Flavor Contents of the Vacuum from the Dirac Spectrum of Overlap Fermions

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11/01/2021 @ 中国格点QCD第一届年会

arXiv: 2102.05380

Background and Motivation

The Banks-Casher relation

[T. Banks and A. Casher 1980]

$$\lim_{m_i \to 0} \lim_{V \to \infty} \rho_\nu(\lambda = 0) = \frac{\Sigma}{\pi} \qquad \rho_\nu(\lambda) \equiv \frac{1}{V} \sum_k \langle \delta(\lambda + \lambda_k) \rangle_\nu$$

connects the spectrum density (near zero modes) of the Dirac operator with the chiral condensate.

The λ dependence reveals the N_f of the vacuum, and the sea quark masses.

$$\lim_{m_i \to 0} \lim_{V \to \infty} \rho_\nu(\lambda) = \frac{\Sigma}{\pi} \left[1 + \left(\frac{N_f^2 - 4}{N_f} \right) \frac{\Sigma}{32\pi F^4} |\lambda| \right]$$

[A. V. Smilga and J. Stern 1993]

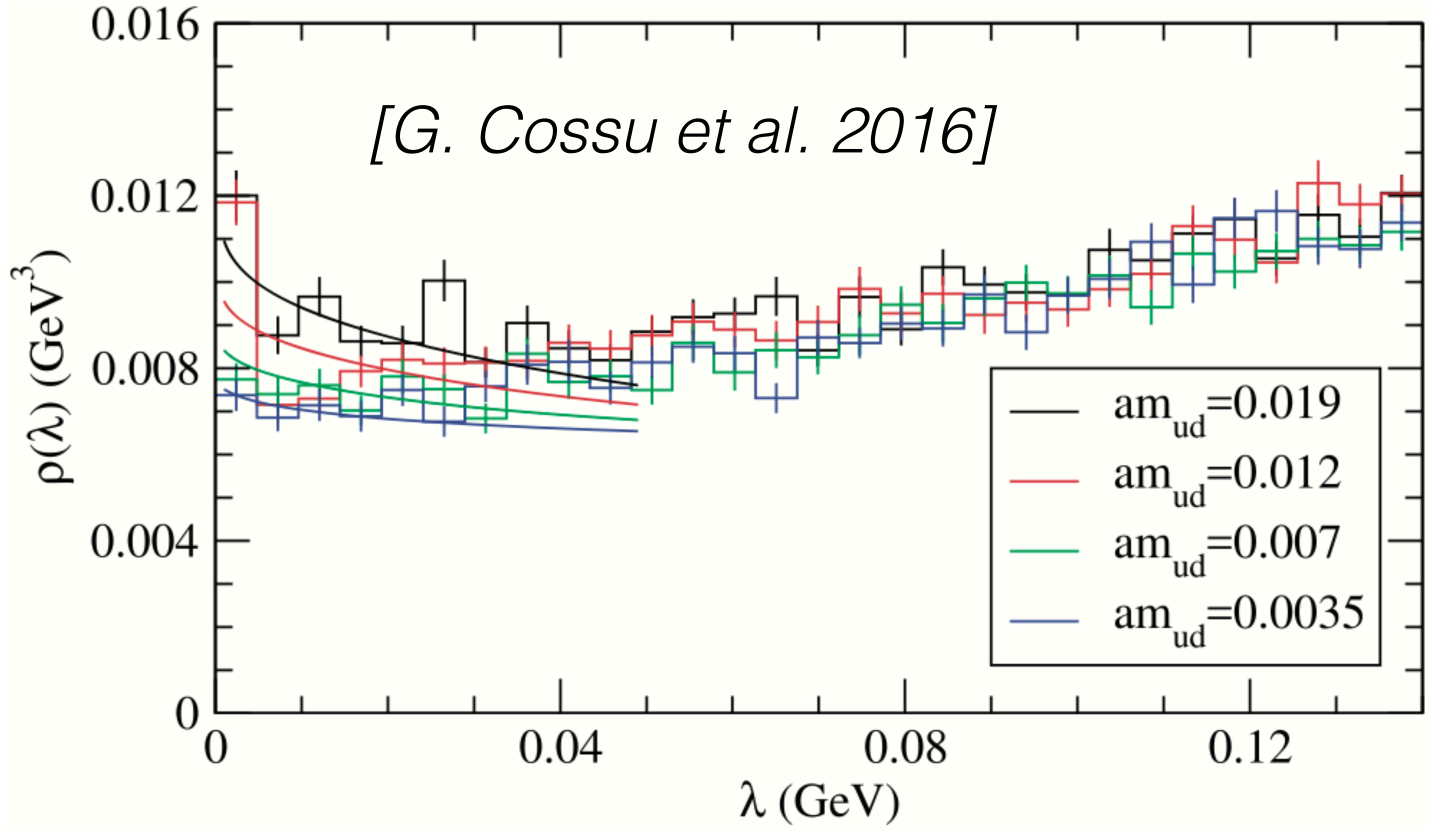
[J. C. Osborn et al. 1999]

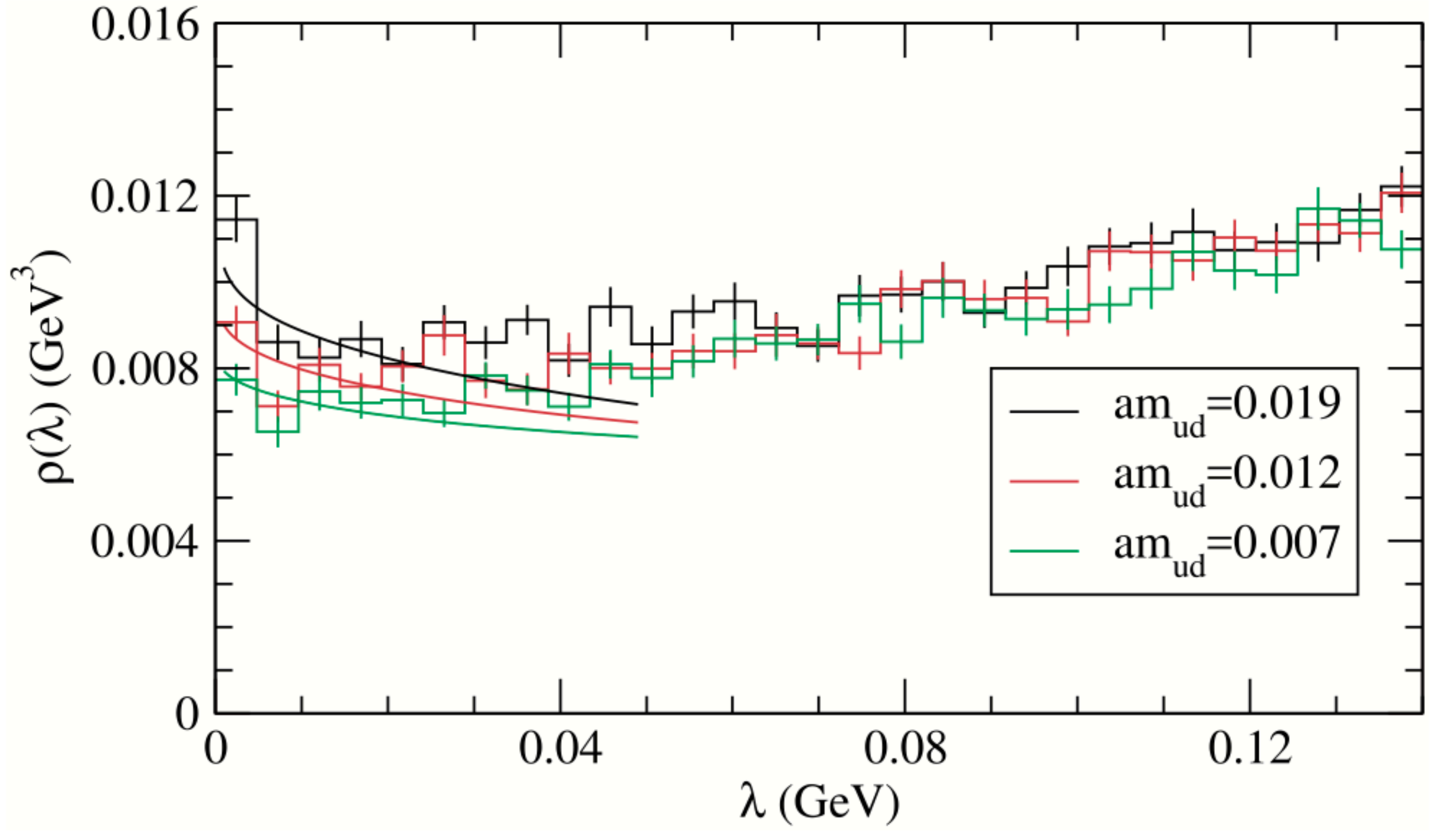
$$\lim_{V \to \infty} \rho_\nu(\lambda) = \frac{\Sigma}{\pi} \Bigg[1 + \frac{\Sigma}{32\pi^2 N_f F^4} \Bigg(2N_f^2 |\lambda| \arctan \frac{|\lambda|}{m} - 4\pi |\lambda| - N_f^2 m \ln \frac{\Sigma^2}{F^4} \frac{m^2 + \lambda^2}{\mu_{sub}^4} - 4m \ln \frac{\Sigma}{F^2} \frac{|\lambda|}{\mu_{sub}^2} \Bigg) + \frac{32 N_f L_6^r(\mu_{sub}) \Sigma m}{F^4} \Bigg]$$

