Lattice realization of ξ gauge

中国格点 QCD 第一届年会

施春江 (IHEP)
In collaborating with 杨一玻 (ITP), 陈莹 (IHEP).

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Content

- **ξ gauge and lattice implement**
- Results in ξ gauge

ξ gauge

Due to gauge invariance $A_\mu(x) \to A_\mu(x) + \frac{1}{g} D_\mu \alpha(x)$, path integral adds gauge fixing ξ-term.

$$\begin{aligned} Z &= \int \mathcal{D}A^\alpha e^{iS} = \int \mathcal{D}A\mathcal{D}\alpha \delta(G[A]) \det(G) e^{iS} \\ &= N \int \mathcal{D}A\mathcal{D}\Lambda \exp(-i\frac{\Lambda^2}{2\xi})\delta(\partial_\mu A^\mu - \Lambda)e^{iS} \\ &= N \int \mathcal{D}A \exp(-i\frac{(\partial_\mu A^\mu)^2}{2\xi})e^{iS} \end{aligned} \tag{1}$$

$$\mathcal{L}_{GF} = -\frac{G[A,\alpha]^2}{2\xi} = -\frac{(\partial_\mu A^\mu)^2}{2\xi}. \tag{2}$$

Choose gauge fixing functional $G[A,\alpha] = \partial_\mu A^\mu(x) - \Lambda(x)$.

The gluon field propagators,

$$\langle A^a_\mu(k) A^b_\nu(-k)\rangle = -i\delta^{ab}\overbrace{\frac{1}{k^2}[\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)}^{\Delta_T} + \overbrace{\xi\frac{k_\mu k_\nu}{k^2}}^{\Delta_L}]. \tag{3}$$

Motivation

1. 强子矩阵元, 重整化常数计算[Zhaofeng Liu, PhysRevD.2014] [Yujiang Bi, PhysRevD. 2017] .
2. Chiral condensite/ sysmetry breaking mass [Yi-Bo Yang, PhysRevLett. 2018] .
3. Check other gauge dependency in numerical calculation.

Renormalizable-ξ gauge (R_ξ gauge, ξ gauge) is a more general linear covariant gauge.

$$\begin{cases} \xi = 0, \text{Lorentz / Landau gauge} \\ \xi = 1, \text{Feymann-t'Hooft gauge.} \end{cases}$$

Landau gauge condition

$$\partial^\mu A_\mu^m = 0. \tag{4}$$

ξ gauge condition

$$\Lambda^m \equiv \partial^\mu A_\mu^m. \tag{5}$$

$$\Lambda(x) = \frac{1}{2}\sum_{m=0}^{8} \Lambda^m(x)\cdot\lambda_m \text{ in SU}(N_c)\text{, where } \Lambda^m(x) \sim \exp(-\frac{(\Lambda^m(x))^2}{2\xi}). \tag{6}$$

- Cucchieri et. al. (2009) 在 SU($N_c = 2$) 规范场提出了在格点 ξ 规范实现；
- Bicudo et. al. (2015) 在 SU($N_c = 3$) 格点规范场对 ξ 规范固定算法作了实现。

ξ-gauge fixing implement on lattice

For Landau gauge, $g(x)$ satisfies the minimumized functional:

$$\mathcal{F}_{LG}[U,g] = -\Re\mathfrak{e}\mathrm{Tr}\sum_{x,\mu} g(x)U_\mu(x)g^\dagger(x+\hat{e}_\mu), \tag{7}$$

For ξ gauge, the minimumized functional(Nielson-Ninomiya):

$$\mathcal{F}_\xi[U,g] = \mathcal{F}_{LG}[U,g] + \Re\mathfrak{e}\mathrm{Tr}\sum_{x} ig(x)\Lambda(x). \tag{8}$$

We define functional under ξ-gauge by using relaxation method:

$$\mathcal{F}'_\xi[U,g](x) = \sum_\mu \left(g(x)U_\mu(x)g^\dagger(x+\hat{e}_\mu) + g(x)U^\dagger_\mu(x-\hat{e}_\mu)g^\dagger(x-\hat{e}_\mu)\right) - i\Lambda(x). \tag{9}$$

