



Probing BSM physics with Magnetic Fields and Gravitational waves observations

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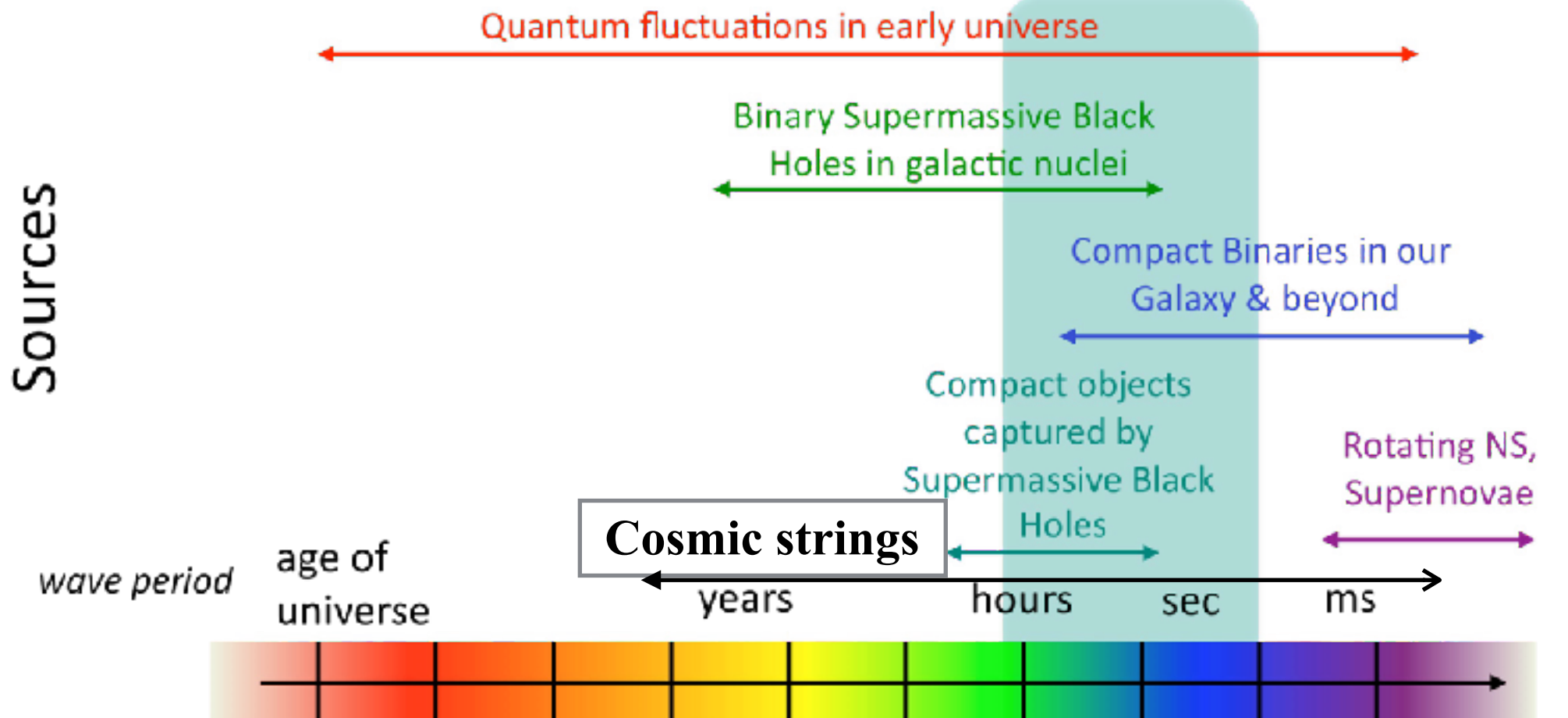
中国格点QCD第一届年会, 10/30-11/02, 2021

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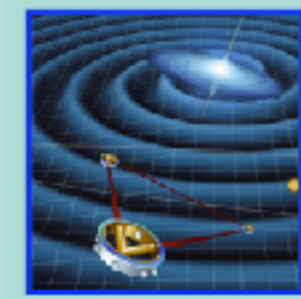
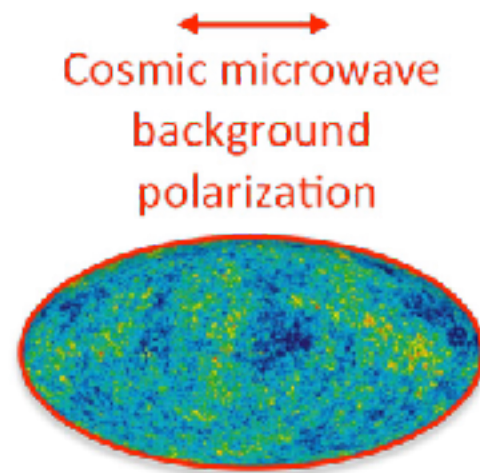
- **GW from first-order phase transition**
- **B violation and sphaleron**
- **PMF from Bubble Collisions**
- **Future prospect**

The Gravitational Wave Spectrum

Sources

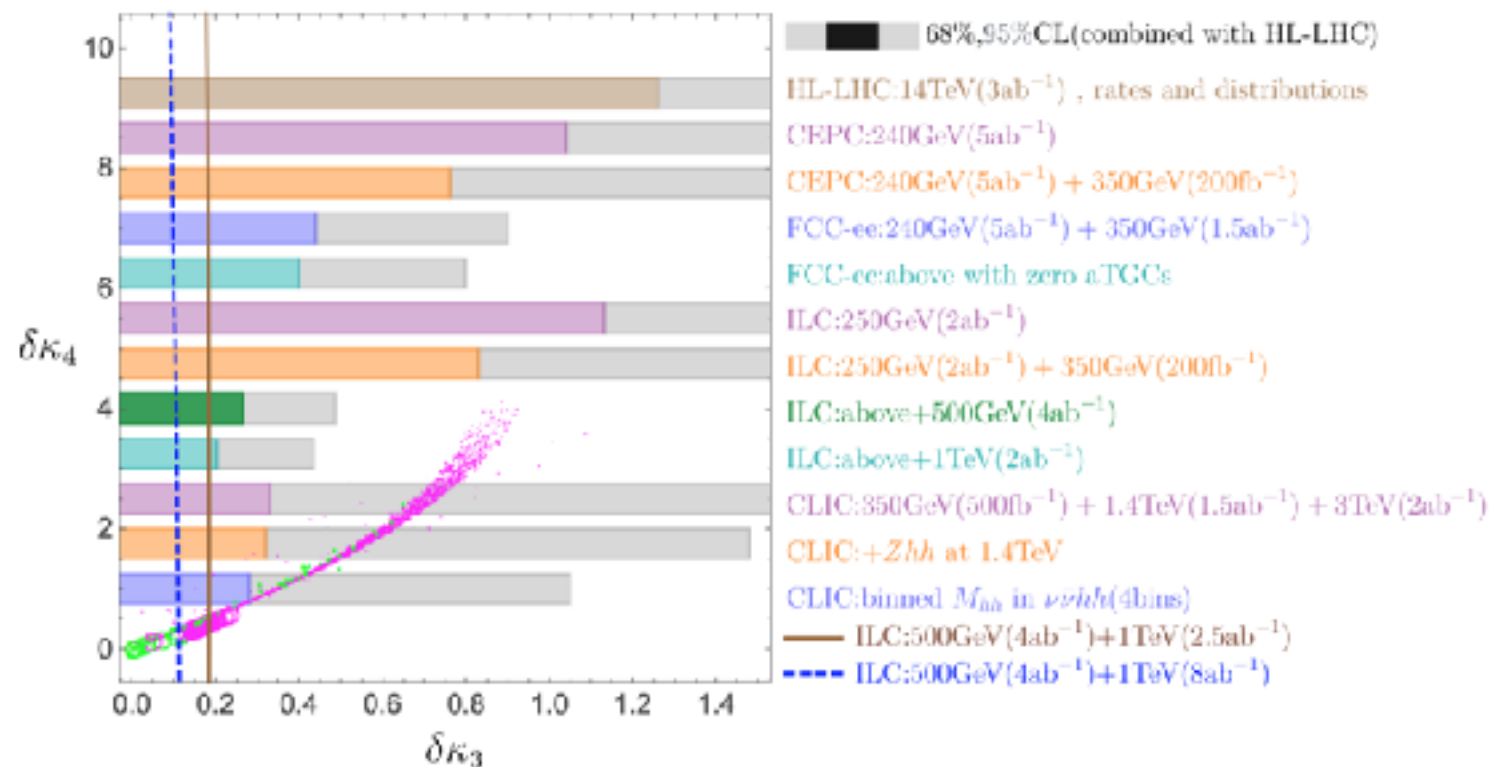
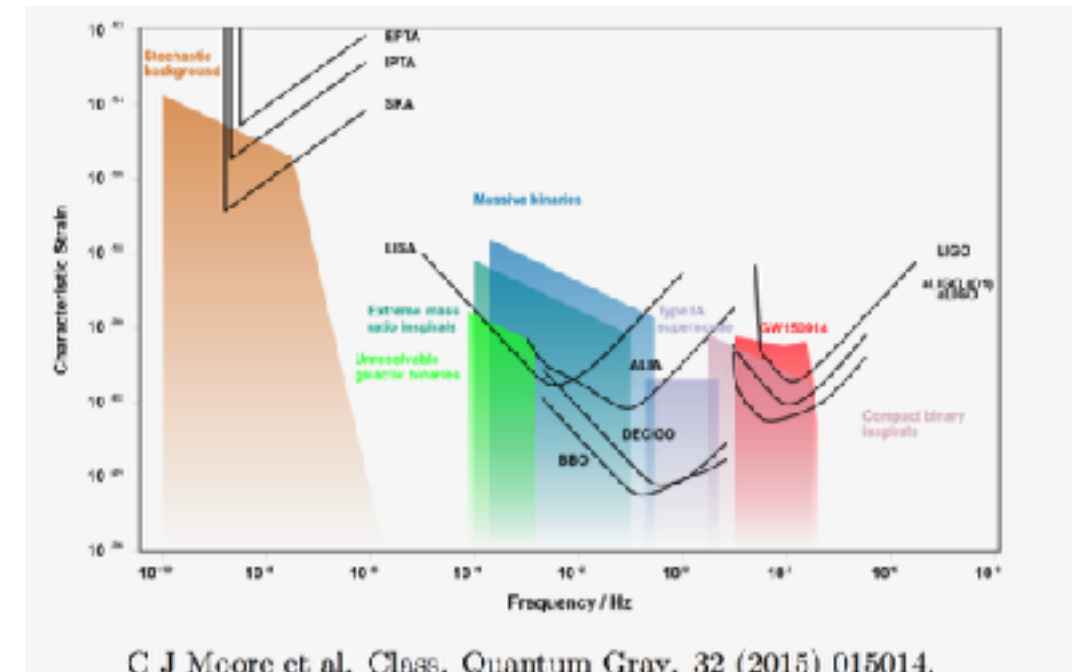
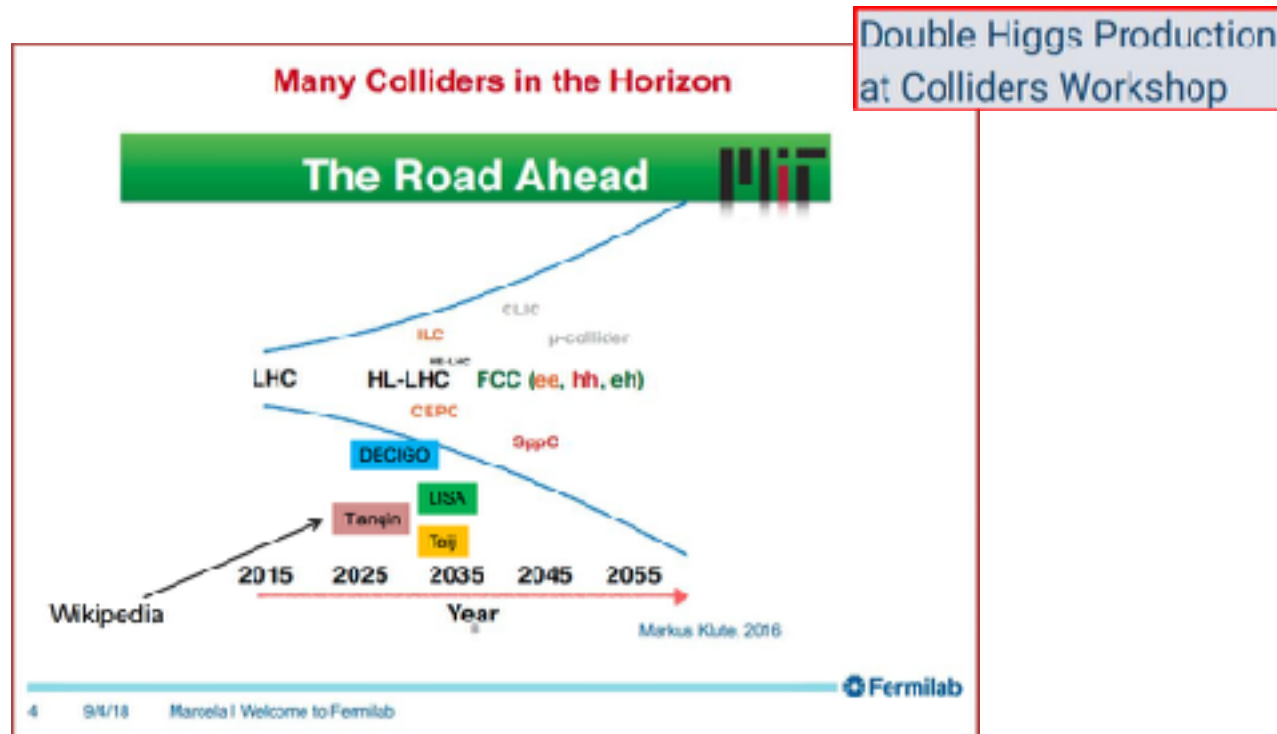


Detectors



LISA, TianQin, Taiji, ...

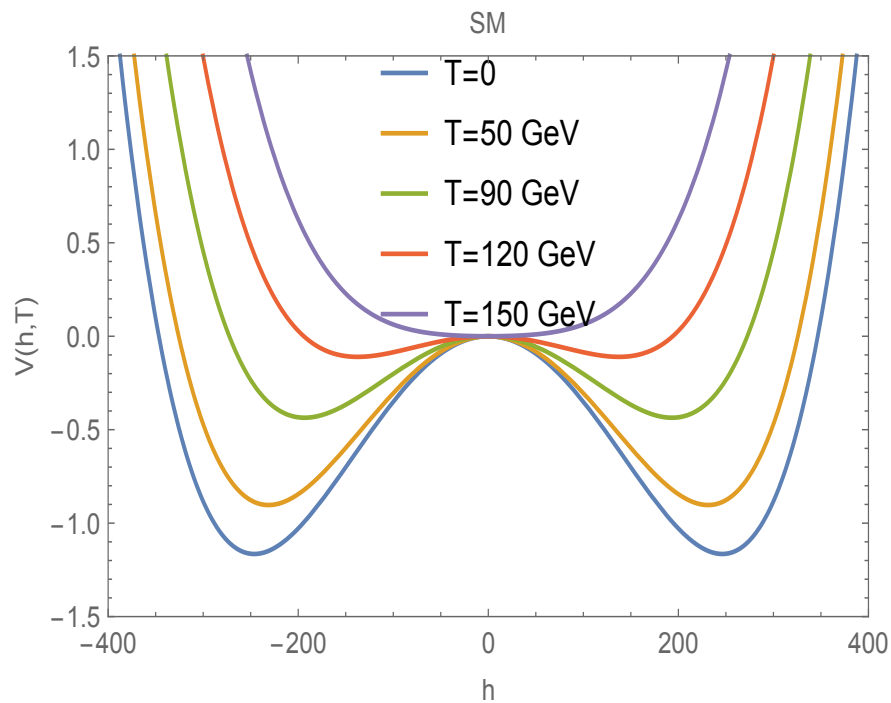
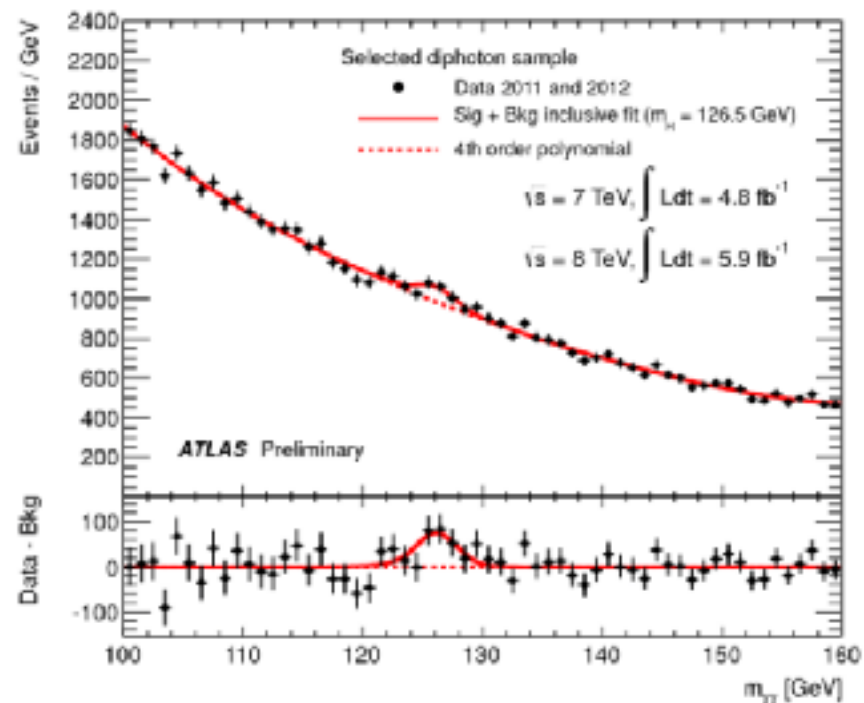
PTGW and collider search



