QCD at finite temperature and density within functional renormalization group approach

Wei-jie Fu

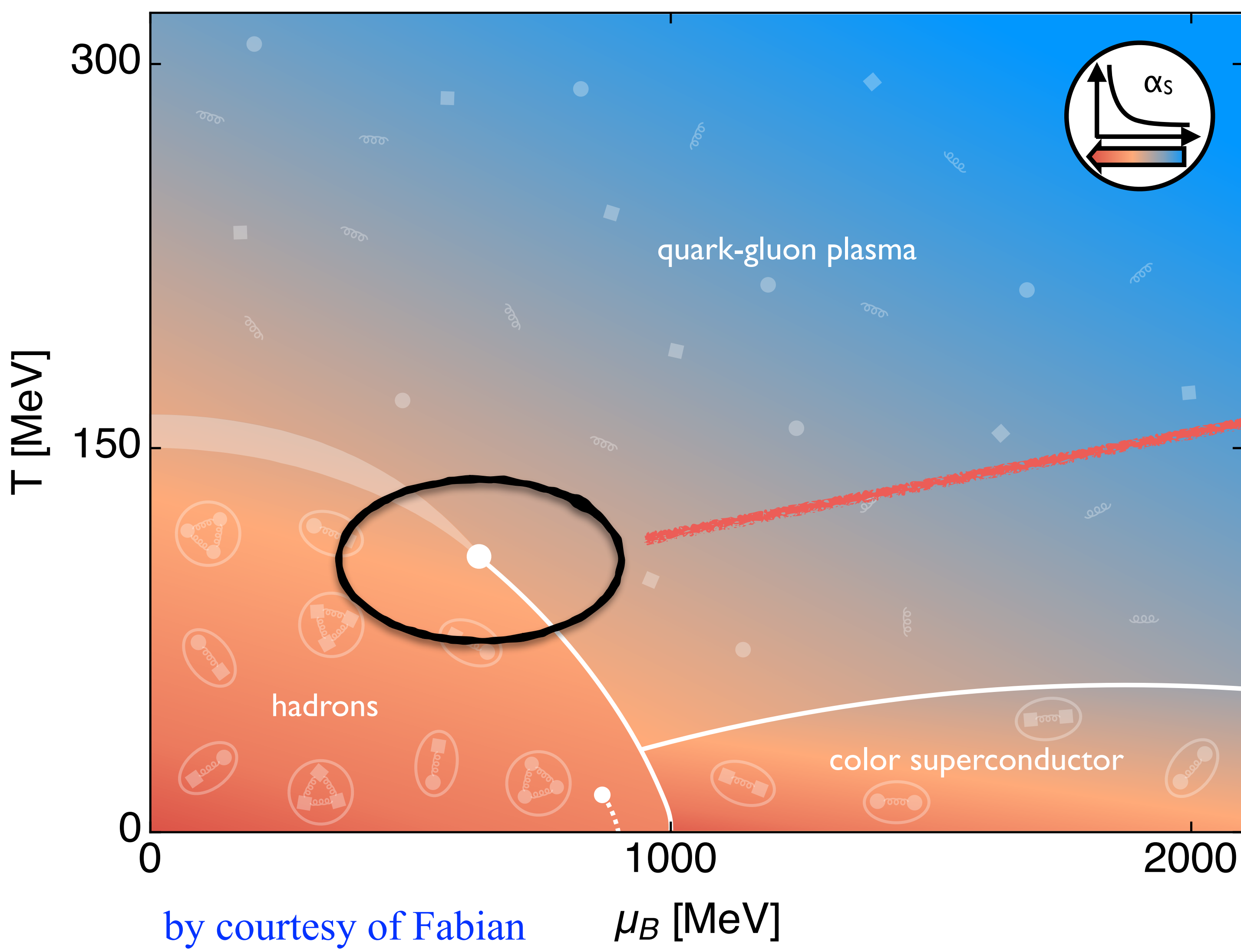
Dalian University of Technology

中国格点QCD第一届年会

online，2021年10月30日-11月2日

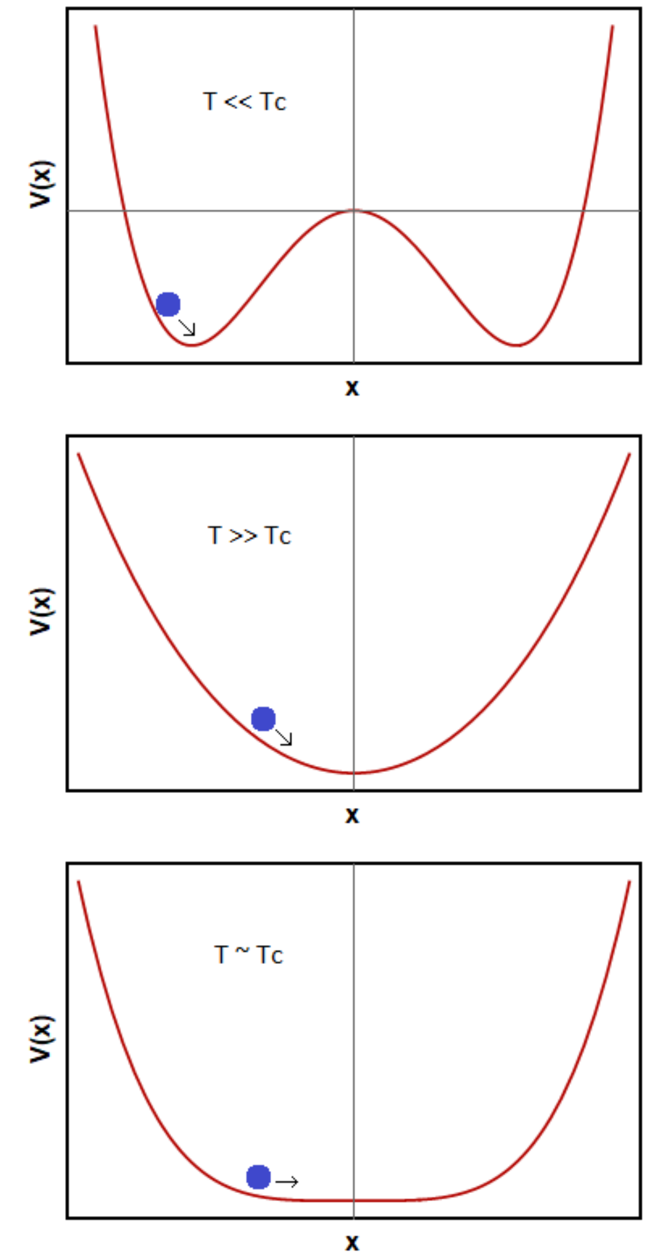
fQCD collaboration:
J. Braun, Y.-r. Chen, WF, F. Gao, J. Horak, C. Huang, F. Ihssen, J. M. Pawlowski, F. Rennecke, D. Rosenblüh, F. Sattler, B. Schallmo, C. Schneider, Y.-y. Tan, S. Töpfel, R. Wen, J. Wessely, N. Wink, S. Yin

Critical end point in QCD phase diagram



by courtesy of Fabian Rennecke

cf. Hengtong Ding's talk



Critical slowing down

Calls for:

- Nonperturbative approach of QCD.
- Errors controllable at finite densities.
- Real-time description of strongly interacting systems.

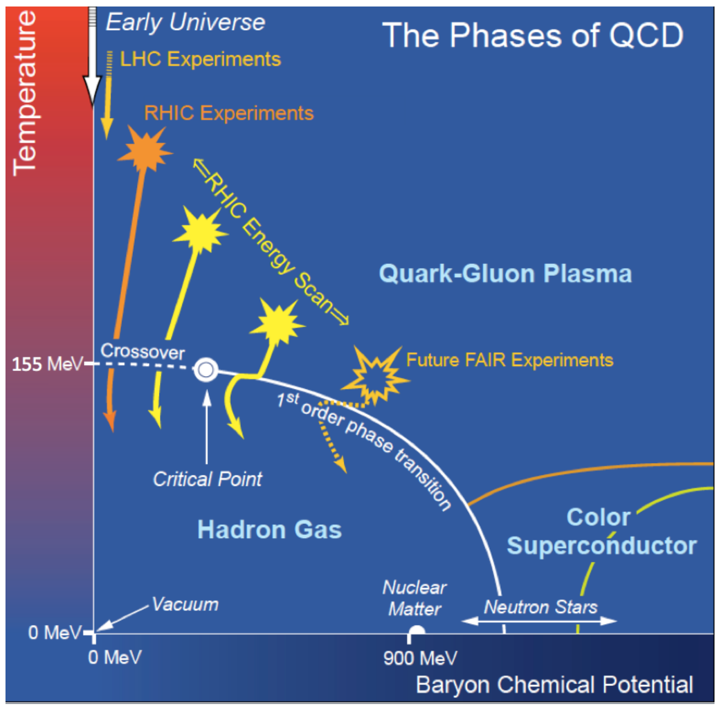
$$\tau = \xi^z f(k\xi)$$

ξ: correlation length

τ: relaxation time

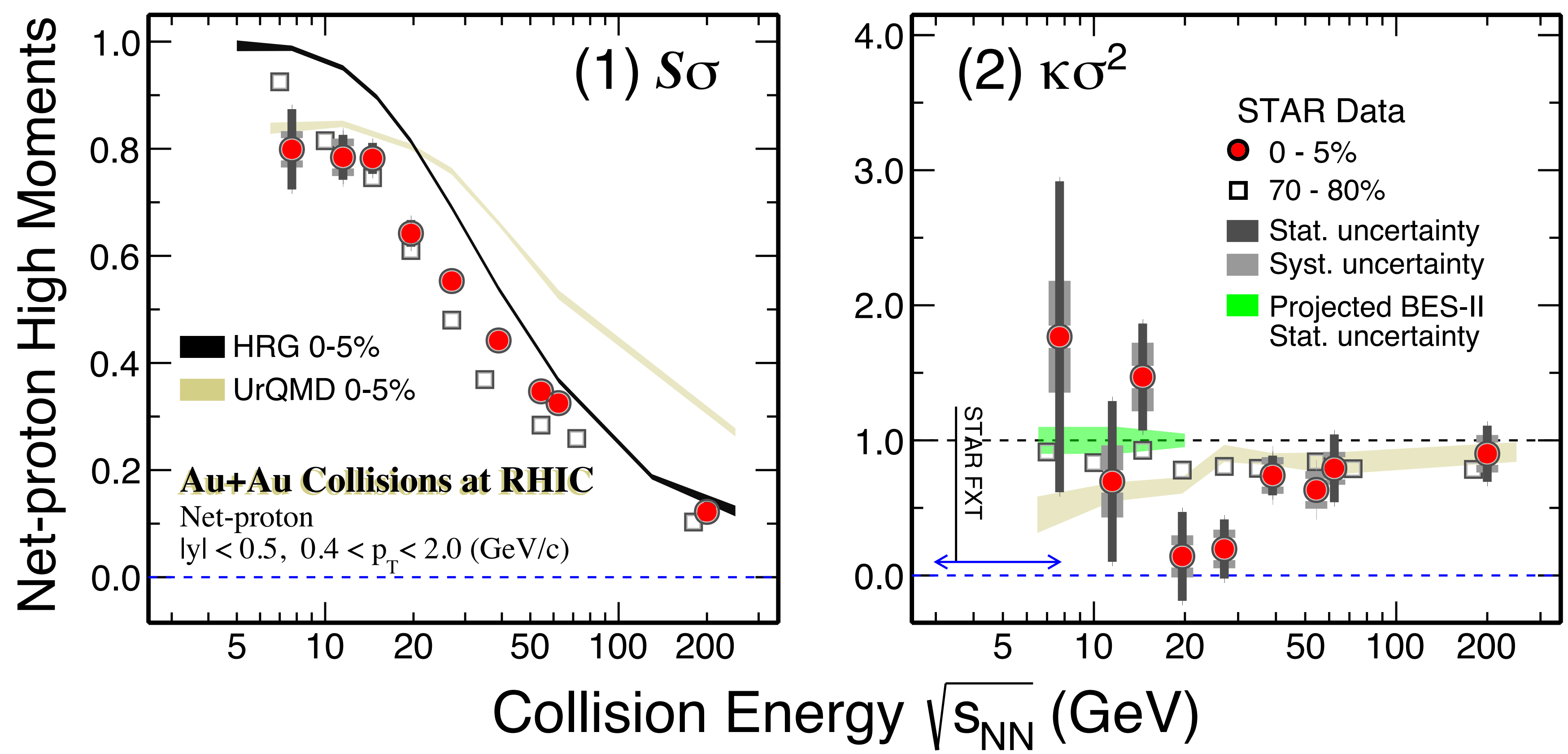
z: dynamical critical exponent

Recent experimental results at RHIC



The Hot QCD White Paper (2015)

Skewness and kurtosis of net-proton distributions:



J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301;
M. Abdallah *et al.* (STAR), *PRC* 104 (2021), 024902.

- The non-monotonicity of the kurtosis is observed with 3.1σ significance.
- Hint of entering critical region?

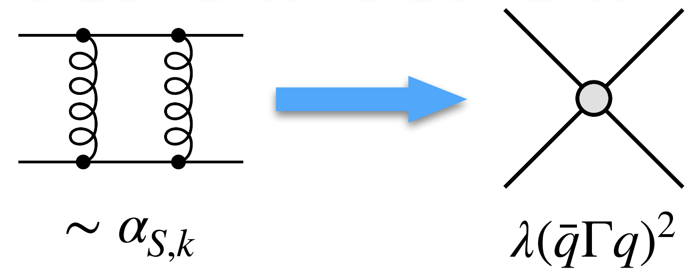
cf. Xiaofeng Luo's talk

Outline

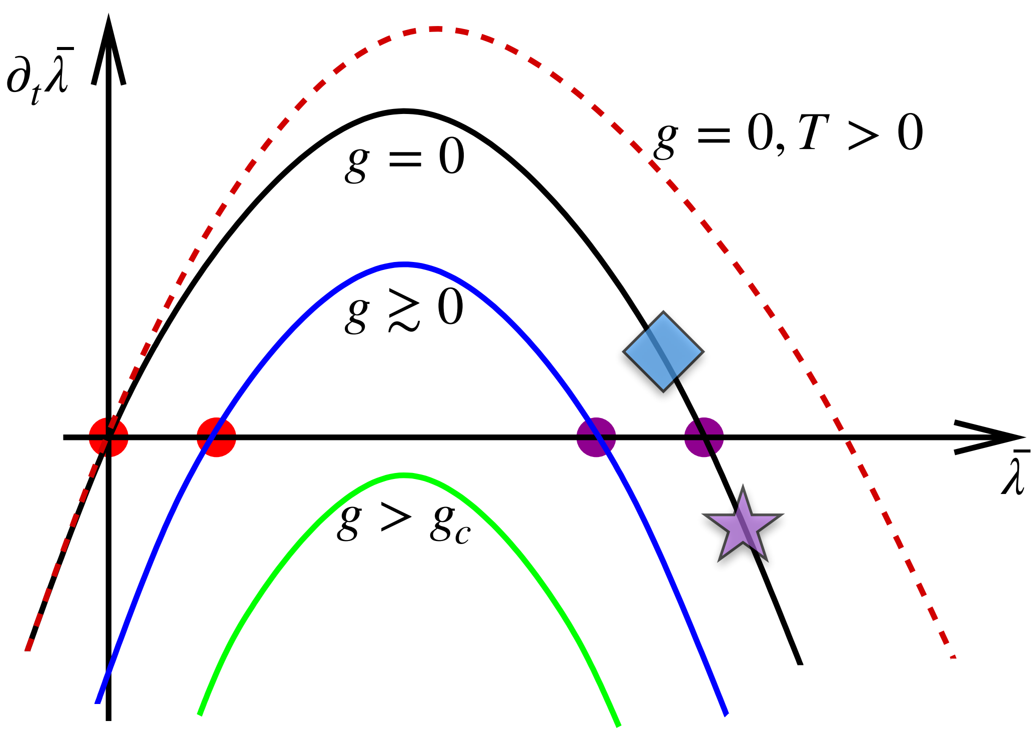
- Introduction
- Perspective from renormalization group
- fRG approach to QCD
- fRG Results for QCD at finite T and muB
- Summary

Quark mass production in RG

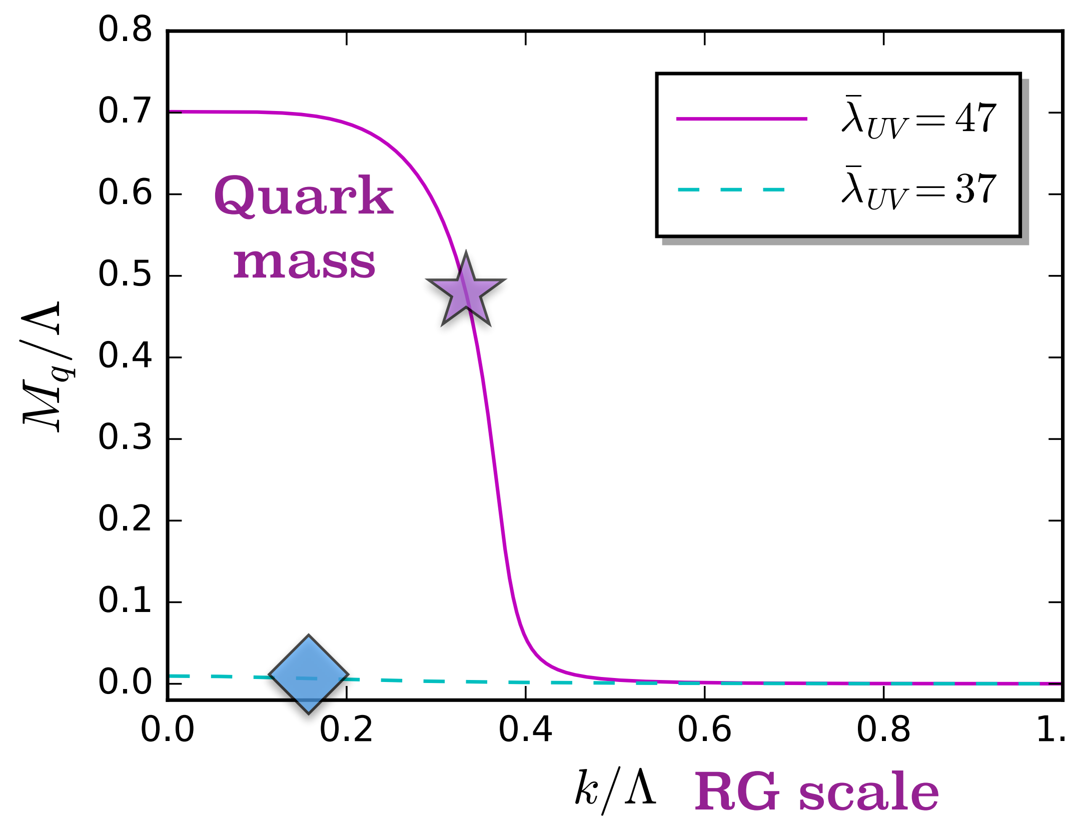
- **Effective 4-quark coupling generated:**



Flow of 4-quark coupling

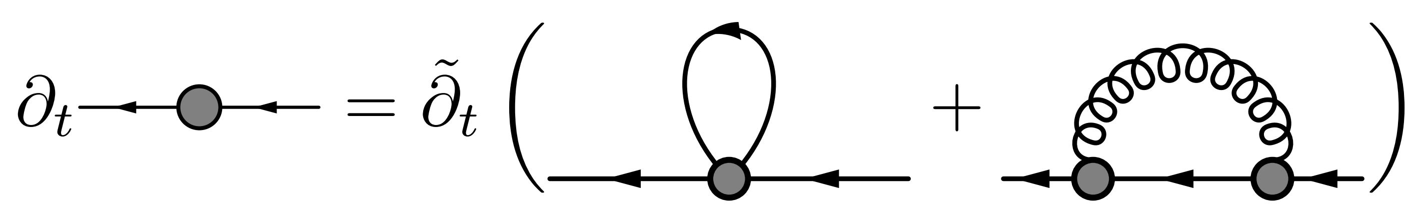


Braun, Gies, *JHEP* 06 (2006) 024.



WF, Huang, Pawlowski, Tan, in preparation.

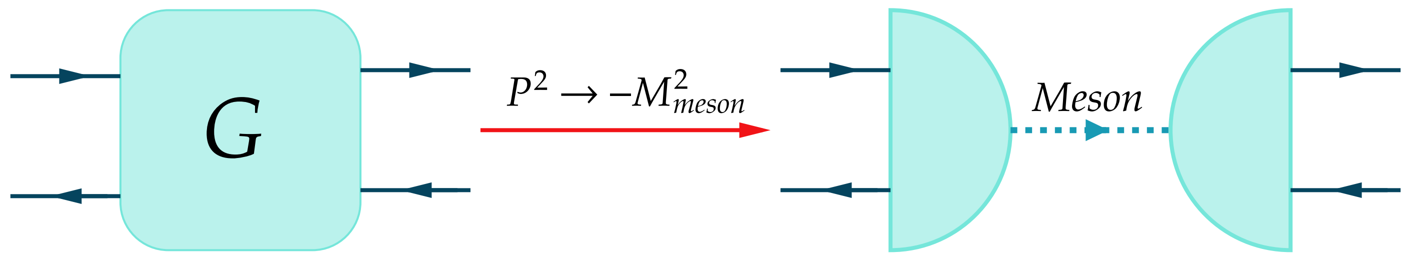
$$\partial_t\bar{\lambda} = (d-2)\bar{\lambda} - a\bar{\lambda}^2 - b\bar{\lambda}g^2 - cg^4\,,$$



- **Understanding quark mass production from the viewpoint of phase transition.**
- **Counterpart of gap equation in terms of RG flow.**

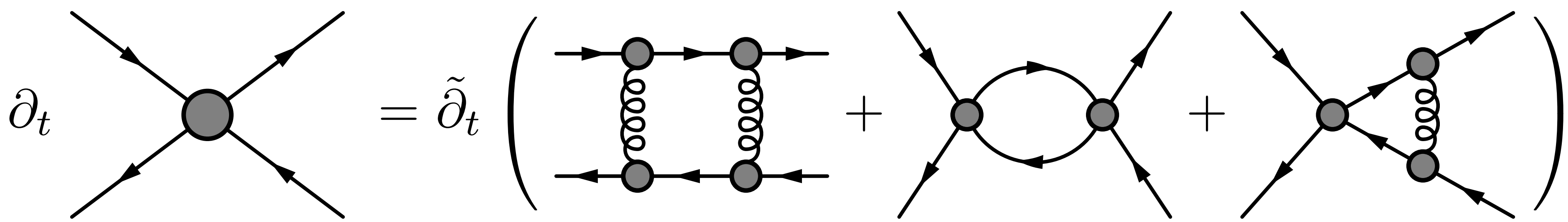
Bound states in RG

- **Bound states encoded in *n*-point correlation functions:**

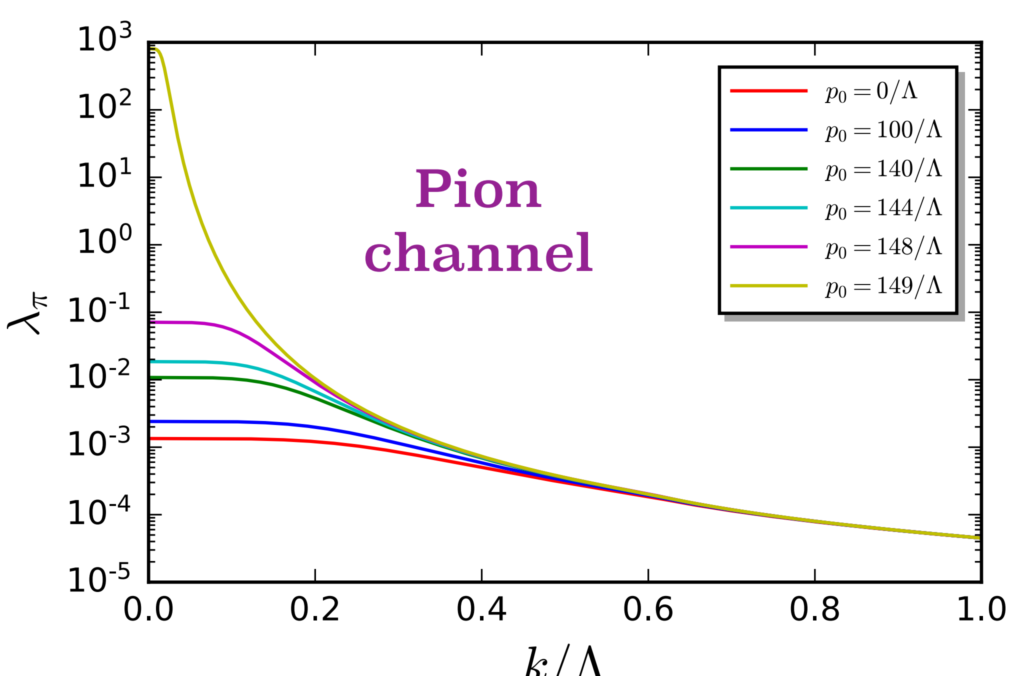


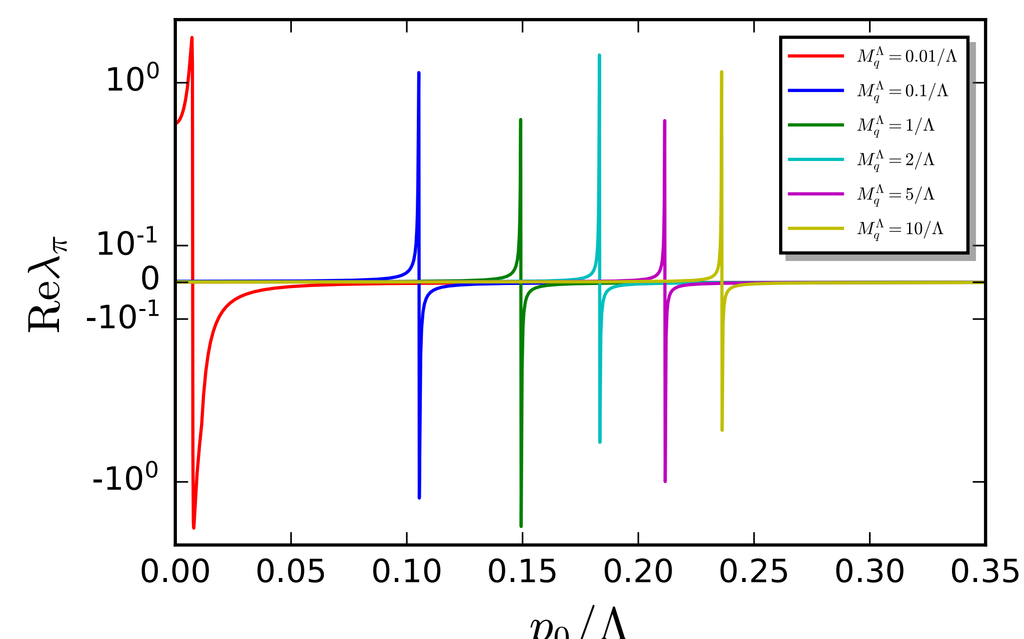
- **Flow equation of 4-quark interaction:**

cf. Lei Chang's talk



Note: playing the same role as the Bethe-Salpeter equation.





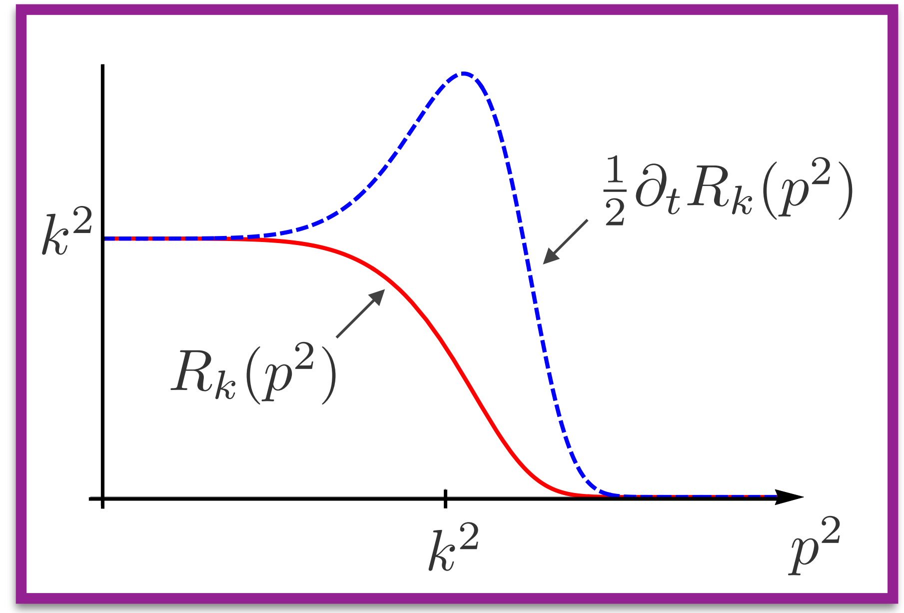
WF, Huang, Pawlowski, Tan, in preparation.

Functional renormalization group

Introduce a IR suppression regulator:

$$W_k[J] = \ln \int [d\varphi] \exp\Big\{ - S[\varphi] - \Delta S_k[\varphi] + J\varphi \Big\}$$

$$\Delta S_k[\varphi] = \frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} \varphi(-q) R_k(q) \varphi(q)$$

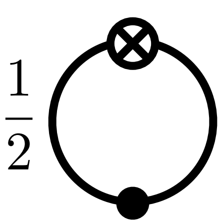


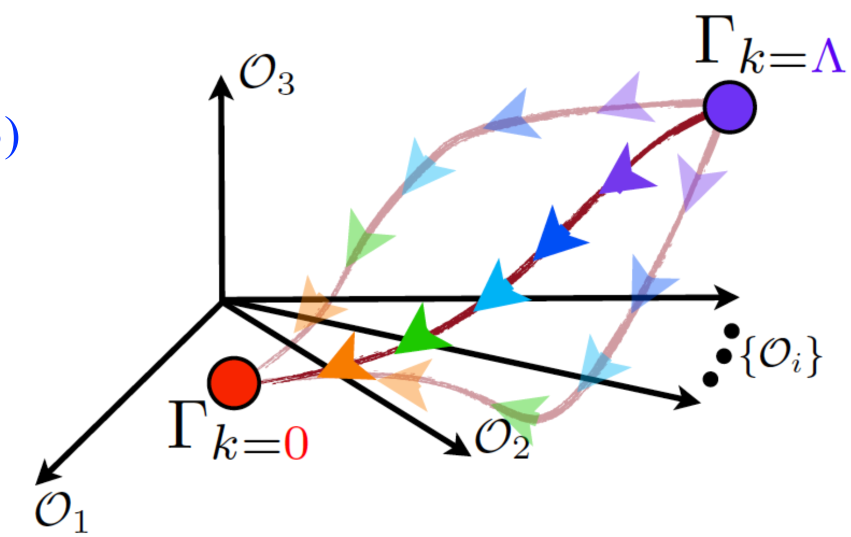
Scale dependent effective action:

$$\Gamma_k[\phi] = -W_k[J] + J\phi - \Delta S_k[\phi]$$

Evolution of the action: C. Wetterich, *PLB*, 301, 90 (1993)

$$\partial_t \Gamma_k[\Phi] = \frac{1}{2} \mathrm{STr}\Big[\partial_t R_k \cdot (\Gamma_k^{(2)}[\phi] + R_k)^{-1}\Big] = \frac{1}{2}$$

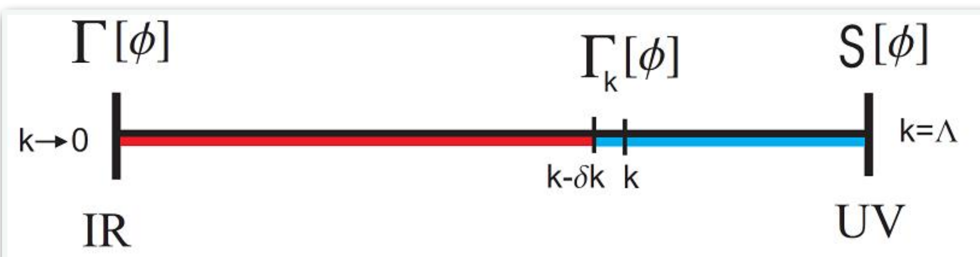




- Evolution in different hierarchies of scales: connecting QCD with LEFTs.
- No sign problem: accessible to finite density and real time.
- Infinite tower of coupled functional ODEs: truncation necessary.

