中国格点QCD第一届年会

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线上

Bridging lattice QCD to experiments using effective field theory for hadron spectroscopy

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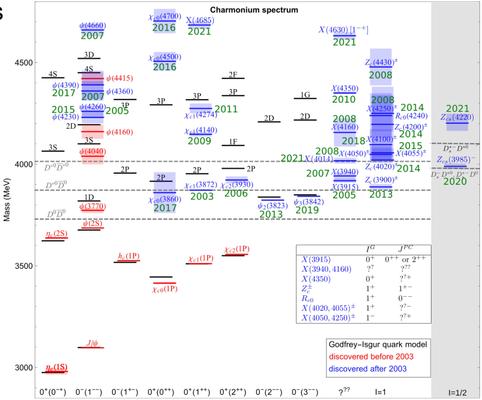
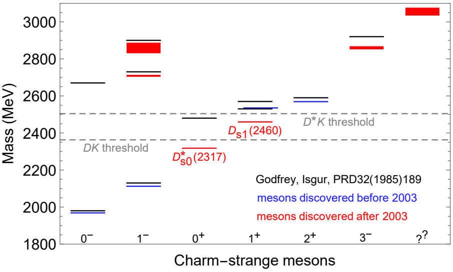
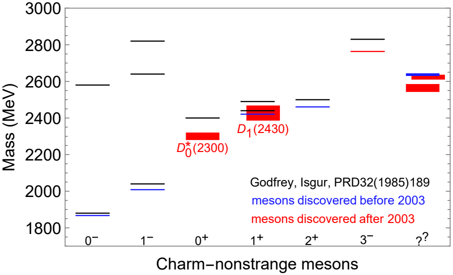
Introduction

- Puzzles of many hadron resonances since 2003, e.g., $D_{s0}^*(2317)$, $X(3872)$; internal structure? hints to confinement mechanism?
- Lattice QCD
  - direct calculations of some systems
  - varying parameters (e.g., quark masses) to get more insights
  - determining low-energy constants in effective field theory (EFT)



- EFT provides a tool for combined analysis of lattice and experimental results
- Examples
  - Positive-parity charmed mesons
  - Wick contractions in meson-meson scattering





- Puzzle 1: Why are $D^*_{s0}(2317)$ and $D_{s1}(2460)$ so light?
- Puzzle 2: Why $\underbrace{M_{D_{s1}(2460)} - M_{D^*_{s0}(2317)}}_{(141.8\pm0.8)\text{ MeV}} \simeq \underbrace{M_{D^{*\pm}} - M_{D^\pm}}_{(140.67\pm0.08)\text{ MeV}}$?
- Puzzle 3: Why $M_{D^*_0(2300)} \gtrsim M_{D^*_{s0}(2317)}$ and $M_{D_1(2430)} \sim M_{D_{s1}(2460)}$?

  $D^*_0(2300)$ was denoted as $D^*_0(2400)$ up to PDG2018

Lattice studies of the charmed scalar mesons: strange

- Early studies using only $c\bar{s}$-type interpolators typically give mass larger than that for $D_{s0}^*(2317)$ Bali (2003); UKQCD (2003); HadSpec (2013); ...
- $c\bar{s} + DK$ interpolators: ∼right mass Mohler et al., PRL111(2013)222001

$M_{D_{s0}^*} - \frac{1}{4}\left(M_{D_s} + 3M_{D_s^*}\right)$:

Mohler et al.	PDG2018
(266 ± 16) MeV	(241.5 ± 0.8) MeV

- Calculation with $M_\pi = 150$ MeV Bali et al. [RQCD Col.], PRD96(2017)074501

	Energy [MeV]	Expt [MeV]
m_{0-}	1976.9(2)	1966.0(4)
m_{1-}	2094.9(7)	2111.3(6)
m_{0+}	2348(4)(+6)	2317.7(0.6)(2.0)
m_{1+}	2451(4)(+1)	2459.5(0.6)(2.0)

Lattice studies of the charmed scalar mesons: nonstrange (1)

- $(S,I) = (0, \frac{1}{2})$: $c\bar{q} + D\pi$
  interpolators:

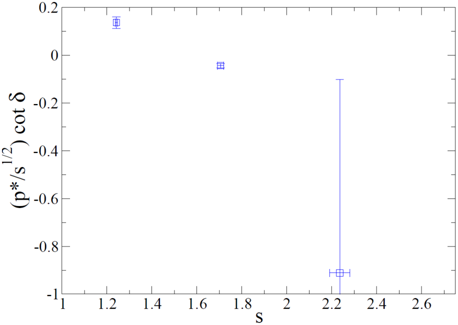
  Mohler et al., PRD87(2013)034501

  $M_\pi \approx 266$ MeV,

  $M_D \approx 1558$ MeV,

  $M_{D^*} \approx 1690$ MeV

  Lüscher's formula $\Rightarrow D\pi$ phase shifts



$\Rightarrow$ BW parameters of $D_0^*(2400)$ [the name of $D_0^*(2300)$ up to PDG18] consistent with PDG values