ξ-gauge fixing implement on lattice

Relaxation iteration algothm:

```
tgfold = F[U];
g=1;
While (conver > GfAccu) && (n_gf < GFmax) Do:
     for cb :           //checkboards loop
          for 3 subgroup SU(2):
//extract SU(3) loop
               grelax();
          end for;
     end for;
     Reunitarize(g);   // new g(x) minimumizes F[U]
     U_new = g U g*;
     tgfnew = F[g U g*];
     conver = calc_theta(U_new) ;
//convergence criterion
end while
```

The iteration algothm

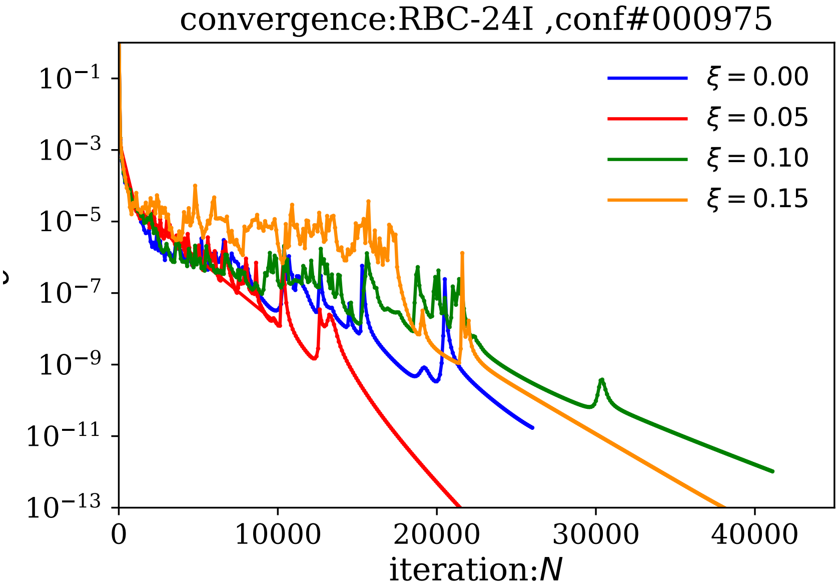
- Over relaxation
- FFT accelerated relaxation

The convergence parameter θ:

$$\Delta(x) \equiv \partial_\mu A^\mu(x) - \Lambda(x),$$

$$\theta = \frac{1}{N_c L^4} \sum_x \mathrm{Tr}\left[\Delta(x)\Delta^\dagger(x)\right] \tag{10}$$

Realize ξ gauge when $\theta < 10^{-14}$.



convergence:RBC-24I ,conf#000975
10^{-1}
10^{-3}
10^{-5}
10^{-7}
10^{-9}
10^{-11}
10^{-13}
0
10000
20000
30000
40000
iteration:N
$\xi = 0.00$
$\xi = 0.05$
$\xi = 0.10$
$\xi = 0.15$

Efficiency of ξ-gauge fixing implement(1)

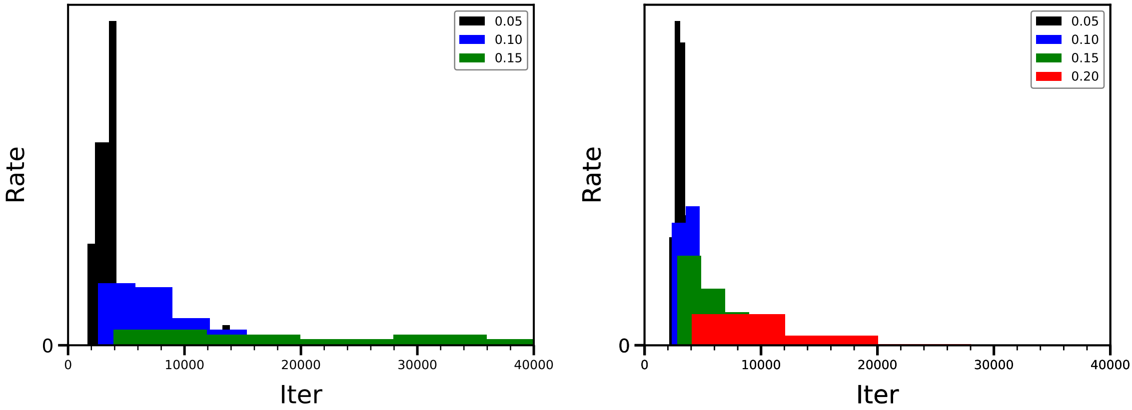


Figure: (a)RBC-24I confs，(b)RBC-32I confs.