Related Lattice Studies

Condensate from lattice QCD
(table from FLAG19)

	Collaboration	Ref.	N_f	publication status	chiral extrapolation	continuum extrapolation	finite volume	renormalization	$\Sigma^{1/3}$
	ETM 17E	[43]	2+1+1	A	○	★	○	★	318(21)(21)
TM	ETM 13	[42]	2+1+1	A	○	★	★	★	280(8)(15)
	JLQCD 17A	[48]	2+1	A	○	★	★	★	274(13)(29)
DW	JLQCD 16B	[47]	2+1	A	○	★	★	★	270.0(1.3)(4.8)
	RBC/UKQCD 15E	[46]	2+1	A	★	★	★	★	274.2(2.8)(4.0)
	RBC/UKQCD 14B	[10]	2+1	A	★	★	★	★	275.9(1.9)(1.0)
	BMW 13	[45]	2+1	A	★	★	★	★	271(4)(1)
	Borsanyi 12	[44]	2+1	A	○	○	★	★	272.3(1.2)(1.4)
OL	JLQCD/TWQCD 10A	[387]	2+1	A	★	■	■	★	234(4)(17)
	MILC 10A	[14]	2+1	C	○	★	★	○	$281.5(3.4)(^{+2.0}_{-5.9})(4.0)$
	RBC/UKQCD 10A	[158]	2+1	A	○	○	■	★	256(5)(2)(2)
OL	JLQCD 09	[386]	2+1	A	★	■	■	★	$242(4)(^{+19}_{-18})$
	MILC 09A, $SU(3)$-fit	[18]	2+1	C	○	★	★	○	279(1)(2)(4)
	MILC 09A, $SU(2)$-fit	[18]	2+1	C	○	★	★	○	$280(2)(^{+4}_{-8})(4)$
	MILC 09	[128]	2+1	A	○	★	★	○	$278(1)(^{+2}_{-3})(5)$
	TWQCD 08	[389]	2+1	A	■	■	■	★	259(6)(9)
	PACS-CS 08, $SU(3)$-fit	[160]	2+1	A	★	■	■	■	312(10)
	PACS-CS 08, $SU(2)$-fit	[160]	2+1	A	★	■	■	■	309(7)
	RBC/UKQCD 08	[161]	2+1	A	○	■	○	★	255(8)(8)(13)





Early overlap studies used quite small lattices with fixed topology. The JLQCD 16B work fitted only the small eigenvalues.

Overlap Fermions

The overlap Dirac operator and the chiral operator:

$$D_{\rm ov} = M_0\Big(1+\gamma_5\epsilon\big(H_{\rm w}(-M_0)\big)\Big)$$

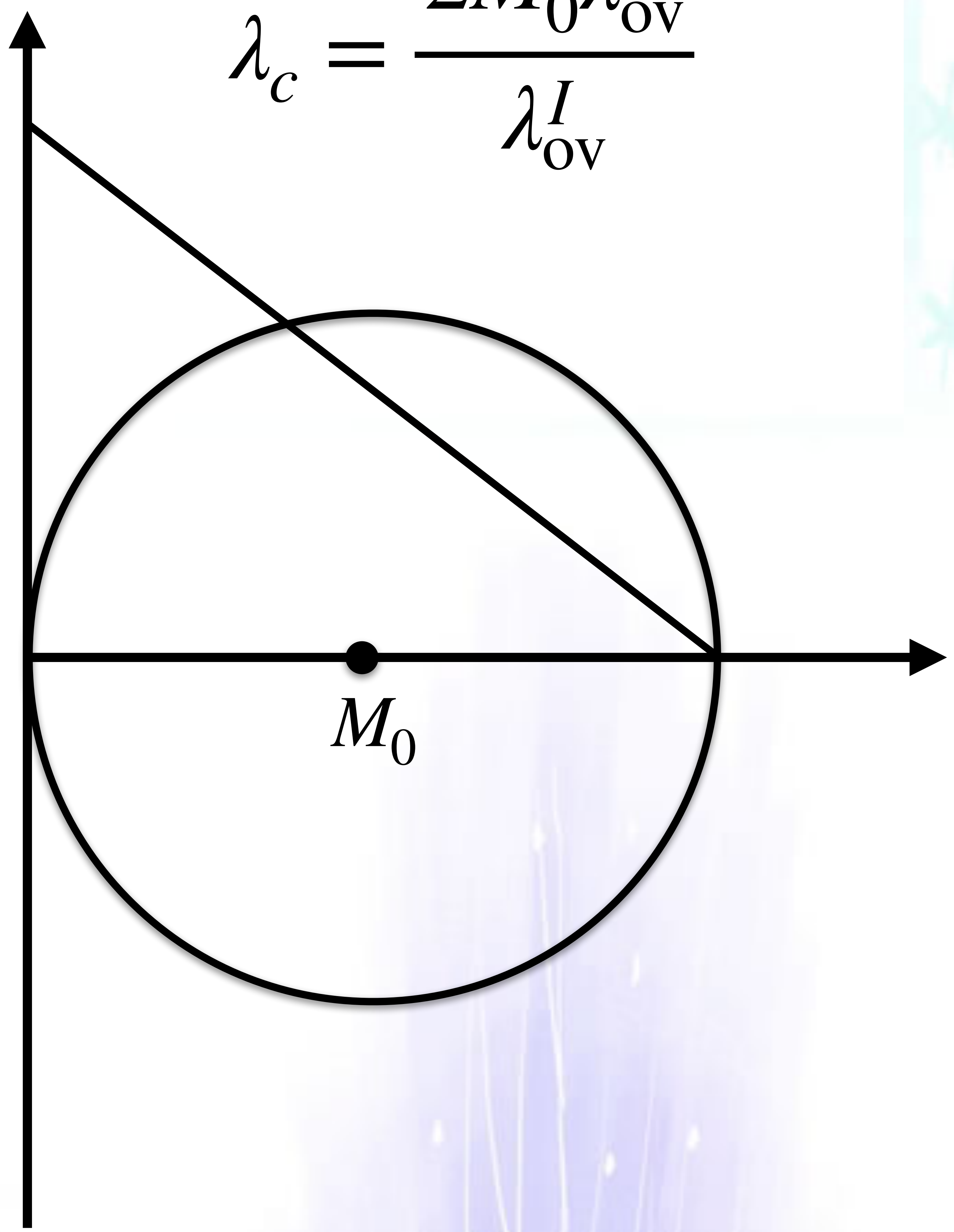
$$D_{\rm c} = \frac{D_{\rm ov}}{1-\frac{1}{2M_0}D_{\rm ov}} = \frac{M_0}{2}\frac{1+\gamma_5\epsilon(\gamma_5 D_{\rm w}(M_0))}{1-\gamma_5\epsilon(\gamma_5 D_{\rm w}(M_0))}$$