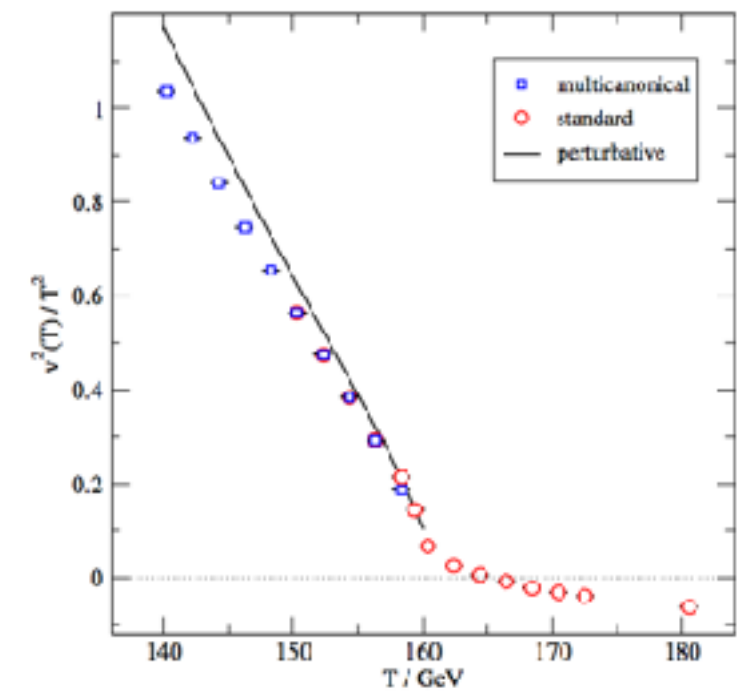
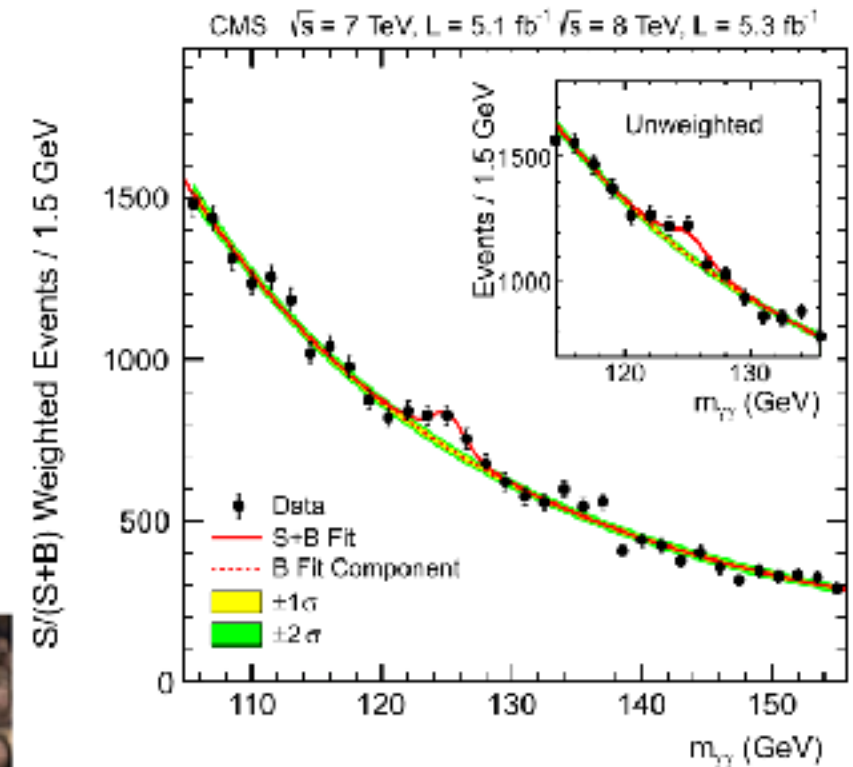
$$\Delta\mathcal{L} = -\frac{1}{2}\frac{m_h^2}{v}(1+\delta\kappa_3)h^3 - \frac{1}{8}\frac{m_h^2}{v^2}(1+\delta\kappa_4)h^4$$

**SNR > 10 for two-step
and one-step SFOEWPT**

Implication of 125 GeV Higgs



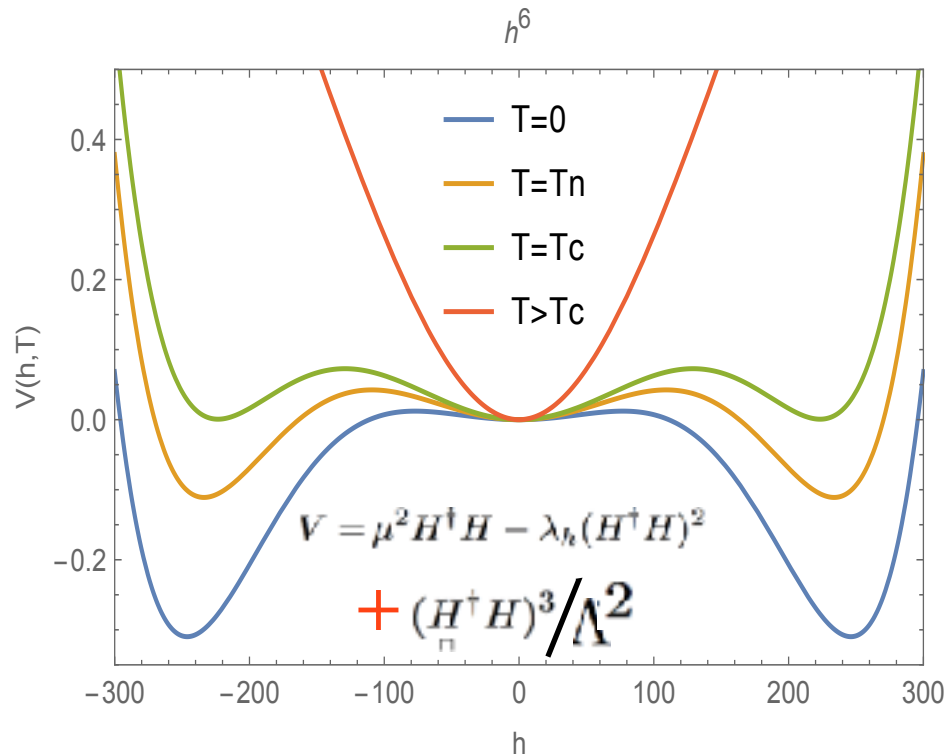
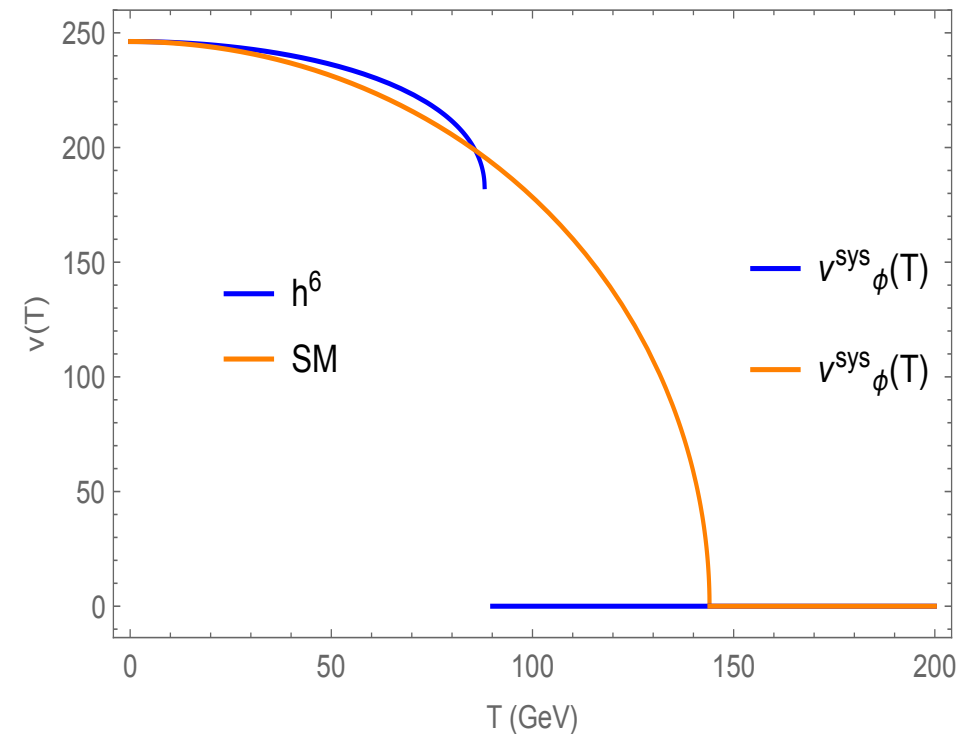
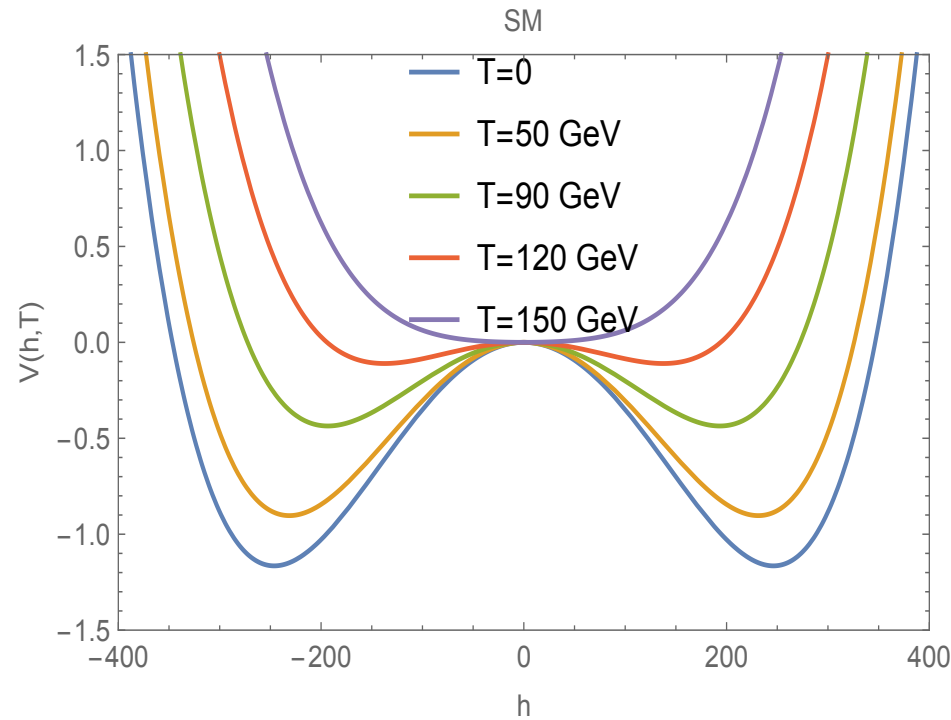
Higgs announcement seminar on 4 July 2012



PRL 113, 141602 (2014)

Higgs Potential Shape??? EFT or ???

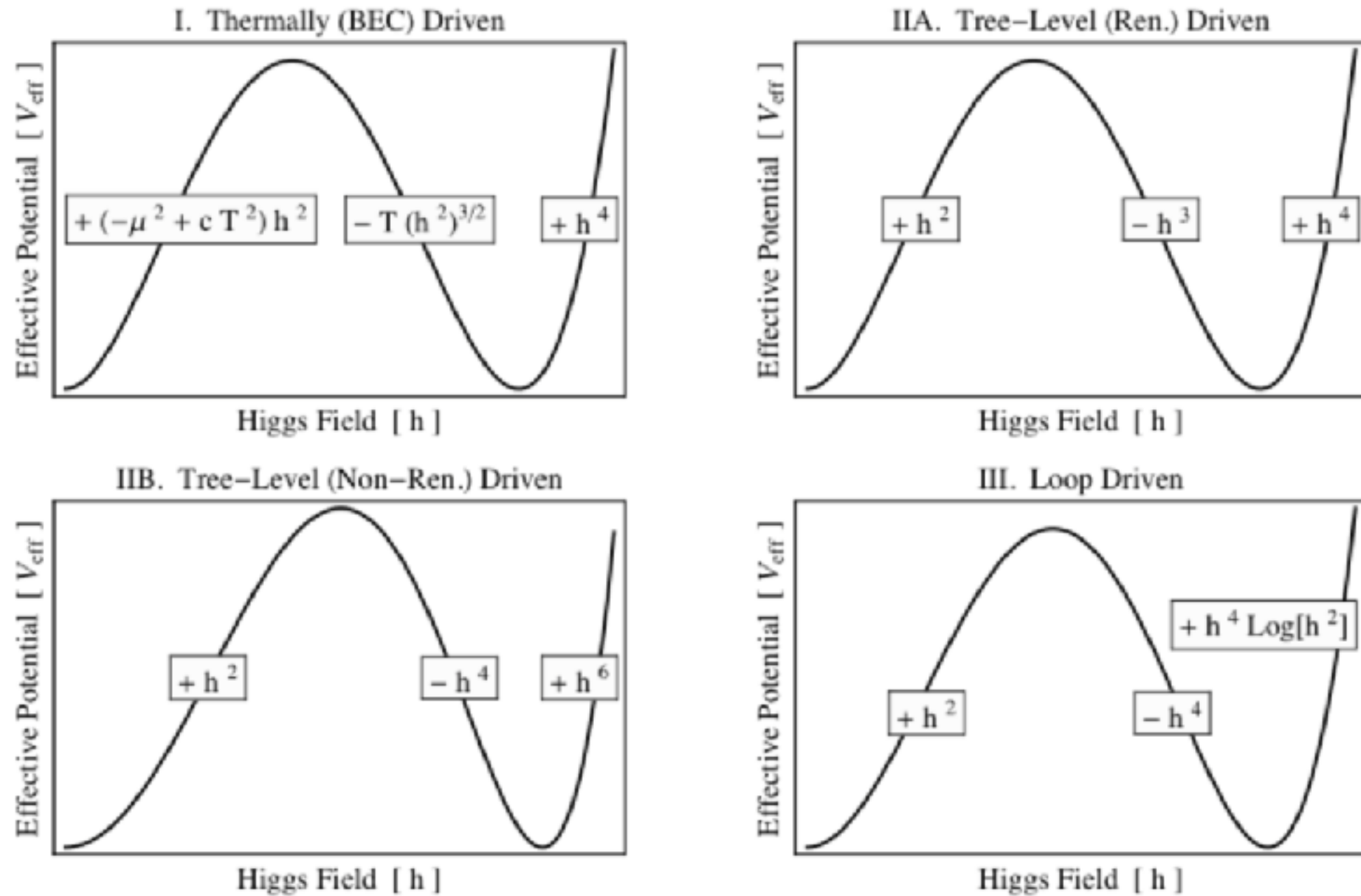
First or second order



Grojean, Servant, Wells 05, P. Huang, Jokelar, Li, Wagner 15, Cao, F.P. Huang, Xie, & Zhang 17, Zhou, **Bian**, Guo 19

LHC say the quantum fluctuation (quadratic oscillation) around $h=v$ with $m_h=126 \text{ GeV}$, not sensitive to the specifically potential shape

Model classes for catalyzing a strongly first order electroweak phase transition



PRD87, 023509 (2013)

- **SM+Scalar Singlet**

Espinosa, Quiros 93, Benson 93, Choi, Volkas 93, Vergara 96, Branco, Delepine, Emmanuel- Costa, Gonzalez 98, Ham, Jeong, Oh 04, Ahriche 07, Espinosa, Quiros 07, Profumo, Ramsey-Musolf, Shaughnessy 07, Noble, Perelstein 07, Espinosa, Konstandin, No, Quiros 08, Barger, Langacker, McCaskey, Ramsey-Musolf, Shaughnessy 09, Ashoorioon, Konstandin 09, Das, Fox, Kumar, Weiner 09, Espinosa, Konstandin, Riva 11, Chung, Long 11, Barger, Chung, Long, Wang 12, Huang, Shu, Zhang 12, Fairbairn, Hogan 13, Katz, Perelstein 14, Profumo, Ramsey-Musolf, Wainwright, Winslow 14, **Jiang, Bian, Huang, Shu 15**, Kozaczuk 15, Cline, Kainulainen, Tucker-Smith 17, Kurup, Perelstein 17, Chen, Kozaczuk, Lewis 17, **Cheng, Bian 17, Bian, Tang 18, Chen, Li, Wu, Bian, 19...**

- **SM+Scalar Doublet**

Turok, Zadrozny 92, Davies, Froggatt, Jenkins, Moorhouse 94, Cline, Lemieux 97, Huber 06, Froome, Huber, Seniuch 06, Cline, Kainulainen, Trott 11, Dorsch, Huber, No 13, Dorsch, Huber, Mimasu, No 14, Basler, Krause, Muhlleitner, Wittbrodt, Wlotzka 16, Dorsch, Huber, Mimasu, No 17, **Bernon, Bian, Jiang 17, Bian, Liu 18,...**

- **SM + Scalar Triplet**

Profumo, Ramsey-Musolf 12, Chiang 14, **Zhou, Cheng, Deng, Bian, Wu 18, Zhou, Bian, Guo, Wu 19,...**

- **NMSSM**

Pietroni 93, Davies, Froggatt, Moorhouse 95, Huber, Schmidt 01, Ham, Oh, Kim, Yoo, Son 04, Menon, Morrissey, Wagner 04, Funakubo, Tao, Yokoda 05, Huber, Konstandin, Prokopec, Schmidt 07, Chung, Long 10, Kozaczuk, Profumo, Stephenson Haskins, Wainwright 15, **Bi, Bian, Huang, Shu, Yin 15, Bian, Guo, Shu 17,...**

