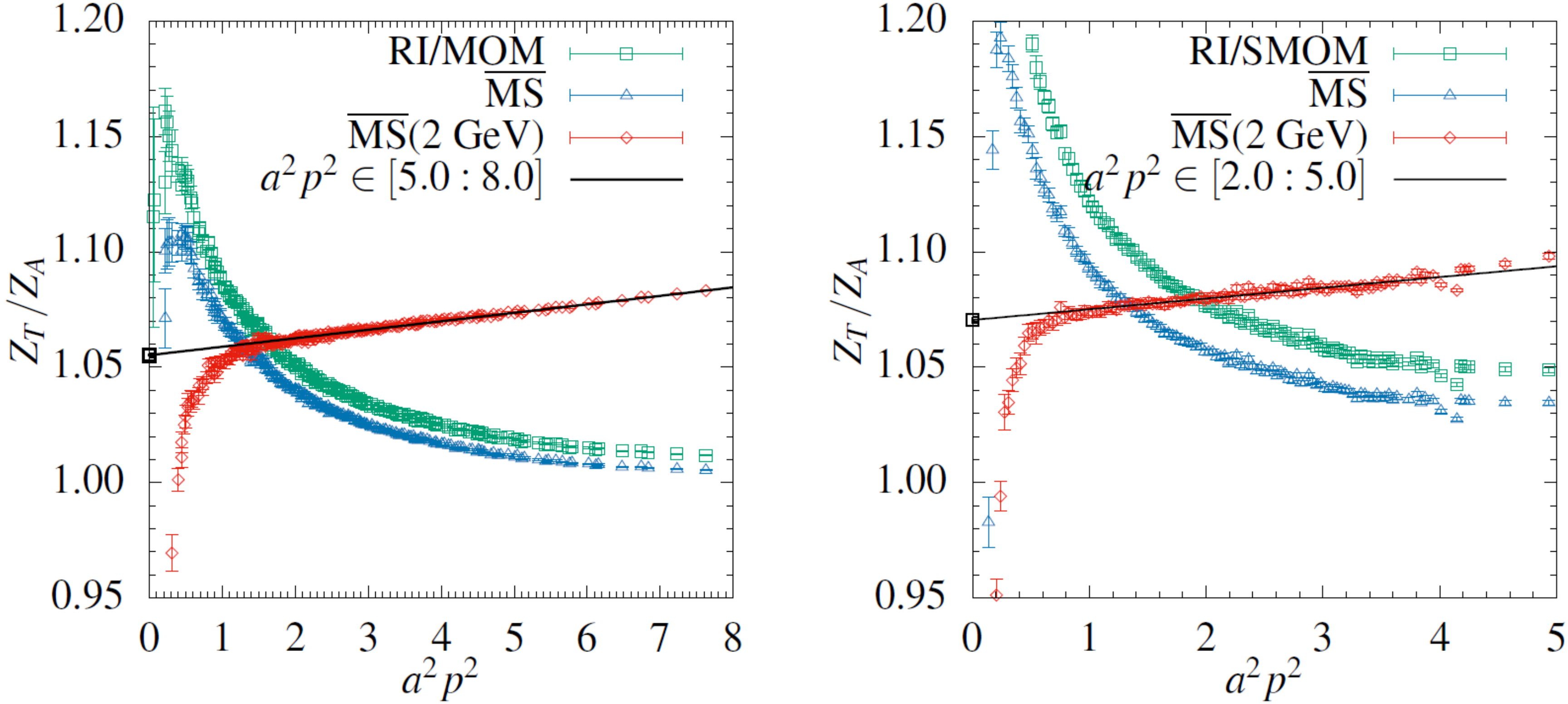
$y = y_0 + c_1 a^2p^2 + c_2(a^2p^2)^2 + c_3/(a^2p^2)^2$

- Z_S in the RI/MOM, RI/SMOM and $\overline{\text{MS}}$ schemes
- $Z_S^{\overline{\text{MS}}}/Z_S^{\text{SMOM}}$ VS. $Z_S^{\overline{\text{MS}}}/Z_S^{\text{MOM}}$ (truncation error: 1.5%→0.2%)
- Larger $O((a^2p^2)^2)$ effects in Z_S through SMOM than MOM?