QCD within the fRG approach

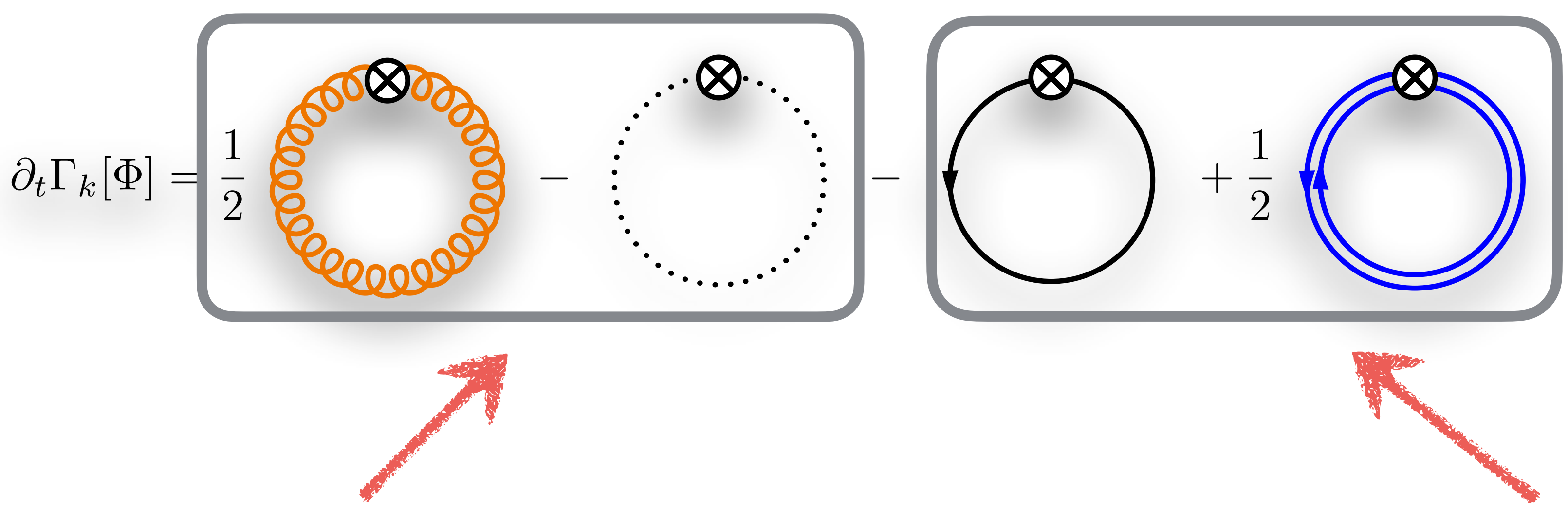
FRG



Rebosonized QCD Effective action:

$$
\begin{aligned}
\Gamma_k = \int_x \Big\{ & \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} + Z_c\,(\partial_\mu \bar{c}^a) D^{ab}_\mu c^b + \frac{1}{2\xi}(\partial_\mu A^a_\mu)^2 \\
& + \frac{1}{2}\int_p A^a_\mu(-p)\left(\Gamma^{(2)\,ab}_{AA\mu\nu}(p) - Z_A \Pi^{\perp}_{\mu\nu}\delta^{ab} p^2\right) A^b_\nu(p) \\
& + \bar{q}\left[Z_q\,(\gamma_\mu D_\mu - \gamma_0 \hat{\mu}) + m_s(\sigma_s)\right] q \\
& - \lambda_q \left[(\bar{q}\,\tau^0 q)^2 + (\bar{q}\,\vec{\tau} q)^2\right] + h\,\bar{q}\,\left(\tau^0 \sigma + \vec{\tau}\cdot\vec{\pi}\right) q \\
& + \frac{1}{2} Z_\phi\,(\partial_\mu \phi)^2 + V_k(\rho, A_0) - c_\sigma\,\sigma - \frac{1}{\sqrt{2}}\,c_{\sigma_s}\,\sigma_s \Big\},
\end{aligned}
$$

Flow equation:

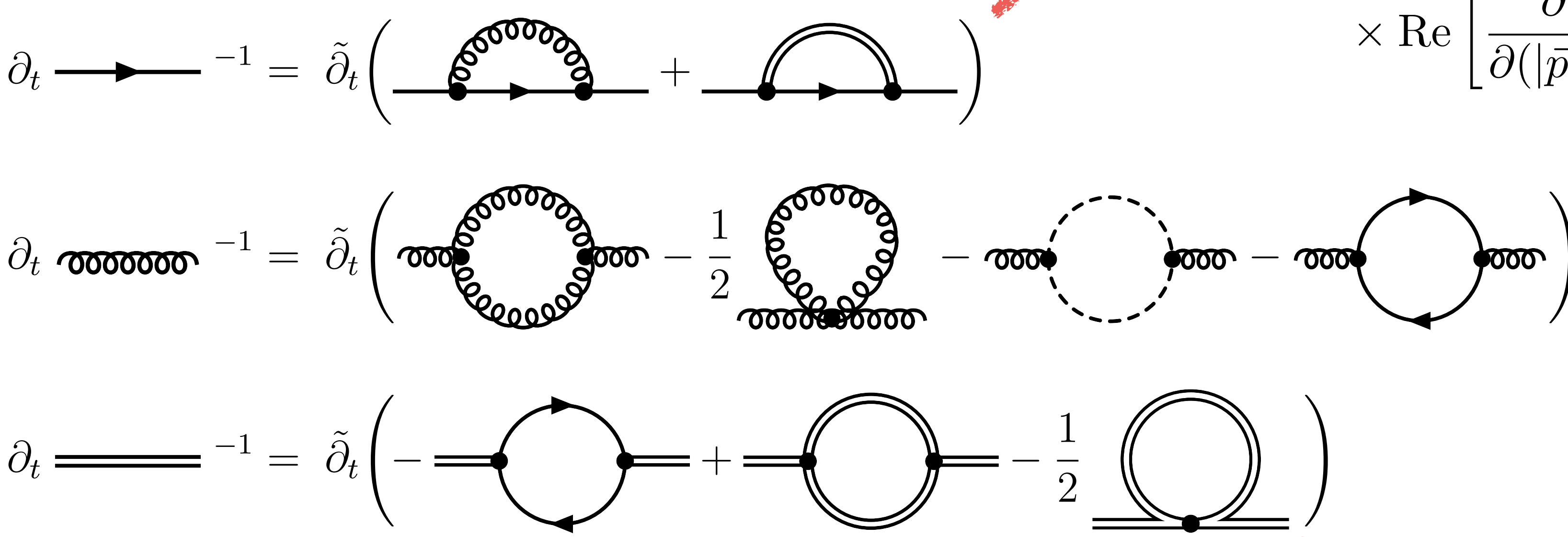


glue sector

matter sector

Propagators and anomalous dimensions

Flow equation:



Quark anomalous dimension:

$$\eta_{q,k}(p_0,\vec{p}) = \frac{1}{4Z_{q,k}(p_0,\vec{p})}$$

$$\times \,\mathrm{Re}\left[\frac{\partial}{\partial(|\vec{p}|^2)}\mathrm{tr}\left(i\vec{\gamma}\cdot\vec{p}\left(-\frac{\delta^2}{\delta\bar{q}(p)\delta q(p)}\partial_t\Gamma_k\right)\right)\right] ,$$

Glue sector:

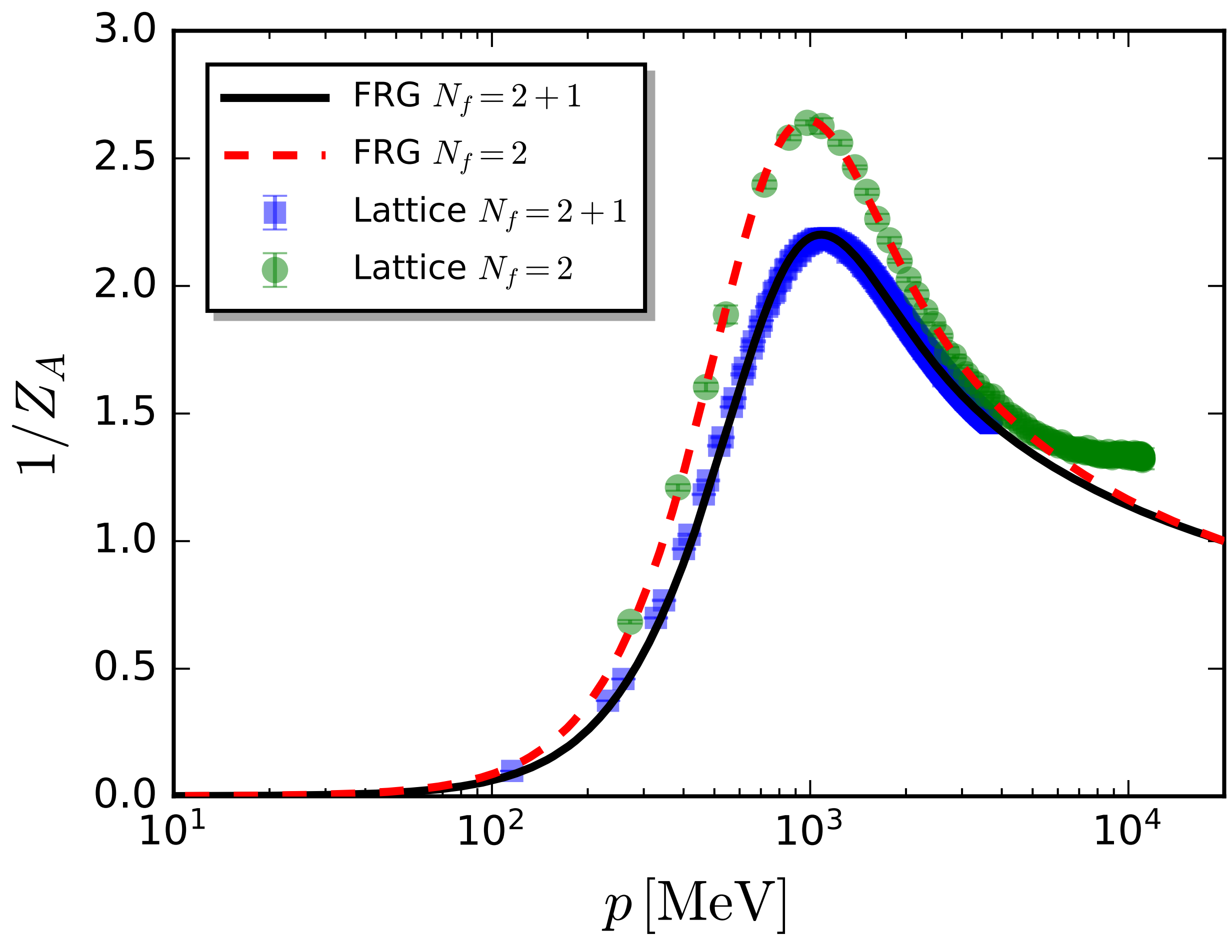
$$\eta_A = \eta_{A,\mathrm{vac}}^{\mathrm{QCD}} + \Delta\eta_A^{\mathrm{glue}} + \Delta\eta_A^{q}$$

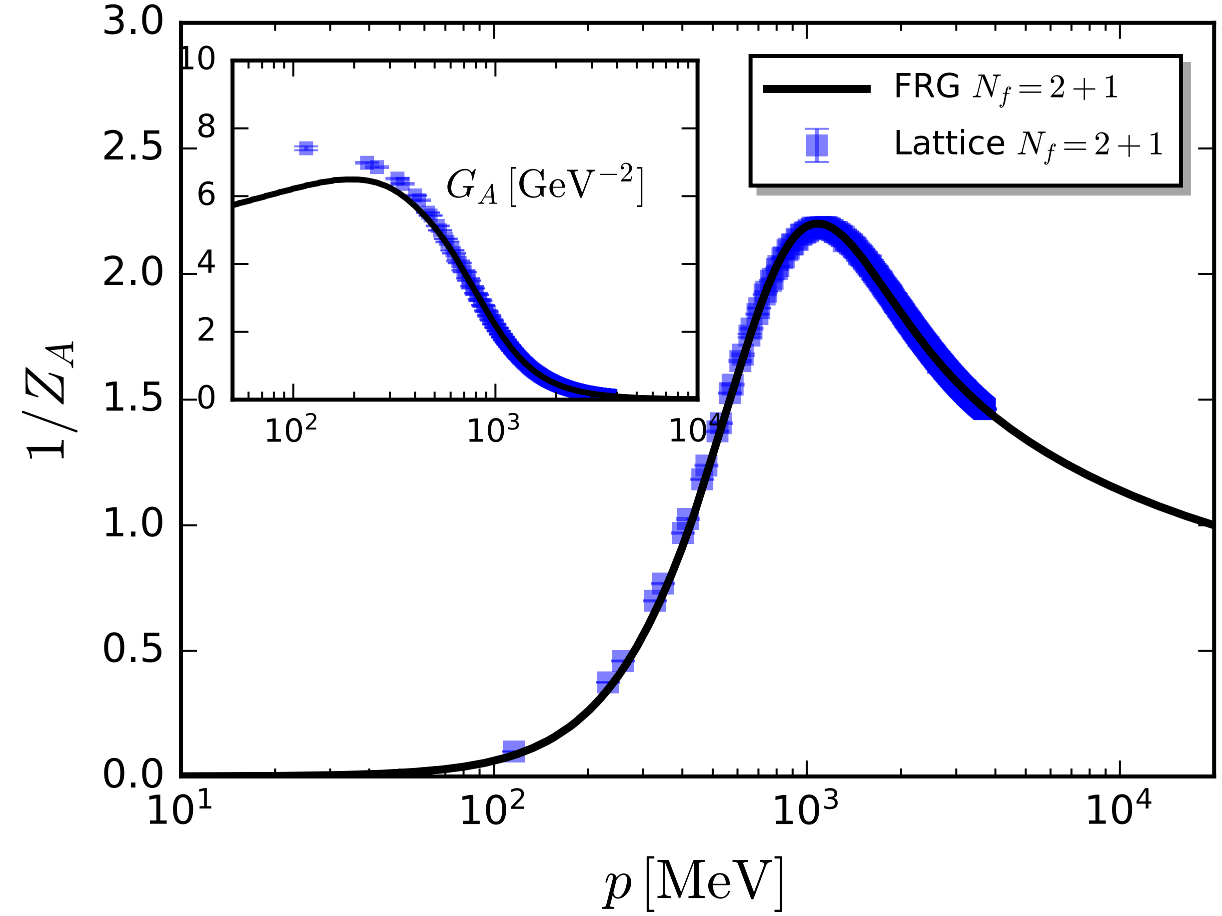
Meson anomalous dimension:

$$\eta_{\phi,k}(p_0,\vec{p}) = -\frac{1}{Z_{\phi,k}}\frac{1}{\delta_{ij}}\frac{\partial}{\partial(|\vec{p}|^2)}\frac{\delta^2\partial_t\Gamma_k}{\delta\pi_i(-p)\delta\pi_j(p)} ,$$

Gluon dressing function

fRG: WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032





fRG $N_f = 2$: Cyrol, Mitter, Pawlowski,Strodthoff, *PRD* 97 (2018), 054006

Lattice $N_f = 2$: Sternbeck *et al.*, *PoS* (2012) LATTICE2012, 243

Lattice $N_f = 2+1$: Boucaud *et al.*, *PRD* 98 (2018), 114515

For the moment we adopt:

$$\eta_A = \eta_{A,\mathrm{vac}}^{\mathrm{QCD}} + \Delta\eta_A^{\mathrm{glue}} + \Delta\eta_A^{q}$$

$$\eta_{A,\mathrm{vac}}^{\mathrm{QCD}} = \eta_{A,\mathrm{vac}}^{\mathrm{QCD}}\Big|_{N_f=2} + \eta_{A,\mathrm{vac}}^{s}\,,$$

Thermal quark loop:

$$\Delta\eta_{A,T}^{q} = -\frac{1}{2(N_c^2-1)}\delta_{ab}\Pi^{\mathrm{M}}_{\mu\nu}(p)\left(\left[\overline{\mathrm{Flow}}^{(2)}{}^{ab}_{AA\mu\nu}(p)\right]^{(q)}_{T} - \left[\overline{\mathrm{Flow}}^{(2)}{}^{ab}_{AA\mu\nu}(p)\right]^{(q)}_{T=0}\right)_{\substack{p_0=0\\|\vec{p}|=k}},$$

Vacuum QCD within fRG

Input: fundamental parameters of QCD at a large momentum scale: $\Lambda = 20$ GeV

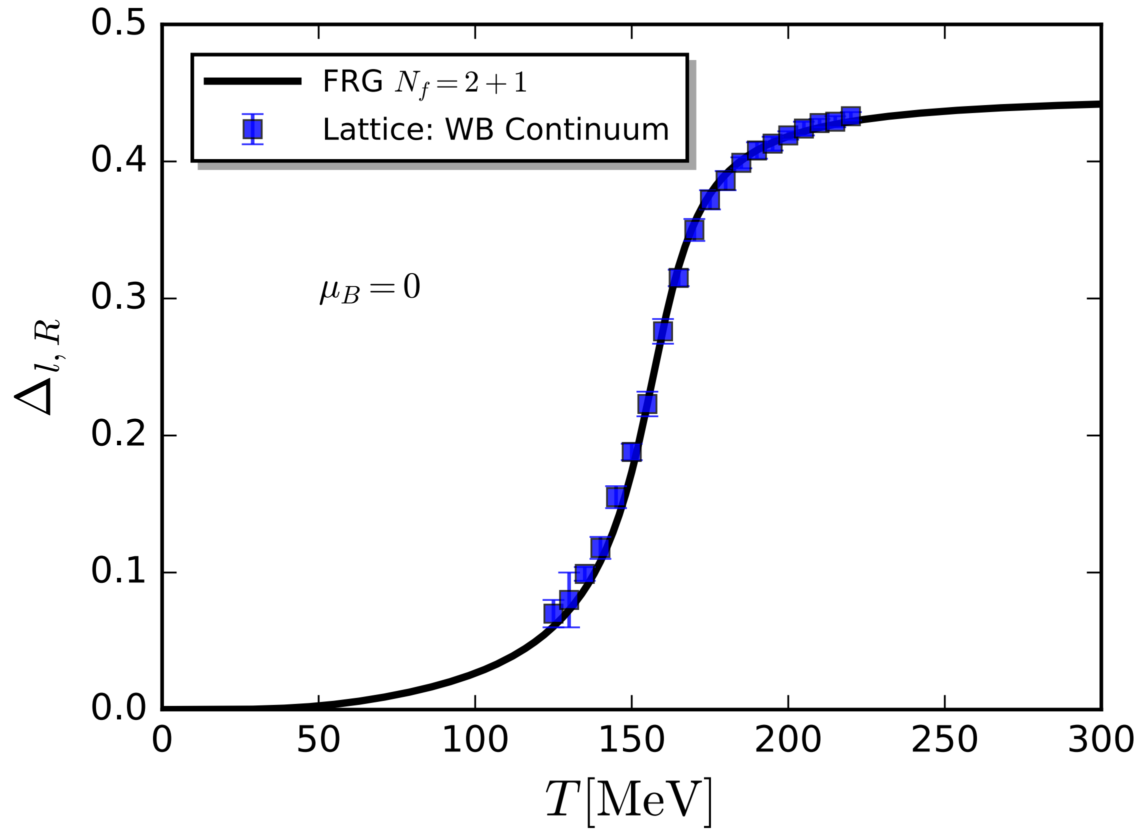
2-flavor QCD

- $\alpha_{s,k=\Lambda}$
- $m_{u,k=\Lambda} = m_{d,k=\Lambda} = m_{l,k=\Lambda}(m_\pi)$ $\qquad$ $m_\pi = 138$ **MeV**

2+1-flavor QCD

- $\alpha_{s,k=\Lambda}$
- $m_{u,k=\Lambda} = m_{d,k=\Lambda} = m_{l,k=\Lambda}(m_\pi)$ $\qquad$ $m_\pi = 138$ **MeV**
- $\dfrac{m_{s,k=\Lambda}}{m_{l,k=\Lambda}} = 27$ $\qquad$ $m_K = 498$ **MeV**

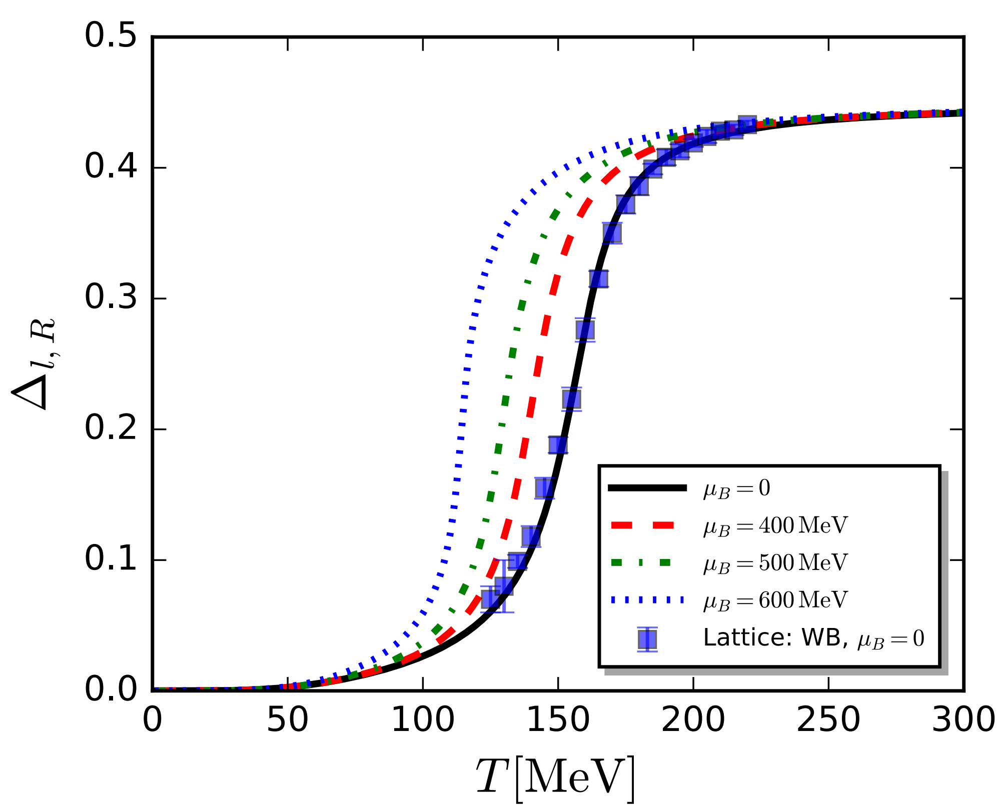
Renormalized light quark condensate

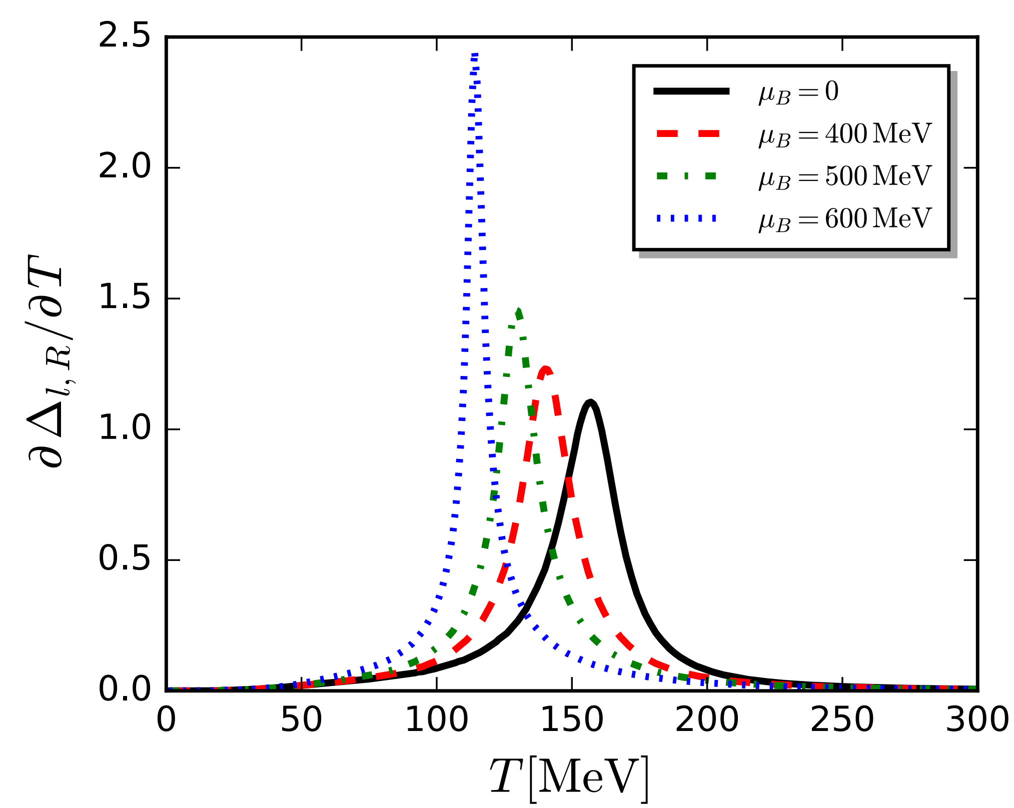


$$\Delta_{q_i} \simeq -m_{q_i}^0 T \sum_{n\in\mathbb{Z}} \int \frac{d^3q}{(2\pi)^3} \mathrm{tr}\, G_{q_i\bar{q}_i}(q)\,.$$

$$\Delta_{q_i,R} = \frac{1}{\mathcal{N}_R}\left[\Delta_{q_i}(T,\mu_q) - \Delta_{q_i}(0,0)\right]\,.$$

WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032



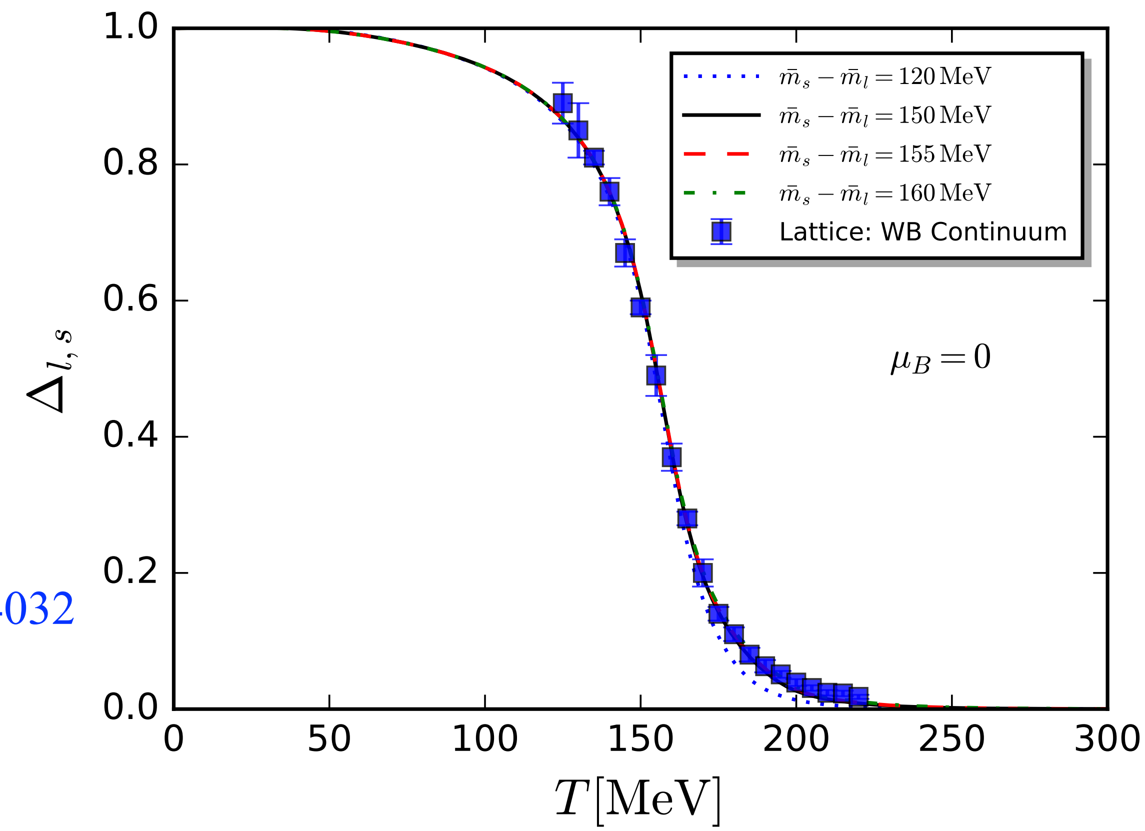


Other fermionic observables

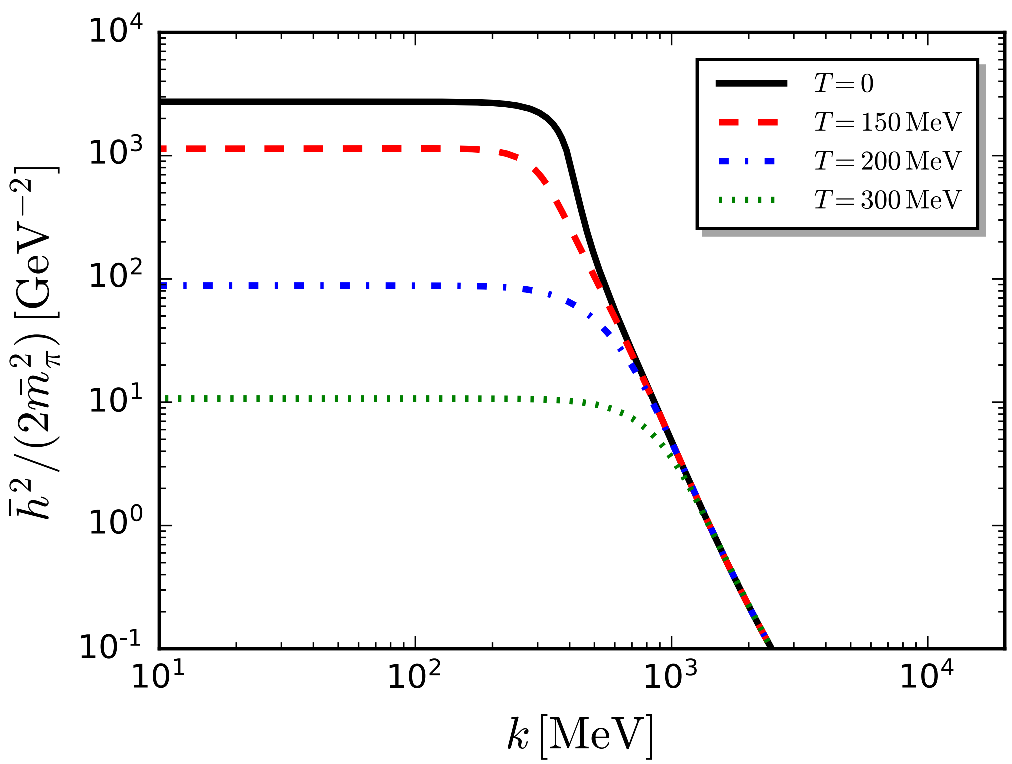
Reduced condensate:

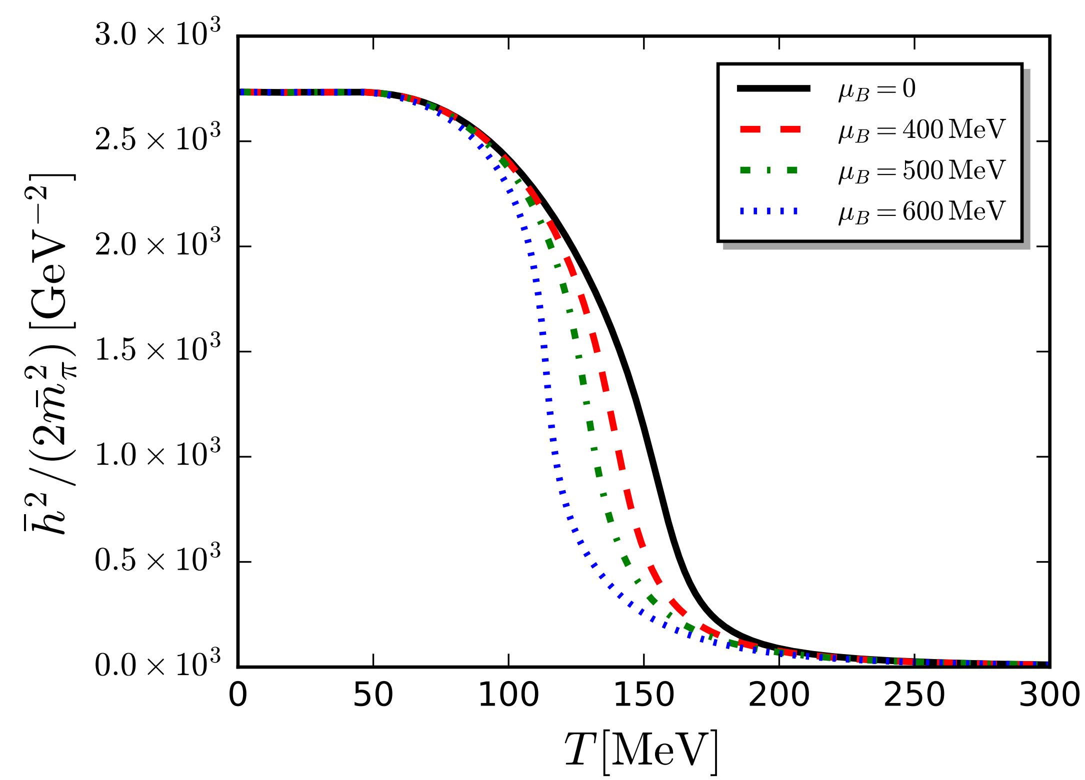
$$\Delta_{l,s} = \frac{\Delta_l(T,\mu_q) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T,\mu_Q)}{\Delta_l(0,0) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(0,0)} \,.$$

WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032



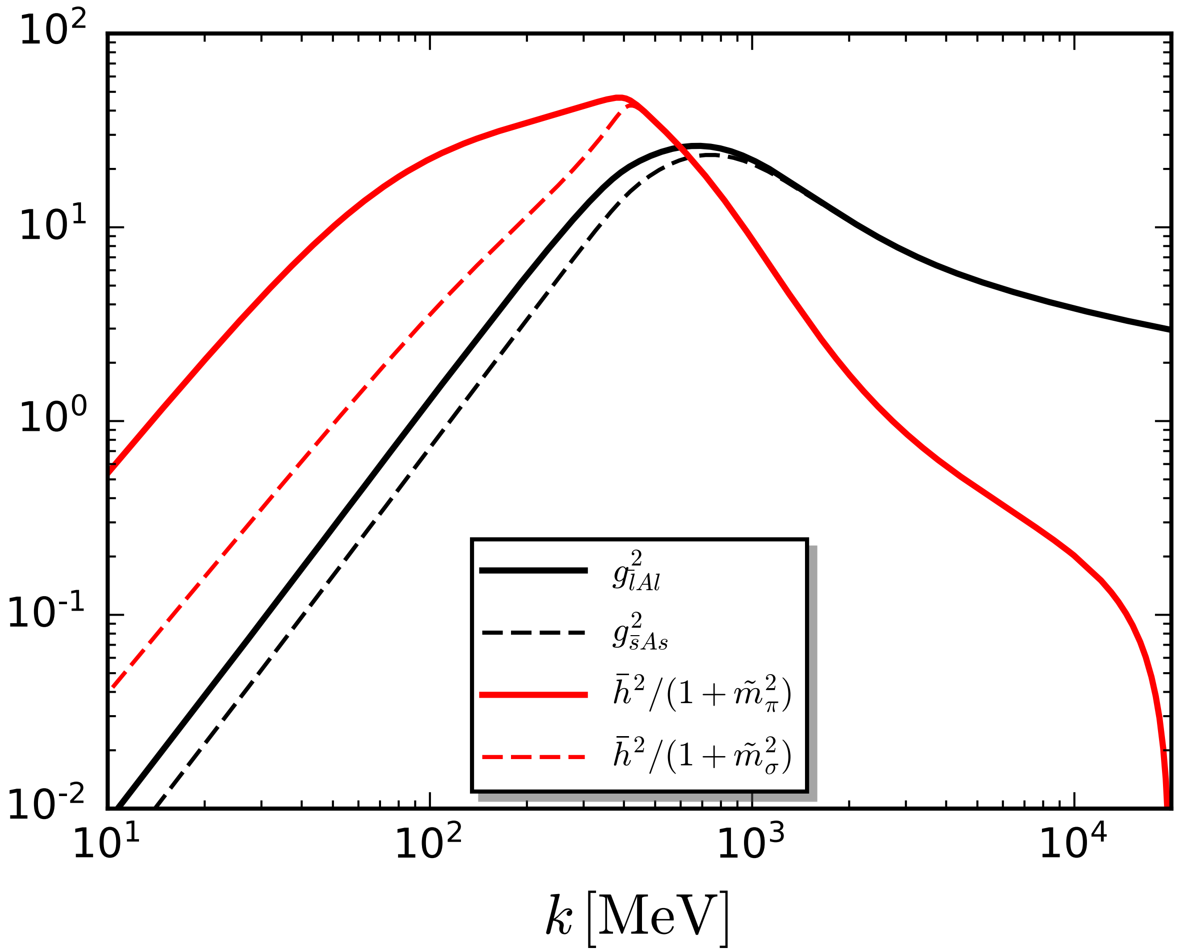
Effective four-quark coupling:



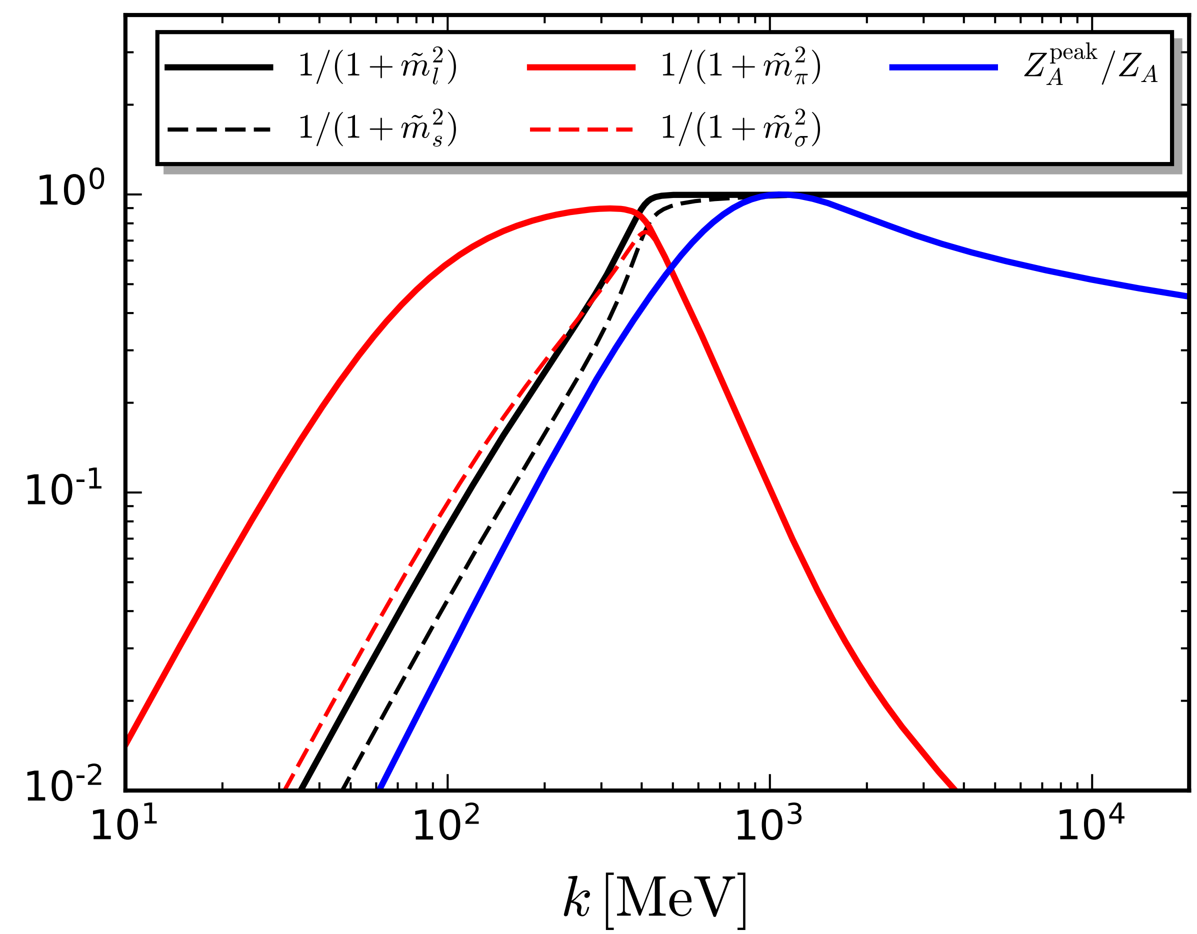


Sequential decoupling and natural emergence of low energy effective theories

Exchange couplings



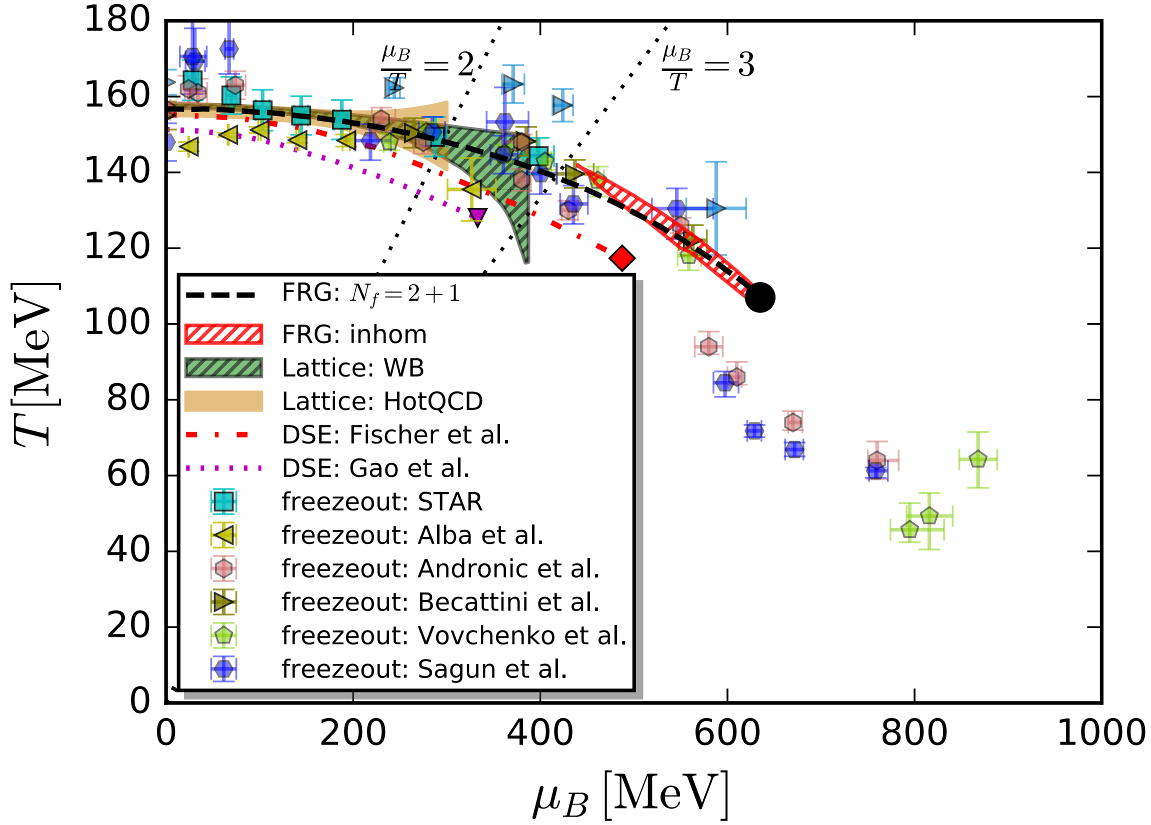
Propagator gapping

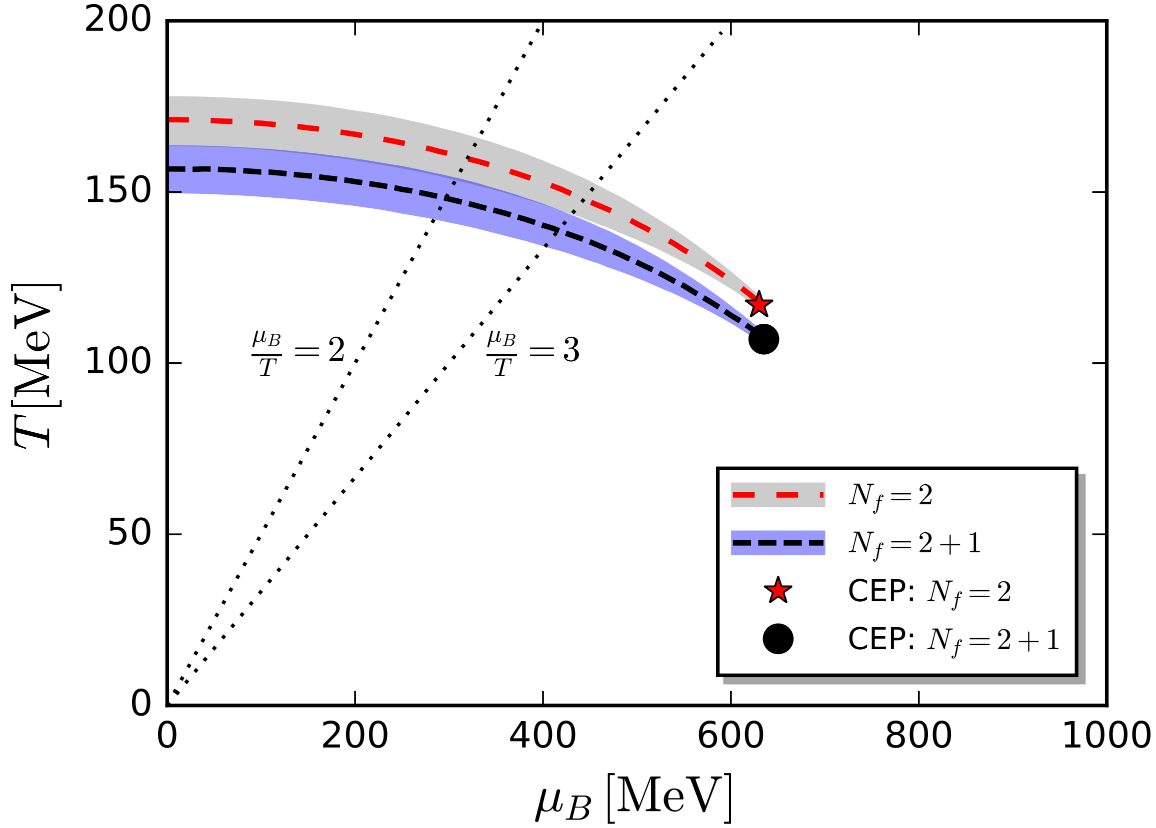


WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

Phase diagram and curvature

WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032





CEP:

$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}})_{N_f=2+1} = (107\,\mathrm{MeV}, 635\,\mathrm{MeV})\,,$$

$$(T_{\mathrm{CEP}}, \mu_{B_{\mathrm{CEP}}})_{N_f=2} = (117\,\mathrm{MeV}, 630\,\mathrm{MeV})\,,$$

FRG curvature of the phase boundary:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \lambda \left(\frac{\mu_B}{T_c}\right)^4 + \cdots\,,$$

$$\kappa_{N_f=2+1} = 0.0142(2)$$

$$\kappa_{N_f=2} = 0.0176(1)$$

Lattice result:

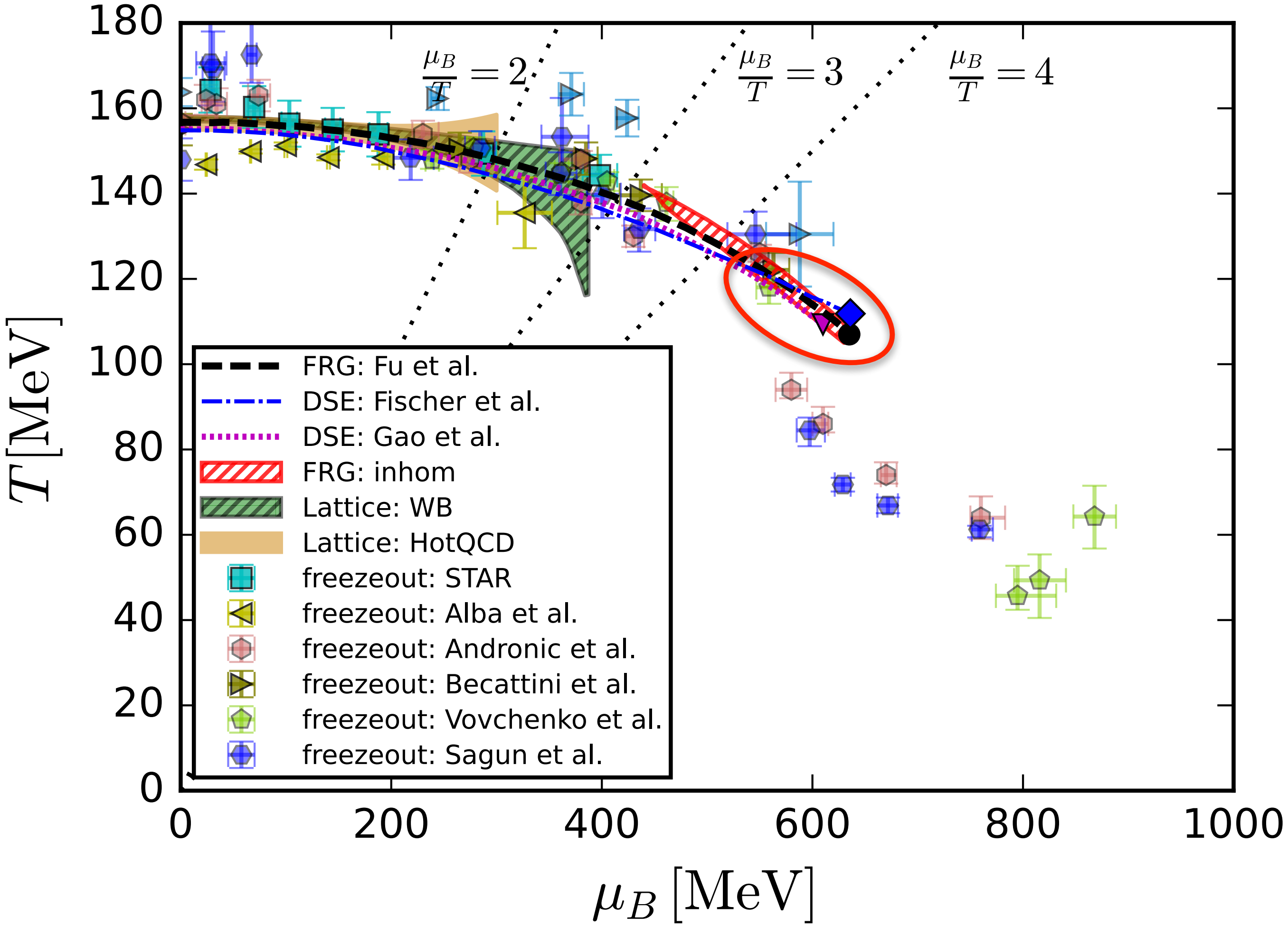
$$\kappa = 0.0149 \pm 0.0021$$

Bellwied *et al.* (WB), *PLB* 751 (2015) 559.

$$\kappa = 0.015 \pm 0.004$$

Bazavov *et al.* (HotQCD), *PLB* 795 (2019) 15.

QCD phase structure



Prediction of location of CEP from functional QCD

fRG:

$$(T, \mu_B)_{\text{CEP}} = (107, 635)\mathbf{MeV}$$

WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

DSE:

$$(T, \mu_B)_{\text{CEP}} = (109, 610)\mathbf{MeV}$$

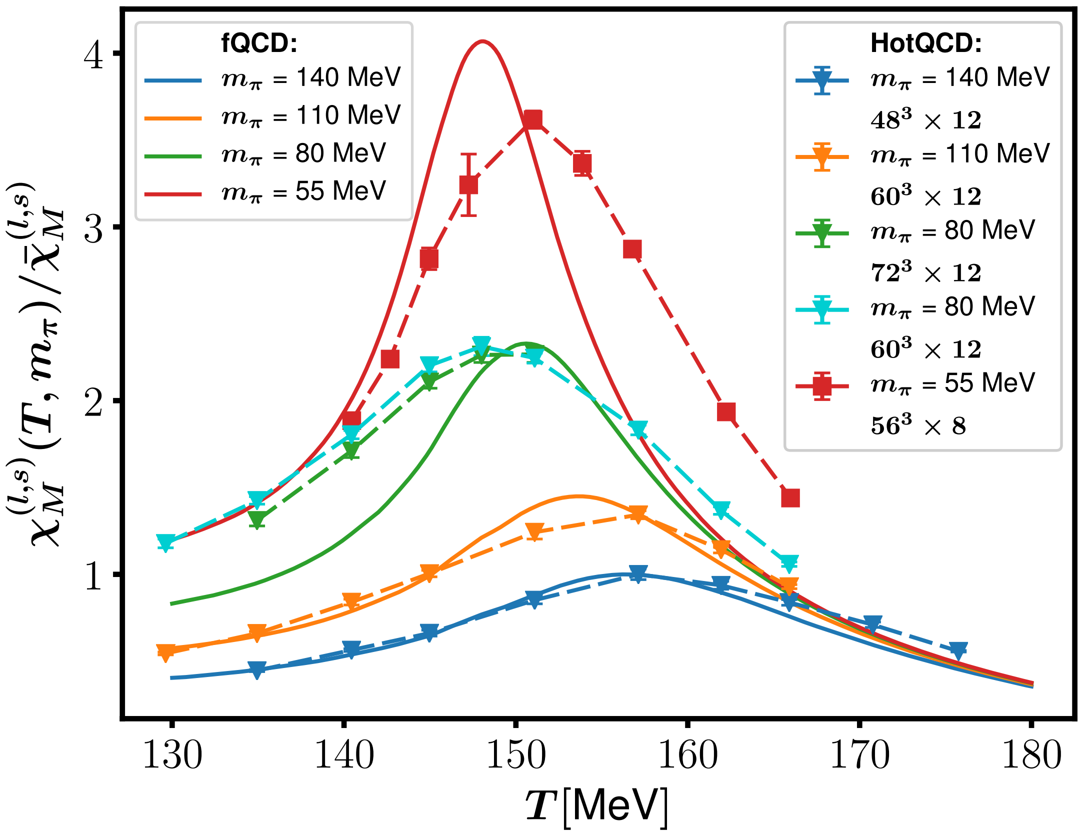
Gao, Pawlowski, PLB 820 (2021) 136584

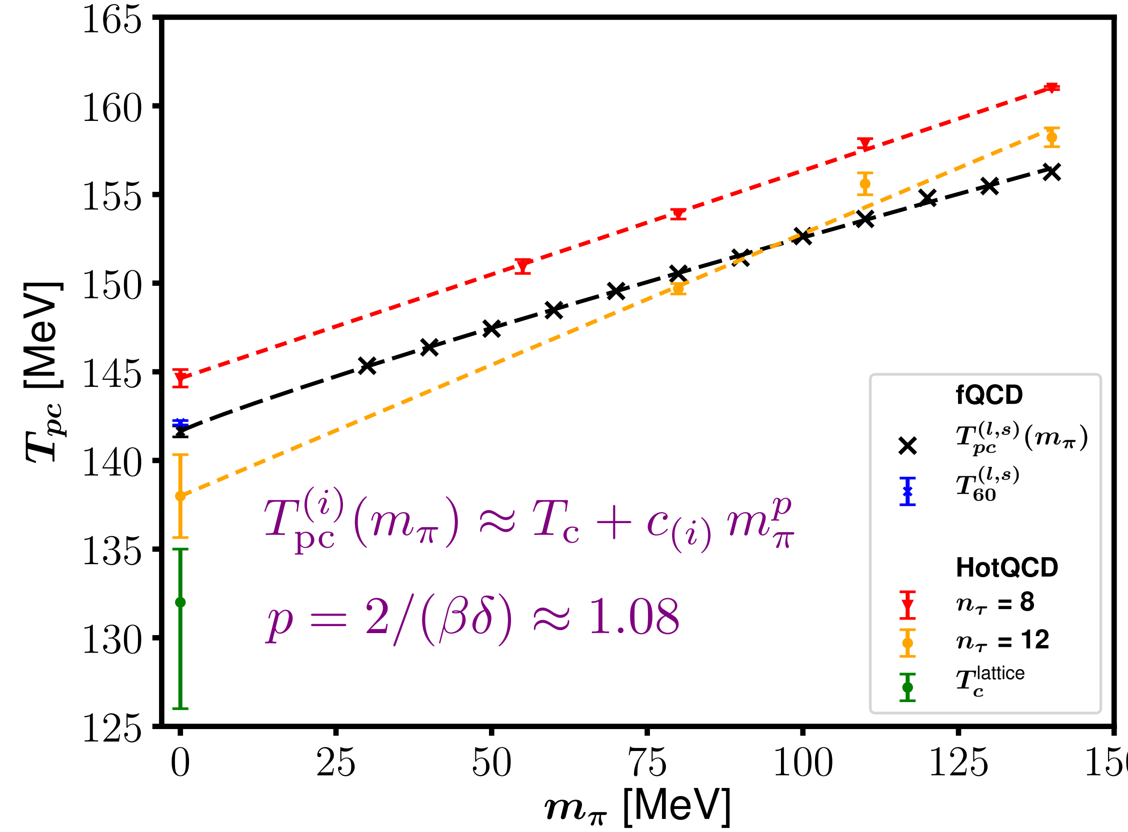
$$(T, \mu_B)_{\text{CEP}} = (112, 636)\mathbf{MeV}$$

Gunkel, Fischer, *PRD* 104 (2021) 5, 054022

- Recent studies of QCD phase structure from both fRG and DSE have shown convergent prediction for the location of CEP.
- Considering relatively larger errors when $\mu_B/T \gtrsim 4$, one arrives at a reasonable estimation : $450\ \text{MeV} \lesssim \mu_{B_{\text{CEP}}} \lesssim 650\ \text{MeV}$.

Critical temperature in the chiral limit





Braun, WF, Pawlowski, Rennecke, Rosenblüh, Yin, *PRD* 102 (2020), 056010.

Lattice (HotQCD):

$$T_c^{\text{lattice}} = 132^{+3}_{-6}\,\text{MeV},$$

Ding *et al.*, *PRL* 123 (2019), 062002.

fRG:

$$T_c^{\text{fRG}} \approx 142\,\text{MeV},$$

Braun, WF, Pawlowski, Rennecke, Rosenblüh, Yin, *PRD* 102 (2020), 056010.