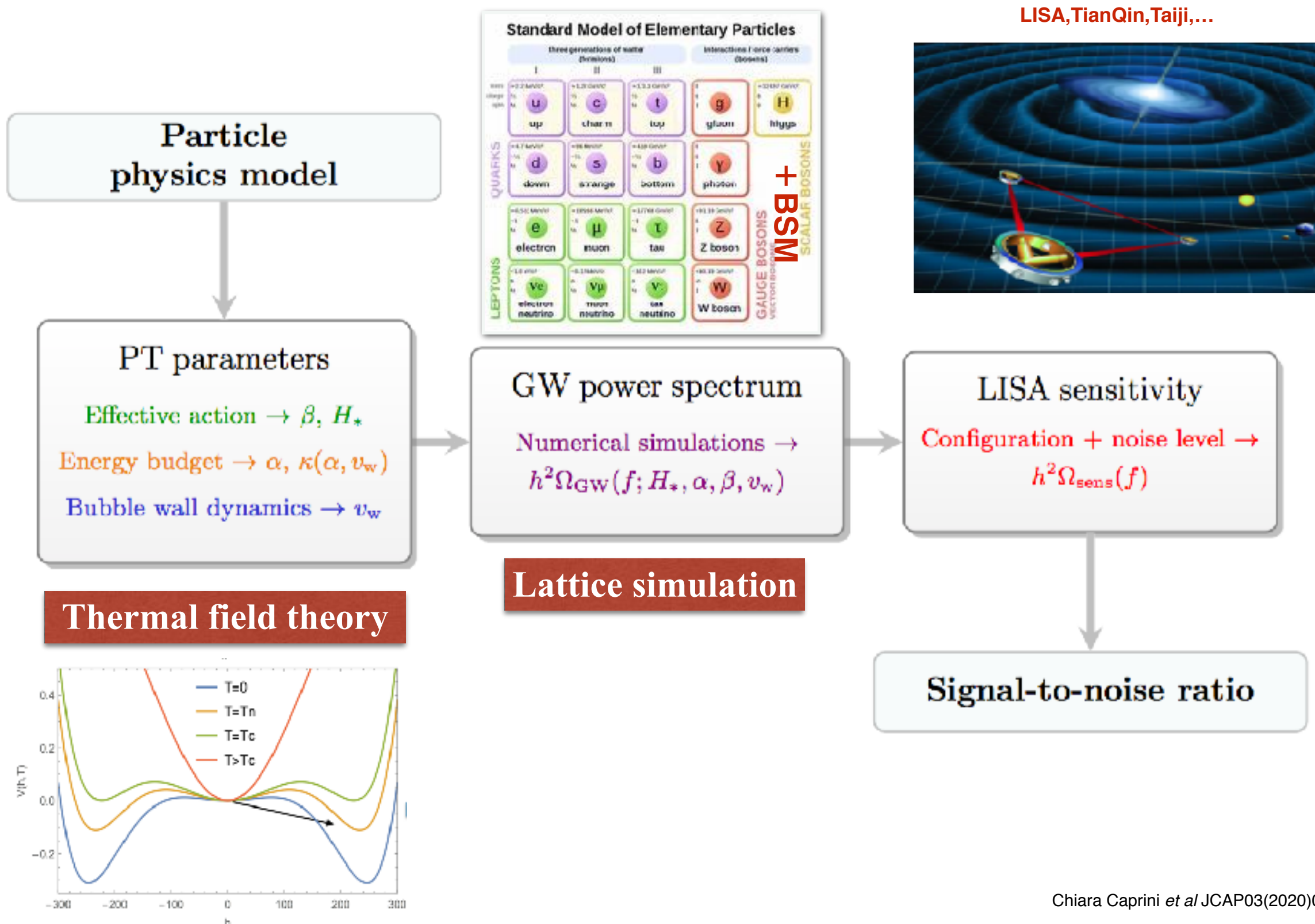
- **Composite Higgs**

Espinosa, Gripaos, Konstandin, Riva 11, Bruggisser, Von Harling, Matsedonskyi, Servant 18, **Bian, Wu, Xie 19**, De Curtis, Delle Rose, Panico 19, **Bian, Wu, Xie 20,...**

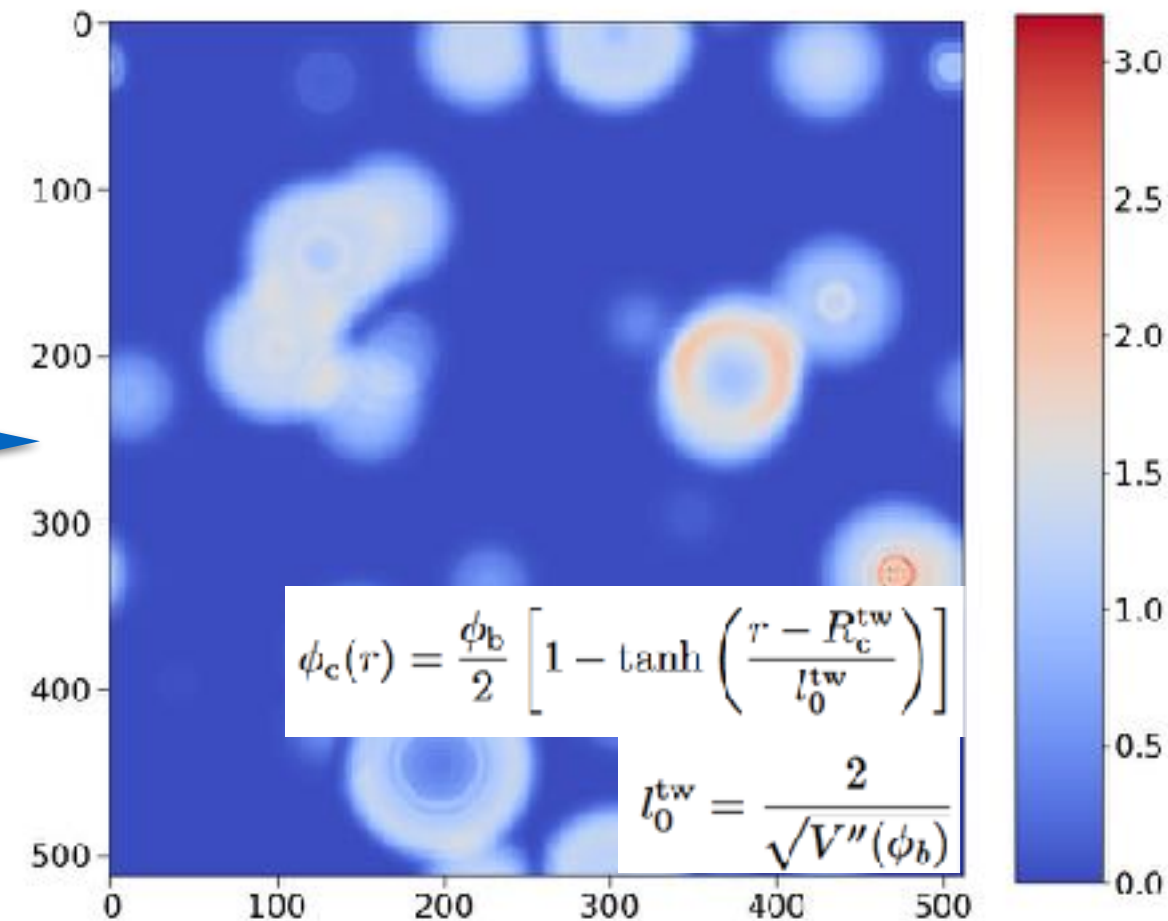
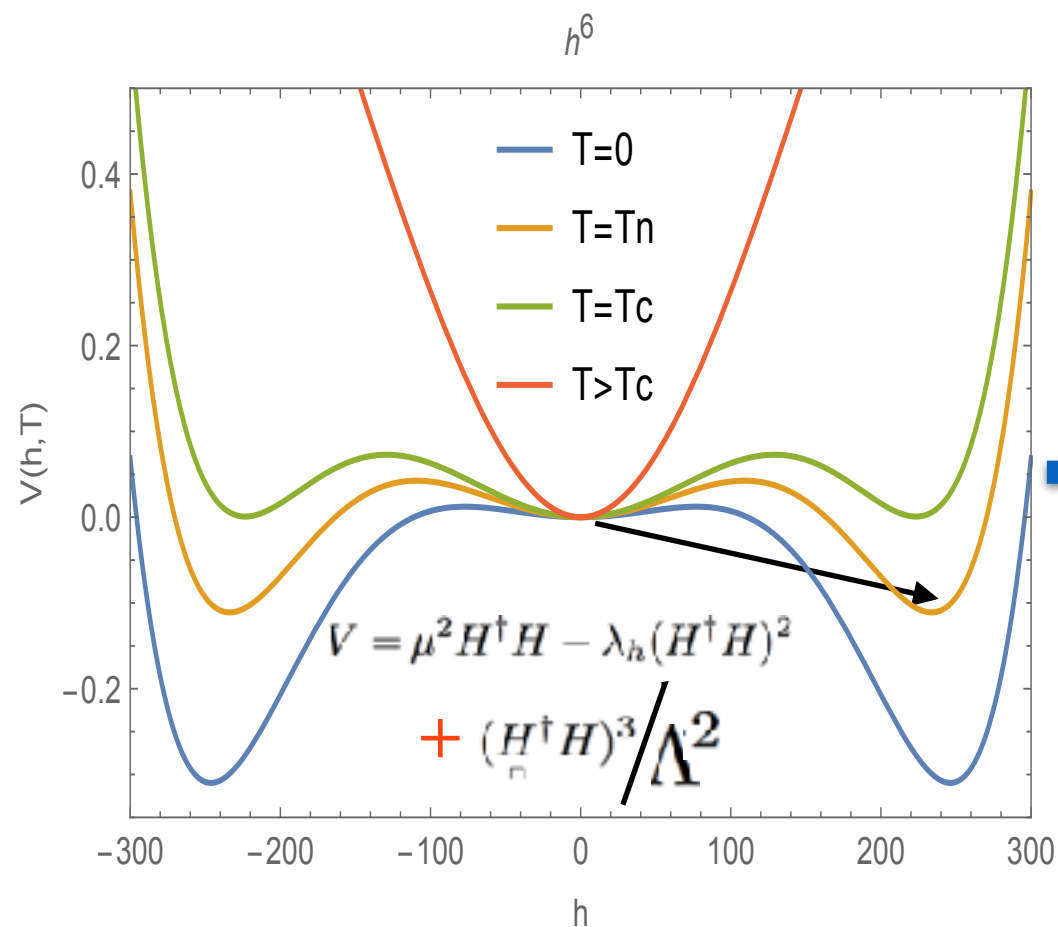
- **EFT**

Grojean, Servant, Wells 05, Bodeker, Froome, Huber, Seniuch 05, Huang, Joglekar, Li, Wagner 15, Cai, Sasaki, Wang 17, **Zhou, Bian, Guo 19, ...**

BSM for FOPT GW



Higgs Potential Shape and the Bubble picture



$$\alpha = \frac{\Delta\rho}{\rho_R} = \frac{1}{\rho_R} \left[-V(\vec{\phi}_b, T) + T \frac{\partial V(\vec{\phi}_b, T)}{\partial T} \right] \Big|_{T=T_n}$$

$$S(t_N) \sim 4 \ln(m_P / T_N)$$

$$\frac{\beta}{H_*} \simeq \frac{2S(t_N)}{(1 - T_N/T_c)}$$

$$p(t_N) = H^4(t_N)$$

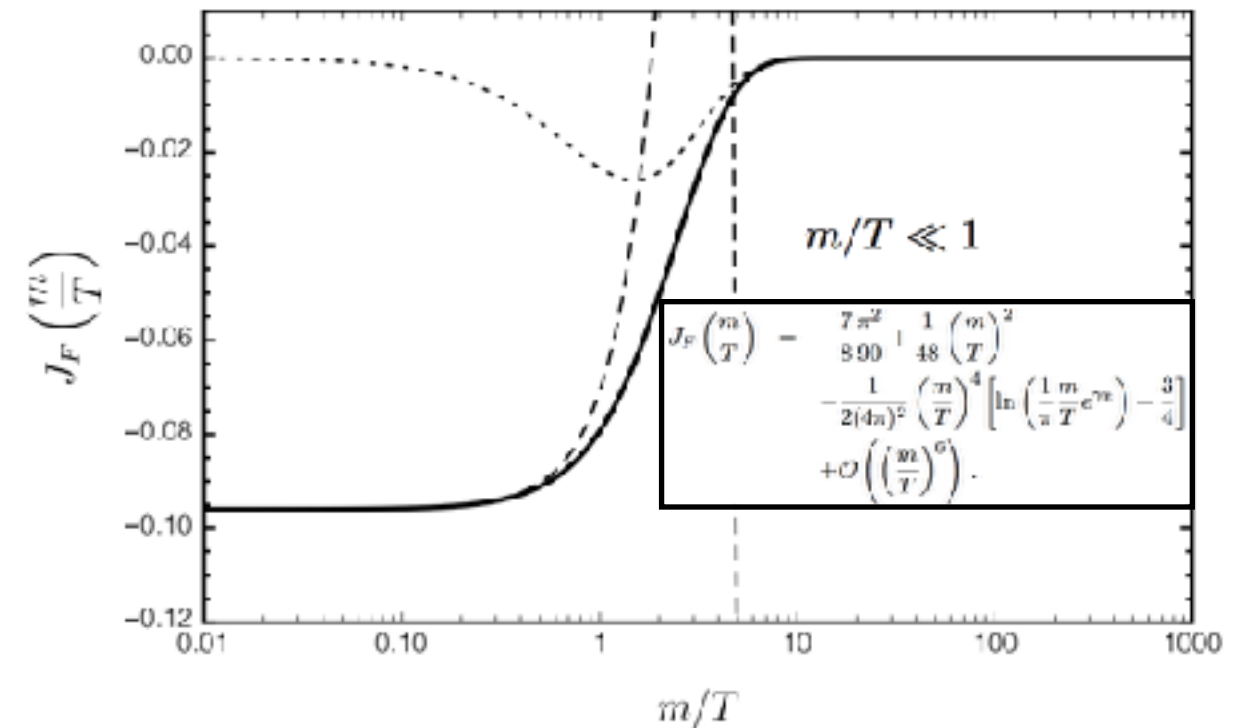
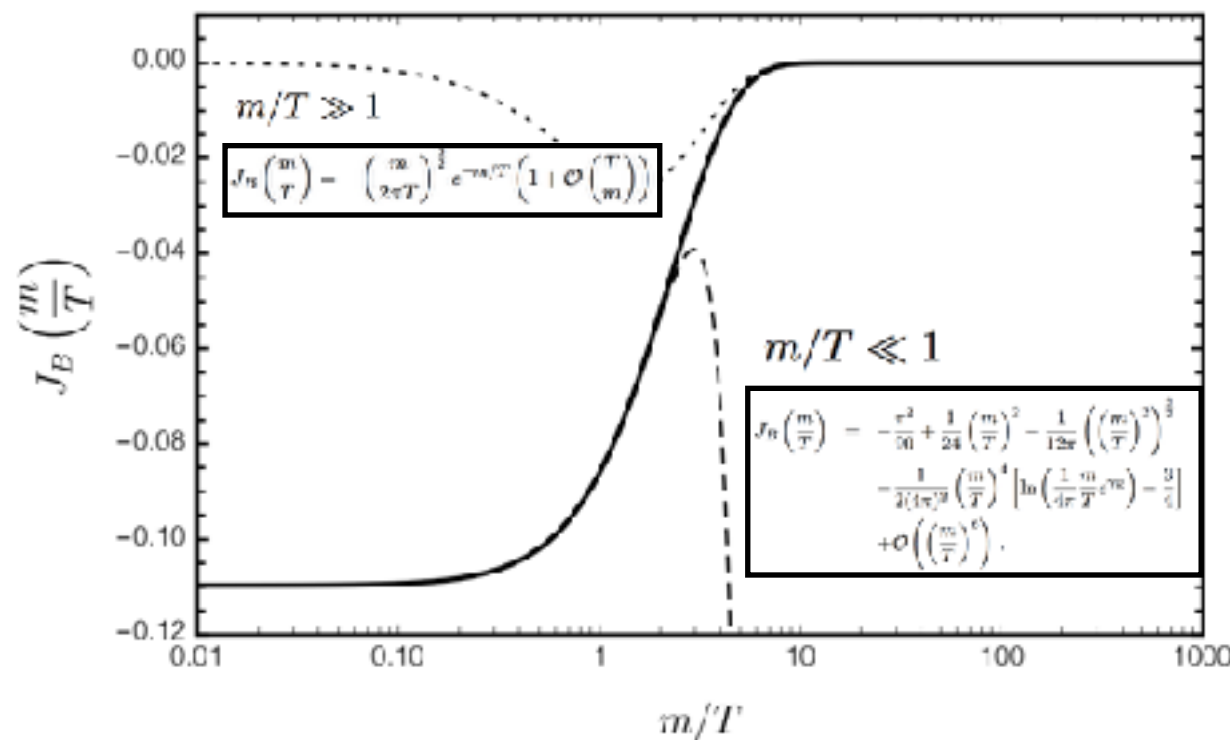
$$p(t) \simeq \Gamma_0 e^{-S(t_N) + \beta(t - t_N)}, \quad \Gamma_0 \sim \alpha^5_{\text{w}} T_N^4$$

$$\beta = - \left. \frac{d \ln p(t)}{dt} \right|_{t_f}$$

Thermal effective scalar potential for PT study

$$V_T(\phi, T) = V_0(\phi) + T^4 \left[\sum_B J_B \left(\frac{M_B}{T} \right) + \sum_F J_F \left(\frac{M_F}{T} \right) \right]$$

all fermions F and bosons B that are relativistic at temperature T



High-T expansion

$$m/T \ll 1$$

$$V_T(\phi) = V_0(\phi) + \frac{T^2}{24} \left(\sum_S M_S^2(\phi) + 3 \sum_V M_V^2(\phi) + 2 \sum_F M_F^2(\phi) \right) - \frac{T}{12\pi} \left(\sum_S \left(M_S^2(\phi) \right)^{\frac{3}{2}} + \sum_V \left(M_V^2(\phi) \right)^{\frac{3}{2}} \right) + \text{higher order terms.}$$

MS, MV, MF are the masses of the scalar fields S, vector fields V and fermionic fields F