Z_T (MOM vs. SMOM)

$y = y_0 + c_1 a^2 p^2$

$O((a^2p^2)^2)$ effects not apparent



Z_T in the RI/MOM, RI/SMOM and $\overline{\text{MS}}$ schemes

Y. Bi et al. (χQCD), PRD97.094501(2018)

Arthur, Boyle, Hashimoto, Hudspith, PRD88.114506.2013

$O((a^2p^2)^2)$ discretization effects and smearing

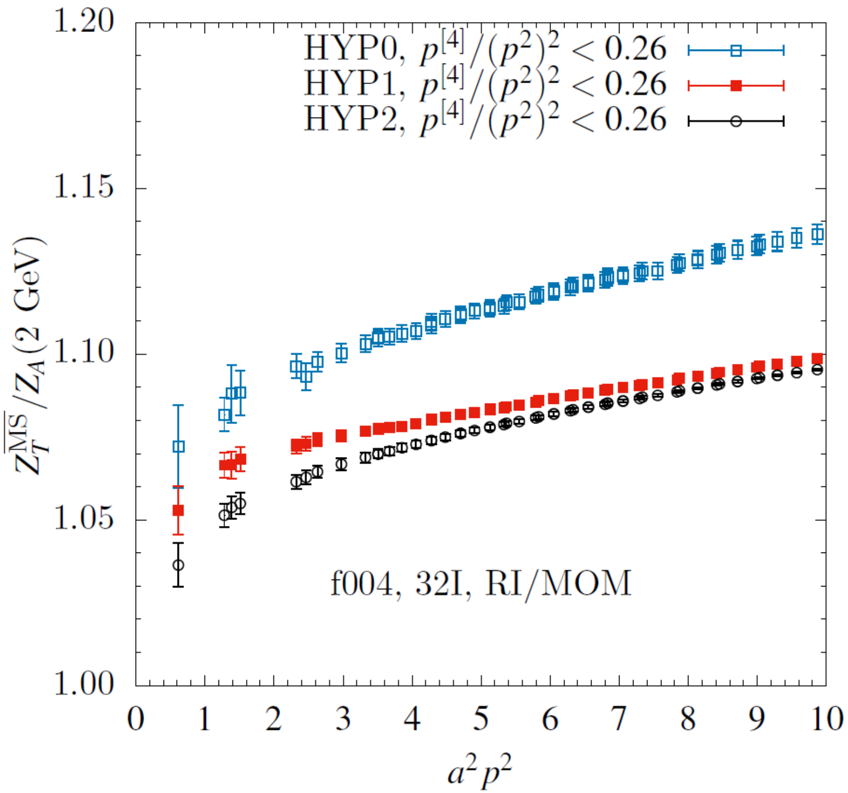
- Is this behavior related to the HYP smearing that we use?
  - Smearing is used to speed up matrix inversions
- How does it depend on specific operators?