	Mohler et al.	PDG2018
$M_{D_0^*} - \frac{1}{4}(M_D + 3M_{D^*})$	(351 ± 21) MeV	(347 ± 29) MeV
$M_{D_1} - \frac{1}{4}(M_D + 3M_{D^*})$	(380 ± 21) MeV	(456 ± 40) MeV

Lattice studies of the charmed scalar mesons: nonstrange (2)

- $(S,I)=(0,\frac{1}{2})$: first coupled-channel lattice calculation including interpolating fields for $c\bar{q}+D\pi+D\eta+D_s\bar{K}$: Moir et al. [HadSpec], JHEP1610(2016)011
- $M_\pi = 391$ MeV, $M_D = 1885$ MeV: $D\pi$ threshold (2276.4 ± 0.9) MeV
- for coupled channels:

  parameterizing the T-matrix with the K-matrix formalism

$$T_{ij}^{-1}(s) = K_{ij}^{-1}(s) + I_{ij}(s)$$

  $I_{ij}(s)$: 2-point loop function evaluated with a subtracted dispersion integral

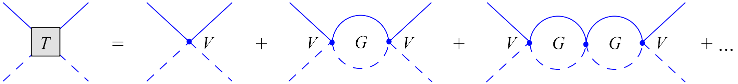
  $K_{ij}(s)$: different forms of the K-matrix were used, summarized as

$$K_{ij}(s) = \left(g_i^{(0)} + g_i^{(1)} s\right)\left(g_j^{(0)} + g_j^{(1)} s\right)\frac{1}{m^2 - s} + \gamma_{ij}^{(0)} + \gamma_{ij}^{(1)} s$$

- $\Rightarrow$ a pole below threshold (2275.9 ± 0.9) MeV. relation to $D_0^*(2300)$?

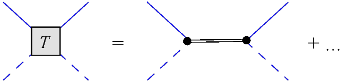
Interactions between charm and light mesons

- S-wave interactions between charm mesons (D, D_s) and light pseudoscalar mesons (π, K, η)



- not far from the thresholds ⇒ chiral EFT for matter field
- D^*_{s0}/D^*_0 should appear as poles in scattering amplitudes:

$$D^*_{s0} : DK, D_s\eta; \qquad D^*_0 : D\pi, D\eta, D_s\bar{K}$$



⇒ needs a nonperturbative treatment: ChPT + unitarization

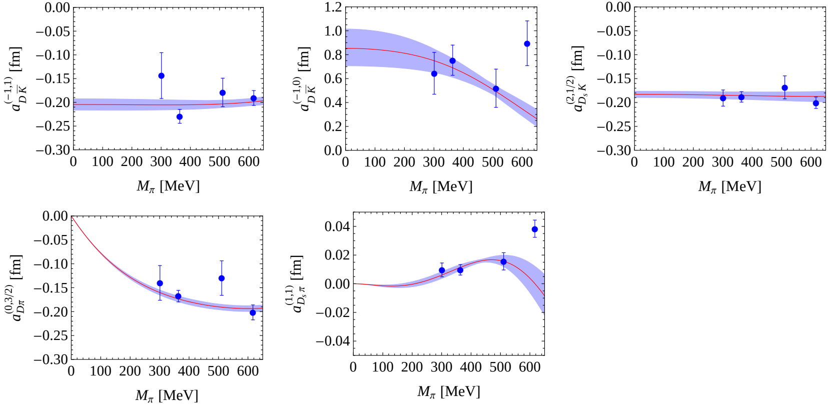
Truong (1988); Oller, Oset (1997); Oller, Oset, Peláez (1998); Nieves, Ruiz Arriola (1999); Oller, Meißner (2001); . . .

$$T^{-1}(s) = V^{-1}(s) - G(s)$$

$V(s)$: from SU(3) chiral Lagrangian, 6 LECs up to NLO

$G(s)$: 2-point scalar loop functions, regularized with a subtraction constant $a(\mu)$

- Fit to lattice data on scattering lengths in 5 simpler channels:
  $D\bar{K}(I=1, I=0)$, D_sK, $D\pi(I=3/2)$, $D_s\pi$: no disconnected contribution
  5 parameters: h_2, h_3, h_4, h_5 and $a(\mu)$



- N_c counting fulfilled: $\underbrace{h_2 \simeq 0.2,\ h_4\, M_D^2 \simeq -0.3,}_{\mathcal{O}(N_c^{-1})}$ $\underbrace{h_3 \simeq 2.1,\ h_5\, M_D^2 \simeq -1.9}_{\mathcal{O}(N_c^0)}$

Predictions for 0^+ & 1^+ heavy mesons

M.-L. Du et al., PRD98(2018)094018

- Heavy-strange

meson	J^P	prediction (MeV)	PDG2020 (MeV)	lattice (MeV)
D_{s0}^*	0^+	2315^{+18}_{-28}	2317.8 ± 0.5	2348^{+7}_{-4}[1]

- Heavy-nonstrange, two $I = 1/2$ states $(M, \Gamma/2)$:

	Lower (MeV)	Higher (MeV)	PDG2020 (MeV)
D_0^*	$(2105^{+6}_{-8}, 102^{+10}_{-11})$	$(2451^{+36}_{-26}, 134^{+7}_{-8})$	$(2300 \pm 29, 137 \pm 20)$
D_1	$(2247^{+5}_{-6}, 107^{+11}_{-10})$	$(2555^{+47}_{-30}, 203^{+8}_{-9})$	$(2427 \pm 40, 192^{+65}_{-55})$
B_0^*	$(5535^{+9}_{-11}, 113^{+15}_{-17})$	$(5852^{+16}_{-19}, 36 \pm 5)$	–
B_1	$(5584^{+9}_{-11}, 119^{+14}_{-17})$	$(5912^{+15}_{-18}, 42^{+5}_{-4})$	–

[1] Bali, Collins, Cox, Schäfer, PRD96(2017)074501

[2] Lang, Mohler, Prelovsek, Woloshyn, PLB750(2015)17

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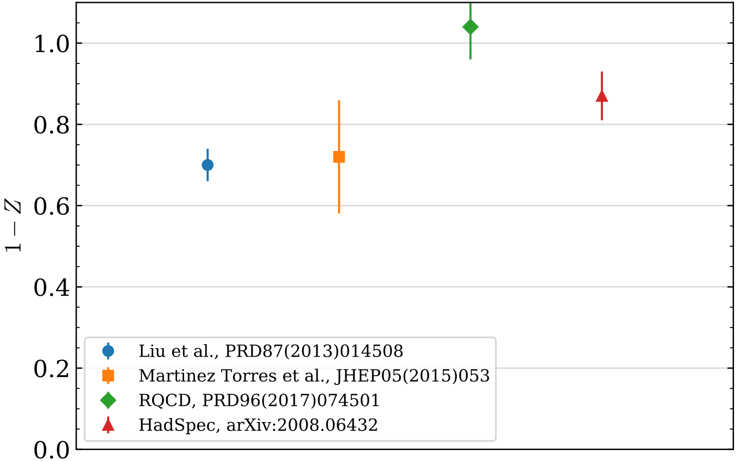
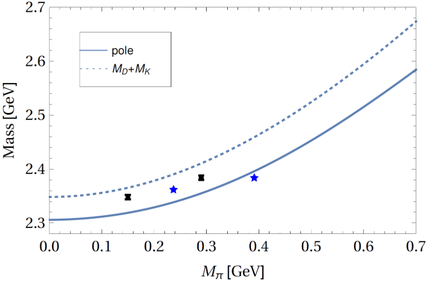
DK component from lattice QCD (1)

- Postdicted mass for $D^*_{s0}(2317)$: 2315^{+18}_{-28} MeV
- Compositeness (DK component) for $D^*_{s0}(2317)$ from (in)direct lattice calculations: DK as the main component, it is in this sense we recognize it as a hadronic molecule



DK component from lattice QCD (2)

- Lattice results in G. Bali et al., PRD96(2017)074501



M_π [MeV]	150	290
$M_{D^*_{s0}(2317)}$ [MeV]	2348 ± 4	2384 ± 3
M_{D_s} [MeV]	1977 ± 1	1980 ± 1

strong M_π dependence!

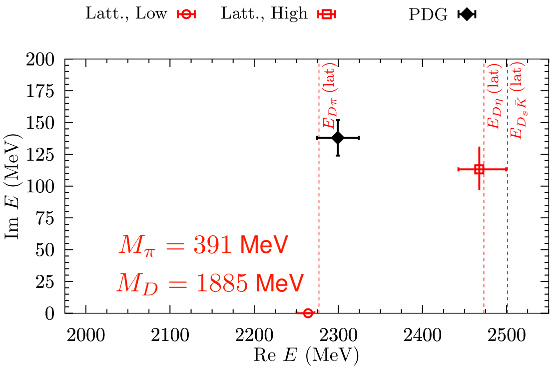
curves: prediction in Du et al., EPJC77(2017)728

- Lattice results in HadSpec, JHEP 02 (2021) 100

M_π [MeV]	239	391
$M_{D^*_{s0}(2317)}$ [MeV]	2362 ± 3	2380 ± 3

Pion mass dependence

Masses	M (MeV)	$\Gamma/2$ (MeV)	RS	$\lvert g_{D\pi}\rvert$	$\lvert g_{D\eta}\rvert$	$\lvert g_{D_s\bar{K}}\rvert$
lattice	$2264^{+\ 8}_{-14}$	0	$(+++)$	$7.7^{+1.2}_{-1.1}$	$0.3^{+0.5}_{-0.3}$	$4.2^{+1.1}_{-1.0}$
	2468^{+32}_{-25}	113^{+18}_{-16}	$(--+)$	$5.2^{+0.6}_{-0.4}$	$6.7^{+0.6}_{-0.4}$	$13.2^{+0.6}_{-0.5}$



