Application of Quasi-Geoid and Vertical model in CEPC alignment

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Introduction

CEPC consist of a 1.2km linac, a 1.6km beam transport line and a ring in which the collider ring and booster will be installed. The circumference of the ring tunnel is 100km and the diameter is about 32km, the overall measurement range is very large. The task of CEPC alignment is to adjust all the components to the designed positions according to their theoretical coordinates in the CEPC coordinate system. The precision requirement of Dipole, Quadrupole and Sextupole in arc area is 0.1mm and the precision requirement of magnets in IR is 0.5mm. In such a large area to achieve such high alignment accuracy, it brings new challenges to the measurement and data processing.

The measurement is generally carried out reference to the local geoid and vertical. For common accelerators, the dimensions are generally from tens of meters to several kilometers. In such a small area, we can suppose the geoid is a plane or a sphere and the vertical is parallel with the normal of the plane or the sphere. As the radius of the earth is very big (6371km), the earth curvature has little effect in the horizontal direction but significant effect in the elevation direction. So, for this kind of accelerator alignment, the horizontal observations can be solved based on a plane and the elevation observations can be solved based on a sphere.

When it comes to large range measurement, the actual shape of the earth must be considered. In fact, the geoid and topography are irregular, the vertical changes with the gravimetric equipotential surface. So, for CEPC alignment, the reference datum cannot simply rely on a standard geometry alone, it needs to apply new datums, coordinate system and data processing methods for the control points spatial position calculation.

CEPC geodetic datum

It needs to know the actual geometrical shape of the geoid and the vertical to carry out the data processing. As the geoid is not a regular geometry and the vertical directions are not uniform, so we need to establish the geoid model and the vertical model as the reference datum to do data processing.

1. Reference Ellipsoid

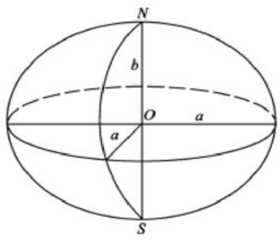

Figure 1: Reference ellipsoid

The reference ellipsoid is the mathematical representation of the earth. It is the base to establish geodetic coordinate system, quasi-geoid model and vertical deflection model. To define a reference ellipsoid, it needs to determine the ellipsoid parameters, ellipsoid location and ellipsoid directional.

Ellipsoid parameters include the semi-major axis, flattening and so on. These can be selected according to the relevant standards and make the shape of the ellipsoid as similar as possible with the local geoid.

Ellipsoid location is to locate the center of the ellipsoid. There are two kinds of location, one is to use the earth center as the ellipsoid center, and the other is the local location. The objective of earth center location is to make the ellipsoid and the geoid best match on a global scale. The objective of local location is to make the ellipsoid has the best agreement with the geoid in a certain range. For CEPC case, we should select the local location method, to make the ellipsoid best agreement with the geoid within CEPC area. Ellipsoid location needs to carry out the Astro-Geodetic measurement on the CEPC surface control network points, then we can get the measuring result of these Laplace points. According to the generalized radian measurement equation, applying the multipoint location method to calculate the center point position.

Ellipsoid directional is to determine the ellipsoid axis direction. To facilitate the conversion between the geodetic coordinates and the astronomical coordinates, the ellipsoid semi-minor axis should parallel with the earth's rotation axis and the geodetic prime meridian plane of the ellipsoid should parallel with the prime astronomical meridian plane.

2. Quasi-Geoid

As the internal mass of the earth cannot be accurately measured, it is impossible to get the accurate geoid, so people use the Quasi-Geoid to substitute the geoid. Quasi-Geoid is a closed surface formed by the end points which are from the ground points along the normal gravity lines take the normal heights and its shape is very close to that of the geoid. Quasi-Geoid is the height datum for level measurement, in this paper the Quasi-Geoid will be used to provide the global height datum, reference to it, the level observations can be conversed to the height coordinates in the CEPC coordinate system.