Bounce solution

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2} \left(\frac{d\phi_b}{dr} \right)^2 + V(\phi_b, T) \right]$$

$$\lim_{r \rightarrow \infty} \phi_b = 0, \quad \left. \frac{d\phi_b}{dr} \right|_{r=0} = 0$$

Bubble nucleation

$$\Gamma \approx A(T) e^{-S_3/T} \sim 1$$

Latent heat

$$\alpha = \frac{1}{\rho_R} \left[-(V_{EW} - V_f) + T \left(\frac{dV_{EW}}{dT} - \frac{dV_f}{dT} \right) \right] \Big|_{T=T_*}$$

Phase transition inverse duration

$$\frac{\beta}{H_n} = T \frac{d(S_3(T)/T)}{dT} \Big|_{T=T_n}$$

- Bubble collisions**

$$\Omega_{\text{col}} h^2 = 1.67 \times 10^{-5} \left(\frac{H_*}{\beta} \right)^2 \left(\frac{\kappa \alpha}{1 + \alpha} \right)^2 \left(\frac{100}{g_*} \right)^{1/3} \left(\frac{0.11 v_b^3}{0.42 + v_b^2} \right) \frac{3.8 (f/f_{\text{env}})^{2.8}}{1 + 2.8 (f/f_{\text{env}})^{3.8}}$$

peak frequency: $f_{\text{env}} = 16.5 \times 10^{-6} \left(\frac{f_*}{H_*} \right) \left(\frac{T_*}{100 \text{ GeV}} \right) \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}$

- Sound Wave**

$$\Omega h_{\text{sw}}^2(f) = 2.65 \times 10^{-6} (H_* \tau_{\text{sw}}) \left(\frac{\beta}{H} \right)^{-1} v_b \left(\frac{\kappa_\nu \alpha}{1 + \alpha} \right)^2 \left(\frac{g_*}{100} \right)^{-1/3} \left(\frac{f}{f_{\text{sw}}} \right)^3 \left(\frac{7}{4 + 3 (f/f_{\text{sw}})^2} \right)^{7/2}$$

phase transition duration: $\tau_{\text{sw}} = \min \left[\frac{1}{H_*}, \frac{R_*}{U_f} \right], H_* R_* = v_b (8\pi)^{1/3} (\beta/H)^{-1}$

Root-mean-square four-velocity of the plasma:

$$\bar{U}_f^2 \approx \frac{3}{4} \frac{\kappa_\nu \alpha}{1 + \alpha}$$

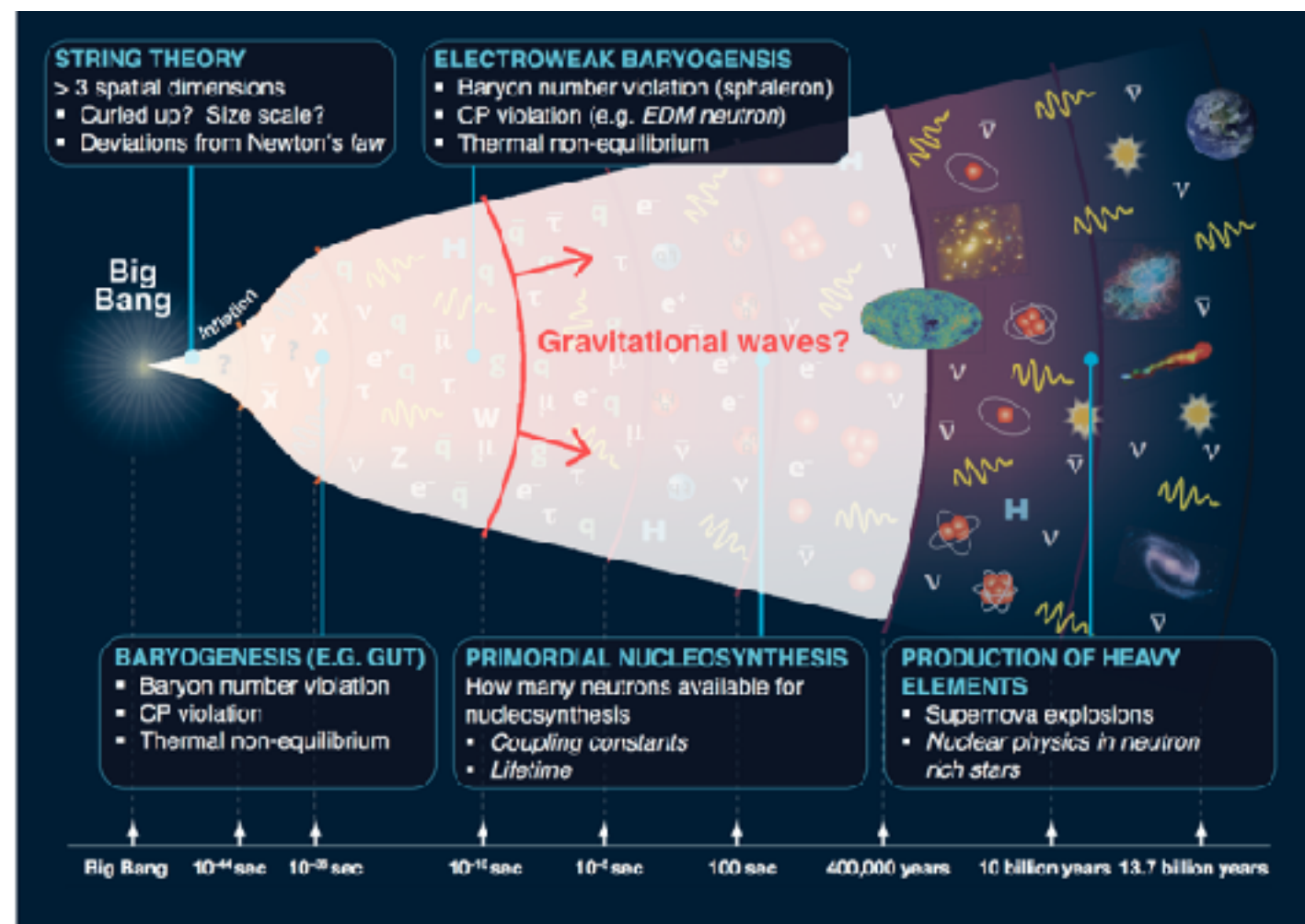
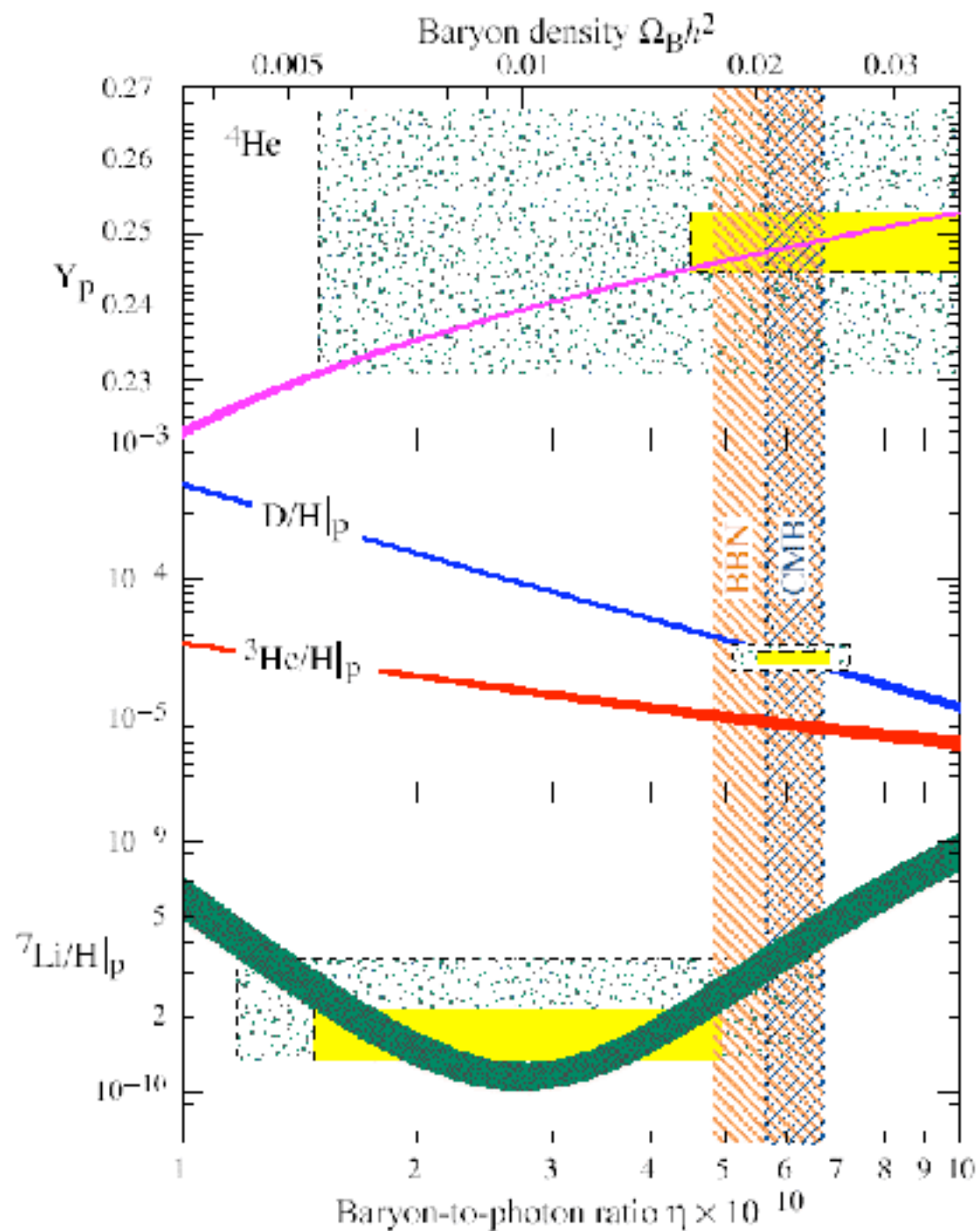
peak frequency: $f_{\text{sw}} = 1.9 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \frac{T_*}{100} \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}$

- MHD turbulence**

$$\Omega h_{\text{turb}}^2(f) = 3.35 \times 10^{-4} \left(\frac{\beta}{H} \right)^{-1} \left(\frac{\epsilon \kappa_\nu \alpha}{1 + \alpha} \right)^{3/2} \left(\frac{g_*}{100} \right)^{-1/3} v_b \frac{(f/f_{\text{turb}})^3 (1 + f/f_{\text{turb}})^{-11/3}}{[1 + 8\pi f a_0 / (a_* H_*)]}$$

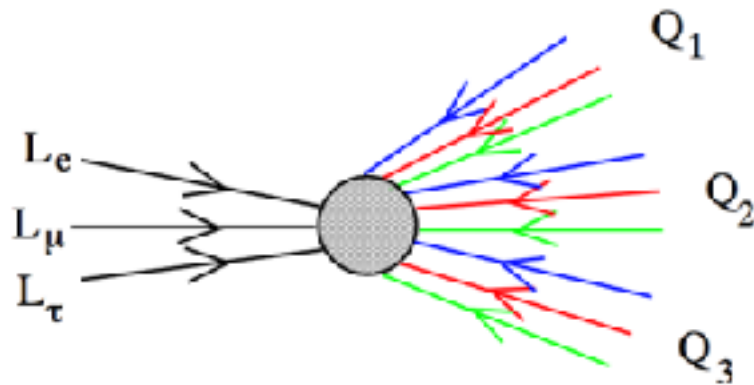
peak frequency: $f_{\text{turb}} = 2.7 \times 10^{-5} \frac{\beta}{H} \frac{1}{v_b} \frac{T_*}{100} \left(\frac{g_*}{100} \right)^{1/6} \text{ Hz}$

Baryon Asymmetry of the Universe

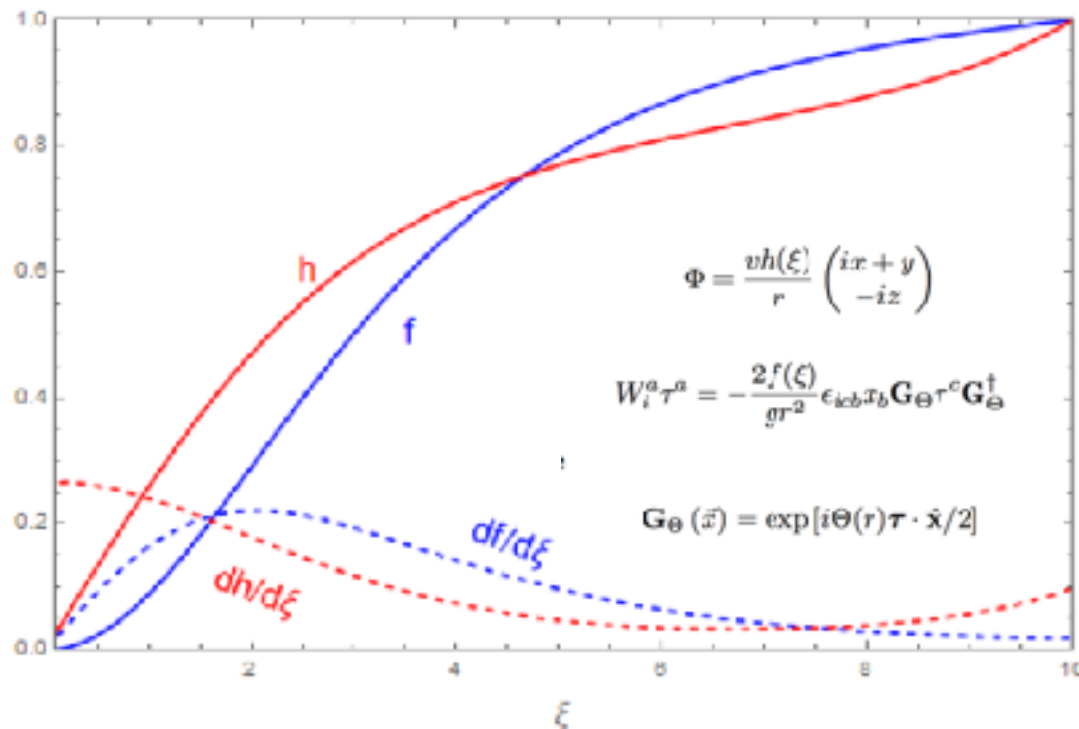
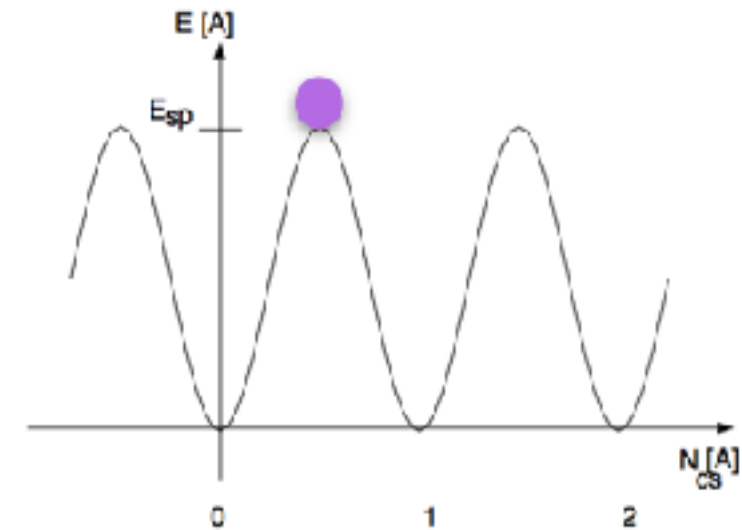


B violation and sphaleron

The Standard Model already contains a process that violates B-number. It is known as the electroweak sphaleron ("sphaleron" is Greek for "ready to fall").