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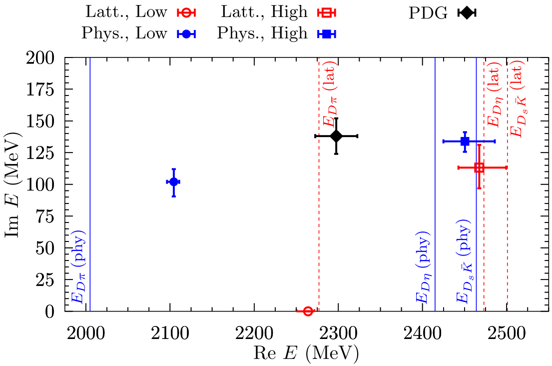
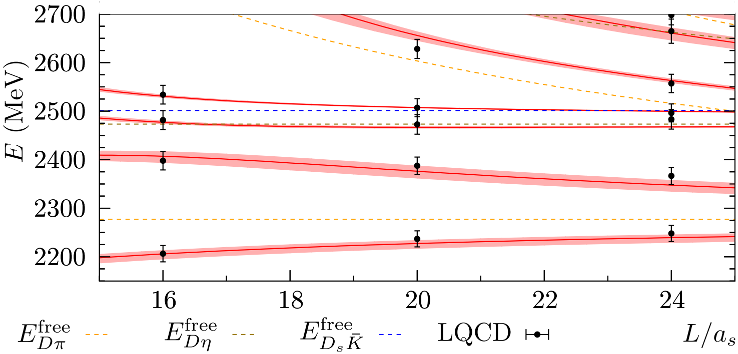
physical	2105^{+6}_{-8}	102^{+10}_{-11}	$(-++)$	$9.4^{+0.2}_{-0.2}$	$1.8^{+0.7}_{-0.7}$	$4.4^{+0.5}_{-0.5}$
	2451^{+36}_{-26}	134^{+7}_{-8}	$(--+)$	$5.0^{+0.7}_{-0.4}$	$6.3^{+0.8}_{-0.5}$	$12.8^{+0.8}_{-0.6}$



Postdictions versus recent lattice results: charm-nonstrange

- Postdicted $I = 1/2\ D\pi, D\eta, D_s\bar{K}$ finite volume energy levels in the c.m. frame versus lattice QCD results by [G. Moir *et al.* [HadSpec], JHEP1610(2016)011]

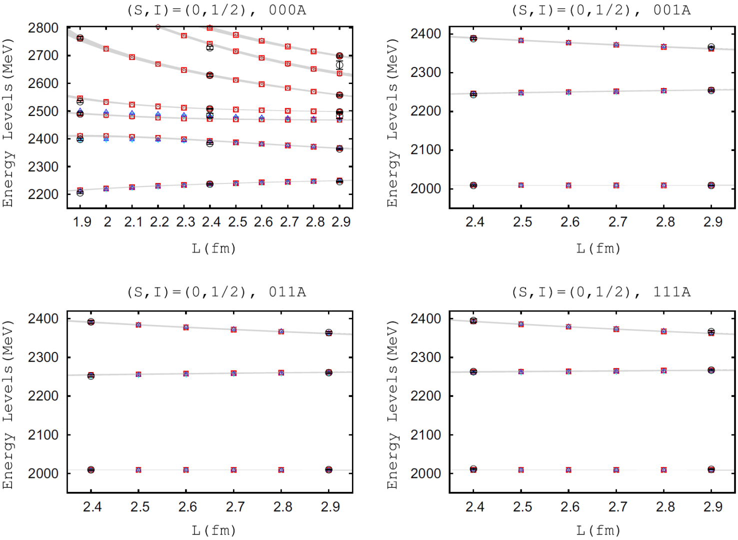
NOT a fit ! M. Albaladejo, P. Fernandez-Soler, FKG, J. Nieves, PLB767(2017)465



consequence of SU(3) + chiral

A recent fit to lattice data including moving frame ones

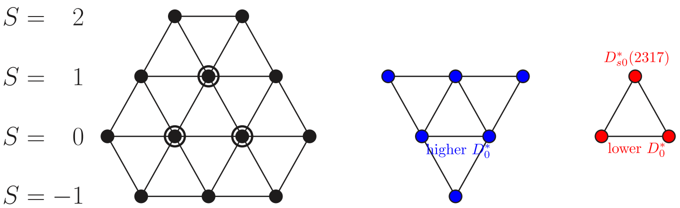
Z.-H. Guo et al., EPJC79(2019)13



Determined parameters (from Fit IIB) are similar

Lattice data: Moir et al., JHEP1610,011

- SU(3) irreps: $\overline{\mathbf{3}} \otimes \mathbf{8} = \overline{\mathbf{15}} \oplus \mathbf{6} \oplus \overline{\mathbf{3}}$ Albaladejo et al., PLB767(2017)465