Efficiency of ξ-gauge fixing implement(2)

Volume	β	ξ	$\leq$0.8	1.0	1.2
16^4	6.0	成功率（%)	$> 80\%$	$\sim 70\%$	$\sim 60\%$
		ξ	$\leq$0.5	0.8	1.0
24^4	6.0	成功率（%)	$> 80\%$	$\sim 40\%$	$\sim 14\%$

Table: 24I/32I 成功率

Volume	β	ξ	0.05	0.10	0.15
$24^3 \times 64$	2.13	成功率（%)	100%	74.6%	31.5%
Volume	β	ξ	0.10	0.15	0.20
$32^3 \times 64$	2.25	成功率（%)	100%	94.2%	61.7%

Table: 规范场组态参数

label	β	a^{-1}(GeV)	Volume	$m_s a / m_l a$	N_{conf}
24I	2.13	1.785(5)	$24^3 \times 64$	0.04/ 0.005	150
32I	2.25	2.383(9)	$32^3 \times 64$	0.03/0.004	150
32If	2.37	1.378(7)	$32^3 \times 64$	0.0186/0.0047	50

Content

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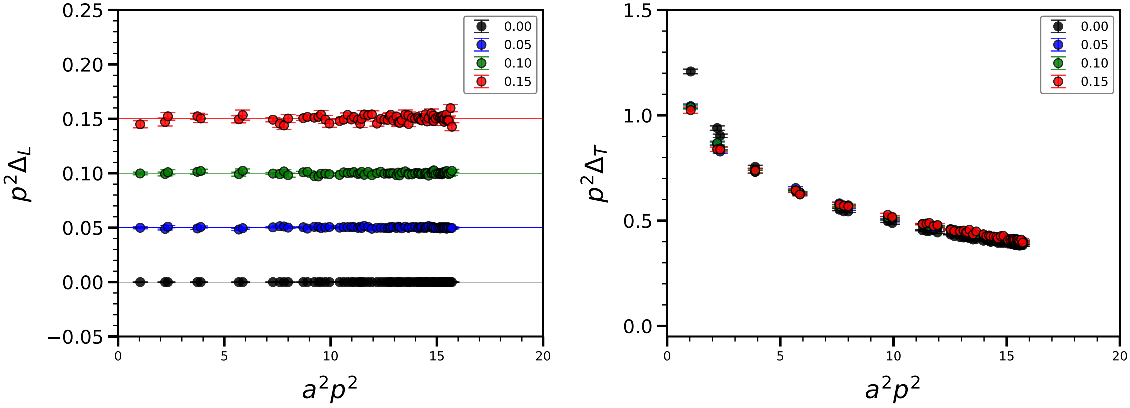
- **Results in ξ gauge**

Gluon propagators

$$\langle A^a_\mu(p) A^b_\nu(-p)\rangle = \frac{-i\delta^{ab}}{p^2}[\overbrace{\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)}^{\Delta_T} + \xi \overbrace{\frac{p_\mu p_\nu}{p^2}}^{\Delta_L}], \begin{cases} \Delta_T \sim \frac{1}{p^2} \\ \Delta_L \sim \frac{1}{p^2}\cdot\xi \end{cases}$$