DSE:

$$T_c^{\text{DSE}} = 142.8 \pm 0.8\,\text{MeV},$$

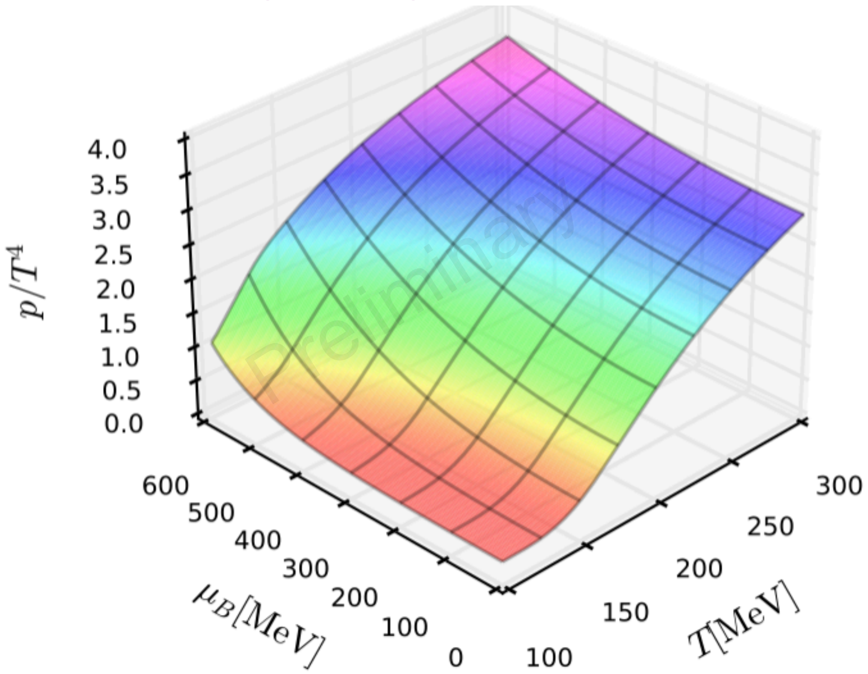
Bai, Chang, Chao, Gao, Liu, *PRD* 104 (2021), 014005.

$$T_c^{\text{DSE}} \approx 141\,\text{MeV},$$

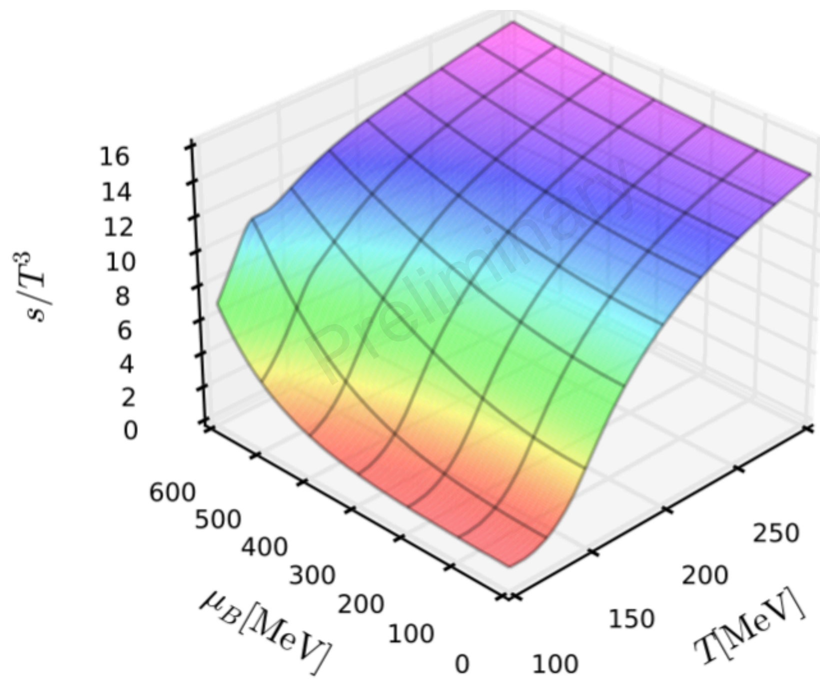
Gao, Pawlowski, in preparation.

Equation of state at $\mu_B \neq 0$

Pressure:



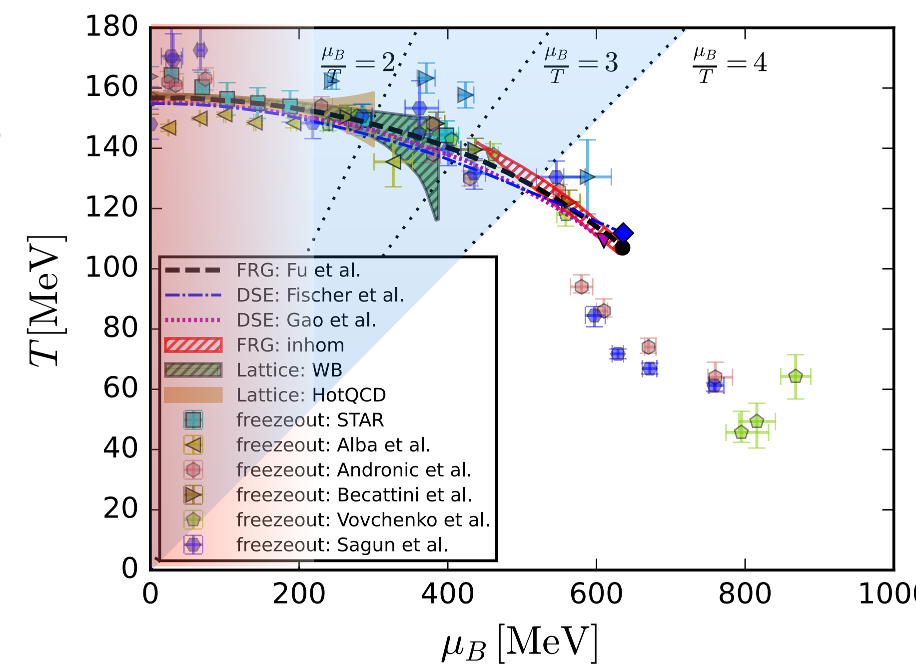
Entropy:



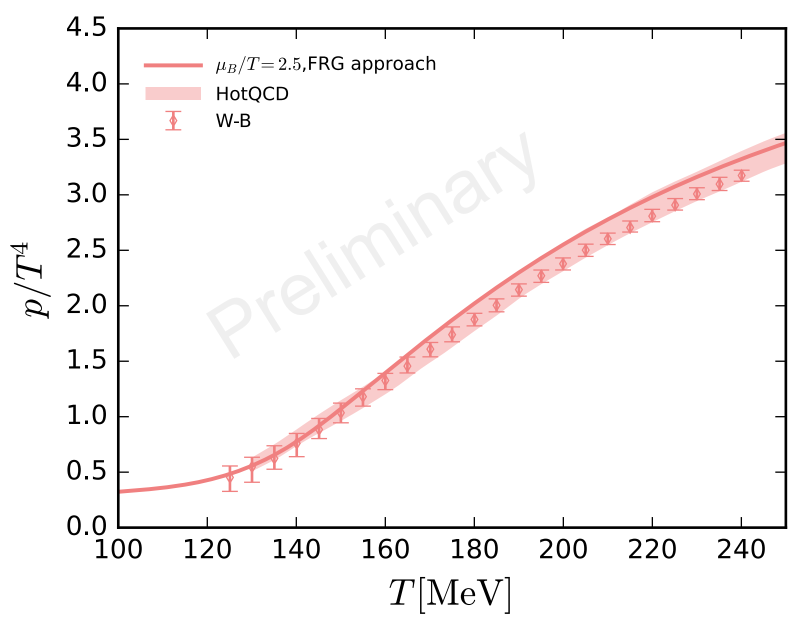
WF, Pawlowski, Rennecke, Wen, Yin, in preparation

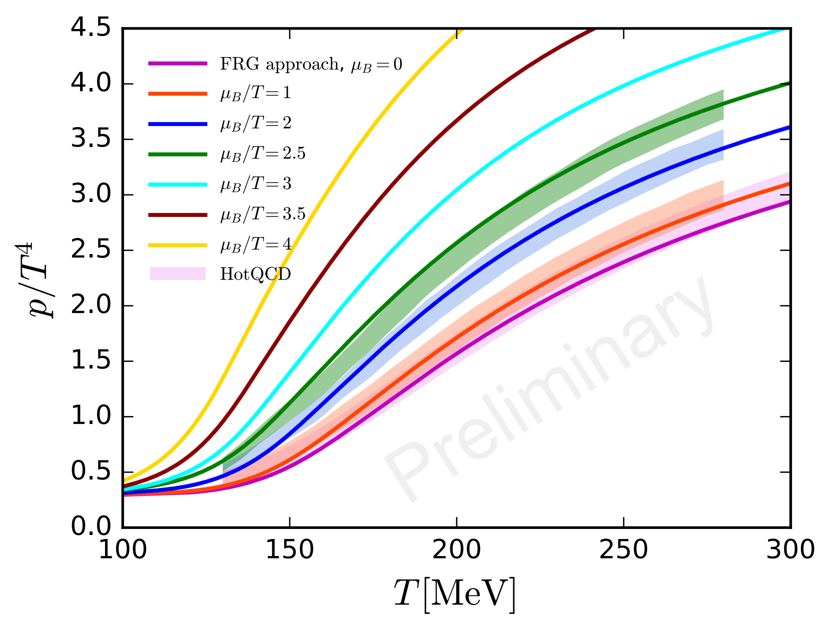
- **Self-consistent truncations to functional relations define 'analytic' functions in μ_B:**

$$\partial_t \langle q(x)\bar{q}(x)\rangle(\mu_B)$$
$$=\mathrm{Flow}[\langle q(x)\bar{q}(x)\rangle(\mu_B),\ \langle q(x)A_\mu\bar{q}(x)\rangle(\mu_B),\ \ldots;\mu_B]$$



Comparison to lattice: $\mu_B \neq 0$

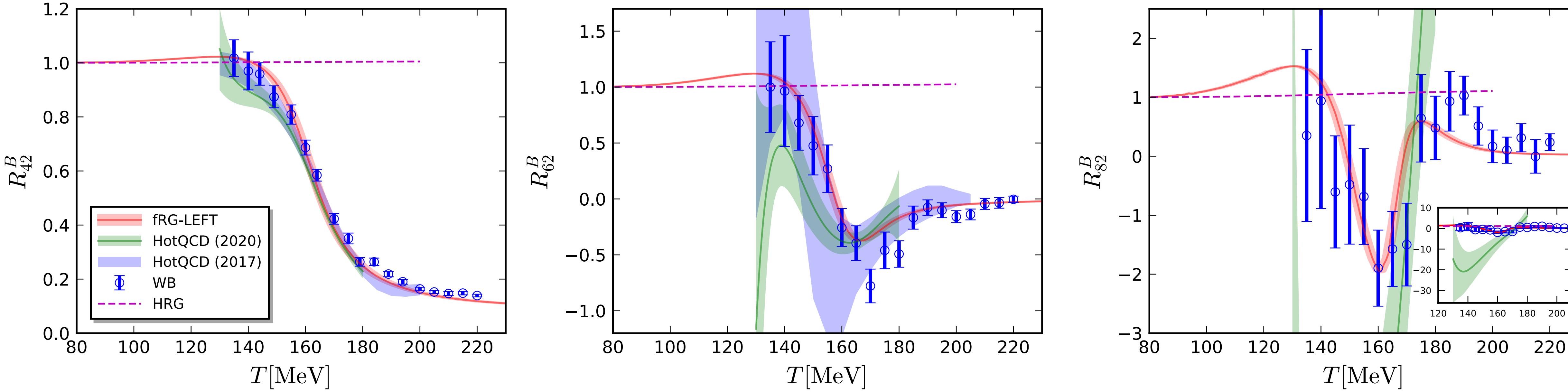




Passing lattice benchmark tests at vanishing μ_B.

Regime of reliability of current best truncation.

Baryon number fluctuations



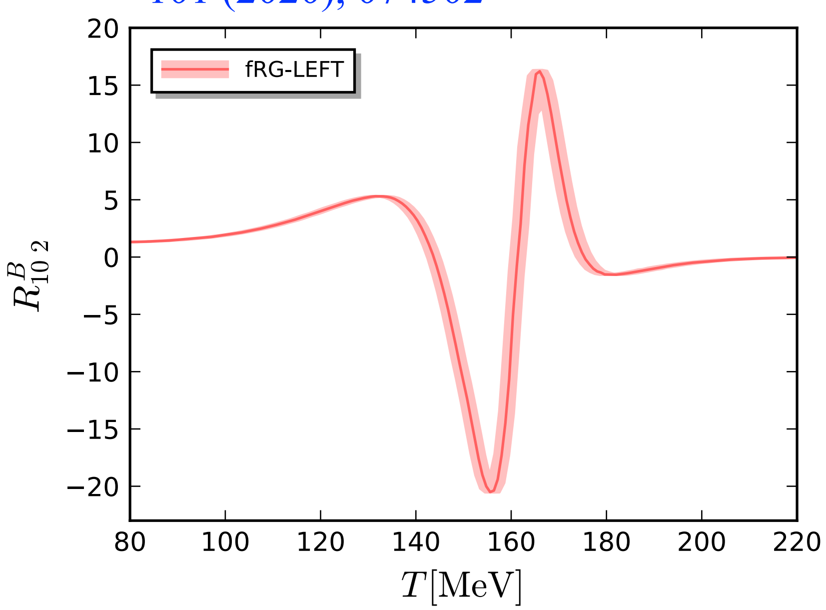
$\mu_B = 0$

in comparison to lattice results and HRG

fRG: WF, Luo, Pawlowski, Rennecke, Wen, Yin, arXiv:2101.06035

HotQCD: A. Bazavov *et al.*, arXiv: *PRD* 95 (2017), 054504; *PRD* 101 (2020), 074502

WB: S. Borsanyi *et al.*, arXiv: *JHEP* 10 (2018) 205



baryon number fluctuations

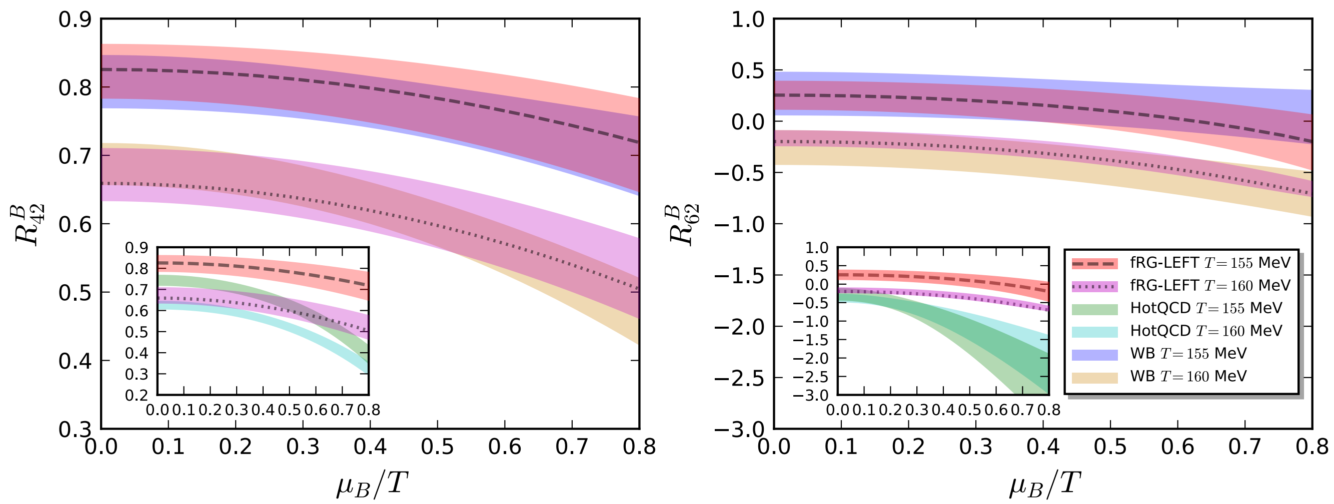
$$\chi_n^B = \frac{\partial^n}{\partial(\mu_B/T)^n}\frac{p}{T^4}\,, \qquad R^B_{nm} = \frac{\chi^B_n}{\chi^B_m}$$

relation to the cumulants

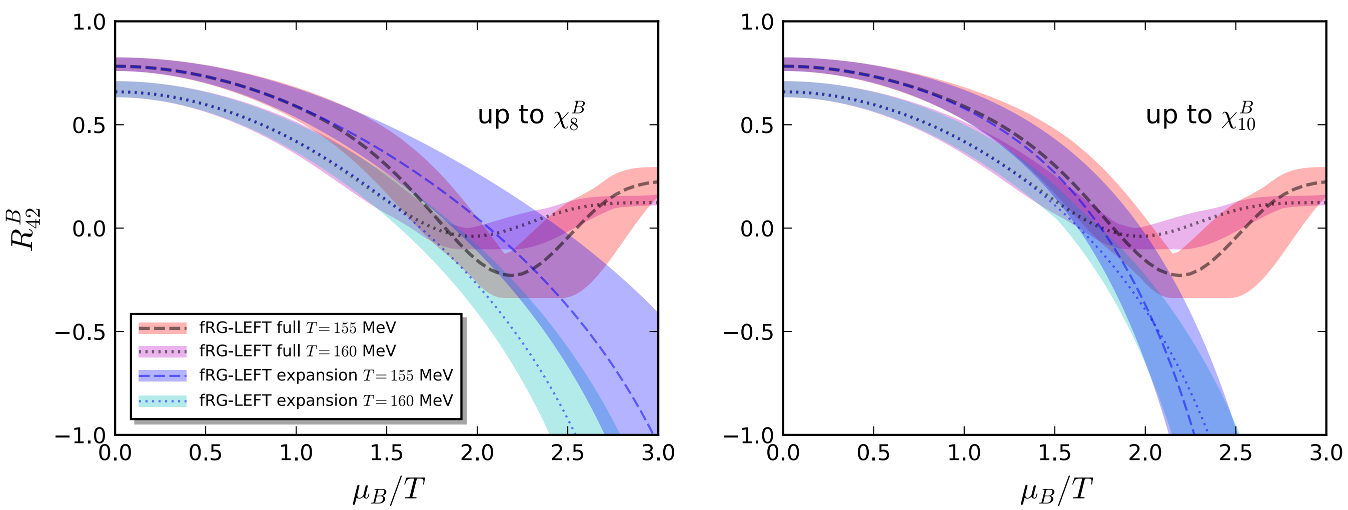
$$M = VT^3\chi_1^B\,, \qquad \sigma^2 = VT^3\chi_2^B$$

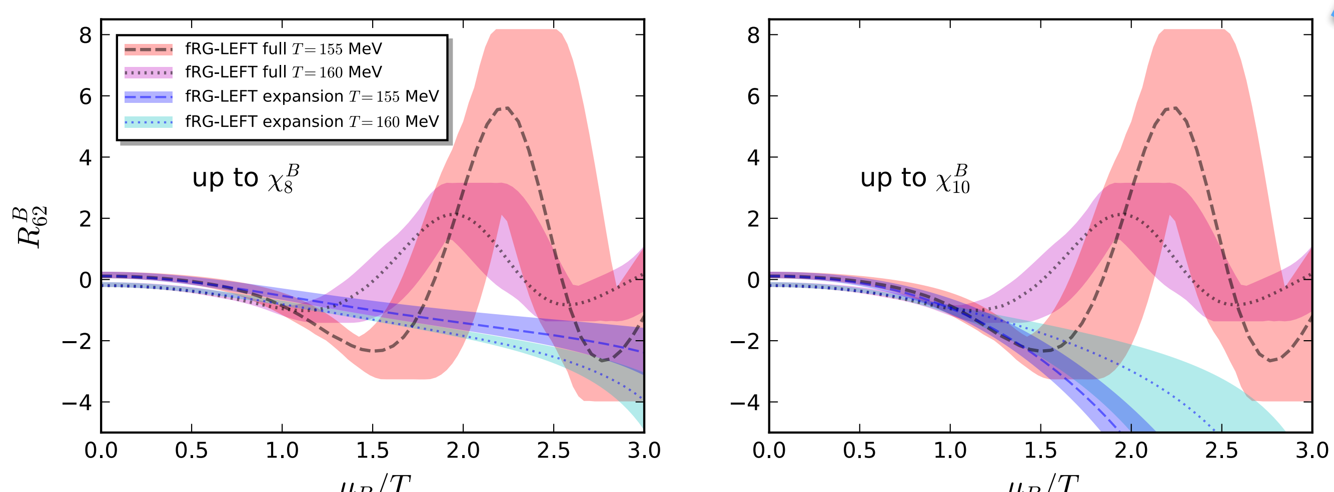
$$S = \chi_3^B/(\chi_2^B\sigma)\,, \qquad \kappa = \chi_4^B/(\chi_2^B\sigma^2)$$

Convergence radius of Taylor expansion?



$\mu_B \neq 0$ **LEFT vs Lattice**





full vs expanded

expanding the pressure at $\mu_B = 0$

$$\frac{p(\mu_B)}{T^4} = \frac{p(0)}{T^4} + \sum_{i=1}^{\infty} \frac{\chi^B_{2i}(0)}{(2i)!}\,\hat{\mu}_B^{2i}$$

expanded fluctuations

$$\chi^B_2(\mu_B) \simeq \chi^B_2(0) + \frac{\chi^B_4(0)}{2!}\hat{\mu}_B^2 + \frac{\chi^B_6(0)}{4!}\hat{\mu}_B^4 + \frac{\chi^B_8(0)}{6!}\hat{\mu}_B^6\,,$$

$$\chi^B_4(\mu_B) \simeq \chi^B_4(0) + \frac{\chi^B_6(0)}{2!}\hat{\mu}_B^2 + \frac{\chi^B_8(0)}{4!}\hat{\mu}_B^4\,,$$

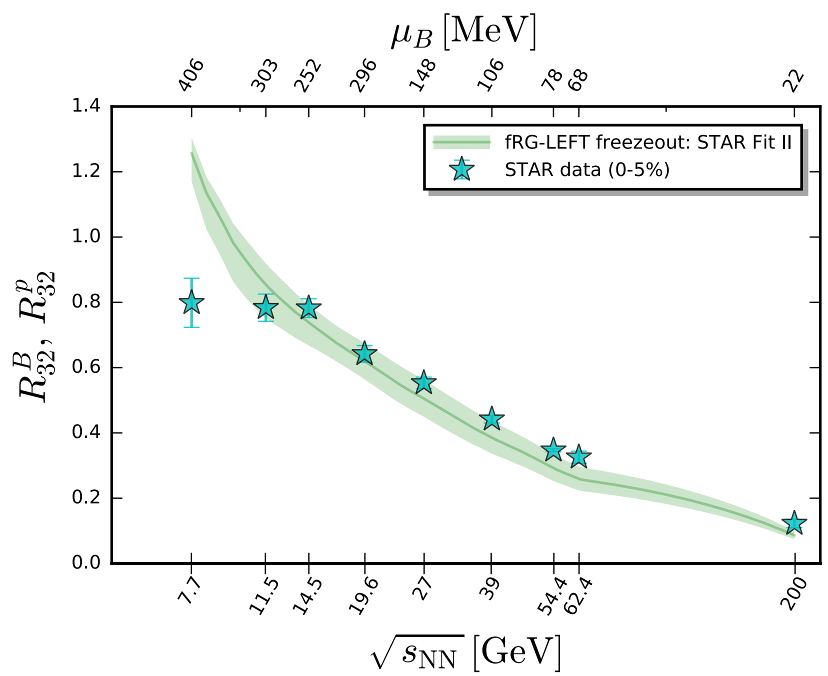
$$\chi^B_6(\mu_B) \simeq \chi^B_6(0) + \frac{\chi^B_8(0)}{2!}\hat{\mu}_B^2$$

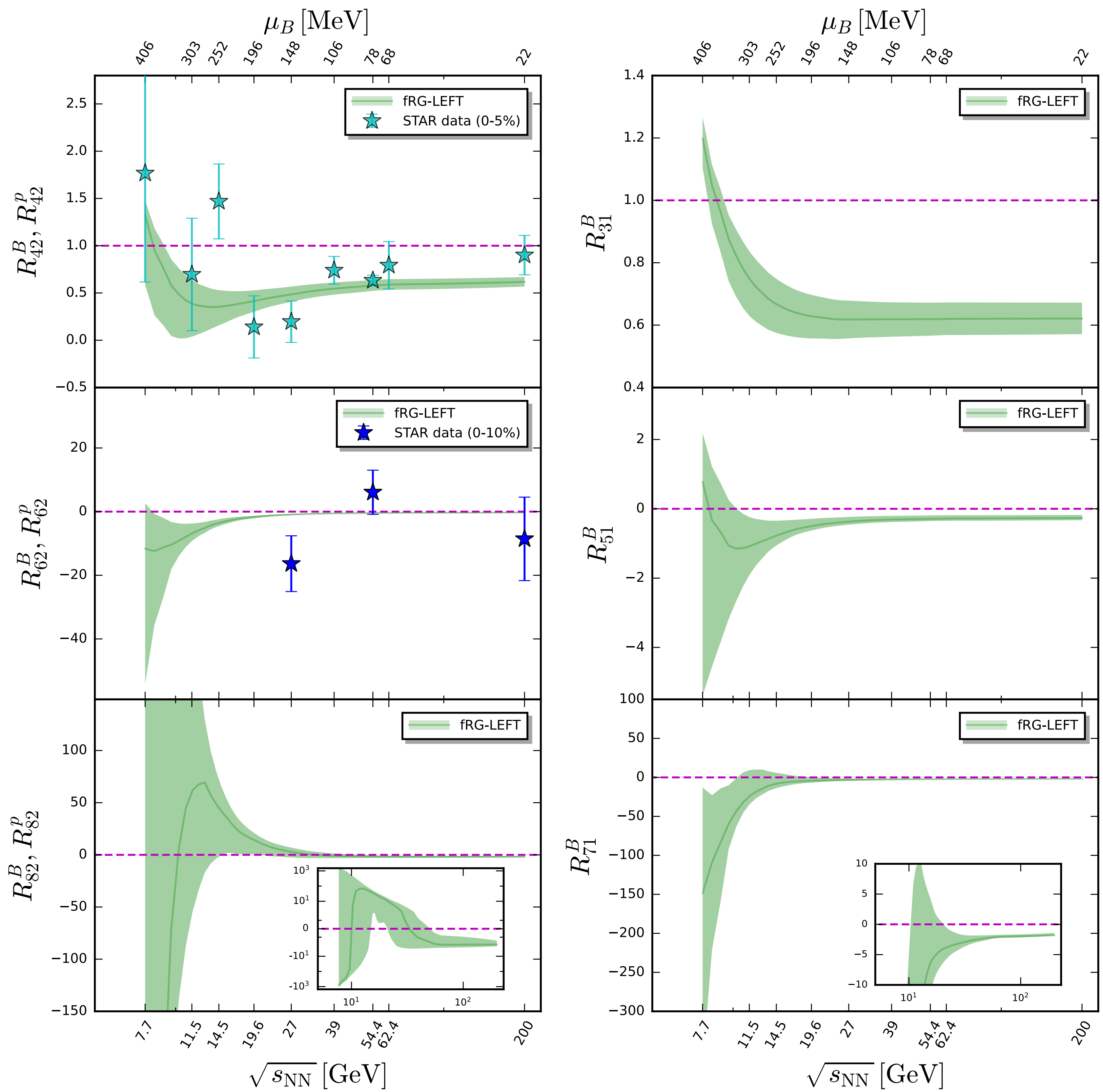
$$T = 155\,\mathrm{MeV}: \quad [\mu_B/T]_{\mathrm{Max}} \approx 1.5\,,$$

$$T = 160\,\mathrm{MeV}: \quad [\mu_B/T]_{\mathrm{Max}} \approx 1.2$$

Yang-Lee edge singularity?

Fluctuations on the freeze-out curve



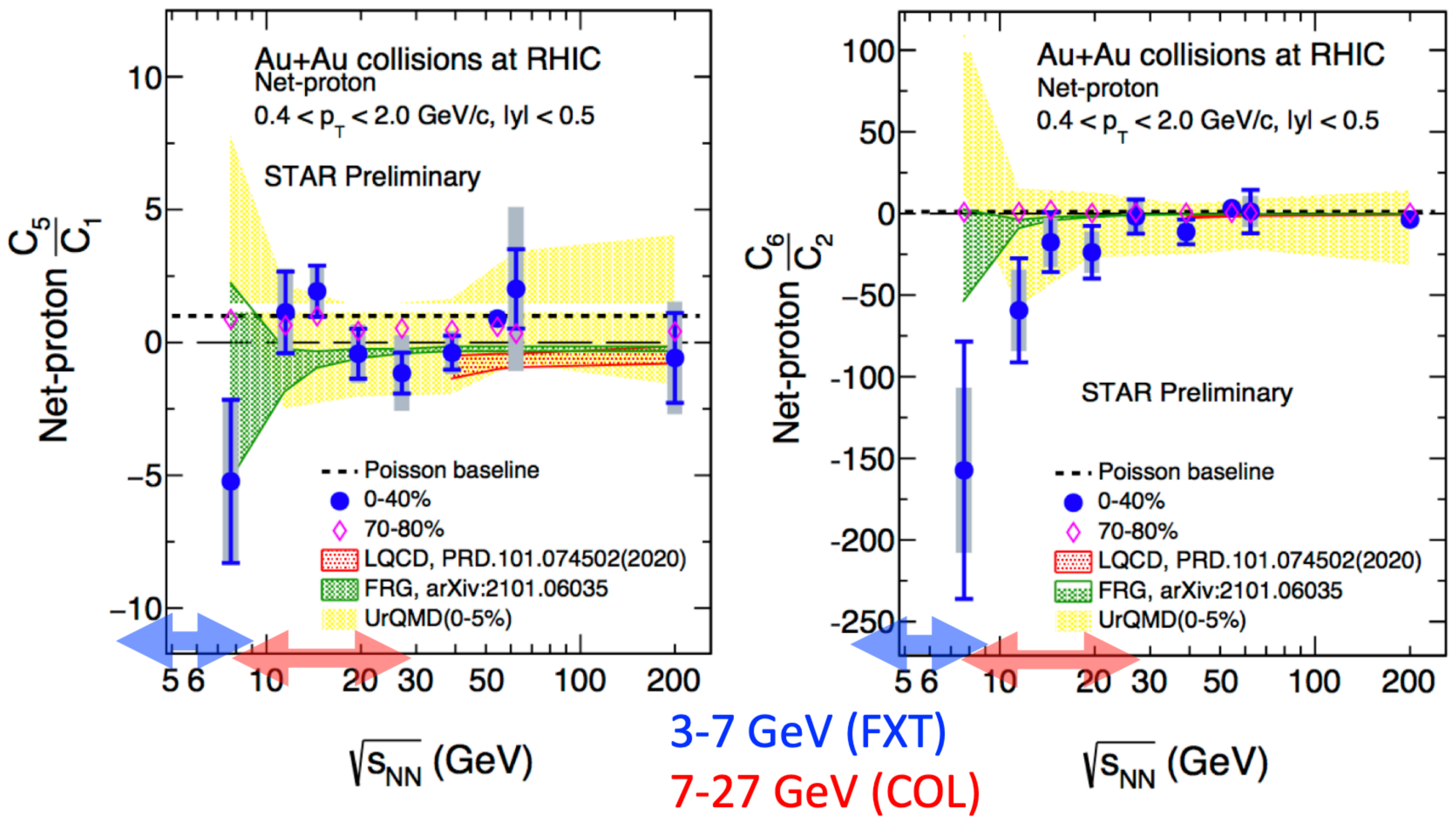


WF, Luo, Pawlowski, Rennecke, Wen, Yin, arXiv:2101.06035.

J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301
M. Abdallah *et al.* (STAR), arXiv:2105.14698.

