# Charmonia Decays on the Lattice

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# Outline

- Motivation
- Recent works
  - Two-body sector: $\eta_c \to 2\gamma, J/\psi \to \gamma\eta_c$
  - Three-body sector: $J/\psi \to 3\gamma$
- Summary and outlook

# Motivation

## Why charmonia decays ?

- Test the interplay of pert and non-pert QCD
  — intermediate energy scale
- More possibilites for the search of new physics
  — invisible decay of charmoinum
- cooperation,cooperation,cooperation
  — BESIII

# Toward precision test for QCD

## Two-photon decay of charmonia[$\eta_c \to 2\gamma$]

- Tension between therories and experiments
  - Inconsistency of different theories NRQCD,DSE, Lattice $\cdots$
  - Major uncertainties of experiments $\sim$ 50% errors BESIII,2013

## Three-photon decay of charmonia[$J/\psi \to 3\gamma$]

- Model-dependent errors[unknown matrix element]
  - CLEO, $\mathcal{B} = (1.2 \pm 0.3 \pm 0.2) \times 10^{-5}$, PRL 101,101801(2008)
  - BESIII, $\mathcal{B} = (1.13 \pm 0.18 \pm 0.2) \times 10^{-5}$, PRD 87,032003(2013)
- Pert troubles[soft photon], crucial role of lattice QCD

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Necessity of high precision study on charmonia decays

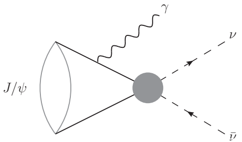
## Invisible decay of charmoinum

- BSM predict CP-odd pseudoscalar higgs boson $A_0$, which can be coupled with SM particle
- Dark matter from collider

$$q\bar{q} \to \gamma + \text{invisible}$$

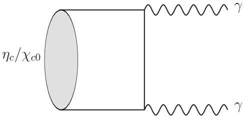
CLEO,PRD.**81**,091101(2010) BaBar,PRL.**107**,021804(2011)
Belle,PRL.**122**,011801(2019) BESIII,PRD.**101**,112005(2020)

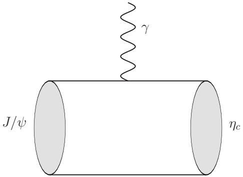
- SM background: $J/\psi \to \gamma + \nu\bar{\nu}$
  - Perturbative calculation
    D.-N.Gao,Phys.Rev.D.**90**,077501(2014)
  - No lattice study yet



# Recent works

- Two-body

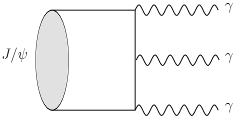


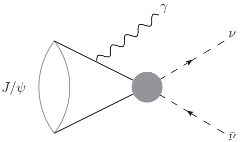




C.Liu,YM, K-L.Zhang, Phys.Rev.D.**102**,034502(2020)    YM *et al*,in preparation

YM *et al*,2109.09381;  Z-H.Zou,YM,C.Liu,2111.00768

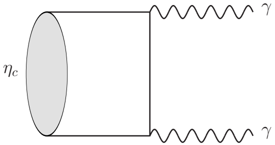
- Three-body




YM,C.Liu, K-L.Zhang,Phys.Rev.D.**102**,054506(2020)    ongoing

# $\eta_c \to 2\gamma$




- Amplitude:

$$\mathcal{M} = e^2 \epsilon_\mu(p)\epsilon_\nu(p')H_{\mu\nu}(p,q)$$
$$H_{\mu\nu}(p,q) = \int d^4x e^{-ipx}\mathcal{H}_{\mu\nu}(x,q), \quad \mathcal{H}_{\mu\nu} = \langle 0|\mathrm{Tr}[J_\mu(x)J_\nu(0)]|\eta_c(q)\rangle$$

- Form factor: $$H_{\mu\nu}(p,q) = \epsilon_{\mu\nu\alpha\beta}p_\alpha q_\beta F(p^2)$$