The chiral operator behaviors like the continuum one

$$\{D_c, \gamma_5\} = 0$$

$$i\lambda_c = \frac{\lambda_{\rm ov}}{1-\frac{\lambda_{\rm ov}}{2M_0}}$$

$$\lambda_c = \frac{2M_0\lambda^R_{\rm ov}}{\lambda^I_{\rm ov}}$$



$$(\lambda^R_{\rm ov} - M_0)^2 + \lambda^I_{\rm ov} = M_0^2$$

Low-Lying Modes of Overlap Fermions on Domain Wall Seas

Symbol	$L^3 \times T$	a (fm)	m_π	m_K	Z_S	$N_{\rm cfg}$
48I	$48^3 \times 96$	0.1141(2)	139	499	1.135(1)(17)	303
64I	$64^3 \times 128$	0.0837(2)	139	508	1.020(1)(13)	304
48IF	$48^3 \times 96$	0.0711(3)	234	564	0.986(1)(15)	185

# of eigenvectors	Max eigenvalues
1000 pairs	~120 MeV
1600 pairs	~180 MeV
1000 pairs	~ 400 MeV

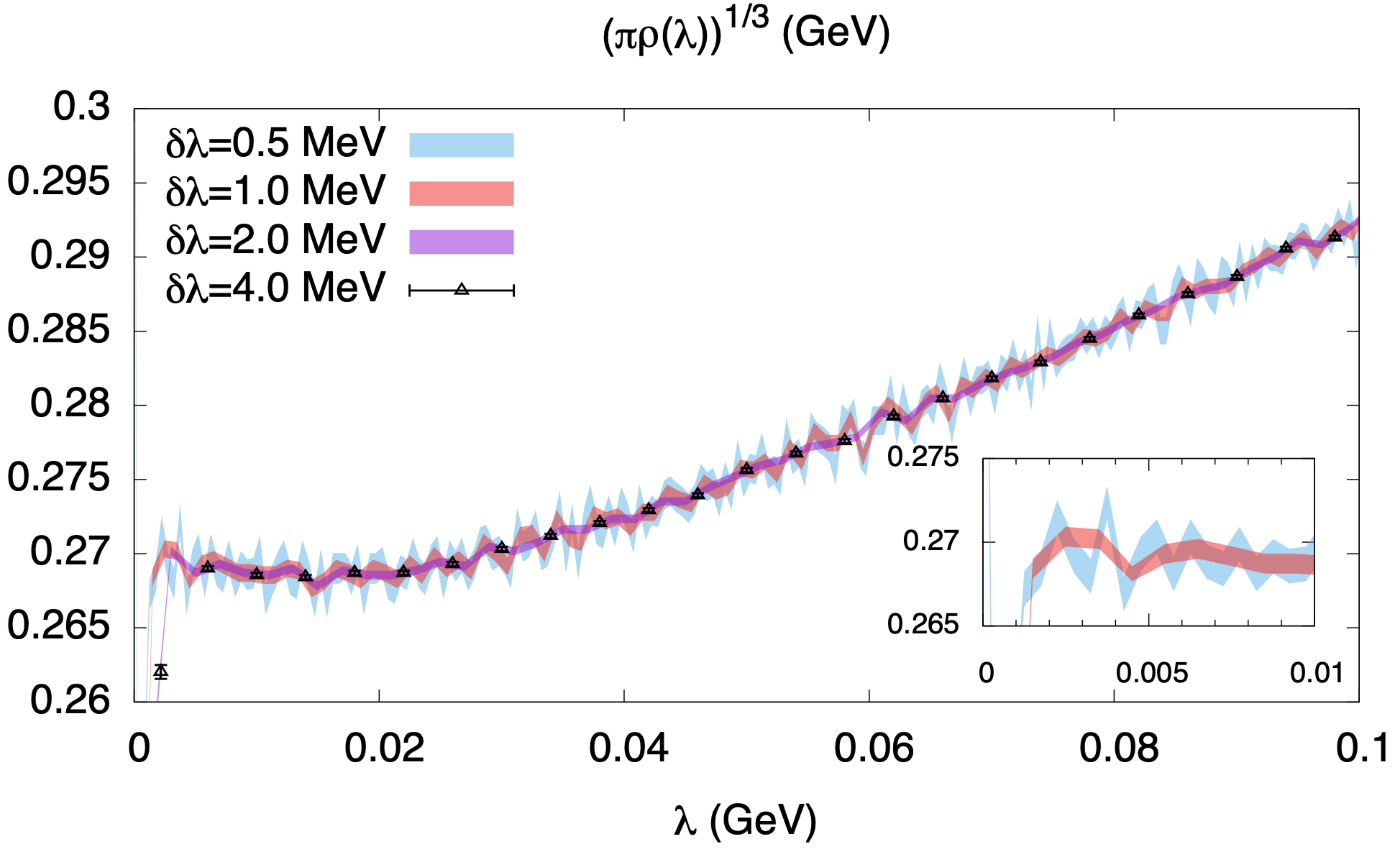
[T. Blum et al. 2016)]
[R. D. Mawhinney 2019]

Although the computations are very costly, we solved the low-lying modes of the overlap Dirac operators to a very high precision (thanks the powerful Machine 先导一号) on two large lattices at the physical pion mass point.

During investigations in the last decade, the near-zero eigen-pairs have been found to be extremely beneficial in the calculations of the correlators and quark loops to improve the signal-to-noise ratio. We actually have the low-lying modes for more ensembles than the ones listed in the table.