Klinkhammer & Manton (1984); Kuzmin, Rubakov, & Shaposhnikov (1985); Harvey & Turner (1990)
but also identified earlier by Dashen, Hasslacher, & Neveu (1974) and Boguta (1983)



$$\partial_\mu J_B^\mu = i \frac{N_F}{32\pi^2} \left(-g_2^2 F^{a\mu\nu} \tilde{F}_{\mu\nu}^a + g_1^2 f^{\mu\nu} \tilde{f}_{\mu\nu} \right),$$

$$\Delta B = N_F (\Delta N_{CS} - \Delta n_{CS}),$$

$$N_{CS} = -\frac{g_2^2}{16\pi^2} \int d^3x \, 2\epsilon^{ijk} \text{Tr} \left[\partial_i A_j A_k + i \frac{2}{3} g_2 A_i A_j A_k \right],$$

$$n_{CS} = -\frac{g_1^2}{16\pi^2} \int d^3x \, \epsilon^{ijk} \partial_i B_j B_k,$$

Lattice EW field foundation

$\Phi(t, \mathbf{x})$: Higgs field doublet defined on sites;

$U_i(t, \mathbf{x})$ and $V_i(t, \mathbf{x})$: SU(2) and U(1) link fields, defined on the link between the neighboring sites $\mathbf{x}$ and $\mathbf{x} + \mathbf{i}$, $\Phi(t, \mathbf{x})$, $U_i(t, \mathbf{x})$ and $V_i(t, \mathbf{x})$ are defined at time steps $t + \Delta t, t + 2\Delta t, \dots$;

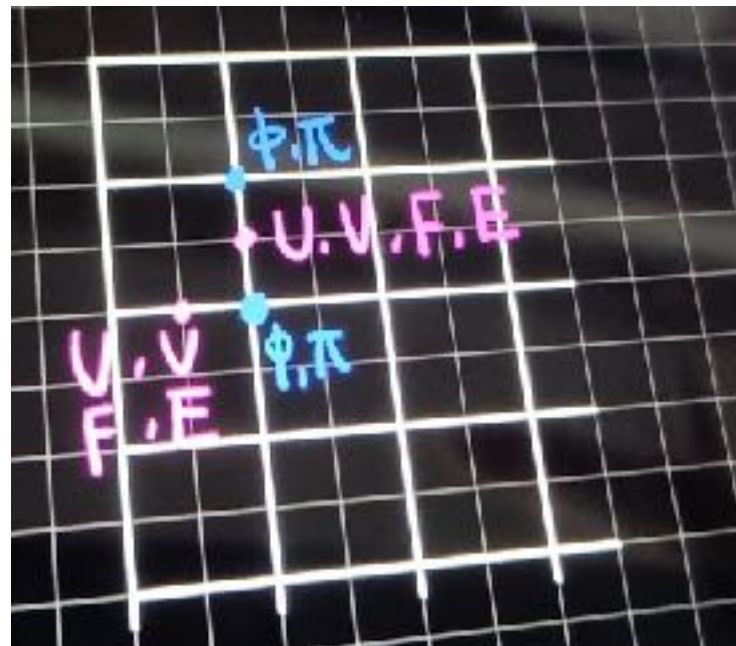
Conjugate momentum fields: $\Pi(t + \Delta t/2, \mathbf{x})$, $F(t + \Delta t/2, \mathbf{x})$ and $E(t + \Delta t/2, \mathbf{x})$, are defined at time steps $t + \Delta t/2, t + 3\Delta t/2$.

$$U_i(t, \mathbf{x}) = \exp \left(-\frac{i}{2} g \Delta x \sigma^a W_i^a \right)$$

$$U_0(t, \mathbf{x}) = \exp \left(-\frac{i}{2} g \Delta t \sigma^a W_0^a \right)$$

$$V_i(t, \mathbf{x}) = \exp \left(-\frac{i}{2} g \Delta x B_i \right)$$

$$V_0(t, \mathbf{x}) = \exp \left(-\frac{i}{2} g \Delta t B_0 \right).$$



$$D_i \Phi = \frac{1}{\Delta x} [U_i(t, \mathbf{x}) V_i(t, \mathbf{x}) \Phi(t, \mathbf{x} + \mathbf{i}) - \Phi(t, \mathbf{x})]$$

$$D_0 \Phi = \frac{1}{\Delta t} [U_0(t, \mathbf{x}) V_0(t, \mathbf{x}) \Phi(t + \Delta t, \mathbf{x}) - \Phi(t, \mathbf{x})].$$

$$\Phi(t + \Delta t, \mathbf{x}) = \Phi(t, \mathbf{x}) + \Delta t \Pi(t + \Delta t/2, \mathbf{x})$$

$$V_i(t + \Delta t, \mathbf{x}) = \frac{1}{2} g' \Delta x \Delta t E_i(t + \Delta t/2, \mathbf{x}) V_i(t, \mathbf{x})$$

$$U_i(t + \Delta t, \mathbf{x}) = g \Delta x \Delta t F_i(t + \Delta t/2, \mathbf{x}) U_i(t, \mathbf{x}),$$

Temporal gauge
 $U_0(t, \mathbf{x}) = I_2, V_0(t, \mathbf{x}) = 1$

leapfrog

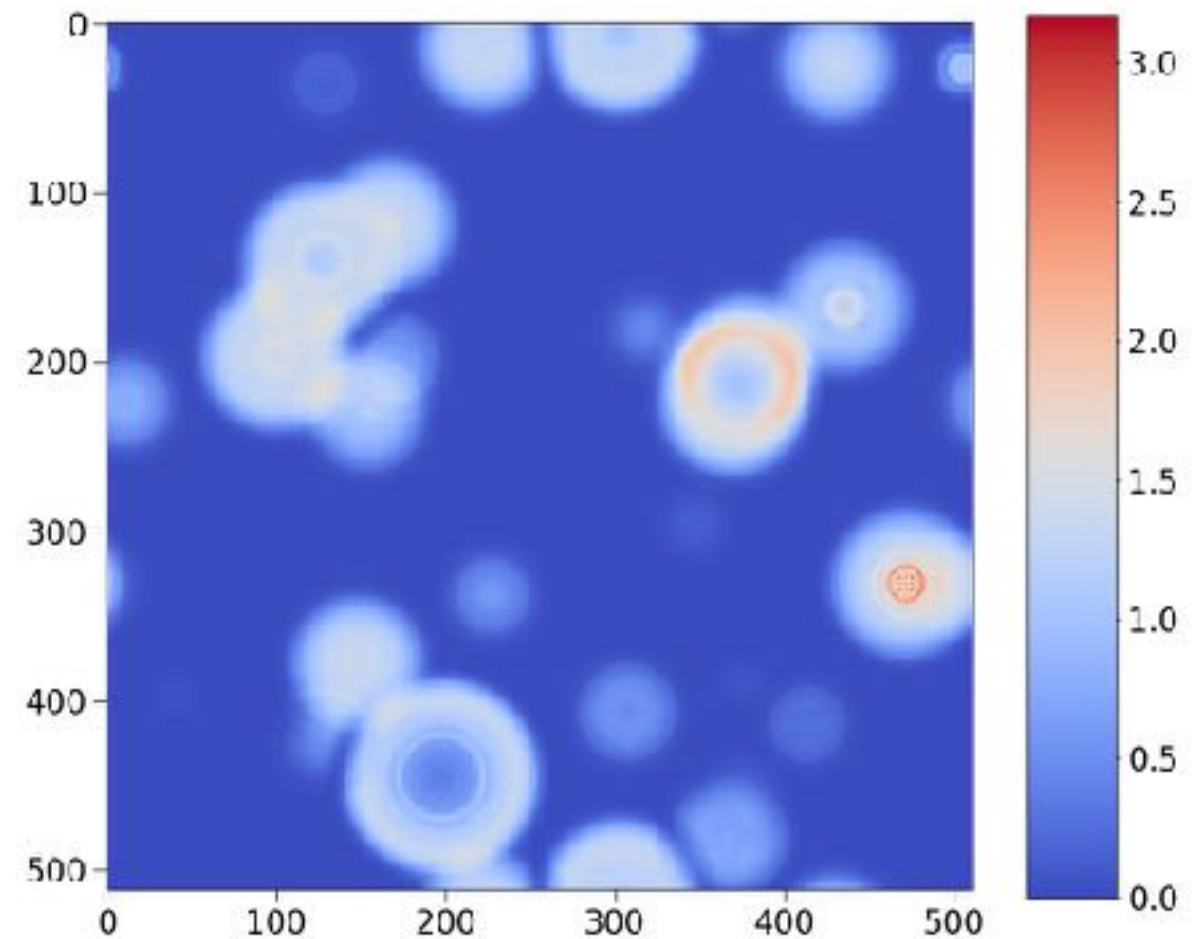
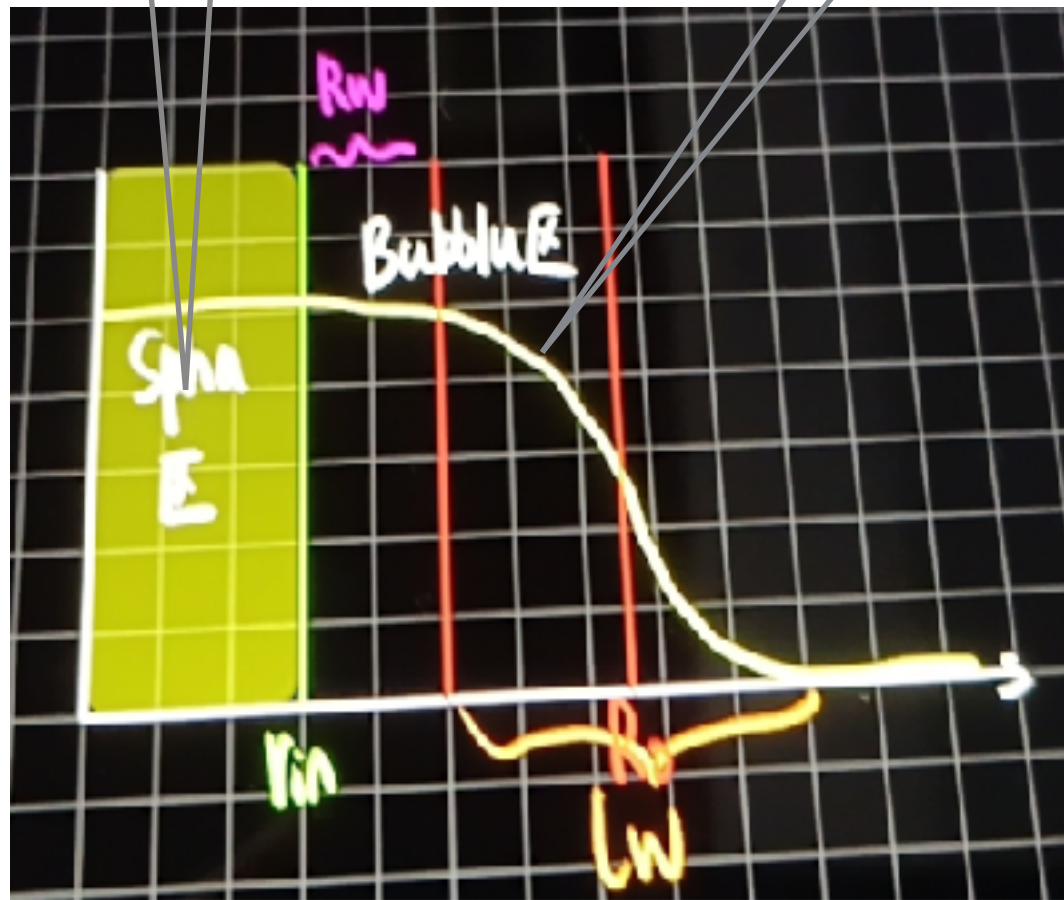
Bubble with sphaleron

$$\Phi = \frac{vh(\xi)}{r} \begin{pmatrix} ix + y \\ -iz \end{pmatrix}$$

$$W_i^a \tau^a = -\frac{2f(\xi)}{gr^2} \epsilon_{acb} x_b \mathbf{G}_\Theta \tau^c \mathbf{G}_\Theta^\dagger$$

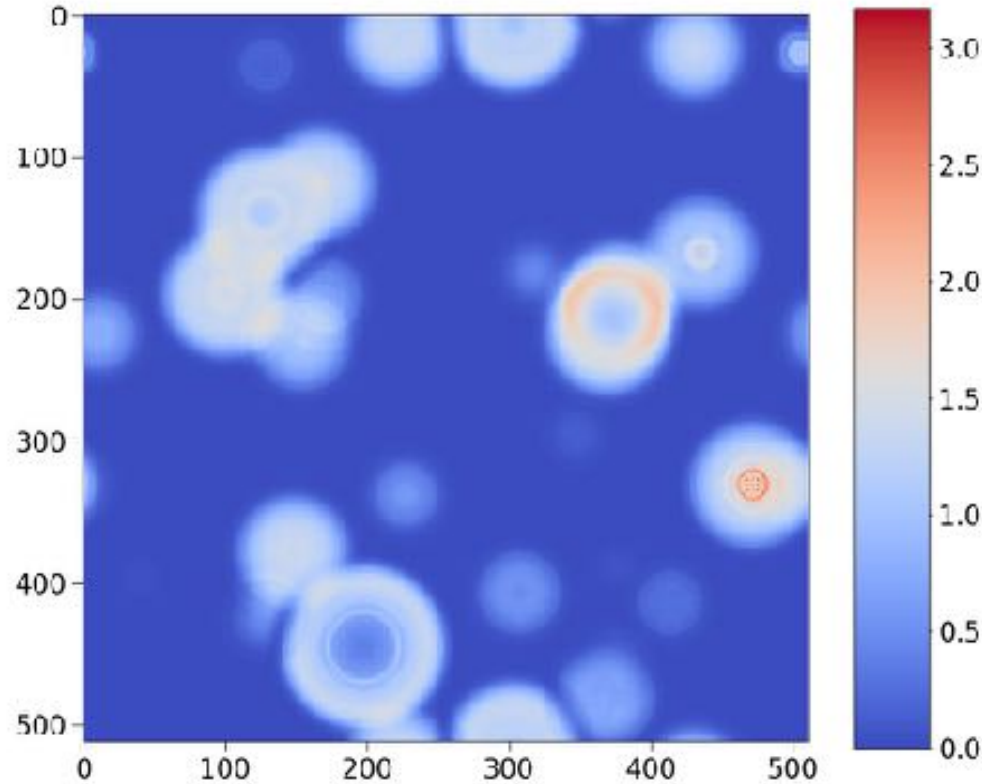
$$\mathbf{G}_\Theta(\vec{x}) = \exp[i\Theta(r)\boldsymbol{\tau} \cdot \hat{\mathbf{x}}/2]$$

$$\phi_c(r) = \frac{\phi_b}{2} \left[1 - \tanh \left(\frac{r - R_c^{\text{tw}}}{l_0^{\text{tw}}} \right) \right]$$



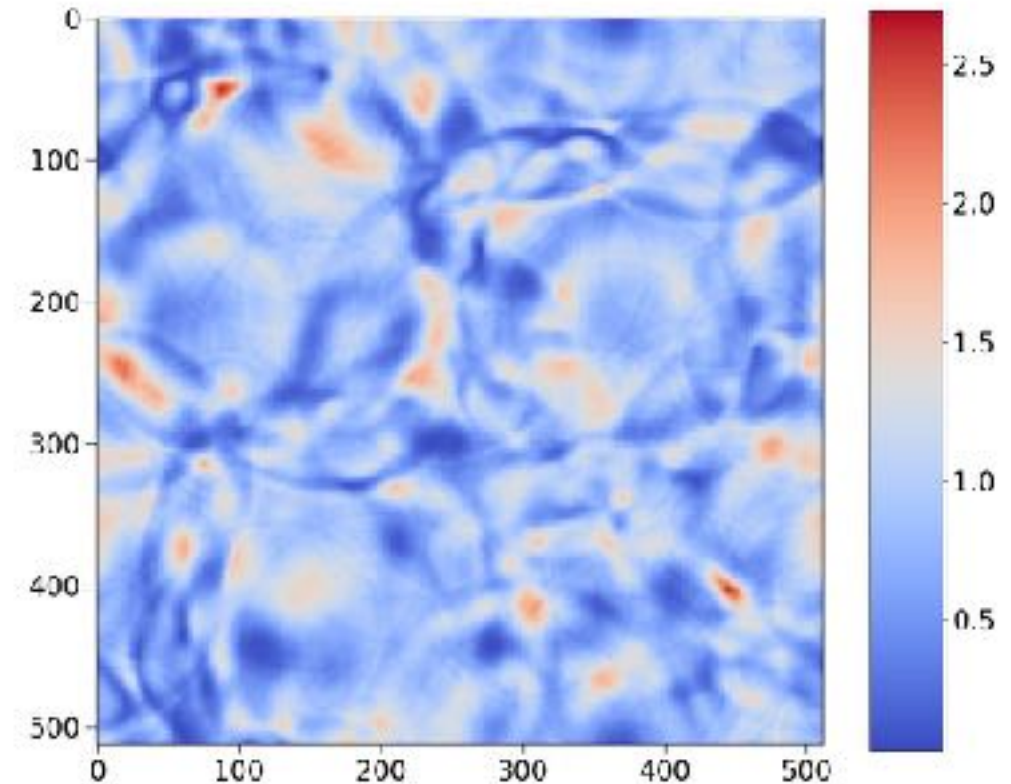
Field basis

$$\begin{aligned}\partial_0^2 \Phi &= D_i D_i \Phi - 2\lambda(|\Phi|^2 - \eta^2)\Phi - 3(\Phi^\dagger \Phi)^2 \Phi / \Lambda^2, \\ \partial_0^2 B_i &= -\partial_j B_{ij} + g' \text{Im}[\Phi^\dagger D_i \Phi], \\ \partial_0^2 W_i^a &= -\partial_k W_{ik}^a - g \epsilon^{abc} W_k^b W_{ik}^c + g \text{Im}[\Phi^\dagger \sigma^a D_i \Phi], \\ \partial_0 \partial_j B_j - g' \text{Im}[\Phi^\dagger \partial_0 \Phi] &= 0, \\ \partial_0 \partial_j W_j^a + g \epsilon^{abc} W_j^b \partial_0 W_j^c - g \text{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] &= 0.\end{aligned}$$



Lattice implementation

$$\begin{aligned}\Pi(t + \Delta t/2, x) &= \Pi(t - \Delta t/2, x) + \Delta t \left\{ \frac{1}{\Delta x^3} \sum_i [U_i(t, x) V_i(t, x) \Phi(t, x + i) \right. \\ &\quad \left. - 2\Phi(t, x) + U_i^\dagger(t, x - i) V_i^\dagger(t, x - i) \Phi(t, x - i)] - \frac{\partial U}{\partial \Phi^\dagger} \right\} \\ \text{Im}[E_k(t + \Delta t/2, x)] &= \text{Im}[E_k(t - \Delta t/2, x)] + \Delta t \left\{ \frac{g'}{\Delta x} \text{Im}[\Phi^\dagger(t, x + k) U_k^\dagger(t, x) V_k^\dagger(t, x) \Phi(t, x)] \right. \\ &\quad \left. - \frac{2}{g' \Delta x^3} \sum_i \text{Im}[V_k(t, x) V_i(t, x + k) V_k^\dagger(t, x + i) V_i^\dagger(t, x) \right. \\ &\quad \left. + V_i(t, x - i) V_k(t, x) V_i^\dagger(t, x + k - i) V_k^\dagger(t, x - i)] \right\} \\ \text{Tr}[i\sigma^m F_k(t + \Delta t/2, x)] &= \text{Tr}[i\sigma^m F_k(t - \Delta t/2, x)] + \Delta t \left\{ \frac{g}{\Delta x} \text{Re}[\Phi^\dagger(t, x + k) U_k^\dagger(t, x) V_k^\dagger(t, x) i\sigma^m \Phi(t, x)] \right. \\ &\quad \left. - \frac{1}{g \Delta x^3} \sum_i \text{Tr}[i\sigma^m U_k(t, x) U_i(t, x + k) U_k^\dagger(t, x + i) U_i^\dagger(t, x) \right. \\ &\quad \left. + i\sigma^m U_k(t, x) U_i^\dagger(t, x + k - i) U_k^\dagger(t, x - i) U_i(t, x - i)] \right\},\end{aligned}$$