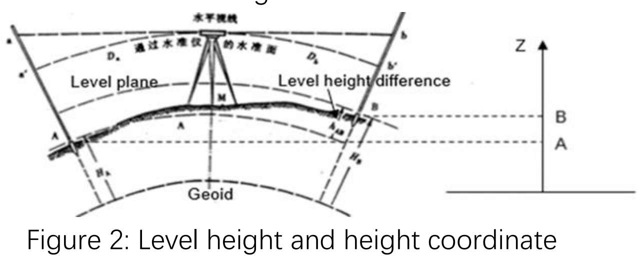

Figure 2: Level height and height coordinate

To establish the Quasi-Geoid, it needs to carry out a series of observations, includes satellite gravity, ground gravity, topographic data, precise GNSS data and precise Leveling data. The calculation method includes geometric method, gravity method and combined method. After calculation, we can get the Quasi-Geoid model in the CEPC geodetic coordinate system. It is a spherical cap harmonic function; use it we can calculate the height anomaly of any point in the CEPC area.

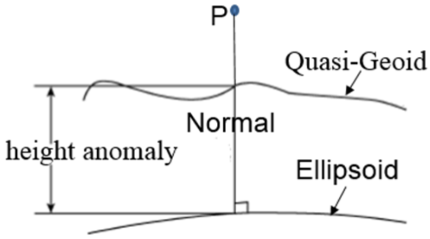

Figure 3: Height anomaly

3. Vertical deflection model

For a point in space, its normal and vertical are not parallel, the angle between them is named vertical deflection.

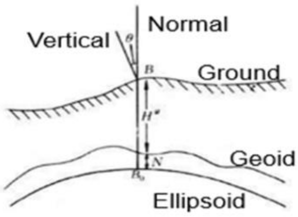

Figure 4: Vertical deflection

In this paper, the function of the vertical deflection model is to provide a global datum to realize the height axis of the instrument coordinate system directional in the CEPC coordinate system. To establish the Vertical deflection model, it needs to carry out a series of observations and calculation, includes Astro-Geodetic measurement, gravity measurement, Astro-Gravity measurement, EGM2008 model calculation and GNSS measurement. The vertical deflection model is a grid mathematic model, through interpolation we can get the deflection components in the meridian circle and the prime vertical of any point.

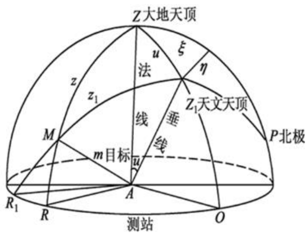

Figure 5: Vertical deflection components

The function relationship between the vertical deflection components, the geodetic coordinates and astronomical coordinates of point is

$$\begin{cases} \xi = \varphi - B \\ \eta = (\lambda - L)\cos\varphi \end{cases} \qquad (1)$$

ξ, η are vertical deflection components; L is longitude, B is latitude in geodetic coordinate system; λ is astronomical longitude, φ is astronomical latitude in astronomical coordinate system.

CEPC coordinate system

To do the data processing, it needs to know the spatial position relationship between the instrument stations and the control points. We designed three coordinate systems to realize the location of instruments and control points.

1. CEPC Geodetic Coordinate System

CEPC Geodetic Coordinate System is the global reference frame to locate the Quasi-Geoid model and the vertical deflection model, and it is also the link between the instrument coordinate system and the CEPC coordinate system. The origin is in the center of reference ellipsoid, Z points to the North Pole, X points to the intersection point of the prime meridian and the equator. It has two forms, one is the B L H coordinate form and the other is the x y z coordinate form, there are formulas to calculate the transformation between them.

2. CEPC Coordinate System

CEPC Coordinate System is the coordinate system of the whole complex, the result of data processing is in the CEPC coordinate system. The origin in the center of the main ring, XY plane parallel with the best-fit plane of the intersection points of the quasi-geoid and the ellipsoid normal at the surface control network points. The purpose is to make the XY plane normal as consistent as possible with the vertical in the CEPC area. Z perpendicular to XY plane, points to the up, Y points to the north.