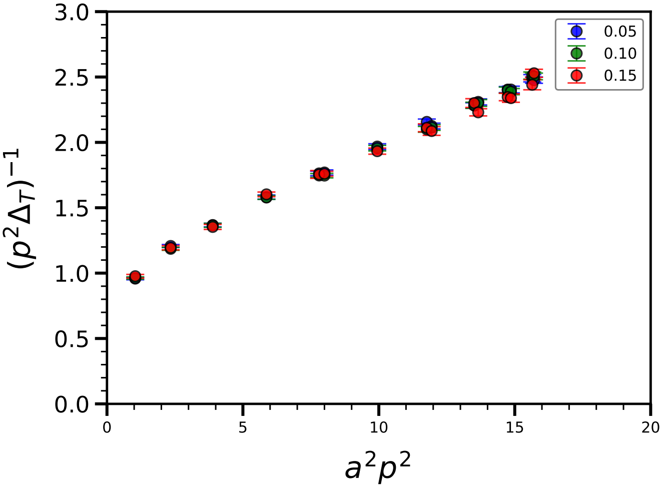
ξ renormalization effect

$$\xi = \xi_0 Z_\xi$$

$$\langle A_\mu A_\nu(p^2)\rangle = Z_A \langle A_\mu^{\text{bare}} A_\nu^{\text{bare}}\rangle$$

$$Z_\xi = Z_A$$

$$\to Z_A(p^2) \sim \frac{1}{p^2\Delta_T} Z_A(p^2) \qquad = Z_A(0)$$



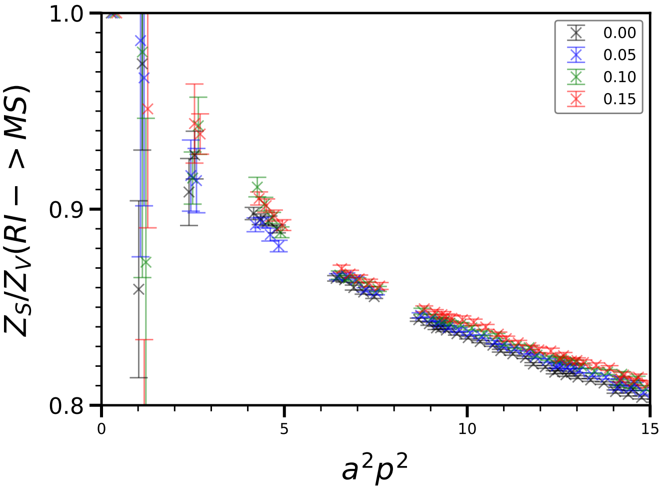
ξ gauge fixing results

ξ gauge perturbative matching from RI/MOM $\to$ $\overline{\text{MS}}$.

$$Z^{\text{RI/MOM}}(Q) = \left.\frac{\langle p \,|\, O^{(0)} \,|\, p\rangle}{\langle p \,|\, O^{\text{bare}} \,|\, p\rangle}\right|_{p^2=-Q^2}$$

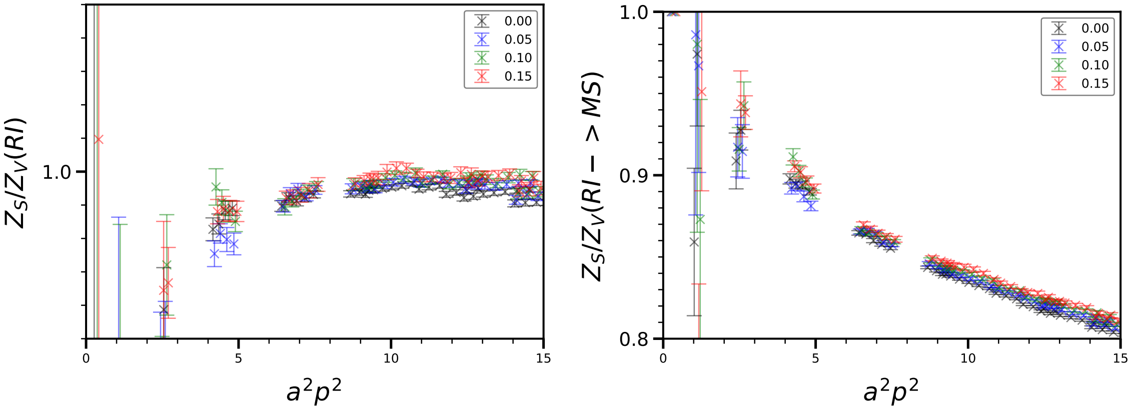
Zhaofeng Liu, PhysRevD.2014
Yujiang Bi, PhysRevD. 2017