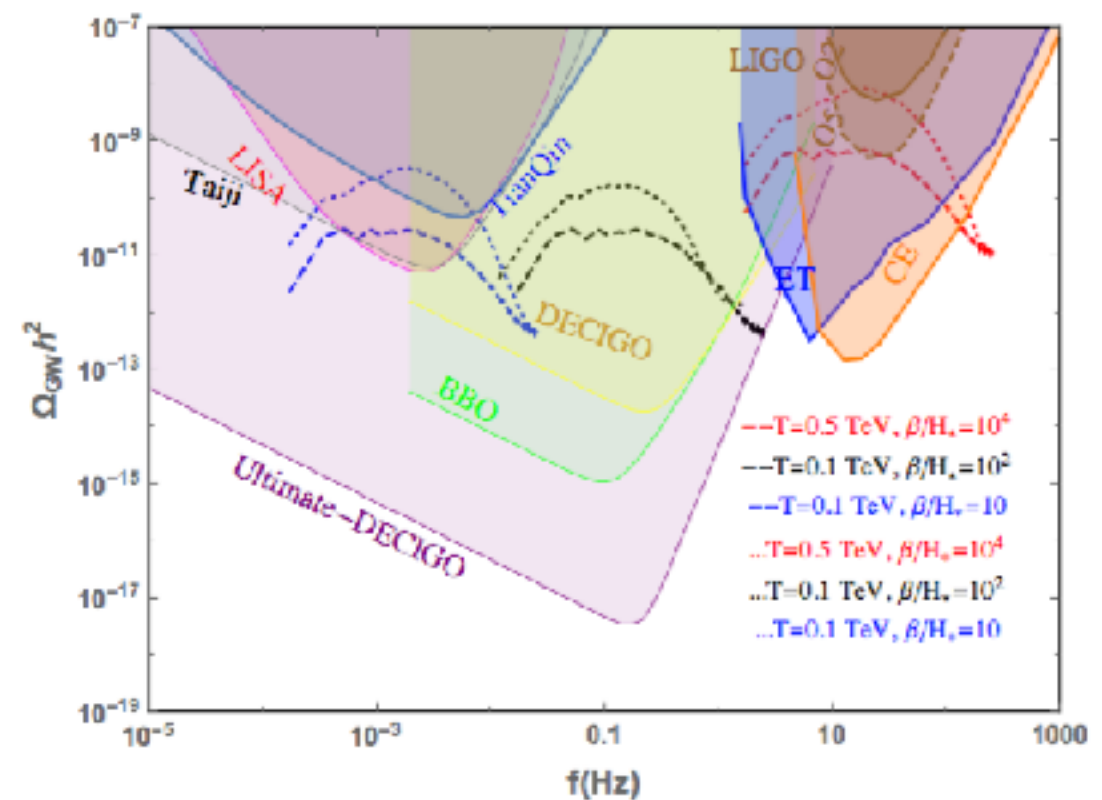
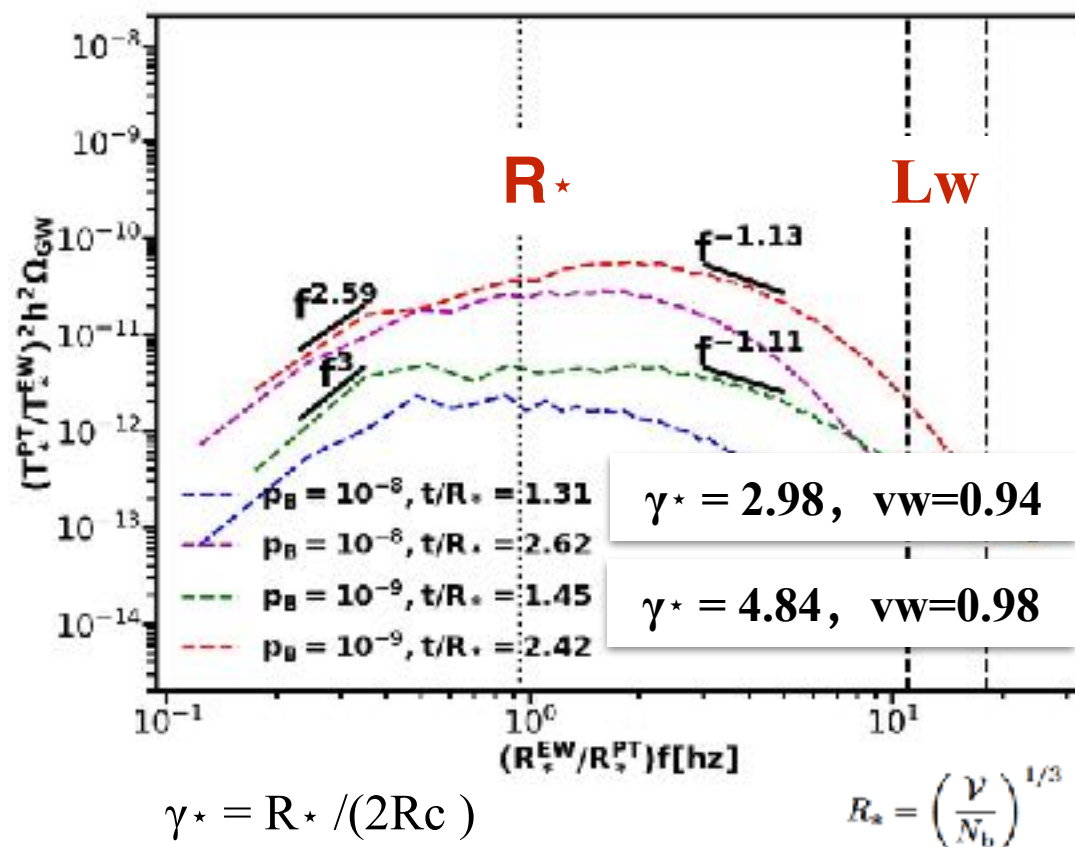
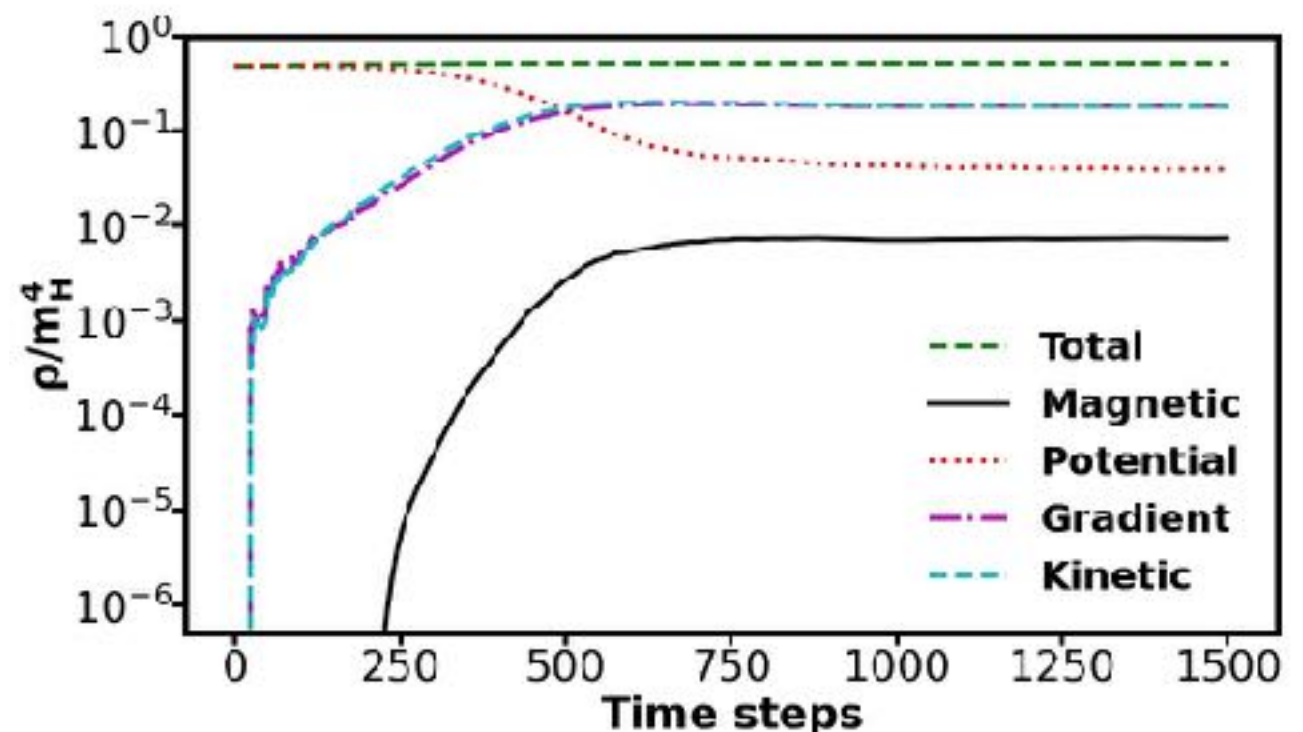
GW from Bubble collisions

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{TT}$$

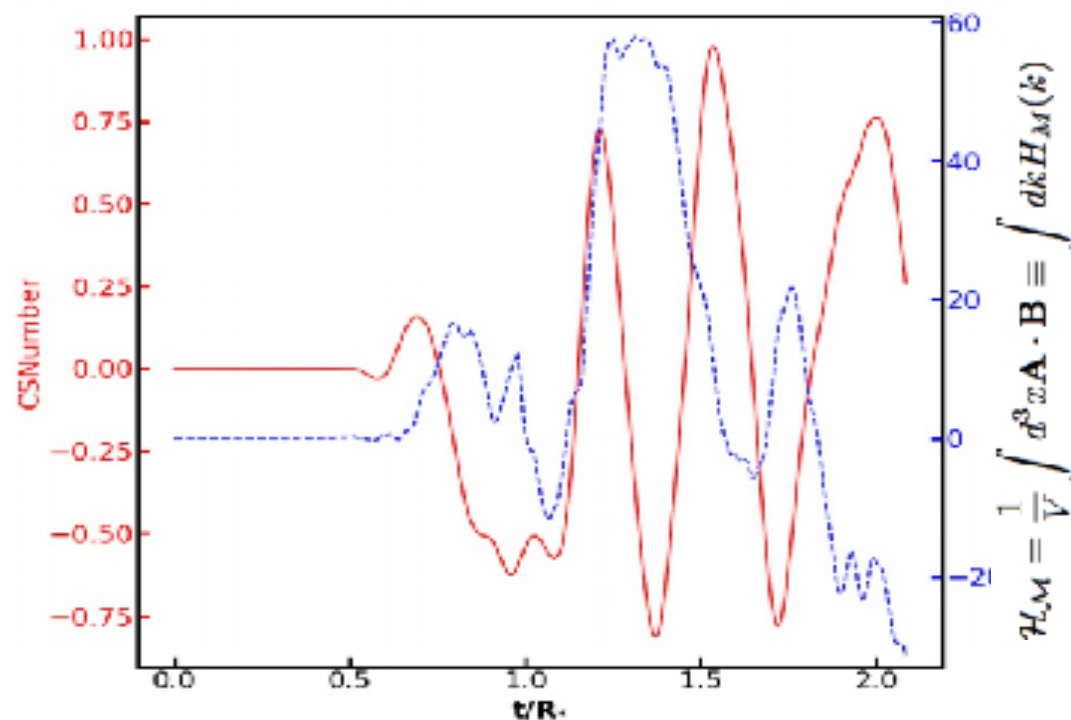
$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu} \frac{1}{2} \text{Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

$$\langle \dot{h}_{ij}^{TT}(\mathbf{k}, t) \dot{h}_{ij}^{TT}(\mathbf{k}', t) \rangle = P_h(\mathbf{k}, t) (2\pi)^3 \delta(\mathbf{k} + \mathbf{k}')$$

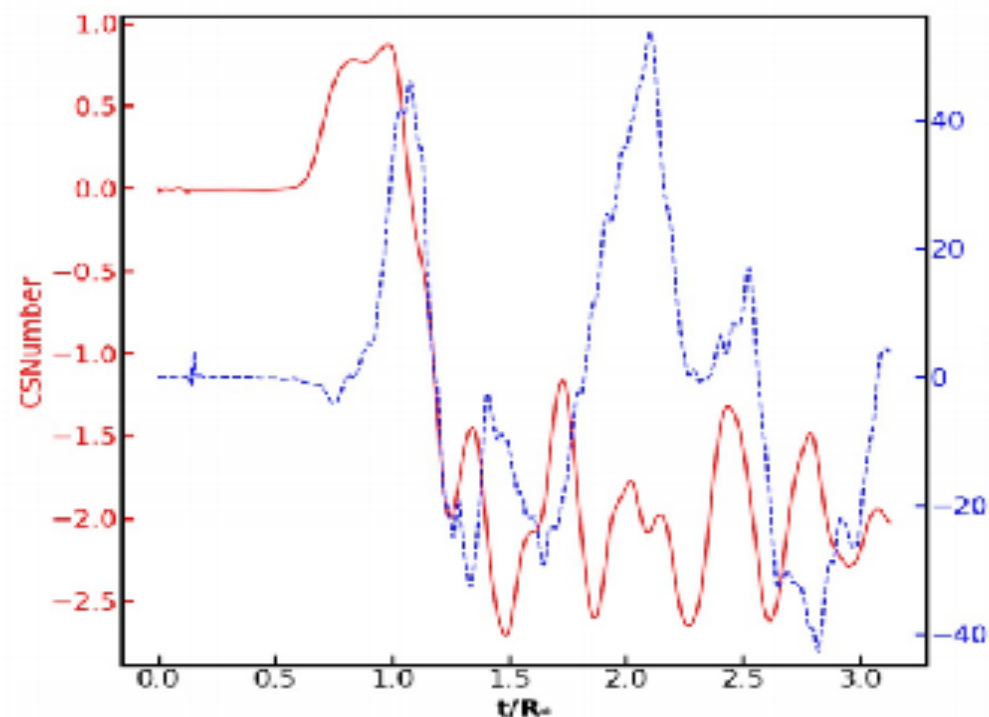
$$\frac{d\Omega_{\text{gw}}}{d\ln(k)} = \frac{1}{32\pi G \rho_c} \frac{k^3}{2\pi^2} P_h(\mathbf{k}, t)$$



CS number and the magnetic helicity



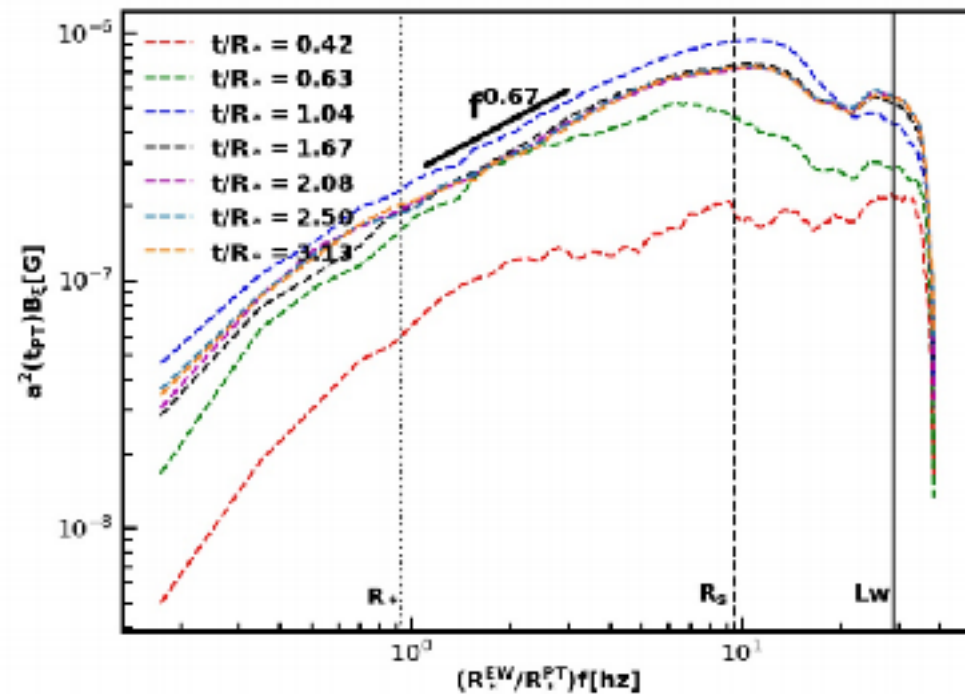
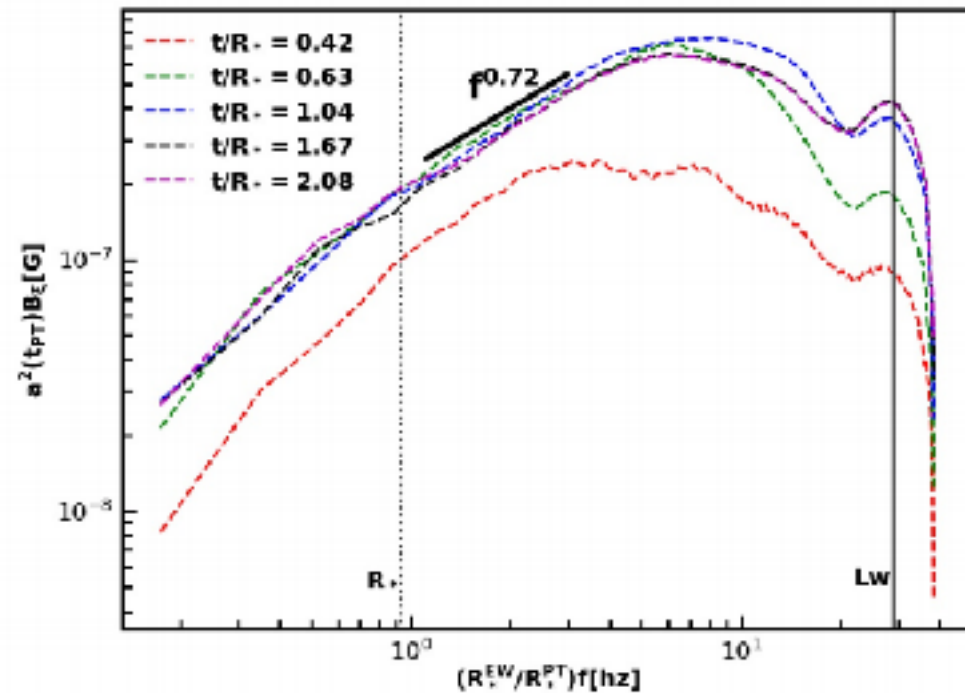
Without sphaleron, the CS number oscillates around zero throughout the PT process.