- Decay width:

$$\Gamma_{\eta_c\gamma\gamma} = \alpha_{\mathrm{em}}^2 \frac{\pi}{4} m_{\eta_c}^3 |F_{\eta_c\gamma\gamma}|^2, \quad F_{\eta_c\gamma\gamma} = F(0)$$

# Traditional method for on-shell form factor

- Off-shell form factors by projecting discrete momentum $\vec{p} = \frac{2\pi\vec{n}}{L}$

$$F(p^2) \xrightarrow{\text{Cont.Limit}} F(0)$$

  - Leading to additional computation cost and systematic source.

CLQCD,Eur.Phys.J.C 7**6**,358 (2016)

C.Liu,YM, K-L.Zhang Phys.Rev.D.**102**,034502(2020)

- Without considering various systematic effects
  - Lattice discretization effect: only one or two lattice spacings used
  - Excited-state contamination: without considering the excited-state effect on the lattice

# Direct approach to on-shell form factor

- Calculate the form factor at $|\vec{p}| = \frac{m_{\eta_c}}{2}$ directly[infinite volume]

$$F_{\eta_c\gamma\gamma}(\Delta t) = -\frac{1}{2m_{\eta_c}} \int dt e^{m_{\eta_c} t/2} \int d^3\vec{\boldsymbol{x}} \frac{j_1(|\vec{\boldsymbol{p}}||\vec{\boldsymbol{x}}|)}{|\vec{\boldsymbol{p}}||\vec{\boldsymbol{x}}|} \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(x, \Delta t)$$

- Replaced by finite-volume version

$$\epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}_{\mu\nu}(x, q) \to \epsilon_{\mu\nu\alpha 0} x_\alpha \mathcal{H}^L_{\mu\nu}(x, q)$$

  - Exponentially supressed with the distance

- Remove excited-state effects

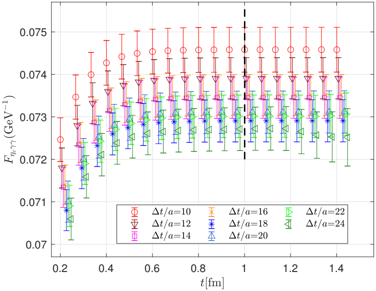
$$F_{\eta_c\gamma\gamma}(\Delta t) = F_{\eta_c\gamma\gamma} + \xi \times e^{-(m_1 - m)\Delta t}$$

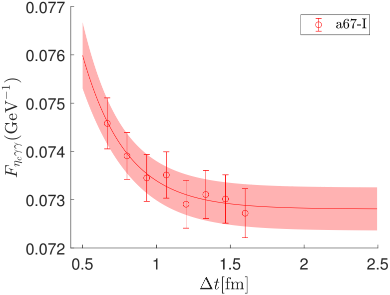
- Finite-volume effects: $R$-dependence of $F_{\eta_c\gamma\gamma}(\Delta t) \Leftrightarrow \sum_{\vec{\boldsymbol{x}}} \to \sum_{|\vec{\boldsymbol{x}}| \le R}$

# $N_f = 2$ twisted-mass

| Ens | $a$(fm) | $V/a^4$ | $a\mu_{sea}$ | $N_{\rm conf} \times T_s$ | $\Delta t/a$ |
|---|---|---|---|---|---|
| a98 | 0.098 | $24^3 \times 48$ | 0.0060 | $235 \times 48$ | 7:1:17 |
| a85 | 0.085 | $24^3 \times 48$ | 0.0040 | $197 \times 48$ | 8:1:18 |
| a67 | 0.0667 | $32^3 \times 64$ | 0.0030 | $197 \times 64$ | 10:2:24 |