New data of R_{51} and R_{62} from RHIC

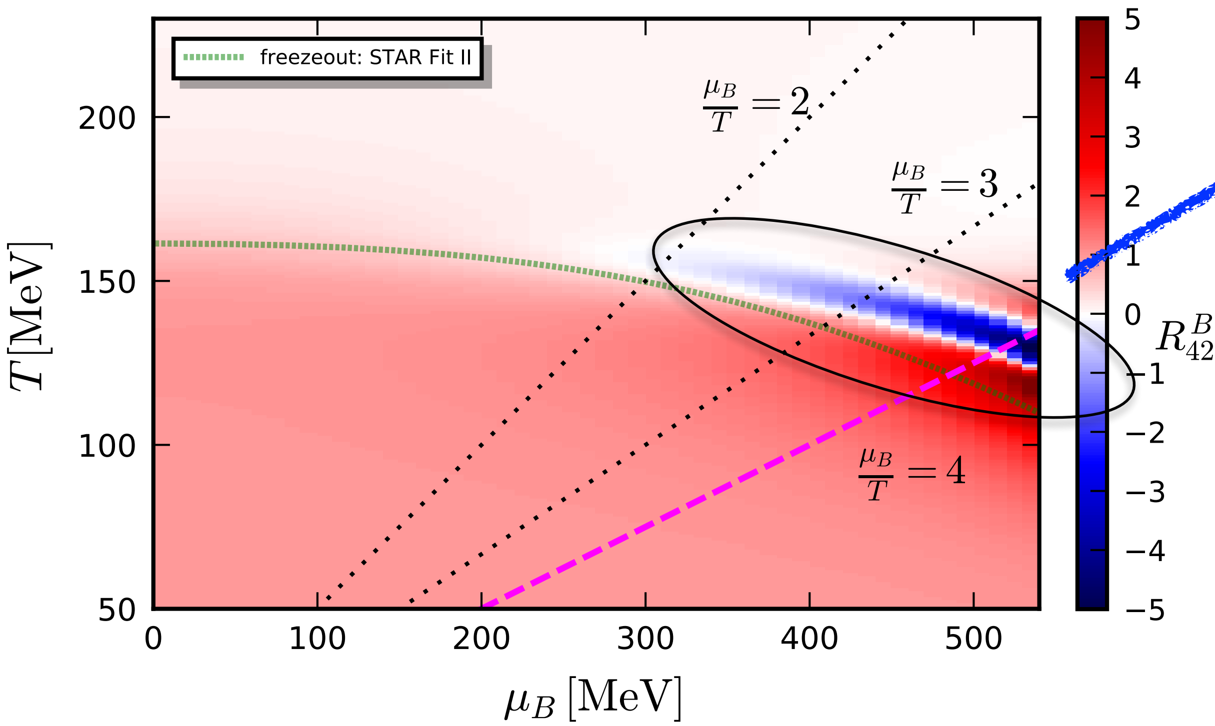
New data of net-proton C_5, C_6 at BES-I will be shown by Ashish P.



from ShinIchi Esumi's talk at CPOD2021 on behalf of the STAR Collaboration

Crossover vs critical scaling

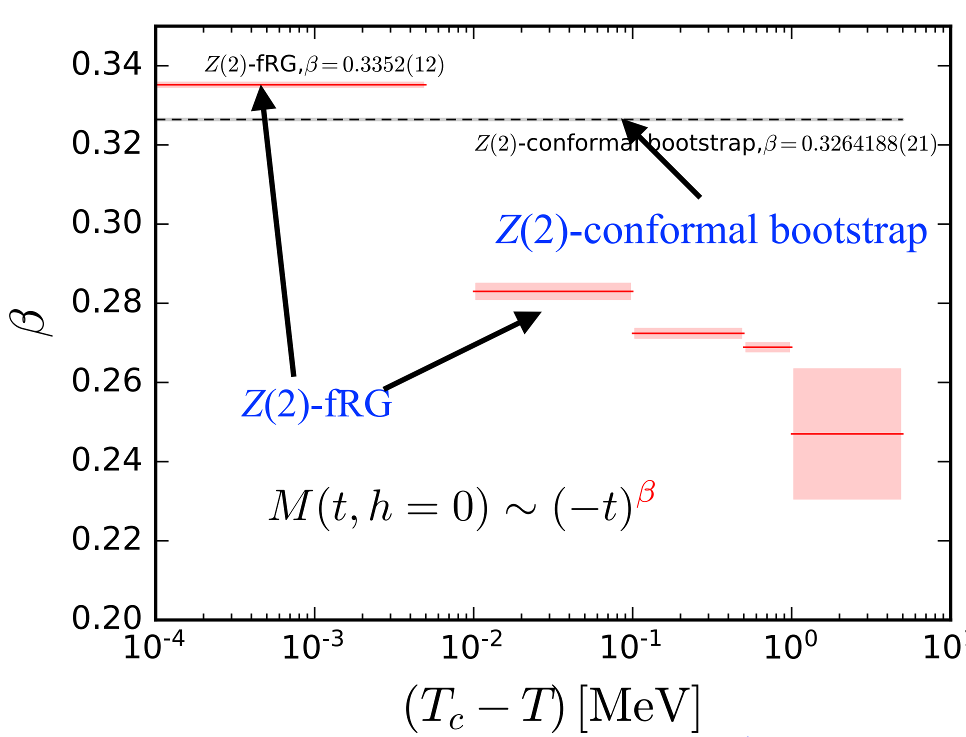
R_{42}^B on the phase diagram

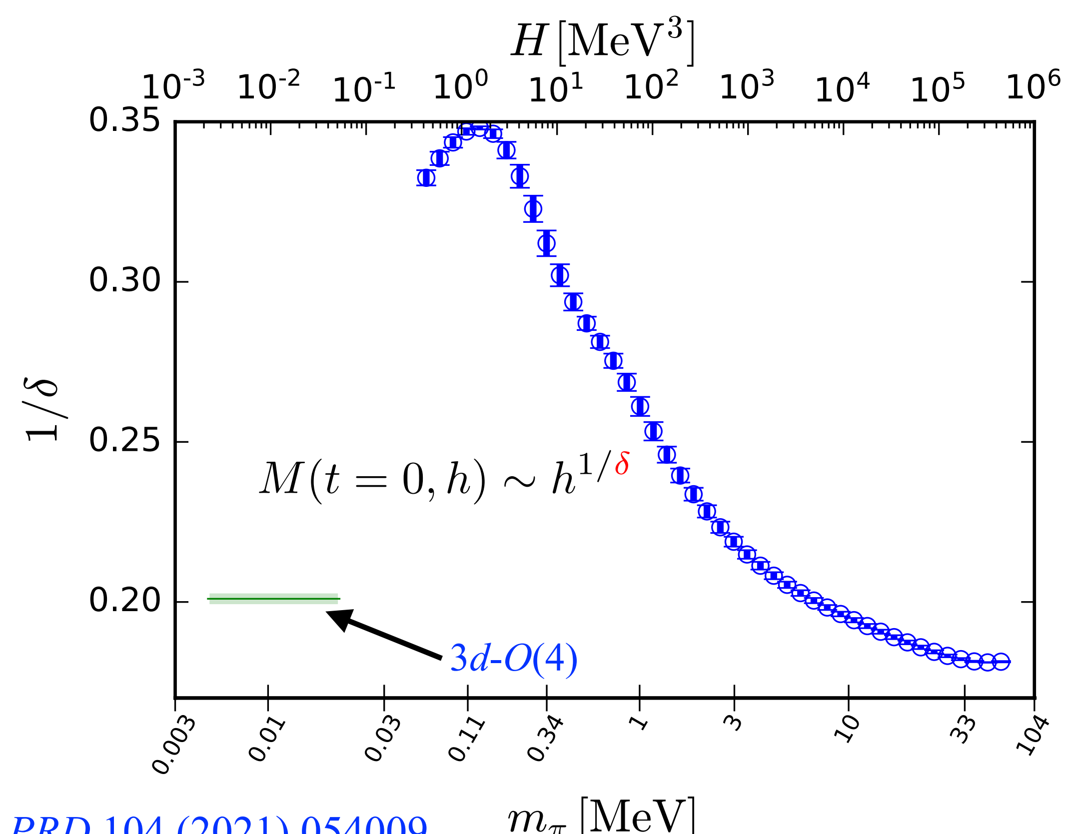


Note: this is a region of increasingly sharp crossover, not a critical scaling in this LEFT

For the moment, most of calculations in effective theories indicate that the region of critical scaling is extremely small.

Schaefer, Wambach, *PRD* 75 (2007) 085015;
Braun, Klein, *PRD* 77 (2008) 096008;
Braun, Klein, Piasecki, *EPJC* 71 (2011) 1576.

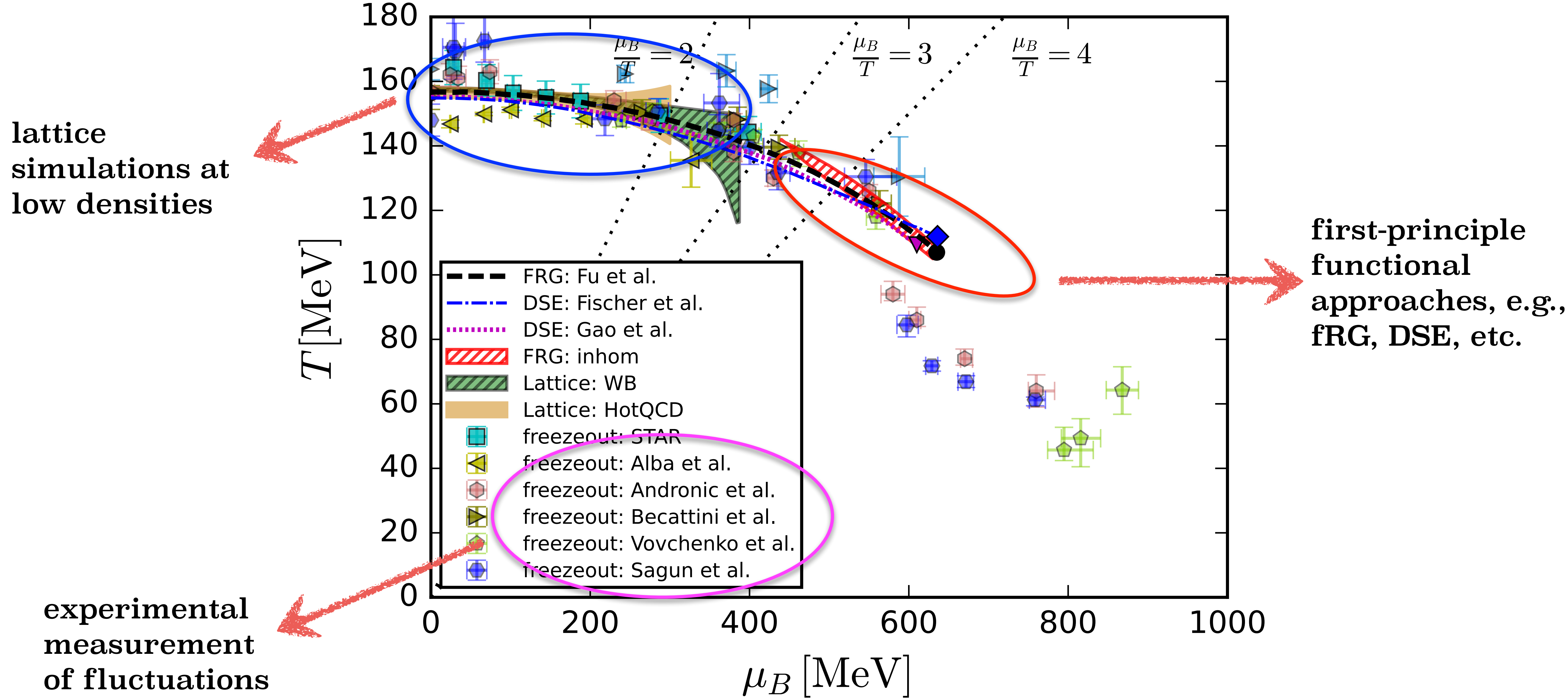




Chen, Wen, WF, *PRD* 104 (2021) 054009.

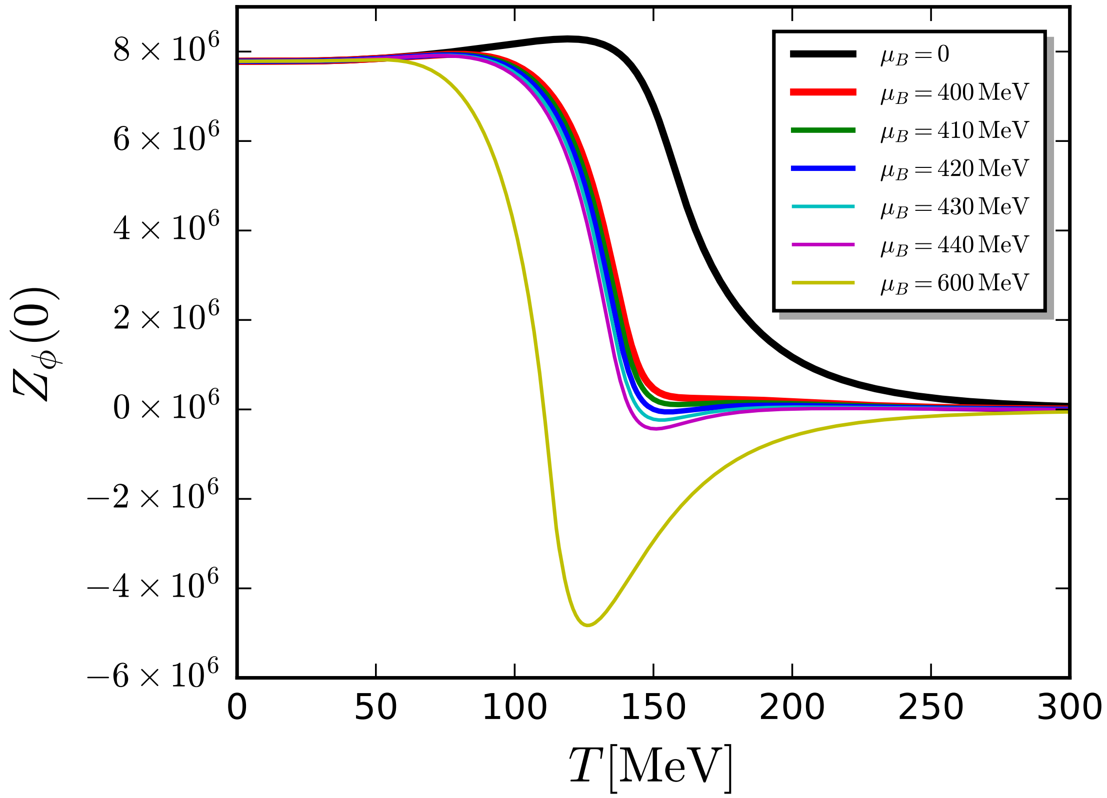
The critical region is very small ($\Delta T, \Delta m_\pi \ll 1\,\mathrm{MeV}$)!

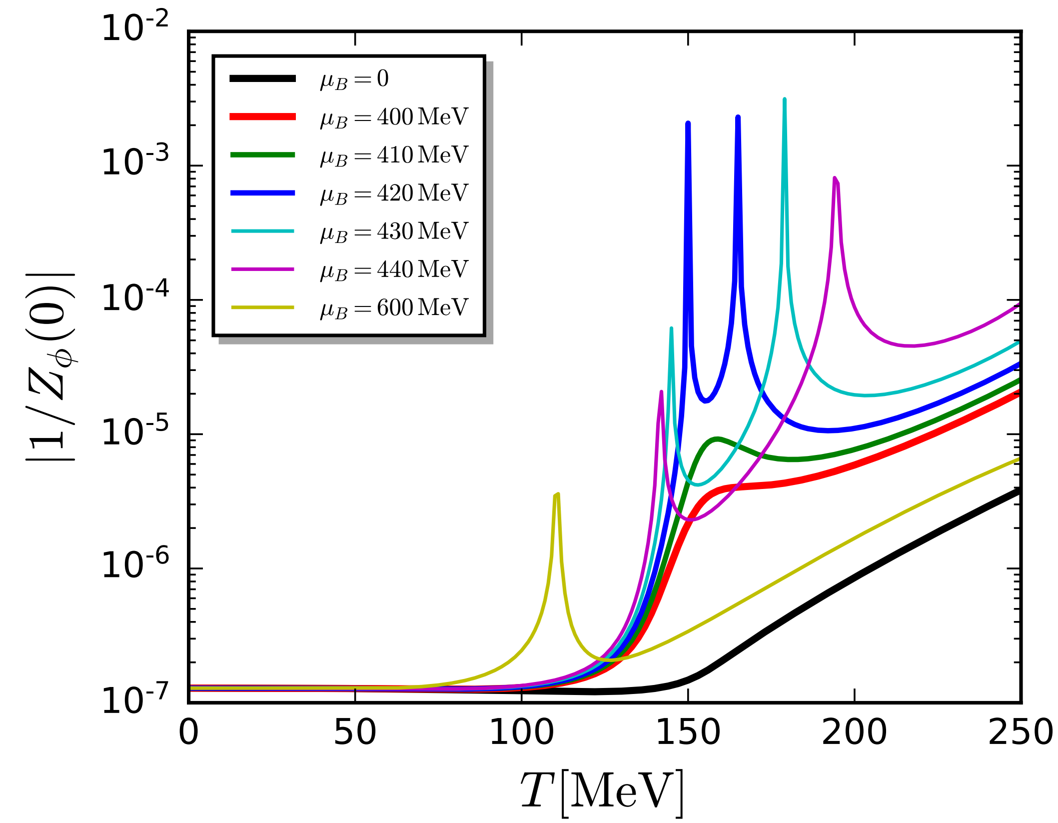
Prospect for the search of CEP

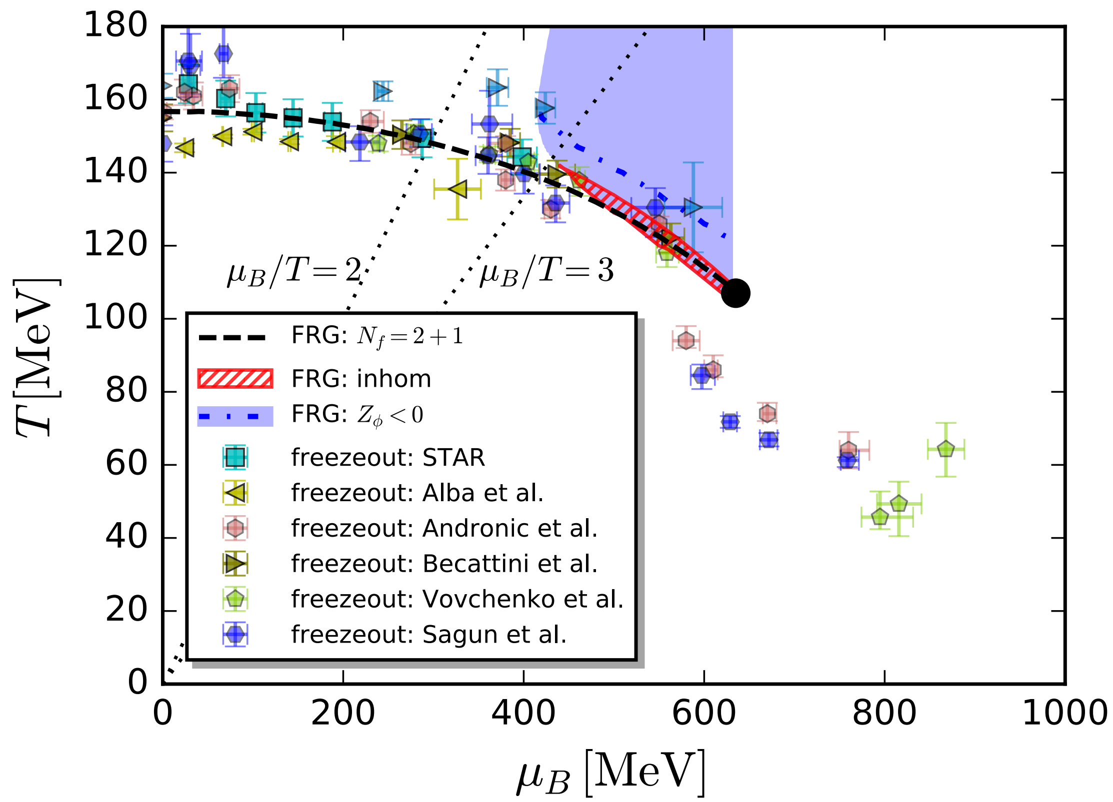


- lattice and functional methods provide predication of $\text{Loc}_{\text{CEP}}(R_{nm})$
- experiments provide fluctuation observables R_{nm} as input

Inhomogeneous phase?







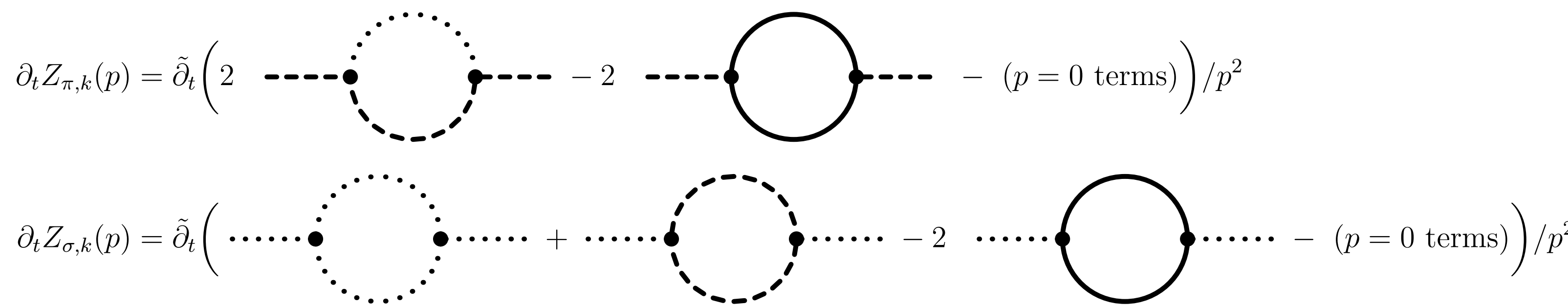
Two point function for the meson:

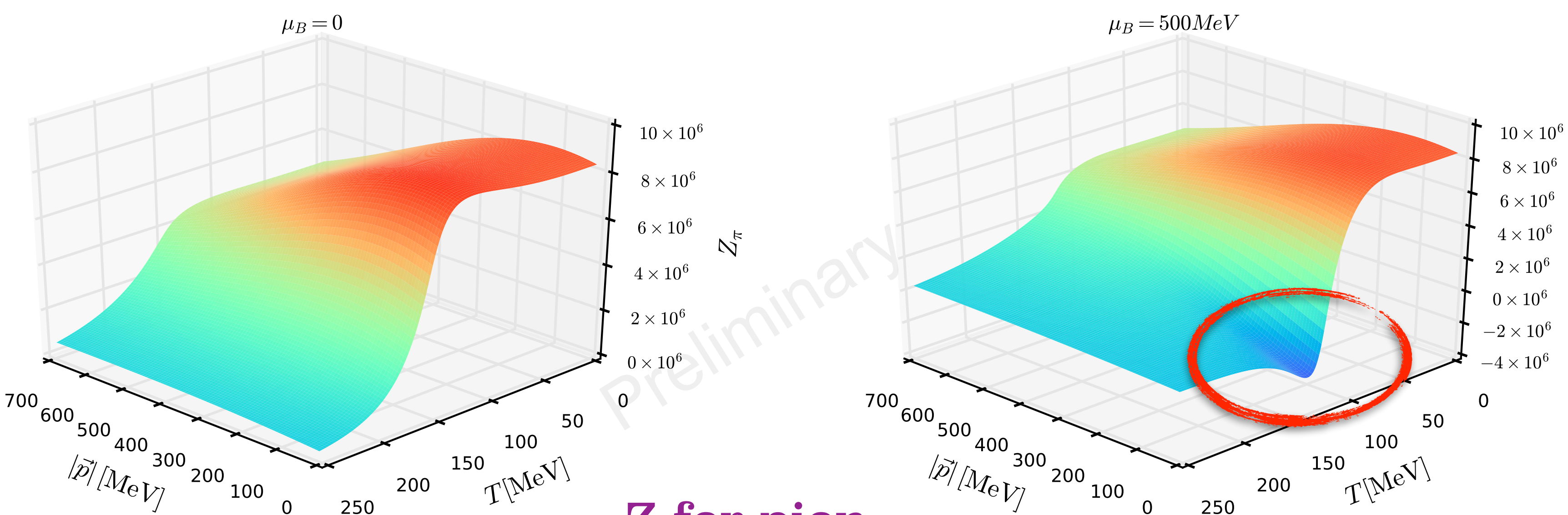
$$\Gamma^{(2)}_{\phi\phi}(p) = Z_\phi(p^2)\,p^2 + m_\phi^2\,,$$

WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

Two-point correlation function for mesons

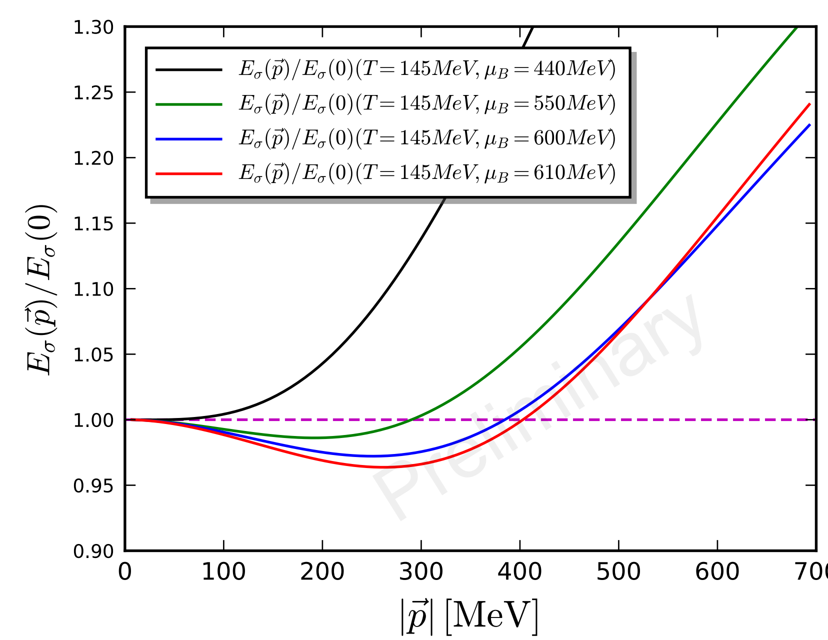
Flow equation for mesonic wave function renormalization:

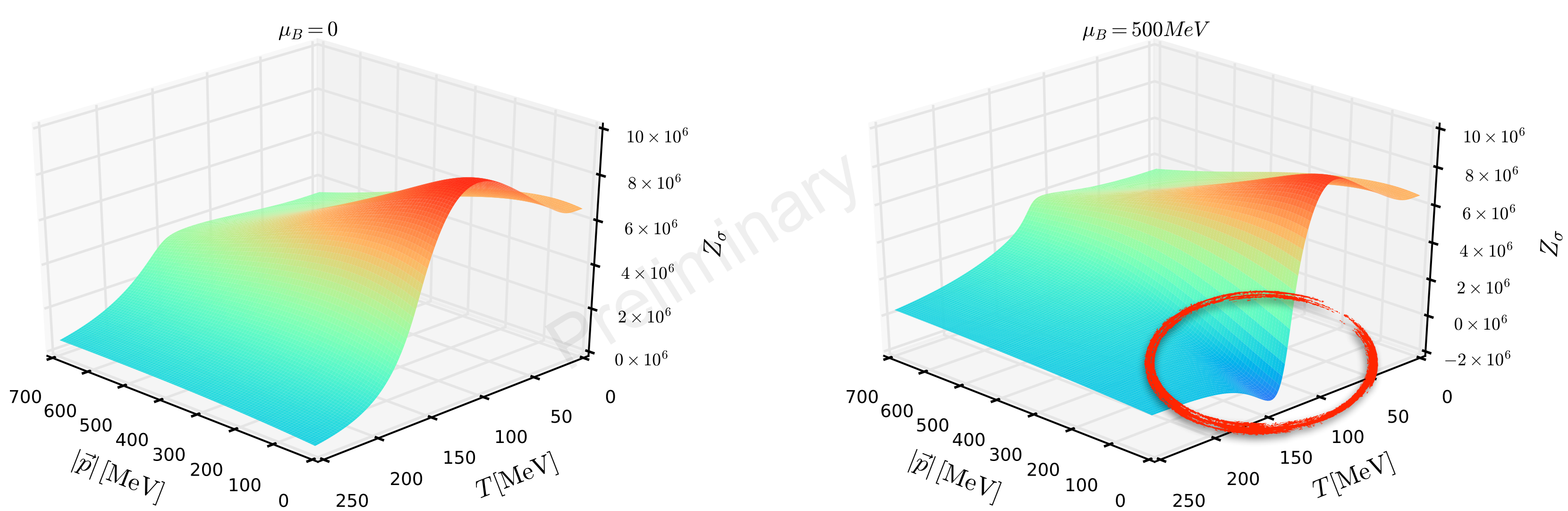




Z for pion

Energy of sigma:





Z for sigma

WF, Pawlowski, Pisarski, Rennecke, Wen, Yin, in preparation.