[A. Li et al. 2010]
[Y.-B. Yang et al. 2016)]
[J. Liang et al. 2017]
[M. Gong et al. 2013]

Extracting the Spectrum Density from the low-modes

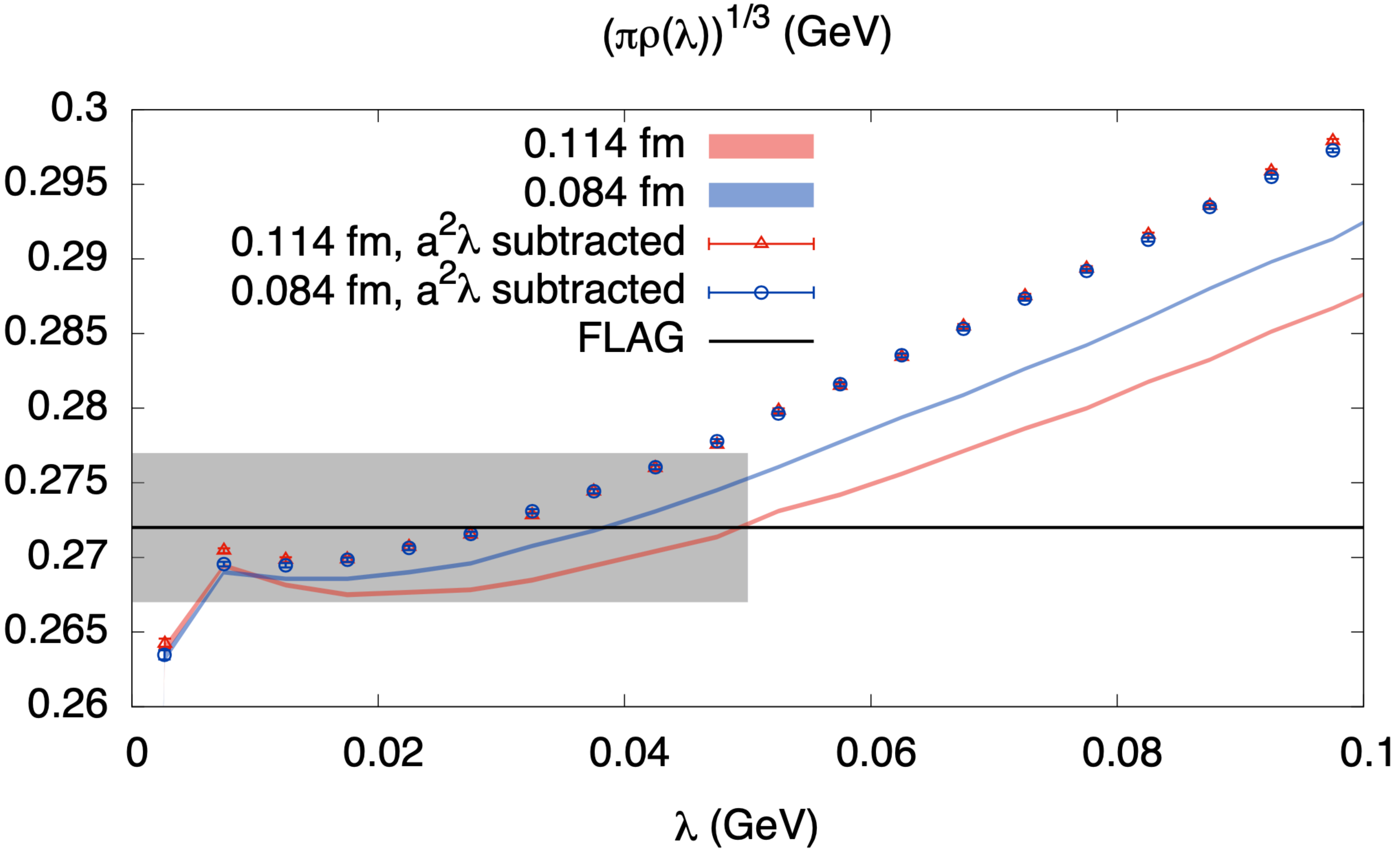


$$\rho(\lambda) = \frac{\delta n(\lambda)}{\delta \lambda}$$

Based on the standard statistical requirement, we choose to use a 5 MeV bin size in the following analysis.

With such a bin size, our statistical uncertainty (~0.2%) is an order of magnitude smaller than those of all the previous calculations.

Qualitative Analysis



Without any fitting/extrapolation, our results of $\pi\rho$ show consistency with the PDG value of the SU(2) chiral condensate at small λ.

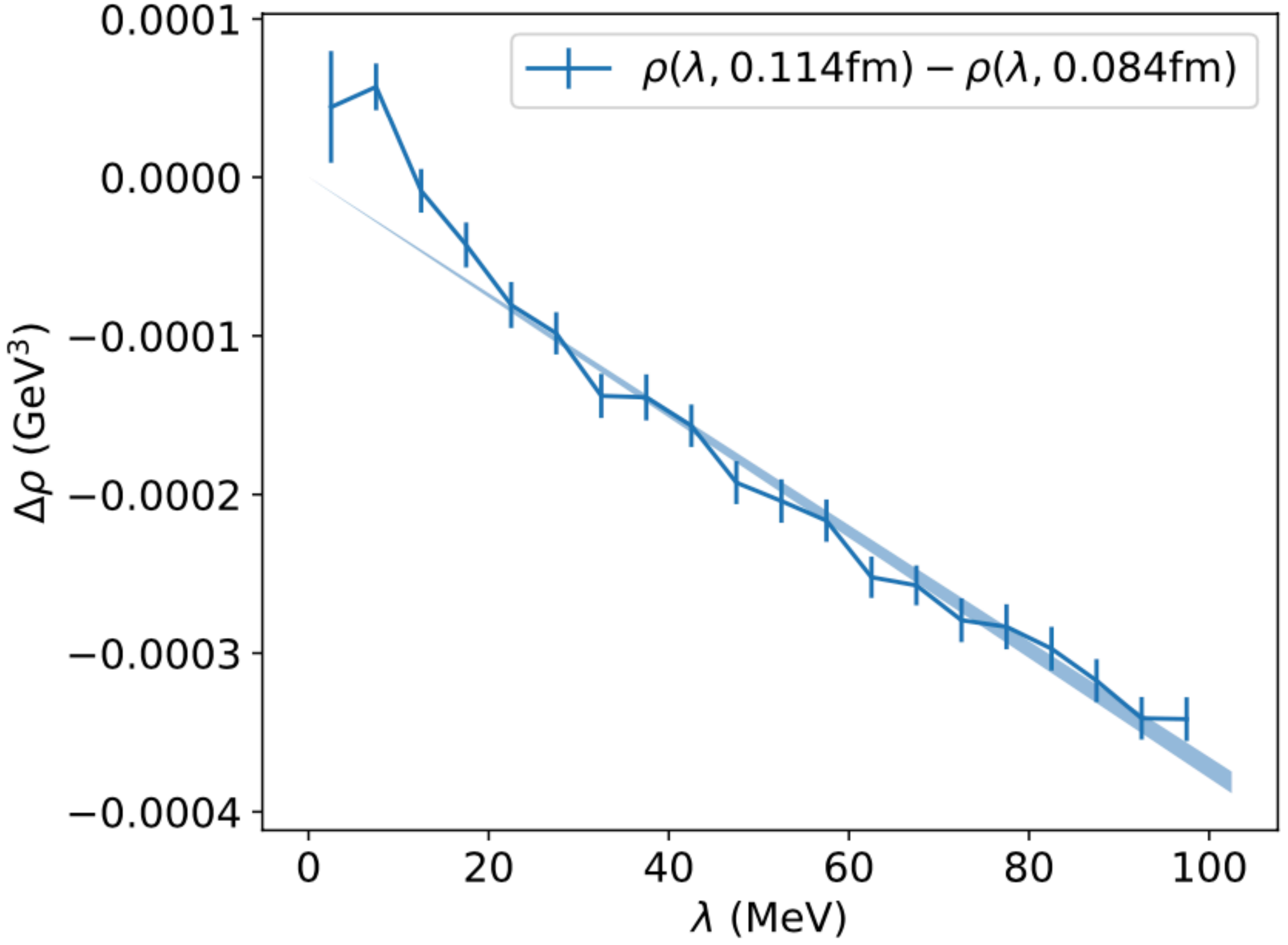
Go to larger λ, we see a linear-like behavior, which indicates the effects of the third flavor (the strange quark).