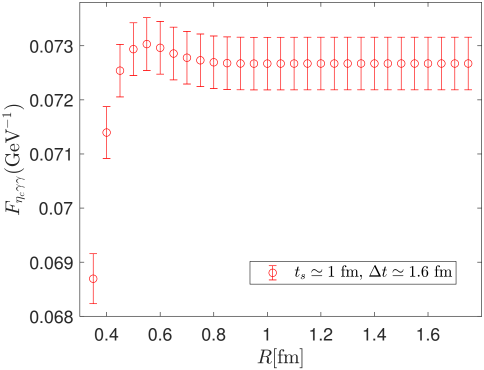
- All ensembles have similar physical spatial volume: $2.04 \sim 2.35$ fm.
- All ensembles have similar pion mass: $300 \sim 365$ MeV.
- A series of $\Delta t(0.7\sim 1.6\text{fm})$ to extract the ground-state contribution.
- Charm quark mass is tuned by physical $\eta_c$ and $J/\psi$ mass, respectively, and take the difference as our systematic error.




- An obvious dependene on excited-state of $\eta_c$.
- Dashed black line: a suitable time truncation $t_{\rm cut} \sim 1$ fm.
- The right: an extrapolation for the form factors at $t_{\rm cut}$.

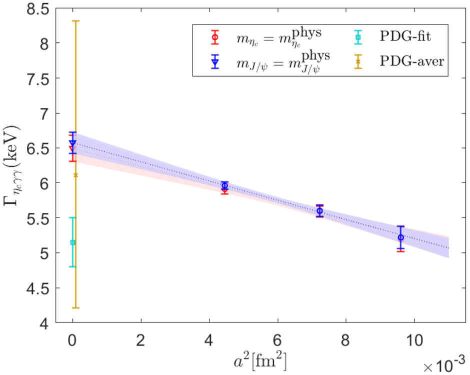
# Finite volume effect




- Examine the possibility $F \propto \int d^4x\omega(x)\epsilon_{\mu\nu\alpha 0}x_\alpha\mathcal{H}_{\mu\nu}(x), \quad \sum_{\vec{x}} \to \sum_{|\vec{x}|\leq R}$

# Continuous limit

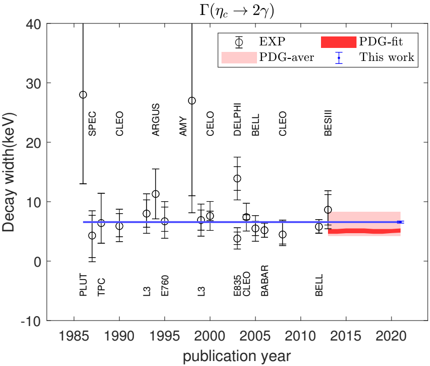
- NRQCD $\sim 6.2$
  K.-T.Chao *et al*, PRD**56**,368(1997)
- NRQCD(NNLO) $\sim 10$
  F.Feng *et al*, PRL**119**,252001(2017)
- Soft gluon factorization $\sim$
  Y.-Q.Ma,K.-T.Chao, PRD**100**,094007(2019)
- DSE $\sim 6.4$
  J.Chen *et al*, PRD**95**,016010(2017)

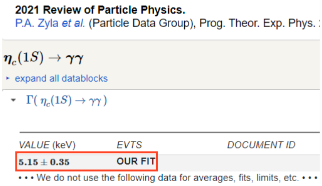



$$\Gamma(\eta_c \to 2\gamma) = \begin{cases} 6.57(15)(8)\,\text{keV} & \text{YM } et\ al, \text{ 2109.09381} \\ 5.15(35)\,\text{keV} & \text{PDG-fit} \\ 6.11^{+2.2}_{-1.9}\,\text{keV} & \text{PDG-aver} \end{cases}$$

- The tension with PDG-fit is beyond 3 $\sigma$

# Lattice & Experiments





**2021 Review of Particle Physics.**
P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys.

$\eta_c(1S) \to \gamma\gamma$

expand all datablocks

$\Gamma(\eta_c(1S) \to \gamma\gamma)$

| *VALUE* (keV) | *EVTS* | *DOCUMENT ID* |
|---|---|---|
| $\mathbf{5.15 \pm 0.35}$ | OUR FIT | |

• • • We do not use the following data for averages, fits, limits, etc. • • •

- OUR FIT: a minimum $\chi^2$-fit for the branching ratios from lots of experimental measurements on different decay channels.
- Other fitting errors are consistent with world average, except for $\eta_c \to 2\gamma$.