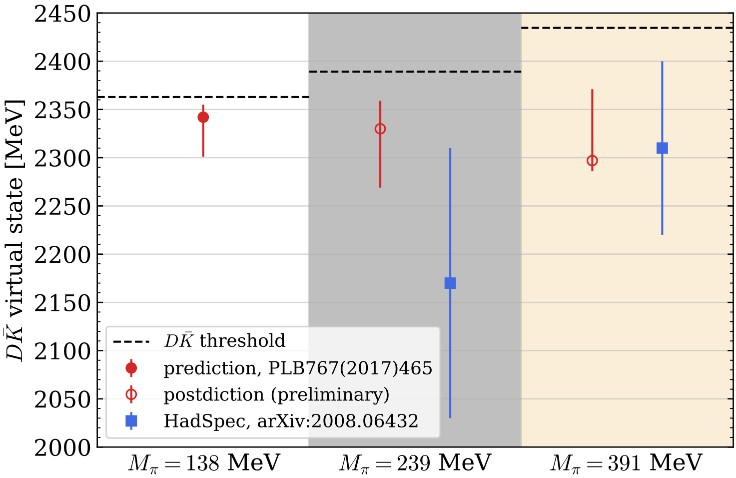


- WT term: $\overline{\mathbf{15}}$: repulsive; $\mathbf{6}$: attractive; $\overline{3}$: most attractive
  $(S, I) = (1, 1)$: deep in the complex plane on wrong Riemann sheets
  $(S, I) = (-1, 0)$: virtual state at 2342^{+13}_{-41} MeV at the physical mass

SU(3) analysis (2)

- Lattice QCD also found a virtual state for the sextet state $(S, I) = (-1, 0)\ D\bar{K}$

  HadSpec, JHEP 02 (2021) 100



Solution to the puzzles

- Solution to Puzzle 1: not quark model $c\bar{s}$ mesons:

  $D_{s0}^*(2317)[\simeq DK(I=0)]$, $D_{s1}(2460)[\simeq D^*K(I=0)]$

  Barnes, Close, Lipkin (2003); van Beveren, Rupp (2003); Kolomeitsev, Lutz (2004); FKG et al. (2006); FKG, Hanhart, Meißner (2009); . . .

- Solution to Puzzle 2: HQSS $\Rightarrow$ similar binding energies

  $M_D + M_K - M_{D_{s0}^*} \simeq 45$ MeV

  $$M_{D_{s1}(2460)} - M_{D_{s0}^*(2317)} \simeq M_{D^*} - M_D \text{ is natural}$$

- Solution to puzzle 3: the SU(3) nonstrange partner of $D_{s0}^*(2317)$ is the lower D_0^* state with a mass of about 2.1 GeV

Searching for the higher nonstrange state: lattice

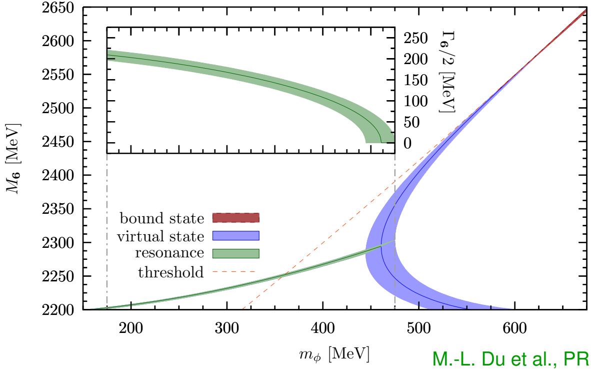
- Tuning interaction strength by varying quark masses:

  Expectation: WT term $\propto E_\pi$, increasing M_π leads to stronger interaction

  increasing S-wave interaction strength $\Rightarrow$ resonance $\to$ below-th. resonance $\to$ virtual state $\to$ bound state, then easier for lattice to get a signal

- SU(3) symmetric, then the sextet decouples from the triplet;

  prediction (qualitative for large m_q), to check with large m_q on lattice:

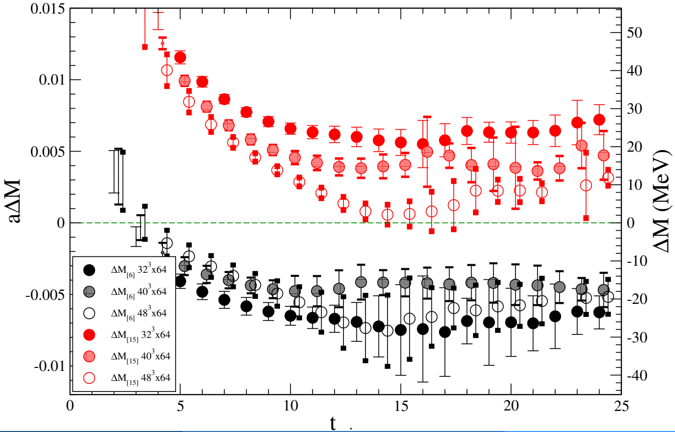


M.-L. Du et al., PRD98(2018)094018

E. B. Gregory, FKG, C. Hanhart, S. Krieg, T. Luu, arXiv.2106.15391

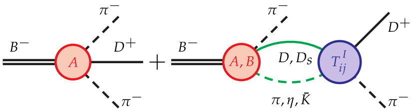
Lattice results of the energy shift at SU(3) symmetric point with $M_\pi = 612(90)$ MeV

- Evidence for a bound state in the **sextet**
- The $\overline{15}$ has a repulsive interaction (in a diquark-antidiquark tetraquark model, $\overline{15}$ would also exist)