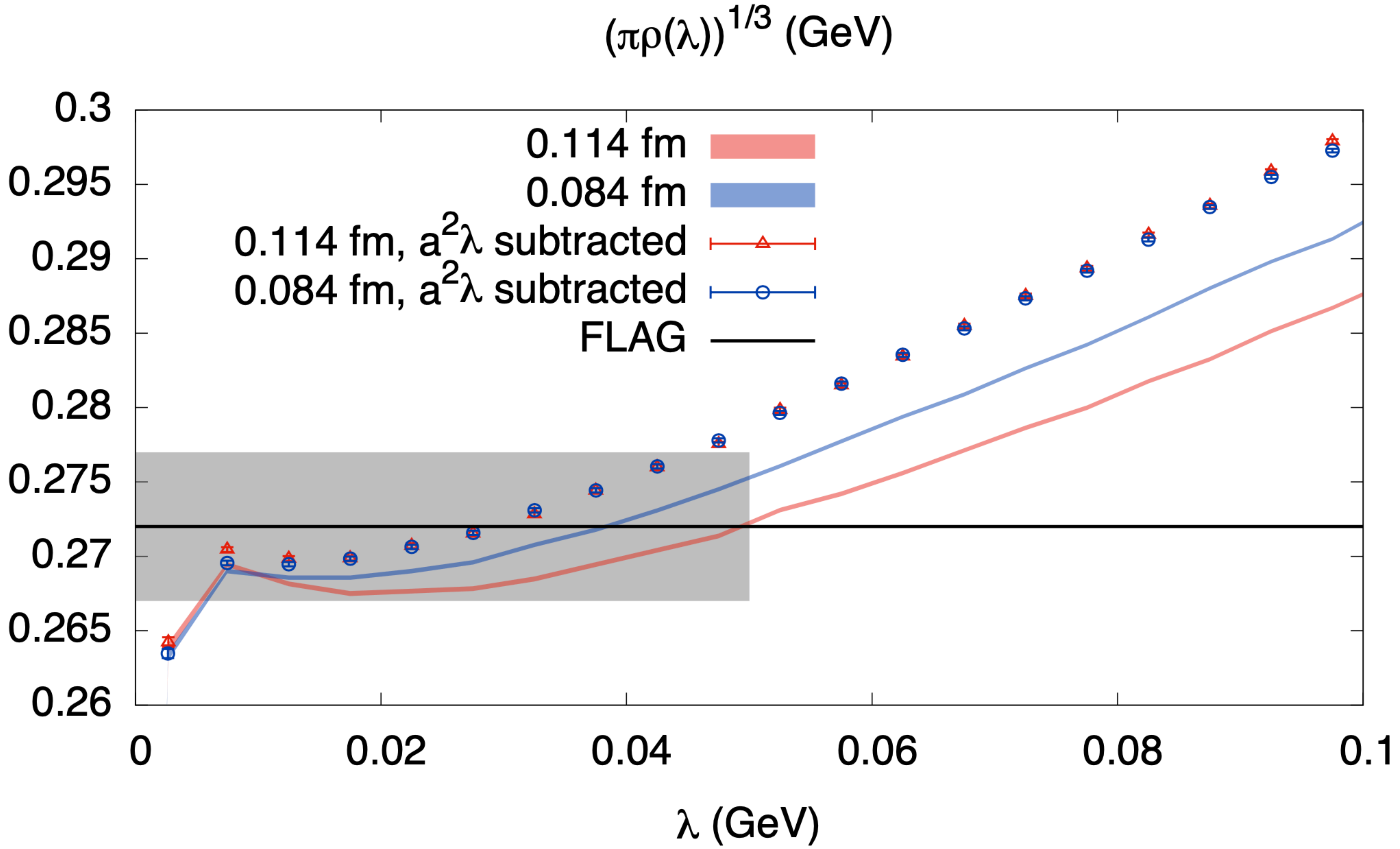
Obvious discrete effects.

$$\Sigma \sim \pi\rho(\lambda \sim 0)$$

$$\lim_{m_i \to 0} \lim_{V \to \infty} \rho_\nu(\lambda) = \frac{\Sigma}{\pi}\left[1 + \left(\frac{N_f^2 - 4}{N_f}\right) \frac{\Sigma}{32\pi F^4}|\lambda|\right]$$

Subtracting Lattice Spacings Effects





The discretization error and is found to be proportional to λ in the range $\lambda \in (20, 100)$ MeV.

After the subtraction, the remaining differences at vary small λ may be due to the residual mixed action effect, which should vanish in the continuum limit. In further analysis the first two points are dropped.

Fitting of the Dirac Spectrum

A NLO chiral formula that works in both p-regime and epsilon-regime with N_f non-degenerate flavors in a finite volume.

[Poul H. Damgaard , Hidenori Fukaya 2009]

$$\langle \bar{q}_v q_v \rangle |_{m_v} = \frac{1}{V} \sum_k \left\langle \frac{1}{m_v + i\lambda_k} \right\rangle$$

$$\begin{aligned}
\rho_\nu(\lambda) &\equiv \frac{1}{V} \sum_k \langle \delta(\lambda + \lambda_k) \rangle_\nu \\
&= -\frac{1}{2\pi V} \sum_k \lim_{\epsilon \to 0} \left\langle \frac{1}{i(\lambda + \lambda_k) - \epsilon} - \frac{1}{i(\lambda + \lambda_k) + \epsilon} \right\rangle_\nu \\
&= \lim_{\epsilon \to 0} \frac{1}{2\pi} \left(\langle \bar{q}_v q_v \rangle_\nu |_{m_v = i\lambda - \epsilon} - \langle \bar{q}_v q_v \rangle_\nu |_{m_v = i\lambda + \epsilon} \right) \\
&= \frac{\Sigma}{\pi} \left[Z_v^0 \, \mathrm{Re} \hat{\Sigma}_\nu^{\mathrm{PQ}}(i\lambda \Sigma V Z_v^0, \{\mu'_{sea}\}) + \mathrm{Re}\left(\delta Z_v(i\lambda)\right) \right],
\end{aligned}$$

Z_v^0 and δZ_v are the NLO corrections, the latter one has λ dependence.

The full expression is quite lengthy.

Fitting of the Dirac Spectrum

Thanks to the precise spectrum data, the complicated chiral formula can be really fitted.

	N_f=2		N_f=2+1			N_f=2+1, fit m_s			
Lattice spacing	$\Sigma^{1/3}$	m_l	$\Sigma_0^{1/3}$	F_0	m_l	$\Sigma_0^{1/3}$	F_0	m_l	m_s
0.114 fm	261.7(0.2)	5.14(15)	229.6(0.3)	62.3(0.4)	3.030(05)	231(1)	64(1)	3.08(07)	74(04)
0.084 fm	261.0(0.3)	4.77(15)	230.9(0.3)	64.7(0.6)	2.993(06)	231(4)	65(4)	3.00(17)	84(12)
0.071 fm	260.5(0.7)		233.2(1.2)	65.8(1.0)					
Continuum	260(1)(2)	4.3(4)(4)	232(1)(2)	68(1)(2)	2.95(1)(36)	231(8)(2)	66(8)(2)	2.89(35)(12)	97(24)(13)
FLAG [36]	272(5)	3.36(4)	245–290	66–84	3.36(4)	245–290	66–84	3.36(4)	92.0(1.1)

F = 86.2(5) MeV
$\lambda \in$ (12,30) MeV

m_s fixed by the Kaon mass
$\lambda \in$ (15, 100)