ξ-matching(up to 3-loop):
RI' $\to$ $\overline{\text{MS}}$:Gracey,2003,hep-ph/0304113
Gracey,2003,hep-ph/0306163



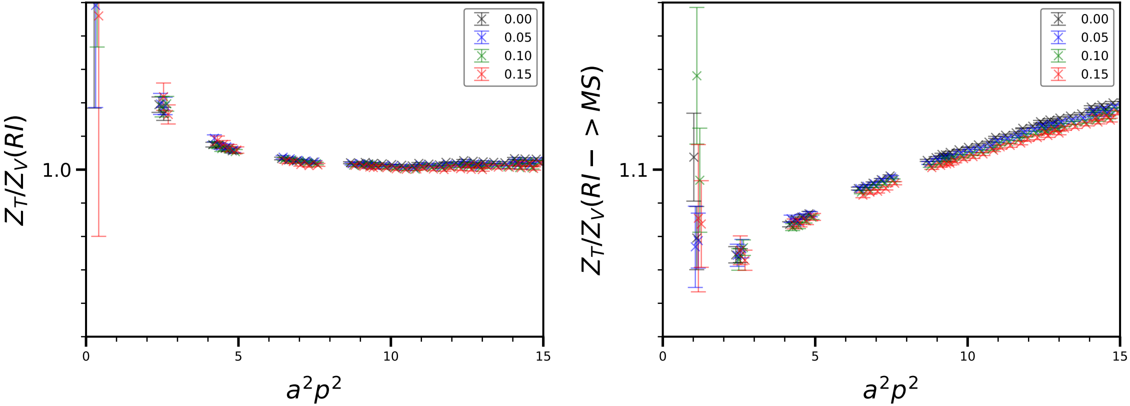
Gockeler,2010,hep-lat/1003.5756

ξ gauge fixing results



$$Z_S(p^2) = Z_A(0) + C_1 a^2 p^2 \text{at large } a^2 p^2$$

ξ gauge fixing results



$$Z_T(p^2) = Z_A(0) + C_1 a^2 p^2 \text{at large } a^2p^2$$

Renormalization constants under ξ gauge

Table: 24I 上 $Z^{\overline{\text{MS}}}$ (2.0GeV) 外推到零动量, χ^2/d.o.f. < 1.5(preliminary)

ξ	$Z_S/Z_V(\overline{\text{MS}})$	$Z_T/Z_V(\overline{\text{MS}})$
Landau	0.9180(33)	1.0724(10)
0.05	0.9199(38)	1.0737(15)
0.10	0.9238(43)	1.0716(15)
0.15	0.9209(53)	1.0689(15)

Summary

1. 较 Bicudo (2015) 的 ξ-gauge fixing 有所提升。
2. 发现影响 ξ 规范的因素 a)Lattice Volumn, b)coulping g_0 and plaq U_P
3. 期待 ξ 规范拓展领域内对规范的选择，为领域内其他研究提供新途径。初步程序共享https://github.com/ChunJiang-Shi/xi-gauge-fixing。
4. ξ 规范下的重整化常数计算初步简单地校验了 $\overline{\text{MS}}$ 方案下的规范不变性。

Outlook

1. ξ 规范计算热点是可并行化的，在更大的格子上，预计实现 GPU+FFT 并行后提高并行度。
2. IR behaviors，Chiral sysmetry breaking contribution...