# $J/\psi \to \gamma\eta_c$

- Matrix element

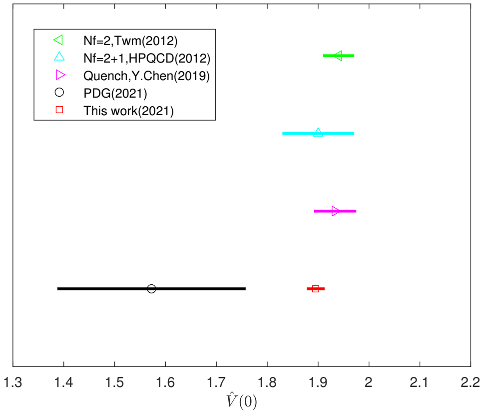
$$\langle \eta_c(\vec{\boldsymbol{p}}_2)|j_\nu(0)|[J/\psi]_\mu(\vec{\boldsymbol{p}}_1)\rangle = \frac{4q_c e}{m_{\eta_c}+m_{J/\psi}}\epsilon_{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}\hat{V}(Q^2)$$

- Traditional: extrapolation for various $\hat{V}(Q^2 \neq 0)$ CLQCD,2011
  - leads to model-dependent uncertainty;
  - $Q_0^2 = -\delta^2 m$ at $|\vec{\boldsymbol{q}}| = |\vec{\boldsymbol{p}}_1| - |\vec{\boldsymbol{p}}_2| = 0$ is unreachable.
- Model-independent: $\hat{V}(Q_0^2) \xrightarrow{\text{correction}} \hat{V}(0)$
  - $Q_0^2 \to 0$, the precision can be much improved;
  - no model-dependent extrapolation.

$$\hat{V}(0) = \hat{V}(Q_0^2) + \frac{m_{\eta_c}}{m_{J/\psi}}\frac{\partial \hat{V}(Q^2)}{\partial|\vec{\boldsymbol{q}}|^2}\bigg|_{|\vec{\boldsymbol{q}}|^2=0} \times \delta^2 m$$

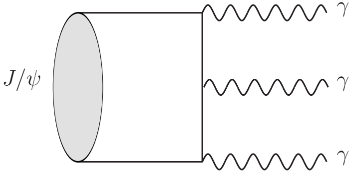
YM *et al*, in preparation

# $J/\psi \to \gamma\eta_c$




- Only show the results using at least three lattice spacings;
- New method reduces the error greatly.

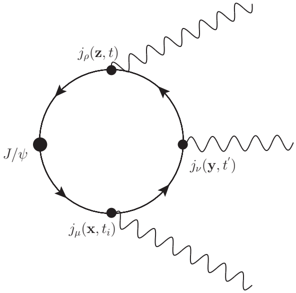
# $J/\psi \to 3\gamma$



# Approach to decay width

- Decay width

$$\Gamma_{3\gamma} = \frac{m_{J/\psi}}{1536\pi^3}\int_0^1 dx \int_{1-x}^1 dy \mathcal{T}(x,y)$$
$$\mathcal{T} = \frac{1}{3}\sum_{\mu\nu\rho\alpha} |\mathcal{M}_{\mu\nu\rho\alpha}|^2$$
$$x \equiv 1 - 2q_2 \cdot q_3/m^2, y \equiv 1 - 2q_1 \cdot q_2/m^2$$