$\lambda \in$ (15, 100)

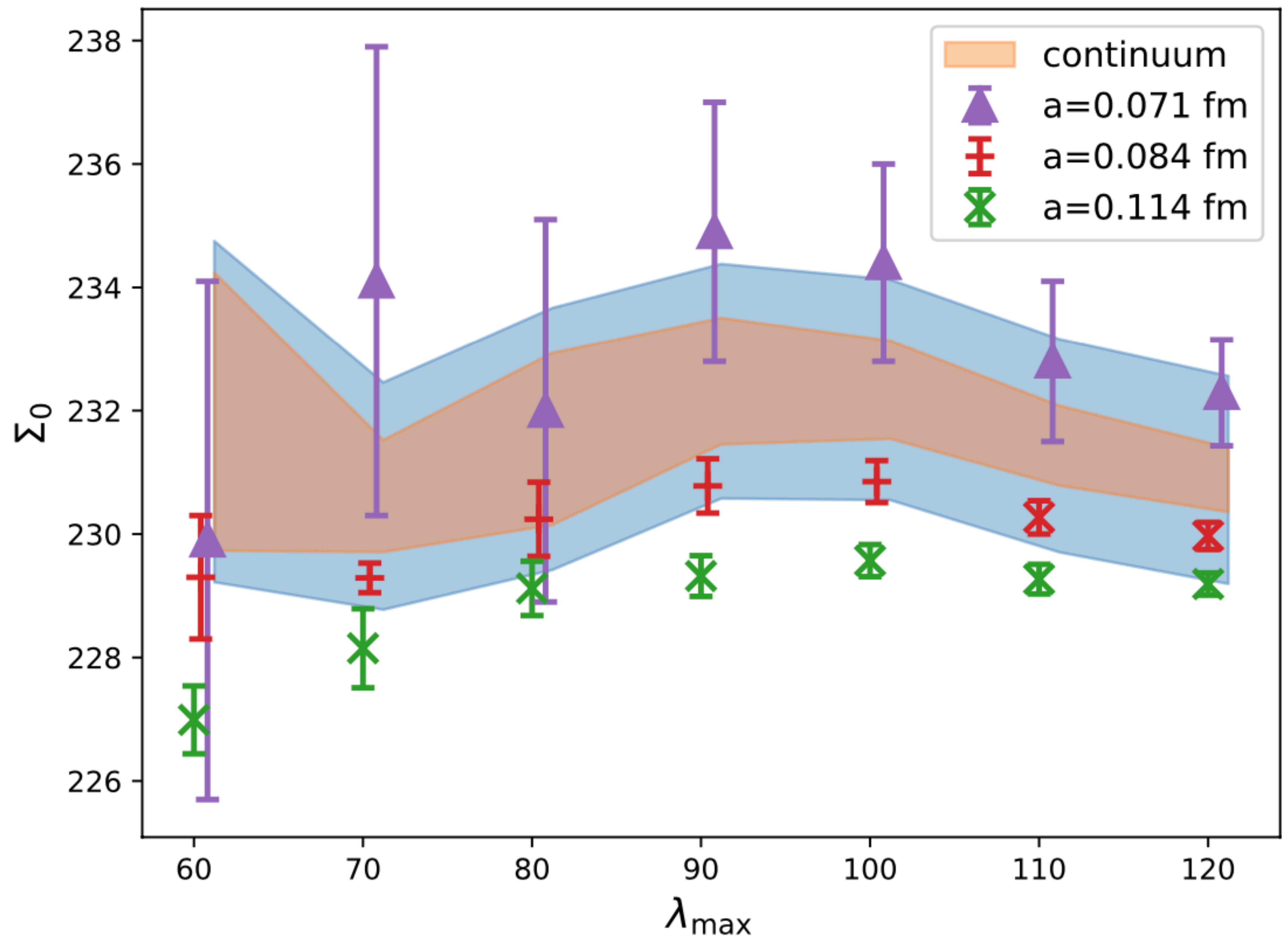
The continuum limit here is to subtract the remaining discrete effects.

Systematic Uncertainties

From the fitting, the SU(2) chiral condensate and light quark mass are determined as 260.3(0.7)(1.3)(0.7)(0.8) MeV and 4.34(35)(6)(43)(1) MeV, respectively.

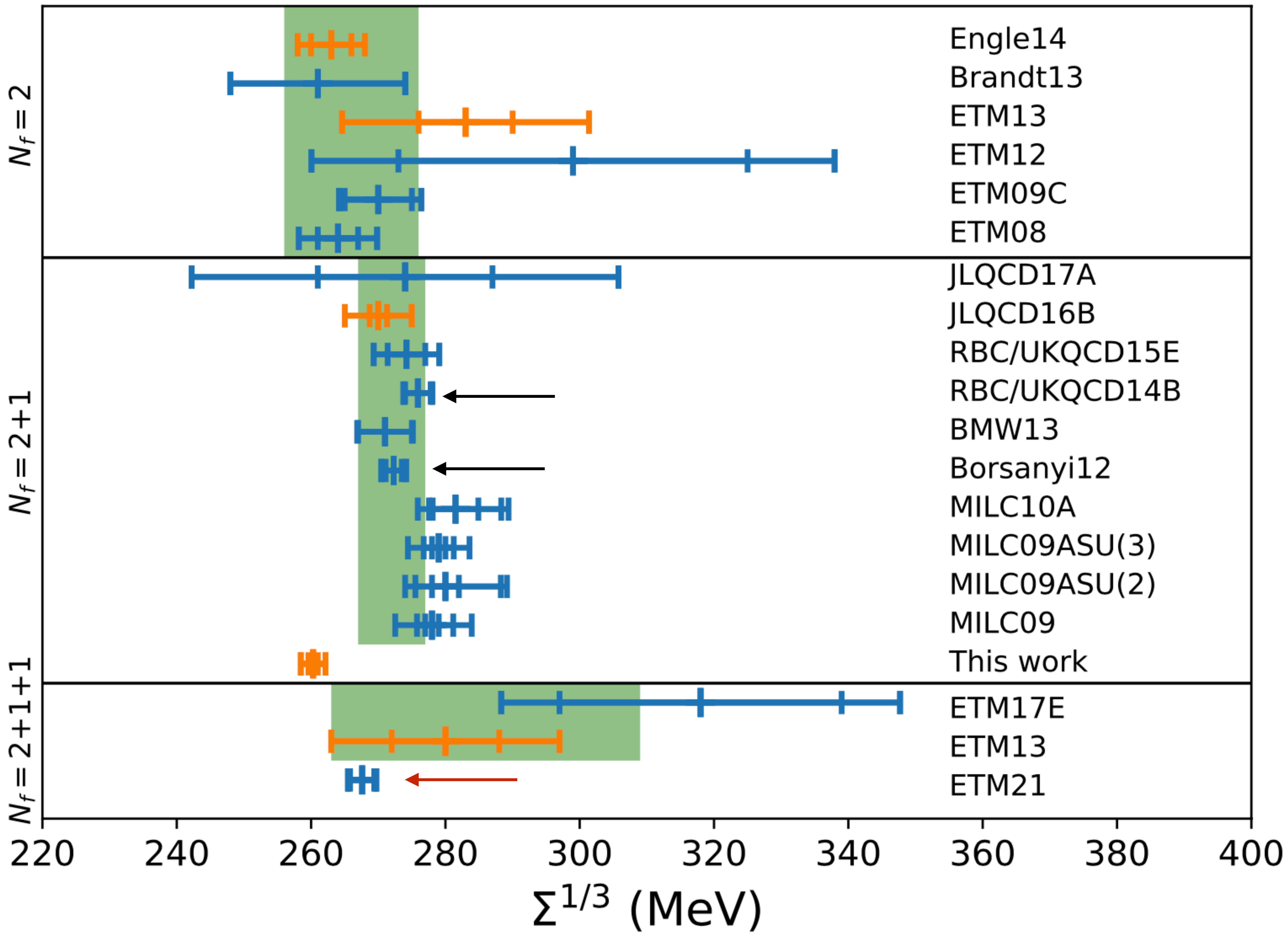
From the SU(3) case, we get the chiral condensate and light quark mass as 67.8(1.2)(0.1)(2.0)(0.8) MeV and 2.95(1)(4)(5)(1), respectively. The strange quark mass is 97(24)(13) MeV.