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注明

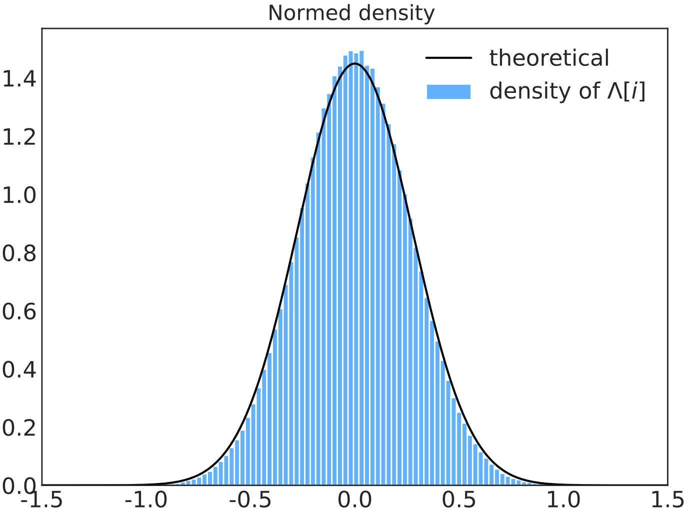
$$A_\mu\left(x+\hat{e}_\mu/2\right)=\left.\frac{U_\mu(x)-U_\mu^\dagger(x)}{2ig_0}\right|_{\text{traceless}}$$

重要的一点是，高斯分布的方差：

$$\sigma=\frac{2N_c}{\beta}\cdot\xi=g_0^2\xi \tag{11}$$

在规范场的数值大小水平下，高斯分布方差直接以 ξ 为基准扩大 g_0^2 倍，规范场组态的**耦合强度（g_0 和 β）的大小将能影响到程序能够实现的 ξ 水准。**

ξ 规范固定程序效率


Normed density
theoretical
density of $\Lambda[i]$
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
-1.5
-1.0
-0.5
0.0
0.5
1.0
1.5

规范固定程序实现

程序实现步骤如下：

1. 采用 Box-Muller 方法在每个 site 上产生一个 8 维的 gaussian 分布数组 $\Lambda^m(x) \in \mathbb{R}$ 作为 SU(3) 生成元系数，gaussian 方差为 σ，组合 SU(3) 元：

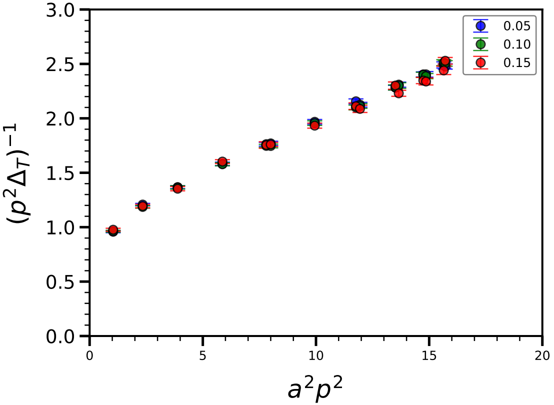
$$\Lambda(x) = \frac{1}{2} \sum_{m=0}^{8} \Lambda^m(x) \cdot \lambda_m, \tag{12}$$

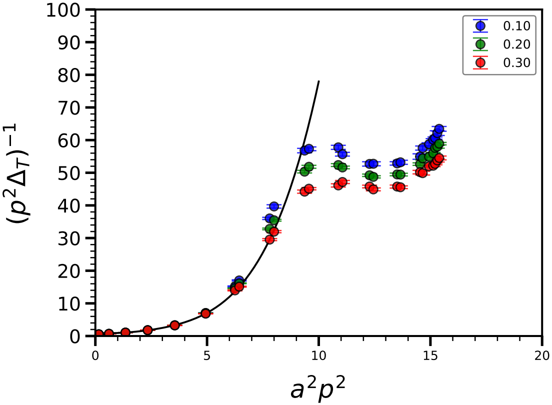
2. 驰豫算法迭代：Relaxation/Over-relaxion iteration.
   算法核心步骤：定义式9的 $K(x)$ 参与驰豫算法的迭代，迭代将寻找 $\mathcal{E}_\xi$ 的极值.
3. 计算收敛参数 θ：

$$\theta = \frac{1}{N_c L^4} \sum_x \mathrm{Tr}\left[\Delta(x)\Delta^\dagger(x)\right] \tag{13}$$

认为当 $\theta < 10^{-14}$ 时实现 ξ 规范固定。

Gluon propagators





Chiral expoloration

Momentum expoloration