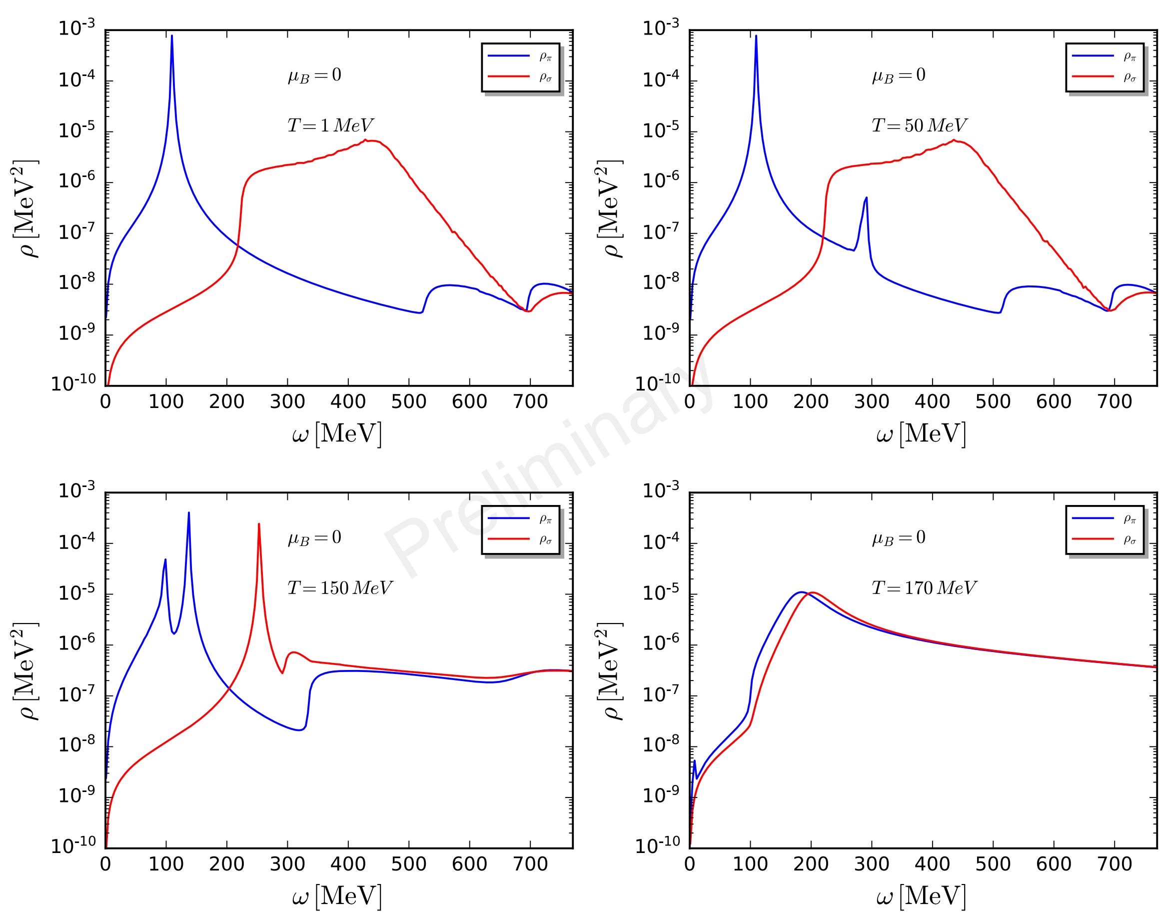
Spectral functions for mesons

Analytic continuation:

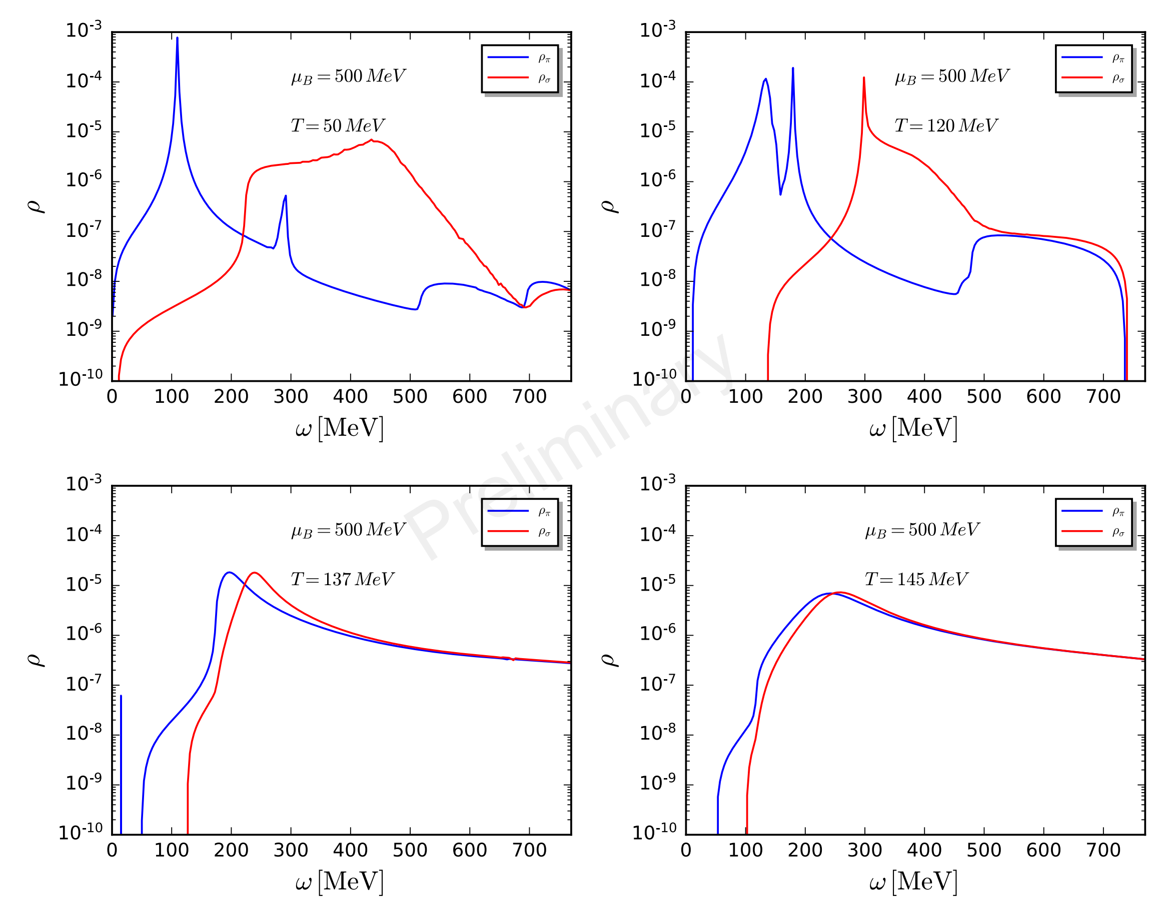
$$\Gamma_k^{(2),R}(\omega) = \lim_{\epsilon \to 0} \Gamma_k^{(2),E}(-i(\omega + i\epsilon), \vec{p}).$$

Spectral function:

$$\rho(\omega, \vec{p}) = -\frac{1}{\pi} \frac{\mathrm{Im}\Gamma^{(2),R}(\omega, \vec{p})}{(\mathrm{Re}\Gamma^{(2),R}(\omega, \vec{p}))^2 + (\mathrm{Im}\Gamma^{(2),R}(\omega, \vec{p}))^2}.$$



$\mu_B = 0$



$\mu_B = 500$ **MeV**

WF, Pawlowski, Pisarski, Rennecke, Wen, Yin, in preparation.

fRG in Keldysh path integral

- **Implement the formalism of fRG in the two time branches :**

$$Z_k[J_c, J_q] = \int (\mathscr{D}\varphi_c \mathscr{D}\varphi_q) \exp\left\{ i\left(S[\varphi] + \Delta S_k[\varphi] + (J_q^i \varphi_{i,c} + J_c^i \varphi_{i,q}) \right) \right\},$$

with

$$\Delta S_k[\varphi] = \frac{1}{2}(\varphi_{i,c}, \varphi_{i,q}) \begin{pmatrix} 0 & R_k^{ij} \\ (R_k^{ij})^* & 0 \end{pmatrix} \begin{pmatrix} \varphi_{j,c} \\ \varphi_{j,q} \end{pmatrix}$$

$$= \frac{1}{2}\left(\varphi_{i,c} R_k^{ij} \varphi_{j,q} + \varphi_{i,q} (R_k^{ij})^* \varphi_{j,c} \right),$$

Keldysh rotation:

$$\begin{cases} \varphi_{i,+} = \frac{1}{\sqrt{2}}(\varphi_{i,c} + \varphi_{i,q}), \\ \varphi_{i,-} = \frac{1}{\sqrt{2}}(\varphi_{i,c} - \varphi_{i,q}), \end{cases}$$

- **Then we derive the flow equation in the closed time path:**

$$\partial_\tau \Gamma_k[\Phi] = \frac{i}{2} \mathrm{STr}\left[\left(\partial_\tau R_k^* \right) G_k \right],$$

$$R_k^{ab} \equiv \begin{pmatrix} 0 & R_k^{ij} \\ (R_k^{ij})^* & 0 \end{pmatrix},$$

$$iG(x,y) = \begin{pmatrix} iG^K(x,y) & iG^R(x,y) \\ iG^A(x,y) & 0 \end{pmatrix},$$

$$iG^R(x,y) = \theta(x^0 - y^0)\langle [\phi(x), \phi^*(y)] \rangle,$$
$$iG^A(x,y) = \theta(y^0 - x^0)\langle [\phi^*(y), \phi(x)] \rangle,$$
$$iG^K(x,y) = \langle \{ \phi(x), \phi^*(y) \} \rangle,$$

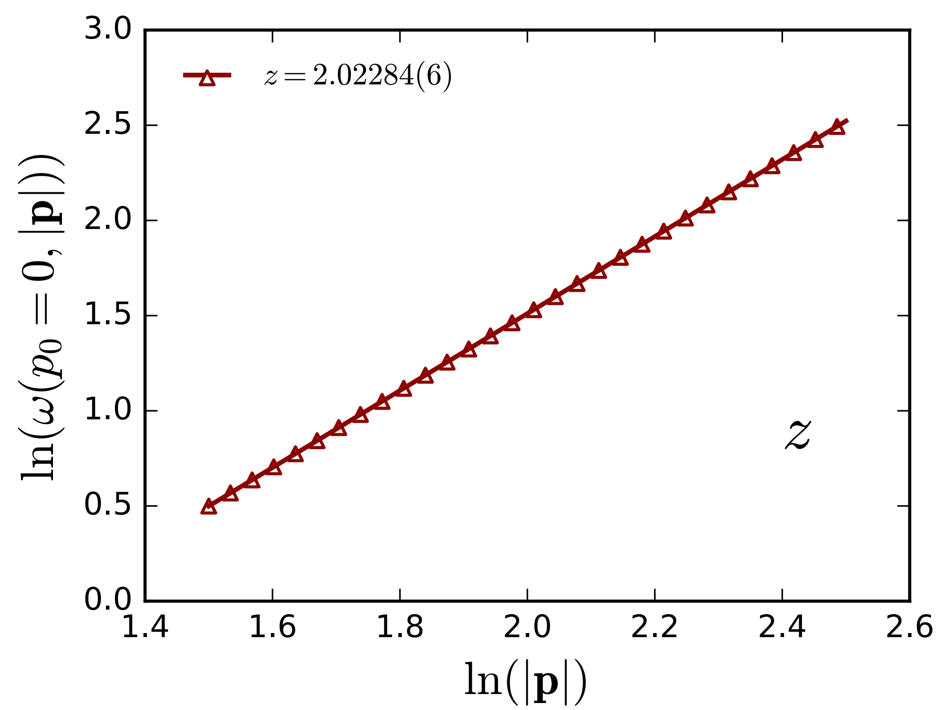
Y.-y. Tan, Y.-r. Chen, WF, arXiv: 2107.06482

Dynamical critical exponent

- **Kinetic coefficient:**

$$\frac{1}{\Gamma(|\vec{p}\,|)} = -i\frac{\partial \Gamma^{(2)}_{\phi_q\phi_c}(p_0,|\vec{p}\,|)}{\partial p_0}\bigg|_{p_0=0} = \frac{\partial \Im\, \Gamma^{(2)}_{\phi_q\phi_c}(p_0,|\vec{p}\,|)}{\partial p_0}\bigg|_{p_0=0},$$

- **Dissipative characteristic frequency:**



$$\omega(|\vec{p}\,|) = \Gamma(|\vec{p}\,|)\left(-\Gamma^{(2)}_{\phi_q\phi_c}(p_0=0,|\vec{p}\,|)\right) = -\Gamma(|\vec{p}\,|)\Re\,\Gamma^{(2)}_{\phi_q\phi_c}(p_0=0,|\vec{p}\,|)\,.$$

$\omega(|\vec{p}\,|) \propto |\vec{p}\,|^z \quad z \simeq 2.023$

Y.-y. Tan, Y.-r. Chen, WF, arXiv: 2107.06482

where we have used

$$\Re\,\Gamma^{(2)}_{\phi_q\phi_c}(-p_0,|\vec{p}|) = \Re\,\Gamma^{(2)}_{\phi_q\phi_c}(p_0,|\vec{p}|)\,,$$

$$\Im\,\Gamma^{(2)}_{\phi_q\phi_c}(-p_0,|\vec{p}|) = -\Im\,\Gamma^{(2)}_{\phi_q\phi_c}(p_0,|\vec{p}|)\,.$$

O(3):

$z \simeq 2.025\,,$

Duclut, Delamotte, *PRE* 95 (2017) 1, 012107.

Model A:

real-time classical-statistical lattice simulations

$z = 1.92(11)\,,$ Schweitzer, Schlichting, von Smekal, *NPB* 960 (2020) 115165.

Model G:

Relativistic O(4) should belong to Model G

$z = 3/2\,,$ Rajagopal and Wilczek, *NPB* 399 (1993) 395.

O(4):

real-time classical-statistical lattice simulations

$z \sim 2\,,$ Schlichting, Smith, L. von Smekal , *NPB* 950 (2020) 114868.

Summary

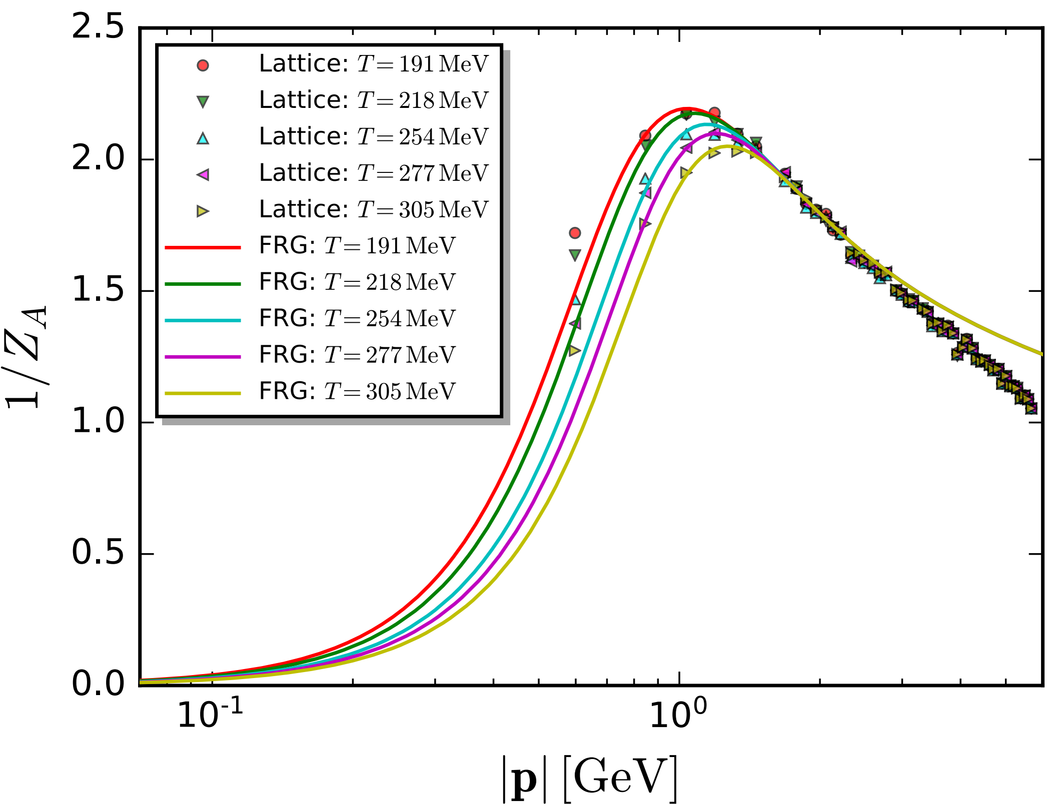
- ★ Functional renormalization group (fRG) is a nonperturbative continuum field approach, which is complementary to other approaches, e.g., lattice, DSE, etc.

- ★ Recent years have seen significant progresses in studies of QCD phase structure, static and dynamical properties of QCD matter in heavy-ion collisions within the fRG approach.

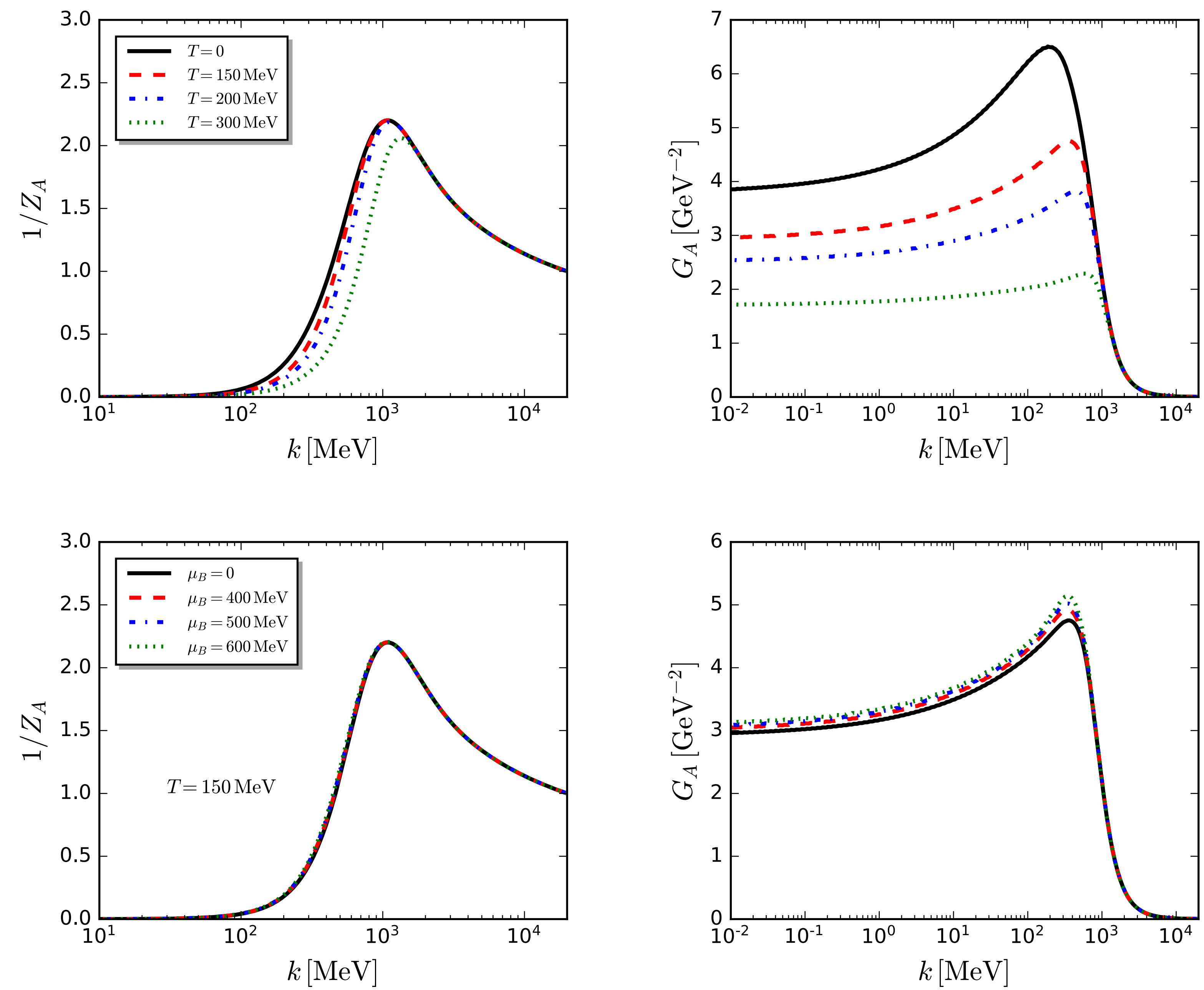
祝中国格点QCD蓬勃发展、蒸蒸日上!

Backup

Gluon propagator at finite T and muB



WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032



Dynamical hadronization

Introduce a scale-dependent meson field:

$$\langle \partial_t \hat{\phi}_k \rangle = \dot{A}_k\, \bar{q}\tau q + \dot{B}_k\, \phi + \dot{C}_k\, \hat{e}_\sigma\,,$$

Gies, Wetterich , *PRD* 65, 065001 (2002); 69, 025001 (2004)
Pawlowski, AP 322, 2831 (2007)
Flörchinger, Wetterich, PLB 680, 371 (2009)

The four-fermion couplings:

$$\partial_t \bar{\lambda}_q - 2\,(1+\eta_q)\,\bar{\lambda}_q - \bar{h}\,\dot{\bar{A}} = \frac{1}{4}\overline{\mathrm{Flow}}^{(4)}_{(\bar{q}q)(\bar{q}q)}\,,$$

Demanding

$$\bar{\lambda}_q \equiv 0\,, \qquad \forall k\,.$$

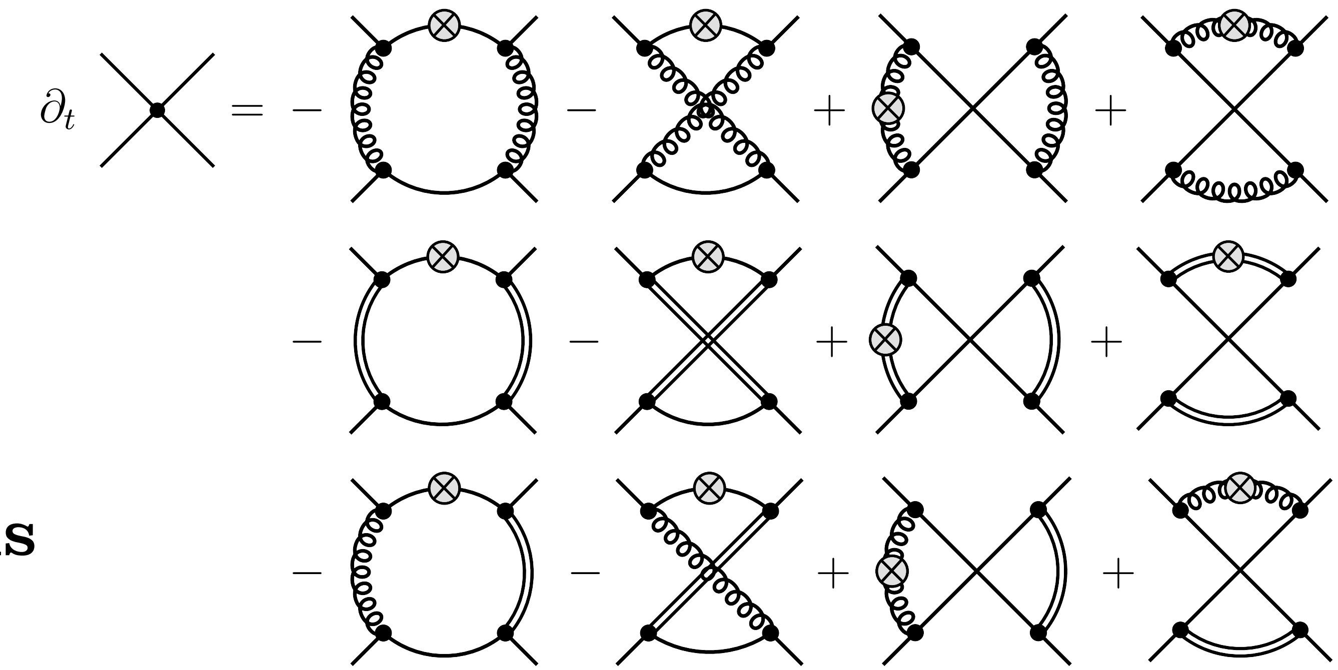
The hadronization function reads

$$\dot{\bar{A}} = -\frac{1}{\bar{h}}\overline{\mathrm{Flow}}^{(4)}_{(\bar{q}q)(\bar{q}q)}/4\,.$$

The Wetterich equation is modified:

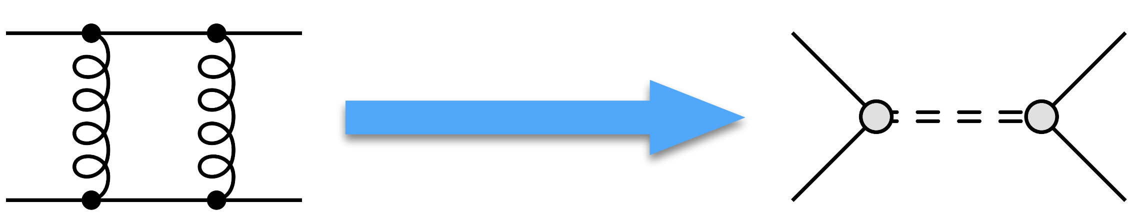
$$\partial_t \Gamma_k[\Phi] = \frac{1}{2}\mathrm{STr}\big(G_k[\Phi]\,\partial_t R_k\big) + \mathrm{Tr}\left(G_{\phi\Phi_j}[\Phi]\frac{\delta\langle\partial_t\hat{\phi}_k\rangle}{\delta\Phi_j}R_\phi\right) - \int \langle \partial_t \hat{\phi}_{k,i}\rangle\left(\frac{\delta\Gamma_k[\Phi]}{\delta\phi_i} + c_\sigma \delta_{i\,\sigma}\right)\,,$$

WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

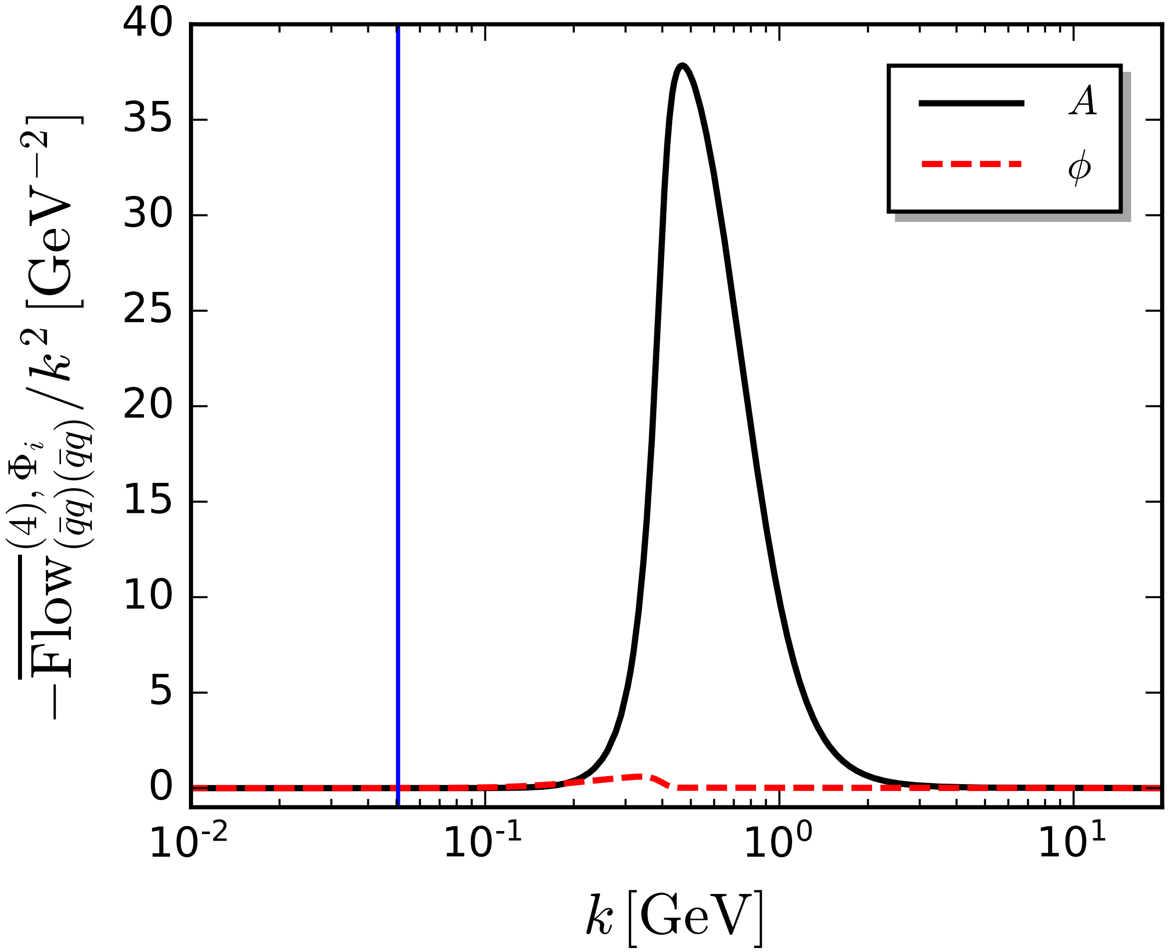


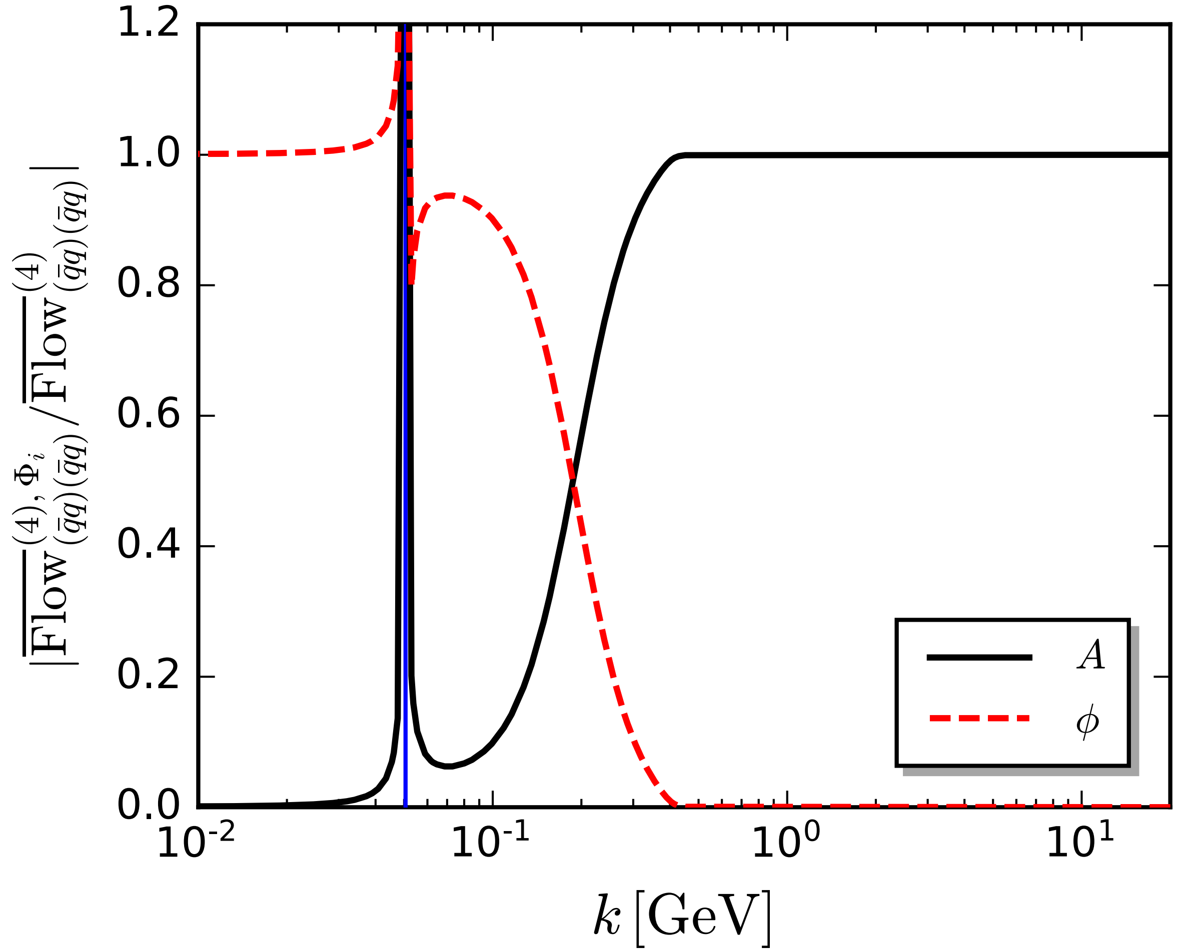
$+$ (gluons $\leftrightarrow$ mesons)
$+$ (permutations of the regulator insertions)