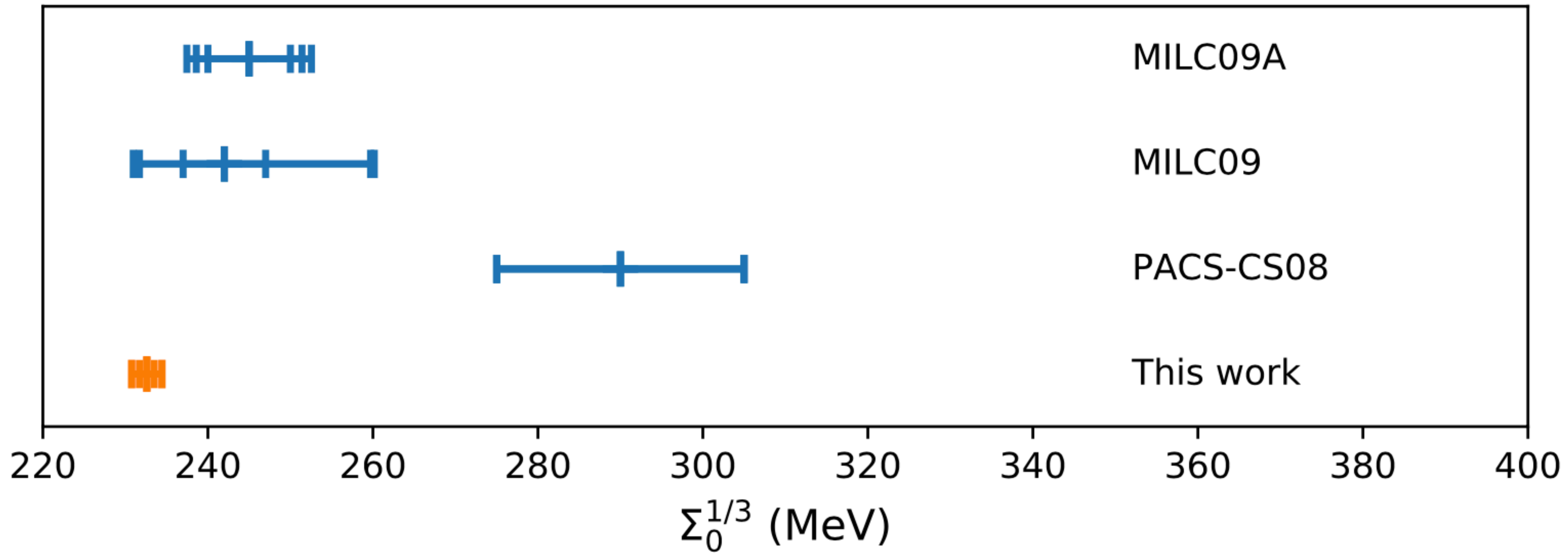
The four uncertainties are from the statistical fluctuations, renormalization constants, continuum extrapolation and the uncertainty of the lattice spacing.

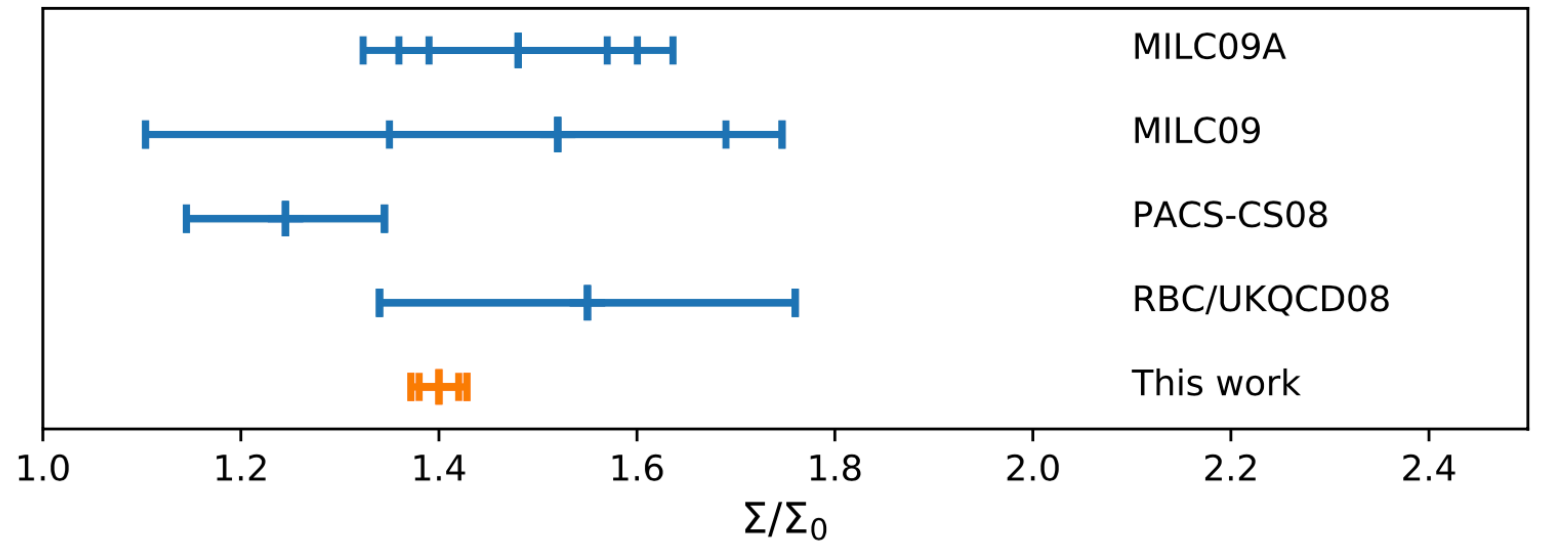


The fitting range does not affect much.
The NNLO correction is not contained.