Lattice setup

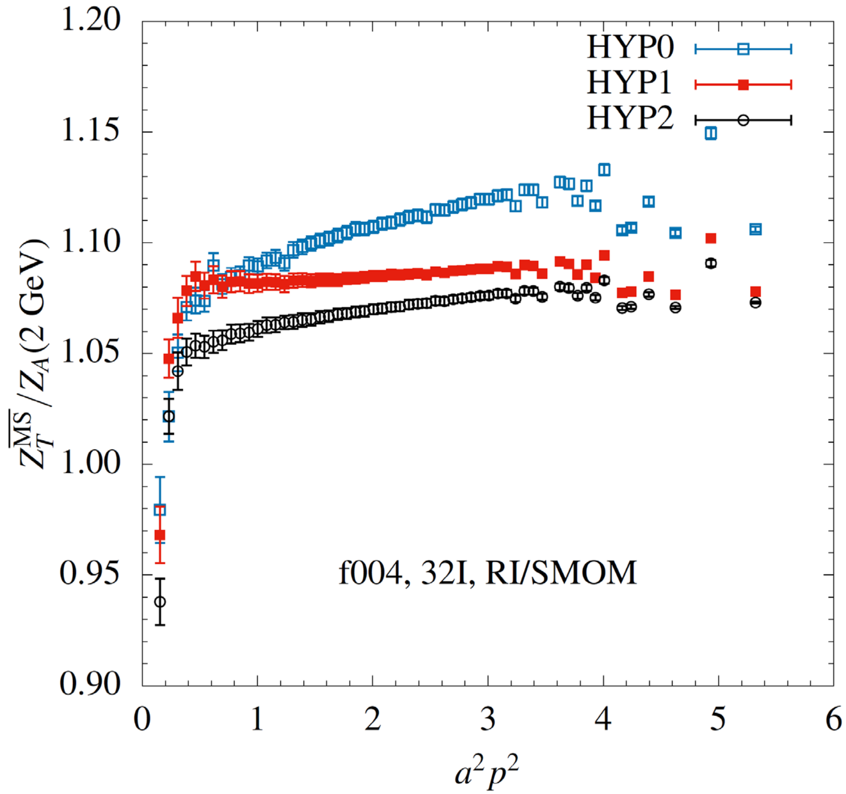
- Overlap fermion on domain wall fermion configurations
- 2+1-flavor, RBC/UKQCD (32I) [RBC/UKQCD, PRD83.074508.2011]
- $1/a = 2.28(3)$ GeV, $m_l/m_s = 0.004/0.03$, $L^3 \times T = 32^3 \times 64$
- Statistics: 42 configurations (fixed to Landau gauge)
- No smearing (HYP0), HYP1 or HYP2 ($\alpha_{1,2,3} = 0.75,\ 0.6,\ 0.3$)
- 10 valence quark masses (pion mass ~ 220-500 MeV)
- Periodic BC in all directions

Z_T : HYP0/1/2 (preliminary)