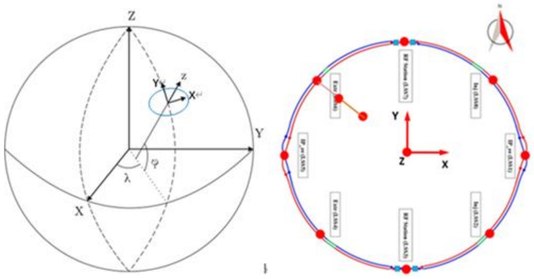

Figure 6: CEPC Coordinate Systems

The process of establish the CEPC coordinate system is as the follow.

1) The civil construction company shall build some civil reference points in the construction area and establish a civil coordinate system.
2) In the civil coordinate system design the positions of the ring center and the surface network control points and build them.
3) Carry out GNSS and level measurement, get the surface network control points' coordinates in CGCS2000 coordinate system and level heights.
4) Do coordinate system transformation that is best-fit the control points' coordinates to their corresponding level height values. After this, the height direction of the transformed coordinate system is what we need.
5) In the transformed coordinate system establish CEPC coordinate system. Origin is the ring center, Z is the height direction, Y points to the north.

The transformation parameters between CEPC geoid coordinate system and CEPC coordinate system can be calculated by use the surface network control points to do the best-fit.

3. Station coordinate system

Station coordinate system is an instrument coordinate system, it is used to carry out measurement. Origin in the center of the instrument, Z is parallel with the vertical, X Y directions are free.

Data processing method

Using the Quasi-Geoid model and the vertical deflection model as the global datum, it needs to know the control point's coordinates. As the change trends of the datum are very slow, it is enough to meet the accuracy requirement by use the approximate coordinates to do the calculation. We can use the surface network control points' coordinates as the known data, according to the observation values of each station, we can calculate the approximate coordinates of all the control points in CEPC coordinate system.

For every measurement station, the instrument must be leveled. When do the data processing we need to know the vertical directions of each station reference to a certain frame that means we need to calculate the vertical direction in CEPC coordinate system. The vertical deflection model can give the deflection components of a point in the CEPC geodetic coordinate system. We can use the approximate coordinates of a point in CEPC coordinate system to calculate the corresponding L and B in CEPC geodetic coordinate system. According to the formula (1), we can calculate the corresponding astronomical coordinate $\lambda\varphi$. Suppose there is an unite vector $S\left(X_{DS} \quad Y_{DS} \quad Z_{DS}\right)$, then the S can be calculated by the following formula

$$\begin{cases} X_{DS} = \cos\varphi \cdot \cos\lambda \\ Y_{DS} = \cos\varphi \cdot \sin\lambda \\ Z_{DS} = \sin\varphi \end{cases} \qquad (2)$$

Suppose M is the rotation transformation matrix which transform from CEPC geodetic coordinate system to CEPC coordinate system, the unite vector in CEPC coordinate system can be calculated by the following formula

$$\begin{bmatrix} X_{CS} \\ Y_{CS} \\ Z_{CS} \end{bmatrix} = M \cdot \begin{bmatrix} X_{DS} \\ Y_{DS} \\ Z_{DS} \end{bmatrix} \qquad (3)$$

In CEPC coordinate system, the angle between the vertical and the Z is $\alpha = \arccos\left(Z_{CS}\right)$, the angle between the projective vector and the X is $\beta = \arccos\left(\frac{X_{CS}}{\sqrt{X_{CS}^2 + Y_{CS}^2}}\right)$.