Results



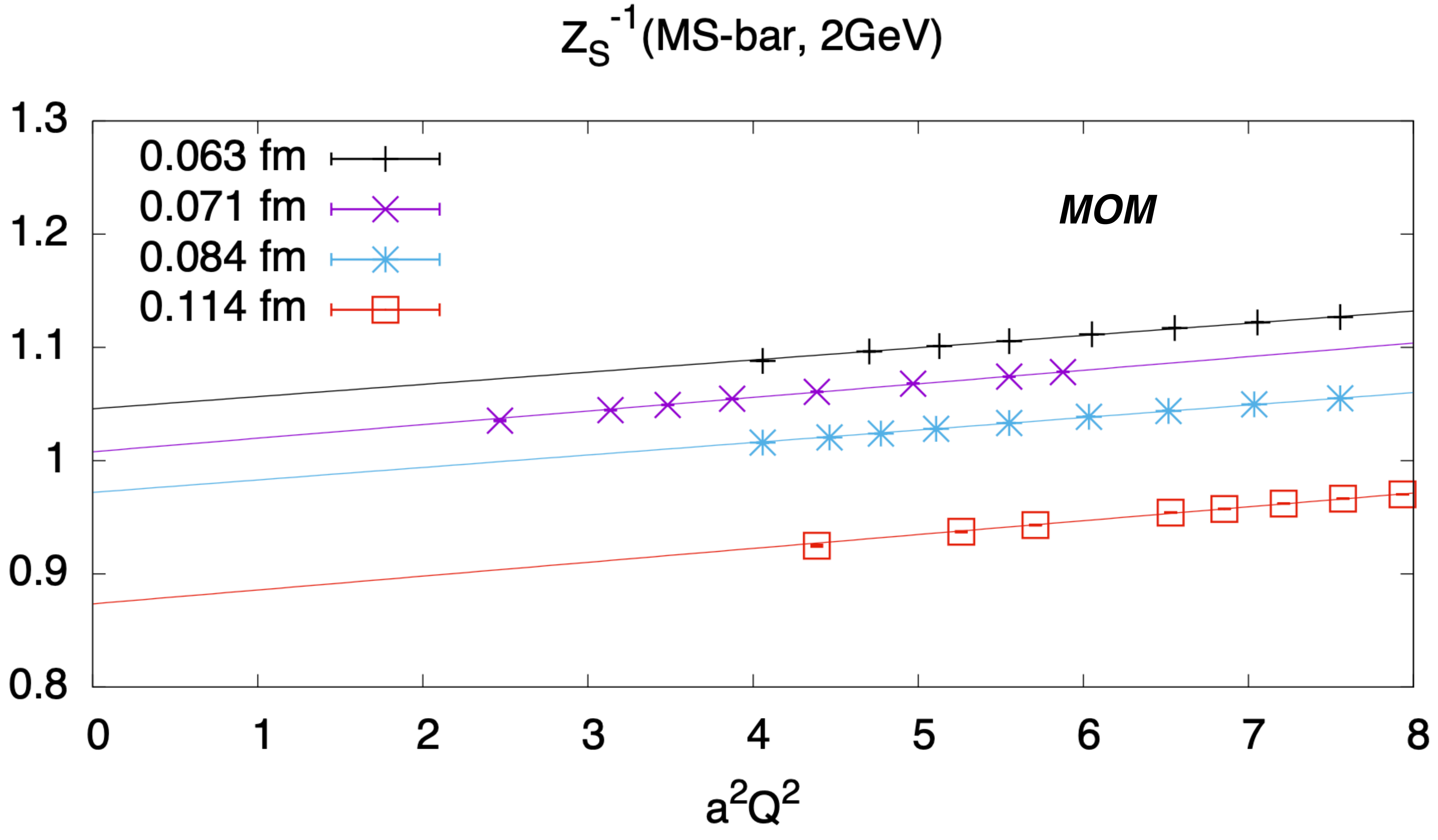


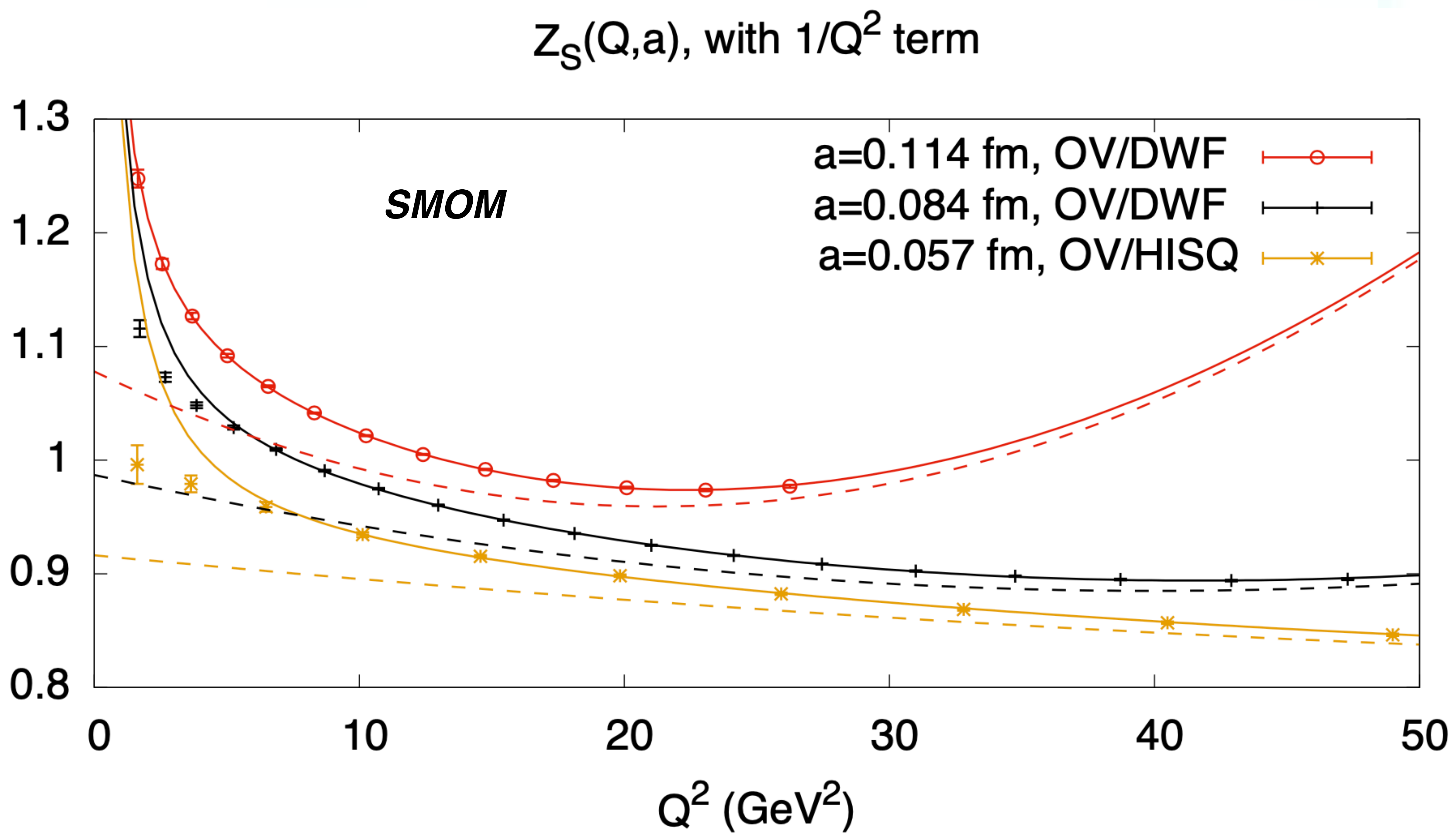


Precise but lower?

much more precise

The Renormalization Issues





The non-linear terms in fitting of the SMOM case can introduce additional systematic uncertainty.

Simple linear extrapolation at small a^2Q^2 leads to the 5~10% larger Z_S and tuns larger Σ.

Summary

Based on the precise spectrum of the overlap Dirac operator with error at 0.2% level, we determined the SU(2) and SU(3) chiral condensates and the light quark mass.

The strange quark mass is only roughly fitted due to the fact that it mainly account for the large λ behavior.

The renormalization issue should be further investigated.

It is fun to have such precise data to play with. Hopefully we can understand more about the Dirac spectrum and the physics related by more systematic analysis on more gauge ensembles.

Thank You