$B^- \to D^+\pi^-\pi^-$

M.-L. Du et al., PRD98(2018)094018



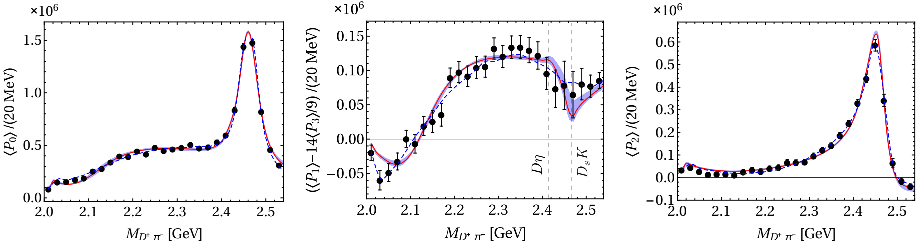
- SU(3) symmetry   Savage, Wise (1989)
- S-wave: FSI, two new parameters
- P, D-wave: BWs from the LHCb fit

Angular moments:

$$\langle P_0 \rangle \propto |\mathcal{A}_0|^2 + |\mathcal{A}_1|^2 + |\mathcal{A}_2|^2 , \quad \langle P_2 \rangle \propto \frac{2}{5}|\mathcal{A}_1|^2 + \frac{2}{7}|\mathcal{A}_2|^2 + \frac{2}{\sqrt{5}}|\mathcal{A}_0||\mathcal{A}_2| \cos(\delta_2 - \delta_0) ,$$

$$\langle P_{13} \rangle \equiv \langle P_1 \rangle - \frac{14}{9}\langle P_3 \rangle \propto \frac{2}{\sqrt{3}}|\mathcal{A}_0||\mathcal{A}_1| \cos(\delta_1 - \delta_0)$$

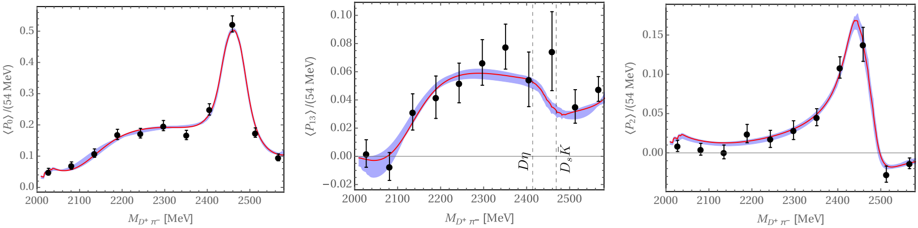
Data: LHCb, PRD94(2016)072001



- Fast variation in [2.4, 2.5] GeV in $\langle P_{13} \rangle$: cusps at $D\eta$ and $D_s\bar{K}$ thresholds

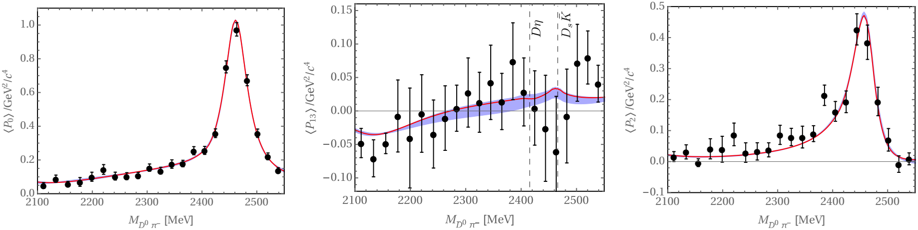
Fit to data of $B^- \to D^+\pi^-K^-$

Data: LHCb, PRD91(2015)092002



Fit to data of $B^0 \to \bar{D}^0\pi^-\pi^+$

Data: LHCb, PRD92(2015)032002

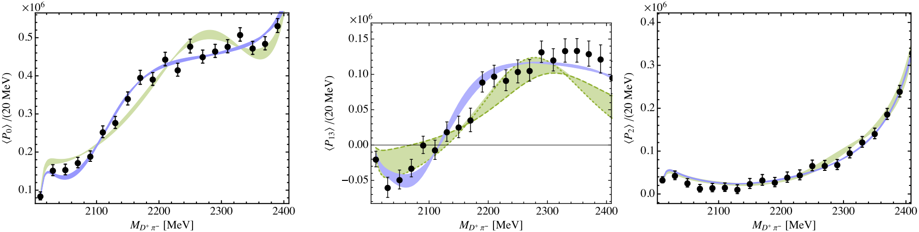


and also $B^0 \to \bar{D}^0\pi^-K^+$, $B^- \to D^+\pi^-K^-$, $B_s^0 \to \bar{D}^0K^-\pi^+$

Where is the lightest D_0^*

M.-L. Du, FKG, Hanhart, Kubis, Meißner, PRL126(2021)192001

- Fits with the Khuri-Treiman equation taking into account three-body unitarity: using S-wave $D\pi$ scattering phase from UCHPT ($\chi^2/\text{d.o.f.} = 1.2$) and from BW ($\chi^2/\text{d.o.f.} = 2.0$)

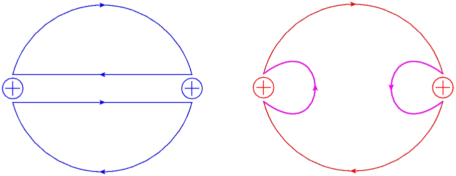


- The LHCb data are well described with UCHPT amplitude with two D_0^* states; the lowest has a mass about 2.1 GeV
- Support from recent lattice results: L. Gayer et al. [HadSpec], JHEP07(2021)123
  D_0^* with $M \approx 2.2$ GeV and $\Gamma \approx 0.4$ GeV obtained using $M_\pi \approx 239$ MeV