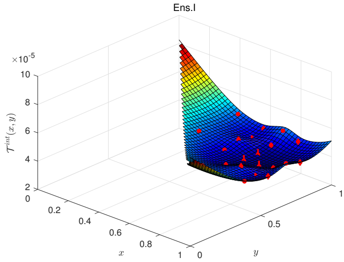


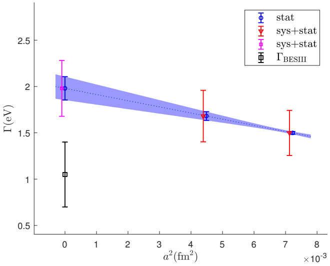

- On-shell fitting

$$\mathcal{T}(x,y,Q_1^2,Q_2^2,Q_3^2) = \mathcal{T}(x,y) + \text{const} \times \sum_i Q_i^2$$

- Choose discrete photon momenta
  - On-shell as possible, $Q_i \to 0$
  - The $(x, y)$ cover the physical region

# The first lattice result

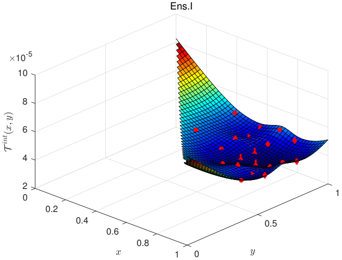


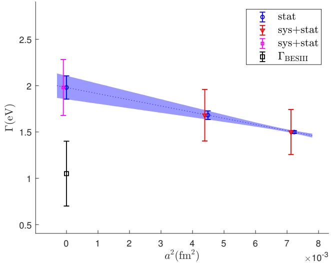




YM,C.Liu, K-L.Zhang Phys.Rev.D.**102**,054506(2020)

- Naive continuum extrapolation;
- The intermediate contribution $J/\psi \to \gamma\eta_c \to 3\gamma$;
- Excited-state contamination;
- Systematical error by the region without data covered;

# Toward systematic study

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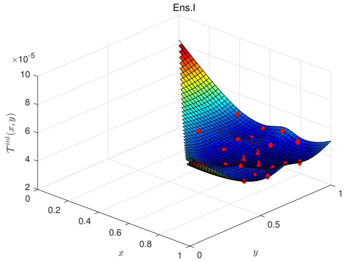


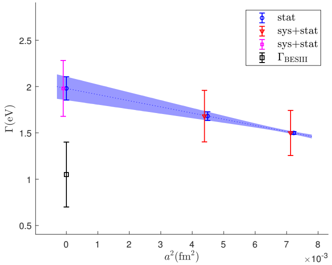


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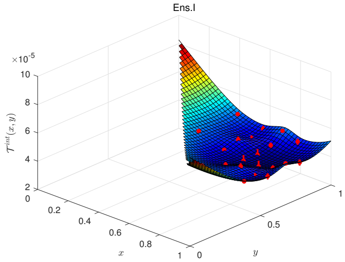


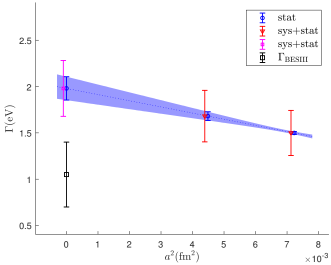


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- Excited-state contamination; ☑ Experienced studies
- Systematical error by the region without data covered; ☑ Challenging

# Summary and outlook

- Summary
  - Model-independent calculation on $\eta_c/\chi_{c_0} \to 2\gamma$ and $J/\psi \to \gamma\eta_c$
  - Various systematic effects included
  - An explorative study on $J/\psi \to 3\gamma$, potential improvements
- Outlook
  - Model-independent calculation on $J/\psi \to \gamma\nu\bar{\nu}$ and $J/\psi \to 3\gamma$
  - Toward high precision calculation in charmonia decays
  - Extensive cooperation with BESIII in charmonia physics
    - Model-independent matix element input for experimental analysis e.g. $J/\psi \to 3\gamma$
    - Test the validity of PDG-fit in the channel $\eta_c \to 2\gamma$
    - Possible theoretical support for DM search

*Thank you!*