B + L anomaly: $\Delta N_B = 3 \Delta N_{CS} \sim 3$;
Magnetic helicity and NCS: $|\mathcal{H}| \sim 18 \Delta N_{CS} \sim 6 \Delta N_B$.

MF versus Sphaleron

Non-vanishing gradients of the Higgs fields can generate magnetic fields when bubbles collide



$$A_{\mu\nu} = \sin \theta_w n^a W_{\mu\nu}^a + \cos \theta_w B_{\mu\nu} - i \frac{2}{g v^2} \sin \theta_w [(D_\mu \Phi)^\dagger (D_\nu \Phi) - (D_\nu \Phi)^\dagger (D_\mu \Phi)] .$$

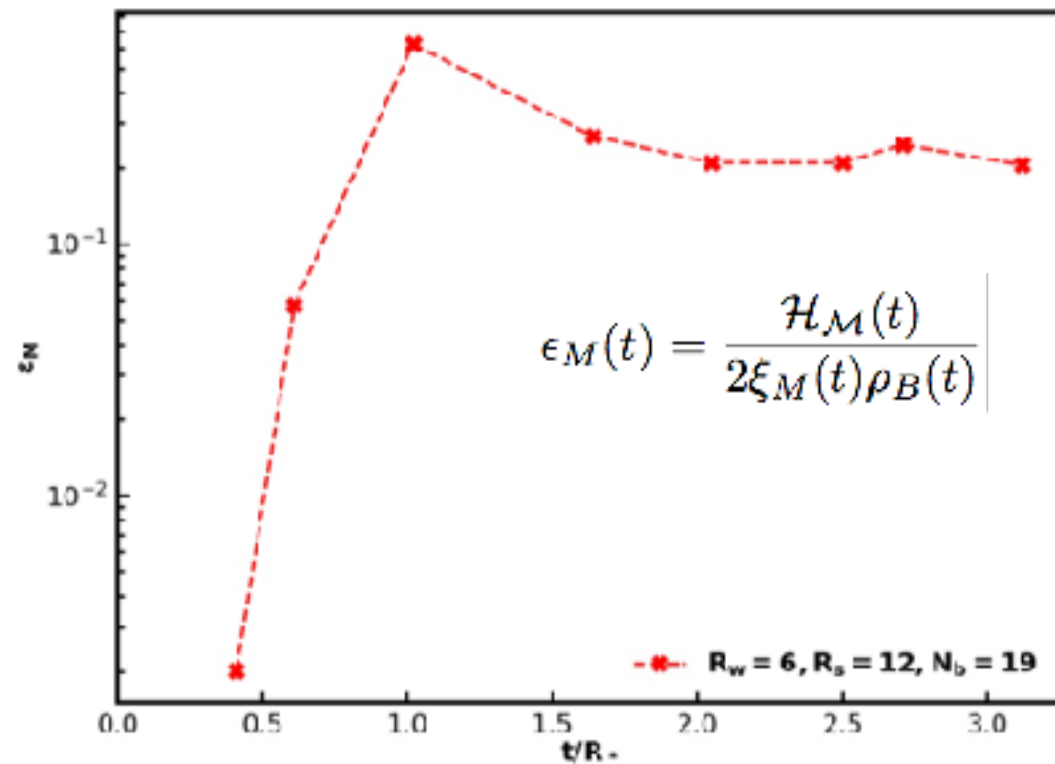
$$\langle B_i^*(\mathbf{k}, t) B_j(\mathbf{k}', t) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') F_{ij}(\mathbf{k}, t)$$

$$F_{ij}(\mathbf{k}, t) = (\delta_{ij} - \hat{k}_i \hat{k}_j) E_M(k, t) / (4\pi k^2) + i \epsilon_{ijl} k_l H_M / (8\pi k^2)$$

$$\rho_B(t) = \int_0^\infty E_M(k, t) dk$$

$$B_\xi = \sqrt{2 d\rho_B / d \log(k)} \quad \xi_M(t) = \int dk k^{-1} E_M(k, t) / \rho_B(t)$$

MF versus Sphaleron

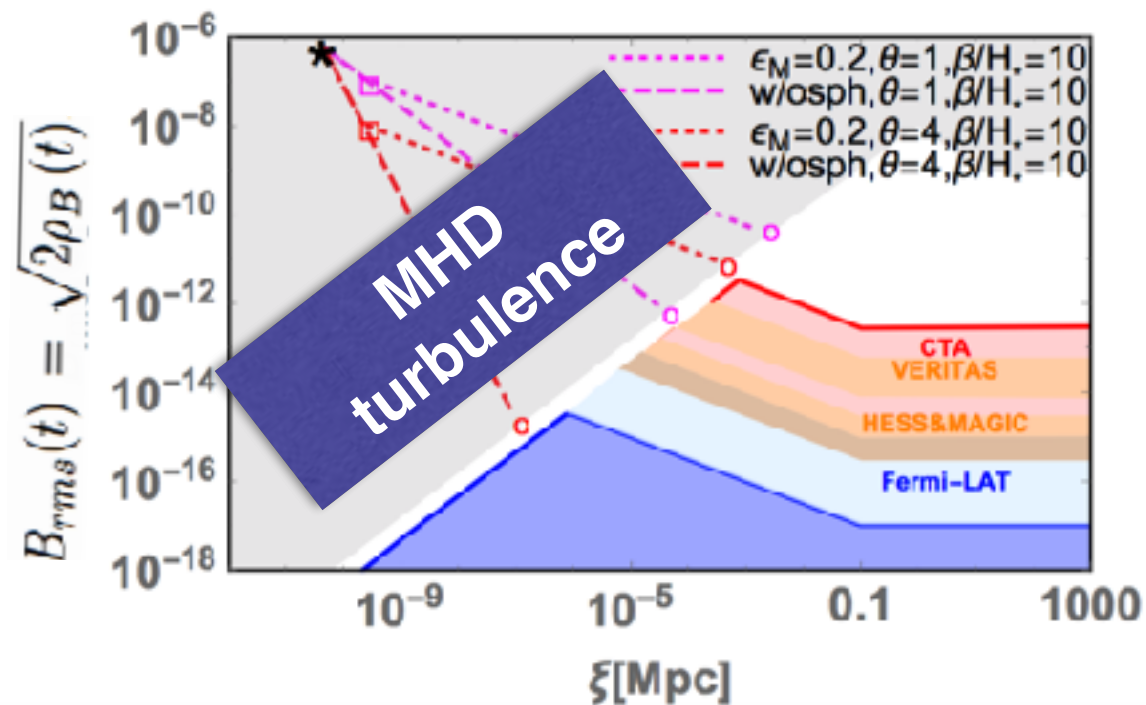


**MF helicity evolution
with PT proceeding**

$$\Phi = \frac{vh(\xi)}{r} \begin{pmatrix} ix + y \\ -iz \end{pmatrix}$$

$$W_i^a \tau^a = -\frac{2f(\xi)}{gr^2} \epsilon_{iab} x_b \mathbf{G}_\Theta \tau^a \mathbf{G}_\Theta^\dagger$$

$$\mathbf{G}_\Theta(\vec{x}) = \exp[i\Theta(r)\boldsymbol{\tau} \cdot \hat{\mathbf{x}}/2]$$



**Cosmic-ray and gamma
ray observations can tell
the helicity of the MF**

Stochastic GW for PTA ???

$$\langle \delta t(t_i, \vec{n}_i) \rangle = 0$$

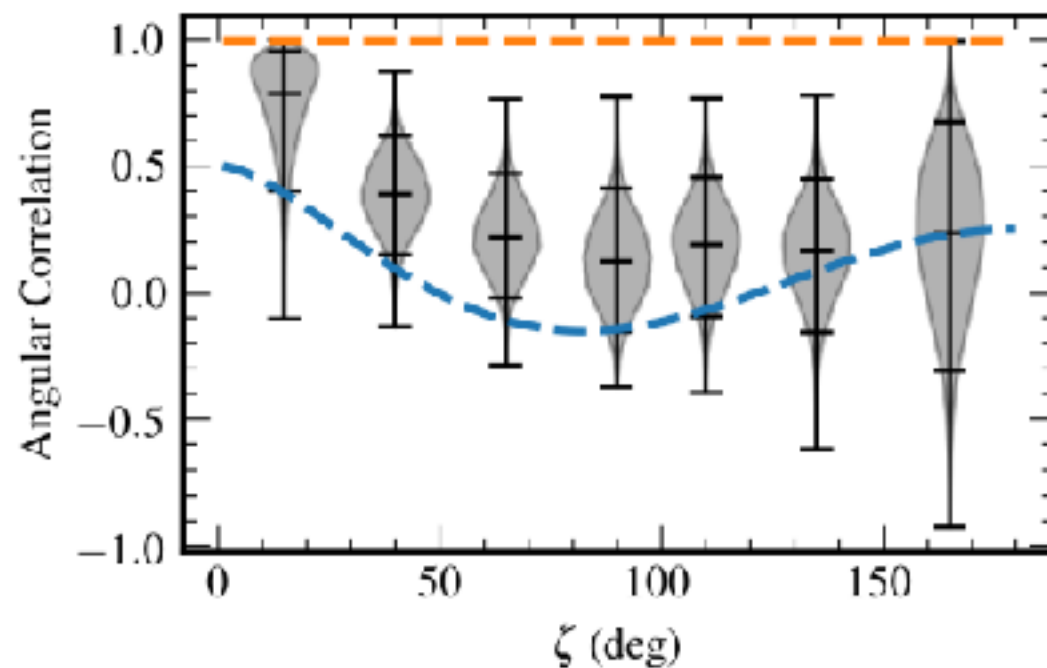
Isotropic, unpolarized

Residual= Data-Model

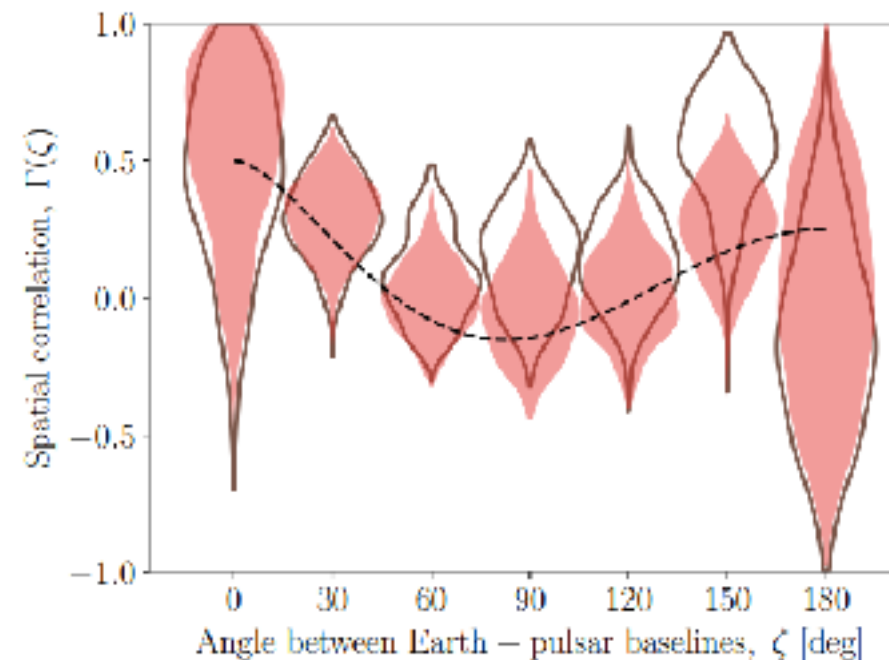
$$\langle \delta t(t_i, \vec{n}_i) \delta t(t_j, \vec{n}_j) \rangle = \int_0 df \Gamma(\vec{n}_i, \vec{n}_j) \cos(2\pi f(t_i - t_j)) \frac{1}{12\pi^2} \frac{1}{f^5} \frac{3H_{100}^2}{2\pi^2} \Omega_{gw}(f) h^2$$

Hellings-Downs Curve

Auto-correlation: $\vec{n}_i = \vec{n}_j$
Cross-correlation: $\vec{n}_i \neq \vec{n}_j$



NANOGrav, 2009.04496

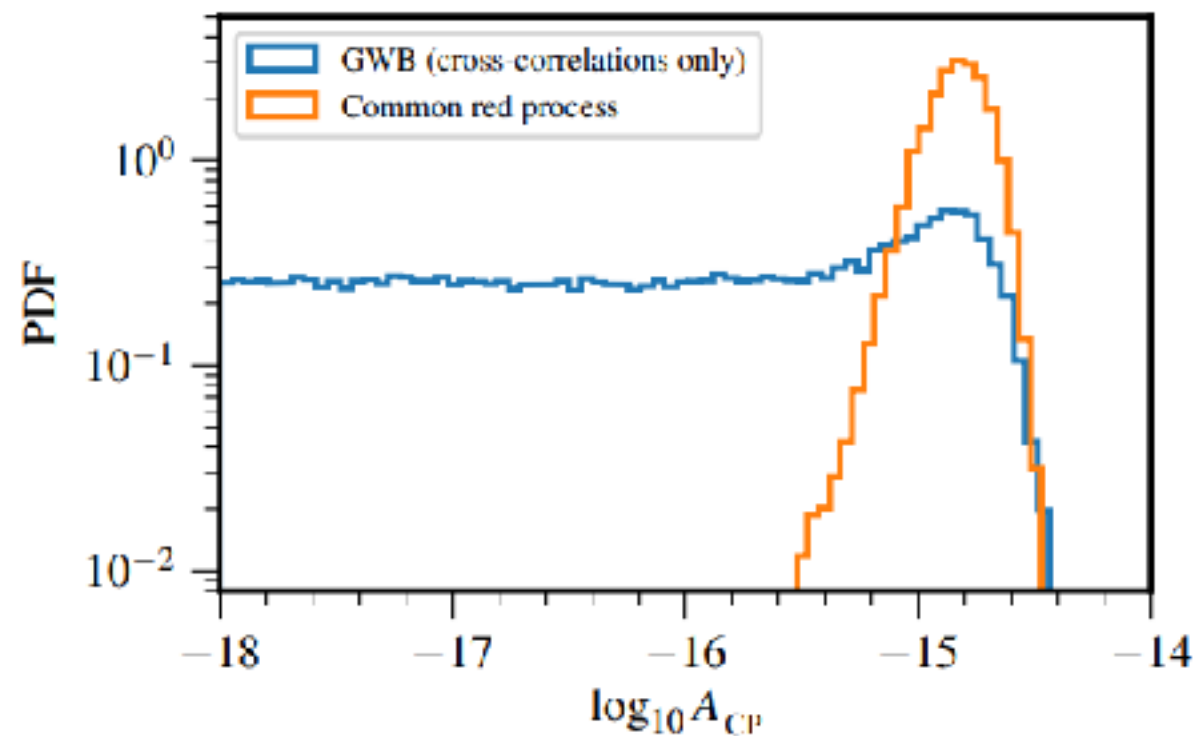


PPTA, 2107.12112

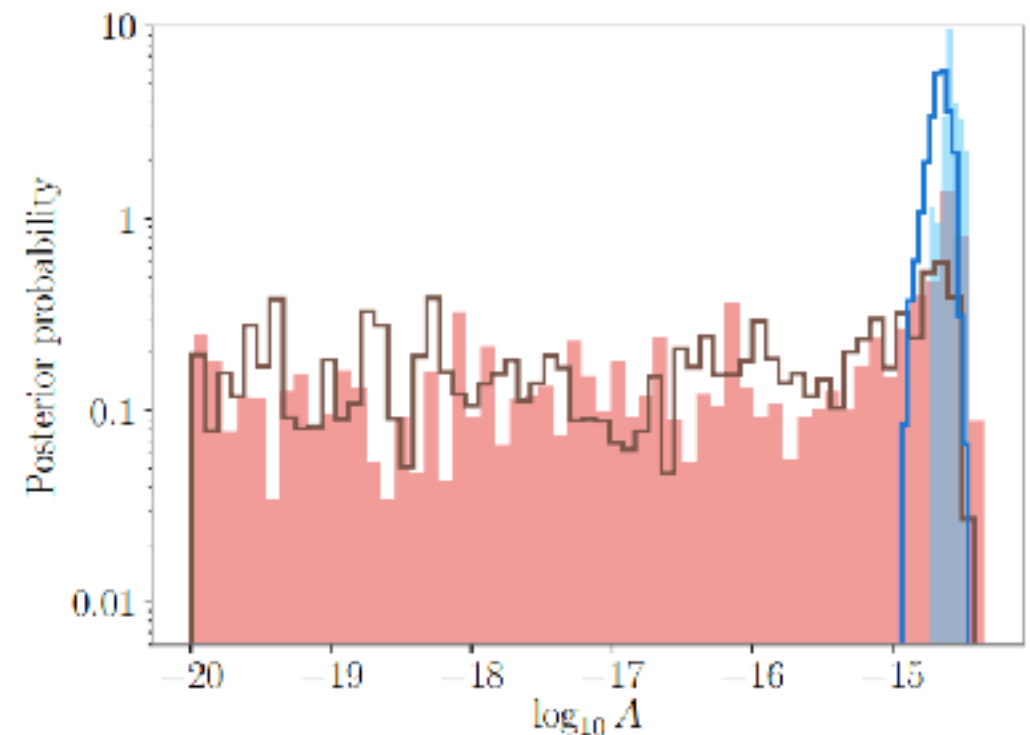
Stochastic GW for PTA ???