$$y = y_0 + c_1 a^2 p^2 + c_2 (a^2 p^2)^2$$



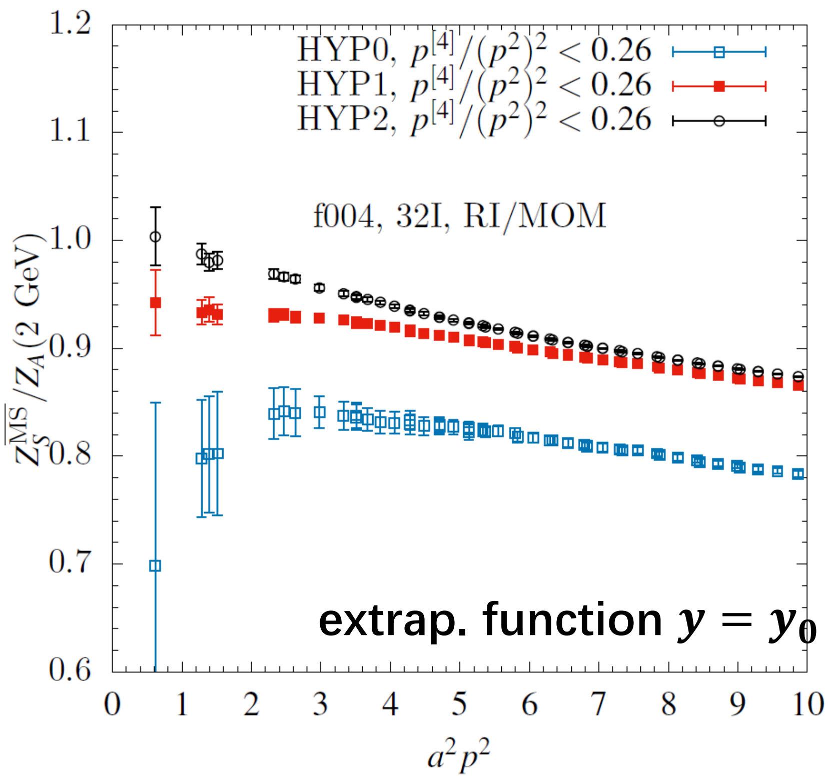
MOM as intermediate scheme

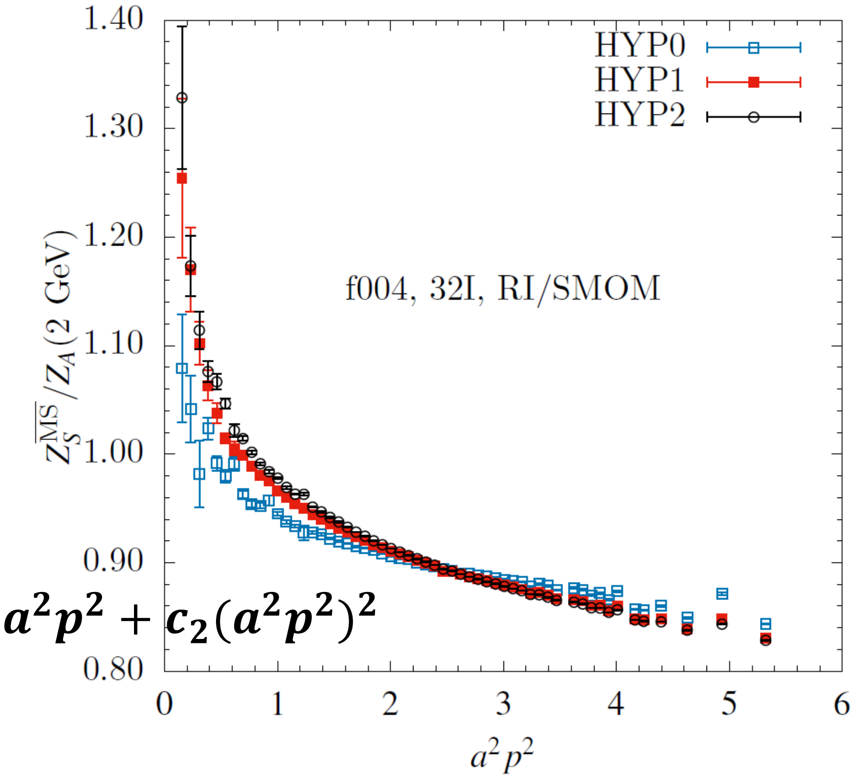


SMOM as intermediate scheme

- **Smearing changes the size of c_2**
- **SMOM gives a small $(a^2p^2)^2$-term for $Z_T^{\overline{\rm MS}}(2\ {\rm GeV})$ in the cases of HYP1 and HYP2 (compared with HYP0)**

Z_S : HYP0/1/2 (preliminary)





extrap. function $y = y_0 + c_1 a^2p^2 + c_2(a^2p^2)^2$

MOM as intermediate scheme **SMOM as intermediate scheme**

- **Smearing changes c_2 (MOM+HYP1 happens to be straight)**
- **SMOM seems to give a bigger $(a^2p^2)^2$-term for $Z_S^{\overline{\rm MS}}(2\ \mathrm{GeV})$ in all cases (HYP0, HYP1, HYP2) compared with MOM (?)**

总结

- 赝标粲介子衰变常数的格点精度与当前实验测量相当或相对更好
- 考虑同位旋破缺效应可进一步提高精度
- 矢量介子衰变常数的格点计算相对较少
- 相关重正化常数的精度需要提高