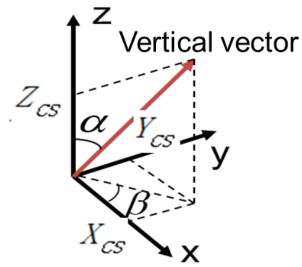

Figure 7: Vertical directional

After calculate the vertical direction in CEPC coordinate system, we can calculate the accurate height coordinates of the control points in CEPC coordinate system. By level measurement we can get the distance h between the control point and the Quasi-Geoid. Suppose a control point in CEPC coordinate system is (x_1, y_1, z_1), its projective point coordinates x_2, y_2 on the Quasi-Geoid can be calculated by the following formula

$$\begin{cases} x_2 = x_1 + h \cdot \sin\alpha \cdot \cos\beta \\ y_2 = y_1 + h \cdot \sin\alpha \cdot \sin\beta \end{cases} \quad (4)$$

According to the Quasi-Geoid model, we can use x_2, y_2 calculate the projective point's height coordinate z_2 in CEPC coordinate system. The accurate height coordinate z_1 of the control point in CEPC coordinate system is $z_1 = h \cdot \cos\alpha + z_2$.

To get the control points' coordinates in CEPC coordinate system need to do adjust calculation use the observation values. In the traditional adjust calculation of accelerator , the height observations and the horizontal observations are calculated separately. According to this way, we need to divide the original observations into the height observations and the horizontal observations in CEPC coordinate system. The directions of the coordinate axis in station coordinate system and CEPC coordinate system are not same. It needs to calculate the rotation matrix R to get the height observations and the horizontal observations in CEPC coordinate system. R can be calculated by best-fit the coordinates of control points in station coordinate system to the corresponding height coordinates in CEPC coordinate system. Suppose $p_{zi}(x_{zi} \quad y_{zi} \quad z_{zi})$ is the coordinate in station coordinate system, $p_{si}(x_{si} \quad y_{si} \quad z_{si})$ is the coordinate after rotation transformation. $p_{ci}(x_{ci} \quad y_{ci} \quad z_{ci})$ is the coordinate in CEPC coordinate system

$$\begin{bmatrix} x_{si} \\ y_{si} \\ z_{si} \end{bmatrix} = R \begin{bmatrix} x_{zi} \\ y_{zi} \\ z_{zi} \end{bmatrix} \quad (5)$$

According to the constraint condition $\sum\left(\Delta z_{si} - \Delta z_{si}\right)^2 = \min$, by least square calculation we can get the parameters of R.

Another way to get the control points' coordinates in CEPC coordinate system is to do 3D adjust. The key point is to determine the rotation angel parameters between CEPC coordinate system and station coordinate system. Suppose the angles between the vertical of the station and the axis of CEPC coordinate system are α and β as mentioned before, the rotation transformation process from CEPC coordinate system to the station coordinate system are as the following: rotate about CEPC z axis by β R_Z^β, rotate about y axis by α R_Y^α, rotate about z axis by γ R_Z^γ.

$$\begin{bmatrix} x_{zi} \\ y_{zi} \\ z_{zi} \end{bmatrix} = R_Z^\gamma R_Y^\alpha R_Z^\beta \begin{bmatrix} x_{ci} \\ y_{ci} \\ z_{ci} \end{bmatrix} \qquad (6)$$

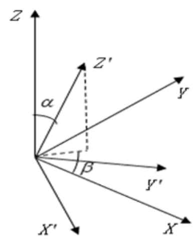

Figure 8: Rotation transformation

γ is the unknown parameter can be solved by 3D adjust calculation. For α and β are known data, 3D directional adjust can avoid the result distortion.

Summary

CEPC measurement range is very large, the undulation of geoid and vertical need to be considered in doing the data processing. Three mathematical models: ellipsoid, Quasi-Geoid and vertical deflection will be used as the global datum in measurement and data processing. Three coordinate systems are defined to describe the posture and position relations of instruments and control points. A data processing method based on Quasi-Geoid model and vertical deflection model is proposed.

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