NANOGrav: cross-correlation only - No evidence

PPTA: auto-correlation + cross correlation - Better explanation than **common red noise**



NANOGrav, 2009.04496



PPTA, 2107.12112

**No decisive conclusion is made by
NANOGrav or PPTA**

GW for NanoGrav ???

Table I: Bayes factors can be interpreted as follows: given candidate models M_i and M_j , a Bayes factor of 20 corresponds to a belief of 95% in the statement “ M_i is true”, this corresponds to strong evidence in favor of M_i [53].

B_{ij}	Evidence in favor of M_i against M_j
1 – 3	Weak
3 – 20	Positive
20 – 150	Strong
≥ 150	Very strong

$$B_{ij} = \begin{pmatrix} 1 & 0.09 & 0.37 & 0.28 & 0.83 & 0.16 & 0.12 & 0.17 \\ 10.8 & 1 & 3.96 & 3.01 & 8.93 & 1.75 & 1.32 & 1.84 \\ 2.73 & 0.25 & 1 & 0.76 & 2.26 & 0.44 & 0.33 & 0.47 \\ 3.6 & 0.33 & 1.32 & 1 & 2.97 & 0.58 & 0.44 & 0.61 \\ 1.21 & 0.11 & 0.44 & 0.34 & 1 & 0.2 & 0.15 & 0.21 \\ 6.18 & 0.57 & 2.26 & 1.72 & 5.11 & 1 & 0.76 & 1.05 \\ 8.17 & 0.76 & 2.99 & 2.27 & 6.75 & 1.32 & 1 & 1.39 \\ 5.86 & 0.54 & 2.15 & 1.63 & 4.85 & 0.95 & 0.72 & 1 \end{pmatrix}$$

A positive evidence in favor of the cosmic strings explanation against SMBHBs, scalar induced GWs, FOPT, and domain walls, and a weak evidence in favor of cosmic strings against cosmic strings+SMBHBs, cosmic strings+scalar induced GWs, and cosmic strings+domain walls.

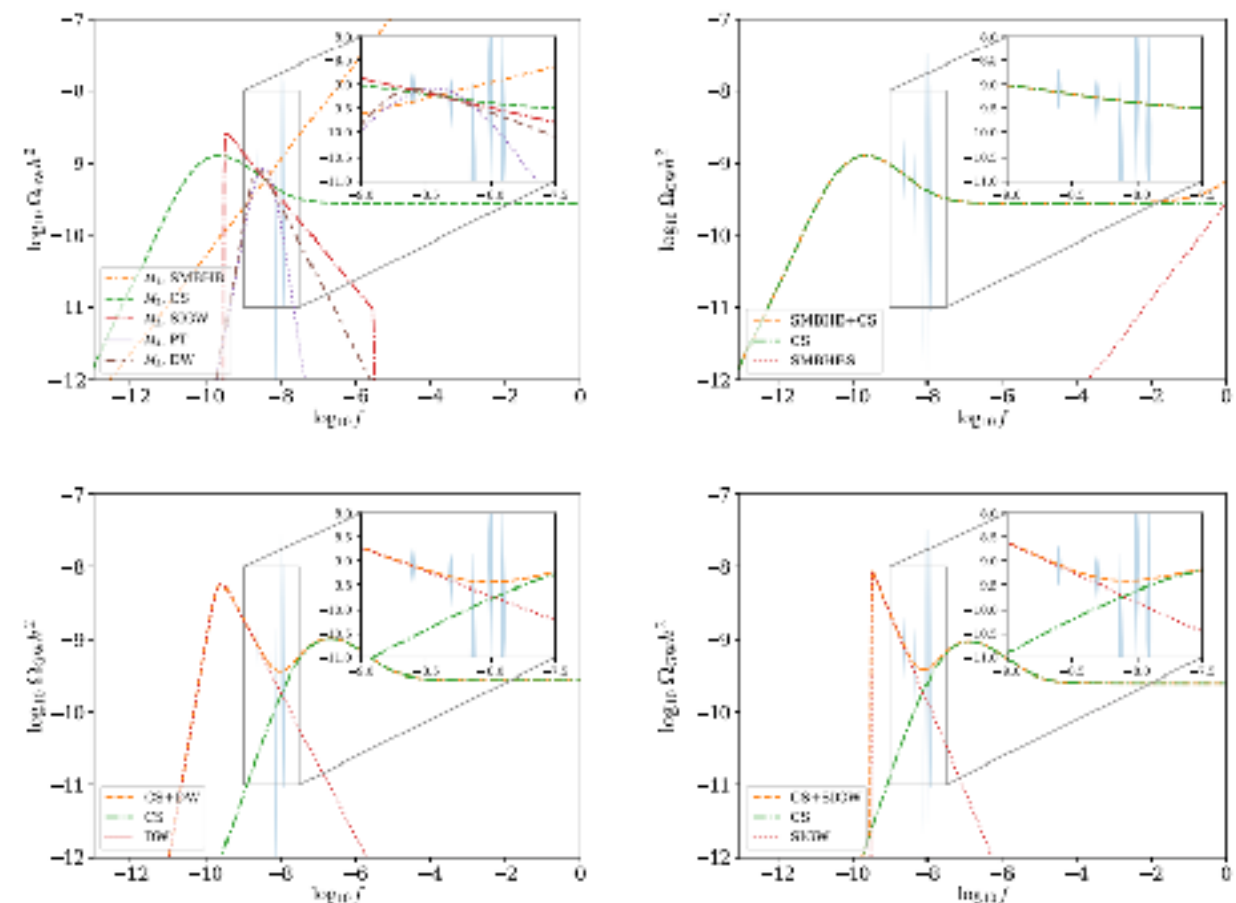
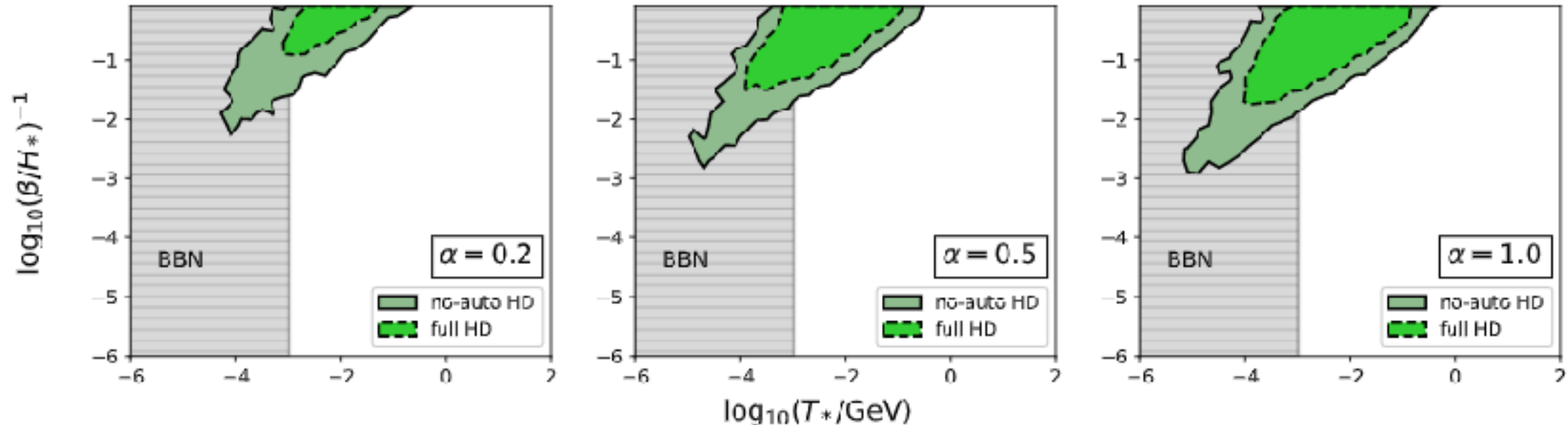


FIG. 5. GW energy spectrum for different models with best-fit value of parameters (which are shown in Supplemental Material [53]). The violin plots show the first five frequency bins of the NANOGrav 12.5-yr data set. The top-left panel shows individual GW source scenario, and the other three plots show the combined explanations with SMBHB + cosmic strings, cosmic strings + domain walls, and cosmic strings + scalar induced GWs.

PTGW for PPTA ???

TABLE I: Description of hypotheses tested in this work and the Bayes factors between them.

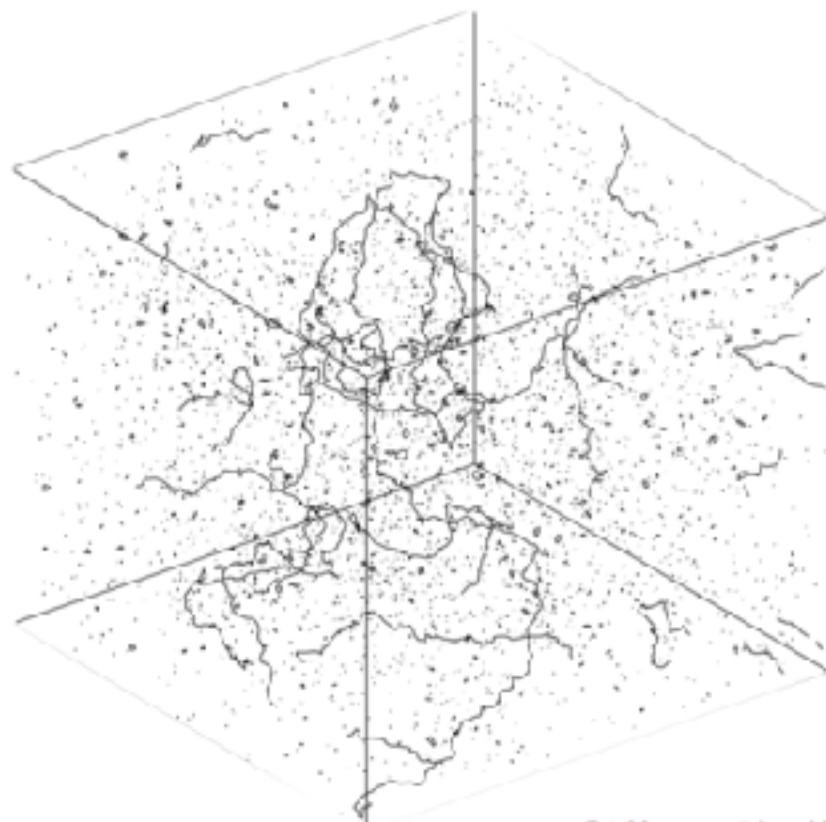
Hypothesis	Pulsar noise	Common red process	HD process FOPT spectrum	Bayes Factors	Parameter Estimation (median and 1- σ interval)	
					$T_*/\text{MeV}, \alpha \times 10^3, \beta/H_*$	$A_{\text{comred}}, \gamma_{\text{comred}}$
H0:Pulsar Noise	yes	no	no			
H1:Common Red	yes	yes	no	$10^{3.5}$ (against H0)		$-14.45^{+0.62}_{-0.64}, 3.31^{+1.36}_{-1.53}$
H2:FOPT	yes	no	yes (full HD)	$10^{1.8}$ (against H0)	$7.4^{+11.9}_{-4.7}, 271^{+165}_{-92}, 9.9^{+11.4}_{-5.4}$	
H3:FOPT1	yes	yes	yes (full HD)	1.04 (against H1)	$9.6^{+232.2}_{-9.2}, 3.8^{+27.9}_{-3.4}, 854^{+9622}_{-782}$	$-14.51^{+0.64}_{-0.68}, 3.36^{+1.39}_{-1.54}$
H4:FOPT2	yes	yes	yes (no-auto HD)	0.96 (against H1)	$10.9^{+290.5}_{-10.6}, 3.2^{+19.9}_{-2.8}, 1053^{+11256}_{-962}$	$-14.45^{+0.62}_{-0.64}, 3.27^{+1.37}_{-1.54}$



95% CL.
Excl.

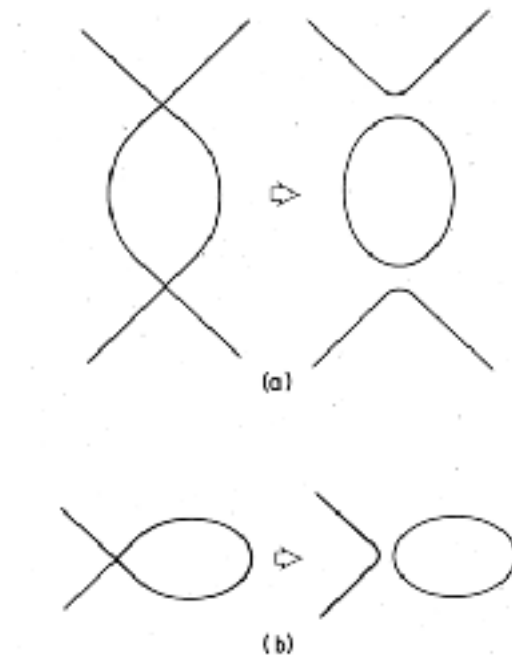
Xue, [Bian*](#), Shu*, Yuan*, Zhu*, Bhat, Dai et al. "Constraining cosmological phase transitions with the Parkes Pulsar Timing Array." *arXiv preprint arXiv:2110.03096* (2021), PRL in press, Editor's suggestion.

CS: SSB of $U(1)$ symmetry



Allen&Shellard PRL 64,119 (1990)

Loop formation



Closed-loop formation by intercommuting strings.

Cosmic string loop formation. A loop forms (a) when two strings interact in 2 separate points or (b) when a string crosses itself.

Tanmay Vachaspati, Alexander Vilenkin

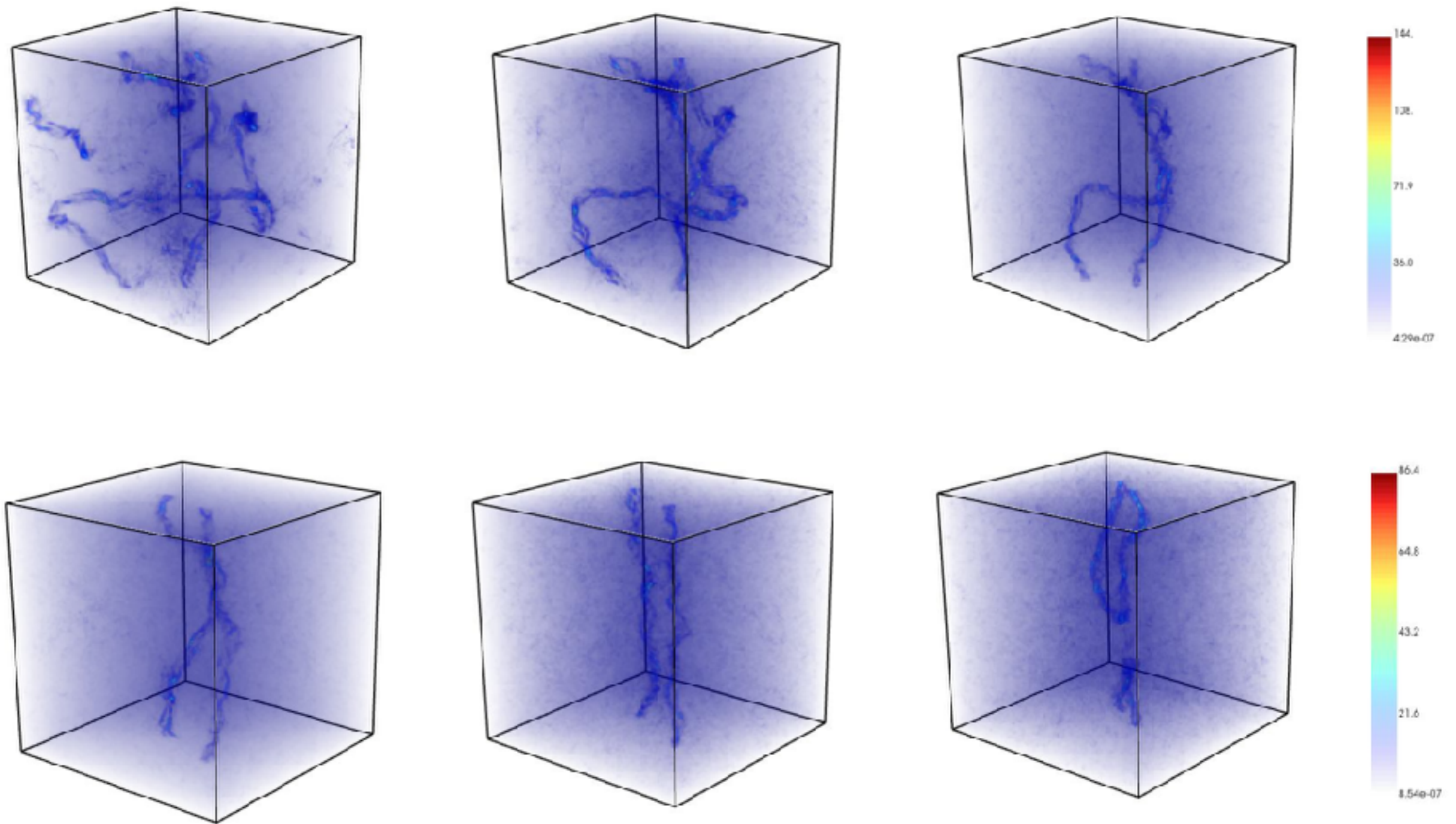
Phys.Rev.D 30 (1984) 2036



GW emission

Cusps: a pointed and highly Lorentz-boosted region which appear few times per oscillation period

CS from first-order PT of abelian U(1)



Summary and future

Observation of the cosmic Magnetic field seeded by phase transition, with GW production may hint the B+L violation

Interaction between bubble wall and Plasma, and interaction among different bubbles are important

- 1) Magnetic field feedback to the phase transition**
- 2) Baryogenesis and/or at fast-wall request by the GW**

Higgs Potential shape

- 1) The future collider prospect, with dihiggs, Zh and/or Zhh production**
- 2) Thin wall or thick wall tell by gravitational wave, wall profile and GW spectrum**

Topological defects at the early Universe and gravitational waves relics

cosmic string, domain wall, ...

Thanks

谢谢！