4-fermion interaction encoded in Yukawa coupling:



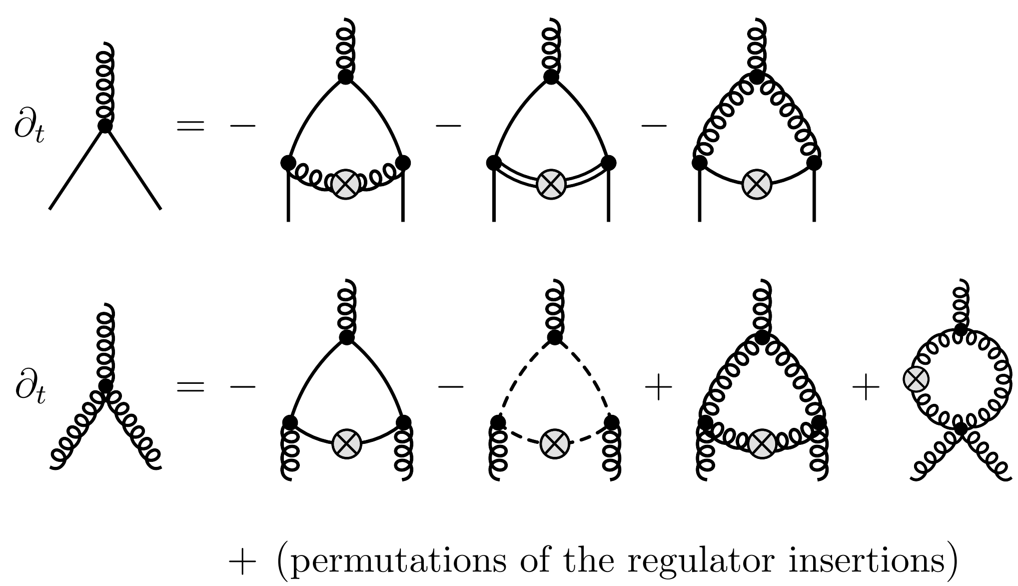
Flow of 4-quark coupling—gluon versus meson

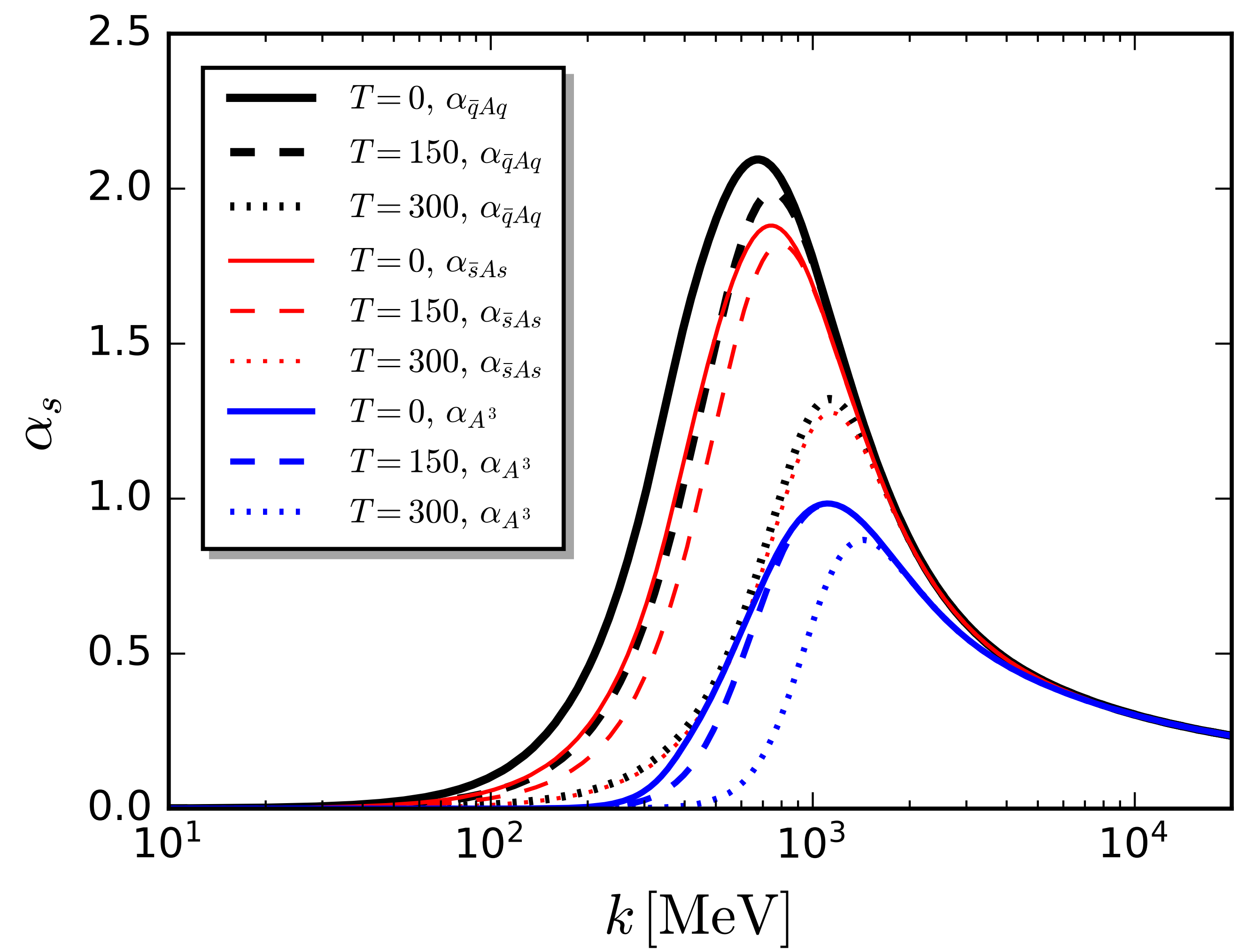




WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

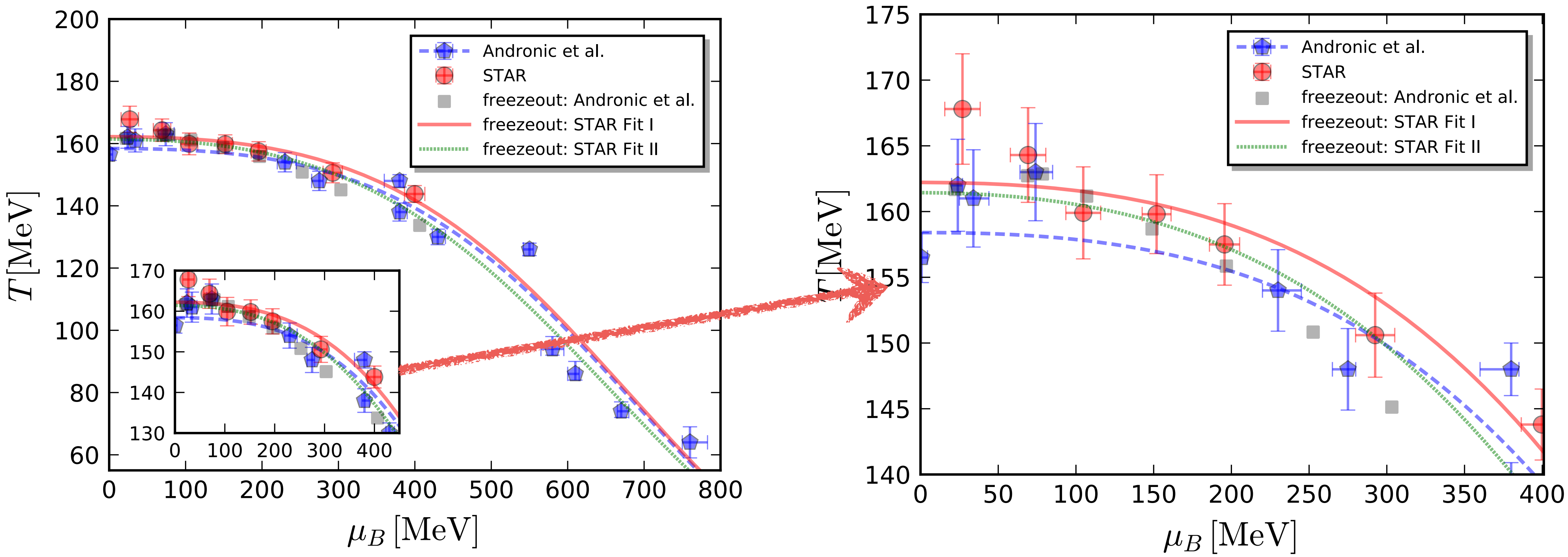
QCD strong couplings among quarks and gluons





WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

Determination of the freeze-out curve



three freeze-out curves

1. freeze-out: Andronic *et al.*

Andronic, Braun-Munzinger, Redlich, *Nature* 561 (2018) 7723, 321

2. freeze-out: STAR Fit I

L. Adamczyk *et al.* (STAR), *PRC* 96 (2017), 044904

$$\mu_{B_{\rm CF}} = \frac{a}{1 + 0.288\sqrt{s_{\rm NN}}}\,,$$

$$T_{\rm CF} = \frac{T^{(0)}_{\rm CF}}{1 + \exp\left(2.60 - \ln(\sqrt{s_{\rm NN}})/0.45\right)}$$

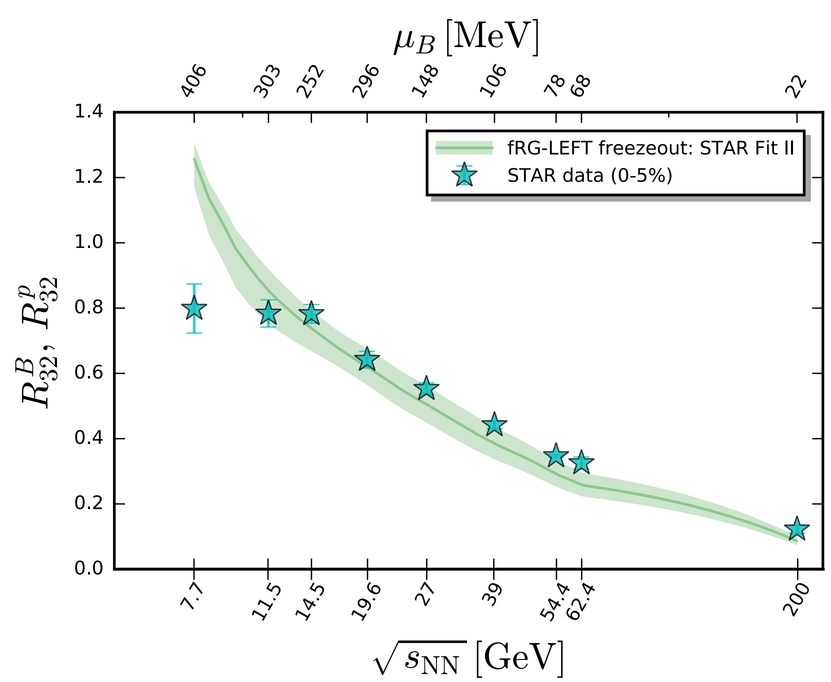
all data points

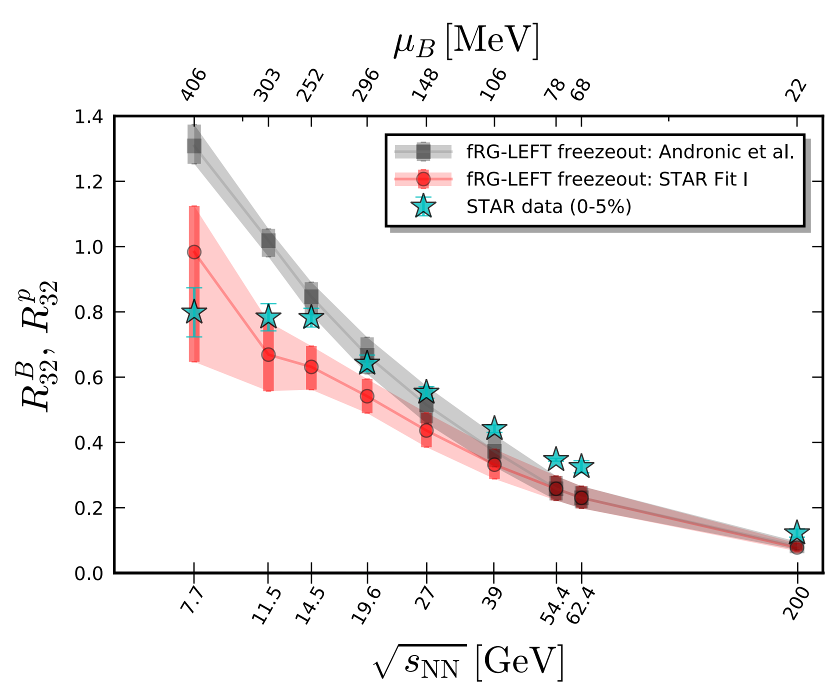
3. freeze-out: STAR Fit II

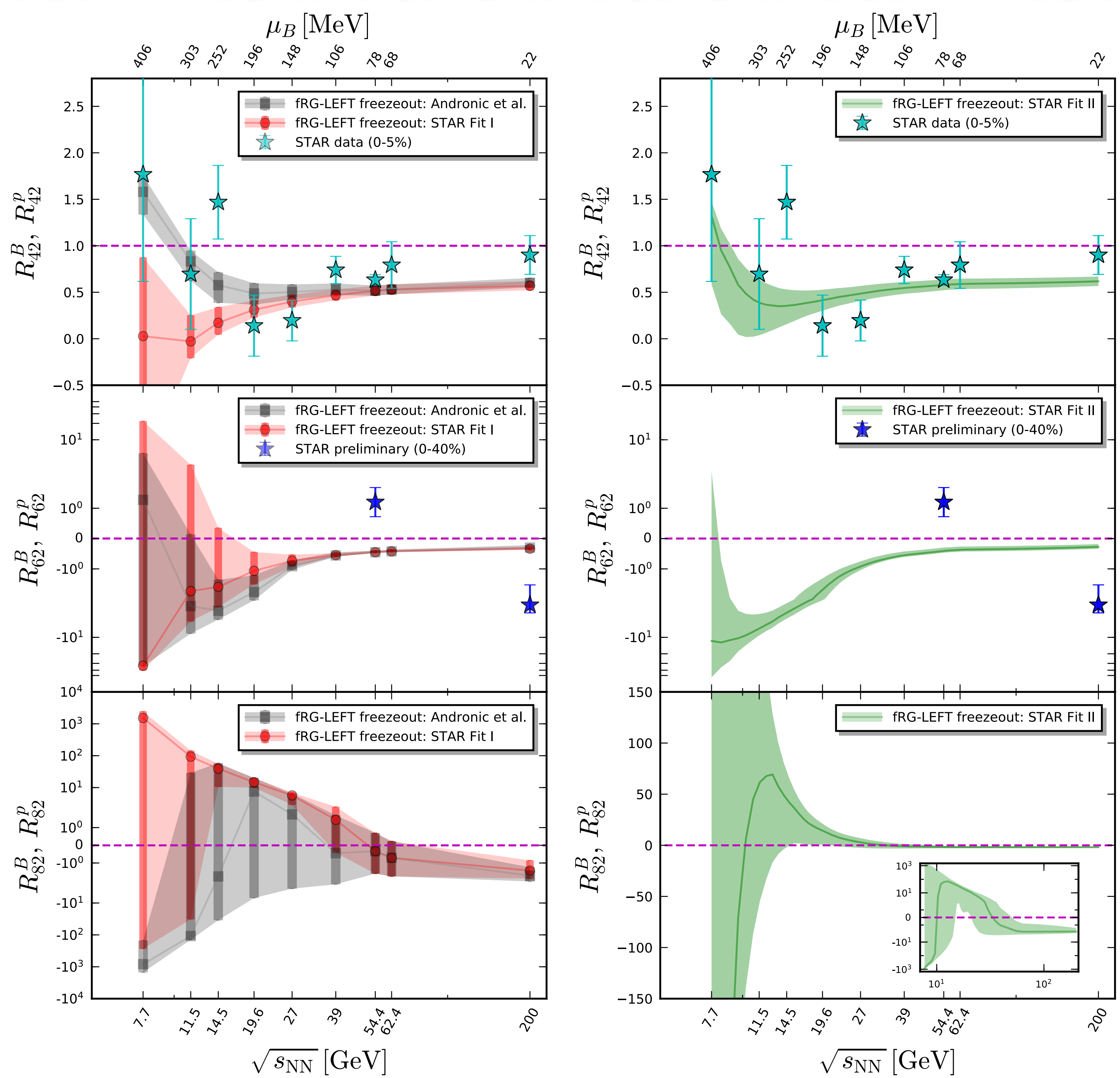
neglecting first two at low μ_B and the last one

- freeze-out curve should not rise with μ_B
- convexity of the freeze-out curve

Fluctuations on the freeze-out curve





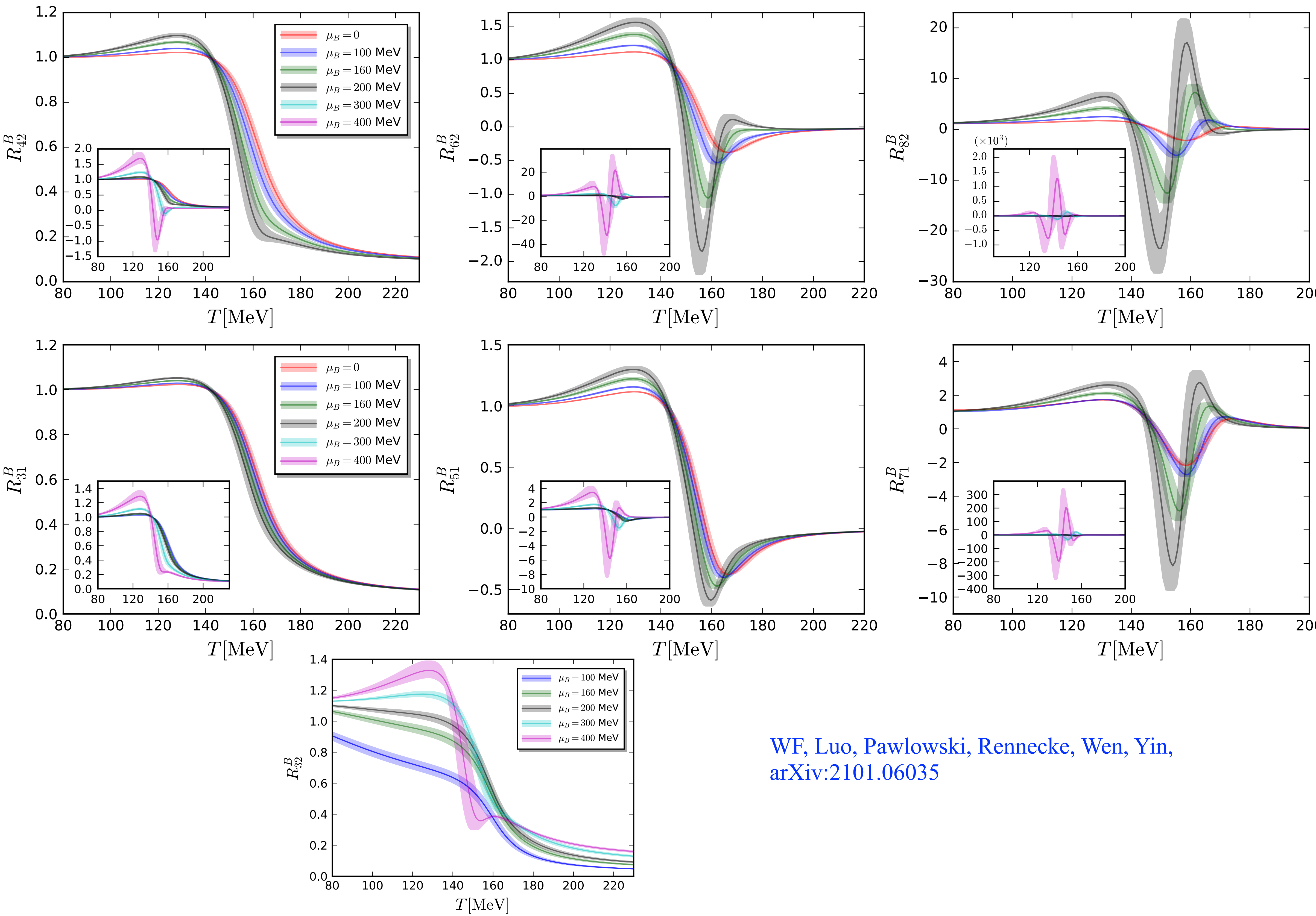


WF, Luo, Pawlowski, Rennecke, Wen, Yin, arXiv:2101.06035

J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301
M. Abdallah *et al.* (STAR), arXiv:2101.12413

T. Nonaka (STAR), *NPA* 1005 (2021) 121882; A. Pandav (STAR), arXiv: *NPA* 1005 (2021) 121936

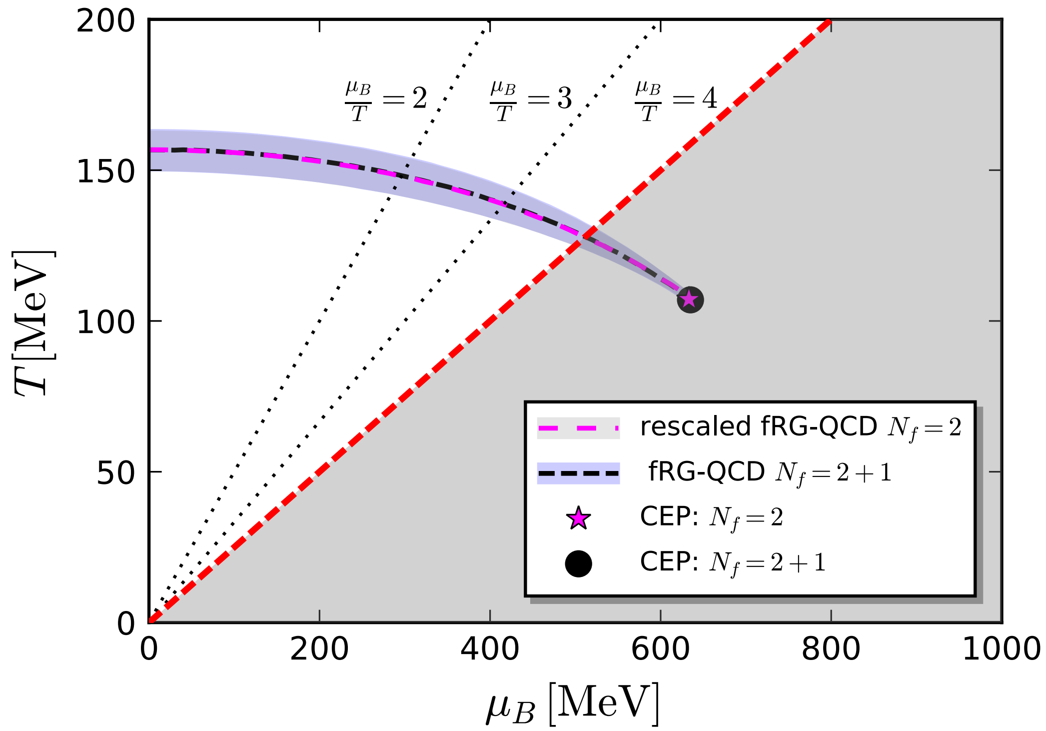
Fluctuations at finite chemical potential



WF, Luo, Pawlowski, Rennecke, Wen, Yin,
arXiv:2101.06035

2- to 2+1-flavour scale-matching in QCD and LEFTs

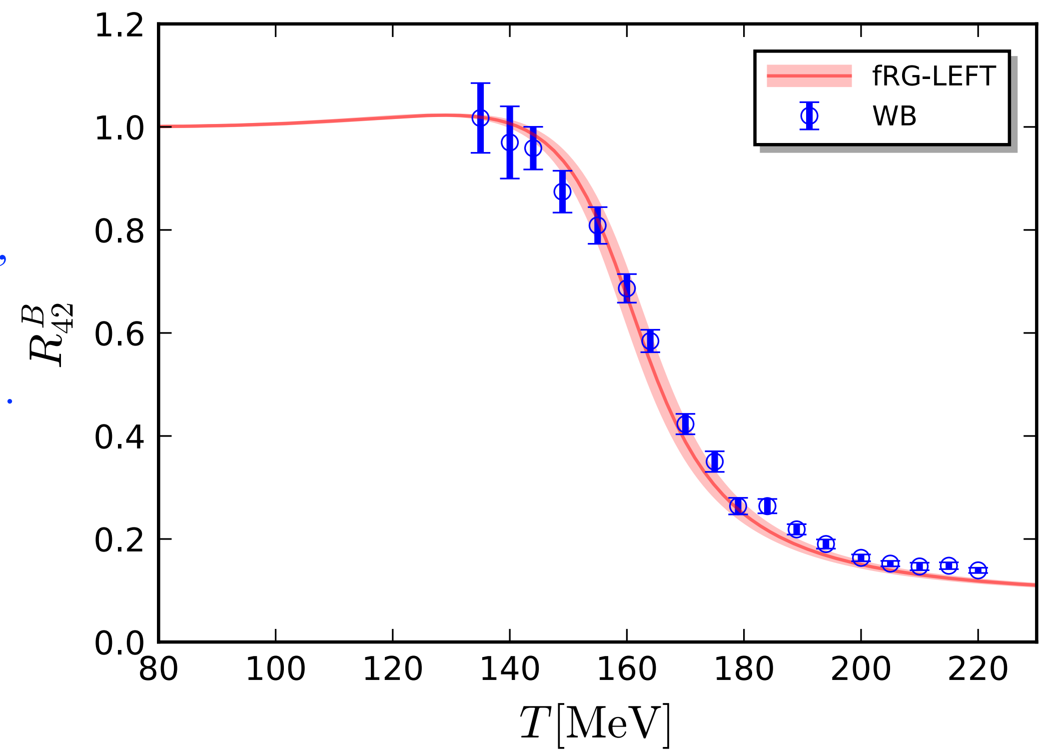
QCD



Data from: WF, Pawlowski, Rennecke, *PRD* 101 (2020), 054032

WF, Luo, Pawlowski, Rennecke, Wen, Yin, arXiv:2101.06035

LEFT



linear scale-matching

$$T^{(N_f=2)} = c_T\, T^{(N_f=2+1)}$$

$$\mu_B^{(N_f=2)} = c_{\mu_B}\, \mu_B^{(N_f=2+1)}$$

curvature of the phase boundary

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \lambda \left(\frac{\mu_B}{T_c}\right)^4 + \cdots$$

$$c_T = 1.247(12)\,, \qquad c_{\mu_B} = c_T \left(\frac{\kappa^{N_f=(2+1)}}{\kappa_{\mathrm{LEFT}}}\right)^{1/2} = 1.110(66)$$

Yukawa Coupling

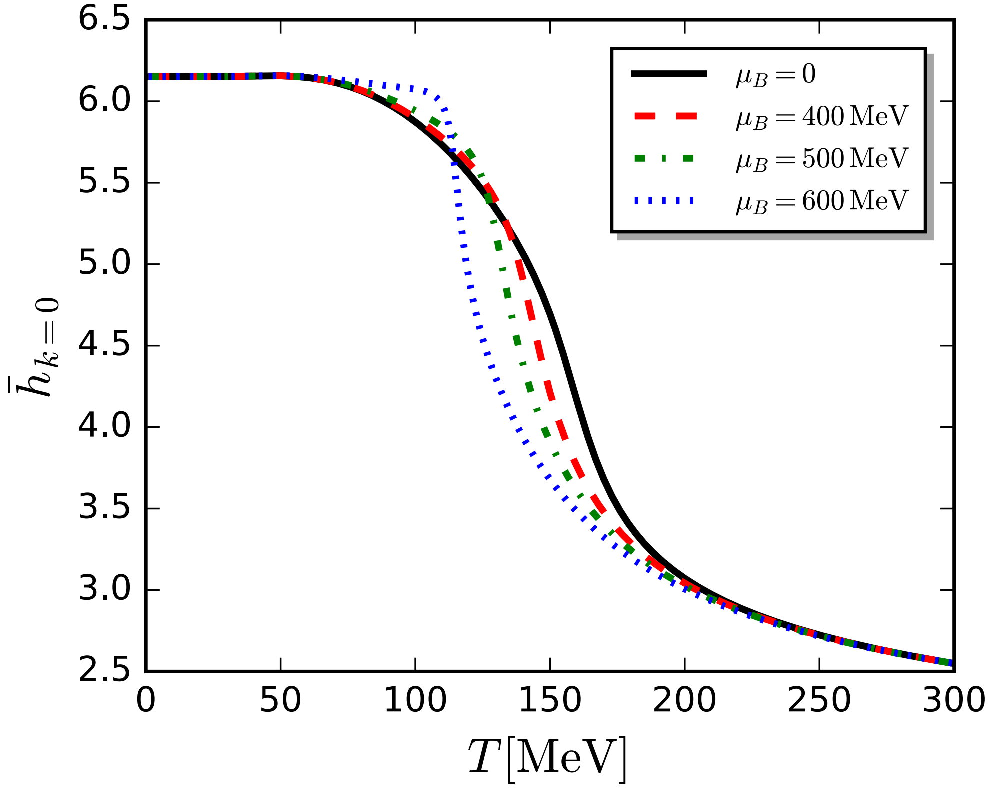
Two different Yukawa coupling:

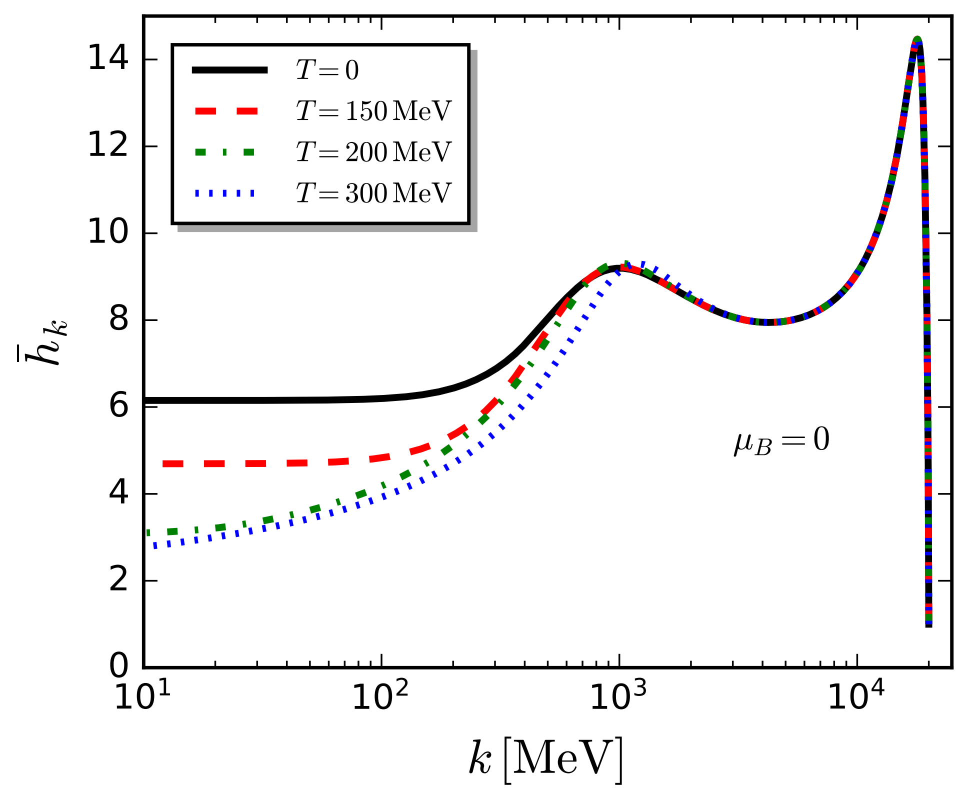
$$h_\pi = h(\rho_0) = \Gamma^{(3)}_{(\bar{q}\vec{\tau}q)\vec{\pi}}[\Phi_0]\,,$$

$$h_\sigma = h(\rho_0) + \rho\, h'(\rho_0) = \Gamma^{(3)}_{(\bar{q}\tau^0 q)\sigma}[\Phi_0]\,.$$