Wick contractions (1)

- Wick contractions of different topology; some are more difficult than others to calculate on lattice
- For instance, consider $\bar{q}c\bar{s}q \; (q = u, d)$ [relevant to the charm-strange isoscalar meson $D^*_{s0}(2317)$]

$$\begin{aligned} & \langle \bar{q}c\bar{s}q(y)\, \bar{q}s\bar{c}q(x) \rangle \\ = & \langle S_q(x,y)S_c(y,x) \rangle \langle S_s(x,y)S_q(y,x) \rangle - \langle S_q(y,y)S_c(y,x)S_q(x,x)S_s(x,y) \rangle \\ \equiv & C_{\text{conn.}} - C_{\text{disc.}} \end{aligned}$$



- For the isospin eigenstates, using isospin symmetry,
  - ☞ $I=0$: $j_0 = \frac{1}{\sqrt{2}}\left(\bar{u}c\bar{s}u + \bar{d}c\bar{s}d\right)$

$$\left\langle j_0^\dagger(y) j_0(x)\right\rangle = \frac{1}{2}\left[\langle \bar{u}c\bar{s}u(y)\,\bar{u}s\bar{c}u(x)\rangle + \left\langle \bar{u}c\bar{s}u(y)\,\bar{d}s\bar{c}d(x)\right\rangle + (u \leftrightarrow d)\right]$$
$$= \langle \bar{u}c\bar{s}u(y)\,\bar{u}s\bar{c}u(x)\rangle + \left\langle \bar{u}c\bar{s}u(y)\,\bar{d}s\bar{c}d(x)\right\rangle$$
$$= C_{\text{conn.}} - C_{\text{disc.}} \qquad - C_{\text{disc.}}$$

  - ☞ $I=1$: $j_1 = \frac{1}{\sqrt{2}}\left(\bar{u}c\bar{s}u - \bar{d}c\bar{s}d\right)$

$$\left\langle j_1^\dagger(y) j_1(x)\right\rangle = \langle \bar{u}c\bar{s}u(y)\,\bar{u}s\bar{c}u(x)\rangle - \left\langle \bar{u}c\bar{s}u(y)\,\bar{d}s\bar{c}d(x)\right\rangle$$
$$= C_{\text{conn.}} - C_{\text{disc.}} \qquad + C_{\text{disc.}} = C_{\text{conn.}}$$

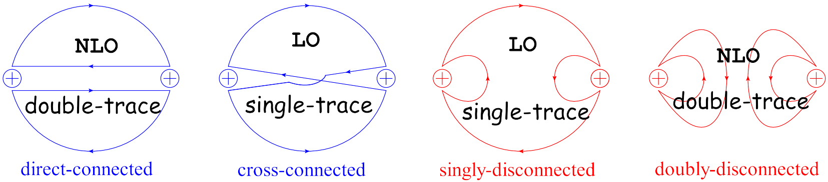
- Can we quantify contributions of the disconnected diagrams using analytical methods?

Qualitative analysis: chiral expansion

FKG, L. Liu, U.-G. Meißner, P. Wang, PRD88(2013)074506

- For $\pi\pi$ scattering $\Rightarrow$ chiral and $1/N_c$-expansion power counting
- Number of quark loops $=$ number of flavor traces

see, e.g., A.Manohar, arXiv:hep-ph/9802419



- The leading order CHPT Lagrangian

$$\mathcal{L}_{\text{LO}} = \frac{F_0^2}{4}\left(\text{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right] + 2B_0\,\text{Tr}\left[\mathcal{M}U^\dagger + \mathcal{M}^\dagger U\right]\right)$$

only contains single-flavor-trace terms

- double trace terms start from next-to-leading order in CHPT, e.g.

$$\text{Tr}\left[\partial_\mu U^\dagger \partial^\mu U\right]\,\text{Tr}\left[\partial_\nu U^\dagger \partial^\nu U\right]$$

Wick contractions for $\pi\pi$ scattering (1)

N.R. Acharya, FKG, U.-G. Meißner, C.-Y. Seng, NPB922(2017)480

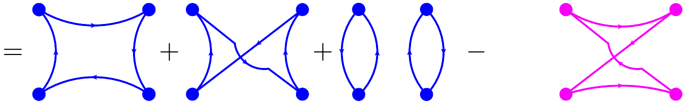
- Contractions can be computed separately in partially quenched CHPT (PQCHPT)
- Isospin symmetry + crossing $\Rightarrow$ only one independent $\pi\pi$ scattering amplitude:

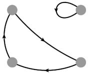
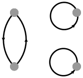
$$T^{I=0}(s,t,u) = 3T(s,t,u) + T(t,s,u) + T(u,t,s)\,,$$
$$T^{I=1}(s,t,u) = T(t,s,u) - T(u,t,s)\,,$$
$$T^{I=2}(s,t,u) = T(t,s,u) + T(u,t,s)\,.$$