The flow equation:

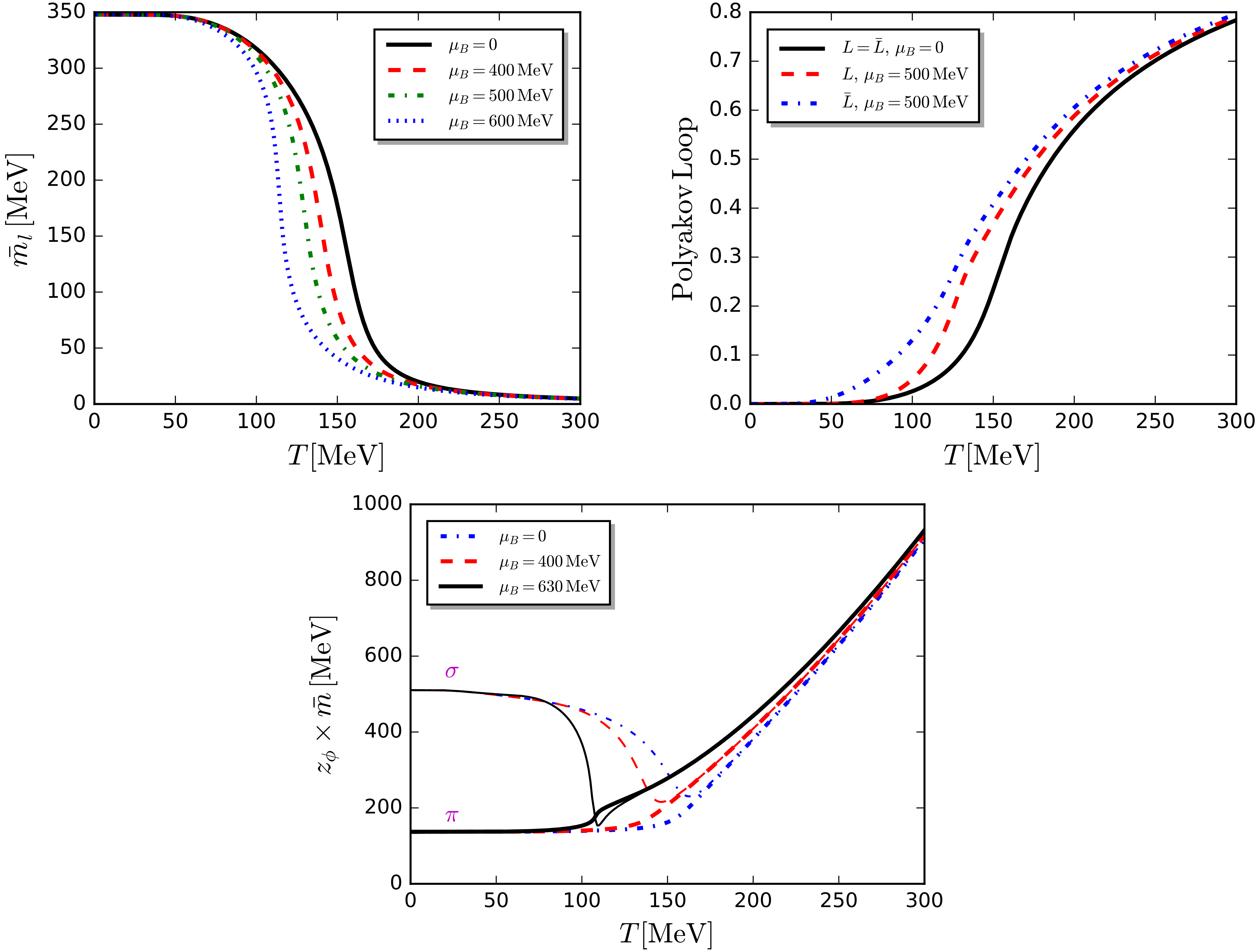
$$\partial_t \bar{h} = \left(\frac{1}{2}\eta_{\phi,k} + \eta_{q,k}\right)\bar{h} - \bar{m}_\pi^2\,\dot{\bar{A}} + \frac{1}{\bar{\sigma}}\mathrm{Re}\,\overline{\mathrm{Flow}}^{(2)}_{(\bar{q}\tau^0 q)}\,,$$





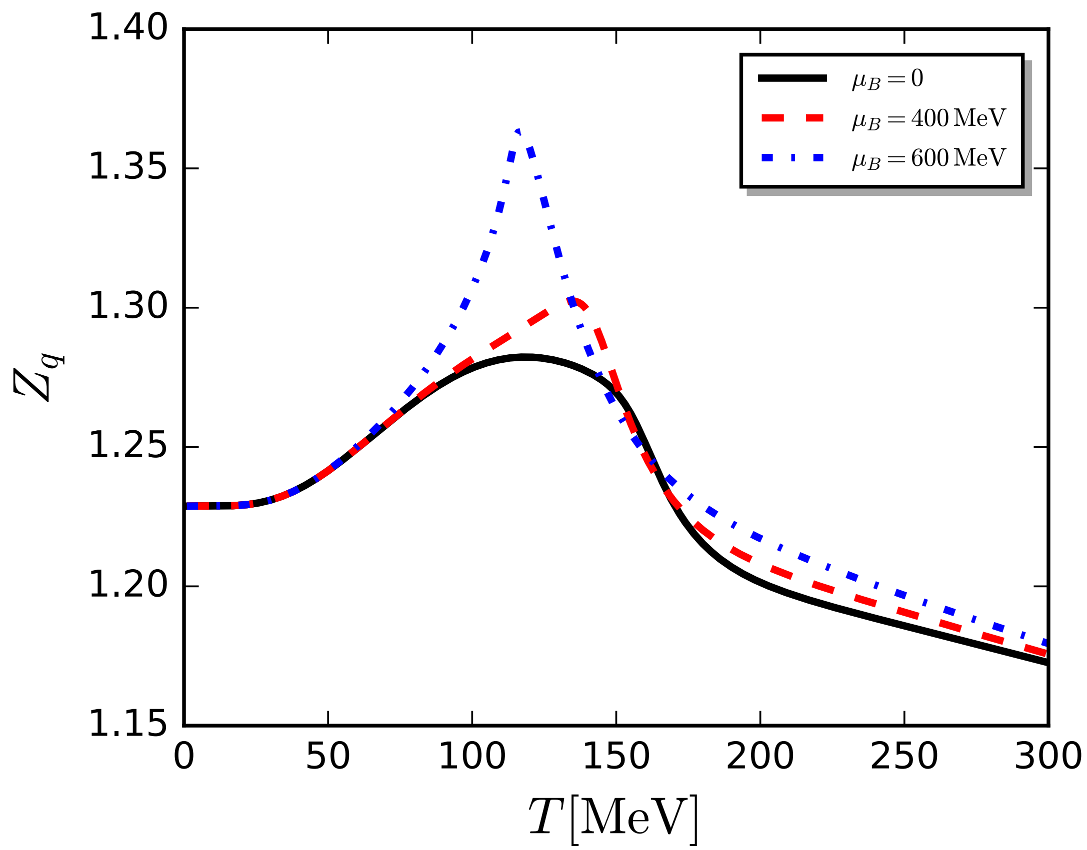
WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

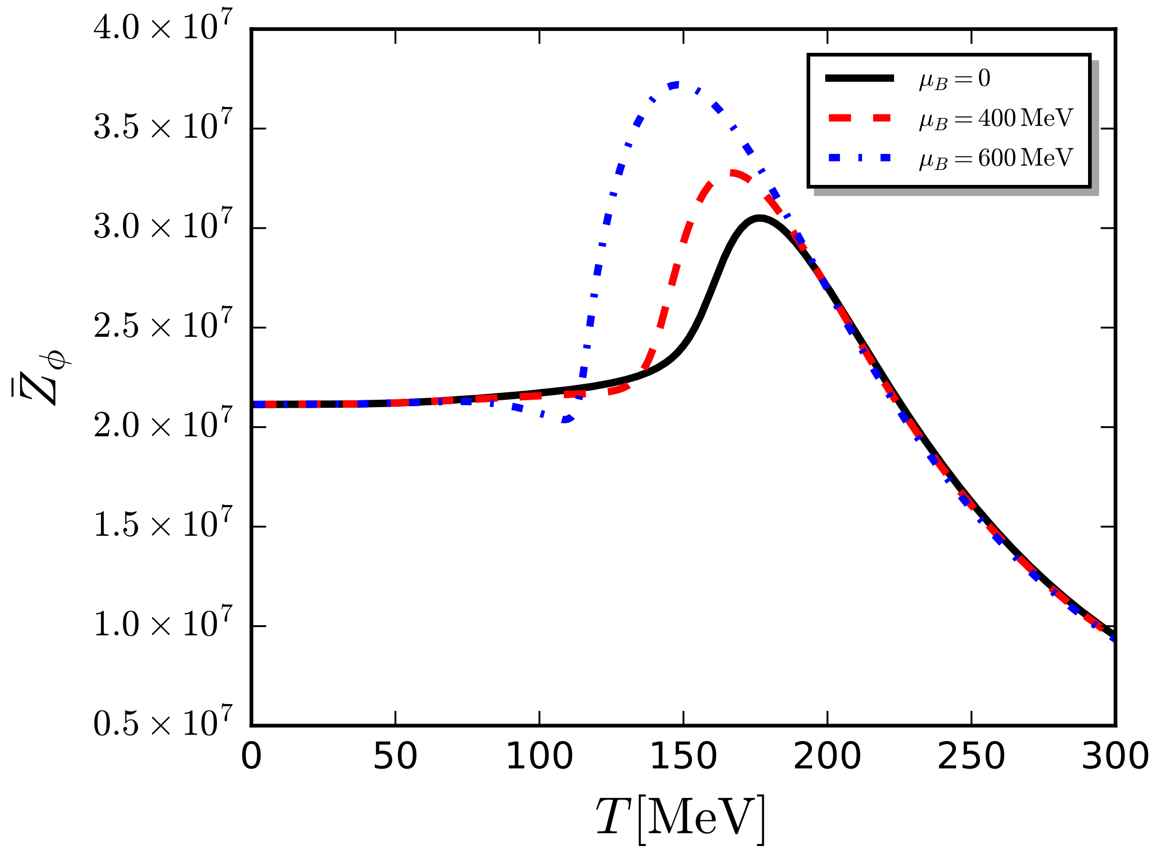
Quark, Meson Mass and Polyakov loop



WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

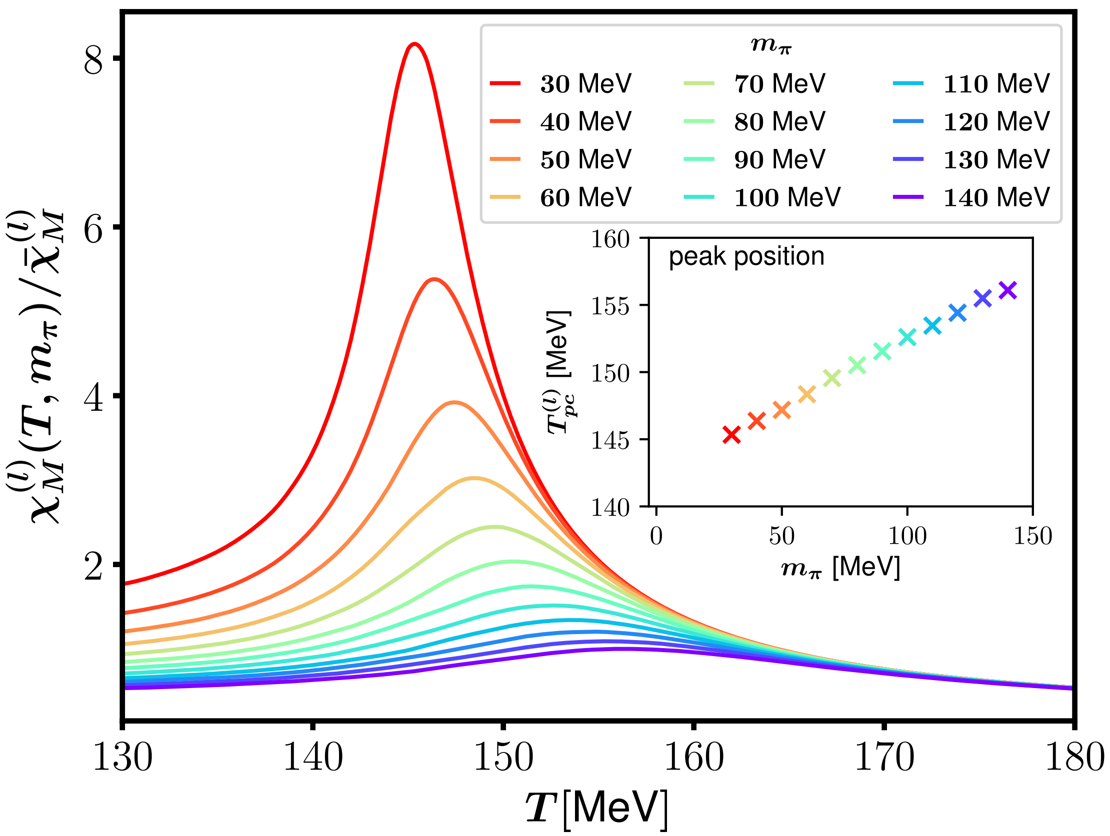
Quark and Meson Wave Function Renormalization

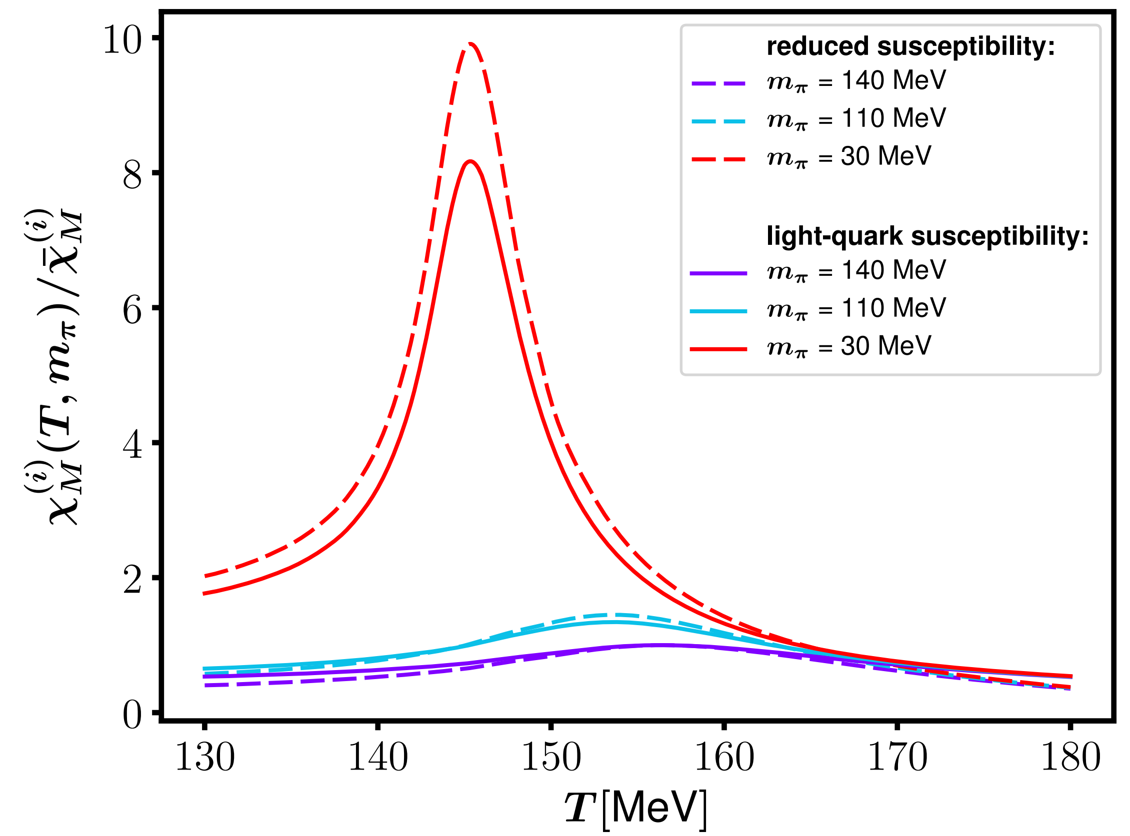




WF, J.M. Pawlowski, F. Rennecke, arXiv:1909.02991

Chiral susceptibility for different pion masses





Light quark condensate:

$$\Delta_{q_i} \simeq -m_{q_i}^0 T \sum_{n\in\mathbb{Z}} \int \frac{d^3q}{(2\pi)^3} \mathrm{tr}\, G_{q_i\bar{q}_i}(q)\,.$$

Susceptibility:

$$\chi_M^{(i)}(T) = -\frac{\partial}{\partial m_l^0}\left(\frac{\Delta_i(T)}{m_l^0}\right)\,,$$

Reduced condensate:

$$\Delta_{l,s} = \frac{1}{\mathcal{N}_{l,s}}\left(\Delta_l(T) - \left(\frac{m_l^0}{m_s^0}\right)^2 \Delta_s(T)\right)\,,$$

Braun, WF, Pawlowski, Rennecke, Rosenblüh, Yin, *PRD* 102 (2020), 056010.

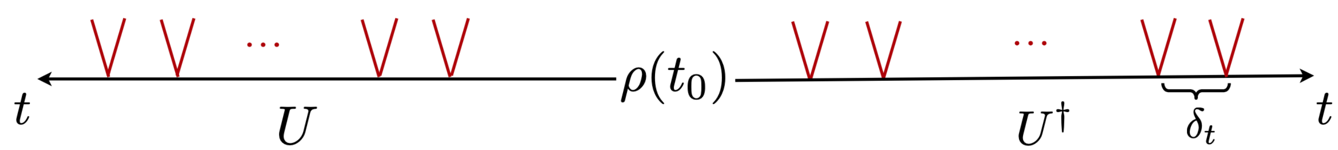
Schwinger-Keldysh path integral

- **Schrödinger equation:**

$$U(t,t_0) = e^{-iH(t-t_0)}$$

$$i\partial_t|\psi(t)\rangle = H|\psi(t)\rangle \quad \longrightarrow \quad |\psi(t)\rangle = U(t,t_0)|\psi(t_0)\rangle,$$

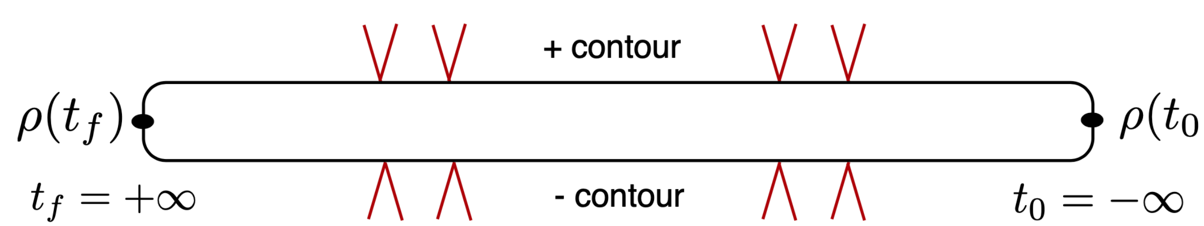
- **von Neumann equation:**



$$\partial_t \rho(t) = -i[H,\rho(t)] \quad \longrightarrow \quad \rho(t) = U(t,t_0)\rho(t_0)U^\dagger(t,t_0),$$

- **Keldysh partition function:**

$$Z = \mathrm{tr}\,\rho(t),$$



- **two-point closed time-path Green's function:**

$$G(x,y) \equiv -i\mathrm{tr}\{T_p(\phi(x)\phi^\dagger(y)\rho)\}$$
$$\equiv -i\langle T_p(\phi(x)\phi^\dagger(y))\rangle\,,$$

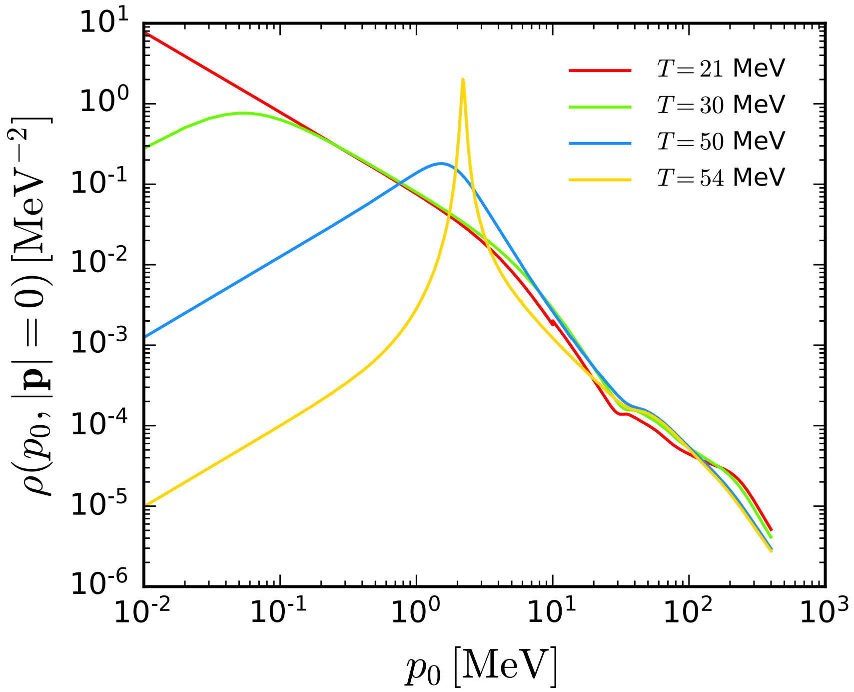
$$G(x,y) = \begin{pmatrix} G_{++} & G_{+-} \\ G_{-+} & G_{--} \end{pmatrix}$$
$$\equiv \begin{pmatrix} G_F & G_+ \\ G_- & G_{\tilde F} \end{pmatrix}\,,$$

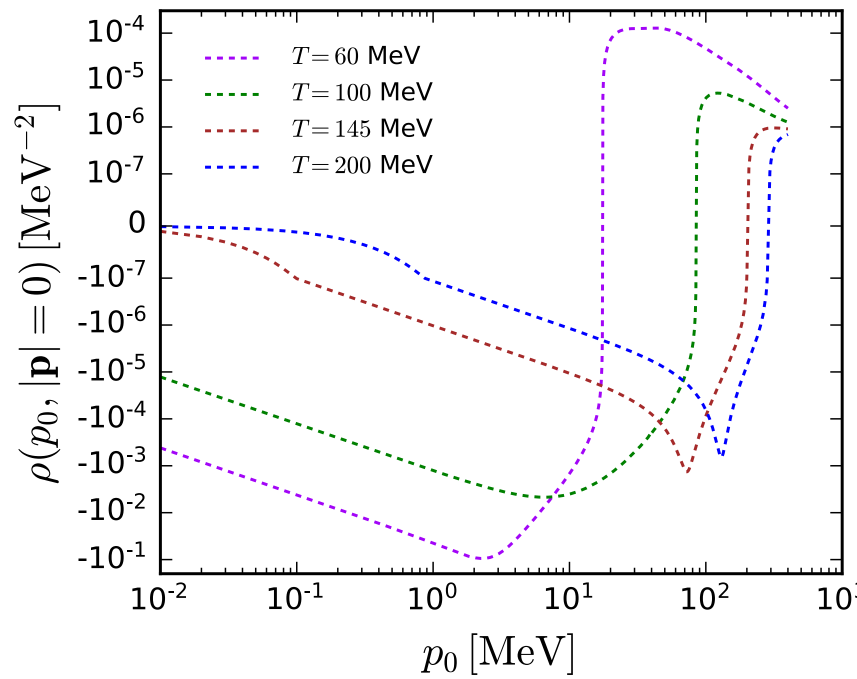
$$G_F(x,y) \equiv -i\langle T(\phi(x)\phi^\dagger(y))\rangle\,,$$
$$G_+(x,y) \equiv -i\langle \phi^\dagger(y)\phi(x)\rangle\,,$$
$$G_-(x,y) \equiv -i\langle \phi(x)\phi^\dagger(y)\rangle\,,$$
$$G_{\tilde F}(x,y) \equiv -i\langle \tilde T(\phi(x)\phi^\dagger(y))\rangle\,,$$

Schwinger, J. Math. Phys. 2, 407 (1961);
Keldysh, Zh. Eksp. Teor. Fiz. 47, 1515 (1964);
Chou, Su, Hao, Yu, Phys. Rept. 118, 1 (1985).

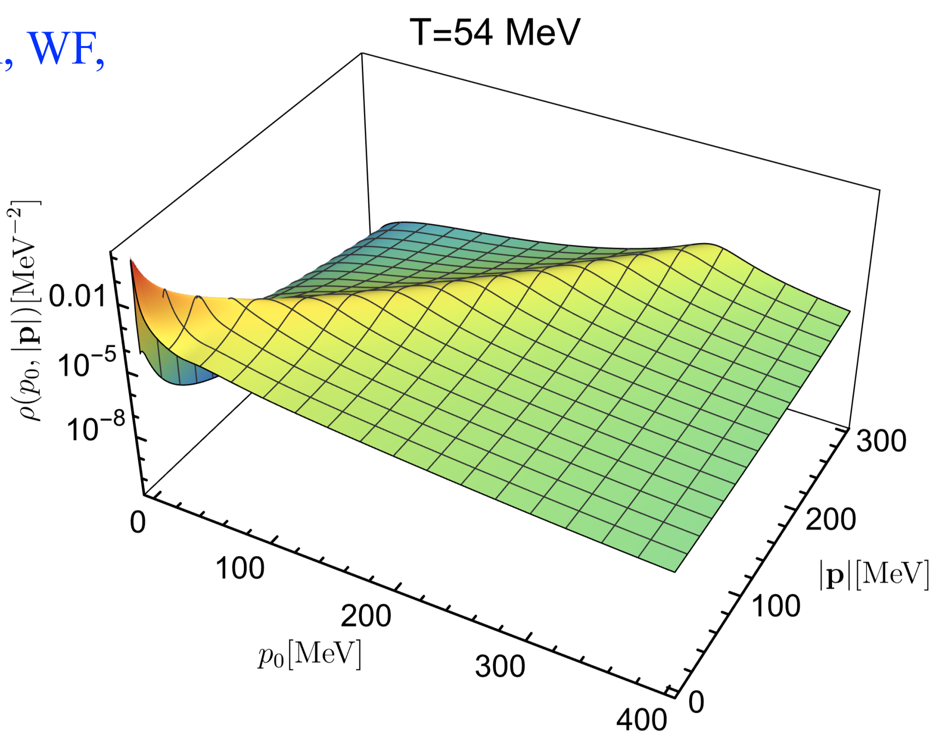
Spectral functions

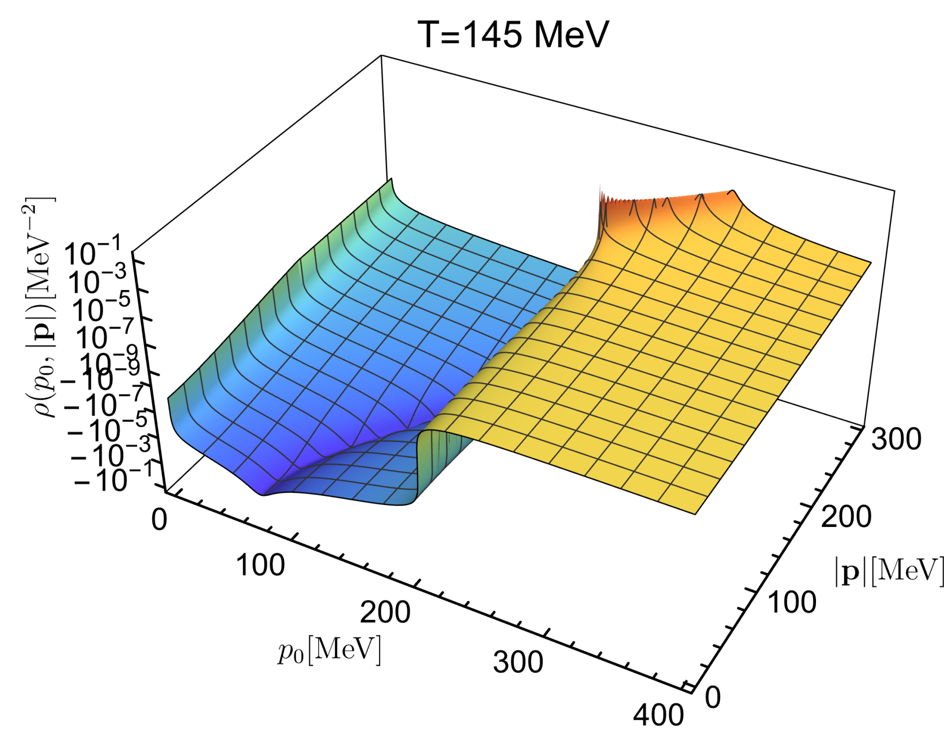
real time:momentum dependent





Y.-y. Tan, Y.-r. Chen, WF, arXiv: 2107.06482





$$G_R(p_0, |\vec{p}\,|) = -\int_{-\infty}^{\infty} \frac{dp_0'}{2\pi} \frac{\rho(p_0', |\vec{p}\,|)}{p_0' - (p_0 + i\epsilon)},$$

Spectral function:

$$\rho(p_0, |\vec{p}\,|) = \frac{2\Im\,\Gamma^{(2)}_{\phi_q\phi_c}(p_0, |\vec{p}\,|)}{\left[\Re\,\Gamma^{(2)}_{\phi_q\phi_c}(p_0, |\vec{p}\,|)\right]^2 + \left[\Im\,\Gamma^{(2)}_{\phi_q\phi_c}(p_0, |\vec{p}\,|)\right]^2}.$$