- Wick contractions for $T(s,t,u) \equiv T_{\pi^+\pi^-\to\pi^0\pi^0}(s,t,u)$

[$\pi^+ = \bar{d}u,\ \pi^- = \bar{u}d,\ \pi^0 = (\eta_u - \eta_d)/\sqrt{2},\ \eta_u \equiv \bar{u}u,\ \eta_d \equiv \bar{d}d$]:

$$T(s,t,u) = T_{\pi^+\pi^-\to\eta_u\eta_u}(s,t,u) - T_{\pi^+\pi^-\to\eta_u\eta_d}(s,t,u)$$

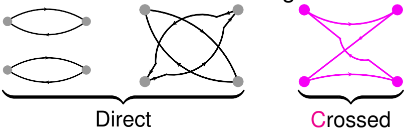


Contractions like  and  do not contribute

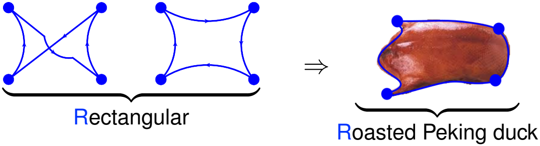
Wick contractions for $\pi\pi$ scattering (2)

- Various possible Wick contractions for $\pi\pi$ scattering:

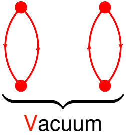
"Connected":



Singly-disconnected:



Doubly-disconnected:



difficult in lattice!

- Isospin symmetric $\pi\pi$ scattering amplitudes:

$$T^{I=0}(s,t,u) = T_D^{I=0}(s,t,u) + T_C^{I=0}(s,t,u) + T_R^{I=0}(s,t,u) + T_V^{I=0}(s,t,u)\,,$$

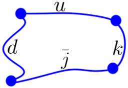
$$T^{I=1}(s,t,u) = T_D^{I=1}(s,t,u) + T_R^{I=1}(s,t,u)\,,$$

$$T^{I=2}(s,t,u) = T_D^{I=2}(s,t,u) + T_C^{I=2}(s,t,u)$$

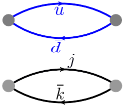
Calculation of contractions in PQCHPT (1)

- Additional (auxiliary) flavors in PQQCD $(u, d, j, k, \tilde{j}, \tilde{k})$

  $\Rightarrow$ Wick contractions can be calculated separately



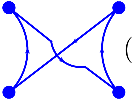
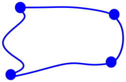
$$(s,t,u) = T_{(u\bar{d})(d\bar{j})\to(u\bar{k})(k\bar{j})}(s,t,u)$$

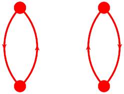
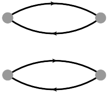


$$(s,t,u) = T_{(u\bar{d})(j\bar{k})\to(u\bar{d})(j\bar{k})}(s,t,u)$$

  calculable as amplitudes of “physical” processes in PQCHPT

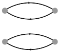
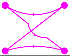
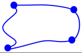
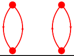
- The other contractions can be obtained by crossing symmetry, e.g.:

 $(s,t,u) =$  $(s,u,t),$

 $(s,t,u) =$  $(t,s,u), \ldots$

Calculation of contractions in PQCHPT (2)

- $\pi\pi$ cattering lengths $a_X^{IJ} = \lim_{q^2\to 0}(q^2)^{-J} T_X^{IJ}(4M_\pi^2 + 4q^2)$

	$10^2 a_X^{00}$	$10^2 a_X^{20}$	$10^2 M_\pi^2 a_X^{11}$	$10^4 M_\pi^4 a_X^{02}$	$10^4 M_\pi^4 a_X^{22}$
D 	$0.35(24)$	$0.35(24)$	$0.02(26)$	$3.5(2.0)$	$3.5(2.0)$
C 	$2.41(12)$	$-4.81(23)$	0	$0.95(96)$	$-1.9(1.9)$
R 	$14.8(7)$	0	$3.59(26)$	$6.7(7.8)$	0
V 	$2.48(38)$	0	0	$0.8(7.3)$	0
Total	$20.0(2)$	$-4.46(7)$	$3.61(4)$	$11.9(8)$	$1.54(71)$

- The R-type contribution dominates as long as it contributes:
  - ☞ expected $\Leftarrow$ leading order in both chiral and $1/N_c$ expansions
  - ☞ neglecting the vacuum-type contribution would reduce the isoscalar S-wave $\pi\pi$ scattering length by about 12%.

Summary

- Lattice results assisted with EFT provide invaluable inputs in understanding hadronic phenomena
- Experimental and lattice data need to be analyzed using EFT with symmetry constraint built in to reach a precision hadron spectroscopy
- Some topics for exotic hadrons:
  - Compute heavy-hadron scattering lengths neglecting coupled channels: for understanding near-threshold structures X.-K.Dong, FKG, B.-S.Zou, PRL126(2021)152001
  - $J/\psi\pi$ scattering: relevant for understanding double-J/ψ system X.-K.Dong et al., Sci.Bull.66(2021)2462
  - $J/\psi N$ scattering: for near-threshold J/ψ photoproduction and P_c
  - ...

Thank you for your attention !